

# Characteristics of Multiple Random Variables

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Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles,Jr. and B. Shi

# Outline

## 1 Simulation of Multiple Random Variables

# Complex Random Variables (1)

$N$  Gaussian random variables

## Definition

$$Z = X + jY$$

$$E[g(Z)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(z) f_{X,Y}(x,y) dx dy$$

$$\bar{Z} = E[Z] = E[X] + jE[Y] = \bar{X} + j\bar{Y}$$

$$\sigma_Z^2 = E[|Z - E[Z]|^2] = E[(Z - E[Z])(Z - E[Z])^*]$$

# Complex Random Variables (2)

$N$  Gaussian random variables

## Definition

$$\sigma_Z^2 = E[|Z - E[Z]|^2] = E[(Z - E[Z])(Z - E[Z])^*]$$

$$g(Z) = |Z - E[Z]|^2$$

$$\tilde{\sigma}_Z^2 = E[(Z - E[Z])^2] = E[(Z - E[Z])(Z - E[Z])]$$

# Complex Random Variables (3)

$N$  Gaussian random variables

## Definition

$$\sigma_X^2 = \frac{1}{2} \operatorname{Re} \{ \sigma_Z^2 - \tilde{\sigma}_Z^2 \}$$

$$\sigma_Y^2 = \frac{1}{2} \operatorname{Im} \{ \sigma_Z^2 - \tilde{\sigma}_Z^2 \}$$

$$\sigma_{XY}^2 = \sigma_{YX}^2 = \frac{1}{2} \operatorname{Im} \{ \tilde{\sigma}_Z^2 \}$$



