Characteristics of Multiple Random Variables

Young W Lim

July 24, 2019

Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

Complex Random Variables (1)

N Gaussian random variables

Definition

$$Z = X + jY$$

$$E[g(Z)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(z) f_{X,Y}(x,y)\,dx\,dy$$

$$\overline{Z} = E[Z] = E[X] + jE[Y] = \overline{X} + j\overline{Y}$$

$$\sigma_Z^2 = E[|Z - E[Z]|^2] = E[(Z - E[Z])(Z - E[Z])^*]$$

Complex Random Variables (2)

N Gaussian random variables

Definition

$$\sigma_Z^2 = E[|Z - E[Z]|^2] = E[(Z - E[Z])(Z - E[Z])^*]$$

$$g(Z) = |Z - E[Z]|^2$$

$$\tilde{\sigma}_Z^2 = E[(Z - E[Z])^2] = E[(Z - E[Z])(Z - E[Z])]$$

Complex Random Variables (3)

N Gaussian random variables

Definition

$$\sigma_X^2 = \frac{1}{2} Re\left\{\sigma_Z^2 - \tilde{\sigma}_Z^2\right\}$$

$$\sigma_Y^2 = \frac{1}{2} Im\left\{\sigma_Z^2 - \tilde{\sigma}_Z^2\right\}$$

$$\sigma_{XY}^2 = \sigma_{YX}^2 = \frac{1}{2} Im\left\{\tilde{\sigma}_Z^2\right\}$$