Background – Trigonmetry (2A)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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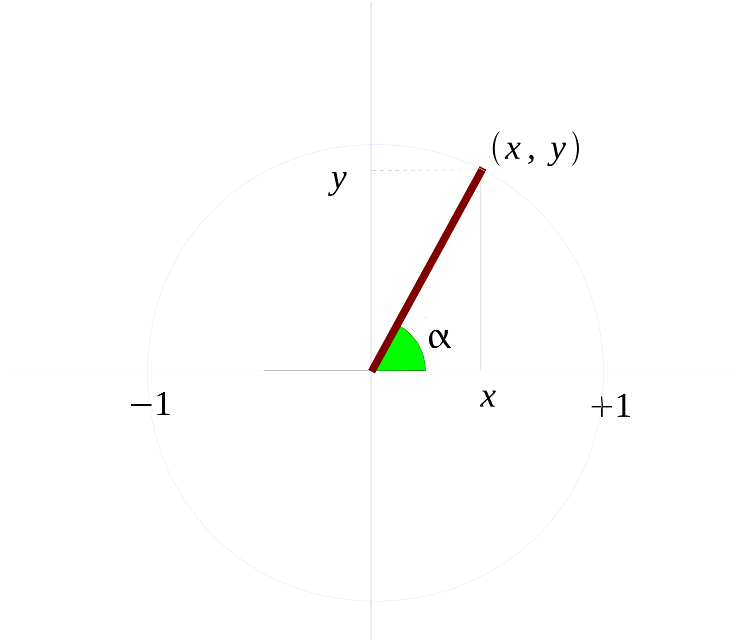
Complex Numbers

Reference Angle (1)

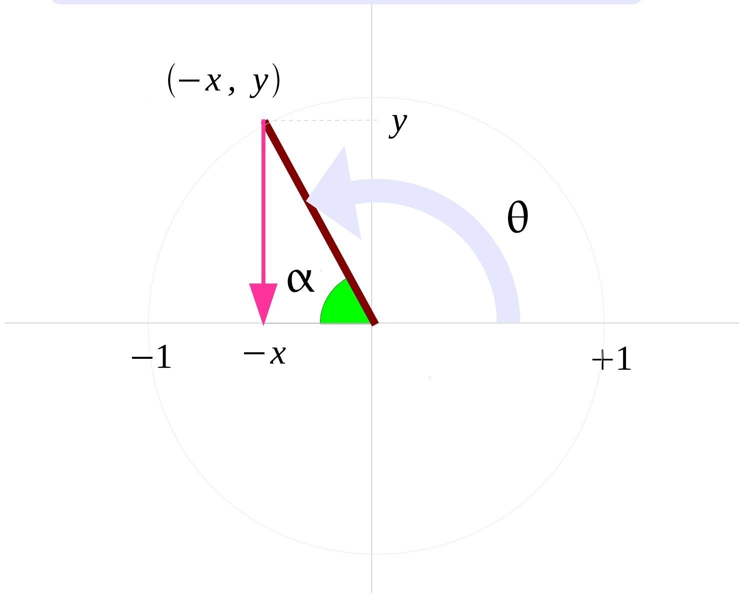
1st Quadrant Angle θ



$$\sin\alpha = y$$
$$\cos\alpha = x$$
$$\tan\alpha = y/x$$

2nd Quadrant Angle θ

$$90^\circ < \theta < 180^\circ$$



Reference Angle α

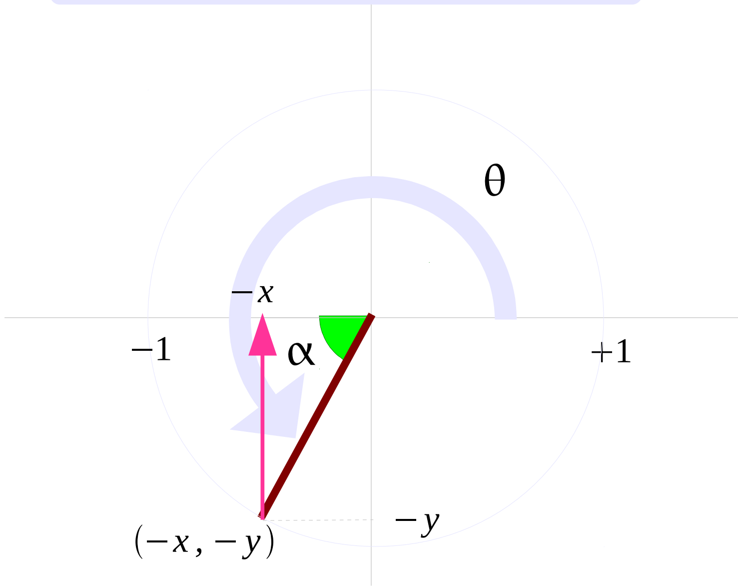
$$\alpha = 180^\circ - \theta$$

$$\sin\theta = +\sin\alpha = (+y)$$
$$\cos\theta = -\cos\alpha = (-x)$$
$$\tan\theta = -\tan\alpha = (+y)/(-x)$$

Reference Angle (2)

3rd Quadrant Angle θ

$$180^\circ < \theta < 270^\circ$$



Reference Angle α

$$\alpha = \theta - 180^\circ$$

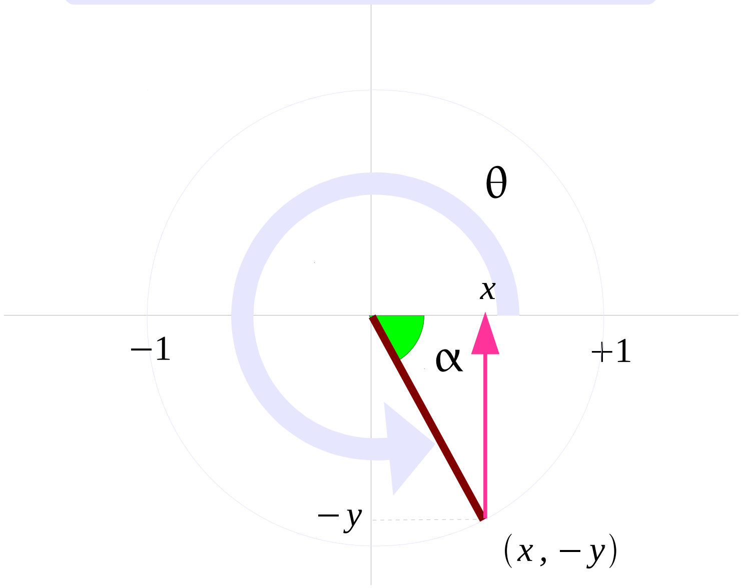
$$\sin\theta = -\sin\alpha = (-y)$$

$$\cos\theta = -\cos\alpha = (-x)$$

$$\tan\theta = +\tan\alpha = (-y)/(-x)$$

4th Quadrant Angle θ

$$270^\circ < \theta < 360^\circ$$



Reference Angle α

$$\alpha = 360^\circ - \theta$$

$$\sin\theta = -\sin\alpha = (-y)$$

$$\cos\theta = +\cos\alpha = (+x)$$

$$\tan\theta = -\tan\alpha = (-y)/(+x)$$

Reference Angle (3)

A Quadrant Angle θ

Reference Angle α

$\alpha = \pi - \theta$

$\sin\theta = +\sin\alpha$
$\cos\theta = -\cos\alpha$
$\tan\theta = -\tan\alpha$

$\alpha = \theta$

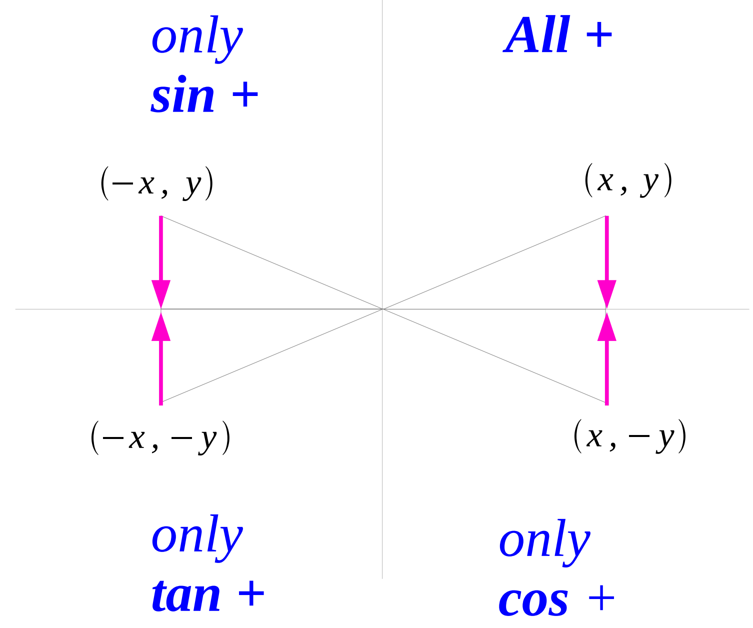
$\sin\theta = \sin\alpha$
$\cos\theta = \cos\alpha$
$\tan\theta = \tan\alpha$

$\sin\theta = -\sin\alpha$
$\cos\theta = -\cos\alpha$
$\tan\theta = +\tan\alpha$

$\alpha = \theta - \pi$

$\sin\theta = -\sin\alpha$
$\cos\theta = +\cos\alpha$
$\tan\theta = -\tan\alpha$

$\alpha = 2\pi - \theta$

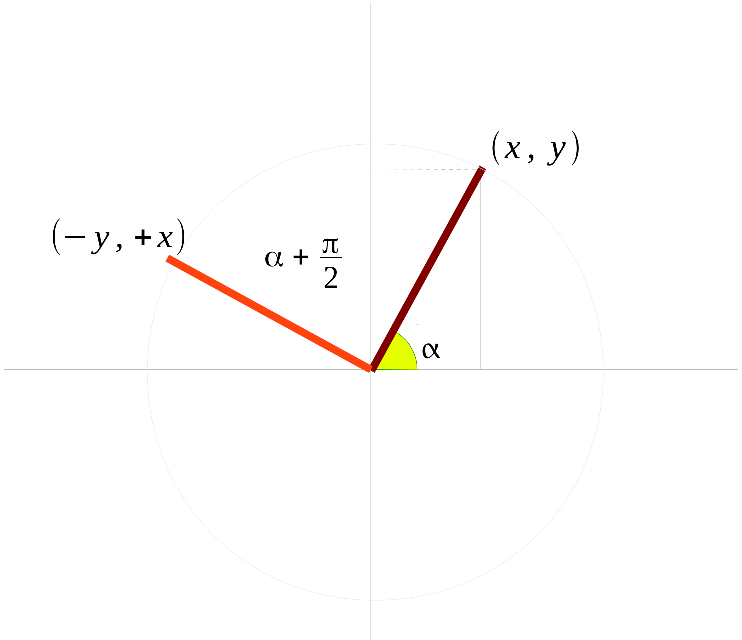


Symmetry

Reflected in $\theta = 0$[4]	Reflected in $\theta = \pi/2$ (co-function identities)[5]	Reflected in $\theta = \pi$
$\sin(-\theta) = -\sin\theta$	$\sin(\frac{\pi}{2} - \theta) = +\cos\theta$	$\sin(\pi - \theta) = +\sin\theta$
$\cos(-\theta) = +\cos\theta$	$\cos(\frac{\pi}{2} - \theta) = +\sin\theta$	$\cos(\pi - \theta) = -\cos\theta$
$\tan(-\theta) = -\tan\theta$	$\tan(\frac{\pi}{2} - \theta) = +\cot\theta$	$\tan(\pi - \theta) = -\tan\theta$
$\csc(-\theta) = -\csc\theta$	$\csc(\frac{\pi}{2} - \theta) = +\sec\theta$	$\csc(\pi - \theta) = +\csc\theta$
$\sec(-\theta) = +\sec\theta$	$\sec(\frac{\pi}{2} - \theta) = +\csc\theta$	$\sec(\pi - \theta) = -\sec\theta$
$\cot(-\theta) = -\cot\theta$	$\cot(\frac{\pi}{2} - \theta) = +\tan\theta$	$\cot(\pi - \theta) = -\cot\theta$

http://en.wikipedia.org/wiki/Derivative

Shift by +π/2



$$\cos\left(\alpha + \frac{\pi}{2}\right) = -\sin\alpha$$

$$\sin\left(\alpha + \frac{\pi}{2}\right) = +\cos\alpha$$

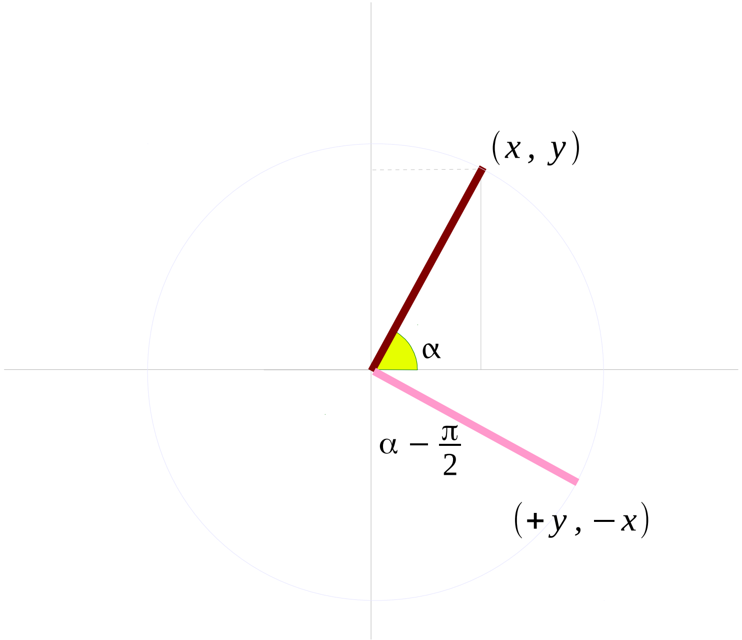
$$\tan\left(\alpha + \frac{\pi}{2}\right) = -\cot\alpha$$

$$\cot\left(\alpha + \frac{\pi}{2}\right) = -\tan\alpha$$

$$csc\left(\alpha + \frac{\pi}{2}\right) = +sec\,\alpha$$

$$sec\left(\alpha + \frac{\pi}{2}\right) = -csc\,\alpha$$

Shift by -π/2



$$\cos\left(\alpha - \frac{\pi}{2}\right) = +\sin\alpha$$

$$\sin\left(\alpha - \frac{\pi}{2}\right) = -\cos\alpha$$

$$\tan\left(\alpha - \frac{\pi}{2}\right) = -\cot\alpha$$

$$\cot\left(\alpha - \frac{\pi}{2}\right) = -\tan\alpha$$

$$csc\left(\alpha - \frac{\pi}{2}\right) = -sec\,\alpha$$

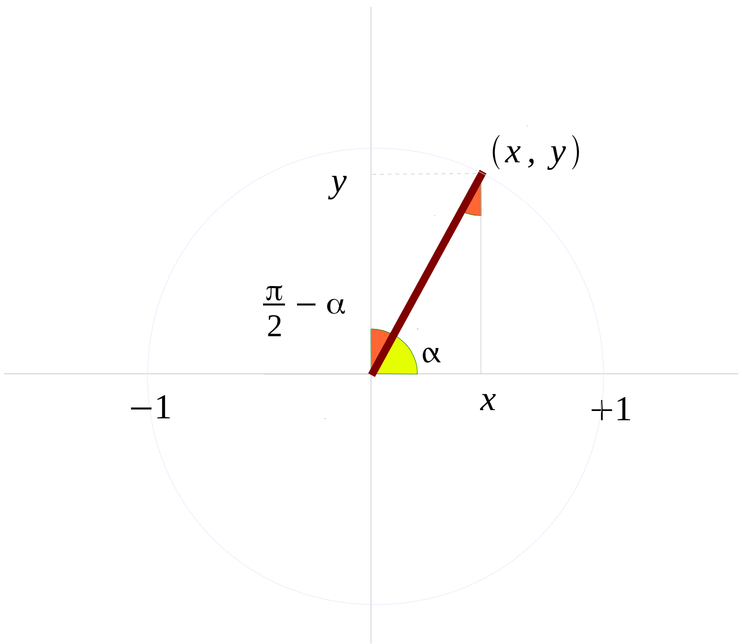
$$sec\left(\alpha - \frac{\pi}{2}\right) = +csc\,\alpha$$

Shifts and periodicity

Shift by π/2	Shift by π Period for tan and cot[6]	Shift by 2π Period for sin, cos, csc and sec[7]
$\sin(\theta + \frac{\pi}{2}) = +\cos\theta$	$\sin(\theta + \pi) = -\sin\theta$	$\sin(\theta + 2\pi) = +\sin\theta$
$\cos(\theta + \frac{\pi}{2}) = -\sin\theta$	$\cos(\theta + \pi) = -\cos\theta$	$\cos(\theta + 2\pi) = +\cos\theta$
$\tan(\theta + \frac{\pi}{2}) = -\cot\theta$	$\tan(\theta + \pi) = +\tan\theta$	$\tan(\theta + 2\pi) = +\tan\theta$
$\csc(\theta + \frac{\pi}{2}) = +\sec\theta$	$\csc(\theta + \pi) = -\csc\theta$	$\csc(\theta + 2\pi) = +\csc\theta$
$\sec(\theta + \frac{\pi}{2}) = -\csc\theta$	$\sec(\theta + \pi) = -\sec\theta$	$\sec(\theta + 2\pi) = +\sec\theta$
$\cot(\theta + \frac{\pi}{2}) = -\tan\theta$	$\cot(\theta + \pi) = +\cot\theta$	$\cot(\theta + 2\pi) = +\cot\theta$

http://en.wikipedia.org/wiki/Derivative

Co-function Identities



$$\sin\alpha = y \Rightarrow \cos\left(\frac{\pi}{2} - \alpha\right)$$

$$\cos\alpha = x \Rightarrow \sin\left(\frac{\pi}{2} - \alpha\right)$$

$$\tan\alpha = y/x \Rightarrow \cot\left(\frac{\pi}{2} - \alpha\right)$$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$$

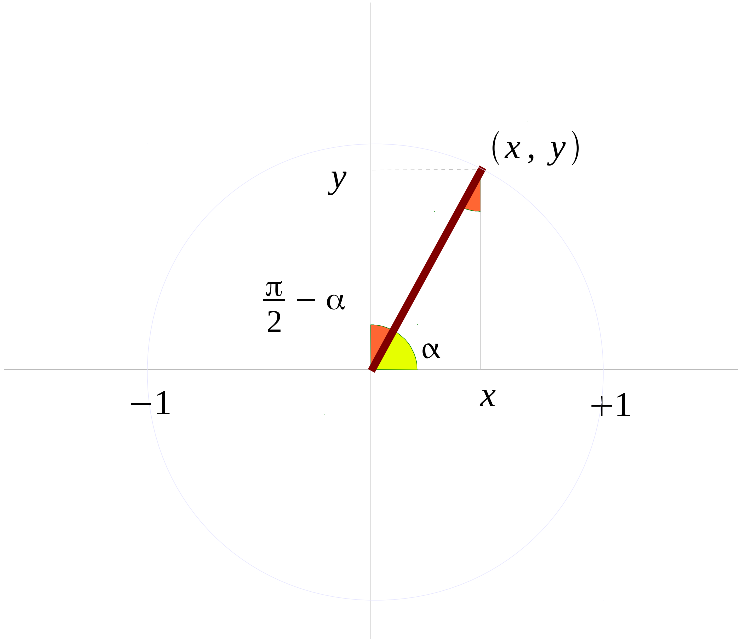
$$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$$

$$\cot\left(\frac{\pi}{2} - \alpha\right) = \tan\alpha$$

$$csc\left(\frac{\pi}{2} - \alpha\right) = sec\,\alpha$$

$$sec\left(\frac{\pi}{2} - \alpha\right) = csc\,\alpha$$

Angle Sum and Difference Identities (1)



$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$
$$\sin(\alpha - \beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$$

$$\sin(60^\circ + 30^\circ) = 1$$

$+ \quad \sin(60^\circ) = \frac{\sqrt{3}}{2} \qquad \sin(30^\circ) = \frac{1}{2}$

$+ \quad \cos(60^\circ) = \frac{1}{2} \qquad \cos(30^\circ) = \frac{\sqrt{3}}{2}$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} = 1$$

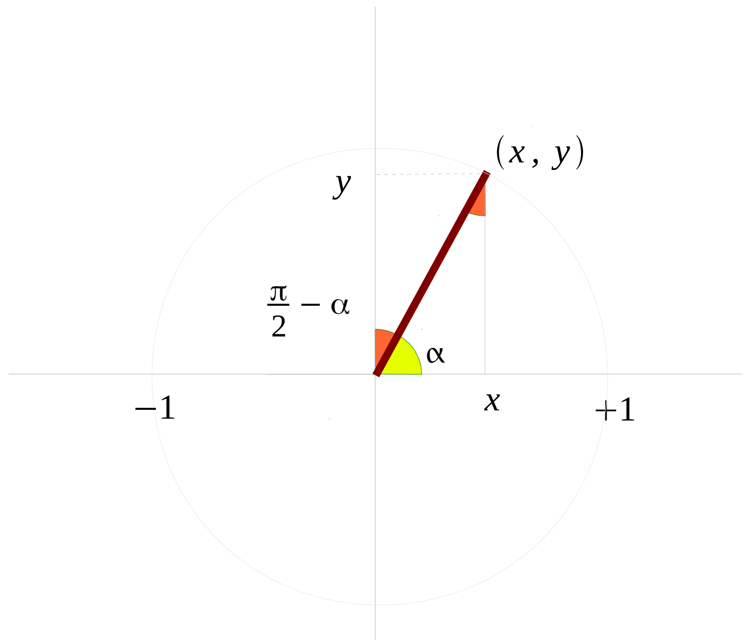
$$\sin(60^\circ - 30^\circ) = \frac{1}{2}$$

$+ \quad \sin(60^\circ) = \frac{\sqrt{3}}{2} \qquad \sin(30^\circ) = \frac{1}{2}$

$- \quad \cos(60^\circ) = \frac{1}{2} \qquad \cos(30^\circ) = \frac{\sqrt{3}}{2}$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$$

Angle Sum and Difference Identities (2)



$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$

$$\cos(30^\circ + 60^\circ) = 0$$

$-$ $\sin(60^\circ) = \frac{\sqrt{3}}{2}$ —— $\sin(30^\circ) = \frac{1}{2}$

$+$ $\cos(60^\circ) = \frac{1}{2}$ —— $\cos(30^\circ) = \frac{\sqrt{3}}{2}$

$$\frac{\sqrt{3}}{2}\cdot\frac{1}{2} - \frac{1}{2}\cdot\frac{\sqrt{3}}{2} = 0$$

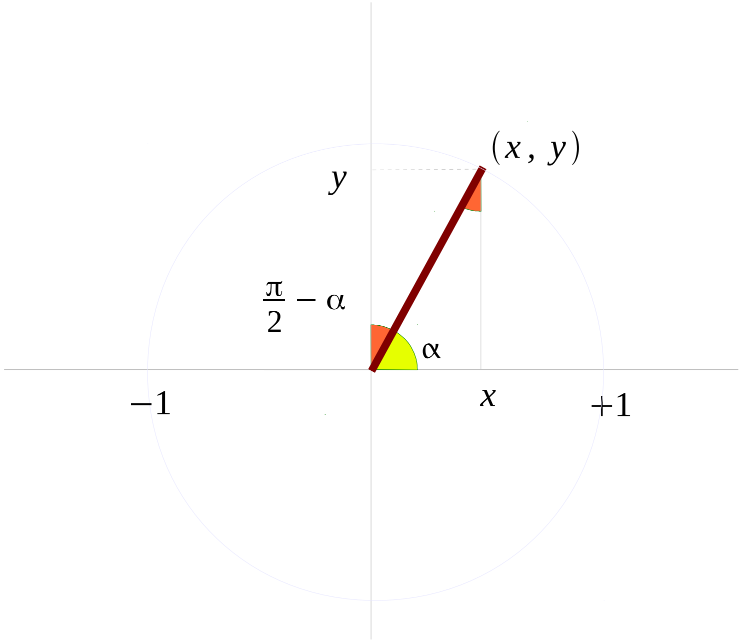
$$\cos(30^\circ - 60^\circ) = \frac{\sqrt{3}}{2}$$

$+$ $\sin(60^\circ) = \frac{\sqrt{3}}{2}$ —— $\sin(30^\circ) = \frac{1}{2}$

$+$ $\cos(60^\circ) = \frac{1}{2}$ —— $\cos(30^\circ) = \frac{\sqrt{3}}{2}$

$$\frac{\sqrt{3}}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

Angle Sum and Difference Identities (3)



$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

$$\tan(30^\circ + 60^\circ) = +\infty$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}} \quad \text{———} \quad \tan(60^\circ) = \sqrt{3}$$

$$\tan(60^\circ) = \sqrt{3} \quad \text{———} \quad \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\frac{\frac{1}{\sqrt{3}} + \sqrt{3}}{1 - \frac{1}{\sqrt{3}} \cdot \sqrt{3}} = +\infty$$

$$\tan(30^\circ - 60^\circ) = -\frac{1}{\sqrt{3}}$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}} \quad \text{———} \quad \tan(60^\circ) = \sqrt{3}$$

$$\tan(60^\circ) = \sqrt{3} \quad \text{———} \quad \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\frac{\frac{1}{\sqrt{3}} - \sqrt{3}}{1 + \frac{1}{\sqrt{3}} \cdot \sqrt{3}} = -\frac{1}{\sqrt{3}}$$

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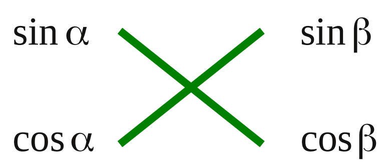
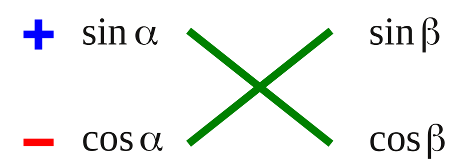
Angle Sum and Difference Identities (4)

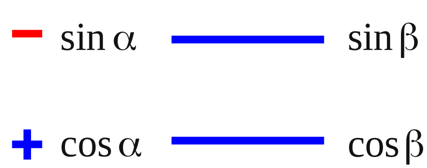
$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$



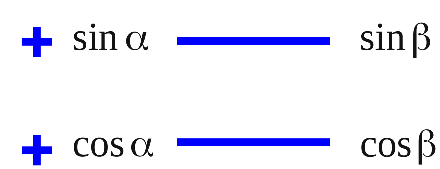
$$\sin(\alpha - \beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$$



$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$



$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$



$$\frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \frac{\sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta}{\cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta}$$

$$\frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} = \frac{\sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta}{\cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta}$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

Product to Sum (1)

$$+\ \sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$$
$$+\ \sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta$$

$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cdot\cos\beta$$

$$\sin\alpha\cdot\cos\beta = \tfrac{1}{2}\{\sin(\alpha+\beta) + \sin(\alpha-\beta)\}$$

$$+\ \sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$$
$$-\ \sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta$$

$$\sin(\alpha+\beta) - \sin(\alpha-\beta) = 2\cos\alpha\sin\beta$$

$$\cos\alpha\cdot\sin\beta = \tfrac{1}{2}\{\sin(\alpha+\beta) - \sin(\alpha-\beta)\}$$

$$+\ \cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$$
$$+\ \cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$$

$$\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha\cdot\cos\beta$$

$$\cos\alpha\cdot\cos\beta = \tfrac{1}{2}\{+\cos(\alpha+\beta) + \cos(\alpha-\beta)\}$$

$$-\ \cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$$
$$+\ \cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$$

$$-\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\sin\alpha\sin\beta$$

$$\sin\alpha\cdot\sin\beta = \tfrac{1}{2}\{-\cos(\alpha+\beta) + \cos(\alpha-\beta)\}$$

Product to Sum (2)

$$\sin(\alpha \pm \beta) = \quad \alpha \cdot \quad \beta \pm \quad \alpha \quad \beta$$

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$$

$$\cos(\alpha \pm \beta) = \quad \alpha \cdot \quad \beta \mp \quad \alpha \cdot \quad \beta$$

$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$

$$\quad \alpha \cdot \quad \beta = \tfrac{1}{2}\{\sin(\quad) + \sin(\quad)\}$$

$$\sin\alpha \cdot \cos\beta = \tfrac{1}{2}\{+\sin(\alpha + \beta) + \sin(\alpha - \beta)\}$$

$$\cos\alpha \cdot \sin\beta = \tfrac{1}{2}\{+\sin(\alpha + \beta) - \sin(\alpha - \beta)\}$$

$$\quad \alpha \cdot \quad \beta = \tfrac{1}{2}\{+\cos(\quad) + \cos(\quad)\}$$

$$\cos\alpha \cdot \cos\beta = \tfrac{1}{2}\{+\cos(\alpha + \beta) + \cos(\alpha - \beta)\}$$

$$\sin\alpha \cdot \sin\beta = \tfrac{1}{2}\{-\cos(\alpha + \beta) + \cos(\alpha - \beta)\}$$

References

[1] http://en.wikipedia.org/
[2] M.L. Boas, “Mathematical Methods in the Physical Sciences”
[3] E. Kreyszig, “Advanced Engineering Mathematics”
[4] D. G. Zill, W. S. Wright, “Advanced Engineering Mathematics”
[5] www.chem.arizona.edu/~salzmanr/480a