Applications of Pointers (1A)

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2/24/24

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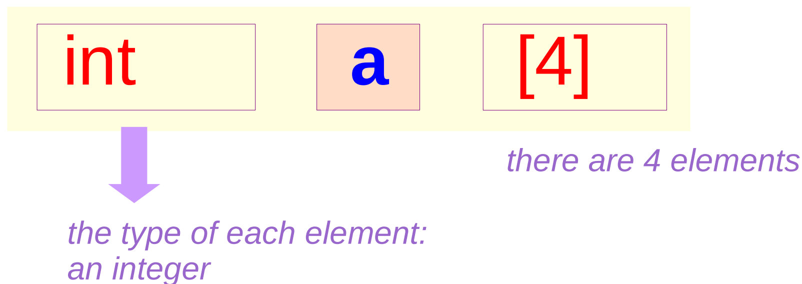
n-d access of a **1-d** array

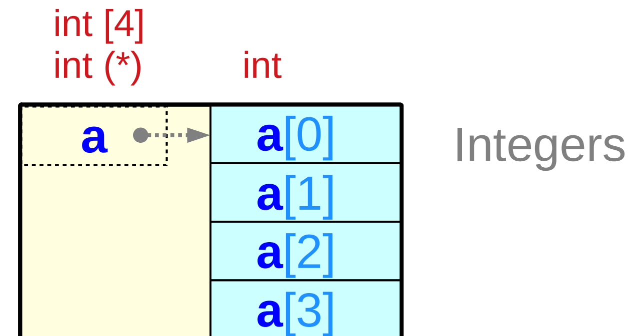
- **2-d** array access of a 1-d array
- Accessing a **contiguous 1-d** array
- Accessing **non-contiguous 1-d** arrays
- Accessing **static** allocated arrays
- Accessing **dynamically** allocated arrays

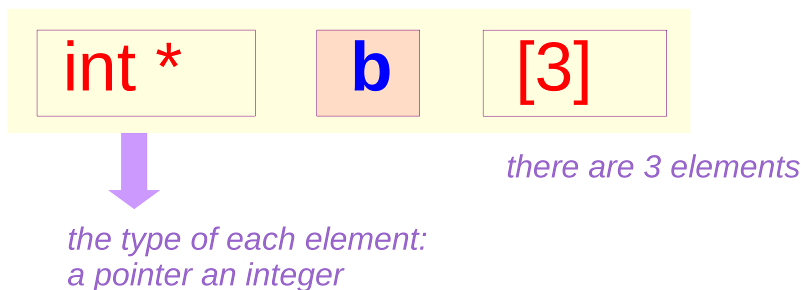
2-d array access of a **1-d** array

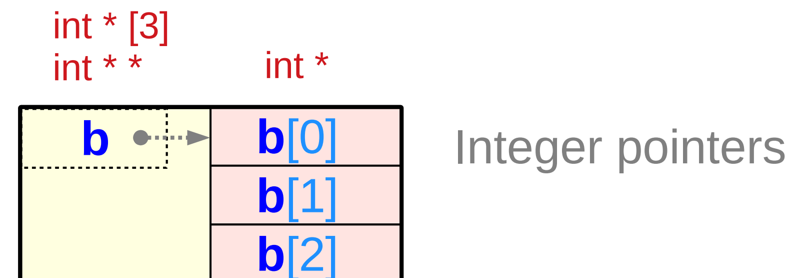
Array of Pointers

int	a [4] ;
int *	b [3] ;

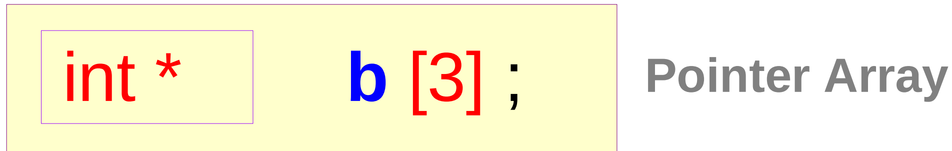




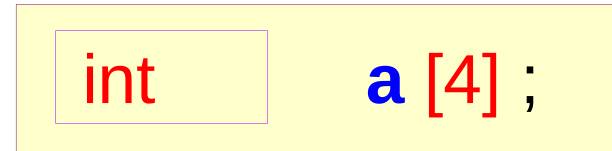


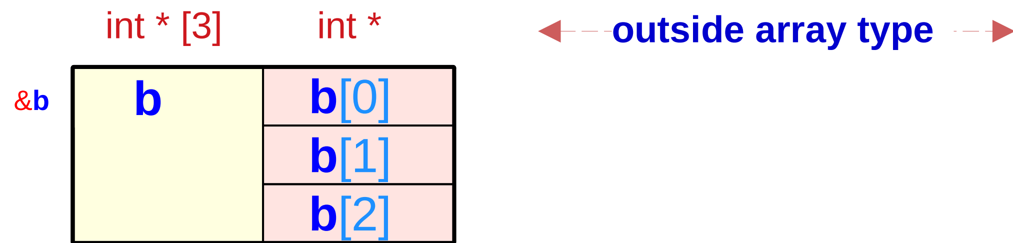


Array of Pointers – a type view

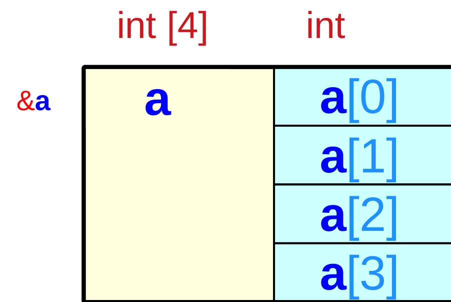


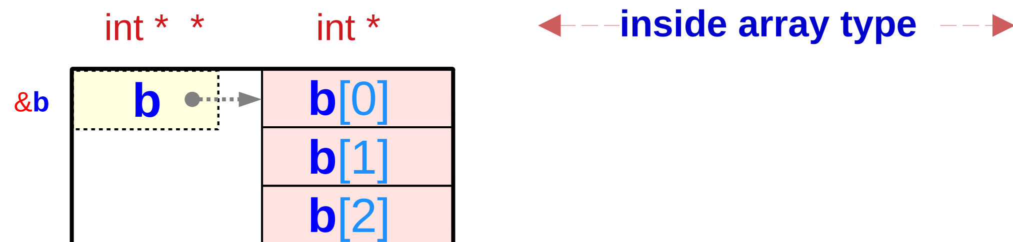
Pointer Array



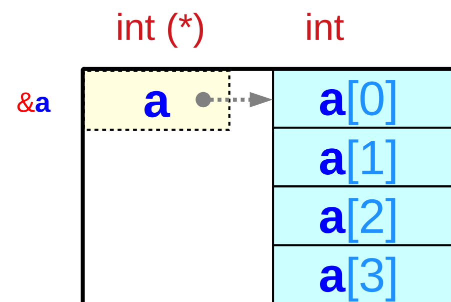


outside array type





inside array type

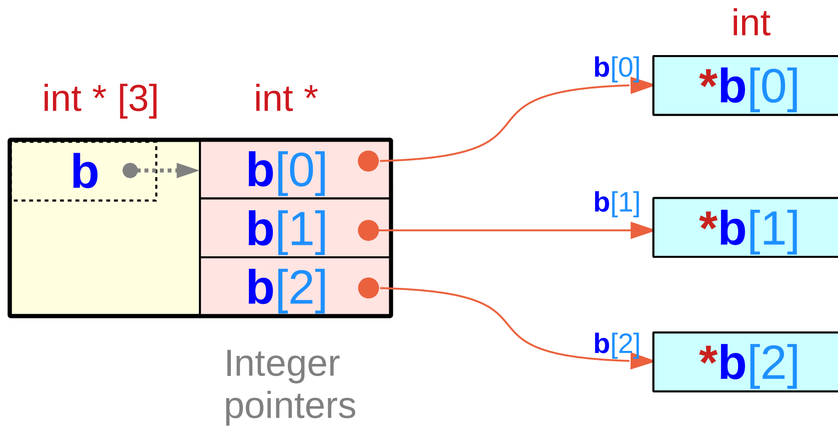


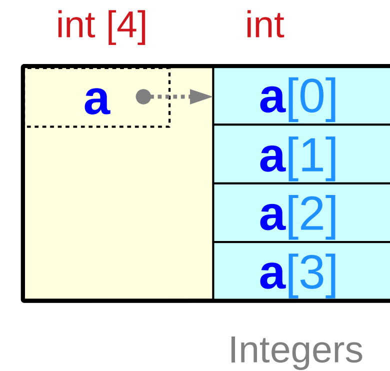
Array of Pointers – a variable view

int * b [3] ;

Pointer Array

int a [4] ;



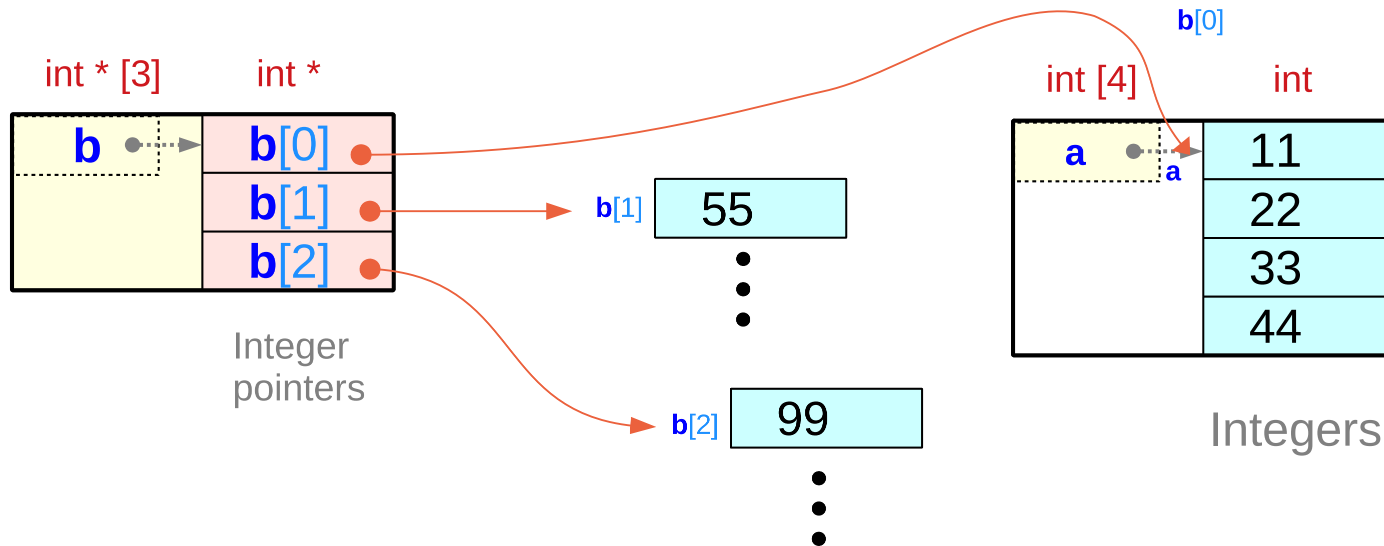


Assigning a **1-d** array name

int * b [3] ; **Pointer Array** int a [4] ;

assignment

b[0] = a (= &a[0])



Assigning a **1-d** array name – equivalence

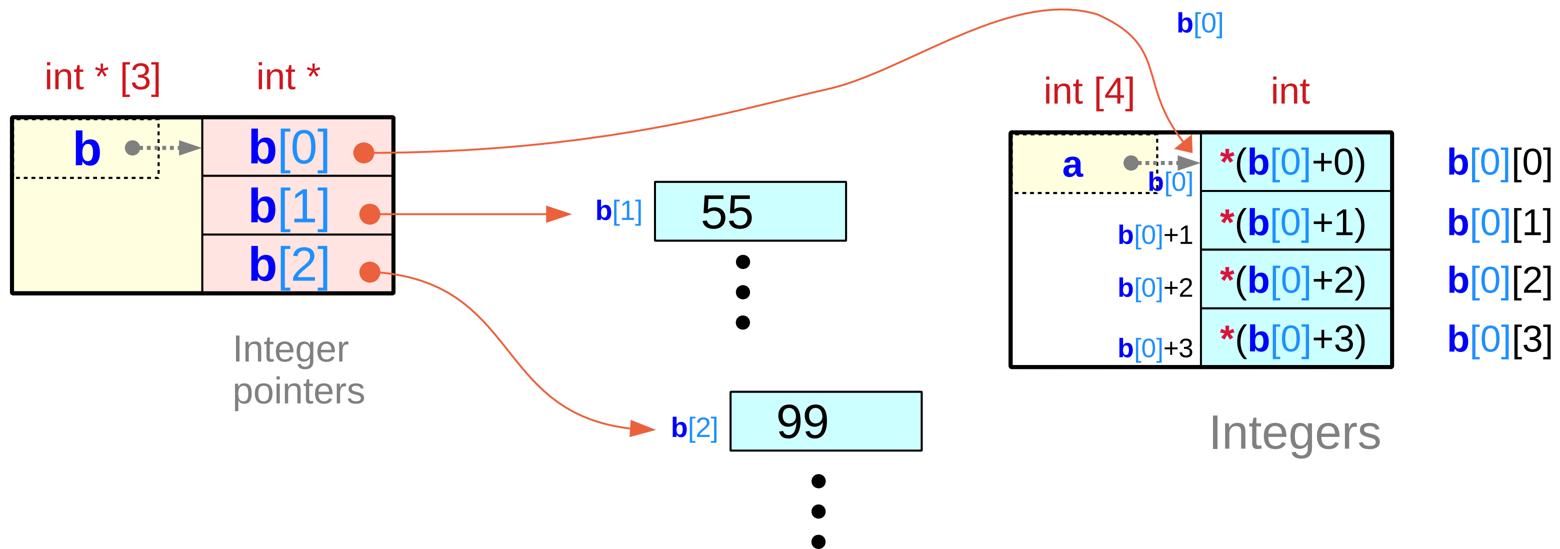
int * b [3] ;

Pointer Array

int a [4] ;

assignment

b[0] = a (= &a[0])



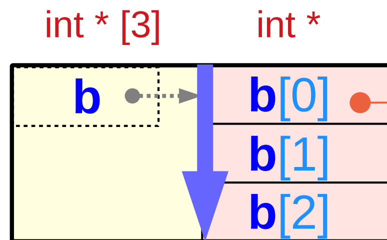
Array of Pointers – extended dimension

int * b [3] ;

Pointer Array

int a [4] ;

array name b

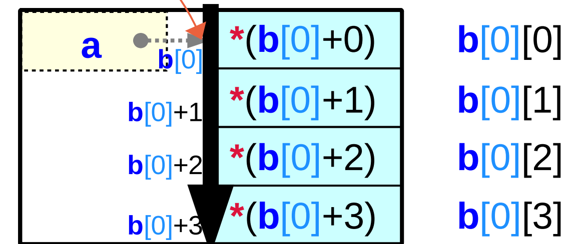


1st dim

assignment

b[0] = a (= &a[0])

array name b[0]

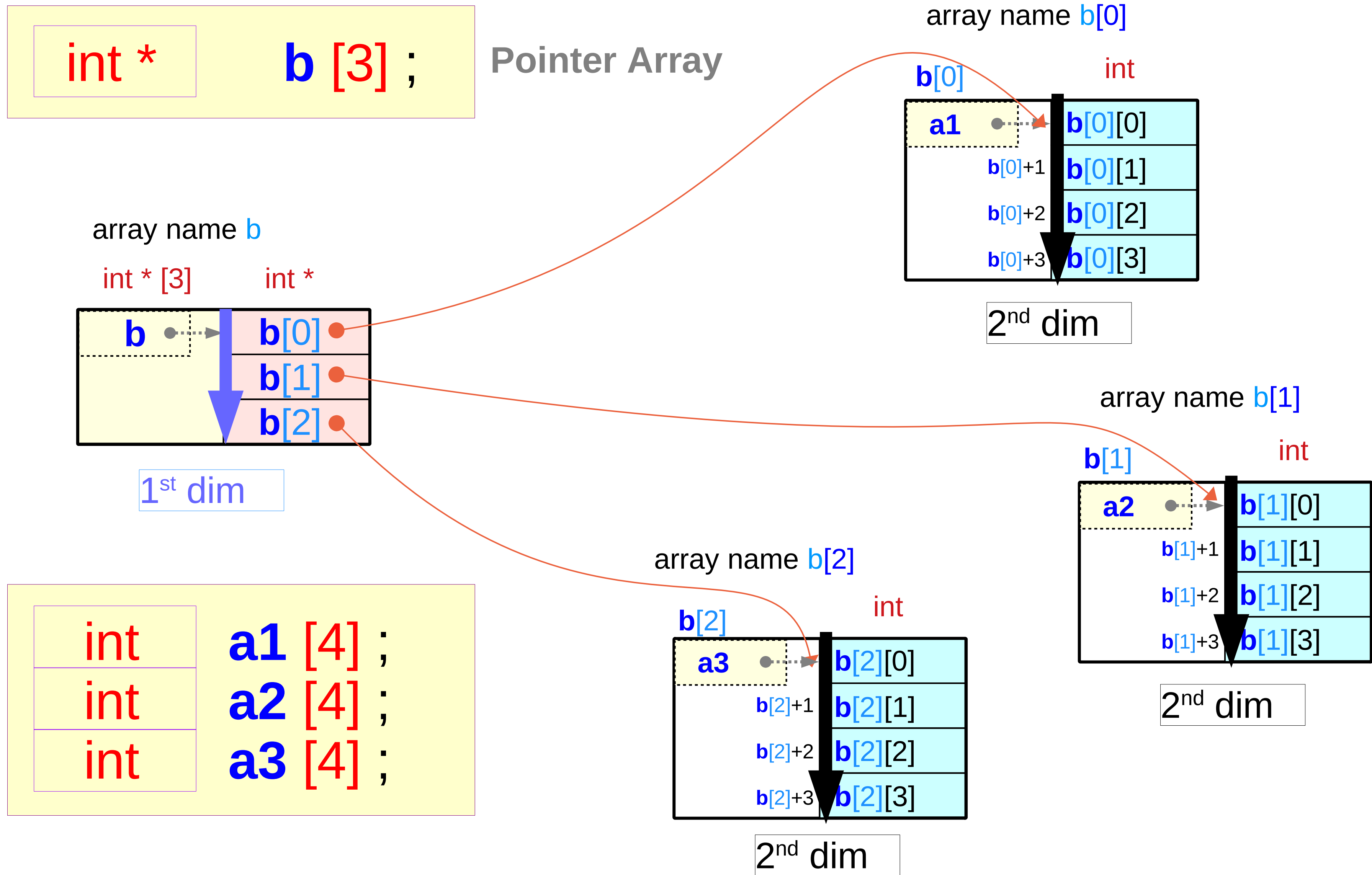


b[0][0]
b[0][1]
b[0][2]
b[0][3]

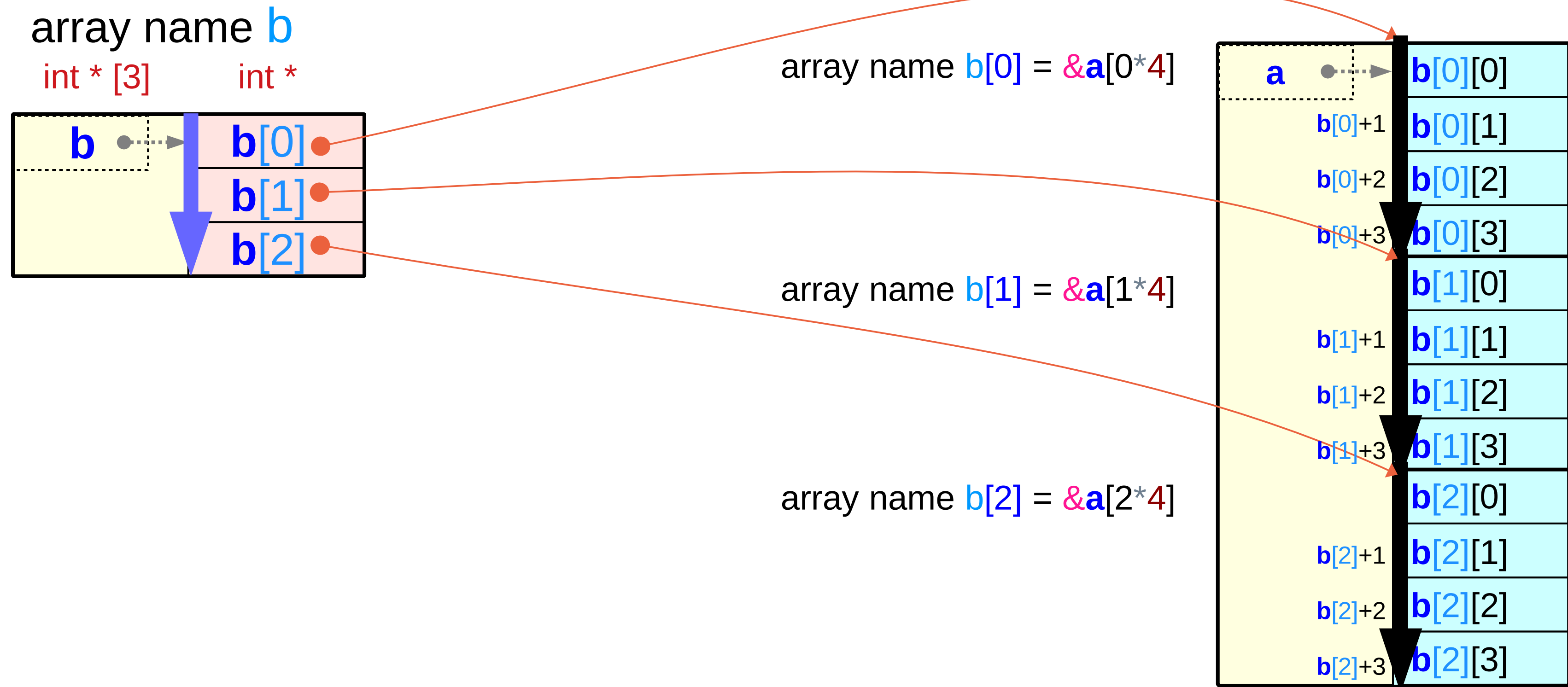
2nd dim

a[0] ≡ b[0][0] ≡ *(*(b+0)+0)
a[1] ≡ b[0][1] ≡ *(*(b+0)+1)
a[2] ≡ b[0][2] ≡ *(*(b+0)+2)
a[3] ≡ b[0][3] ≡ *(*(b+0)+3)

2-d access of 1-d arrays



2-d access of a 1-d array



2-d access of a 1-d array – pointer array assignment

int *	b [2*3] ;
int	a [2*3*4] ;

b[j] = &a[j*4] (= a+j*4)

b[j] + k = a+j*4 + k
*(b[j] + k) = *(a+j*4 + k)

b[j][k] ≡ a[j*4 +k]

j = [0:5] k = [0:3]

j*4+k = [0:23]

constraint : contiguous b[i][j] over j

2-d access of a **1-d** array

b[i][j] ≡ *(b[i] + j)

↕ ↕

a[i*4+j] ≡ *(a+i*4 + j)

1-d access of a **1-d** array

constraint : contiguous a[i*4+j] over j

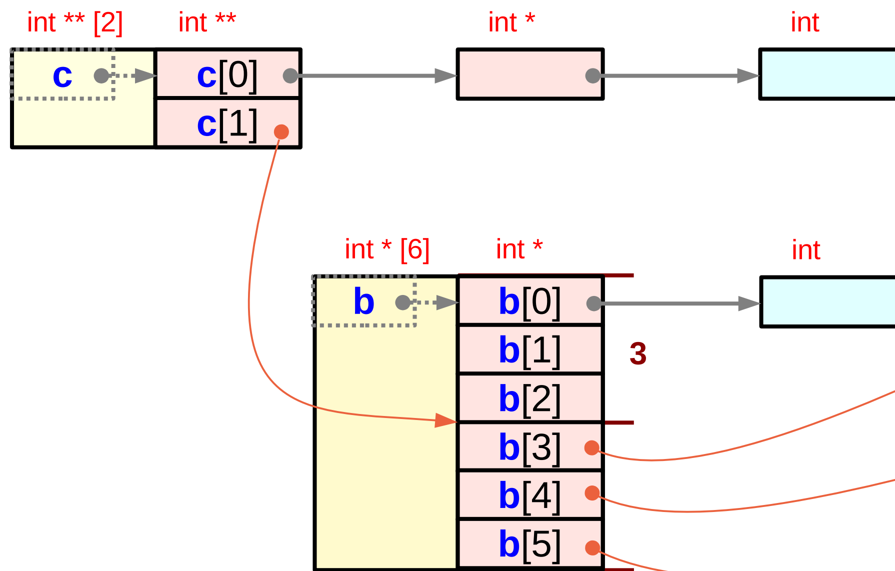
3-d array access of a **1-d** array

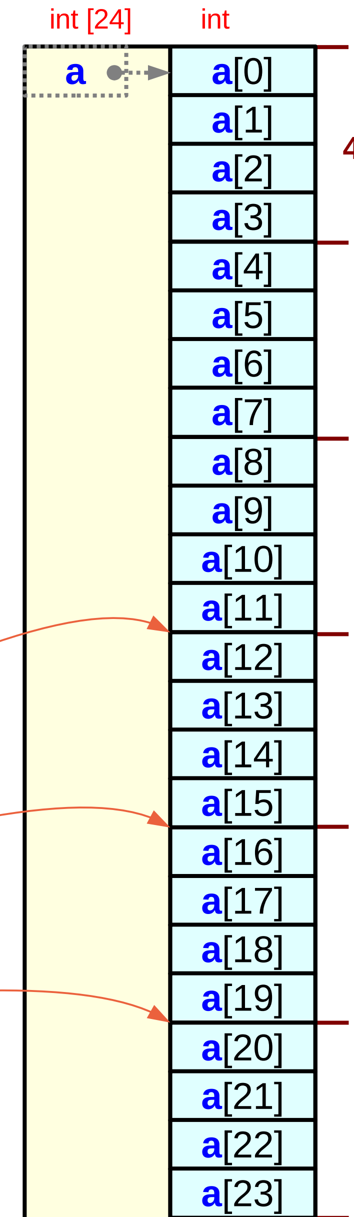
Integer array a and pointer arrays b, c

int **	c [2] ;
int *	b [2*3] ;
int	a [2*3*4] ;

divide 2·3·4 elements of a into
six (2·3) partitions
each partition has with 4 elements

divide 2·3 elements of b into
two (2) partitions
each partition has 3 elements





Pointer array initializations

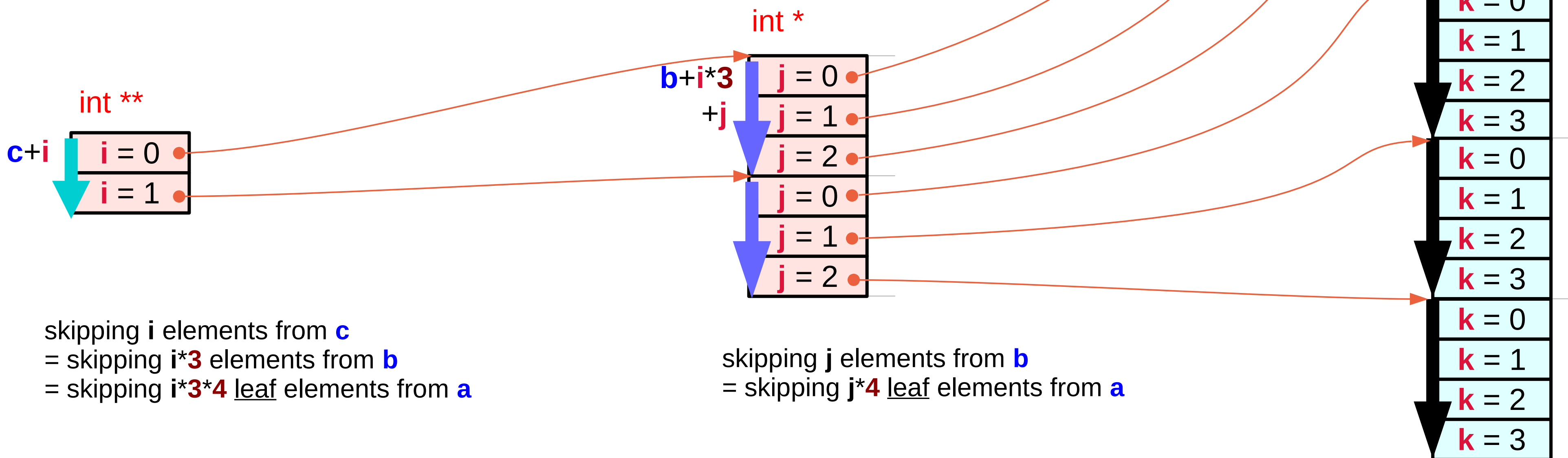
int **	c [2] ;
int *	b [2*3] ;
int	a [2*3*4] ;

c[i] = &b[i*3] (= b+i*3)

b[j] = &a[j*4] (= a+j*4)

each element of c handles 3 elements of b
→ partition size of b = 3 elements

each element of b handles 4 elements of a
→ partition size of a = 4 elements



skipping i elements from c
= skipping i*3 elements from b
= skipping i*3*4 leaf elements from a

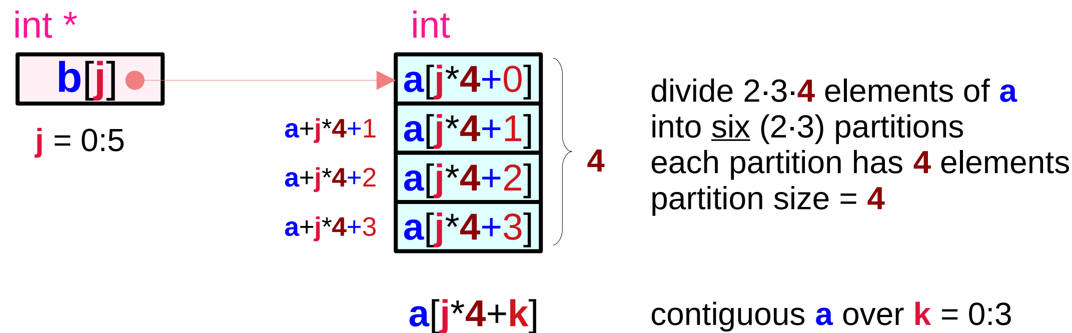
skipping j elements from b
= skipping j*4 leaf elements from a

Partitioning arrays **a** and **b**

int **	c [2] ;
int *	b [2*3] ;
int	a [2*3*4] ;

Assigning pointer array

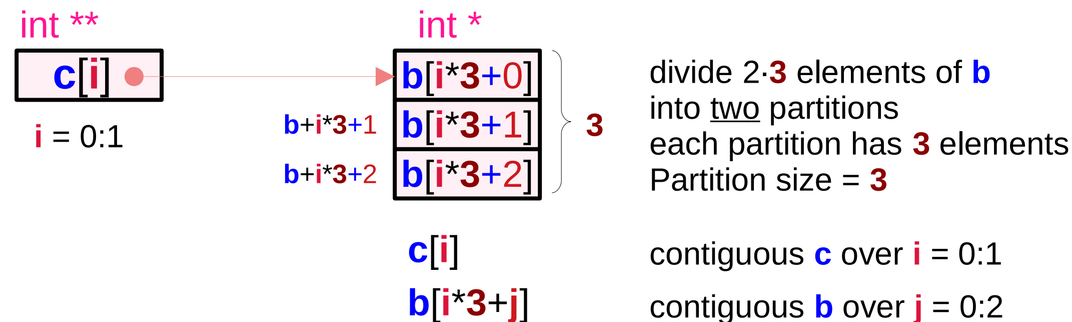
b[j] = &a[j*4] (= a+j*4)
c[i] = &b[i*3] (= b+i*3)



divide 2·3·**4** elements of **a**
into six (2·3) partitions
each partition has **4** elements
partition size = **4**

a[j*4+k] contiguous **a** over **k** = 0:3

b[0] = &a[0*4]; (= a + 0*4)
b[1] = &a[1*4]; (= a + 1*4)
b[2] = &a[2*4]; (= a + 2*4)
b[3] = &a[3*4]; (= a + 3*4)
b[4] = &a[4*4]; (= a + 4*4)
b[5] = &a[5*4]; (= a + 5*4)



divide 2·**3** elements of **b**
into two partitions
each partition has **3** elements
Partition size = **3**

c[i] contiguous **c** over **i** = 0:1

b[i*3+j] contiguous **b** over **j** = 0:2

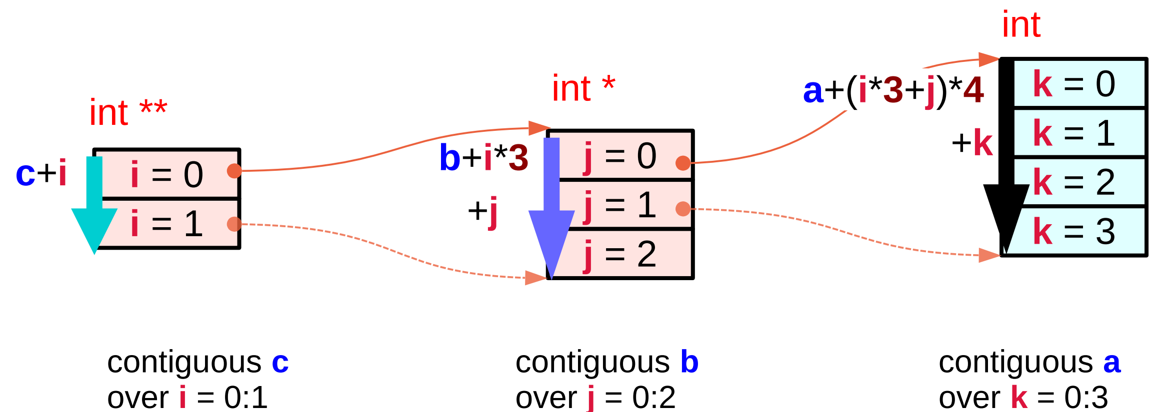
c[0] = &b[0*3]; (= b + 0*3)
c[1] = &b[1*3]; (= b + 1*3)

Contiguous indices

int **	c [2] ;
int *	b [2*3] ;
int	a [2*3*4] ;

b[j] = &a[j*4] (= a+j*4)
c[i] = &b[i*3] (= b+i*3)

c[i][j][k] ≡
a[(i*3 +j)*4+k]



contiguous c
over i = 0:1

contiguous b
over j = 0:2

contiguous a
over k = 0:3

i = 0:1 is an index for the
contiguous 2 elements of c

j = 0:2 is an index for the
contiguous 3 elements of b

k = 0:3 is an index for the
contiguous 4 elements of a

Equivalence relations in pointer array assignments

c[i] = &b[i*3] = b+i*3
b[j] = &a[j*4] = a+j*4

```
c[i][j]        substitute c[i]
               = *(c[i]+j)
               = *(b+i*3+j) = b[i*3+j]

b[m][k]        substitute b[m]
               = *(b[m]+k)
               = *(a+m*4+k) = a[m*4+k]

c[i][j][k]     = b[i*3+j][k] = a[(i*3+j)*4+k]
```

c[i] = &b[i*3] = b+i*3
b[j] = &a[j*4] = a+j*4

```
substitute c[i]
c[i][j]        = (b+i*3)[j]
               = *(b+i*3+j) = b[i*3+j]

substitute b[m]
b[m][k]        = (a+m*4)[k]
               = *(a+m*4+k) = a[m*4+k]

c[i][j][k]     = b[i*3+j][k] = a[(i*3+j)*4+k]
```

Skipping elements

```
int **   c [2] ;
int *    b [2*3] ;
int      a [2*3*4] ;
```

b[j] = &a[j*4] (= a+j*4)

skipping 1 element in **b**
= skipping **4** <u>leaf</u> elements in **a**

c[i] = &b[i*3] (= b+i*3)

skipping 1 element in **c**
= skipping **3** elements in **b**
= skipping **3*4** <u>leaf</u> elements in **a**

c[i][j] ≡ b[i*3 +j]

skipping **i** elements from **c**
+ skipping **j** elements from **b**

= skipping **i*3+j** elements from **b**

c[i][j][k] ≡ a[(i*3 +j)*4+k]

skipping **i*3+j** elements from **b**
+ skipping **k** <u>leaf</u> elements from **a**

= skipping (**i*3+j**)***4+k** <u>leaf</u> elements from **a**

3-d access of a 1-d array (1)

```
int **   c  [2] ;
int *    b  [2*3] ;
int      a  [2*3*4] ;
```

```
int *    b  [2*3] ;
int      a  [2*3*4] ;
```

b[j] = &a[j*4] (= a+j*4)

b[j] + k = a+j*4 + k
*(b[j] + k) = *(a+j*4 + k)

b[j][k] ≡ a[j*4 +k]

j = [0:5]   k = [0:3]
j*4+k = [0:23]

```
int **   c  [2] ;
int *    b  [2*3] ;
```

c[i] = &b[i*3] (= b+i*3)

c[i] +j = b+i*3 +j
*(c[i] +j) = *(b+i*3 +j)

c[i][j] = b[i*3 +j]

c[i][j] +k = b[i*3 +j] +k
*(c[i][j] +k) = b[i*3 +j][k]

c[i][j][k] = a[(i*3 +j)*4 +k]

c[i][j][k] ≡ a[(i*3+j)*4+k]

i = [0:1]   j = [0:2]   k = [0:3]
(i*3+j)*4+k = [0:23]

3-d access of a 1-d array (2)

int **	c [2] ;
int *	b [2*3] ;
int	a [2*3*4] ;

a[k] ≡ *(a+k)
b[j][k] ≡ *(*(b+j)+k)
c[i][j][k] ≡ *(*(*(c+i)+j)+k)

constraint : contiguous a[i], b[i], c[i]

Assignments

c[i] = &b[i*3] (= b+i*3)
b[j] = &a[j*4] (= a+j*4)

Initializing pointer arrays **b** and **c**

3-d access of a **1-d** array

c[i][j][k] ≡ *(c[i][j] +k)

b[i*3+j][k] ≡ *(b[i*3+j] +k)

a[(i*3+j)*4 +k] ≡ *(a+(i*3+j)*4 +k)

1-d access of a **1-d** array

3-d access of a 1-d array (3)

int **	c	[2] ;
int *	b	[2*3] ;
int	a	[2*3*4] ;

a[k] ≡ *(a+k)
b[j][k] ≡ *(*(b+j)+k)
c[i][j][k] ≡ *(*(*(c+i)+j)+k)

(((c[i])[j])[k])
≡ (((b+i*3)[j])[k])
≡ ((b[i*3+j])[k])
≡ ((a+(i*3+j)*4)[k])
≡ a[(i*3+j)*4+k]

c[i] = &b[i*3] = b+i*3

b[j] = &a[j*4] = a+j*4

((*(c+i)+j)+k)
≡ *(*(b+i*3+j)+k)
≡ *(b[i*3+j]+k)
≡ *(a+(i*3+j)*4+k)
≡ a[(i*3+j)*4+k]

Accessing **contiguous 1-d** arrays

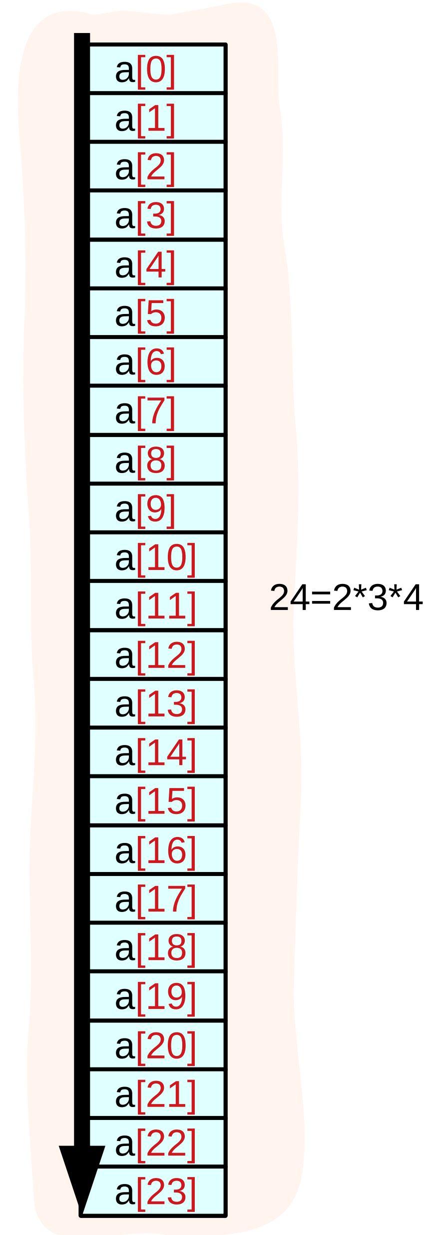
- **1-d** array access
- **2-d** array access
- **3-d** array access

Accessing an int array **a** as a **1-d** array

int **a** [2*3*4] ;

a [**k**]

k = 0,1, …,23



24=2*3*4

c[i][j][k]	≡ *(*(*(c+i)+j)+k)	int c[2][3][4] ;
b[i][j]	≡ *(*(b+i)+j)	int b[2*3][4] ;
a[i]	≡ *(a+i)	int a[2*3*4] ;

Accessing an int array **a** as a **2-d** array using **b**

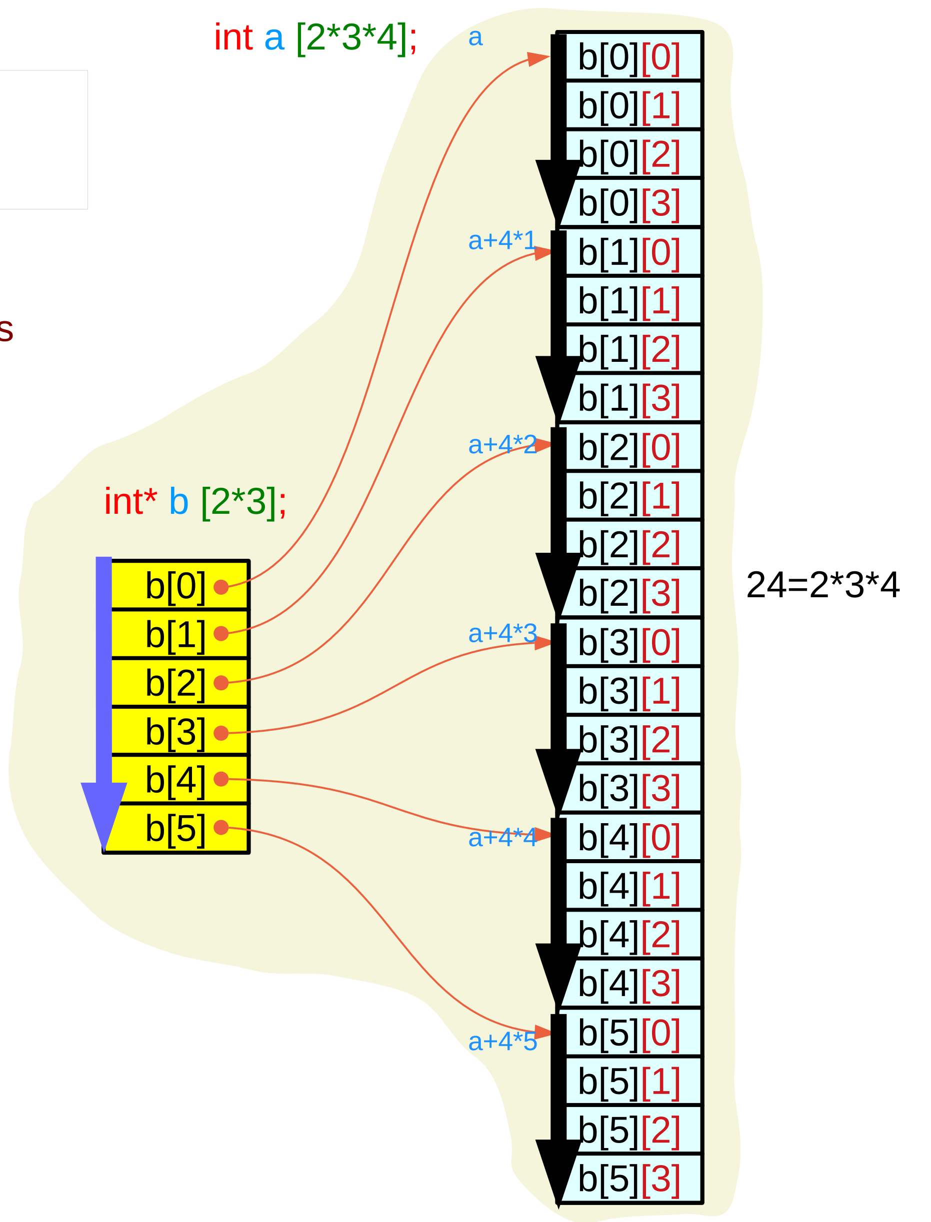
```
int     a [2*3*4] ;
int *   b [2*3] ;
```

```
b[j] = &a[j*4];
```

b take actual memory locations

b [j][k] ≡ a [j*4 +k]

j = 0:5
k = 0:4



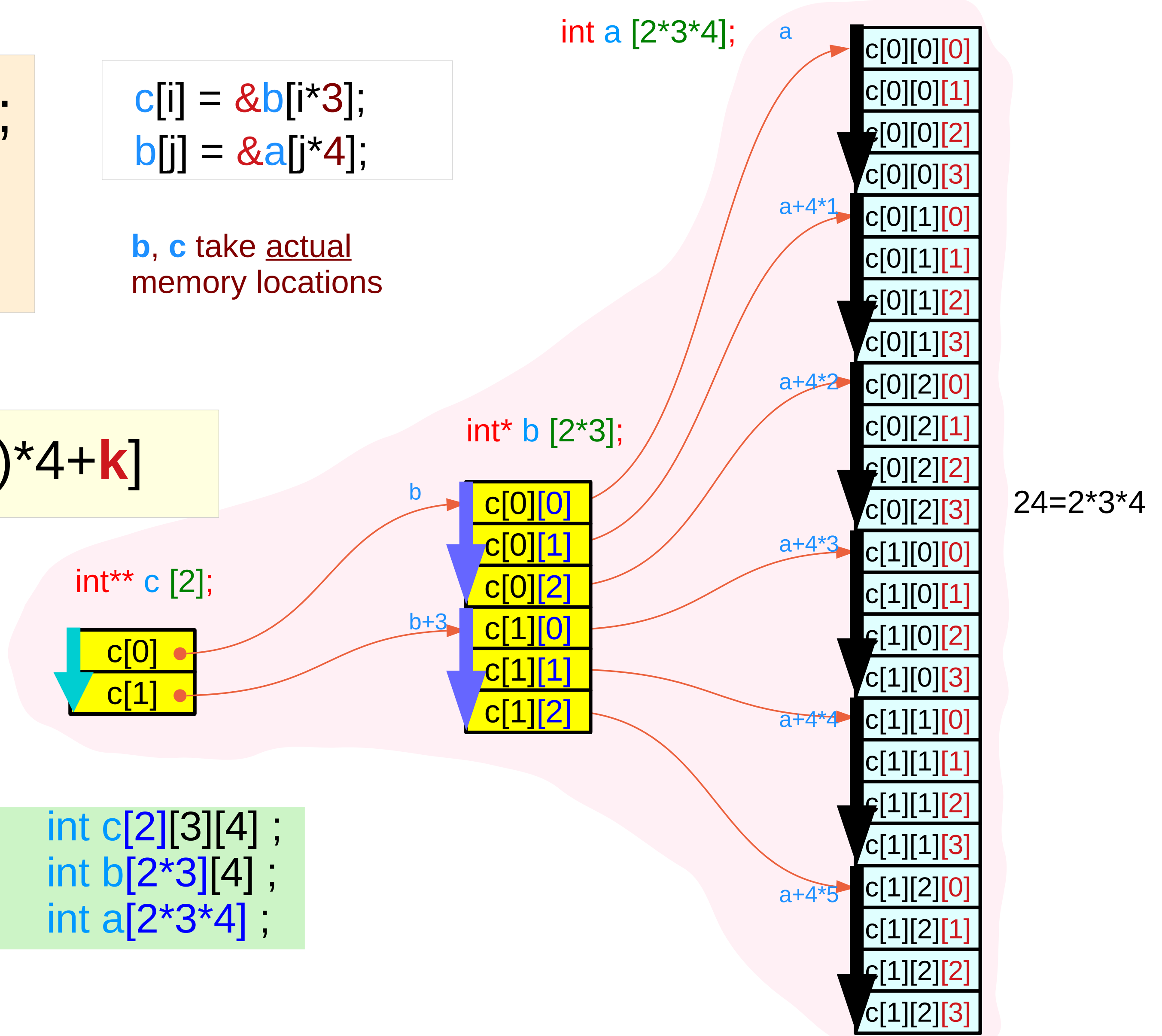
c[i][j][k]	≡ *(*(*(c+i)+j)+k)	int c[2][3][4] ;
b[i][j]	≡ *(*(b+i)+j)	int b[2*3][4] ;
a[i]	≡ *(a+i)	int a[2*3*4] ;

Accessing an int array **a** as a **3-d** array

```
int     a [2*3*4] ;
int *   b [2*3] ;
int **  c [2] ;
```

c [i][j][k] ≡ **a** [(i*3+j)*4+k]

i = 0, 1
j = 0, 1, 2
k = 0, 1, 2, 3



c[i][j][k]	≡ *(*(*(c+i)+j)+k)	int c[2][3][4] ;
b[i][j]	≡ *(*(b+i)+j)	int b[2*3][4] ;
a[i]	≡ *(a+i)	int a[2*3*4] ;

Accessing **non-contiguous 1-d** arrays

- **3-d** array access

Accessing **non-contiguous 1-d** arrays as a **3-d** array (1)

```
int       a [2*3*4] ;
int *     b [2*3] ;
int **    c [2] ;
```

```
c[i] = &b[i*3];
b[j] = &aj;
```

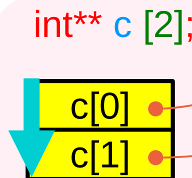
b, **c** take actual memory locations

```
int a1 [4];
int a2 [4];
int a3 [4];
int a4 [4];
int a5 [4];
int a6 [4];
```

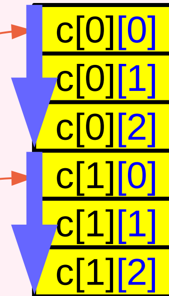
c [i][j][k]

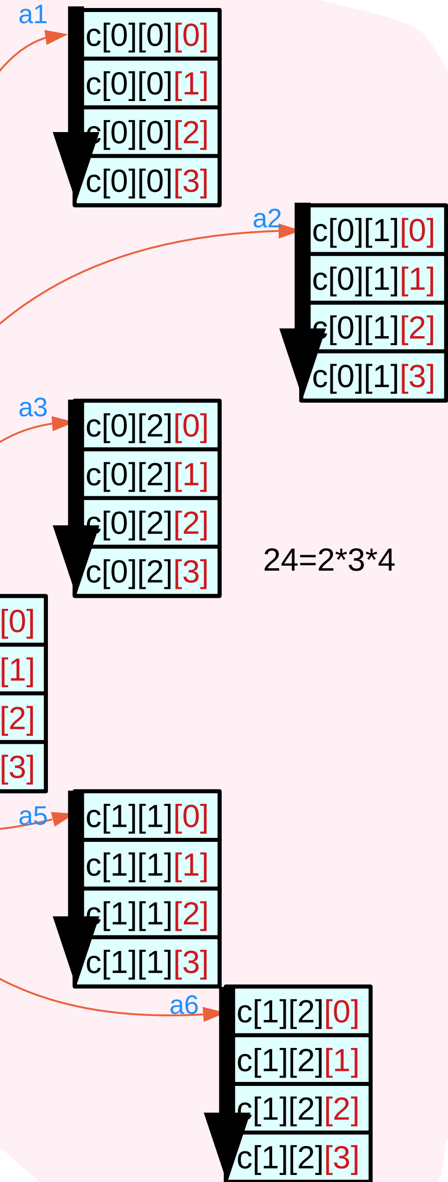
i = 0, 1
j = 0, 1, 2
k = 0, 1, 2, 3

int** c [2];



int* b [2*3];





24=2*3*4

Because the physical **allocation** of array **c** and **b**, the **contiguous constraints** can be **relaxed**
contiguous c[i][j][k] only for k=0,1,2,3

Accessing **non-contiguous 1-d** arrays as a **3-d** array (2)

```
int      a [2*3*4] ;
int *    b [2*3] ;
int **   c [2] ;
```

c [i][j][k]

i = 0, 1
j = 0, 1, 2
k = 0, 1, 2, 3

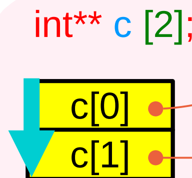
```
c[i] = &bi[i*3];
b[j] = &aj;
```

b, **c** take actual memory locations

```
int a1 [4];
int a2 [4];
int a3 [4];
int a4 [4];
int a5 [4];
int a6 [4];

int b1 [3];
int b2 [3];
```

int** c [2];



b1
c[0][0]
c[0][1]
c[0][2]

b2
c[1][0]
c[1][1]
c[1][2]

a1
c[0][0][0]
c[0][0][1]
c[0][0][2]
c[0][0][3]

a2
c[0][1][0]
c[0][1][1]
c[0][1][2]
c[0][1][3]

a3
c[0][2][0]
c[0][2][1]
c[0][2][2]
c[0][2][3]

a4
c[1][0][0]
c[1][0][1]
c[1][0][2]
c[1][0][3]

a5
c[1][1][0]
c[1][1][1]
c[1][1][2]
c[1][1][3]

a6
c[1][2][0]
c[1][2][1]
c[1][2][2]
c[1][2][3]

24=2*3*4

Because the physical **allocation** of array **c** and **b**,
the **contiguous constraints** can be **relaxed**
contiguous c[i][j][k] only for k=0,1,2,3

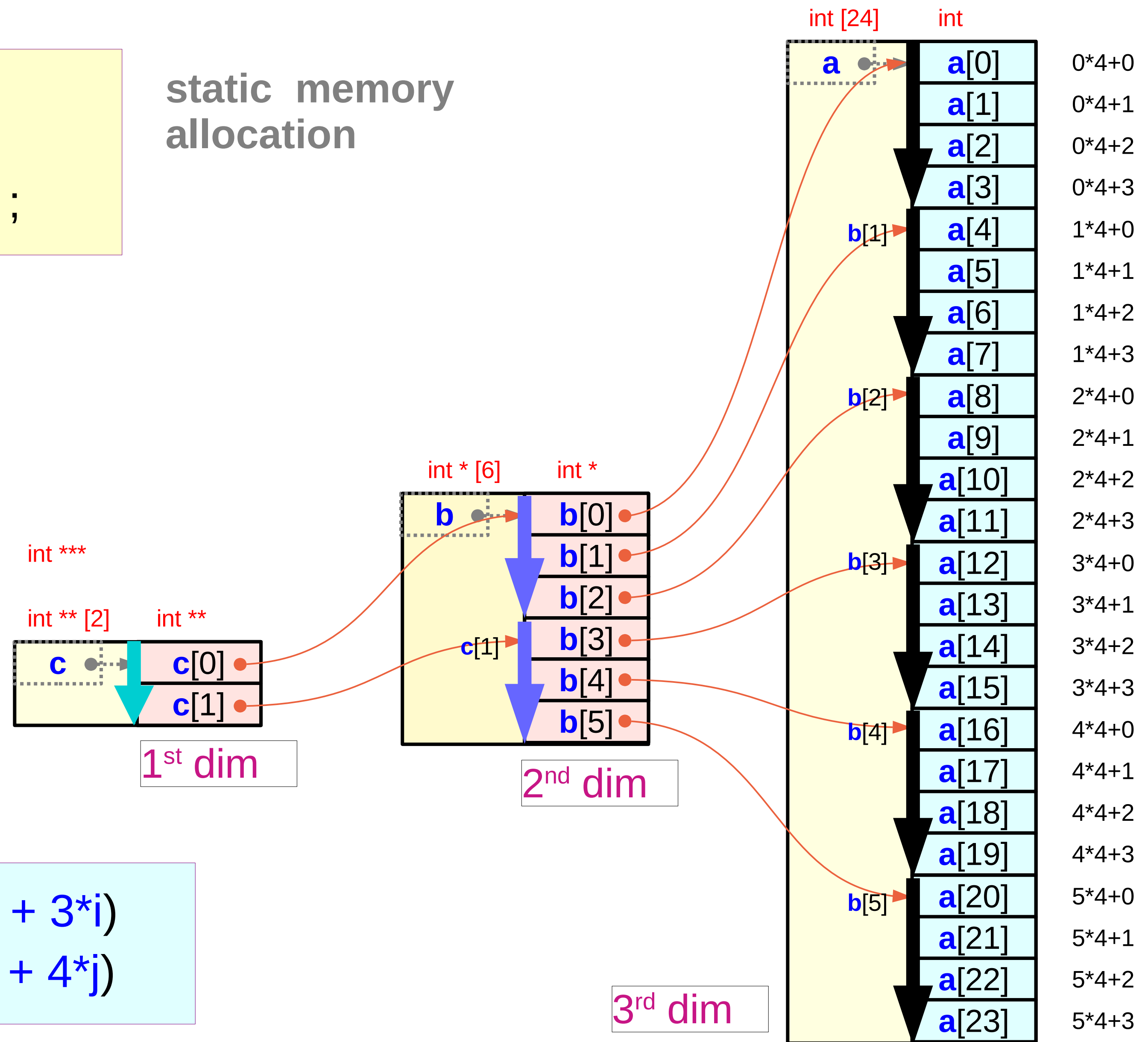
Accessing **statically** allocated arrays

Accessing **dynamically** allocated arrays

Using arrays a, b, c – statically allocated

int **	c [2] ;
int *	b [2*3] ;
int	a [2*3*4] ;

static memory allocation

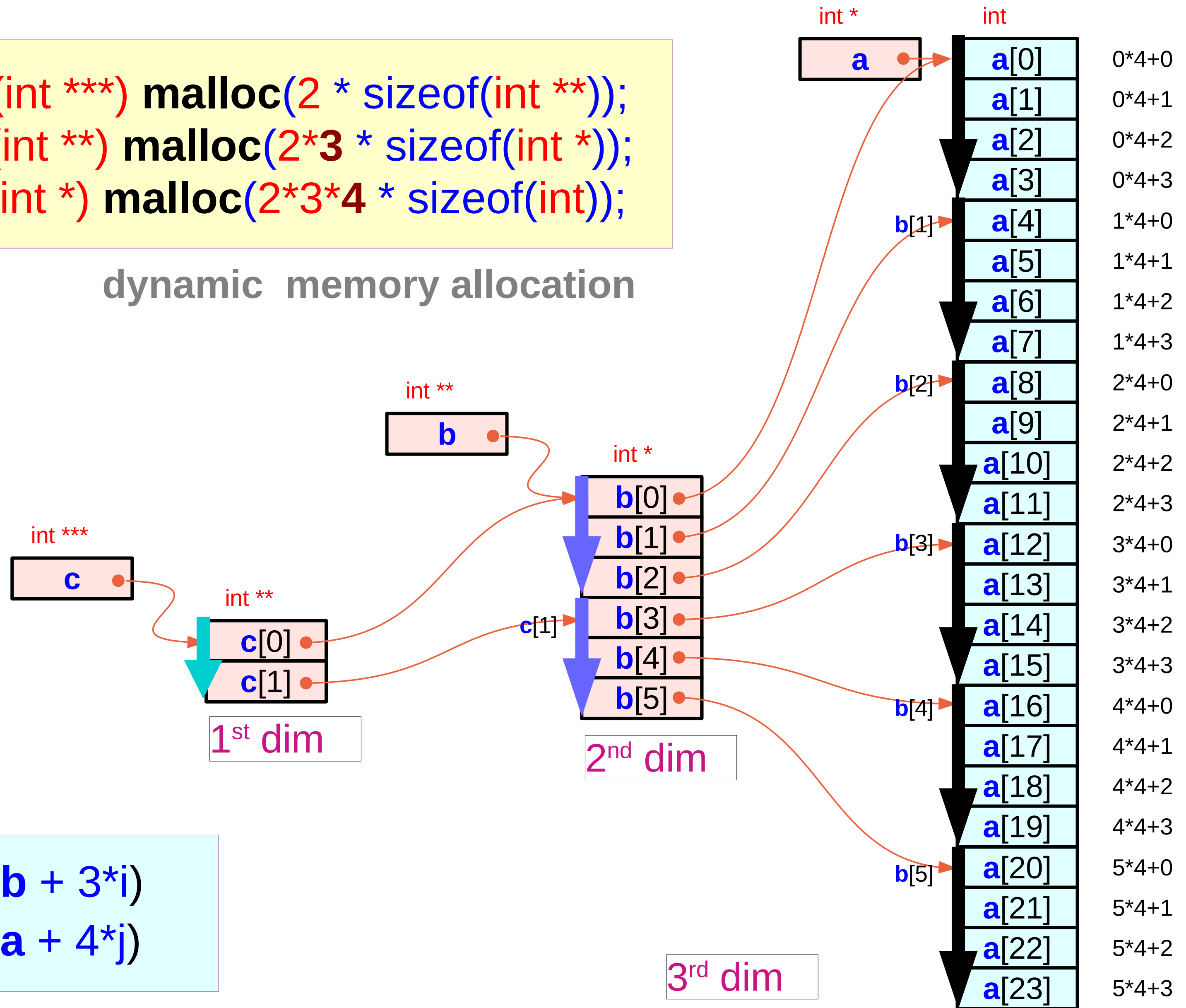


c[i] = &b[3*i] (= b + 3*i)

b[j] = &a[4*j] (= a + 4*j)

Using pointer a, b, c – dynamically allocated

```
int ***  c   = (int ***) malloc(2 * sizeof(int **));
int **   b   = (int **) malloc(2*3 * sizeof(int *));
int *    a   = (int *) malloc(2*3*4 * sizeof(int));
```



c[i] = &b[3*i] (= b + 3*i)

b[j] = &a[4*j] (= a + 4*j)

Static v.s. dynamic allocation (1)

```
int **   c  [2] ;
int *    b  [2*3] ;
int      a  [2*3*4] ;
```

```
int ***  c  = (int ***) malloc(2 * sizeof(int **));
int **   b  = (int **)  malloc(2*3 * sizeof(int *));
int *    a  = (int *)   malloc(2*3*4 * sizeof(int));
```

static memory allocations

type(**c**) = int ** [2] → int ***
type(**b**) = int * [2*3] → int **
type(**a**) = int [2*3*4] → int *

sizeof(**c**) = 2 * sizeof(int **)
sizeof(**b**) = 2*3 * sizeof(int *)
sizeof(**a**) = 2*3*4 * sizeof(int)

value(**c[i]**) = **b** + 3***i**
value(**b[j]**) = **a** + 4***j**

dynamic memory allocations

type(**c**) = int ***
type(**b**) = int **
type(**a**) = int *

sizeof(**c**) = 4 bytes on 32-bit system
sizeof(**b**) = 4 bytes on 32-bit system
sizeof(**a**) = 4 bytes on 32-bit system

value(**c[i]**) = **b** + 3***i**
value(**b[j]**) = **a** + 4***j**

c[**i**] = &**b**[3***i**] (= **b** + 3***i**)
b[j] = &**a**[4***j**] (= **a** + 4***j**)

Static v.s. dynamic allocation (2)

- **static allocation**

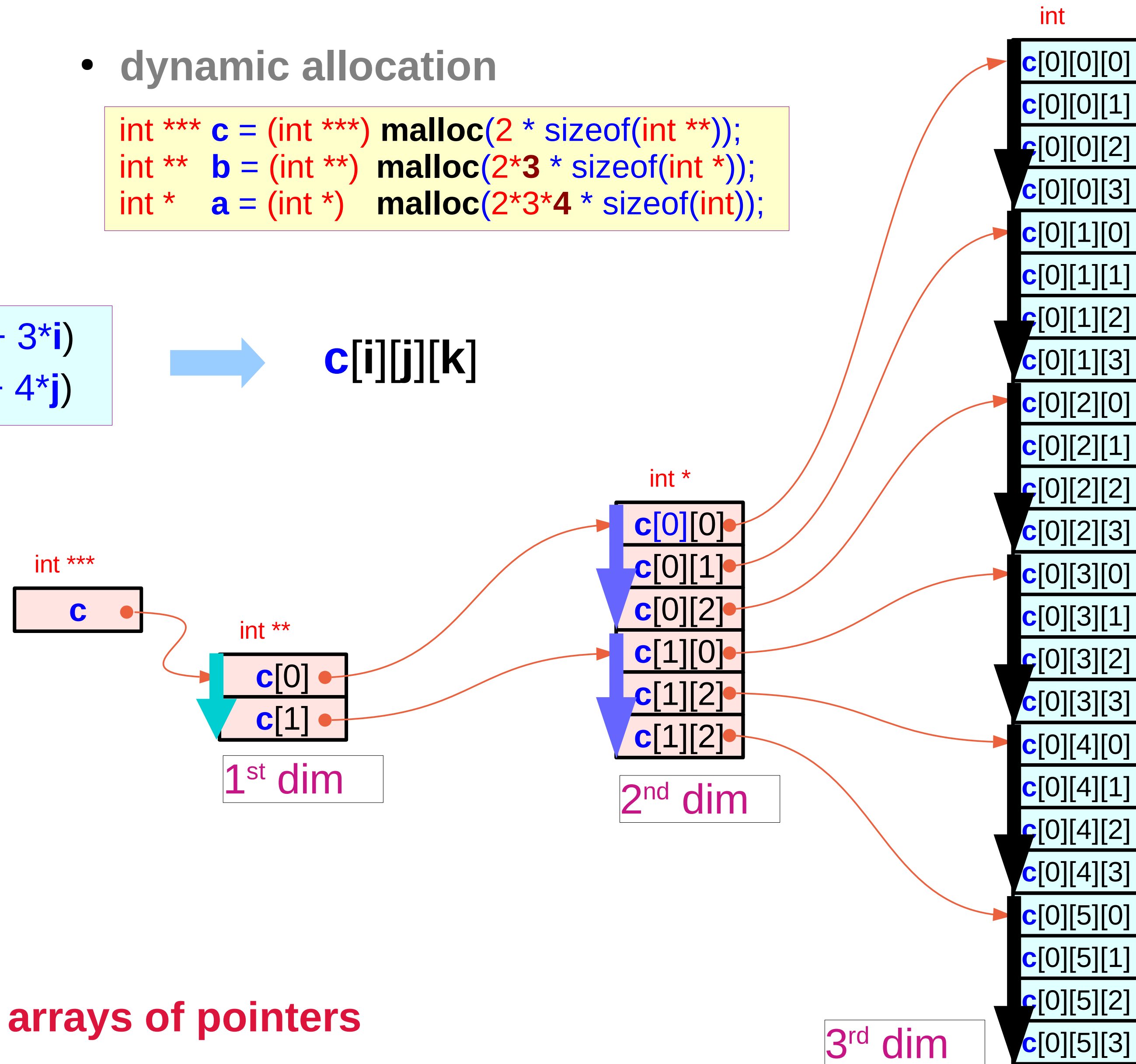
```
int **    c   [2] ;
int *     b   [2*3] ;
int       a   [2*3*4] ;
```

- **dynamic allocation**

```
int *** c = (int ***) malloc(2 * sizeof(int **));
int **  b = (int **)  malloc(2*3 * sizeof(int *));
int *   a = (int *)   malloc(2*3*4 * sizeof(int));
```

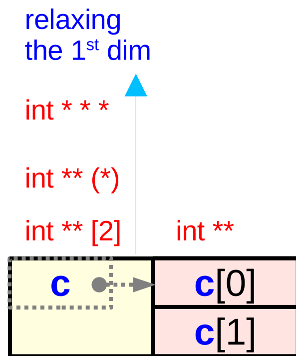
c[i] = &b[3*i] (= b + 3*i)
b[j] = &a[4*j] (= a + 4*j)

c[i][j][k]

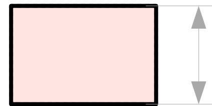


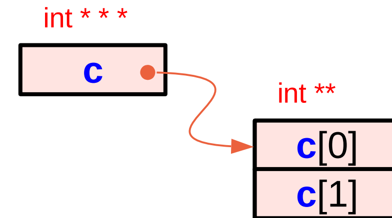
Static v.s. dynamic allocation (3)

```
int ***   c = (int ***)  malloc(2 * sizeof(int **));
```



malloc(2 * sizeof(int **));





int ** c [2] ;

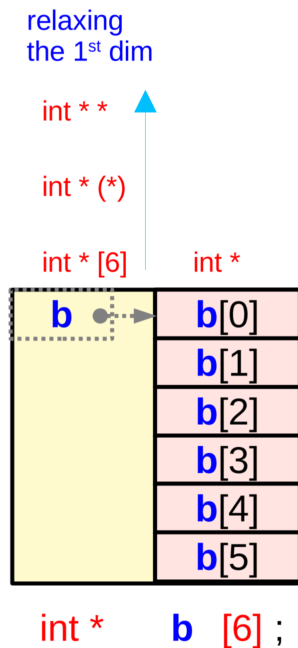
int *** c = (int ***) malloc(2 * sizeof(int **));

static memory allocation

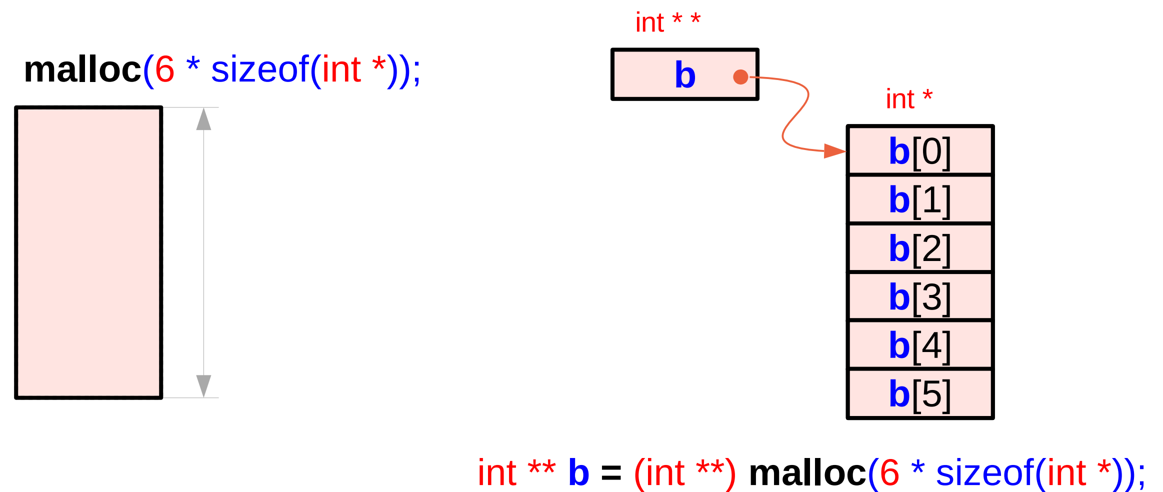
dynamic memory allocation

Static v.s. dynamic allocation (4)

```
int **    b = (int ***)  malloc(6 * sizeof(int *));
```

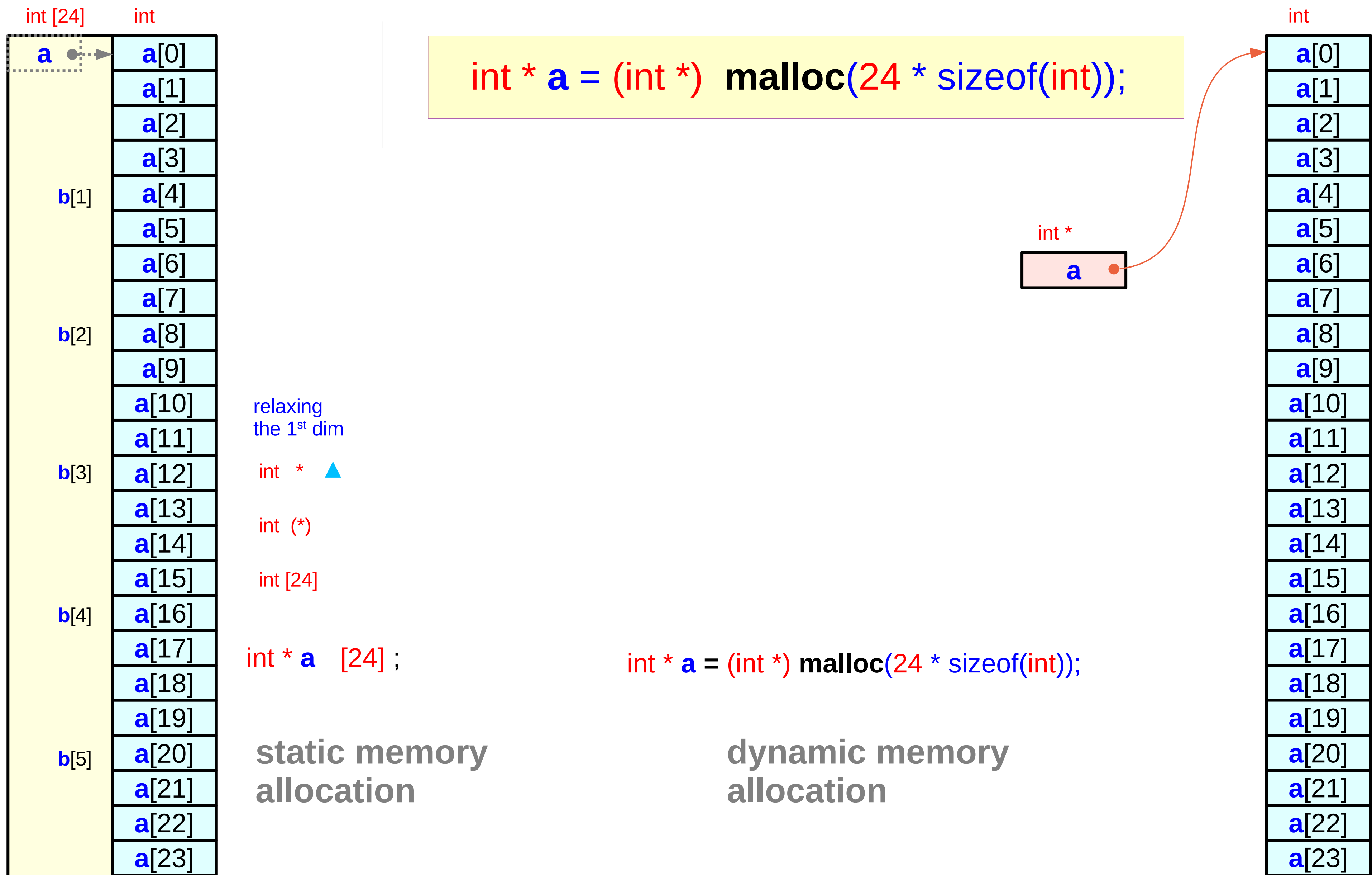


static memory allocation



dynamic memory allocation

Static v.s. dynamic allocation (5)



& address-of operator

***** dereference operator

Address-of operator and dereferencing operator

the address of a variable :
*address-of operator **&***

& variable *:*
returns the address of a variable

variable *has memory locations*
whose value can be changed
by an assignment

(***variable*** *must be an lvalue*)

the content at an address :
*dereferencing operator ******

**** address*** *:*
returns the value at the address

**** address*** *has memory locations*
whose value can be changed
by an assignment

(**** address*** *is an lvalue*)

lvalue and rvalue in assignments

Left Hand Side *Right Hand Side*
LHS **=** ***RHS***

```
int   a,  b = 10 ;
int * p, q = &a ;
```

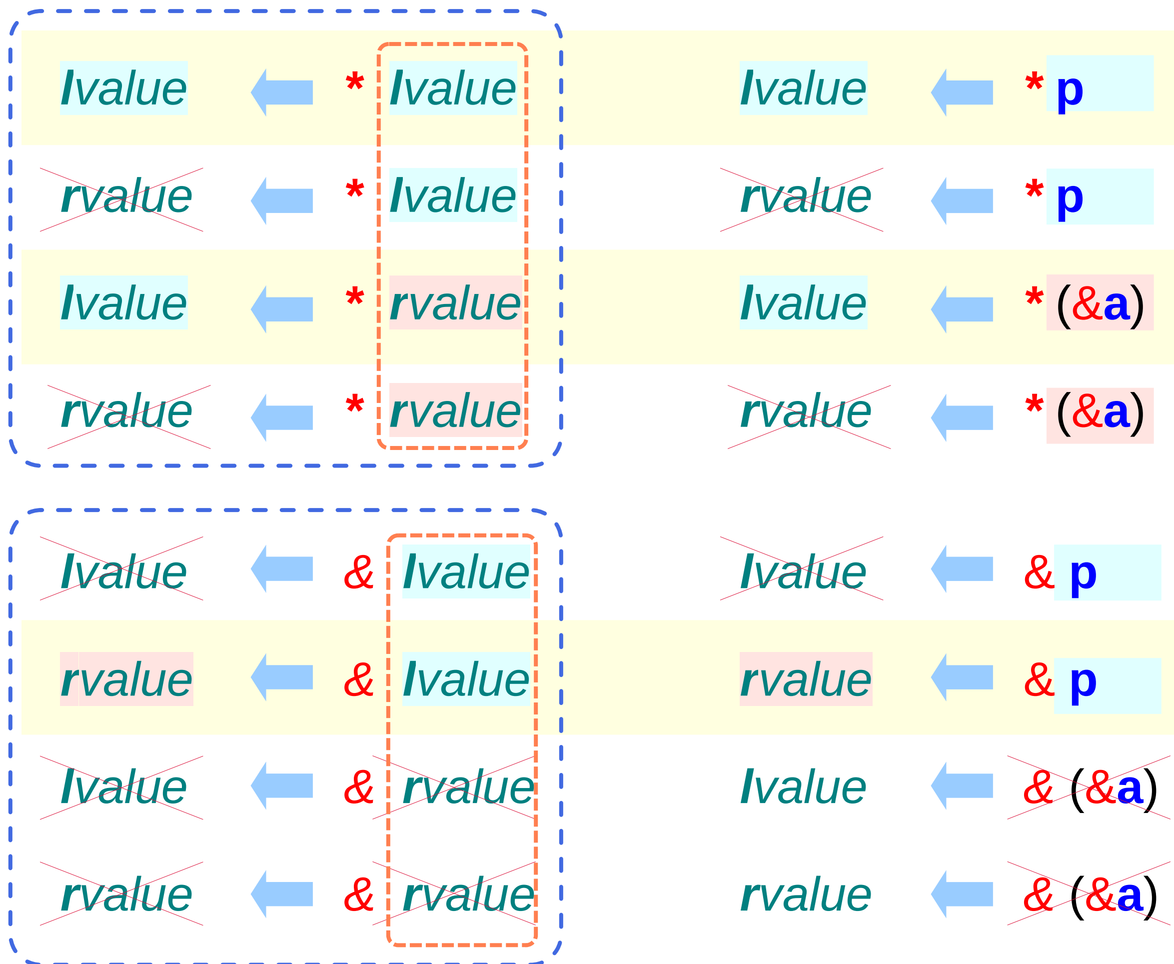
LHS		RHS
***l**value*	=	***l**value*
***l**value*	=	***r**value*
~~***r**value*~~	=	***l**value*
~~***r**value*~~	=	***r**value*

LHS		RHS
p	=	**q** ;
p	=	&**a** ;
~~&**a**~~	=	**p** ;
~~&**a**~~	=	&**b** ;

*in the **LHS,** only **l**value can exist*
r**value can exist only in the **RHS

a, **b**, **p**, **q**	: **l**values	… variables	… RW
***p**, ***q**	: **l**values	… variables	… RW
&**a**, &**b**	: **r**values	… constants	… RO

lvalue and rvalue with * and & operators




```
int   a = 10 ;
int * p = &a ;
```

* can be applied
to either an **lvalue** variable
or a **rvalue** address

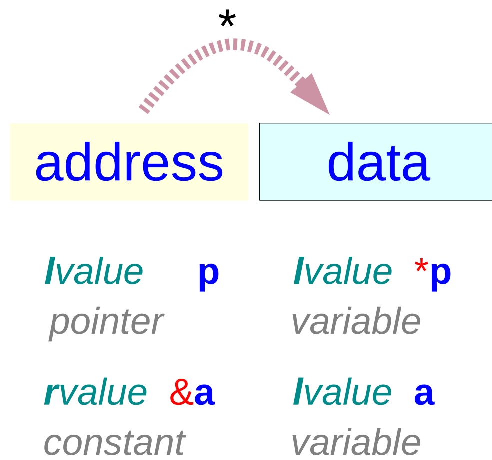
* **operand** becomes
an **lvalue** variable
thus can be applied
successively.

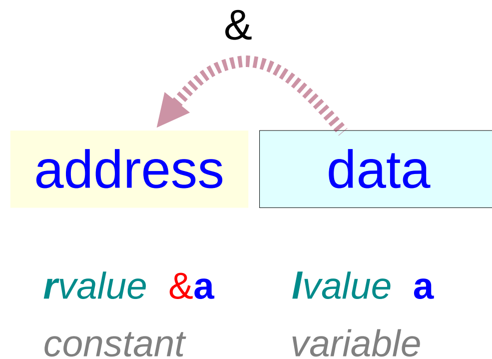
& can be applied
to only an **lvalue** variable and
returns only an **rvalue** address

a, **p**	: **l**values	… variables	… RW
*p	: **l**values	… variables	… RW
&a	: **r**values	… constants	… RO

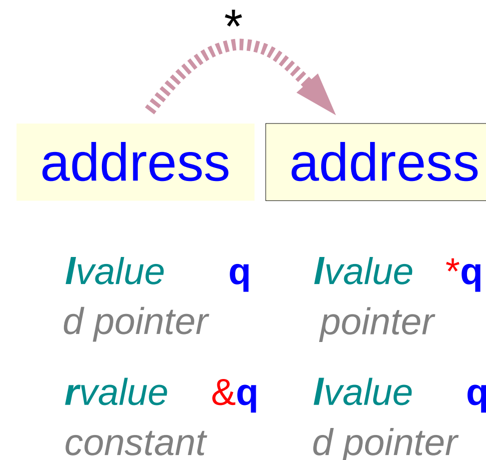
Address-of and dereference operators

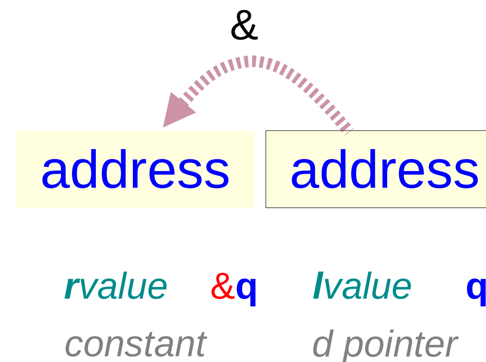
Primitive Data Type



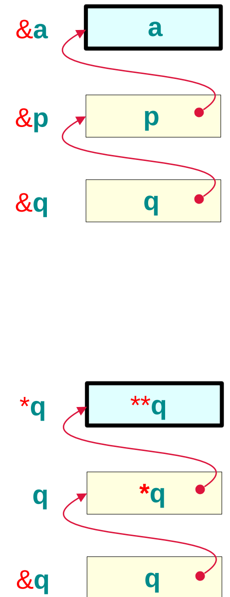


Pointer Data Type






```
int    a ;
int *  p ;
int ** q ;
```



Finding sub-array sizes

int c [2][3][4] ;

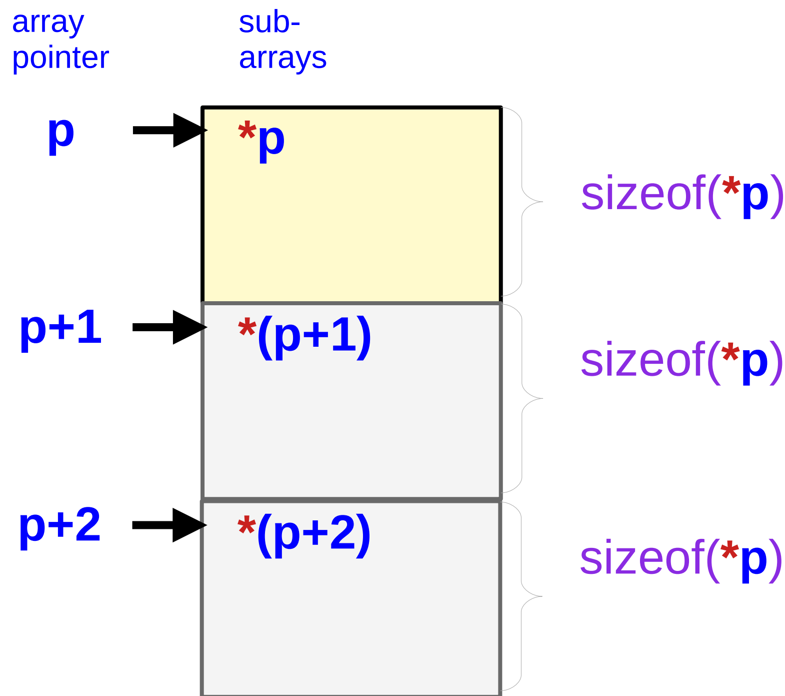
[2] [3] [4]
sizeof(c[i][j][0]) = sizeof(int)

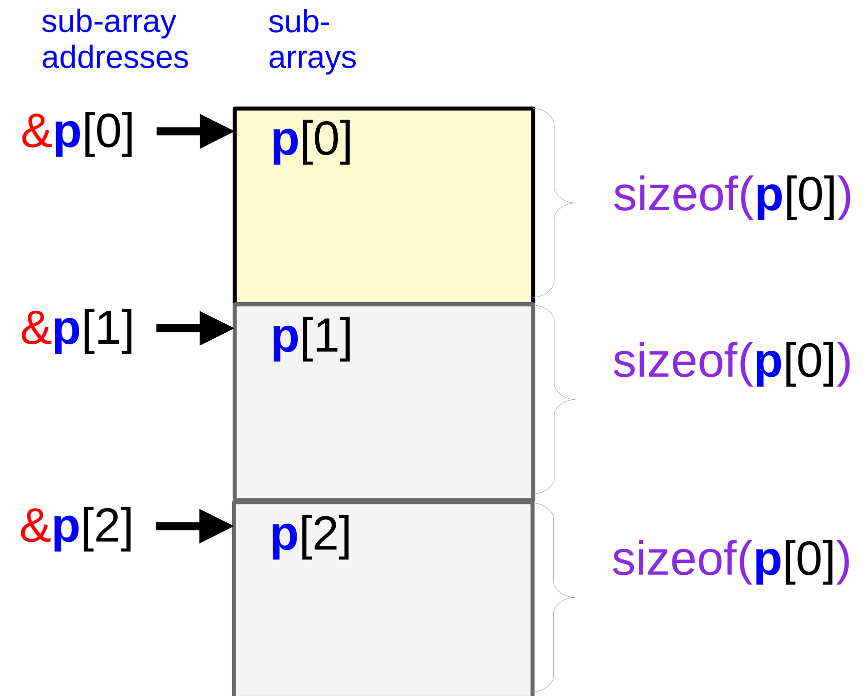
[2] [3] [4]
sizeof(c[i][0]) = 4 * sizeof(int)

[2] [3] [4]
sizeof(c[i]) = 3 * 4 * sizeof(int)

[2] [3] [4]
sizeof(c) = 2 * 3 * 4 * sizeof(int))

Byte addresses in an array





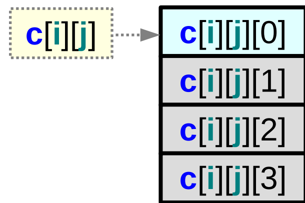
value(p+0) = value(p) + 0 * sizeof(*p)
value(p+1) = value(p) + 1 * sizeof(*p)
value(p+2) = value(p) + 2 * sizeof(*p)

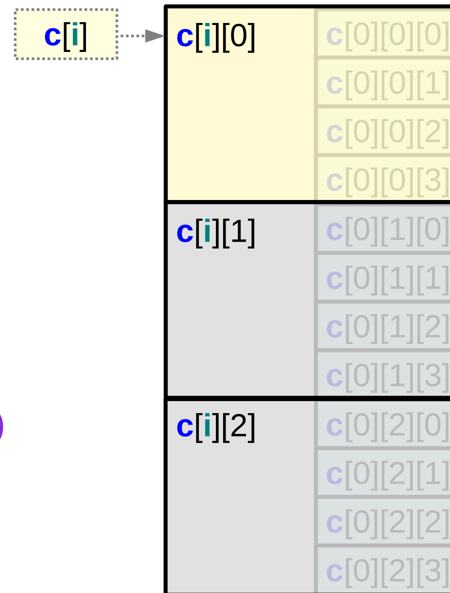
byte address byte address byte size

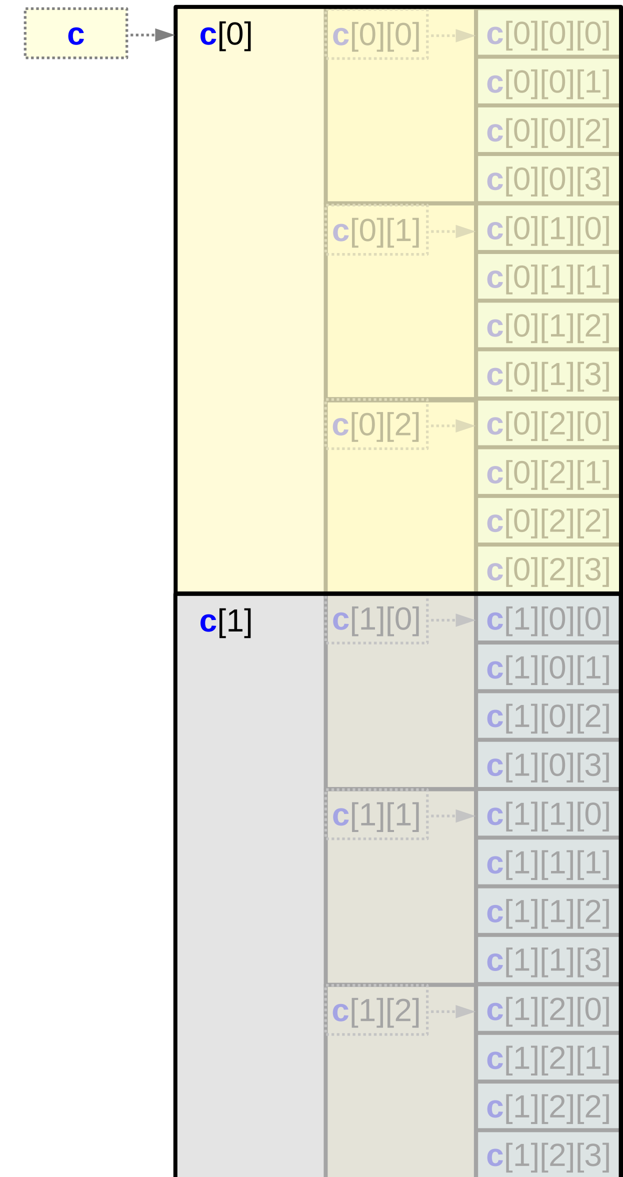
value(&p[0]) = value(p) + 0 * sizeof(p[0])
value(&p[1]) = value(p) + 1 * sizeof(p[0])
value(&p[2]) = value(p) + 2 * sizeof(p[0])

byte address byte address byte size

Byte addresses of &c[i], &c[i][j], &c[i][j][k]







value(&c[i][j][k]) **k** = 0:3
= value(c[i][j]) + **k** * sizeof(*c[i][j])
= value(c[i][j]) + **k** * sizeof(c[i][j][0])
= value(c[i][j]) + **k** * sizeof(int)

value(&c[i][j]) **j** = 0:2
= value(c[i]) + **j** * sizeof(*c[i])
= value(c[i]) + **j** * sizeof(c[i][0])
= value(c[i]) + **j** * sizeof(int) * 4

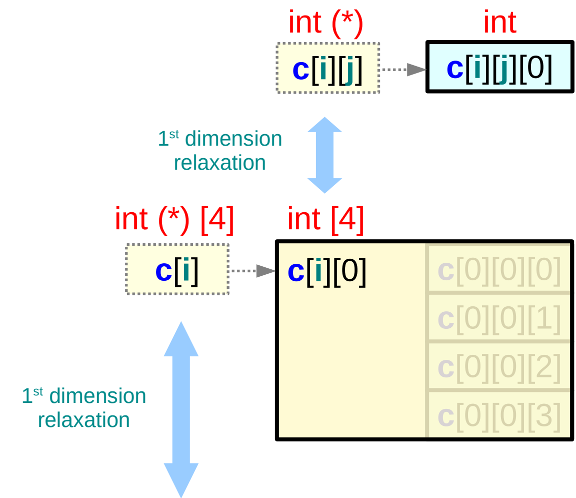
value(&c[i]) **i** = 0 : 1
= value(c) + **i** * sizeof(*c)
= value(c) + **i** * sizeof(c[0])
= value(c) + **i** *sizeof(int) * 3 * 4

Byte addresses of &c[i], &c[i][j], &c[i][j][k]

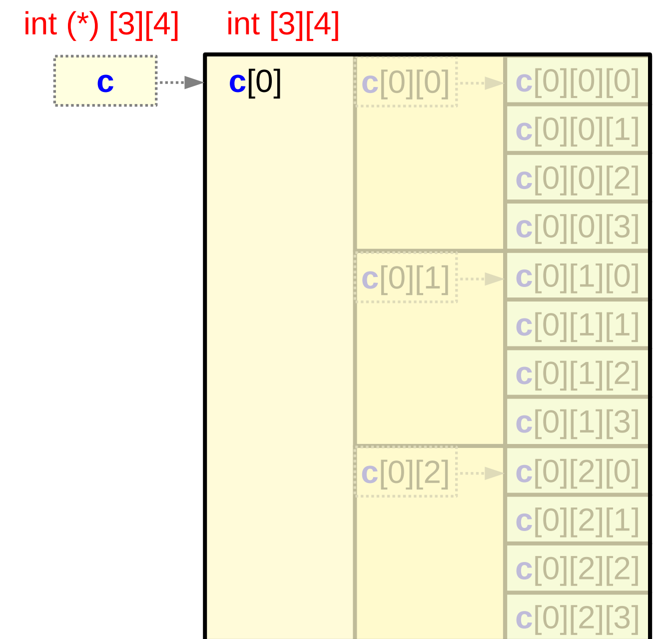
Address Replication

transferring pointing address
to the pointer that references itself

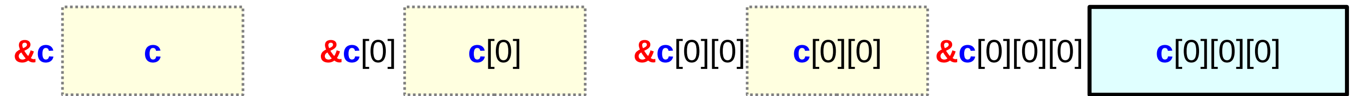
&X = X



int (*) int value(&c[i][j][k])	=	int (*) value(c[i][j]) + k * sizeof(int)
int (*) [4] int [4] value(&c[i][j])	=	int (*) [4] value(c[i]) + j * 4 * sizeof(int)
int (*) [3][4] int [3][4] value(&c[i])	=	int (*) [3][4] value(c) + i * 3 * 4 * sizeof(int)
byte addresses of abstract data		values of virtual pointers

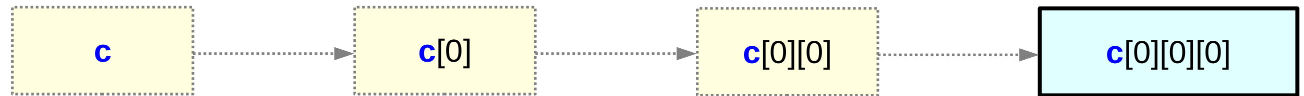


Address Replication (1)

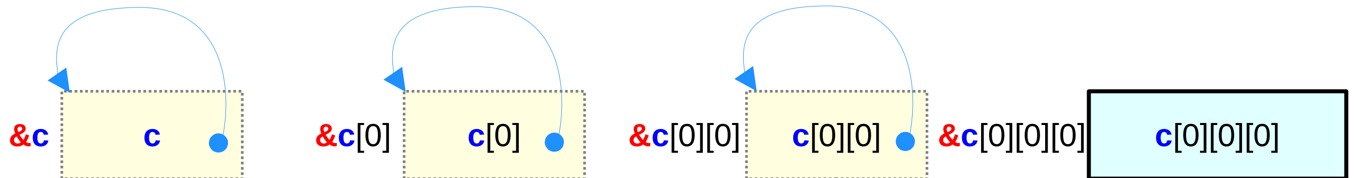


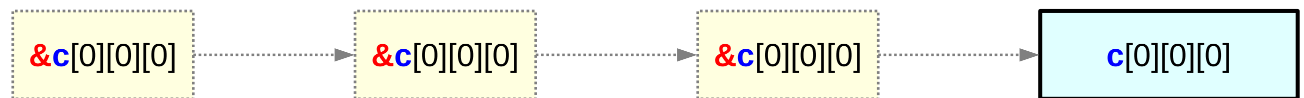
equivalences

c = &c[0] c[0] ≡ &c[0][0] c[0][0] ≡ &c[0][0][0]



address replication
value(c[i][j]) = value(&c[i][j])
value(c[i]) = value(&c[i])
value(c) = value(&c)

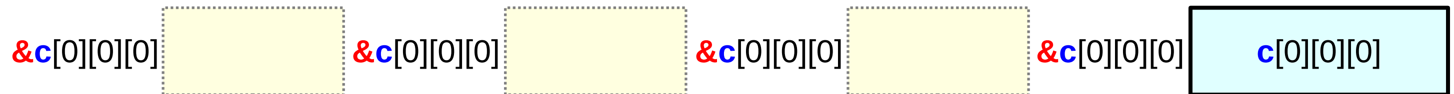




all have the same address value

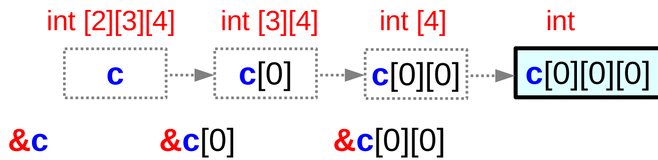
c, c[0], c[0][0] :
virtual pointers
the same address value

a physical location
has a unique address



all have the same starting address

Address Replication (2)



&c &c[0] &c[0][0]

equivalence relations

c[i][j] = *(c[i]+j) &c[i][j] = (c[i]+j)

c[i] = *(c+i) &c[i] = (c+i)

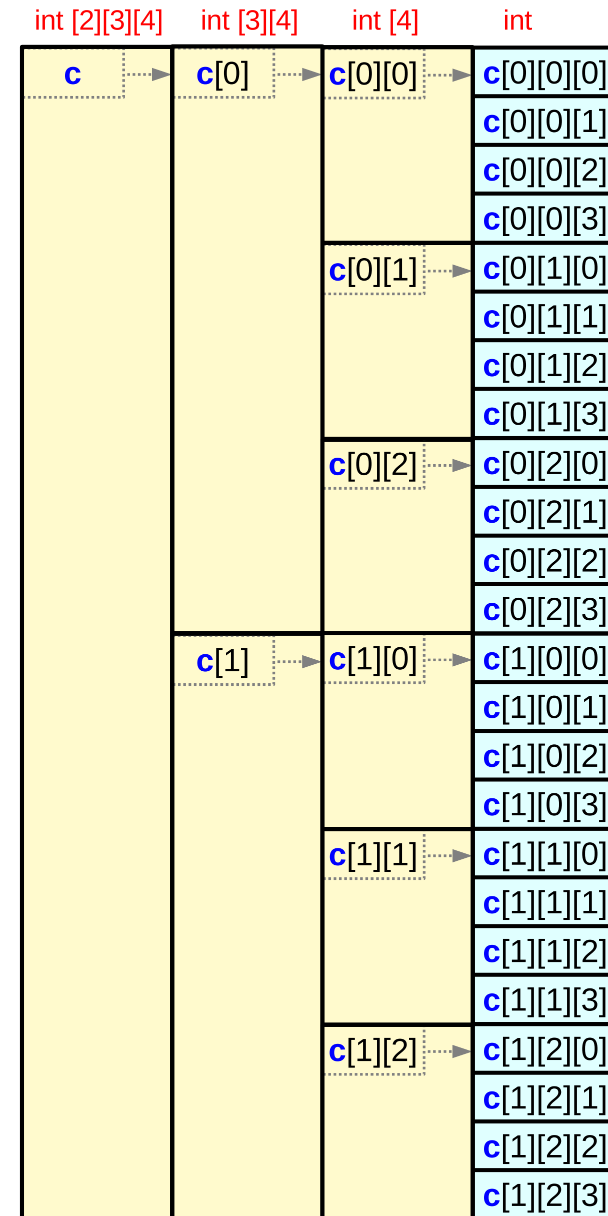
address replicaion

value(c[i][j]) = value(&c[i][j]) = value(c[i]+j)

value(c[i]) = value(&c[i]) = value(c+i)

c[i] and c[i][0] point to the same data c[i][0][0]

c and c[0] point to the same data c[0][0][0]



Address Replication (3)

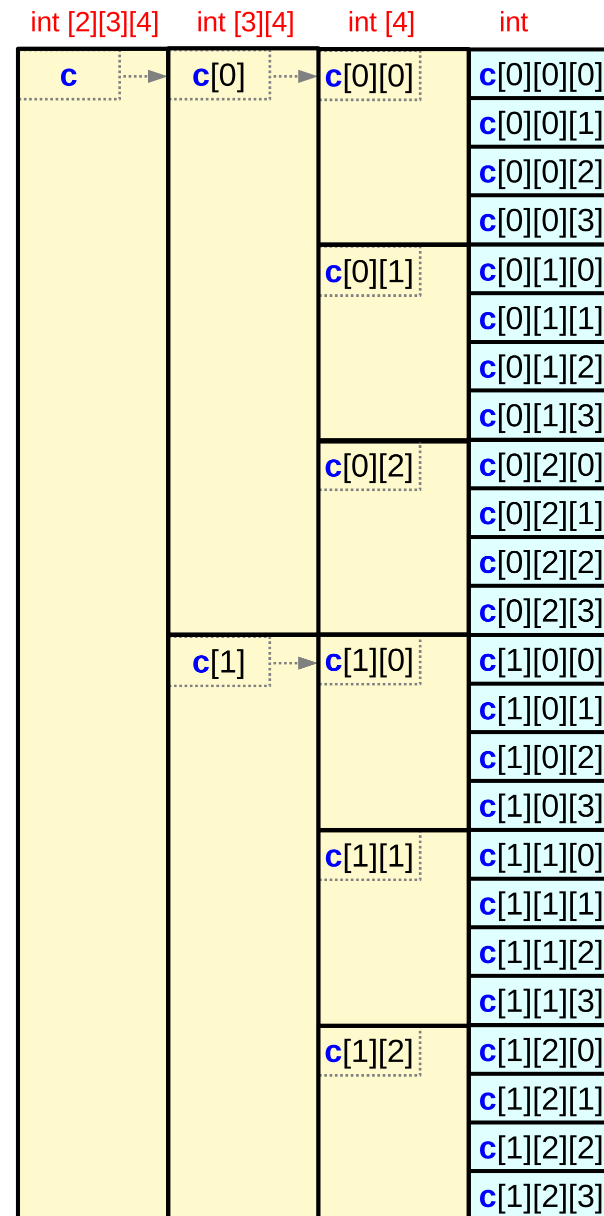
value(c) = value(c[0]) = value(c[0][0]) = value(&c[0][0][0])
value(c[0][1]) = value(&c[0][1][0])
value(c[0][2]) = value(&c[0][1][0])
value(c[1]) = value(c[1][0]) = value(&c[1][0][0])
value(c[1][1]) = value(&c[1][1][0])
value(c[1][2]) = value(&c[1][1][0])

value(c) = value(c[0]) = value(c[0][0]) = value(&c[0][0][0])
value(c[i]) = value(c[i][j]) = value(&c[i][j][0])

address replication
value(c[i][j]) = value(&c[i][j])
value(c[i]) = value(&c[i])
value(c) = value(&c)

equivalences
value(c[i][j]) = value(&c[i][j][0])
value(c[i]) = value(&c[i][0])
value(c) = value(&c[0])

virtual pointers



Using a pointer chain type II

int c [2][3][4] ; ⇒ c[i][j][k]

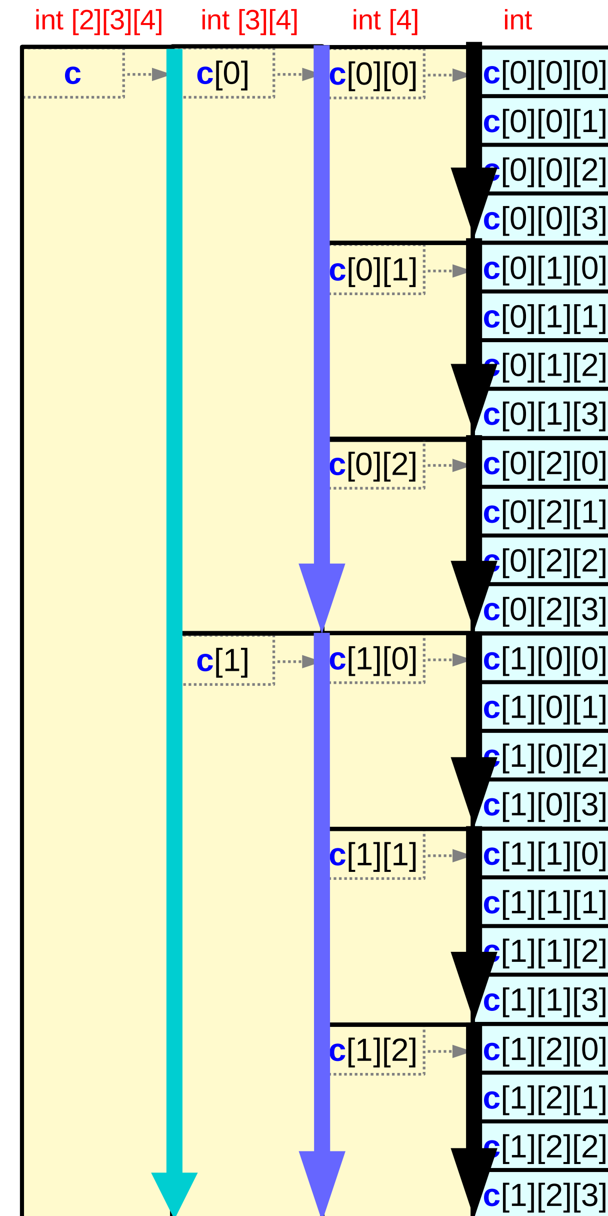
static allocation

value(c) = value(c[0]) = value(c[0][0]) = &c[0][0][0]
value(c[0][1]) = &c[0][1][0]
value(c[0][2]) = &c[0][1][0]
value(c[1]) = value(c[1][0]) = &c[1][0][0]
value(c[1][1]) = &c[1][1][0]
value(c[1][2]) = &c[1][1][0]

sizeof(c) = 2*3*4 * sizeof(int)
sizeof(c[i]) = 3*4 * sizeof(int)
sizeof(c[i][j]) = 4 * sizeof(int)

type(c) = int [2][3][4]
int (*)[3][4]
type(c[i]) = int [3][4]
int (*)[4]
type(c[i][j]) = int [4]
int (*)

pointers to arrays



Byte addresses of sub-arrays in an array

&(&(&(c[i])[j])[k])

& C operator

can be applied to only **lvalue** variable
returns address value
thus, the above expression is not possible

successive application of & is not possible

In constrast, ***p** becomes a lvalue variable
* operator can be applied successively.

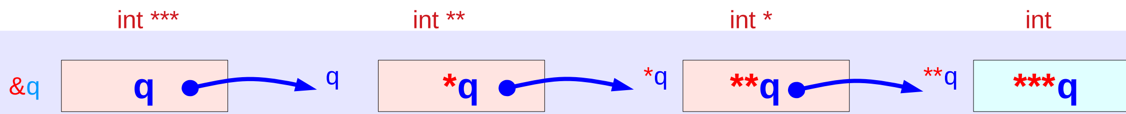
Pointer Array Approach

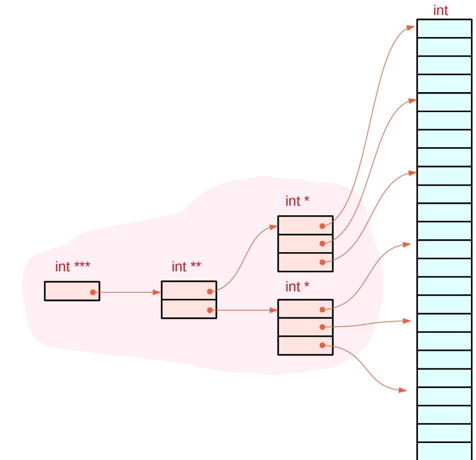
Array Pointer Approach

Two types of 3-d array accesses

Pointer Array Approach (arrays of pointers)

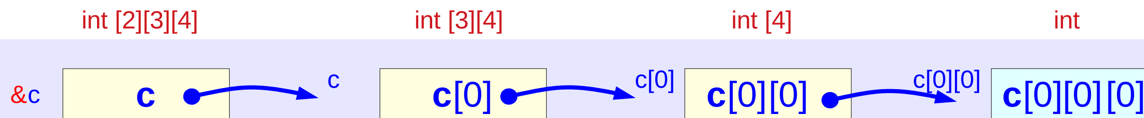
Pointer Chain Type I

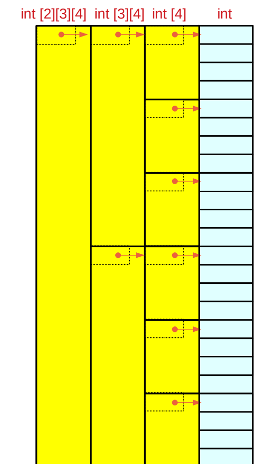




Array Pointer Approach (pointers to arrays)

Pointer Chain Type II





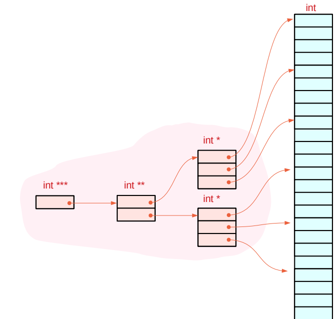
Accessing c[i][j][k] element

Accessing c[i][j][k]

skip i elements in c[i]	skip j elements in c[i][j]	skip k elements in c[i][j][k]

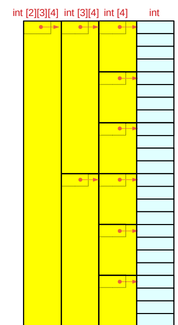
Pointer Array Approach (arrays of pointers)

skip i * sizeof(int **) i * 4 bytes from c	skip j * sizeof(int *) j * 4 bytes from c[i]	skip k * sizeof(int) k * 4 bytes from c[i][j]
$(c + i)_{1 \cdot 4}$	$(c[i] + j)_{1 \cdot 4}$	$(c[i][j] + k)_{1 \cdot 4}$



Array Pointer Approach (pointers to arrays)

skip i * sizeof(int [3][4]) i * 3*4*4 bytes from c	skip j * sizeof(int [4]) j *4*4 bytes from c[i]	skip k * sizeof(int) k * 4 bytes from c[i][j]
$(c + i)_{3 \cdot 4 \cdot 4}$	$(c[i] + j)_{4 \cdot 4}$	$(c[i][j] + k)_{1 \cdot 4}$



Accessing c[i][j][k] – Pointer Array Approach

Pointer Array Approach

skip i * sizeof(int **)
i * 4 bytes from c

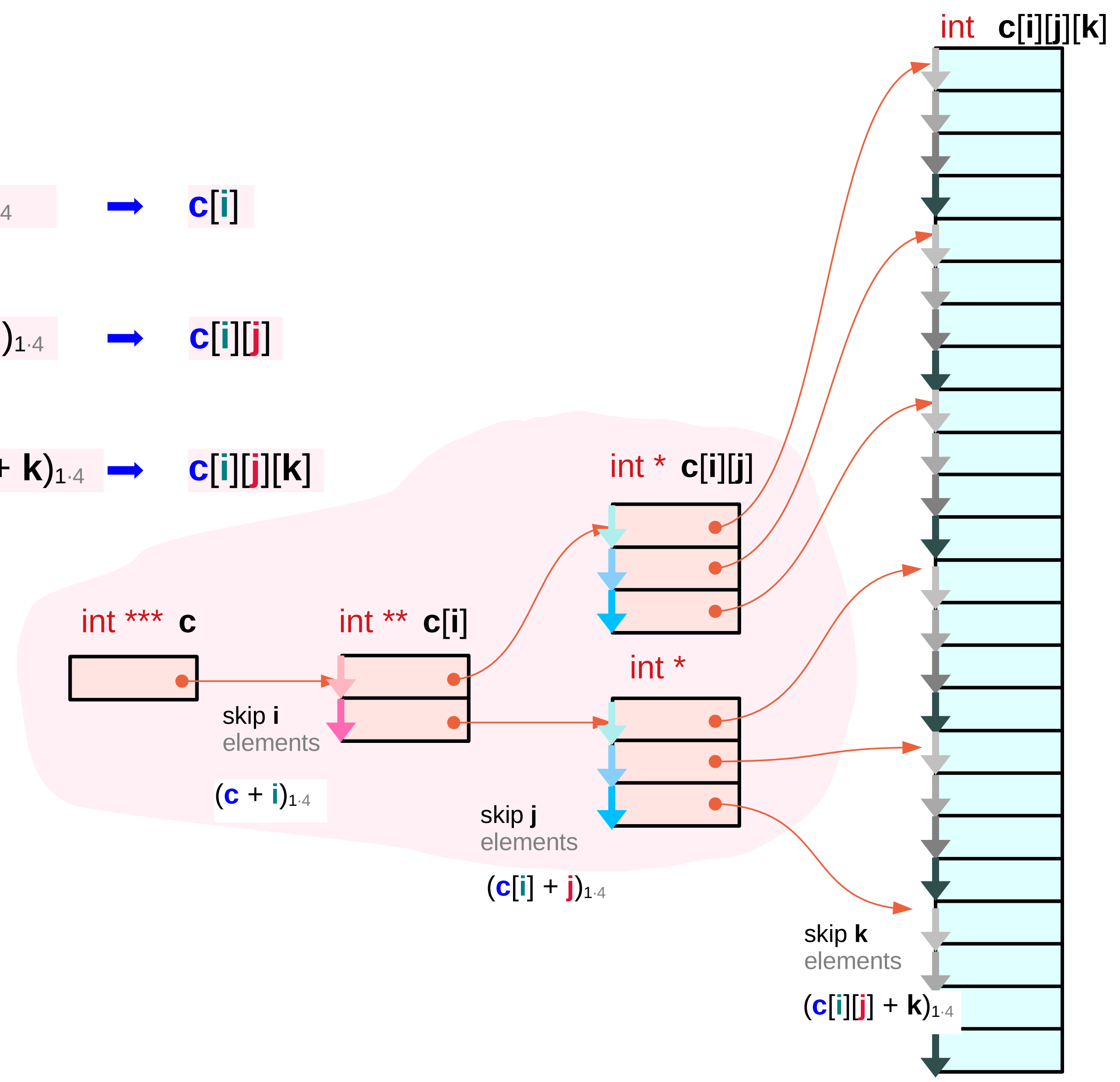
$(c + i)_{1\cdot 4}$ ➡ c[i]

skip j * sizeof(int *)
j * 4 bytes from c[i]

$(c[i] + j)_{1\cdot 4}$ ➡ c[i][j]

skip k * sizeof(int)
k * 4 bytes from c[i][j]

$(c[i][j] + k)_{1\cdot 4}$ ➡ c[i][j][k]



Accessing c[i][j][k] – Array Pointer Approach

Array Pointer Approach

skip **i** * sizeof(int [3][4])
i * 3*4*4 bytes from **c**

$(c + i)_{3 \cdot 4 \cdot 4}$ ➡ c[i]

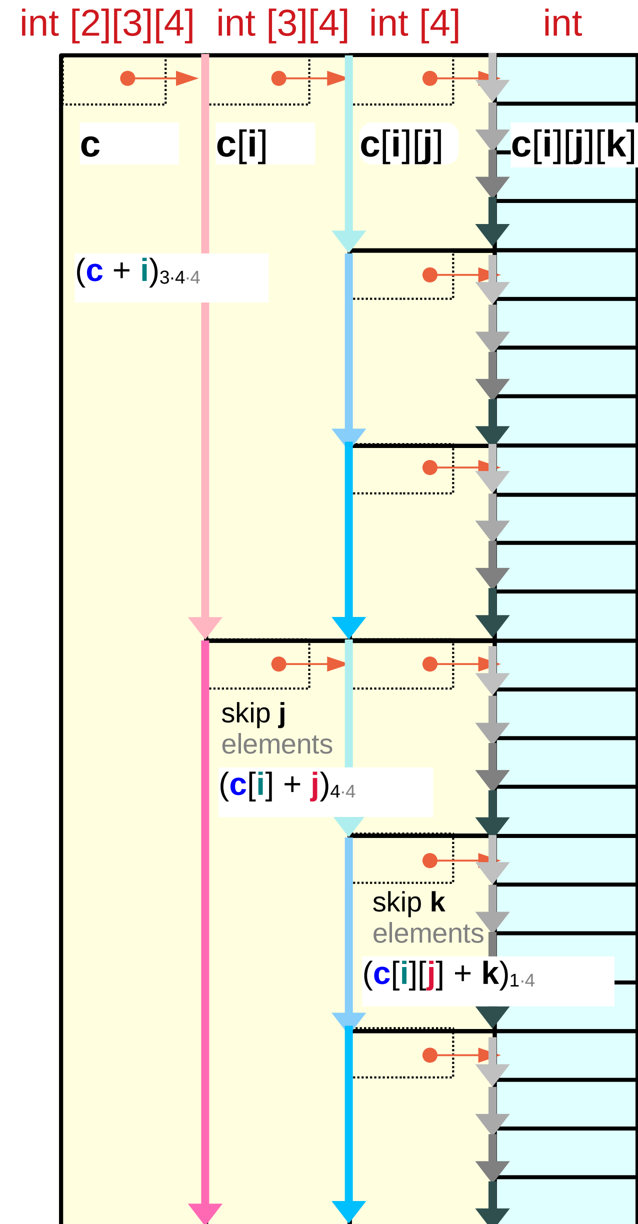
skip **j** * sizeof(int [4])
j *4*4 bytes from **c[i]**

$(c[i] + j)_{4 \cdot 4}$ ➡ c[i][j]

skip **k** * sizeof(int)
k * 4 bytes from **c[i][j]**

$(c[i][j] + k)_{1 \cdot 4}$ ➡ c[i][j][k]

- ***subarray partitioning***
- ***address replication***



Three contiguity constraints

Continuity Constraints

(c[i][j] +k)	➡	c[i][j][k]
(c[i] +j)	➡	c[i][j]
(c +i)	➡	c[i]

contiguous c[i][j][k]	over k=0:3
contiguous c[i][j]	over j=0:2
contiguous c[i]	over i=0:1

Pointer Array Approach (array of pointers)

c[i][j][k]	≡	*(c[i][j] +k)
*(c[i][j] +k)	≡	*(*(c[i] +j) +k)
((c[i] +j) +k)	≡	*(*(*(c +i) +j) +k)

contiguous 4 c[i][j][k]	4 * (int	4 bytes)
contiguous 3 c[i][j]	3 * (int *	4 or 8 bytes)
contiguous 2 c[i]	2 * (int **	4 or 8 bytes)

Array Pointer Approach (pointer to arrays)

c[i][j][k]	≡	*(c[i][j] +k)
*(c[i][j] +k)	≡	*(*(c[i] +j) +k)
((c[i] +j) +k)	≡	*(*(*(c +i) +j) +k)

contiguous 4 c[i][j][k]	4 * (int	4 bytes)
contiguous 3 c[i][j]	3 * (int [4]	4*4 bytes)
contiguous 2 c[i]	2 * (int [3][4]	3*4*4 bytes)

Three contiguity constraints – skipping leaf elements

Continuity Constraints

(c[i][j] +k)	➡	c[i][j][k]
(c[i] +j)	➡	c[i][j]
(c +i)	➡	c[i]

contiguous c[i][j][k]	over k=0:3
contiguous c[i][j]	over j=0:2
contiguous c[i]	over i=0:1

Pointer Array Approach (array of pointers)

$(c[i][j] + k)_{1 \cdot 4}$	skip k * sizeof(int) bytes from c[i][j]
$(c[i] + j)_{1 \cdot 4}$	skip j * sizeof(int *) bytes from c[i]
$(c + i)_{1 \cdot 4}$	skip i * sizeof(int **) bytes from c

k leaf elements	k * sizeof(int)
j * 4 leaf elements	j * 4 * sizeof(int)
i * 3 * 4 leaf elements	i * 3 * 4 *sizeof(int)

Array Pointer Approach (pointer to arrays)

$(c[i][j] + k)_{1 \cdot 4}$	skip k * sizeof(int) bytes from c[i][j]
$(c[i] + j)_{4 \cdot 4}$	skip j * sizeof(int [4]) bytes from c[i]
$(c + i)_{3 \cdot 4 \cdot 4}$	skip i * sizeof(int [3][4]) bytes from c

k leaf elements	k * sizeof(int)
j * 4 leaf elements	j * 4 * sizeof(int)
i * 3 * 4 leaf elements	i * 3 * 4 *sizeof(int)

Accessing c[i][j][k]

```
int     c [L][M][N] ;
```

c[i]	≡ *(c + i)	
c[i][j]	≡ *(c[i] + j)	
c[i][j][k]	≡ *(c[i][j] + k)	

&c[i]	≡ (c + i)
&c[i][j]	≡ (c[i] + j)
&c[i][j][k]	≡ (c[i][j] + k)

equivalence relations		multiple indirections	address replications
c[i]	≡ *(c+i)	≡ *(c+i)	≡ (c+i)
c[i][j]	≡ *(c[i]+j)	≡ *(*(c+i)+j)	≡ ((c+i)+j)
c[i][j][k]	≡ *(c[i][j]+k)	≡ *(*(*(c+i)+j)+k)	≡ (((c+i)+j)+k)
			→ *(c +i +j +k)

Pointer Array Approach →

Array Pointer Approach →

Array element address – Pointer Array Approach

equivalence relations – c expressions

```
c[i][j][k]  = *(c[i][j] + k)
c[i][j]     = *(c[i] + j)
c[i]        = *(c + i)
```

```
&c[i][j][k] = (c[i][j] + k)
&c[i][j]    = (c[i] + j)
&c[i]       = (c + i)
```

address replication – math expressions

value(c[i][j]) = value($(c[i] + j)_{4\cdot 4}$)
value(c[i]) = value($(c + i)_{3\cdot 4\cdot 4}$)

value($(c[i] + j)_{4\cdot 4}$) = value(c[i]) + j * 4*4 …. sizeof(*c[i])
value($(c + i)_{3\cdot 4\cdot 4}$) = value(c) + i * 3*4*4 …. sizeof(*c)

address of c[i][j][k] – math expressions

&c[i][j][k] = value($(c[i][j] + k)_{1\cdot 4}$)
= value($(c[i] + j)_{4\cdot 4}$) + k * 1*4
= value($(c + i)_{3\cdot 4\cdot 4}$) + j * 4*4 + k*1*4
= value(c) + i * 3*4*4 + j * 4*4 + k * 1*4

- ***address replication***
- ***combining <u>size</u> and <u>address</u> information***

Array element address – Array Pointer Approach

equivalence relations – c expressions

```
c[i][j][k]   = *(c[i][j] + k)
c[i][j]      = *(c[i] + j)
c[i]         = *(c + i)
```

```
&c[i][j][k]  = (c[i][j] + k)
&c[i][j]     = (c[i] + j)
&c[i]        = (c + i)
```

address replication – math expressions

address of c[i][j][k] – math expressions

Accessing c[i][j][k] via byte addresses

Pointer Array Approach (array of pointers)

&c[i][j][k] = value($(c[i][j] + k)_{1\cdot 4}$) = value(c[i][j]) + k * 4
&c[i][j] = value($(c[i] + j)_{1\cdot 4}$) = value(c[i]) + j * 4
&c[i] = value($(c + i)_{1\cdot 4}$) = value(c) + i * 4

c[i][j][k] = *value(c[i][j]) + k * 4
c[i][j] = *value(c[i]) + j * 4
c[i] = *value(c) + i * 4

c[i][j][k] = *value(c[i][j]) + k * 4
= *value(*value(c[i]) + j * 4) + k * 4
= *value(*value(*value(c) + i * 4) + j * 4) + k * 4

three memory accesses for **c[i][j][k]**

Array Pointer Approach (pointer to arrays)

&c[i][j][k] = value($(c[i][j] + k)_{1\cdot 4}$) = value(c[i][j]) + k * 4
&c[i][j] = value($(c[i] + j)_{4\cdot 4}$) = value(c[i]) + j * 4*4
&c[i] = value($(c + i)_{3\cdot 4\cdot 4}$) = value(c) + i * 3*4*4

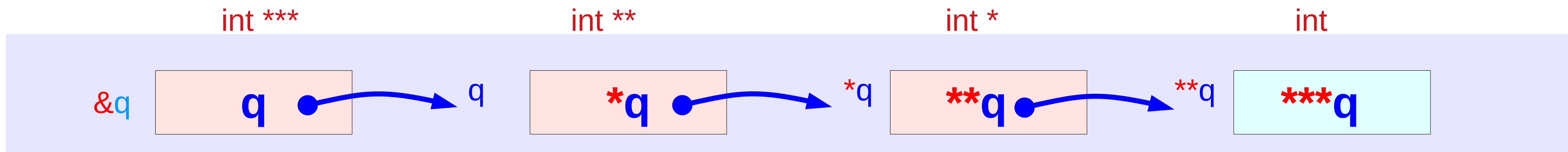
c[i][j][k] = *value(c[i][j]) + k * 4
c[i][j] = *value(c[i]) + j * 4*4
c[i] = *value(c) + i * 3*4*4

c[i][j][k] = *value(c[i][j]) + k * 4
= *value(*value(c[i]) + j * 4*4) + k * 4
= *value(*value(*value(c) + i * 3*4*4) + j * 4*4) + k * 4
= *value(value(value(c) + i * 3*4*4) + j * 4*4) + k * 4 address replication
= *value(c + i * 3*4*4 + j * 4*4 + k * 4) single memory access for **c[i][j][k]**

Pointer Chain Types

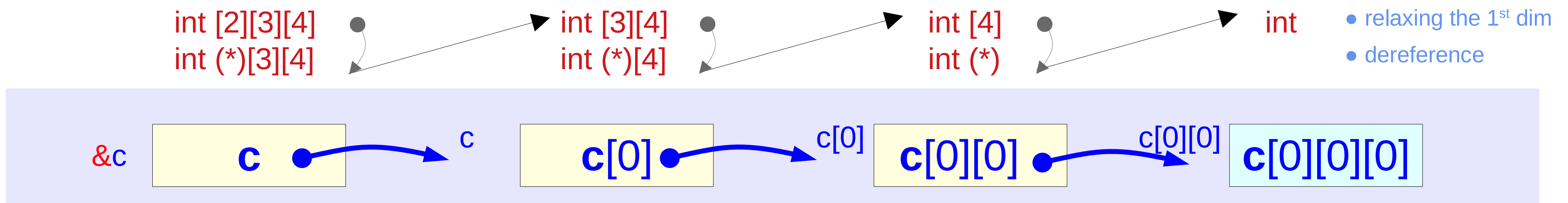
Pointer Chain Types

Pointer Chain Type I



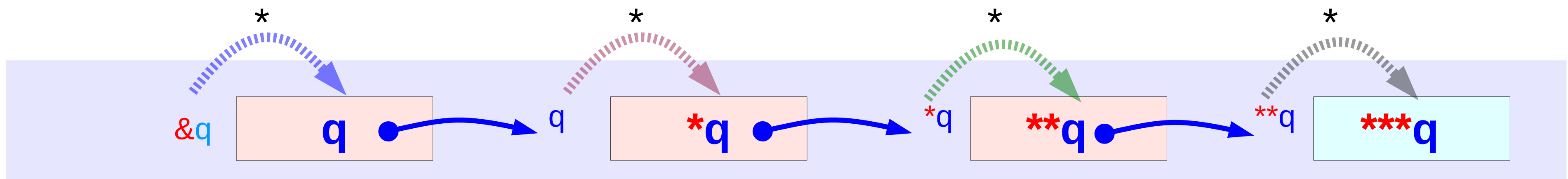
- **Pointer Array Approach** (user handled index operation)

Pointer Chain Type II



- **Array Pointer Approach** (compiler handled index operation)

Pointer Chain Type I – * and & operators



*(&q) ≡ q

C expression *(&q) equals to the variable q

*(q) ≡ *q

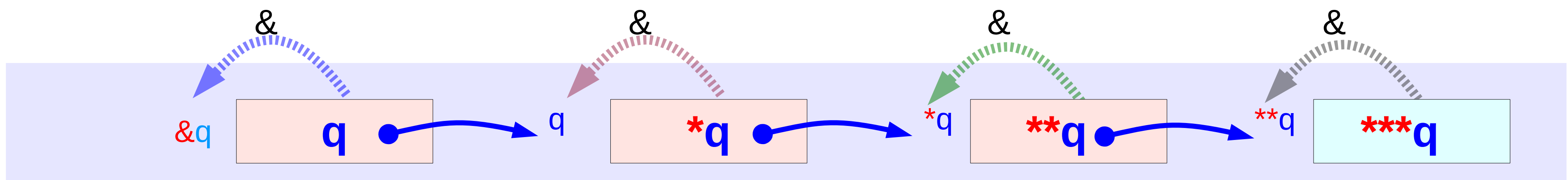
C expression *(q) equals to the variable q

*(*q) ≡ **q

C expression *(*q) equals to the variable **q

*(**q) ≡ ***q

C expression *(**q) equals to the variable ***q



&q

C expression &q equals to value(&q) which is the address value of a variable q

&(*q) ≡ value(q)

C expression &(*q) equals to value(q) which is the address value of a variable *q

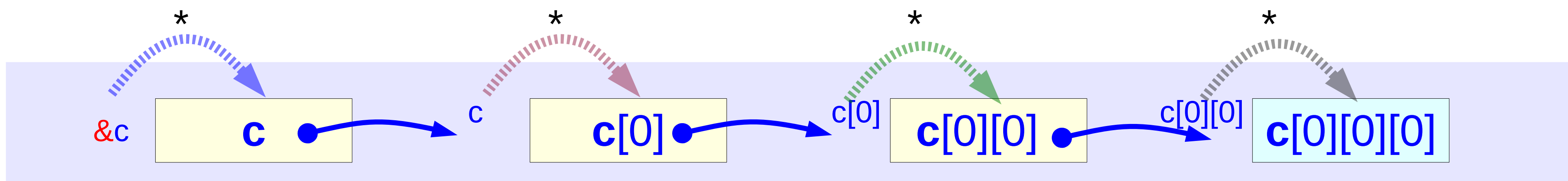
&(**q) ≡ value(*q)

C expression &(**q) equals to value(*q) which is the address value of a variable **q

&(***q) ≡ value(**q)

C expression &(***q) equals to value(**q) which is the address value of a variable ***q

Pointer Chain Type II – * and & operators



*(&c) ≡ c

*(c) ≡ c[0]

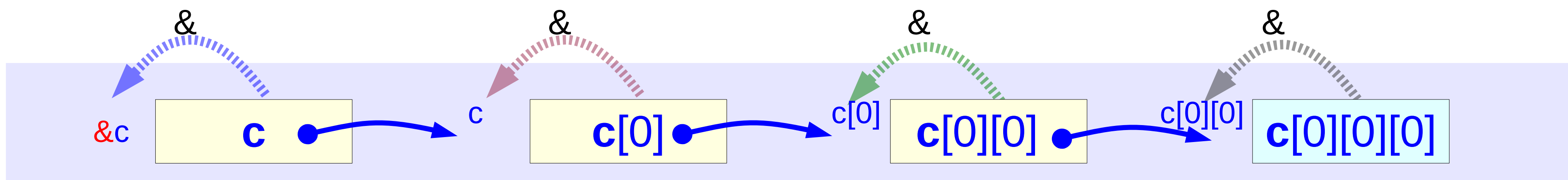
*(c[0]) ≡ c[0][0]

*(c[0][0]) ≡ c[0][0][0]

(int (*)[3][4]) c can be viewed as a pointer to (int [3][4]) c[0]

(int (*)[4]) c[0] can be viewed as a pointer to (int [4]) c[0][0]

(int (*)) c[0][0] can be viewed as a pointer to (int) c[0][0][0]



&c

&(c[0]) ≡ value(c)

&(c[0][0]) ≡ value(c[0])

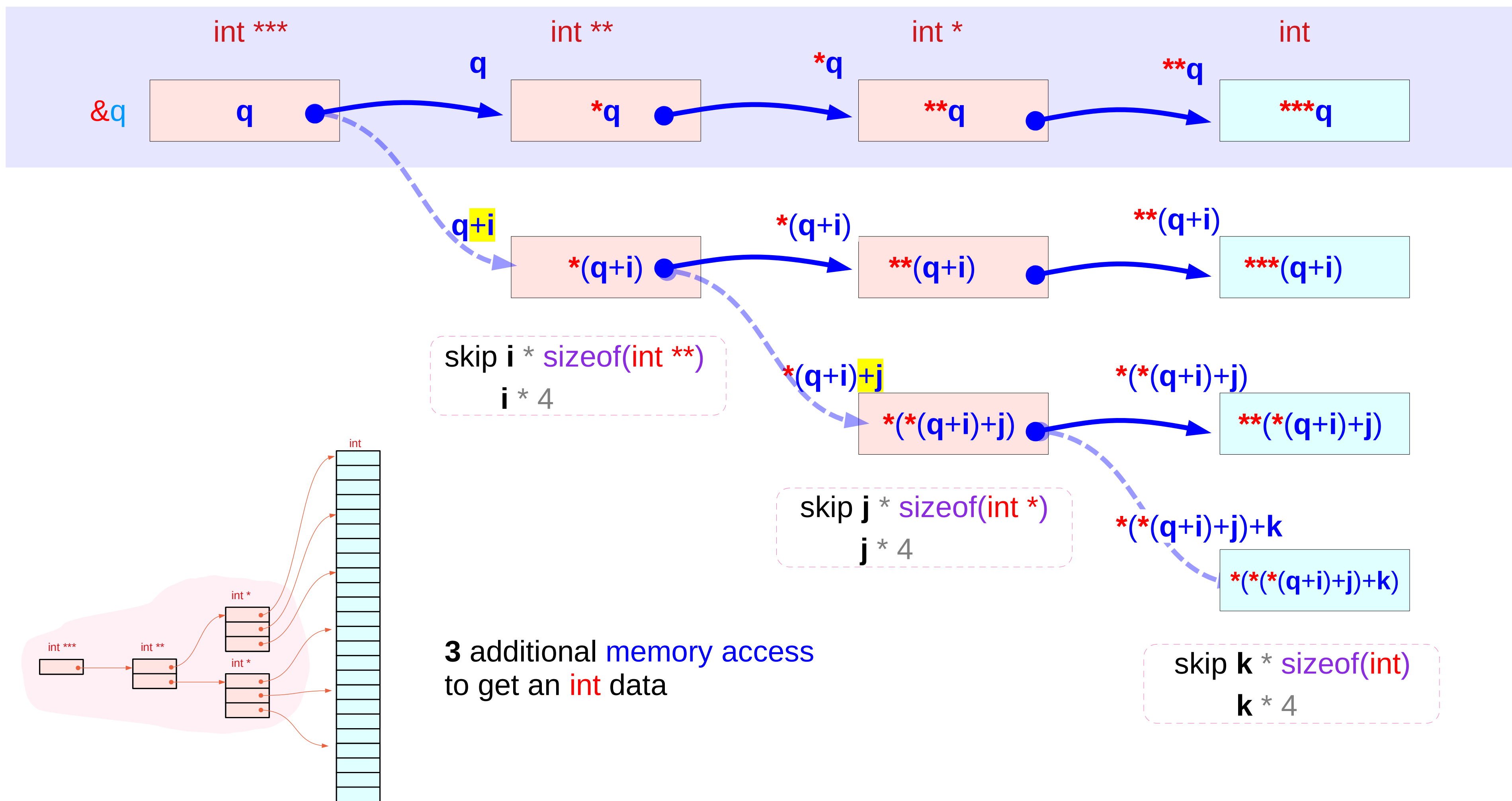
&(c[0][0][0]) ≡ value(c[0][0])

(int (*)[3][4]) c has the address value of (int [3][4]) c[0]

(int (*)[4]) c[0] has the address value of (int [4] c[0][0]

(int (*)) c[0][0] has the address value of (int) c[0][0][0]

Pointer Chain Type II – skipping elements



Pointer Chain Type I – skipping elements

Pointer Chain Type II

virtual pointer to an abstract data

size: abstract data size

int [2][3][4] → int [3][4] → int [4] → int

int (*)[3][4] → int (*)[4] → int (*)

- relaxing the 1st dim
- dereference

&c → c —c→ c[0] —c[0]→ c[0][0] —c[0][0]→ c[0][0][0]

c —c+i→ c[i] —c[i]→ c[i][0] —c[i][0]→ c[i][0][0]

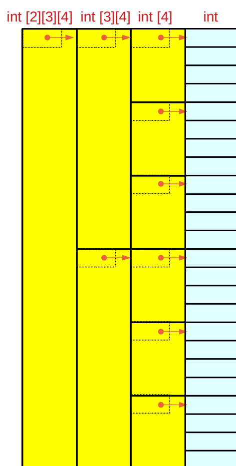
c[i] —c[i]+j→ c[i][j] —c[i][j]→ c[i][j][0]

c[i][j] —c[i][j]+k→ c[i][j][k]

skip **i** * sizeof(int [3][4])
i * 3 * 4 * 4

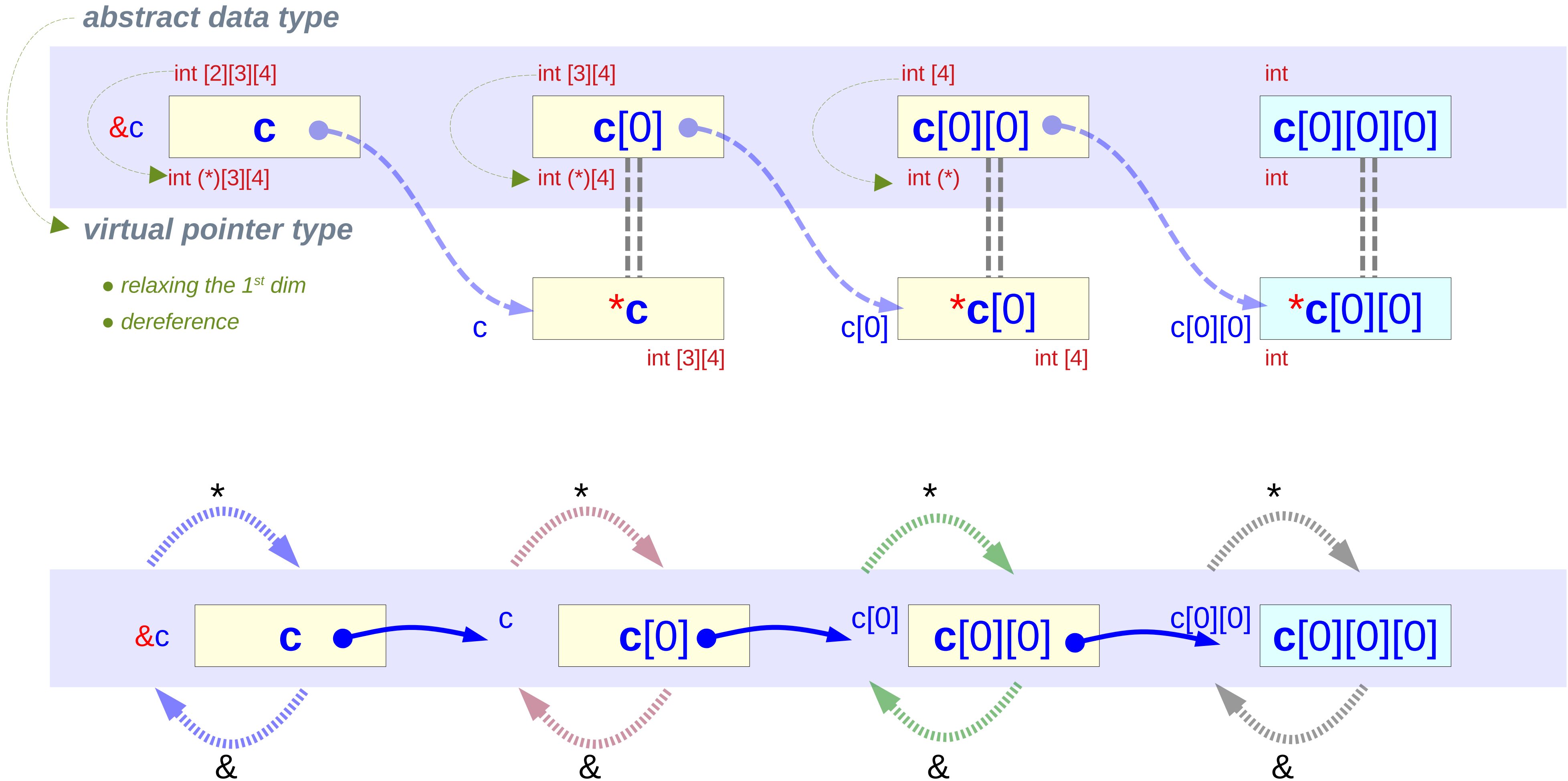
skip **j** * sizeof(int [4])
j * 4 * 4

skip **k** * sizeof(int)
k * 4

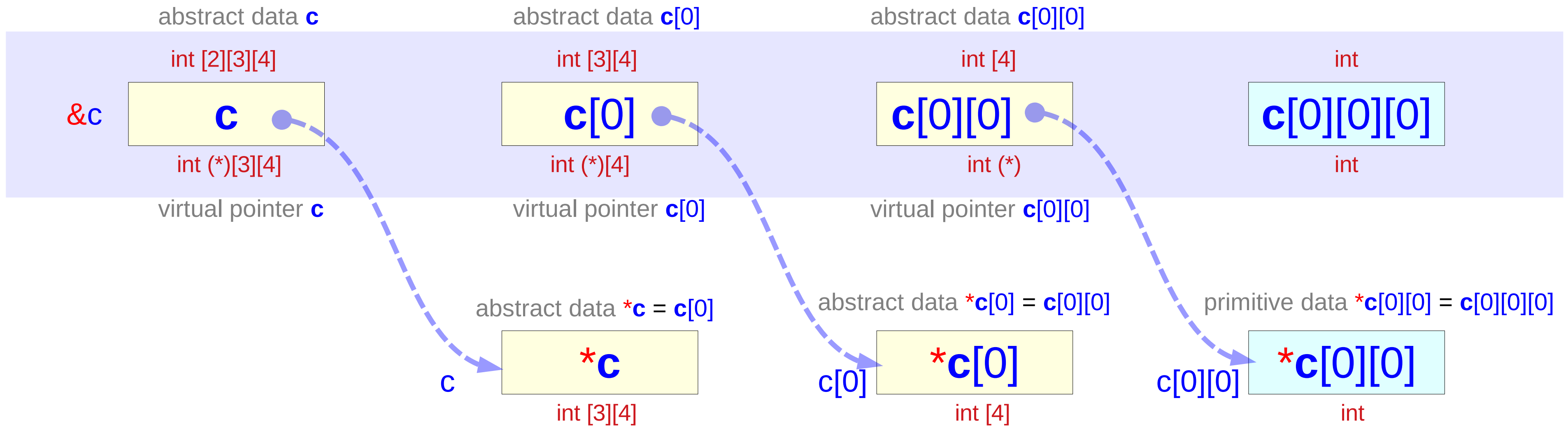


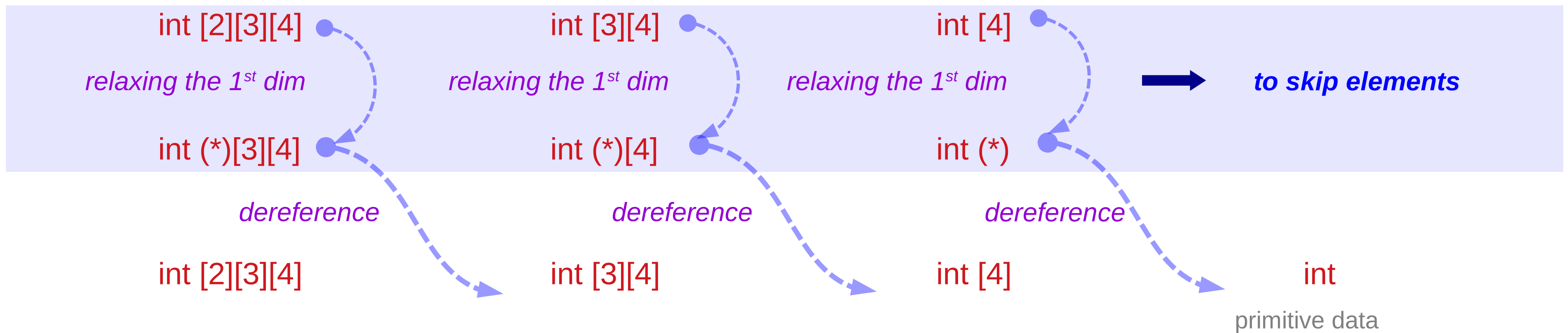
no additional memory access
to get an int data

Two step deferencing in type II (1)

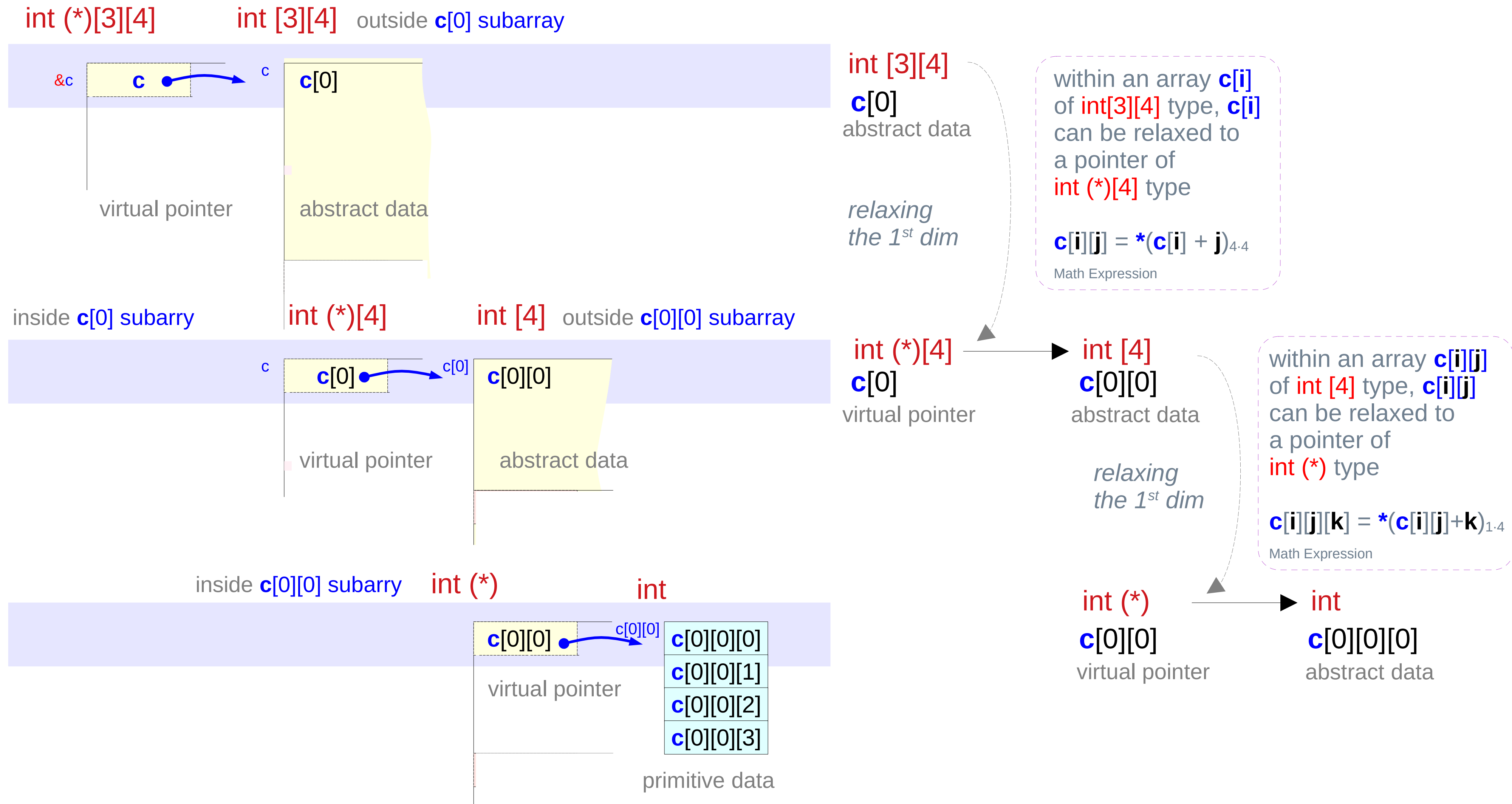


Two step deferencing in type II (2)

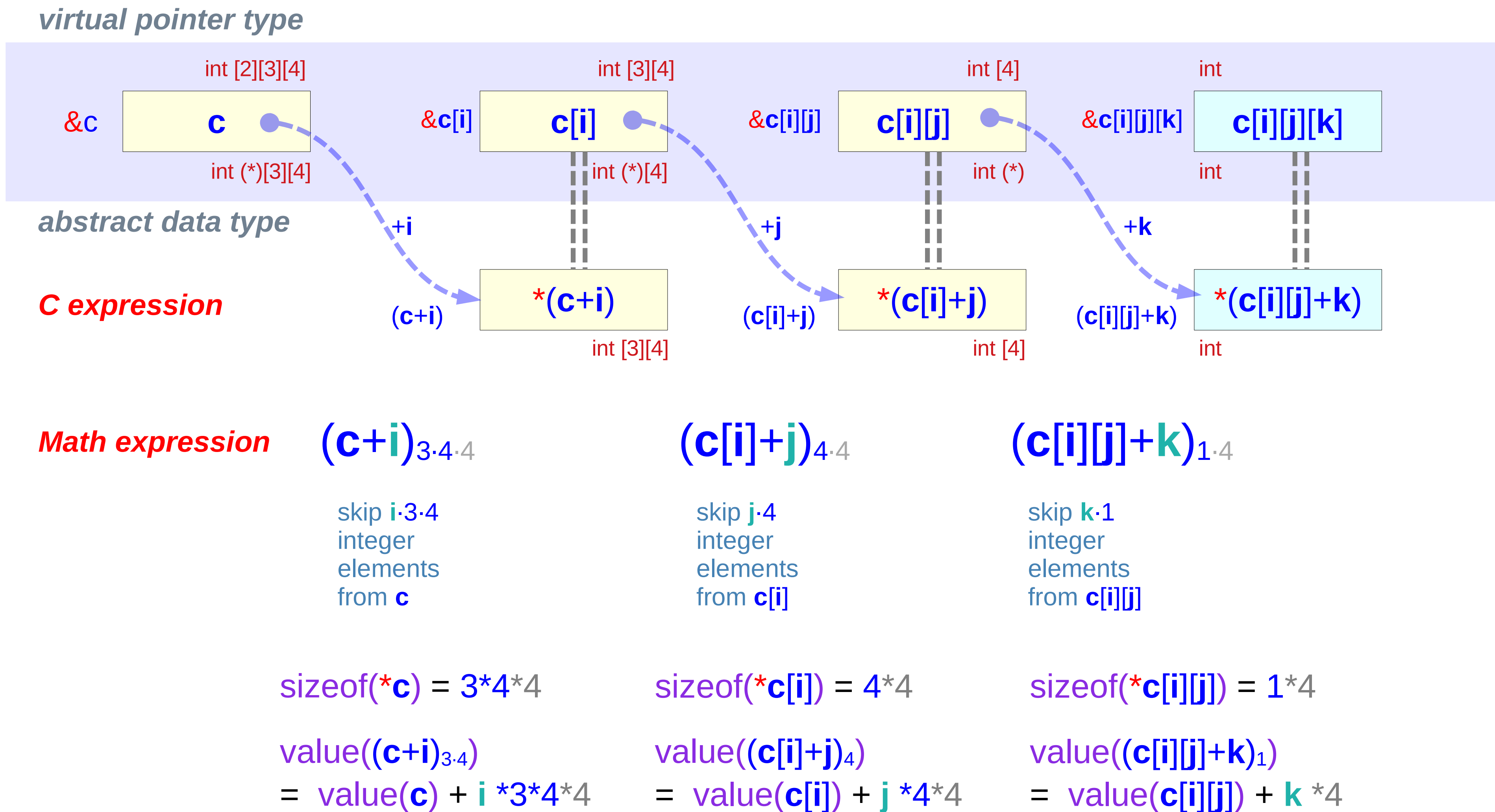




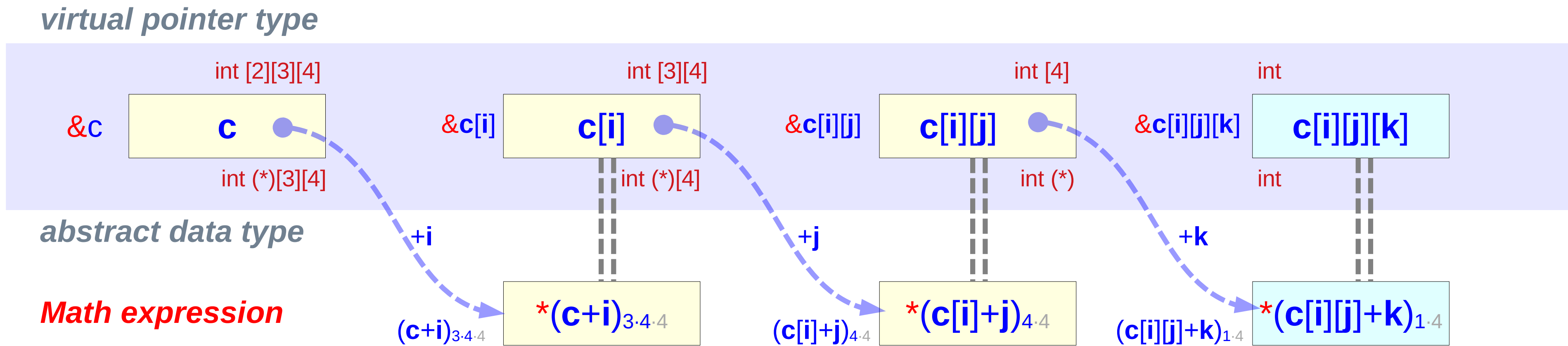
Relaxing the 1st dimension and skip element size



Skipping elements



Address replication



equivalence relations – c expressions

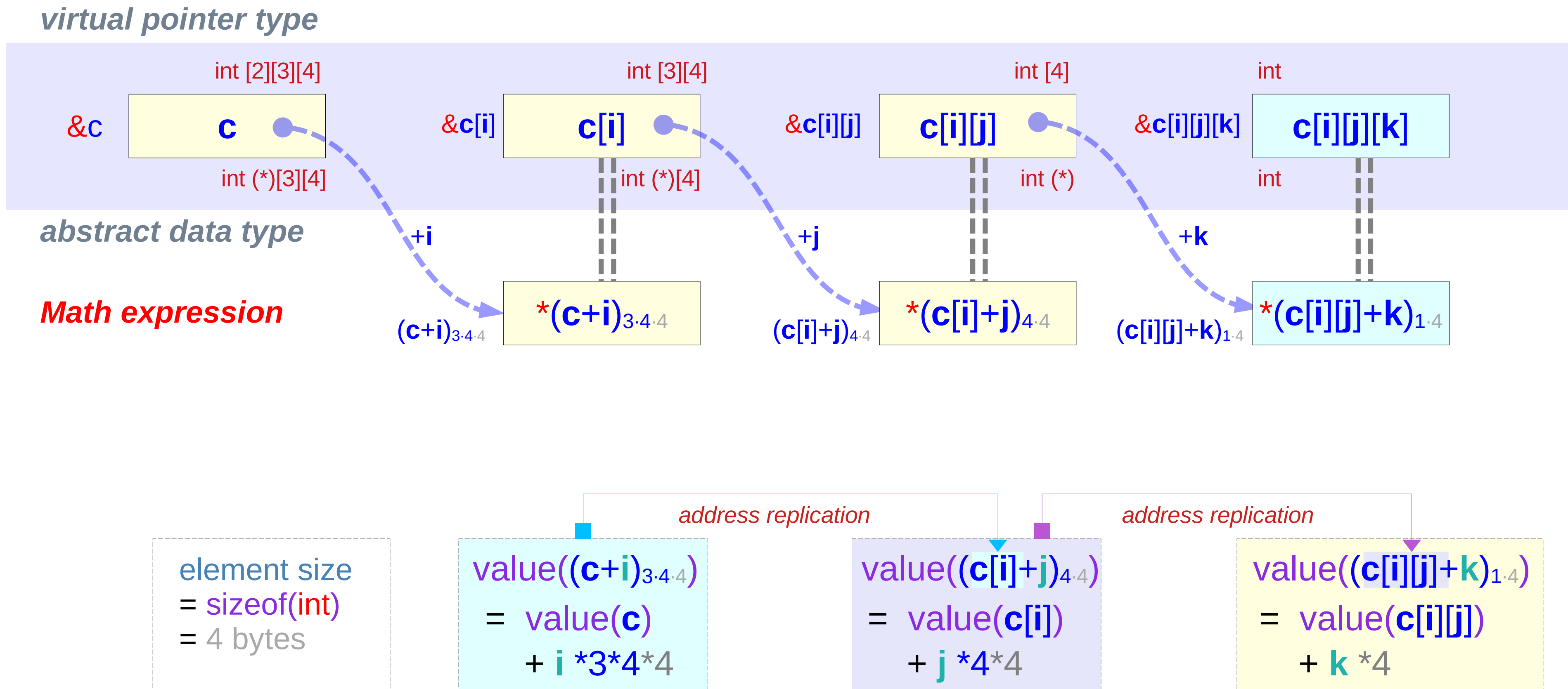
c[i][j][k] = *(c[i][j] + k)
c[i][j] = *(c[i] + j)
c[i] = *(c + i)

&c[i][j][k] = (c[i][j] + k)
&c[i][j] = (c[i] + j)
&c[i] = (c + i)

address replication – math expressions

value(c[i][j]) = value($(c[i] + j)_{4\cdot4}$)
value(c[i]) = value($(c + i)_{3\cdot4\cdot4}$)

Applying address replication



3-d Access **c**[**i**][**j**][**k**]

Accessing **c[i][j][k]** – Conditions

General requirements

&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

Pointer array approach

```
int** c[2];
int*  b[2*3];
int   c[2*3*4];
```

c[i][j][k] :: int
c[i][j] :: int *
c[i] :: int **

c[i] ⟸ &b[i*3]
b[j] ⟸ &a[j*4]

Hierarchical Pointer Array Constraints

Abstract Data Type

Array pointer approach

```
int c[2][3][4];
```

c[i][j][k] :: int
c[i][j] :: int [4]
c[i] :: int [3][4]

c = &c[0][0][0]
c[i] = &c[i][0][0]
c[i][j] = &c[i][j][0]

Virtual Array Pointer Constraints

Abstract Data Type

Accessing c[i][j][k] – Pointer Array Approach (1)

c[i] ⇐ &b[i*3]
b[j] ⇐ &a[j*4]

c[i]	≡ *(c + i)
c[i][j]	≡ *(c[i] + j)
c[i][j][k]	≡ *(c[i][j] + k)

&c[i]	≡ (c + i)
&c[i][j]	≡ (c[i] + j)
&c[i][j][k]	≡ (c[i][j] + k)

[2][3][4]

b[j] ≡ (a+j*4)

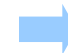
*(b[j]+k) = *(a+j*4+k);

b[j][k] ≡ a[j*4+k]

c[i] ≡ (b+i*3)

*(c[i]+j) = *(b+i*3+j);

c[i][j] ≡ b[i*3+j]



c[i][j] ≡ (a+(i*3+j)*4)

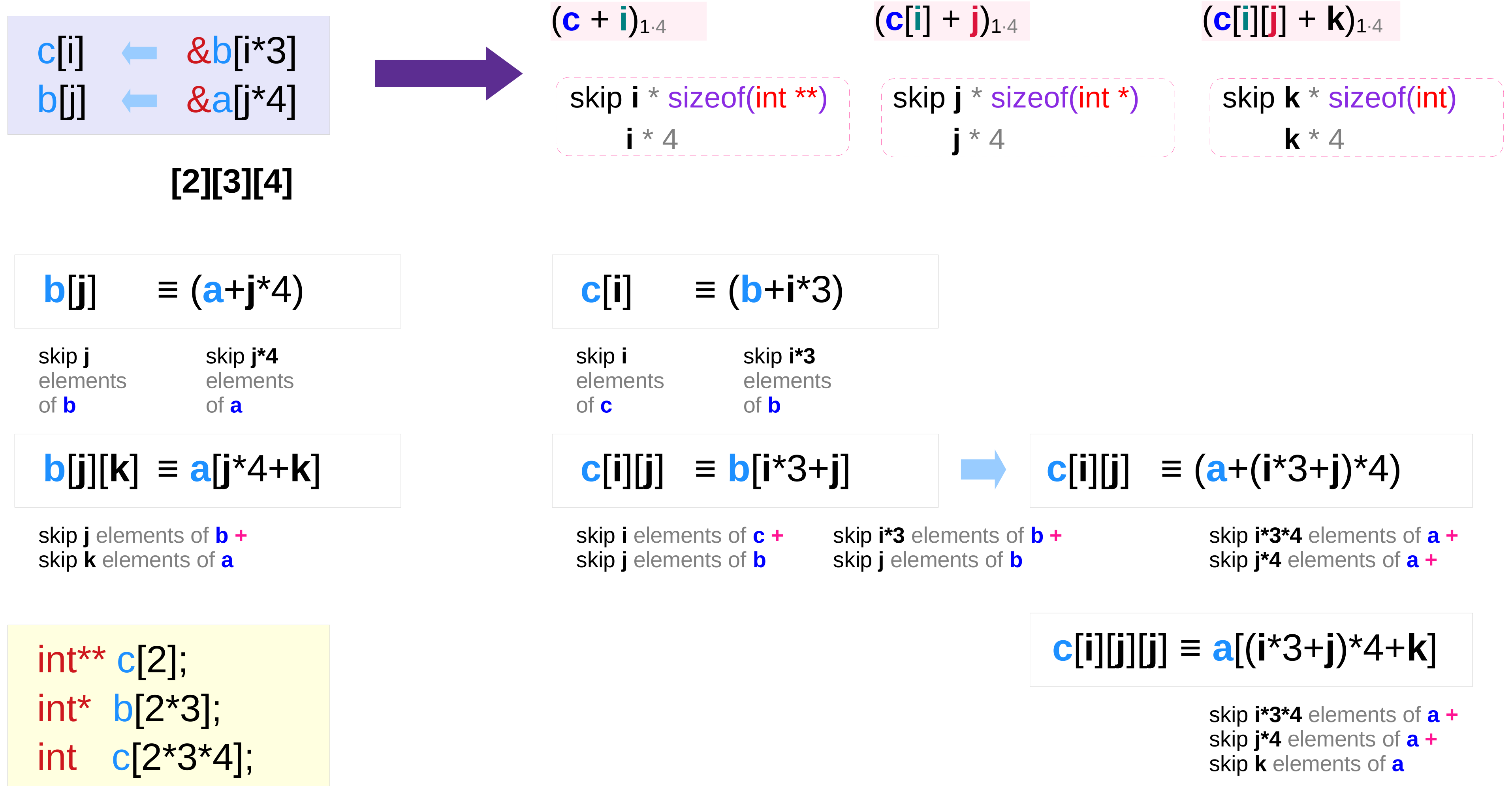
*(c[i][j]+k) = *(b[i*3+j]+k);

*(c[i][j]+k) = *(a+(i*3+j)*4+k);

c[i][j][j] ≡ a[(i*3+j)*4+k]

```
int** c[2];
int*  b[2*3];
int   c[2*3*4];
```

Accessing c[i][j][k] – Pointer Array Approach (2)



const pointers

const type, const pointer type (1)

`const int * p;`		`int * p;`
`int * const q ;`		`int * q ;`
`const int * const r ;`		`int * r ;`

constant

must not be changed
must not be updated
must not be written
must not be assigned

const type, const pointer type (2)

const int * p ;

constant integer

int* const q ;

constant pointer

const int * const r ;

constant integer

const int * const r ;

constant pointer

const [ ]

group with the following

*p : constant integer value

q : constant (int *) pointer

*r : constant integer value

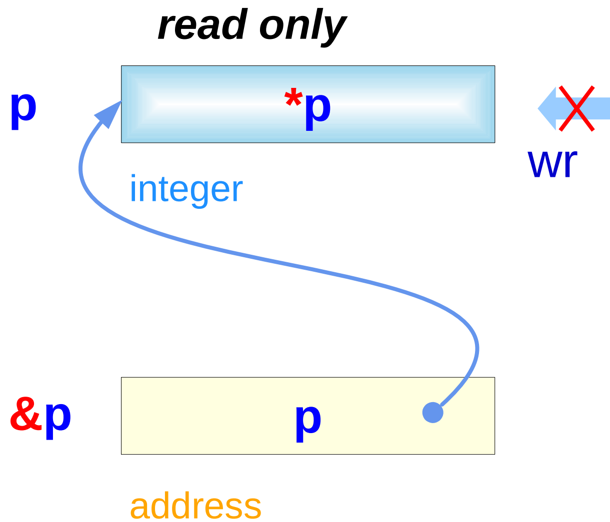
r : constant (int *) pointer

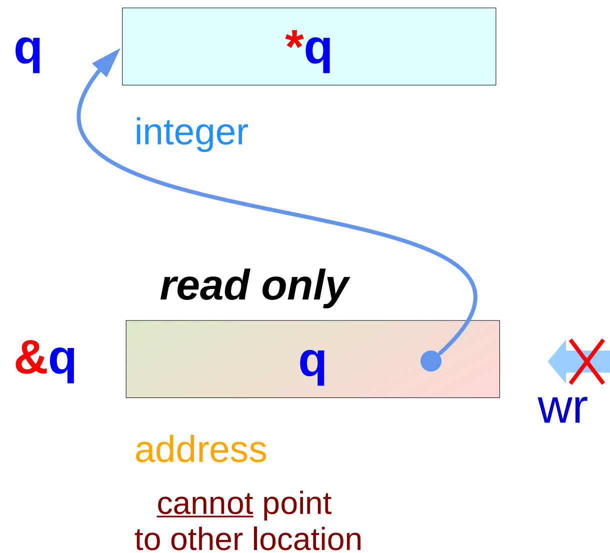
const type, const pointer type (3)

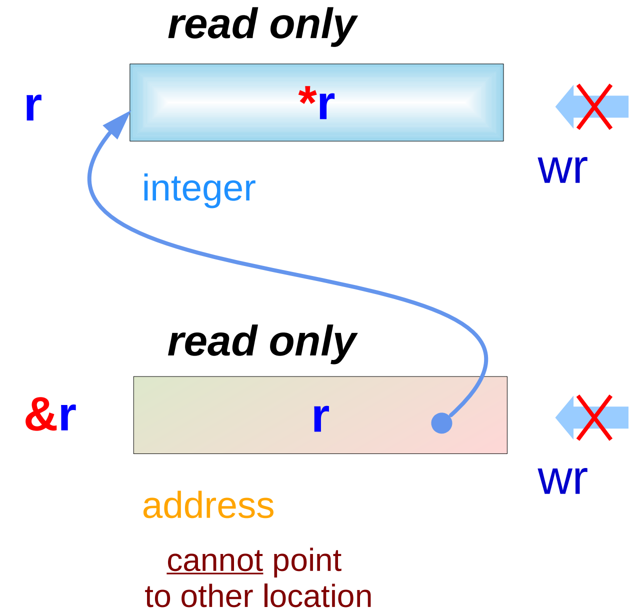
const int *p;

int *const q ;

const int *const r ;

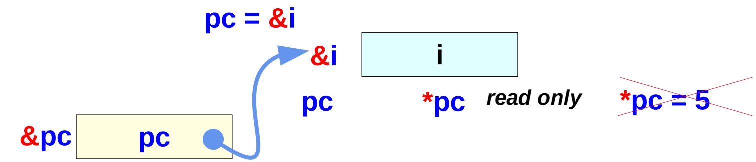






const examples (1)

```
const int * pc;
      int * p, i;
const int   ic;
```



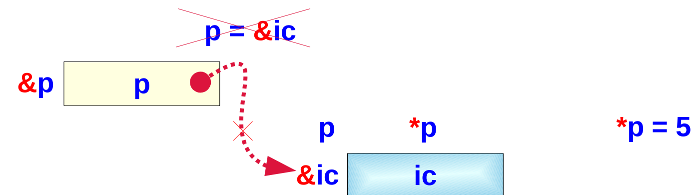
pc can point to i
*pc must be const

the same memory location
that can be written via i
cannot be written via *pc

*pc should not write
the writable memory location

```
 pc = &i;    // (const int *) ← (int *)
*pc  = 5;    // (const int) error
```

Writing to the writable memory location (**i**)
is forbidden via **pc** … (no harm, OK)




```
 p   = &ic;   // (int *) ← (const int *) warning
*p   = 5;     // (int)
```

Writing to the read only memory location (**ic**)
is not forbidden via **p** … (hazardous, not OK)

Assume **p** points to **const ic**

the same memory location
that cannot be written via **ic**,
can be written via ***p**

thus ***p** can write
the **const** memory location

therefore, **p** should not point to **const ic**

C A Reference Manual, Harbison & Steele Jr.

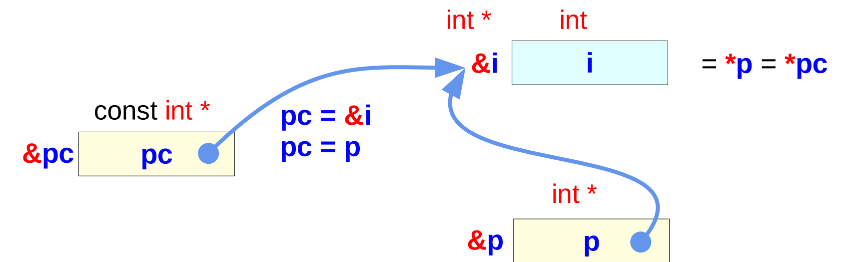
const examples (2)

```
const int * pc;
          int * p, i = 5;
const int    ic = 7;
```

```
p    = &i;
pc   = &ic
```

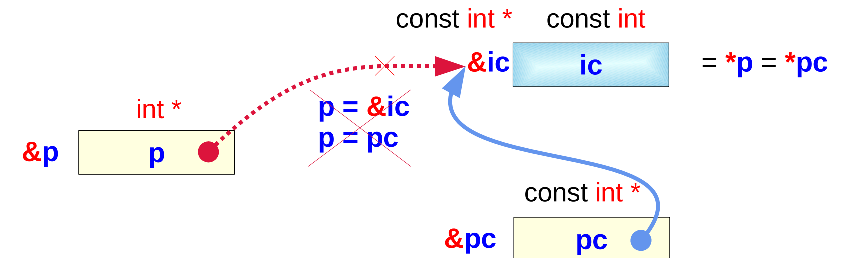
// more constrained type ← general type (O)

```
pc   = &i;     // (const int * ←  int *)
pc   =  p;     // (const int * ←  int *)
```



// general type ← more constrained type (X)

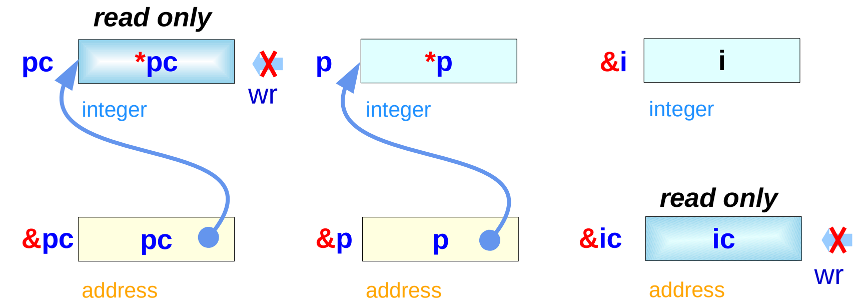
```
p    = &ic;    // (int * ← const int *)   warning
p    =  pc;    // (int * ← const int *)   warning
```



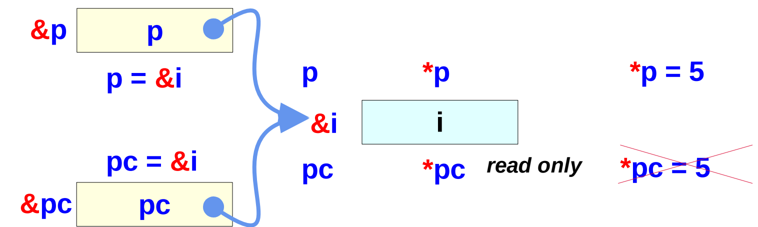
C A Reference Manual, Harbison & Steele Jr.

const examples (3)

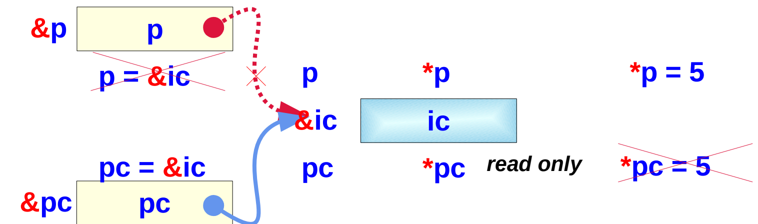
```
const int * pc;
      int * p, i;
const int   ic;
```




```
 p   = &i;    // (int *) ← (int *)
*p   = 5;     // (int)
 pc  = &i;    // (const int *) ← (int *)
*pc  = 5;     // (const int) error
```



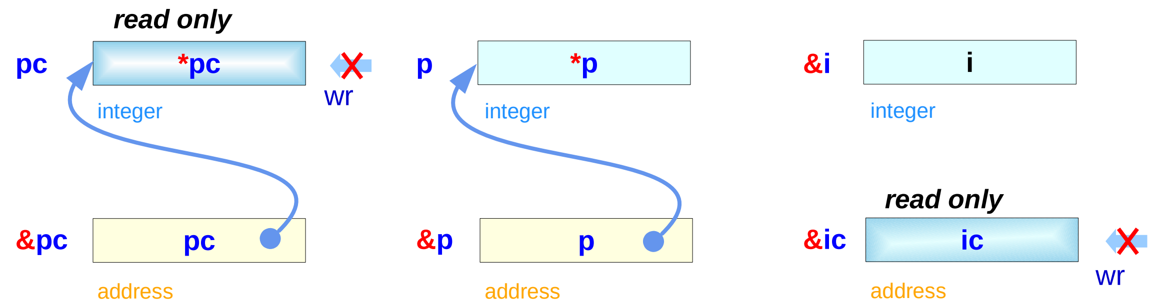

```
 p   = &ic;   // (int *) ← (const int *) warning
*p   = 5;     // (int)
 pc  = &ic;   // (const int *) ← (const int *)
*pc  = 5;     // (const int) error
```



C A Reference Manual, Harbison & Steele Jr.

const examples (4)

```
const int * pc;
          int * p, i;
const int   ic;
```




```
 pc  =   p = &i;
 pc  = &ic
*p   =   5;
*pc  =   5;              // invalid        *pc :: cons int
```

```
pc   = &i;               //                (const int * ←  int *)
pc   =   p;              //                (const int * ←  int *)
p    = &ic;              // invalid        (int * ← const int *)
p    =   pc;             // invalid        (int * ← const int *)
p    = (int *) &ic;      // type cast
p    = (int *)   pc;     // type cast
```

C A Reference Manual, Harbison & Steele Jr.

References

[1] Essential C, Nick Parlante
[2] Efficient C Programming, Mark A. Weiss
[3] C A Reference Manual, Samuel P. Harbison & Guy L. Steele Jr.
[4] C Language Express, I. K. Chun