Applications of Array Pointers (1A)

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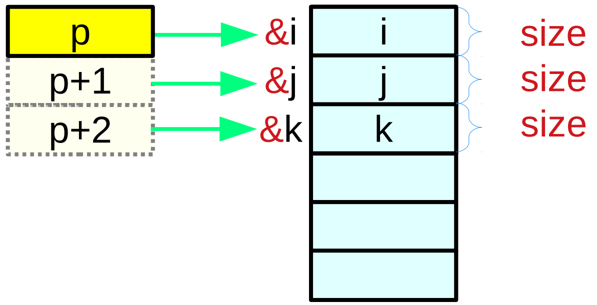
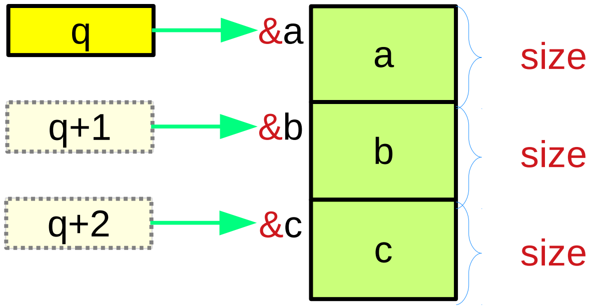
Virtual Array Pointers in Multi-dimensional Arrays

Pointers to various data types

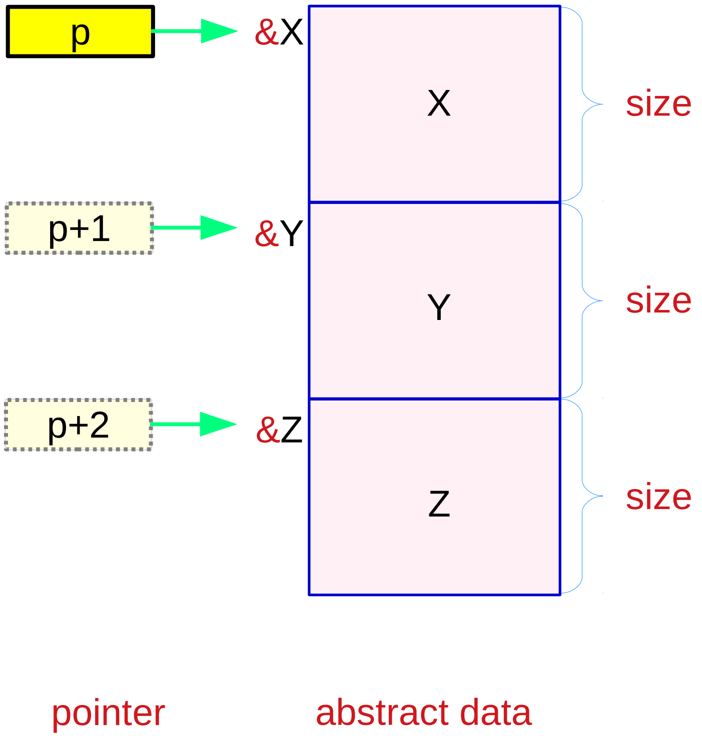
int *p; int i, j, k;



double *q; double a, b, c;

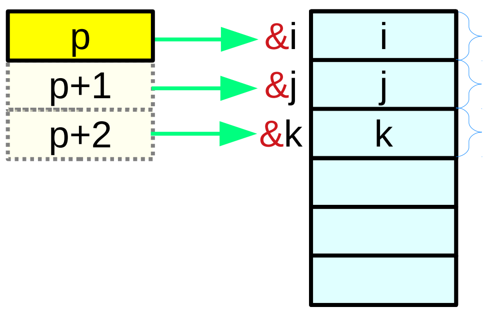


T *p; T X, Y, Z;



Pointers to primitive data

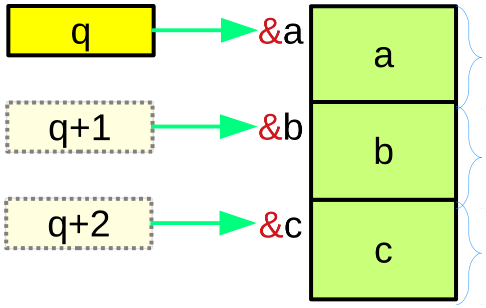
int *p; **int i, j, k;** **sizeof(int) = 4 bytes**



size	=	sizeof(i)	=	sizeof(*p)	≠	sizeof(p)
size	=	sizeof(j)	=	sizeof(*(p+1))	≠	sizeof(p+1)
size	=	sizeof(k)	=	sizeof(*(p+2))	≠	sizeof(p+2)

pointer size
4 or 8 bytes

double *q; **double a, b, c;** **sizeof(double) = 8 bytes**

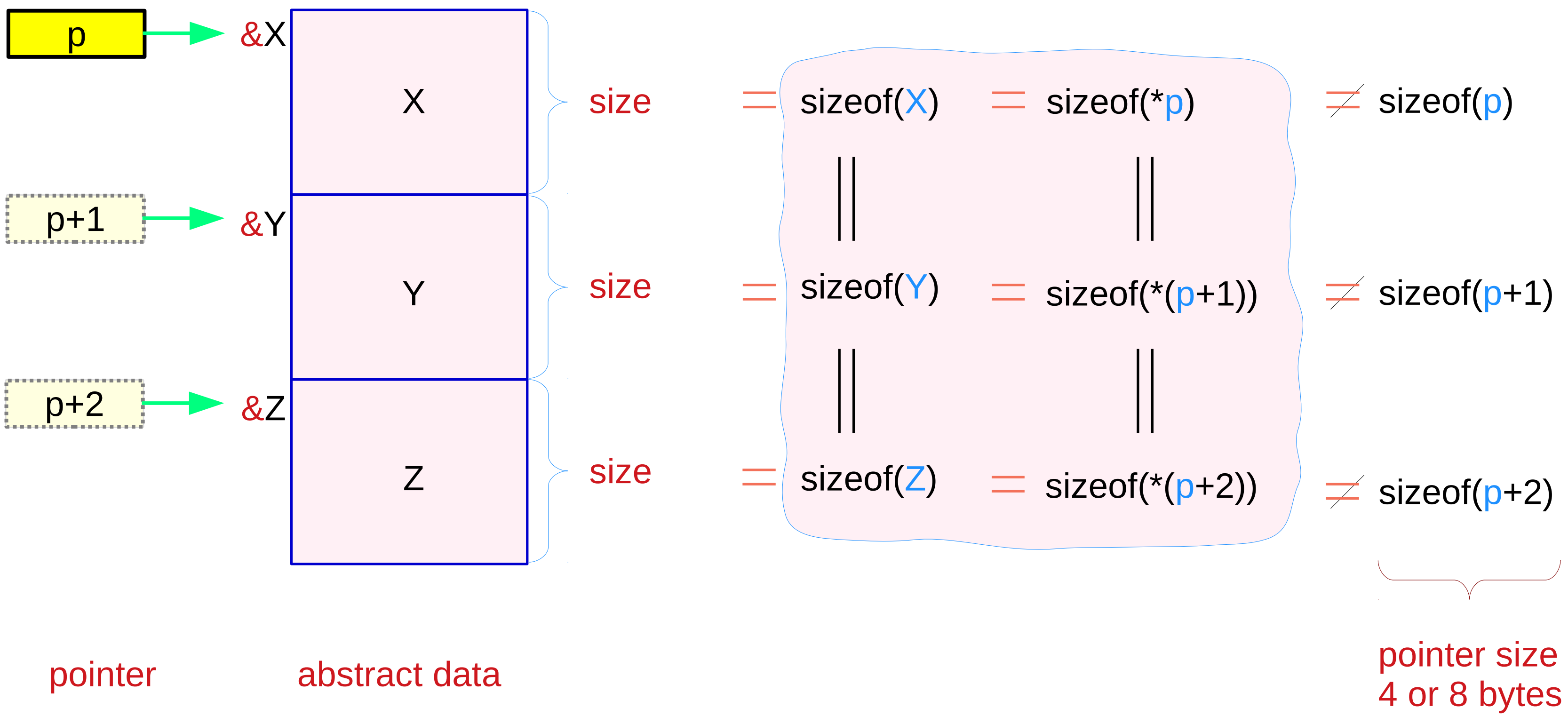


size	=	sizeof(a)	=	sizeof(*q)	≠	sizeof(q)
size	=	sizeof(b)	=	sizeof(*(q+1))	≠	sizeof(q+1)
size	=	sizeof(c)	=	sizeof(*(q+2))	≠	sizeof(q+2)

pointer size
4 or 8 bytes

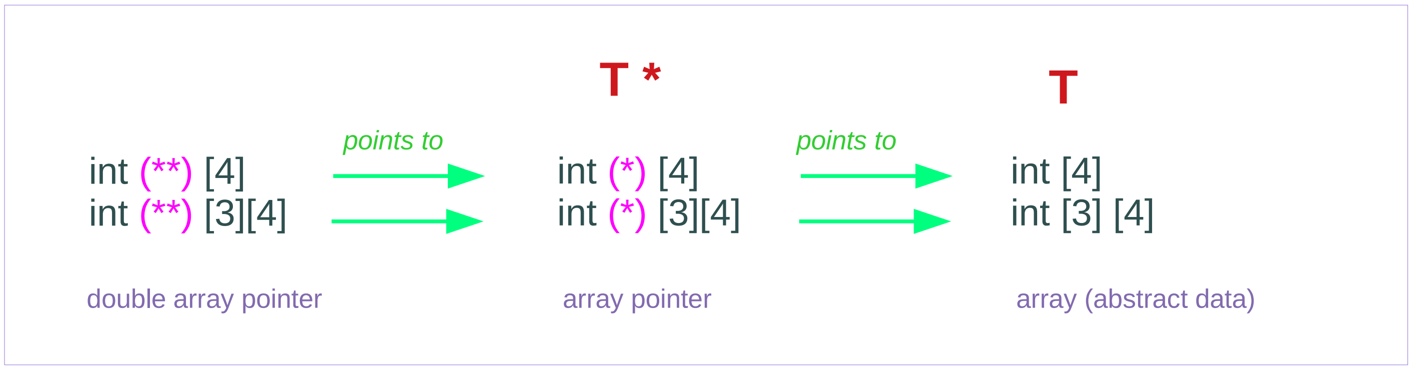
Pointers to abstract data

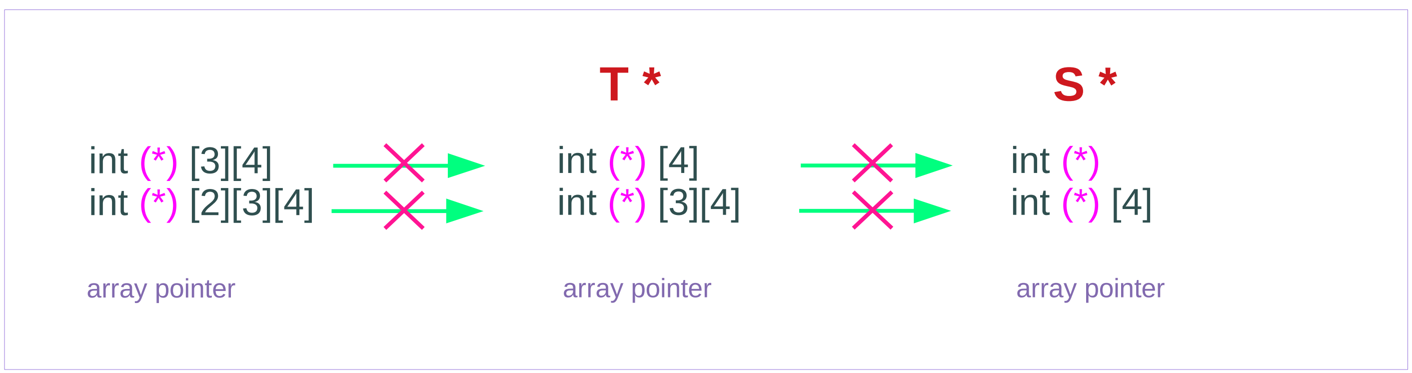
T *p; **T X, Y, Z;** **sizeof(T)**



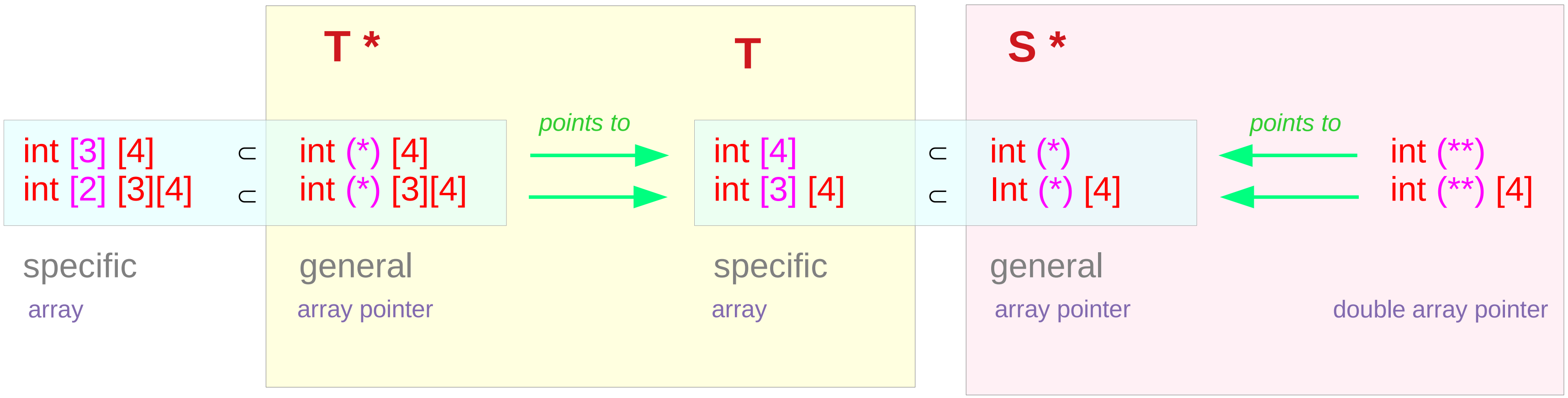
type -------- array
value -------- start address
increment size -------- size

Array pointer and array types





Specific array types v.s. general array pointer types



Array pointers have augmented dimensions

typedef int (*T1) [4];
typedef int (*T1) [3][4];

int (*) [4]
int (*) [3][4]
general

typedef int T2[4];
typedef int T2[3][4];

int [4]
int [3] [4]
specific

T1 a;
T2 b;

T1 is a pointer type
T2 is an array type
T1 has one more dimension than T2

a = &**b**;
***a** = **b**;

a references **b**

b is the dereference of **a**

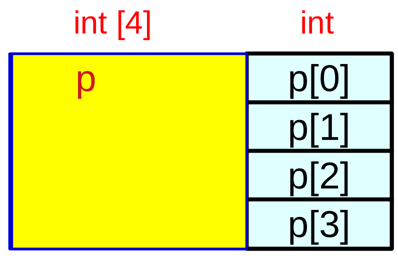
(**a+1**) = ?
* (**a+1**) = ?

(**a+2**) = ?
* (**a+2**) = ?

Virtual pointers in an array of integers

int p[3];

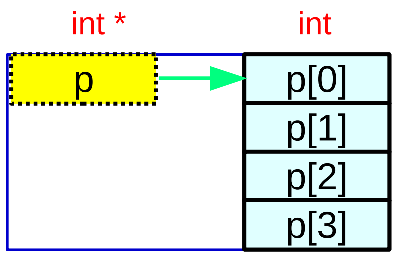
p is an abstract data (array)



p is the name of an array
p has the size of the whole array

p has an array type (abstract data)

p can also be viewed as a pointer

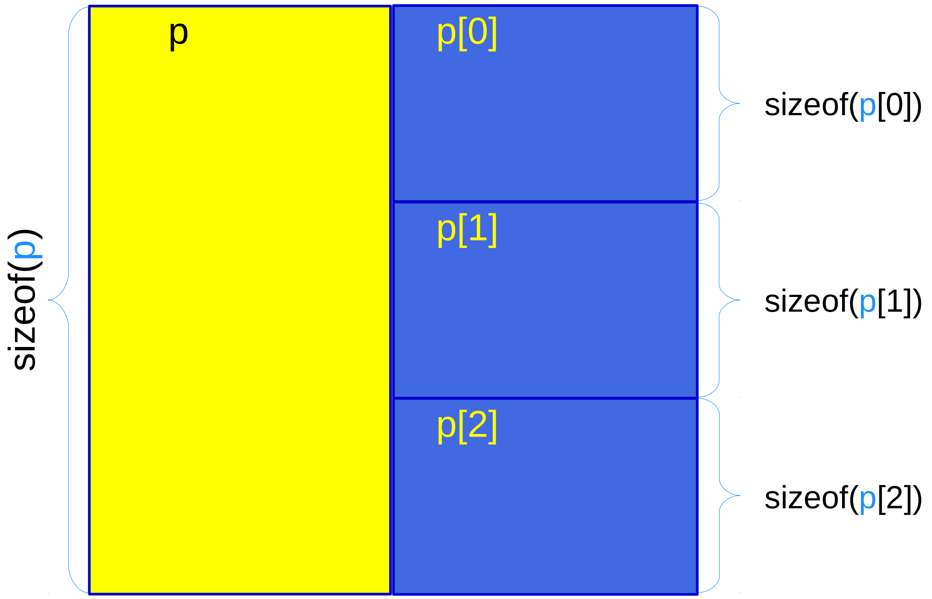


p also has pointer characteristics
p has the value of the starting address

p is a virtual pointer

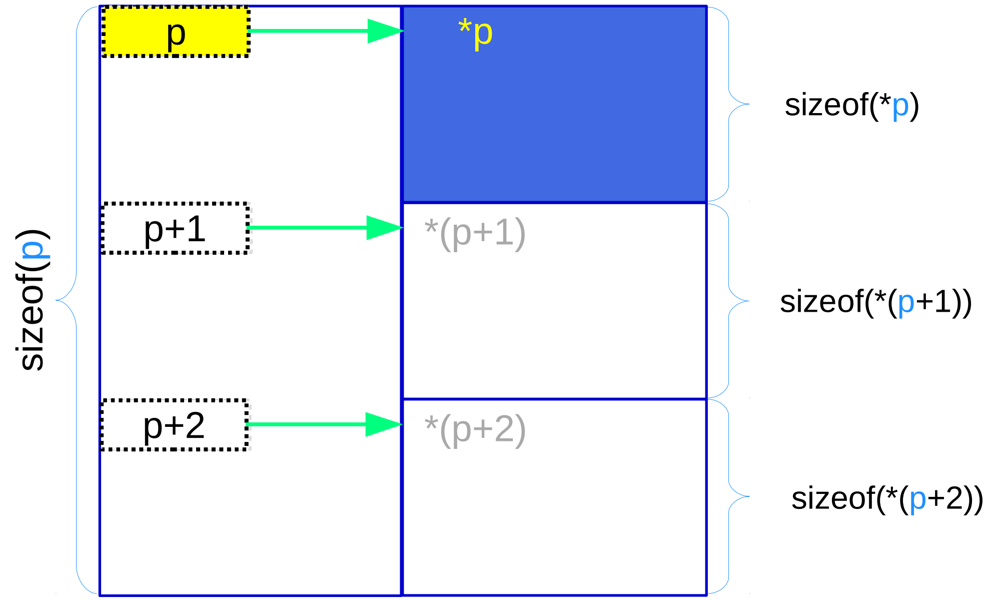
Virtual pointers in an array of abstract data

Abstract data array p



p has an array type (abstract data element)
p is the name of an array
p has the size of the whole array

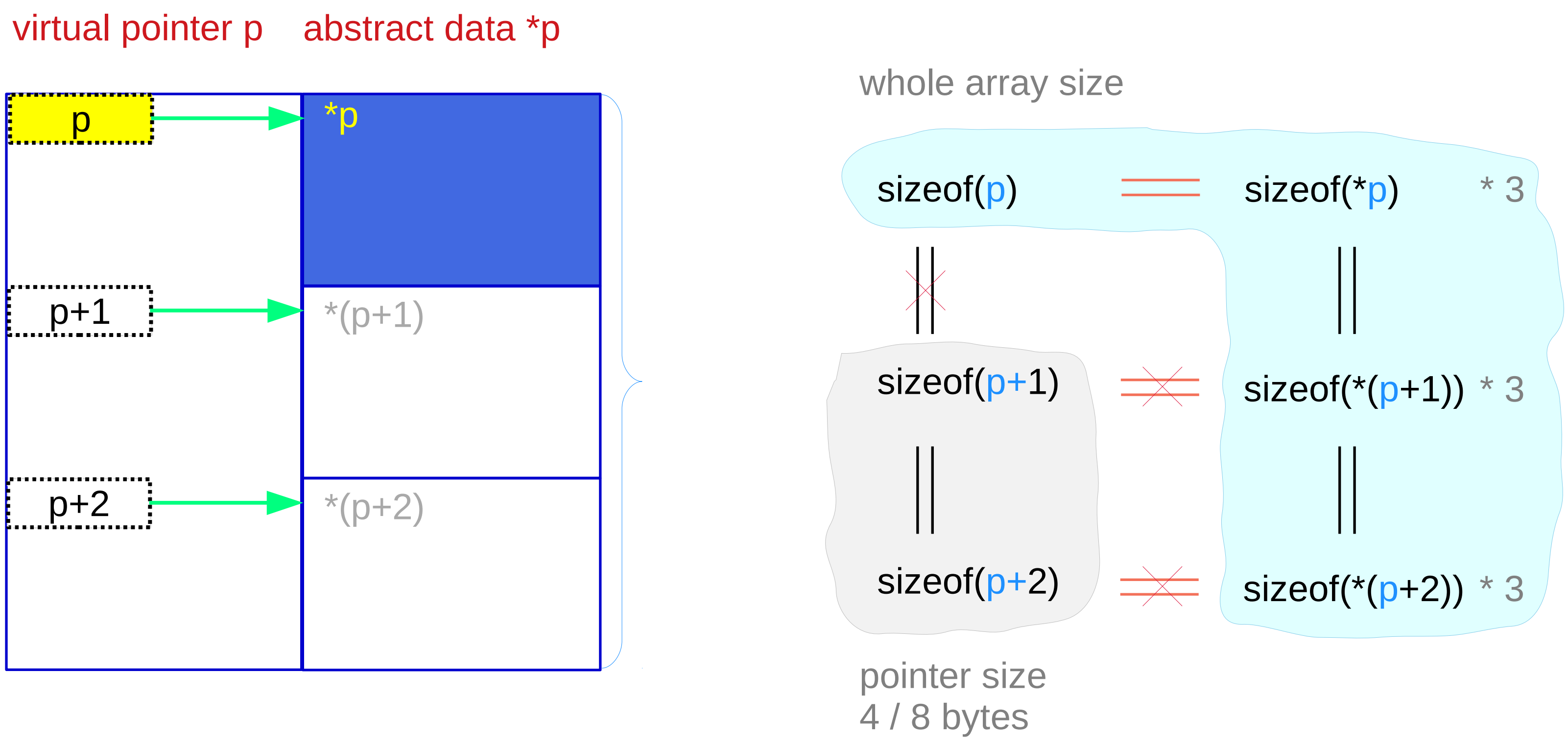
Virtual pointer p



p also has a pointer type
p has the value of the starting address

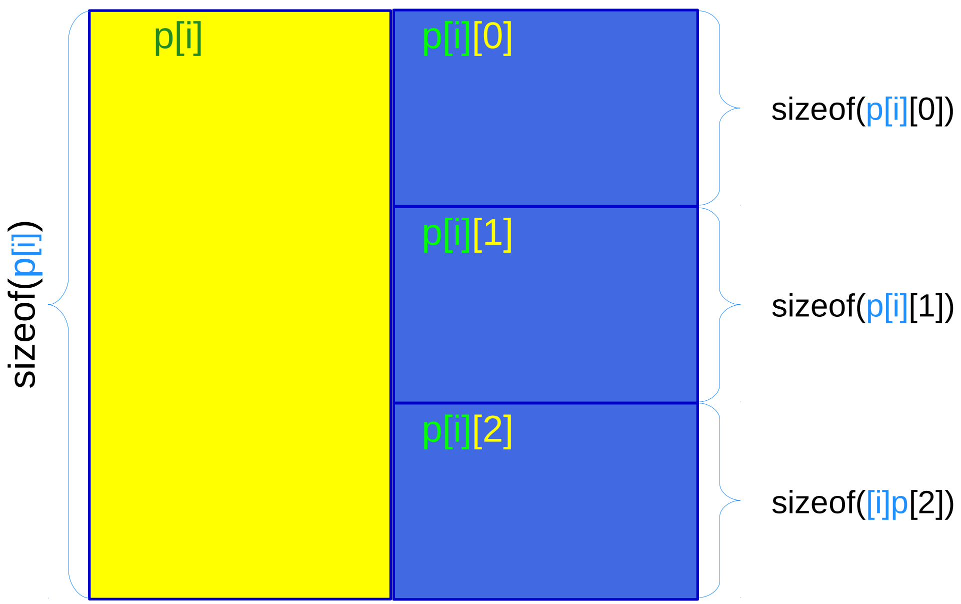
p is a virtual array pointer

Virtual pointer to abstract data

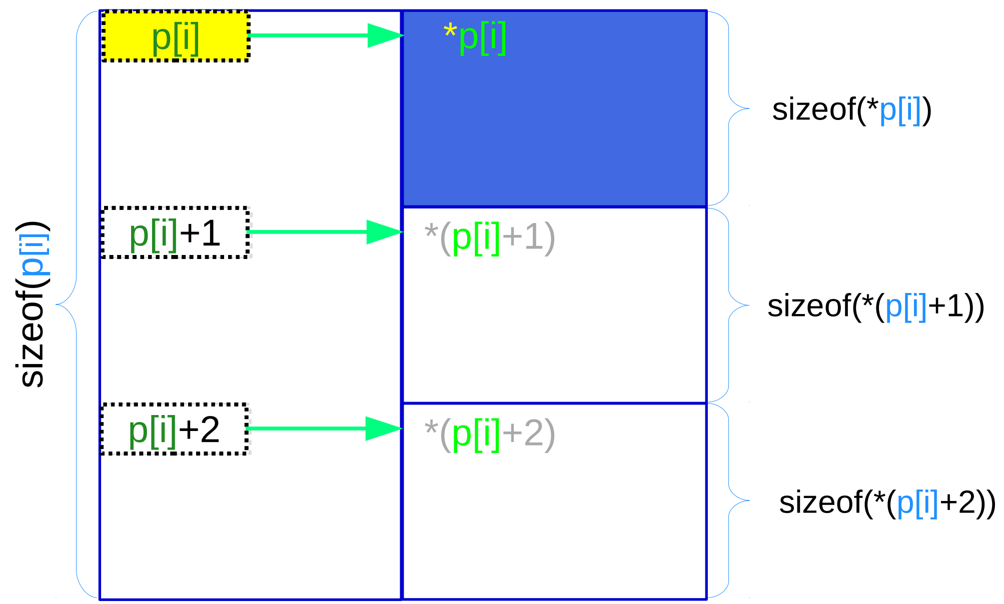


Virtual array pointers in a multi-dimensional array

Abstract data (array) p[i]



Virtual array pointer p[i]



p[i] has an array type (abstract data)
p[i] is the name of an array
p[i] has the size of the whole array

p[i] also has an array pointer type
p[i] has the value of the starting address

p[i] is a virtual array pointer

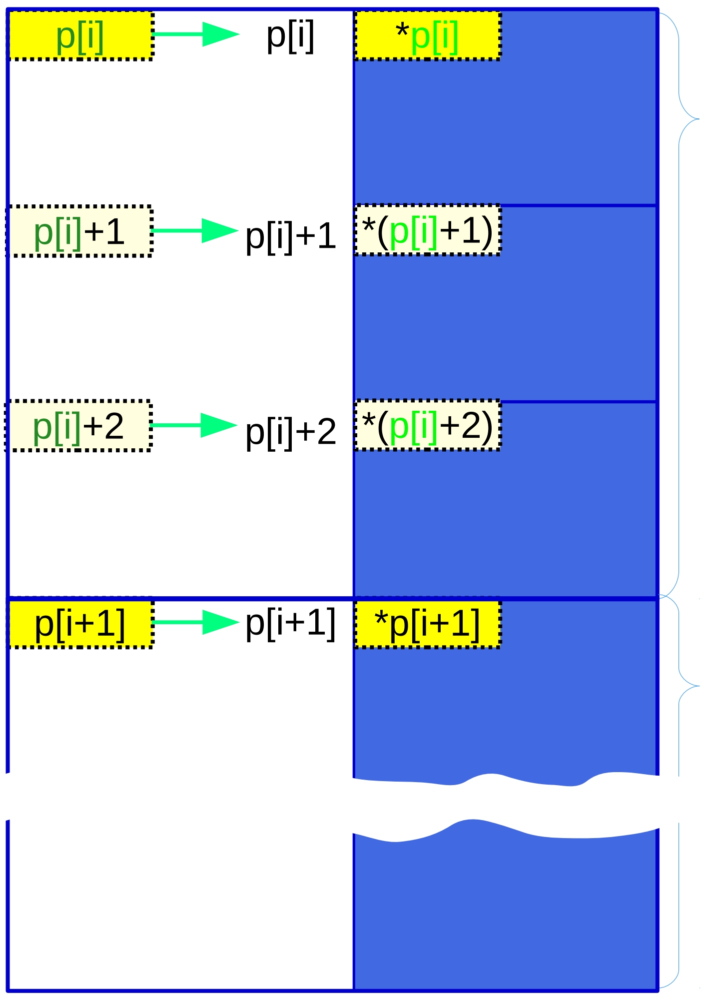
Virtual pointers to a sub array

p[i] :: T *

array pointer type

***p[i], *p[i+1] :: T**

array type



sizeof(p[i])

= sizeof(*p[i]) * N
sizeof(p[i][0]) * N

size = sizeof(*p[i]) =
sizeof(p[i][0])

||

size = sizeof(*(p[i]+1)) =
sizeof(p[i][1])

||

size = sizeof(*(p[i]+2)) =
sizeof(p[i][2])

sizeof(p[i+1])

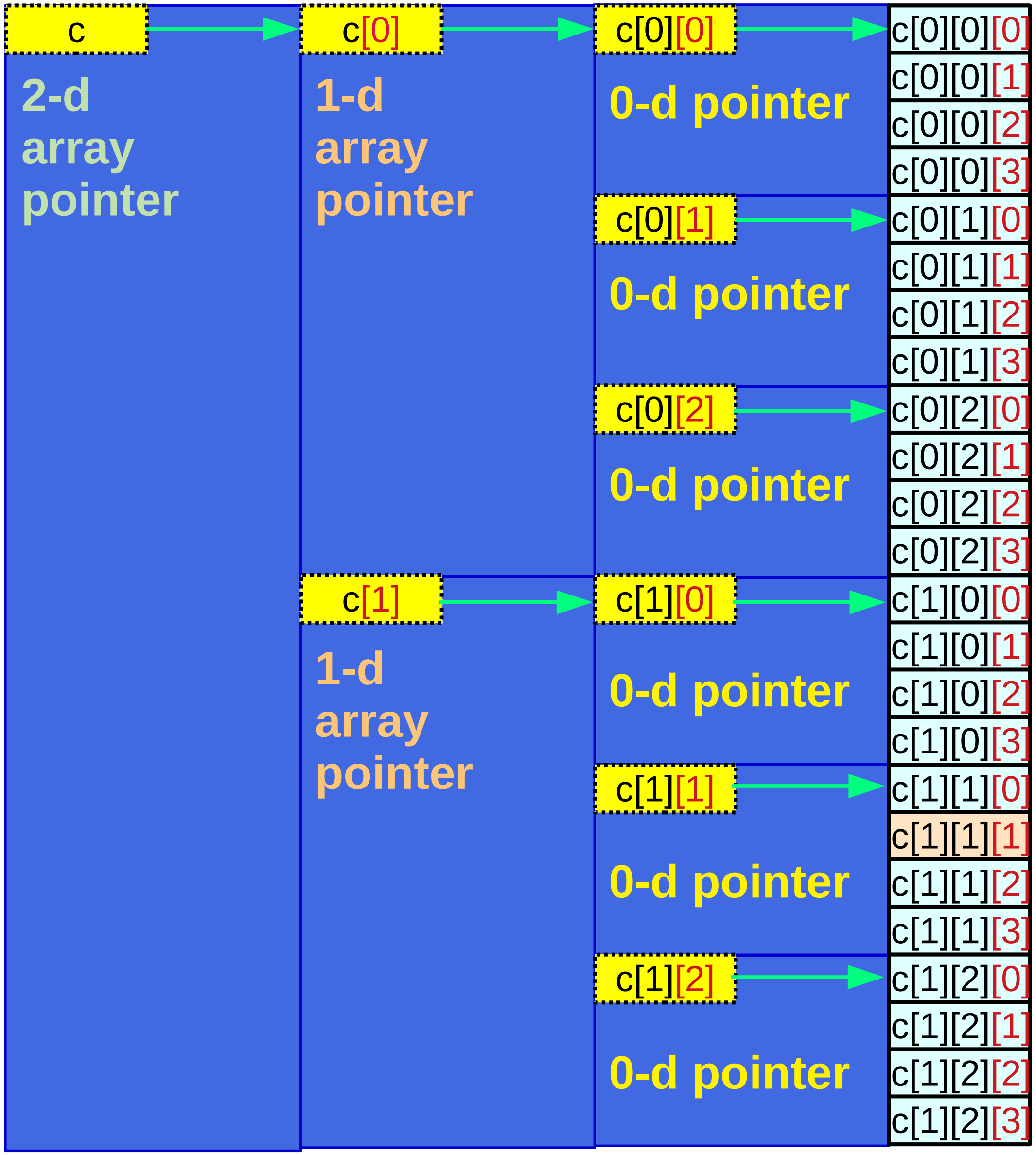
= sizeof(*p[i+1]) * N
sizeof(p[i+1][0]) * N

3-d array structure – pointer representation

int c[2][3][4];

((*(c +i) +j) +k)

- Hierarchical
- Nested Structure
- Virtual Array Pointers to abstract data (subarrays)
- Contiguous and Linear Data Layout
- Row Major Order

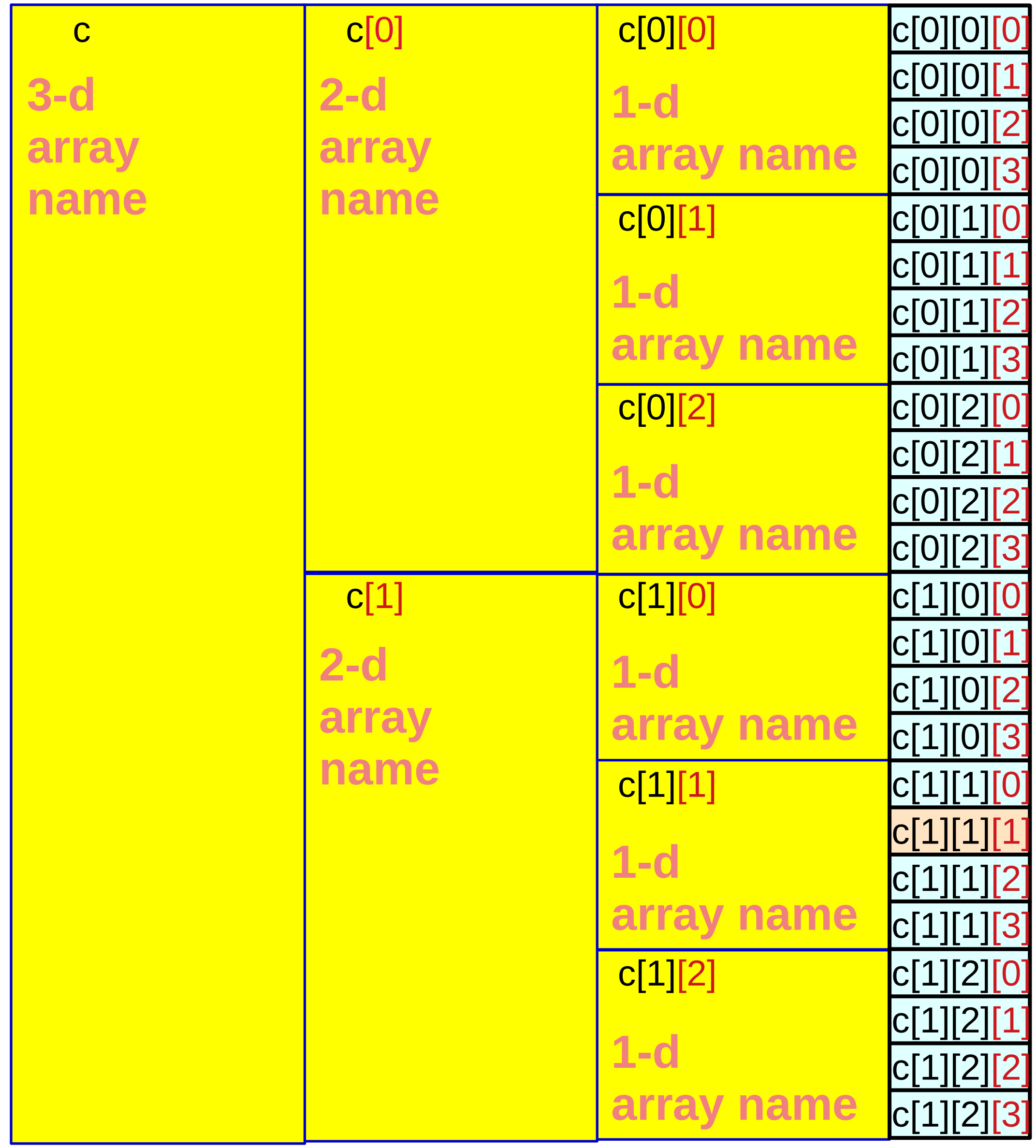


3-d array structure – abstract data representation

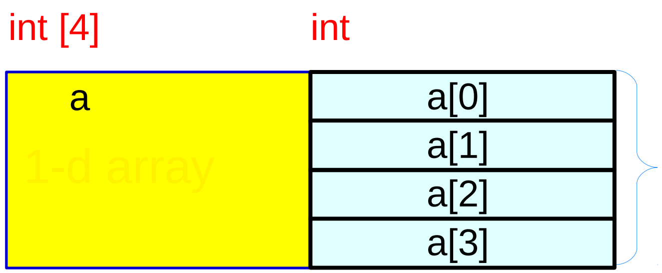
int c[2][3][4];

((c [i])[j])[k]

- Hierarchical
- Nested Structure
- Virtual Array Pointers to abstract data (subarrays)
- Contiguous and Linear Data Layout
- Row Major Order

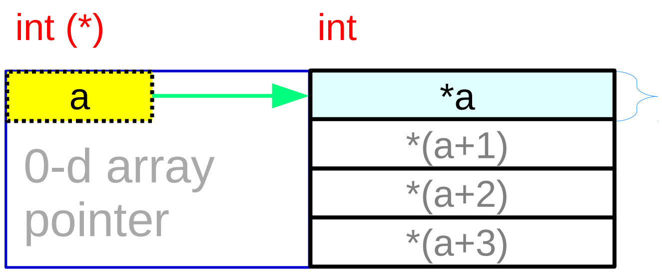


Array **a** and pointer **a**



1-d array a specific array type

sizeof(a)



pointer a general pointer type

sizeof(a) = sizeof(*a) * 4

a is the name of a 1-d array
a also has a pointer type
a has the size of the array
a has the value of the starting address

a is a <u>virtual</u> array pointer

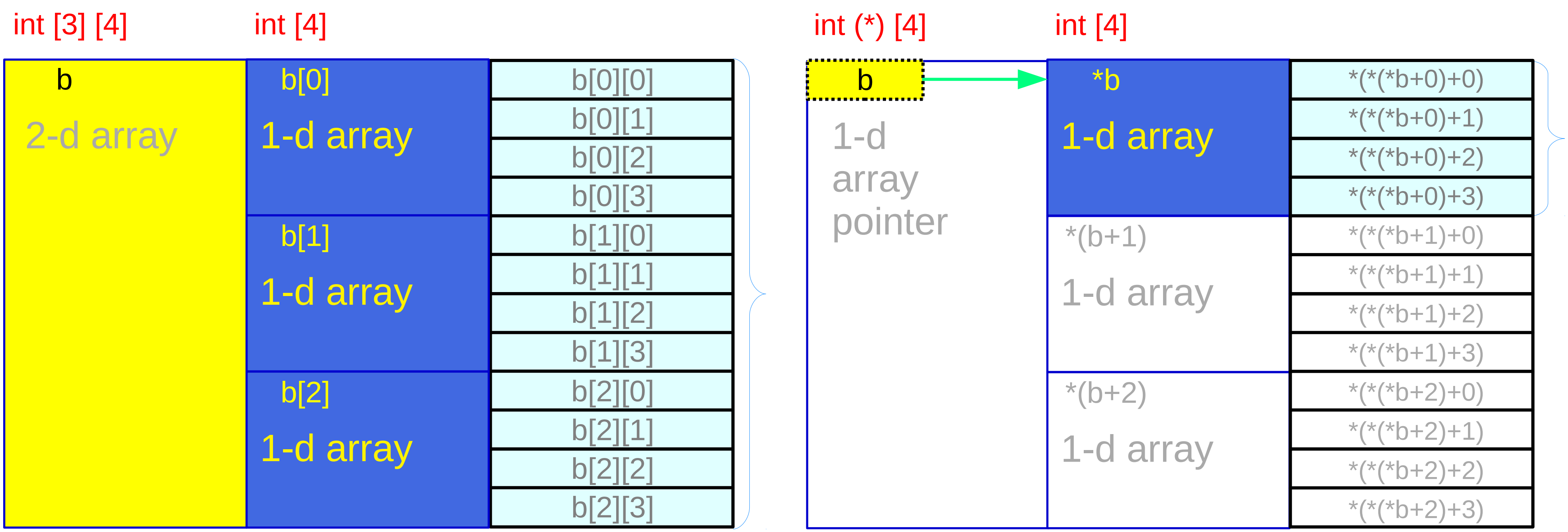
Array **b** and pointer **b**

2-d array b specific array type

sizeof(b)

1-d array pointer b general pointer type

sizeof(b) = sizeof(*b) * 3



b is the name of a 2-d array
b has the size of the array

b also has a 1-d array pointer type
b has the value of the starting address

b is a <u>virtual</u> array pointer

Array **c**

3-d array c

specific array type

sizeof(c)

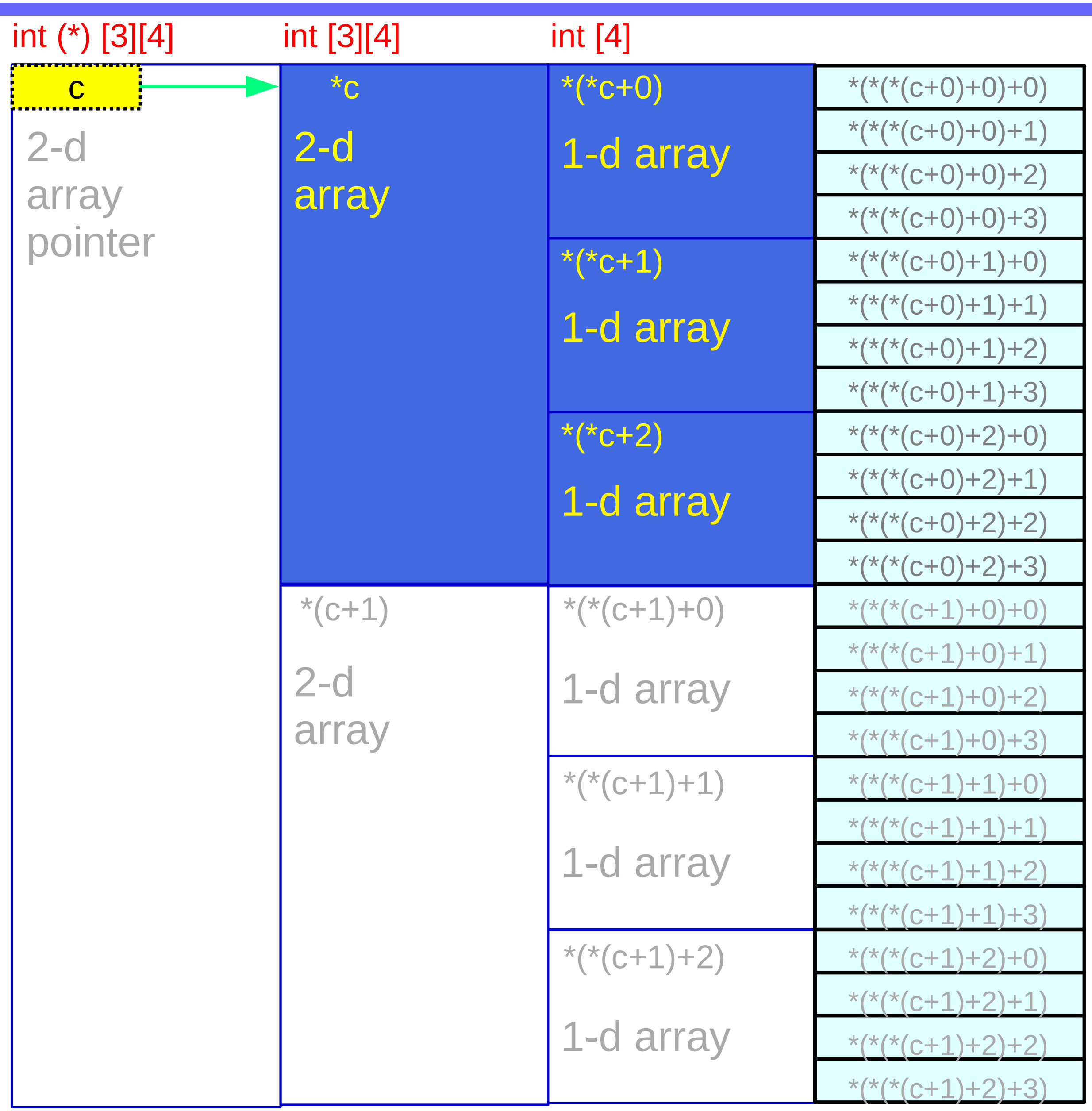
c is the name of a 3-d array
c has the size of the array

Pointer **c**

2-d array pointer c

general pointer type

sizeof(c) = sizeof(*c) * 2



c also has a 2-d array pointer type
c has the value of the starting address

c is a virtual array pointer

Types of virtual array pointers in a 3-d array

```
int c[2][3][4];
```

	array type (name)	array pointer type
c[i][j][k]	int	int
c[i][j] [k]	int [4] [k]	int (*) [k]
c[i] [j] [k]	int [3][4] [j] [k]	int (*)[4] [j] [k]
c [i] [j] [k]	int [2][3][4] [i] [j] [k]	int (*)[3][4] [i] [j] [k]

Sizes of virtual array pointers in a 3-d array

```
int c[2][3][4];
```

sizeof(**c[i][j][k]**) = sizeof(int)

sizeof(**c[i][j]**) = sizeof(int) * 4
[k] [k]

sizeof(**c[i]**) = sizeof(int) * 3 * 4
[j] [k] [j] [k]

sizeof(**c**) = sizeof(int) * 2 * 3 * 4
[i] [j] [k] [i] [j] [k]

Address values of virtual array pointers in a 3-d array

```
int c[2][3][4];
```

c[i][j][k] = &c[i][j][k]

c[i][j]+k = &c[i][j][0] + k * sizeof(c[i][j][k])

sizeof(*c[i][j])
= sizeof(c[i][j][0]) = sizeof(int)

c[i]+j = &c[i][0][0] + j * sizeof(c[i][j])

sizeof(*c[i])
= sizeof(c[i][0]) = sizeof(int) * 4
[k]

c+i = &c[0][0][0] + i * sizeof(c[i])

sizeof(*c)
= sizeof(c[0]) = sizeof(int) * 3 * 4
[j] [k]

Types in a multi-dimensional **3-d** array

```
int     c [2][3][4];
```

abstract data int [2] [3][4]

array pointer int (*) [3][4] | c → *c | int [3] [4] abstract data

[I] [j] [k] | x 2 | [j] [k] | int (*) [4] array pointer

abstract data int [3] [4]

array pointer int (*) [4] | c[i] → *c[i] | int [4] abstract data

[j] [k] | x 3 | [k] | int (*) array pointer

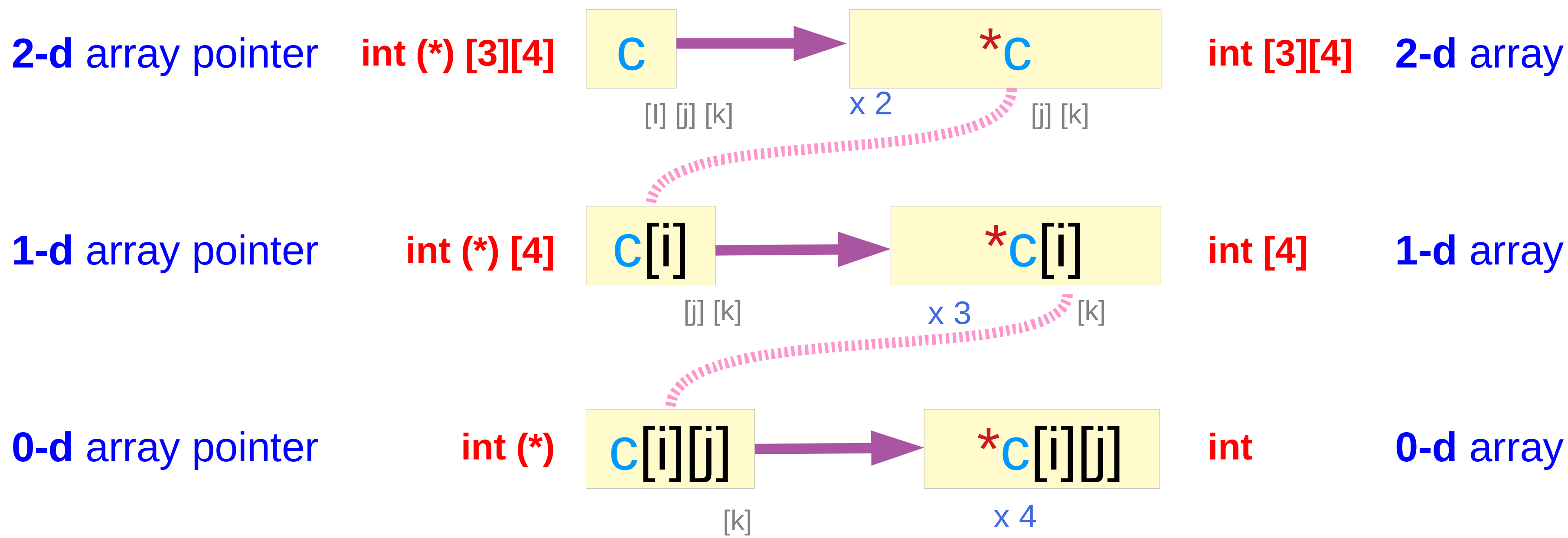
abstract data int [4]

array pointer int (*) | c[i][j] → *c[i][j] | int primitive data

[k] | x 4

Virtual array pointers and abstract data

```
int    c [2][3][4];
```



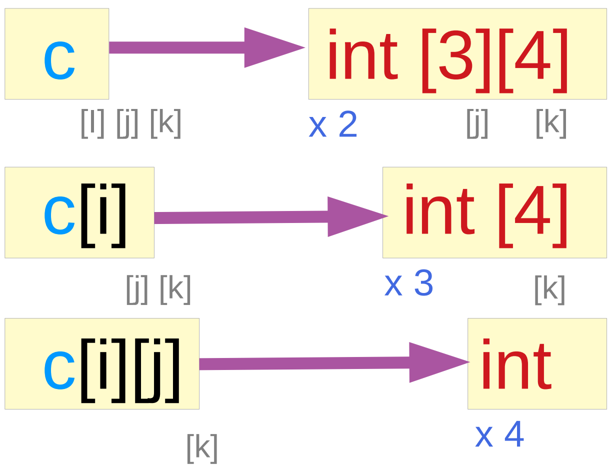
all these pointers are <u>virtual</u>, and take <u>no</u> actual memory locations

exploiting the **contiguity** of allocated memory locations

Abstract data sizes

```
int      c [2][3][4];
```

the size of a pointer type is fixed
Here, the sizes of virtual pointers are shown
i.e, the sizes of different abstract data types




```
sizeof( c)         = sizeof(int [2][3][4])
sizeof(*c)         = sizeof(int [3][4])

sizeof( c[i])      = sizeof(int [3][4])
sizeof(*c[i])      = sizeof(int [4])

sizeof( c[i][j])   = sizeof(int [4])
sizeof(*c[i][j])   = sizeof(int)
```

all are sizes of arrays

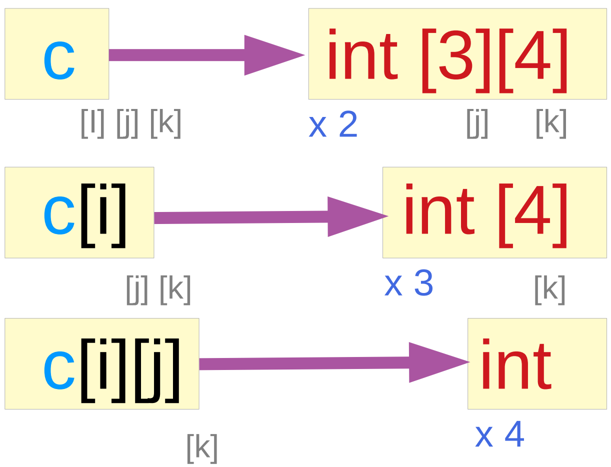
c, c[i], c[i][j] are virtual array pointers
and they are also abstract data (arrays)
[k]

when sizes are considered,
view them as abstract data (arrays)

Virtual array pointer sizes and abstract data sizes

int c [2][3][4];

size of a virtual array pointer = size of the pointed abstract data type * the number of such types



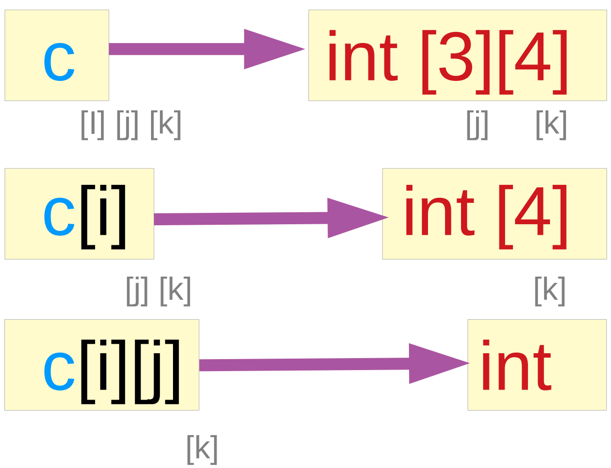
sizeof(c) = sizeof(*c) * **2**

sizeof(c[i]) = sizeof(*c[i]) * **3**

sizeof(c[i][j]) = sizeof(*c[i][j]) * **4**

Sizes of array pointer types

int c [2][3][4];



not real array pointers
virtual array pointers

c int (*)[3][4] = sizeof(c)
sizeof(int (*) [3][4]) = pointer size ≠ sizeof(c)

c[i] int (*) [4] = sizeof(c[i])
sizeof(int (*) [4]) = pointer size ≠ sizeof(c[i])

c[i][j] int [4] = sizeof(c[i][j])
sizeof(int [4]) = pointer size ≠ sizeof(c[i][j])

4 bytes for 32-bit machines
8 bytes for 64-bit machines

Virtual array pointer increment size

```
int     c [2][3][4];
```

c points to a **2-d** array
increment size: sizeof(int[2][3][4])

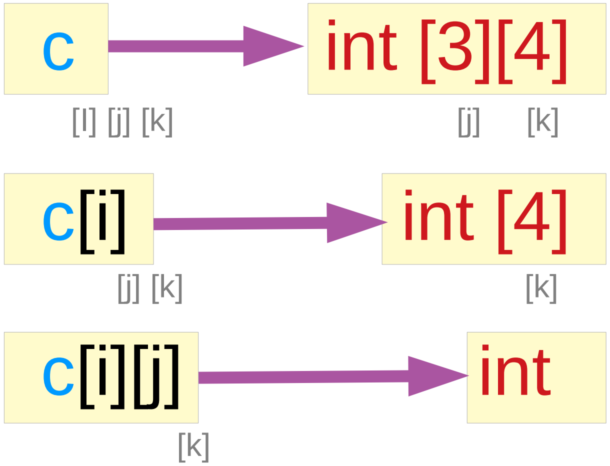
int (*) [3][4]

c[i] points to an **1-d** array
increment size: sizeof(int[3][4])

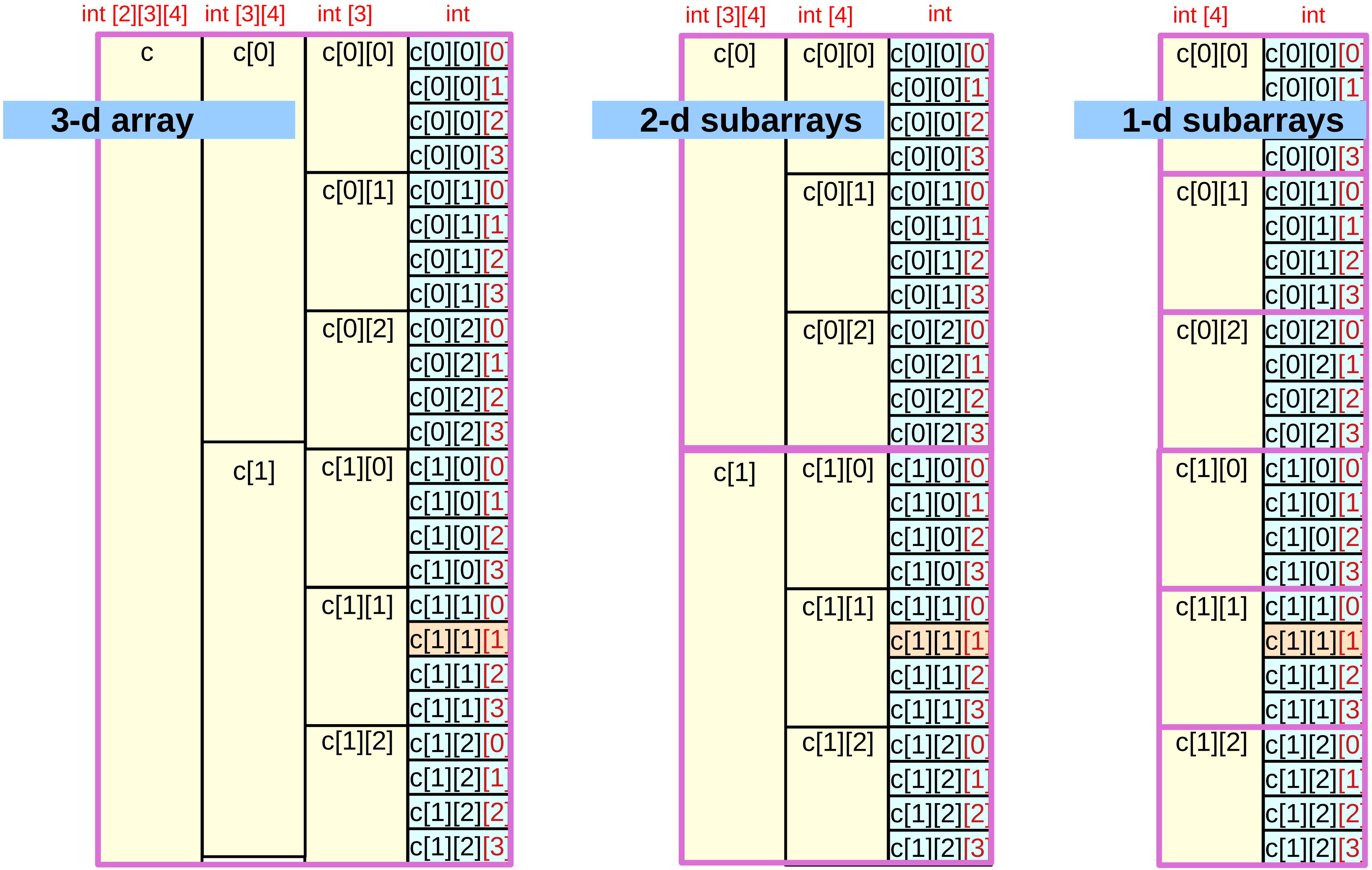
int (*) [4]

c[i][j] points to an integer
increment size: sizeof(int[4])

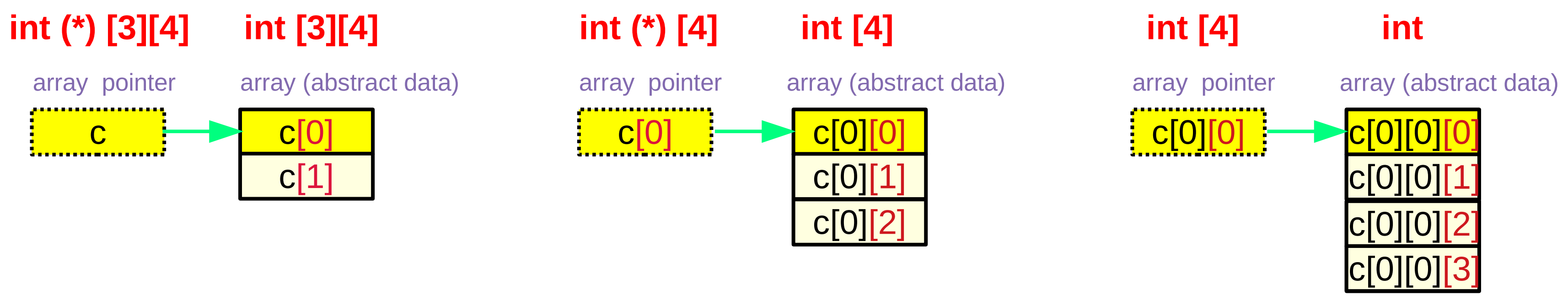
int (*)



Subarrays in a 3-d array



Types – array pointers



Sizes – abstract data

sizeof(**c**)
sizeof(**int [2][3][4]**)
sizeof(**int**) * **2** * **3** * **4**

sizeof(**c[0]**)
sizeof(**int [3][4]**)
sizeof(**int**) * **3** * **4**

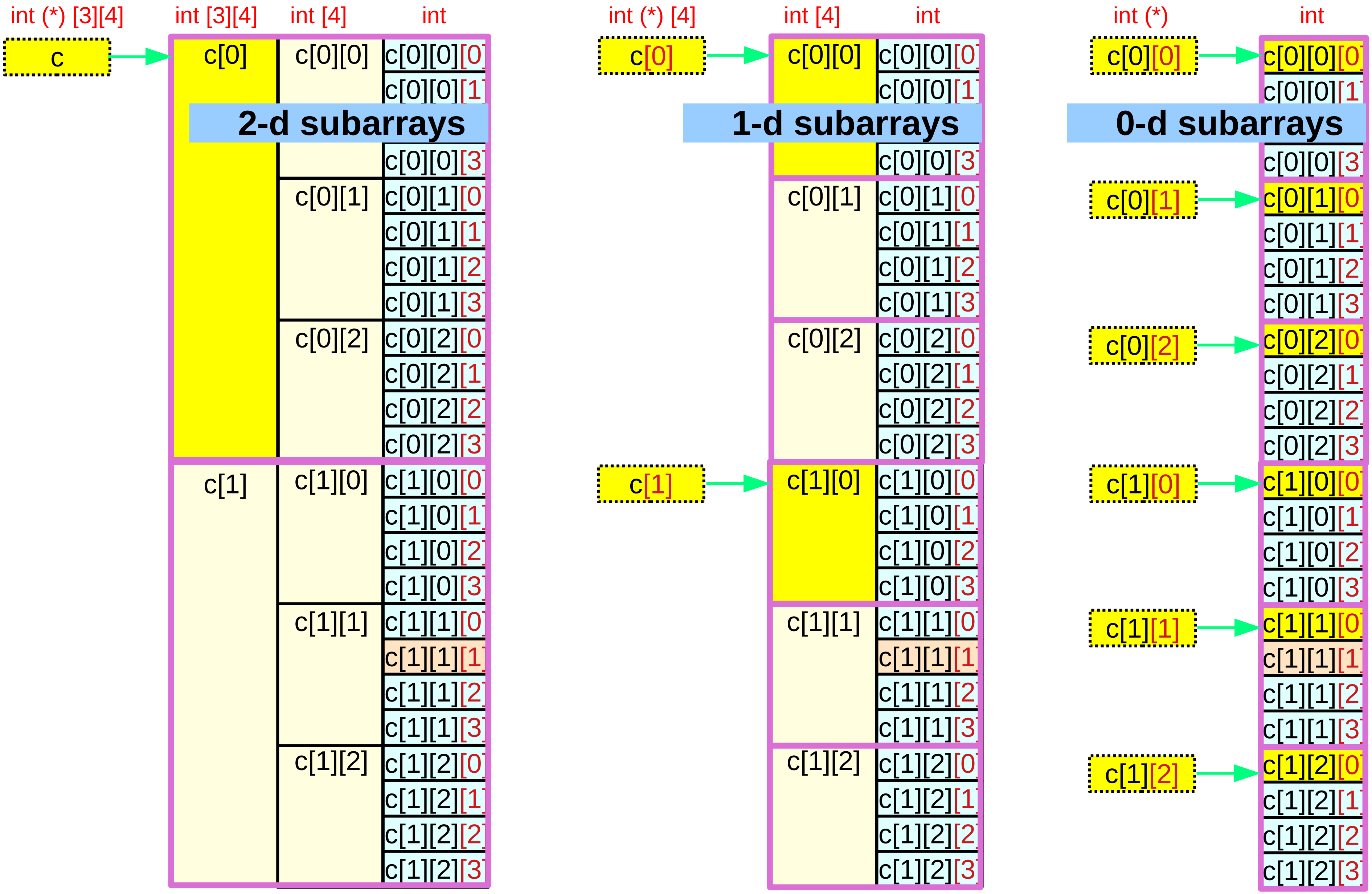
sizeof(**c[0][0]**)
sizeof(**int [4]**)
sizeof(**int**) * **4**

sizeof(**int [2][3][4]**) = 96
sizeof(**int (*)[3][4]**) = 4 / 8

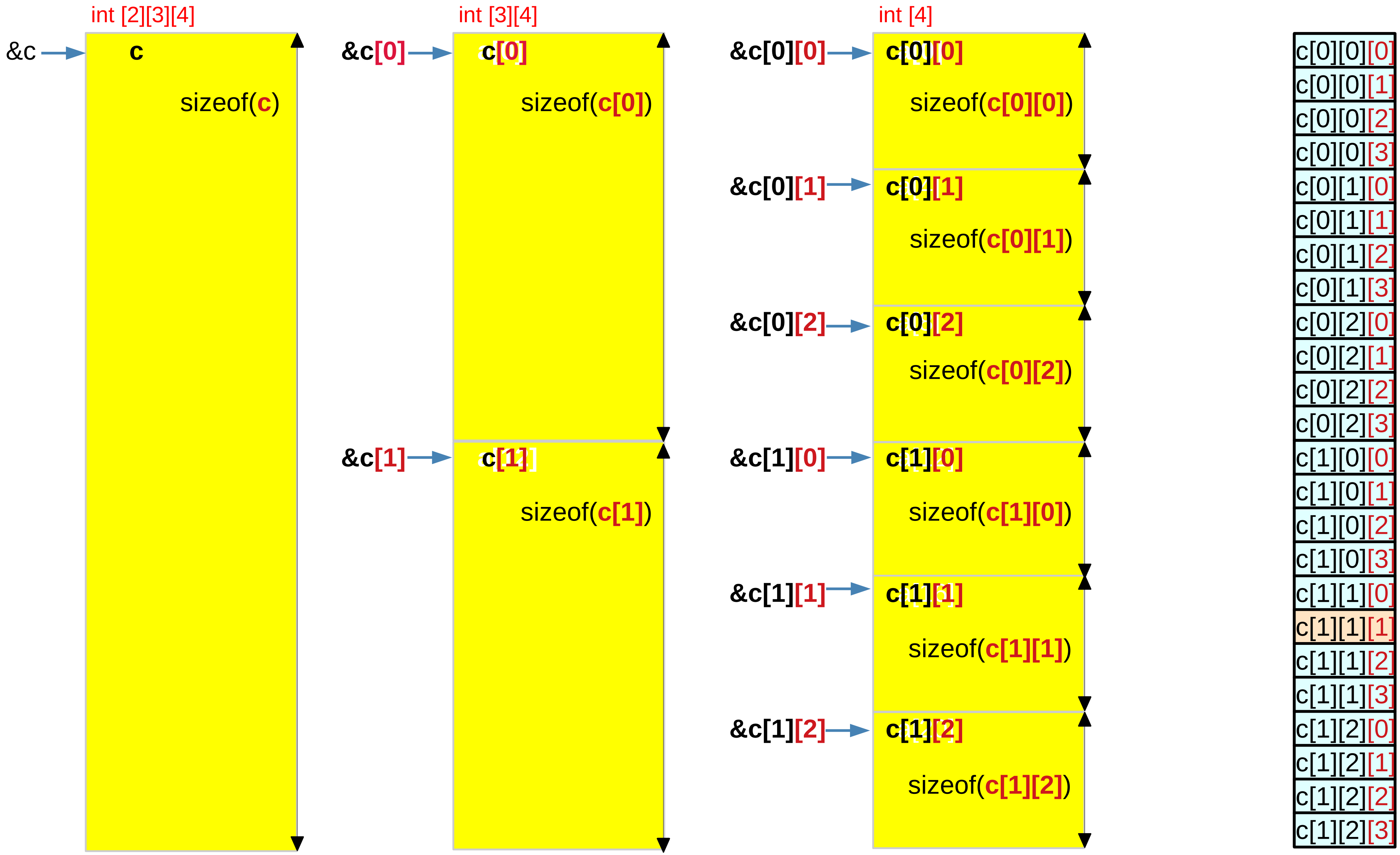
sizeof(**int [3][4]**) = 48
sizeof(**int (*)[4]**) = 4 / 8

sizeof(**int [4]**) = 16
sizeof(**int (*)**) = 4 / 8

Pointers to subarrays in a 3-d array



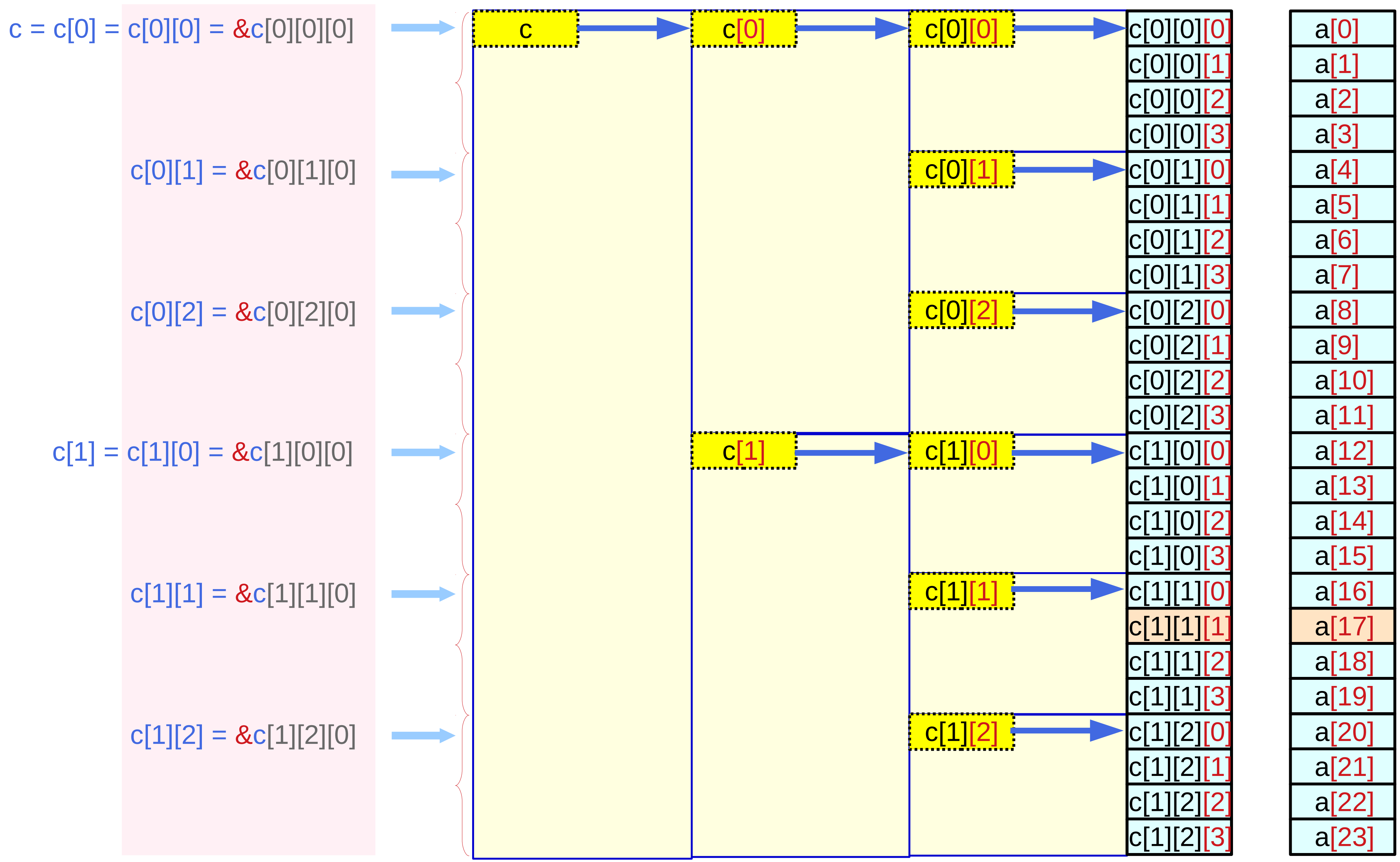
Abstract Data c, c[i], c[i][j] – start addresses and sizes



Virtual array pointers – types, sizes, and values

int c[2][3][4];	**c[i][j]**	**c[i][j][0]**	
type	int [4] int (*)	int int	• abstract data type • array pointer type
size	sizeof(c[i][j]) =	sizeof(c[i][j][0]) * **4**	= sizeof(int) * **4**
value (address)	c[i][j] =	&c[i][j][0]	
int c[2][3][4];	**c[i]**	**c[i][0]**	
type	int [3][4] int (*)[4]	int [4] int (*)	• abstract data type • array pointer type
size	sizeof(c[i]) =	sizeof(c[i][0]) * **3**	= sizeof(int) * **4** * **3**
value (address)	c[i] =	&c[i][0][0]	
int c[2][3][4];	**c**	**c[0]**	
type	int [2][3][4] int (*)[3][4]	int [3][4] int (*)[4]	• abstract data type • array pointer type
size	sizeof(c) =	sizeof(c[0]) * **2**	= sizeof(int) * **4** * **3** * **2**
value (address)	c =	&c[0][0][0]	

Virtual array pointer c, c[i], c[i][j] – values (addresses)



Virtual array pointer c, c[i], c[i][j] – vertical displacement

c = c[0] = c[0][0] = &c[0][0][0]

For address values

Horizontal displacements
are not counted

Only vertical displacements
are considered

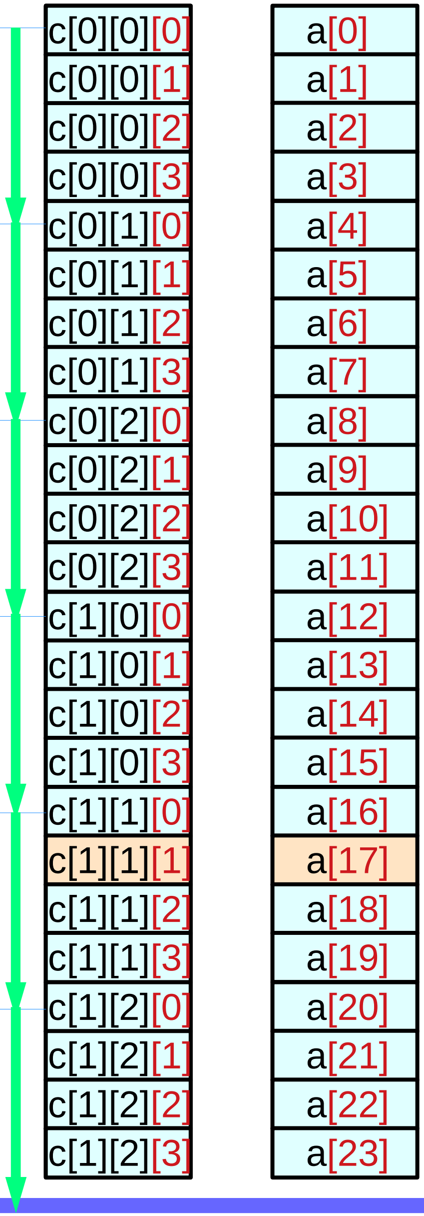
c[0][1] = &c[0][1][0]

c[0][2] = &c[0][2][0]

c[1] = c[1][0] = &c[1][0][0]

c[1][1] = &c[1][1][0]

c[1][2] = &c[1][2][0]



Virtual array pointer c, c[i], c[i][j] – values and types

c = c[0] = c[0][0] = &c[0][0][0]	means	value(c) = value(c[0]) = value(c[0][0]) = value(&c[0][0][0]) type(c) ≠ type(c[0]) ≠ type(c[0][0]) = type(&c[0][0][0]) int (*) [3][4] int (*) [4] int * int *
c[0][1] = &c[0][1][0]	means	value(c[0][1]) = value(&c[0][1][0]) type(c[0][1]) = type(&c[0][1][0]) int * int *
c[0][2] = &c[0][2][0]	means	value(c[0][2]) = value(&c[0][2][0]) type(c[0][2]) = type(&c[0][2][0]) int * int *
c[1] = c[1][0] = &c[1][0][0]	means	value(c[1]) = value(c[1][0]) = value(&c[1][0][0]) type(c[1]) ≠ type(c[1][0]) = type(&c[1][0][0]) int (*) [4] int * int *
c[1][1] = &c[1][1][0]	means	value(c[1][1]) = value(&c[1][1][0]) type(c[1][1]) = type(&c[1][1][0]) int * int *
c[1][2] = &c[1][2][0]	means	value(c[1][2]) = value(&c[1][2][0]) type(c[1][2]) = type(&c[1][2][0]) int * int *

Summary of virtual array pointers in a 3-d array

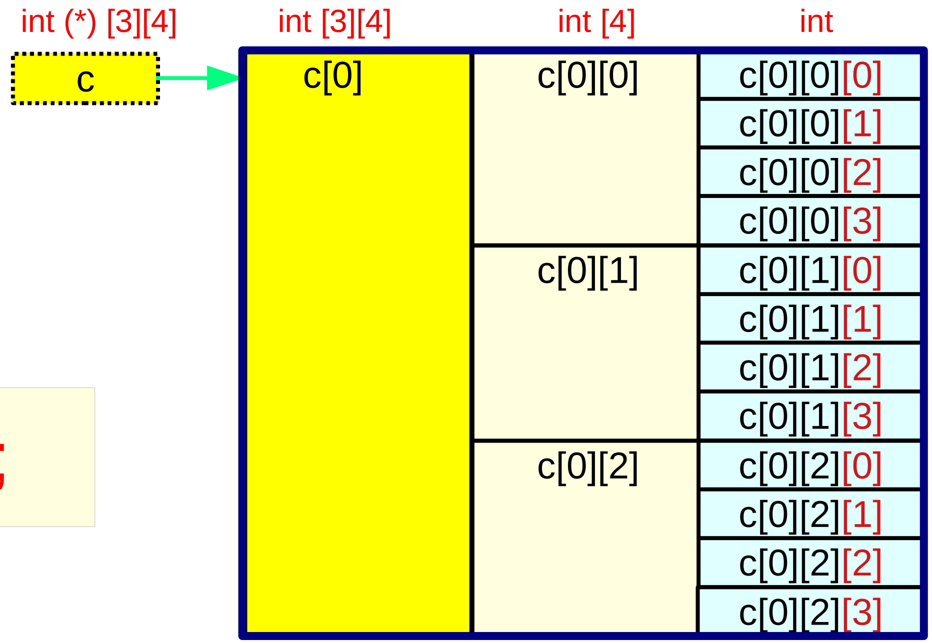
c[i] ≡ *(c +i)	c[i][j] ≡ *(c[i] +j)	c[i][j][k]≡ *(c[i][j] +k)
int (*) [3][4] 2-d array pointer c int [2] [3][4] 3-d array name c	int (*) [4] 1-d array pointers c[i] Int [3] [4] 2-d array names c[i]	int (*) 0-d array pointers c[i][j] int [4] 1-d array names c[i][j]
address value c + i &c[0][0][0] + i * sizeof(*c) &c[0][0][0] + i * sizeof(c[0]) &c[0][0][0] + i * 4 * 3 * 4	**address value** c[i] + j &c[i][0][0] + j * sizeof(*c[i]) &c[i][0][0] + j * sizeof(c[i][0]) &c[i][0][0] + j * 4 * 4	**address value** c[i][j] + k &c[i][j][0] + k * sizeof(*c[i][j]) &c[i][j][0] + k * sizeof(c[i][j][0]) &c[i][j][0] + k * 4
leading elements c[0][0][0]	**leading elements** c[0][0][0] c[1][0][0]	**leading elements** c[0][0][0] c[0][1][0] c[0][2][0] c[1][0][0] c[1][1][0] c[1][2][0]

Dual type constraints in multi-dimensional arrays

Virtual pointers to subarrays in a 3-d array

virtual 2-d array pointer

sizeof(**c**) =
sizeof(**c[0]**) * **2**



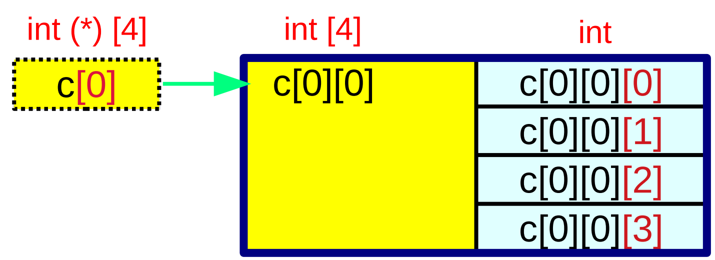
the <u>first</u> 2-d subarray

sizeof(**c[0]**) =
sizeof(**int [3][4]**)

int c [2][3][4];

virtual 1-d array pointer

sizeof(**c[0]**) =
sizeof(**c[0][0]**) * **3**

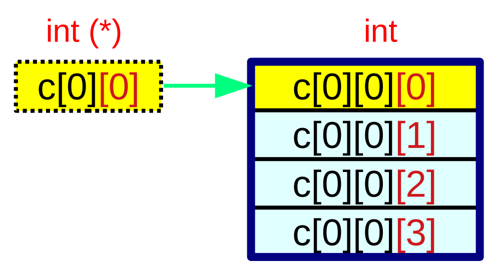


the <u>first</u> 1-d subarray

sizeof(**c[0][0]**) =
sizeof(**int [4]**)

virtual 0-d array pointer

sizeof(**c[0][0]**) =
sizeof(**c[0][0][0]**) * **4**

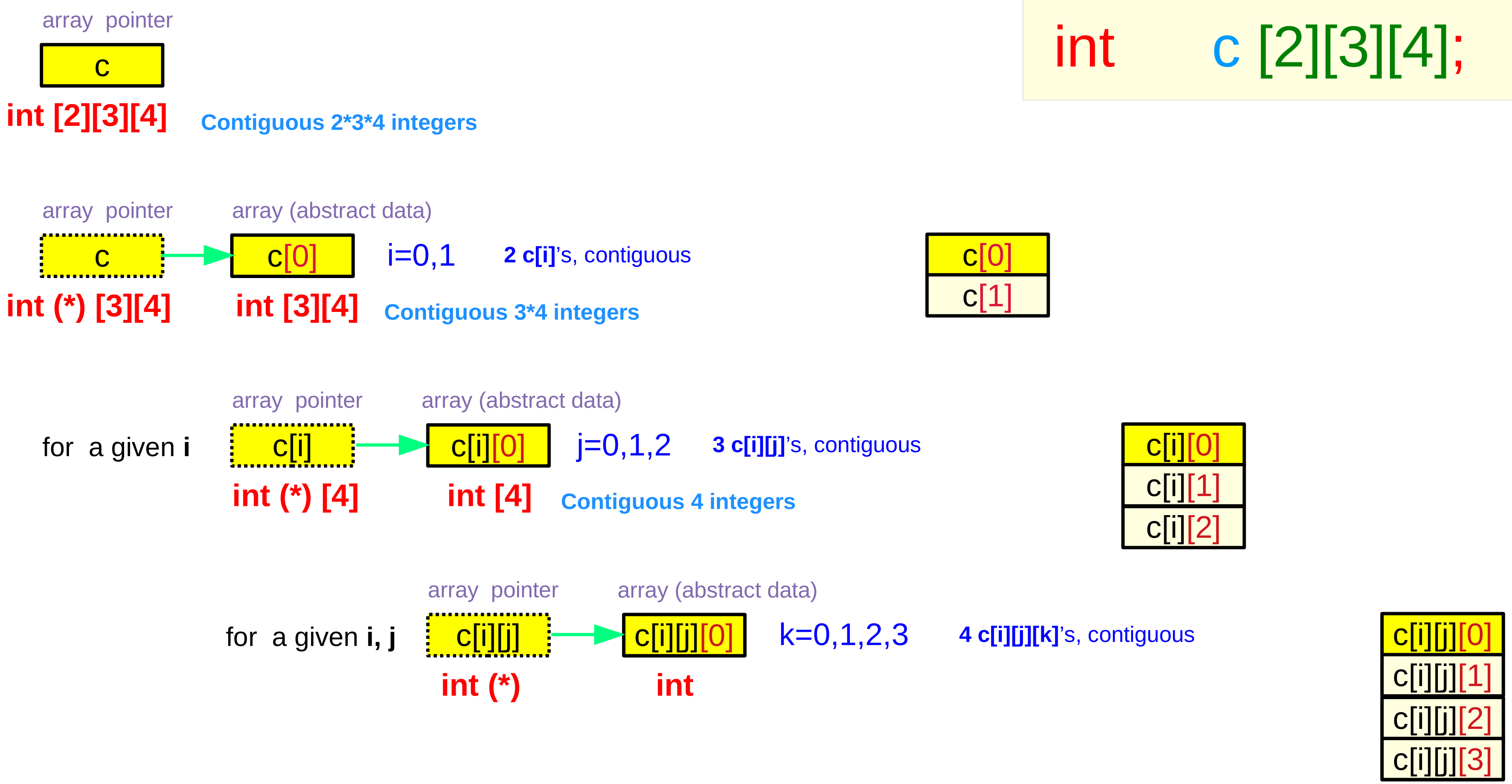


the <u>first</u> 0-d subarray

sizeof(**c[0][0][0]**) =
sizeof(**int**)

Contiguous subarrays

int c [2][3][4];



Virtual assignments

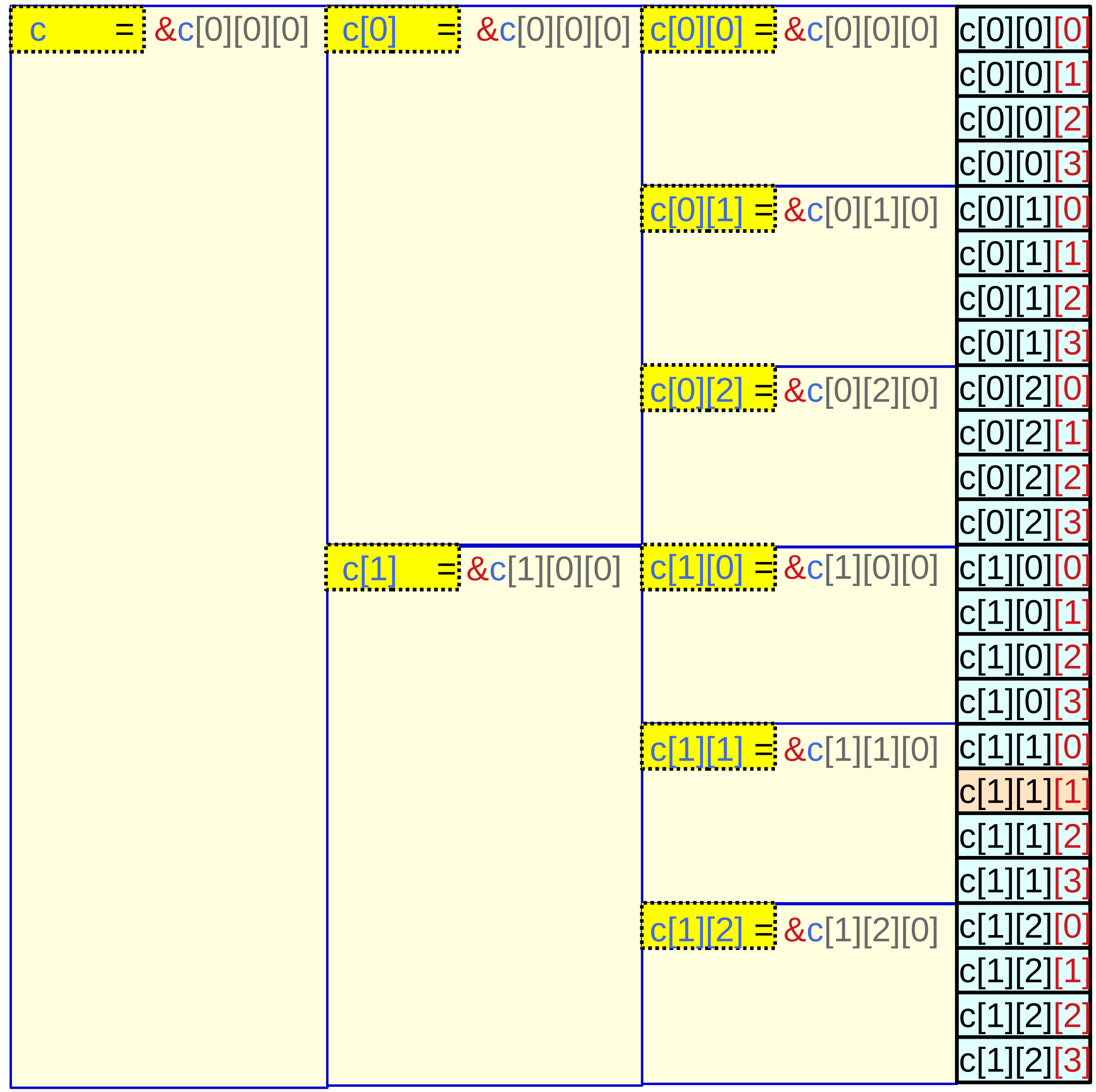
int c [2][3][4];

c [i][j][k];

virtual assignments

c ⟸ &c[0][0][0]
c[i] ⟸ &c[i][0][0]
c[i][j] ⟸ &c[i][j][0]

row major ordering
contiguous linear layout



Virtual assignments and type casts

```
int    c [2][3][4];
```

↓

```
       c [i][j][k];
```

virtual assignments

c	⇐	&c[0][0][0]
c[i]	⇐	&c[i][0][0]
c[i][j]	⇐	&c[i][j][0]

row major ordering
contiguous linear layout

		type casts	address values
c	⇐	(int (*)[3][4])	&c[0][0][0]
c[i]	⇐	(int (*)[4])	&c[i][0][0]
c[i][j]	⇐	(int (*))	&c[i][j][0]

if c, c[i], c[i][j] were real pointer variables,
type casts would be needed

Dual types of **c**, **c[i]**, **c[i][j]**

```
int     c [2][3][4];
```

c :: int (*) [3][4]	2-d array pointers
→ c[i] :: int [3] [4]	2-d arrays
c[i] :: int (*) [4]	1-d array pointers
→ c[i][j] :: int [4]	1-d arrays
c[i][j] :: int (*)	0-d array pointers
→ c[i][j][k] :: int	0-d arrays (integers)

c →	c[0] →	c[0][0] →	c[0][0][0] … c[0][0][3]	
		c[0][1] →	c[0][1][0] … c[0][1][3]	
		c[0][2] →	c[0][2][0] … c[0][2][3]	
	c[1] →	c[1][0] →	c[1][0][0] … c[0][0][3]	
		c[1][1] →	c[1][1][0] … c[0][1][3]	
		c[1][2] →	c[1][2][0] … c[0][2][3]	
int [2] [3][4]	int [3] [4]	int [4]	int … int	
int (*) [3][4]	int (*) [4]	int (*)	int … int	
pointers to a 2-d array	pointers to a 1-d array	1-d array names	leading element of 4-integer array	

Values of **c**, **c[i]**, **c[i][j]**

```
int     c [2][3][4];
```

virtual array pointers have address values
in each row in the following figure
have the same address value

c ═	c[0] ═	c[0][0] ═	&c[0][0][0]
		c[0][1] ═	&c[0][1][0]
		c[0][2] ═	&c[0][2][0]
	c[1] ═	c[1][0] ═	&c[1][0][0]
		c[1][1] ═	&c[1][1][0]
		c[1][2] ═	&c[1][2][0]

Horizontal displacements are not counted
only **vertical displacements** are considered
for address values

virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

Finding address values of **c**, **c[i]**, **c[i][j]**

```
int     c [2][3][4];
```

append [0] to the right

c	+[0]	c[0]	+[0]	c[0][0]	+[0]	&c[0][0][0]
				c[0][1]	+[0]	&c[0][1][0]
				c[0][2]	+[0]	&c[0][2][0]
		c[1]	+[0]	c[1][0]	+[0]	&c[1][0][0]
				c[1][1]	+[0]	&c[1][1][0]
				c[1][2]	+[0]	&c[1][2][0]
int (*) [3][4]		int (*) [4]		int [4]		int

c[0][0][0] **:** leading elements of **c**

&c[0][0][0]

c[i][0][0] **:** leading elements of **c[i]**

&c[0][0][0]

&c[1][0][0]

c[i][j][0] **:** leading elements of **c[i][j]**

&c[0][0][0]
&c[0][1][0]
&c[0][2][0]
&c[1][0][0]
&c[1][1][0]
&c[1][2][0]

Finding sub-array names with the address &c[i][j][0]

```
int     c [2][3][4];
```

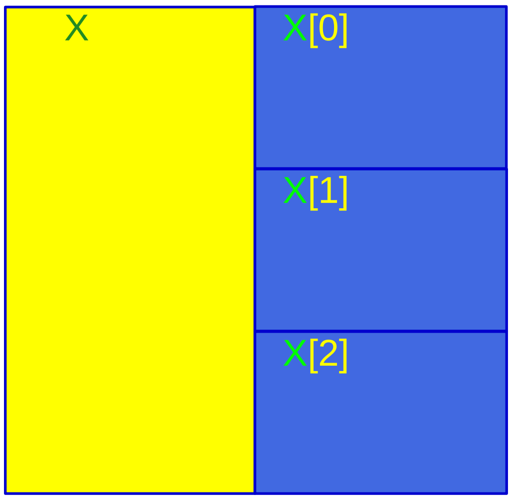
delete [0] from the right

&c[0][0][0]	-[0] =	c[0][0]	-[0] =	c[0]	-[0] =	c
&c[0][1][0]	-[0] =	c[0][1]				
&c[0][2][0]	-[0] =	c[0][2]				
&c[1][0][0]	-[0] =	c[1][0]	-[0] =	c[1]		
&c[1][1][0]	-[0] =	c[1][1]				
&c[1][2][0]	-[0] =	c[1][2]				
int		int [4]		int (*) [4]		int (*) [3][4]

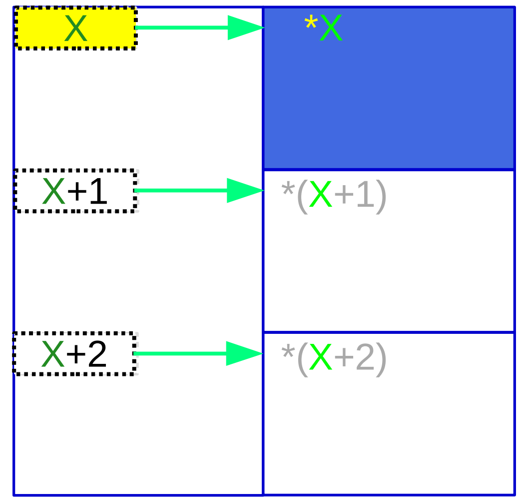
c[0][0][0] is the leading element of c[0][0], c[0], c
c[0][1][0] is the leading element of c[0][1]
c[0][2][0] is the leading element of c[0][2]
c[1][0][0] is the leading element of c[1][0], c[1]
c[1][1][0] is the leading element of c[1][1]
c[1][2][0] is the leading element of c[1][2]

Dual type – virtual array pointer and abstract array

Abstract data (array) p[i]



Virtual array pointer p[i]



array (abstract data)		array (abstract data)
c[i][j]	starts from	&c[i][j][0]
c[i]	starts from	&c[i][0]
c	starts from	&c[0]

&c[i][j]	=	&c[i][j][0]
&c[i]	=	&c[i][0]
&c	=	&c[0]

array pointer		array (abstract data)
c[i][j]	points to	c[i][j][0]
c[i]	points to	c[i][0]
c	points to	c[0]

address value

c[i][j]	=	&c[i][j][0]
c[i]	=	&c[i][0]
c	=	&c[0]

Virtual array pointer values and addresses

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

Virtual array pointer

array pointer		array (abstract data)
c[i][j]	points to	c[i][j][0]
c[i]	points to	c[i][0]
c	points to	c[0]
address value		

&c[i][j][0] = &c[i][j]
&c[i][0] = &c[i]
&c[0] = &c

Abstract data (array)

array (abstract data)		array (abstract data)
c[i][j]	starts from	&c[i][j][0]
c[i]	starts from	&c[i][0]
c	starts from	&c[0]

c[i][j] = &c[i][j]
c[i] = &c[i]
c = &c

array (abstract data)				Address of an array pointer
c[i][j]	pointer value	=	pointer address	&c[i][j]
c[i]	pointer value	=	pointer address	&c[i]
c	pointer value	=	pointer address	&c

c[0] = c[0][0] relation

int c [2][3][4];

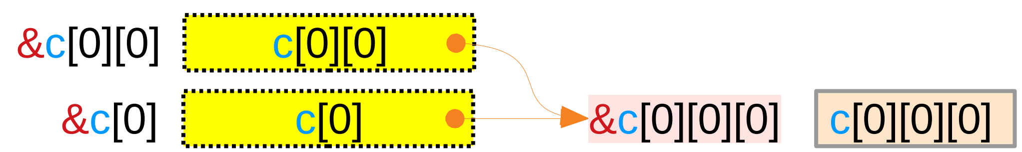
c ═ c[0] ═ c[0][0] ═ &c[0][0][0]

value(c[0]) = &c[0][0][0]
value(c[0][0]) = &c[0][0][0]

type(c[0]) = int (*)[4]
type(c[0][0]) = int [4]

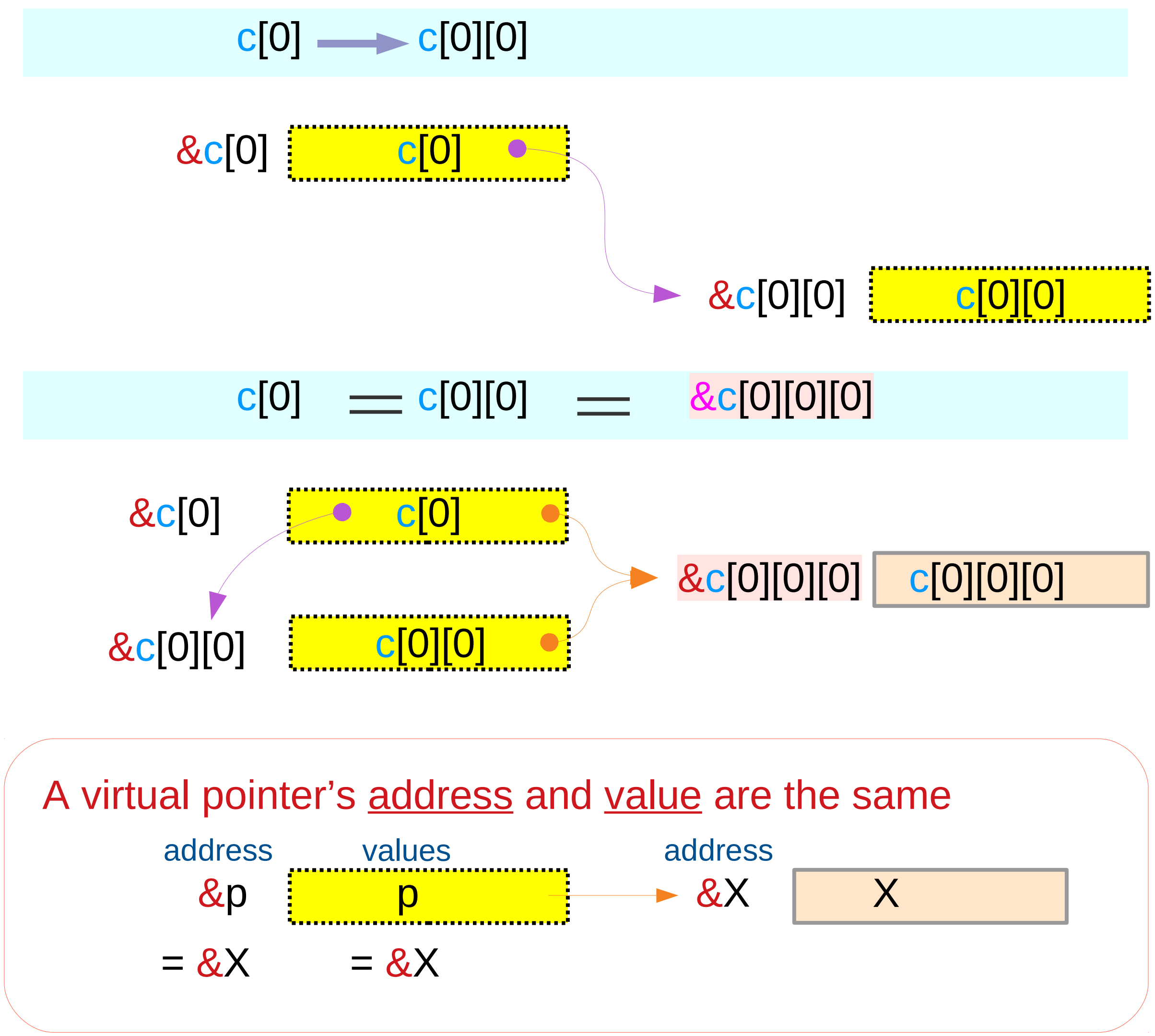
c[0] = c[0][0] *means*
value(c[0]) = value(c[0][0])

c[0] = c[0][0] *does **not** mean*
type(c[0]) = type(c[0][0])



Addresses and Values of **c[0]** and **c[0][0]**

```
int    c [2][3][4];
```

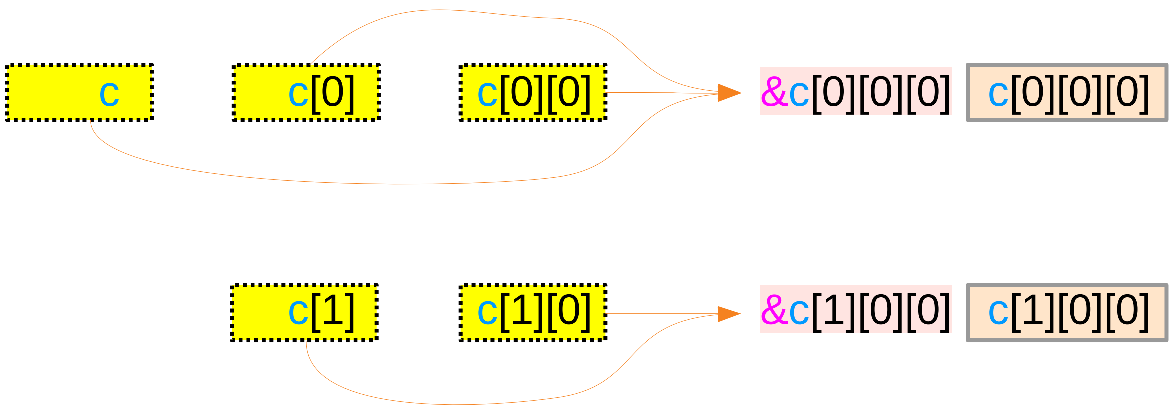


c[i] and **c[i][0]** point to the same **c[i][0][0]**

```
int     c [2][3][4];
```

c =	c[0] =	c[0][0] =	&c[0][0][0]	⟸ value
int(*)[3][4]	int(*)[4]	int(*)	int	⟸ type

	c[1] =	c[1][0] =	&c[1][0][0]	⟸ value
	int(*)[4]	int(*)	int	⟸ type



These virtual pointers have different types
but the same value (address)

&c[i][0] and &c[i][0][0] – equivalence relations

```
int    c [2][3][4];
```

int(*)[3][4]   int(*)[4]   int(*)   int *

c = c[0] = c[0][0] = &c[0][0][0]

&

&c   &c[0]   &c[0][0]

equivalences

c ≡ &c[0],
c[0] ≡ &c[0][0]
c[0][0] ≡ &c[0][0][0]

c[1] = c[1][0] = &c[1][0][0]

&

&c[1]   &c[1][0]

equivalences

c[1] ≡ &c[1][0]
c[1][0] ≡ &c[1][0][0]

Horizontal displacements are <u>not</u> counted
only **vertical displacements** are considered
for **<u>address values</u>**

equivalences

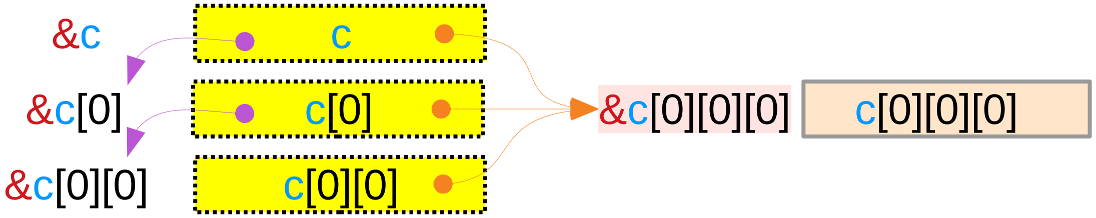
c ≡ &c[0],
c[i] ≡ &c[i][0]
c[i][0] ≡ &c[i][0][0]

c[i] = &c[i] and c[i][0] = &c[i][0]

```
int     c [2][3][4];
```

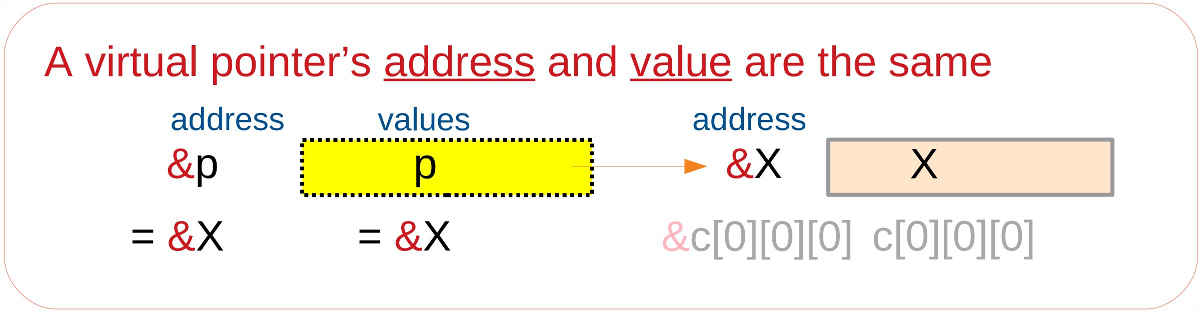
c = c[0] = c[0][0]= &c[0][0][0]
|| || ||
&c = &c[0] = &c[0][0]

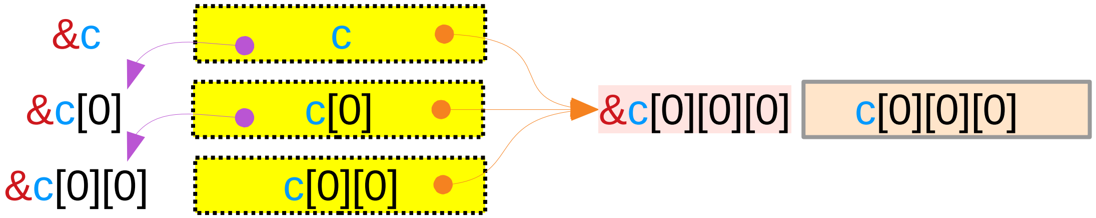
c[1] = c[1][0] = &c[1][0][0]
|| ||
&c[1] = &c[1][0]



c[i] = &c[i] and c[i][0] = &c[i][0]

```
int      c [2][3][4];
```





c[i]

Array Pointers to **c[i][0][0]**

&c[i][0][0] ≡ c[i][0]

&c[i][0] ≡ c[i]

&c[i] ≡ c+i

= c + i*sizeof(*c)

= &c[0][0][0] + i*3*4

virtual pointers:
the address of a pointer is
the same as its value

delete [0] from the right

&c[0][0][0]	$\overset{-[0]}{=}$	c[0][0]	$\overset{-[0]}{=}$	c[0]	$\overset{-[0]}{=}$	c
&c[1][0][0]	$\overset{-[0]}{=}$	c[1][0]	$\overset{-[0]}{=}$	c[1]		

Array Pointers to **c[i][j][0]**

&c[i][j][0] ≡ c[i][j]

||

&c[i][j] ≡ c[i] + j

= c[i] + j*sizeof(*c[i])

= c + i*sizeof(*c) + j*4

= &c[0][0][0] + i*3*4 + j*4

delete [0] from the right

&c[0][0][0]	$\overset{-[0]}{=}$	c[0][0]	$\overset{-[0]}{=}$	c[0]	$\overset{-[0]}{=}$	c
&c[0][1][0]	$\overset{-[0]}{=}$	c[0][1]				
&c[0][2][0]	$\overset{-[0]}{=}$	c[0][2]				
&c[1][0][0]	$\overset{-[0]}{=}$	c[1][0]	$\overset{-[0]}{=}$	c[1]		
&c[1][1][0]	$\overset{-[0]}{=}$	c[1][1]				
&c[1][2][0]	$\overset{-[0]}{=}$	c[1][2]				

Sub-array properties in multi-dimensional arrays

int c [2][3][4]; ➡ 3-d access c **[i][j][k]**

2-d array pointer	c	int (*) [3][4]
1-d array pointers	c[i]	int (*) [4]
0-d array pointers	c[i][j]	int (*)

Hierarchical sub-arrays in a 3-d array

```
int     c [L][M][N];
```

c [i][j][k]

left-to-right associativity

c	[i]	[j][k]
c[i]	[j]	[k]
c[i][j]	[k]	

Array Names and Types		Pointers to hierarchical sub-arrays	
c	3-d array names	int (*) [M][N]	2-d array pointer
c[i]	2-d array names	int (*) [N]	1-d array pointer
c[i][j]	1-d array names	int (*)	0-d array pointer

Associativity and Equivalence Relations

left-to-right associativity		left-to-right associativity
((c [i])[j])[k]	≡	*(*(*(c +i) +j) +k)

X[n]	≡	*(X+n)

given c[i][j]	c[i][j][k]	≡	*(c[i][j]+k)	for all k
given c[i]	c[i][j]	≡	*(c[i]+j)	for all j
given c	c[i]	≡	*(c+i)	for all i

Requirements for the expression **c[i][j][k]**

3 contiguity requirements

for a given c[i][j],	for all k	c[i][j][k] = *(c[i][j]+k)
for a given c[i],	for all j	c[i][j] = *(c[i]+j)
for a given c,	for all i	c[i] = *(c+i)

for a given c[i][j],	contiguous c[i][j][k]'s
for a given c[i],	contiguous c[i][j]'s
for a given c,	contiguous c[i]'s

for a given subarray pointer — contiguous subarrays

Equivalent requirements for the expression **c[i][j][k]**

for all k	c[i][j][k] = *(c[i][j]+k)	
for all j	c[i][j] = *(c[i]+j)	
for all i	c[i] = *(c+i)	

&c[i][j][k] = c[i][j]+k	for all k
&c[i][j] = c[i]+j	for all j
&c[i] = c+i	for all i

c[i][j][0] = *(c[i][j])
c[i][0] = *(c[i])
c[0] = *(c)

with contiguous subarrays

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

3-d access pattern **c[i][j][k]**

General requirements

c [i][j][k]

&c[i][j][k]	= c[i][j]+k	for all k
&c[i][j]	= c[i]+j	for all j
&c[i]	= c+i	for all i

Pointer array approach

```
int** c[2];
int*  b[2*3];
int   c[2*3*4];
```

c[i][j][k]	:: int
c[i][j]	:: int *
c[i]	:: int **
c	:: int ***

c[i]	⇐	&b[i*3]
b[j]	⇐	&a[j*4]

with contiguous a, b, c

Explicit
Arrays of pointers with
Multiple Indirection

N-dim Array approach

```
int c[2][3][4];
```

c[i][j][k]	:: int
c[i][j]	:: int [4]
c[i]	:: int [3][4]
c	:: int [2][3][4]

c[i][j]	⇐	&c[i][j][0]
c[i]	⇐	&c[i][0][0]
c	⇐	&c[0][0][0]

with contiguous c

Implicit
Nested
Virtual Array Pointers

3-d access pattern **c[i][j][k]** – array pointer approach

General requirements

c [i][j][k]

&c[i][j][k]	= c[i][j]+k	for all k
&c[i][j]	= c[i]+j	for all j
&c[i]	= c+i	for all i

N-dim array approach

```
int c[2][3][4];
```

c[i][j][k]	:: int
c[i][j]	:: int [4]
c[i]	:: int [3][4]
c	:: int [2][3][4]

c[i][j]	⟵	&c[i][j][0]
c[i]	⟵	&c[i][0][0]
c	⟵	&c[0][0][0]

with contiguous c

Implicit
Nested
Virtual Array Pointers

multi-dimensional arrays

virtual assignments			constraints		
c[i][j]	=	&c[i][j][0]	&c[i][j][0] = c[i][j]		for all k
c[i]	=	&c[i][0][0]	&c[i][0] = c[i]		for all j
c	=	&c[0][0][0]	&c[0] = c		for all i

→

virtual assignments			constraints		
c[i][j]	=	&c[i][j][0]	&c[i][j][0] = c[i][j]		for all k
c[i]	=	&c[i][0][0]	&c[i][0] = c[i]		for all j
c	=	&c[0][0][0]	&c[0] = c		for all i

←

General requirements for **c[i][j][k]**

int c [2][3][4];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k] = c[i][j]+k   for all k
&c[i][j] = c[i]+j   for all j
&c[i] = c+i   for all i

virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

Virtual assignments

int (*)	c[i][j]	=	(int (*))	&c[i][j][0]
int (*) [4]	c[i]	=	(int (*) [4])	&c[i][0][0]
int (*) [3][4]	c	=	(int (*) [3][4])	&c[0][0][0]

Pointer Types

Sizes of abstract data types

int [4]	c[i][j]	size =4*4
int [3][4]	c[i]	size =3*4*4
int [2][3][4]	c	size =2*3*4*4

Abstract Data Types

Strides of array elements

c[i][j][0]	stride =4*4
c[i][0][0]	stride =3*4*4
c[0][0][0]	stride =2*3*4*4

k=[0:3]	c[i][j][k]	4 integers
j=[0:2], k=[0:3]	c[i][j][k]	3*4 integers
I=[0:1], j=[0:2], k=[0:3]	c[i][j][k]	2*3*4 integers

General requirements for **c[i][j][k]**

```
int     c [2][3][4];
```

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k] = c[i][j]+k	for all k
&c[i][j] = c[i]+j	for all j
&c[i] = c+i	for all i

virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

Pointer Types			
c[i][j]	has an address of	c[i][j][0]	
c[i]	has an address of	c[i][0][0]	
c	has an address of	c[0][0][0]	

Pointer Types			
c[i][j]+1	has an address of	c[i][j][1]	1 integer away
c[i]+1	has an address of	c[i][1][0]	4 integers away
c+1	has an address of	c[1][0][0]	3*4 integers away

Pointer Types			
c[i][j]+k	has an address of	c[i][j][k]	1*k integers away
c[i]+j	has an address of	c[i][j][0]	4*j integers away
c+i	has an address of	c[i][0][0]	3*4*i integers away

General requirements for **c[i][j][k]**

int c [2][3][4];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k] = c[i][j]+k   for all k
&c[i][j] = c[i]+j   for all j
&c[i] = c+i   for all i

virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

c[i][j]	starts from	&c[i][j][0]
c[i]	starts from	&c[i][0][0]
c	starts from	&c[0][0][0]

Abstract Data

c[i][j]	has a size of	c[i][j][k]	4	integers
c[i]	has a size of	c[i][j][k]	3*4	integers
c	has a size of	c[i][j][k]	2*3*4	integers

Abstract Data

General requirements for **c[i][j][k]**

int c [2][3][4];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k] = c[i][j]+k for all k
&c[i][j] = c[i]+j for all j
&c[i] = c+i for all i

virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

c[i][j]	starts from	&c[i][j][0]
c[i]	starts from	&c[i][0][0]
c	starts from	&c[0][0][0]

Abstract Data

c[i]	and	c[i][0]	start from	&c[i][0][0]
c	and	c[0]	start from	&c[0][0][0]

Abstract Data　　Abstract Data

c[i]	starts from	&c[i][0]
c	starts from	&c[0]

Abstract Data　　Abstract Data

General requirements for **c[i][j][k]**

int c [2][3][4];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k] = c[i][j]+k   for all k
&c[i][j] = c[i]+j   for all j
&c[i] = c+i   for all i

virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

c[i] and c[i][0] start from &c[i][0][0]
c and c[0] start from &c[0][0][0]

Abstract Data   Abstract Data

c[i] → c[i][0]

c[i] has a size of 3*4 integers

c[i][0] has a size of 4 integers
c[i][1] has a size of 4 integers
c[i][2] has a size of 4 integers

c[i] is the name of an array which has 3 elements
c[i][0] c[i][1] c[i][2]

points to c[i][0]

&c[i][0] = c[i]

General requirements for **c[i][j][k]**

```
int     c [2][3][4];
```

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k] = c[i][j]+k	for all k
&c[i][j] = c[i]+j	for all j
&c[i] = c+i	for all i

virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

Each	c[i][j]	sub-array contains	4 integers
Each	c[i]	sub-array contains	3*4 integers
Each	c	sub-array contains	2*3*4 integers

Abstract Data Types

Each	c[i][j]+1	starts at	c[i][j][1]	1 integer away
Each	c[i]+1	starts at	c[i][1][0]	4 integers away
Each	c+1	starts at	c[1][0][0]	3*4 integers away

Pointer Types

Each	c[i][j]+k	starts at	c[i][j][k]	1*k integers away
Each	c[i]+j	starts at	c[i][j][0]	4*j integers away
Each	c+i	starts at	c[i][0][0]	3*4*i integers away

Pointer Types

multi-dimensional arrays

assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

constraints

&c[i][j][0] = c[i][j]	for all k
&c[i][0] = c[i]	for all j
&c[0] = c	for all i

assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

constraints

&c[i][j][0] = c[i][j]	for all k
&c[i][0] = c[i]	for all j
&c[0] = c	for all i

Subarray starting addresses

int c [2][3][4];

Virtual array pointer

&c[i][j][0]	=	c[i][j]
&c[i][0]	=	c[i]
&c[0]	=	c

c[i][j]	points to	c[i][j][0]
c[i]	points to	c[i][0]
c	points to	c[0]

Abstract data (array)

&c[i][j][0]	=	&c[i][j]
&c[i][0]	=	&c[i]
&c[0]	=	&c

c[i][j]	starts from	&c[i][j][0]
c[i]	starts from	&c[i][0]
c	starts from	&c[0]

c →	c[0] →	c[0][0] →	c[0][0][0]
		c[0][1] →	c[0][1][0]
		c[0][2] →	c[0][2][0]
	c[1] →	c[1][0] →	c[1][0][0]
		c[1][1] →	c[1][1][0]
		c[1][2] →	c[1][2][0]

c[i][j]	=	&c[i][j][0]
c[0][0]	═	&c[0][0][0]
c[0][1]	═	&c[0][1][0]
c[0][2]	═	&c[0][2][0]
c[1][0]	═	&c[1][0][0]
c[1][1]	═	&c[1][1][0]
c[1][2]	═	&c[1][2][0]

c[i]	=	&c[i][0]		
c[0]	═	&c[0][0]	═	&c[0][0][0]
c[1]	═	&c[1][0]	═	&c[1][0][0]

c	=	&c[0]				
c	═	&c[0]	═	&c[0][0]	═	&c[0][0][0]

General requirements for **c[i][j][k]**

```
int     c [2][3][4];
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

with contiguous subarrays

&c[i][j][k] = c[i][j]+k	for all k
&c[i][j] = c[i]+j	for all j
&c[i] = c+i	for all i

virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

Contiguity Constraints

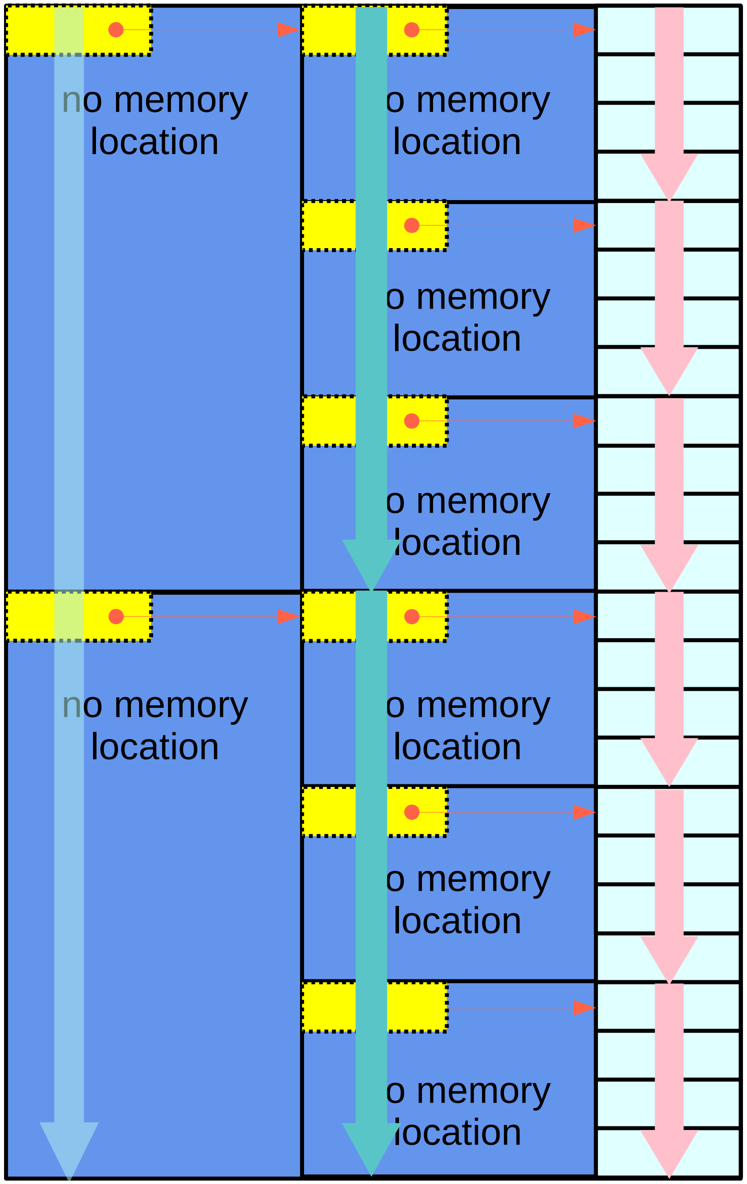
c **[i][j][k]**;

Virtual Array Pointers and Contiguity

Using array pointers

int (*) [N]	int (*) [M][N]	int (*) [L][M][N]	Array Pointer
↓	↓	↓	
int [N]	int [M][N]	int [L][M][N]	Array Name
int (*)	int (*) [N]	int (*)[M][N]	Array Pointer

Array pointer approach – contiguity constraints



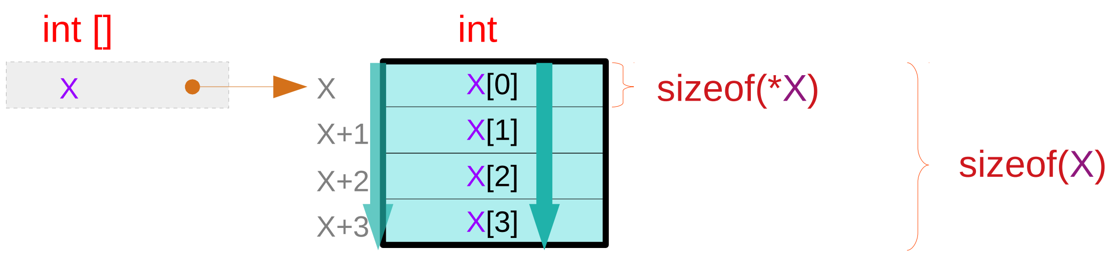
Array Pointer Approach
(pointer to arrays)

Equivalence and contiguity (1)

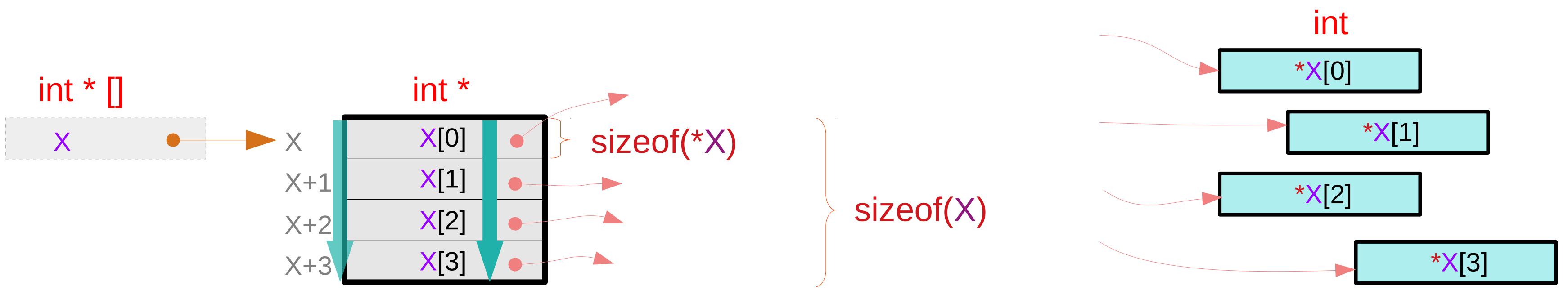
consecutive address　　　consecutive data

*(X+n) ≡ X[n]

contiguous index : n



int X[4]; contiguous X[i] for a given X : **primitive types**



int * X[4]; contiguous X[i] for a given X : **pointer types**

Equivalence and contiguity (2)

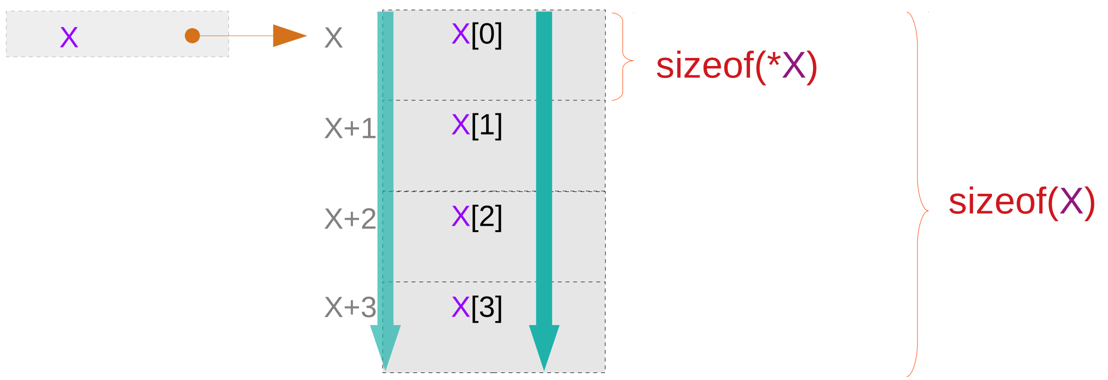
consecutive address

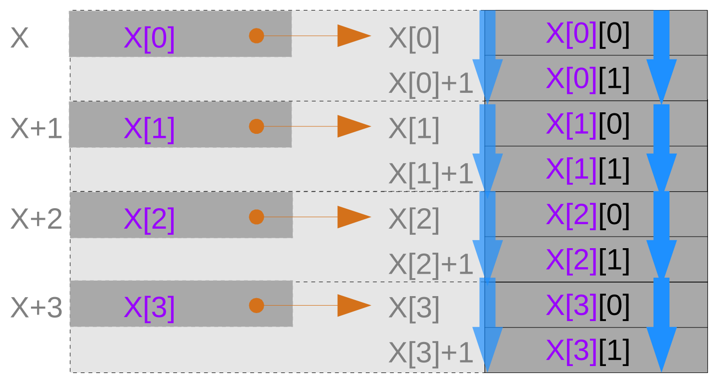
consecutive data

*(X+n) ≡ X[n]

contiguous index : n

can be recursively applied





atype * X[4]; contiguous X[i] for a given X : **abstract data types**

Recursive applications of equivalences

By definition, contiguous memory locations are assumed

consecutive address		consecutive data
*(X+n)	≡	X[n]

contiguous index : n

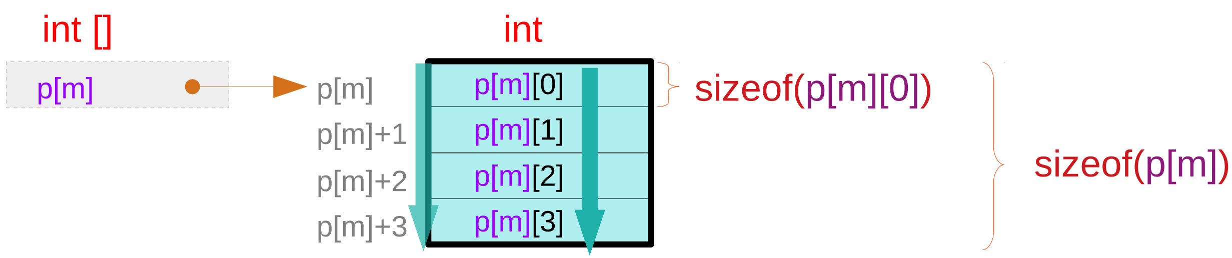
*(p[m]+n)	⟷	p[m][n]	X = p[m] contiguous index : **n**
(*(p+m))[n];	⟷	p[m][n];	X = p contiguous index : **m**

Equivalence for a given p[m] (1)

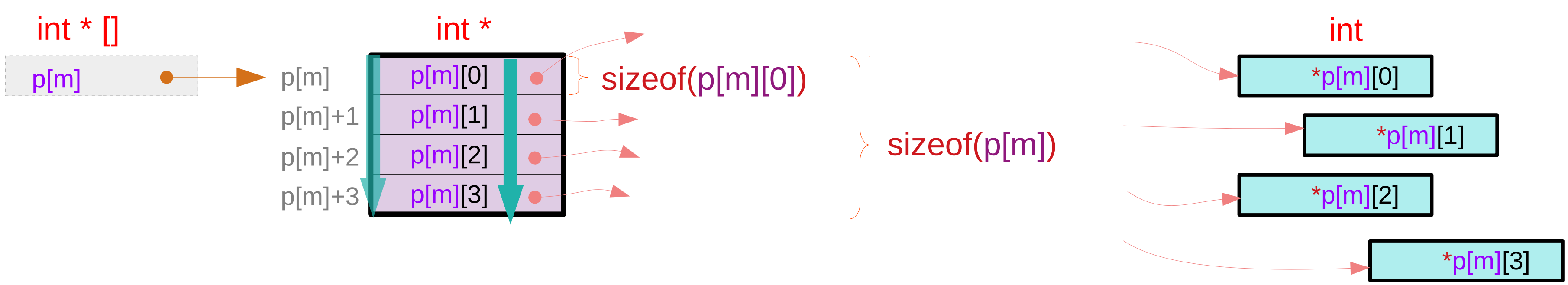
*(p[m]+n) ⟺ p[m][n]

for a given p[m]   contiguous index : **n**

int p[M][4]; contiguous p[m][n] for a given p[m] : **primitive types**   m = 0, 1, …, M-1



int * p[M][4]; contiguous p[m][n] for a given p[m] : **pointer types**   m = 0, 1, …, M-1



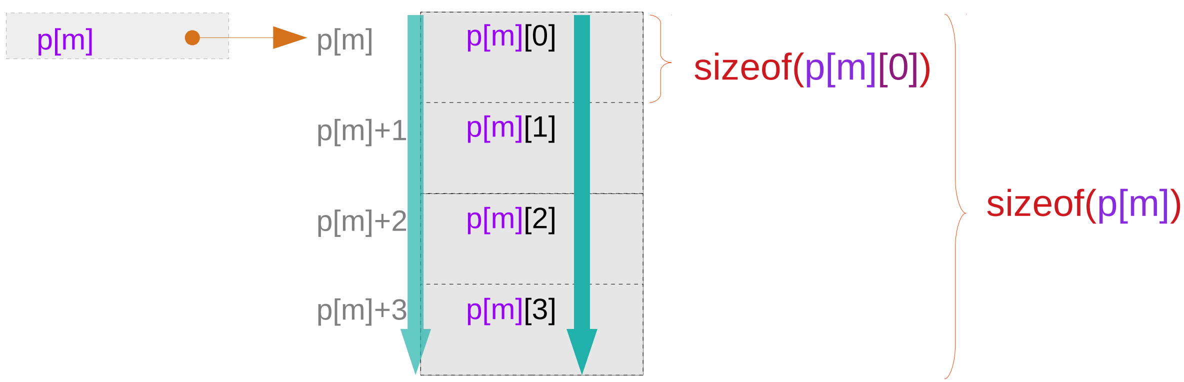
Equivalence for a given p[m] (2)

*(p[m]+n) ⟺ p[m][n]

for a given p[m]

contiguous index : **n**

atype * p[M][4]; contiguous p[m][n] for a given p[m] : **abstract data types** m = 0, 1, …, M-1



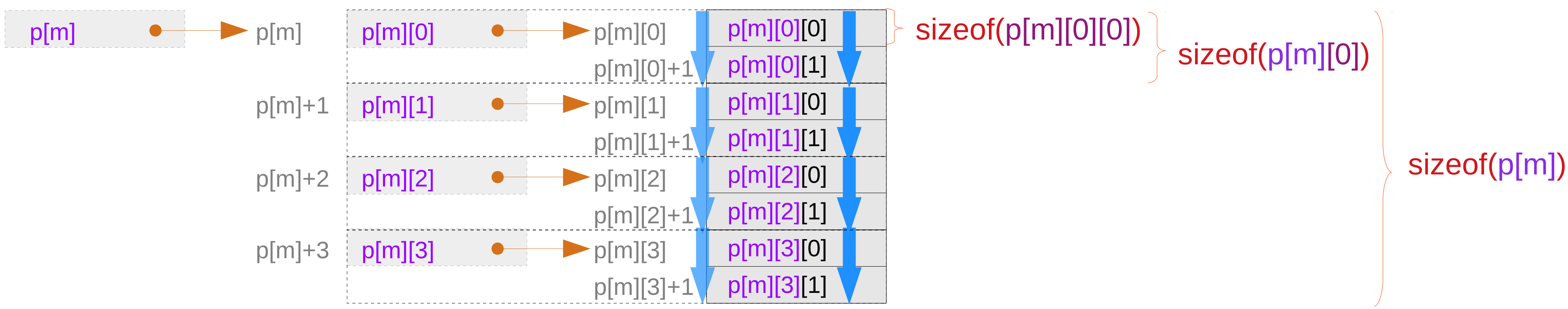
can be recursively applied

Equivalence for a given p[m][n]

*(p[m][n]+k) ⟺ p[m][n][k]

for a given p[m][n] contiguous index : **k**

atype * p[M][4][2]; contiguous p[m][n][k] for a given p[m][n] : **abstract data types** m = 0, 1, …, M-1

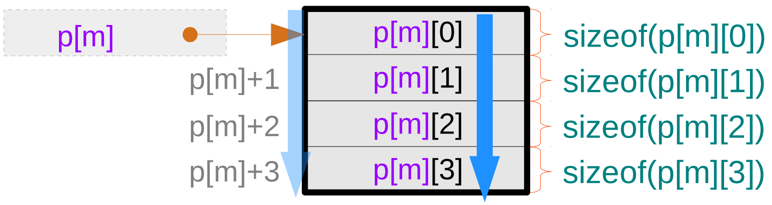


Contiguity constraints in multi-dimensional arrays

*(p[m]+n) ⟺ p[m][n]

for a given p[m], thus for a given p and m,
p[m][n]'s must be contiguous for all n.
p[m][0], p[m][1], … , p[m][N-1]

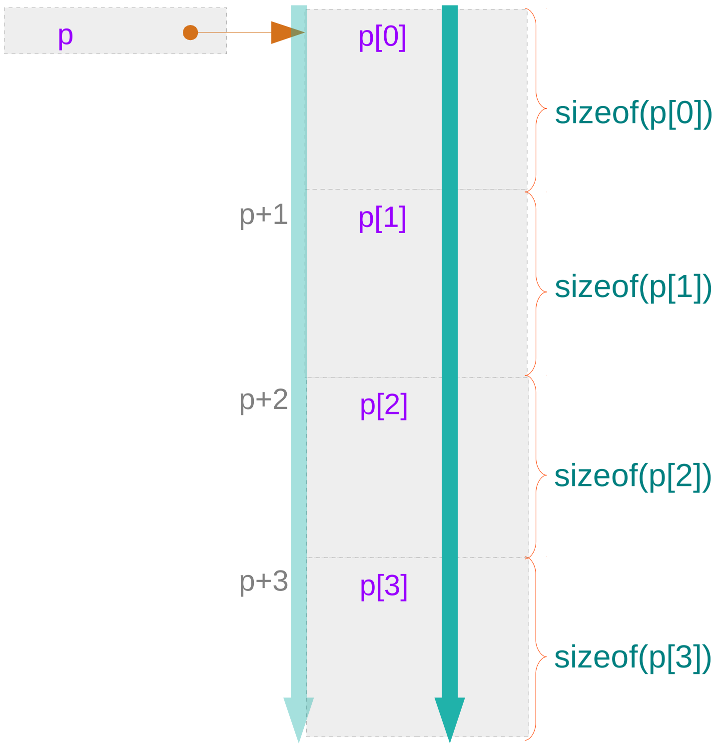
contiguous index : n



*(p+m) ⟺ p[m]

for a given p,
p[m]'s must be contiguous for all m.
p[0], p[1], … , p[M-1]

contiguous index : m



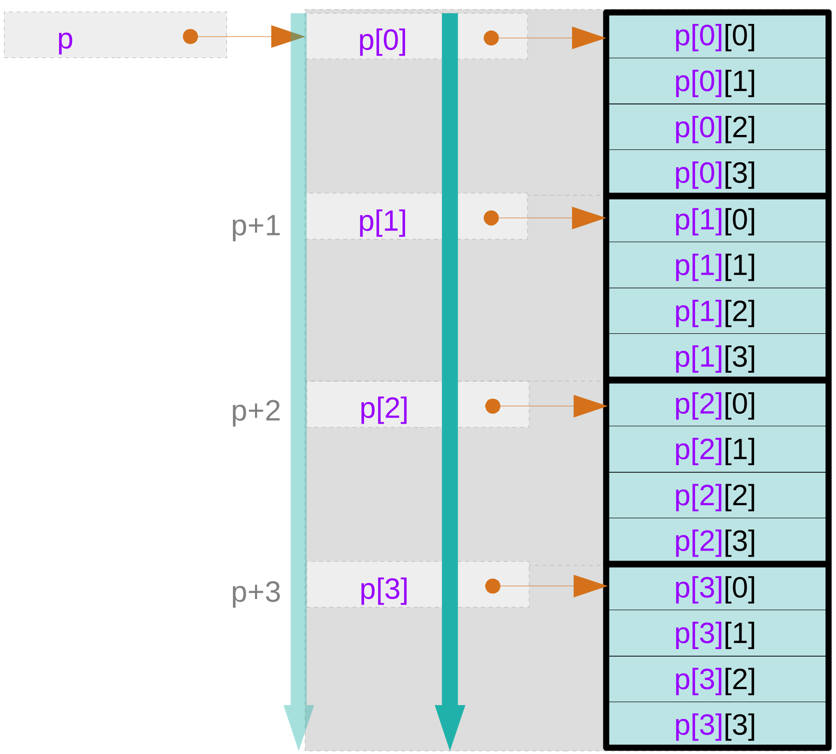
Contiguity constraints for p

$*(p+m) \Longleftrightarrow p[m]$

for a given p

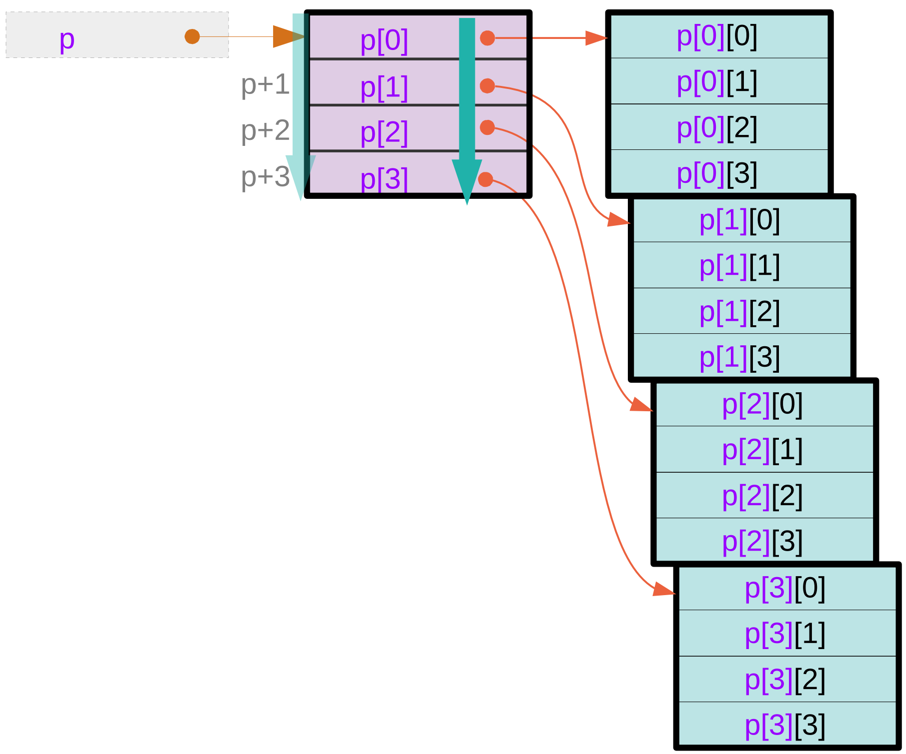
contiguous index : **m**

2-d array name **1-d** array names



contiguous p[m] ➡ contiguous p[m][n]

1-d array of pointers



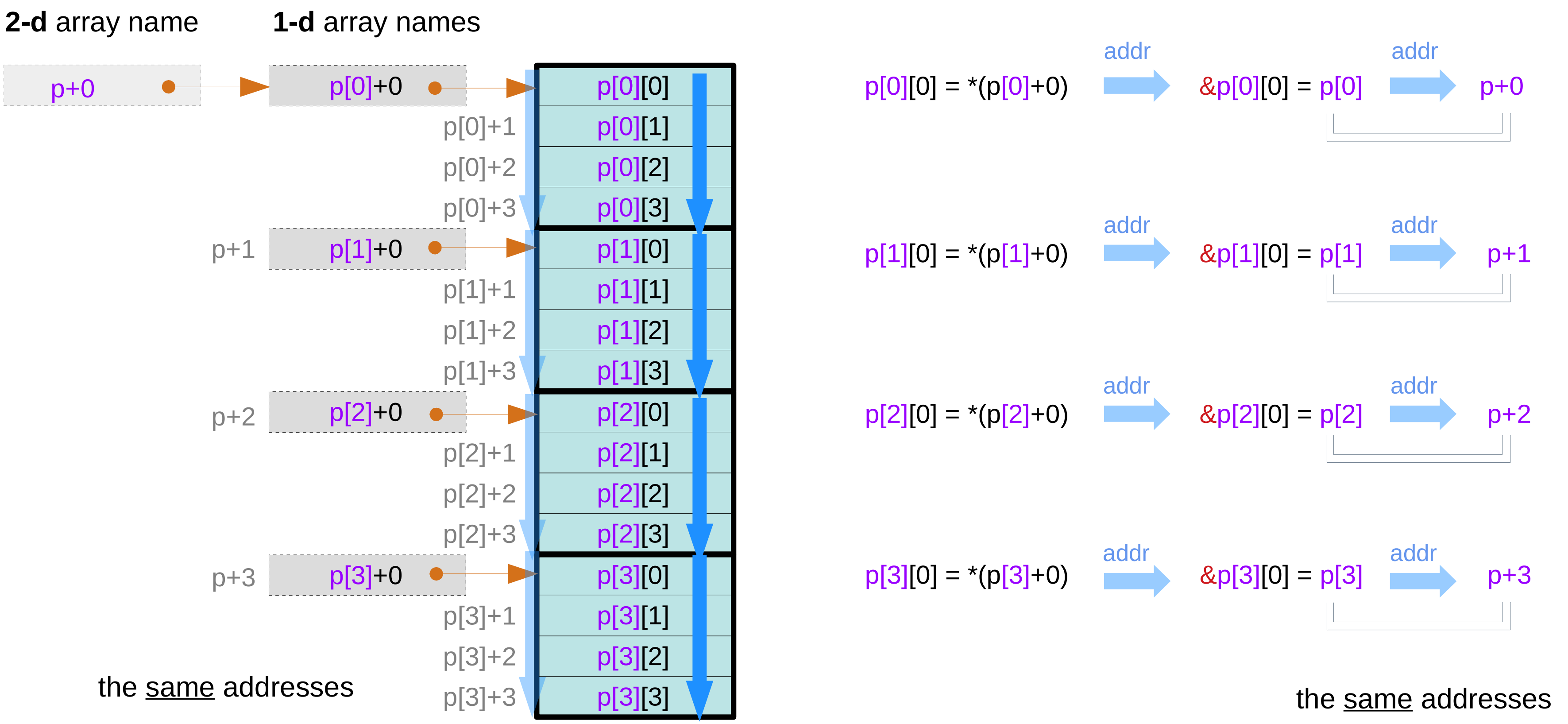
contiguous p[m] ➡ contiguous p[m][n]

Not necessarily

Contiguity constraints for p[m] – using array pointers

*(p[m]+n) ⟺ p[m][n]

for a given p[m]   contiguous index : **n**



contiguous p[m] ➡ contiguous p[m][n]

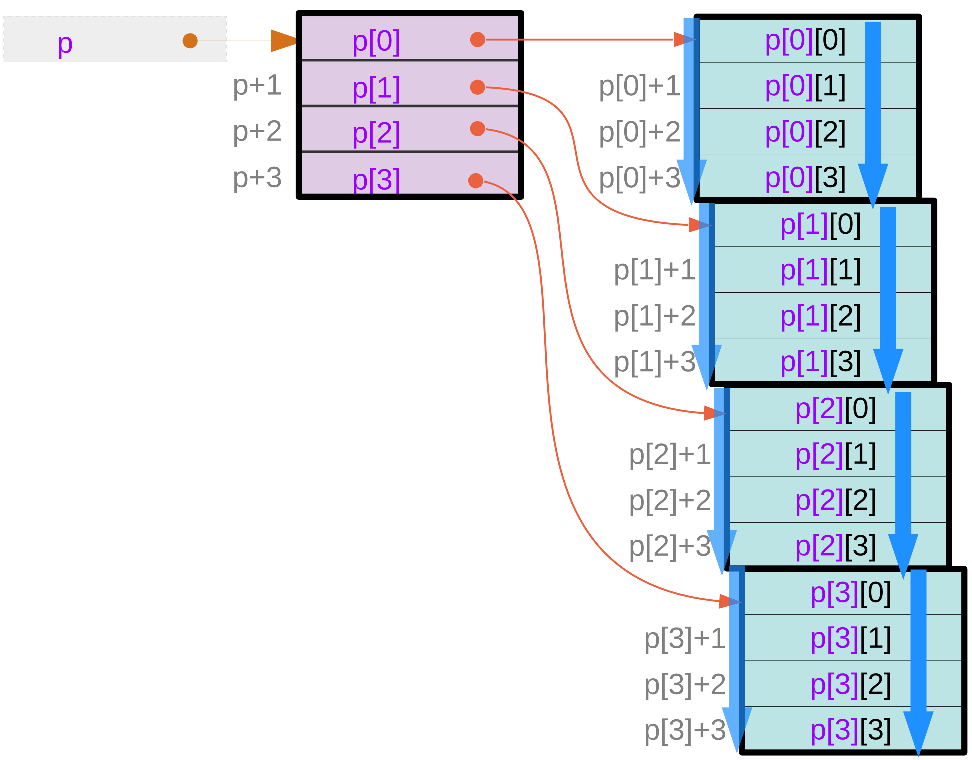
virtual array pointer ⟺ no real memory locations

Contiguity constraints for p[m] – using pointer arrays

*(p[m]+n) ⟺ p[m][n]

for a given p[m] contiguous index : **n**

1-d array of pointers



data: p[0][0] = *(p[0]+0) → addr: &p[0][0] = p[0] → addr: p+0

data: p[1][0] = *(p[1]+0) → addr: &p[1][0] = p[1] → addr: p+1

data: p[2][0] = *(p[2]+0) → addr: &p[2][0] = p[2] → addr: p+2

data: p[3][0] = *(p[3]+0) → addr: &p[3][0] = p[3] → addr: p+3

the different addresses

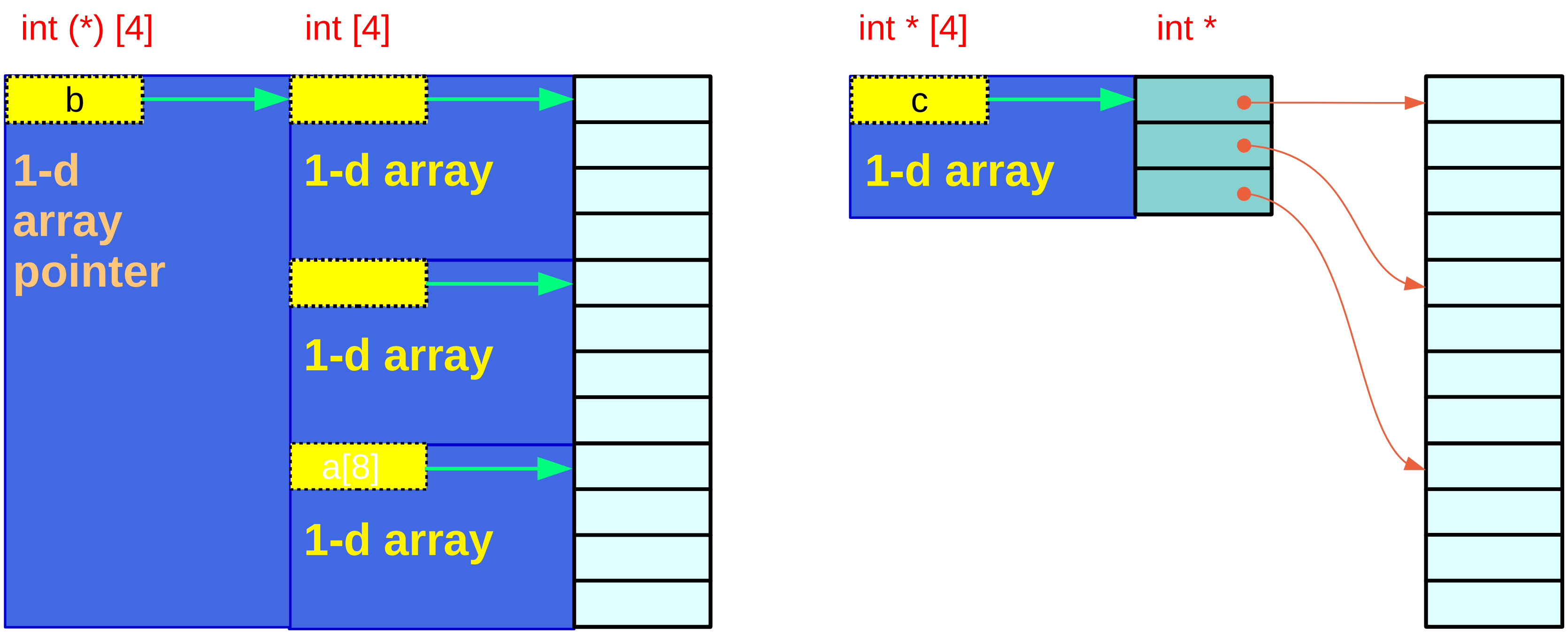
contiguous p[m] ➡ contiguous p[m][n]

Not necessarily

Contiguity constraints for 2-d arrays

int a[M][N] ;	int (*b)[N] ;	int * c[M] ;
*(a+m) ⟺ a[m]	*(b+m) ⟺ b[m]	*(c+m) ⟺ c[m]
a[0], a[1], … , a[M-1] are contiguous	b[0], b[1], … , b[M-1] are contiguous	c[0], c[1], … , c[M-1] are contiguous
*(a[m]+n) ⟺ a[m][n]	*(b[m]+n) ⟺ b[m][n]	*(c[m]+n) ⟺ c[m][n]
a[m][0], a[m][1], … , a[m][N-1] are contiguous	b[m][0], b[m][1], … , b[m][N-1] are contiguous	c[m][0], c[m][1], … , c[m][N-1] are contiguous
		a set of assignments of pointers are necessary for this contiguity

Pointer Arrays vs Array Pointers



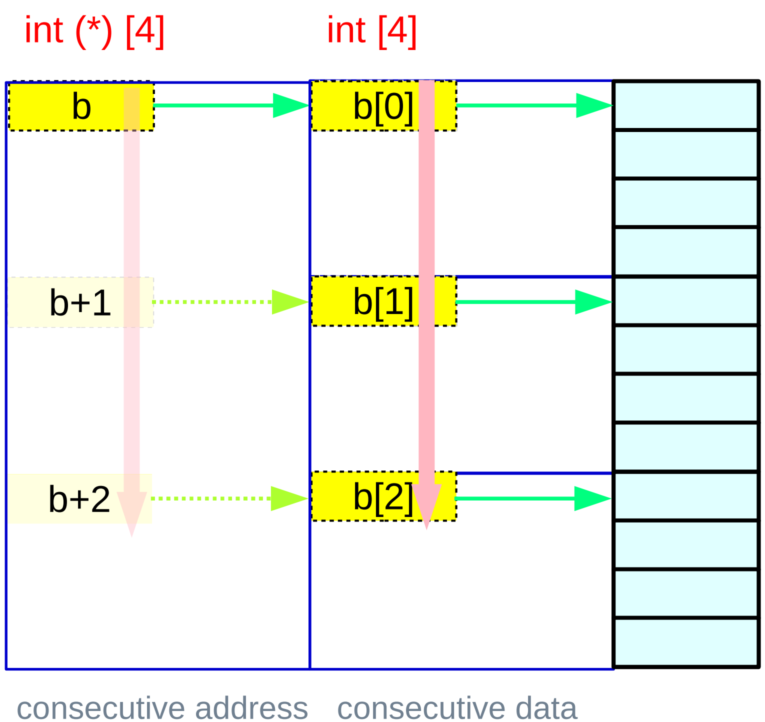
int (*b)[N] ;

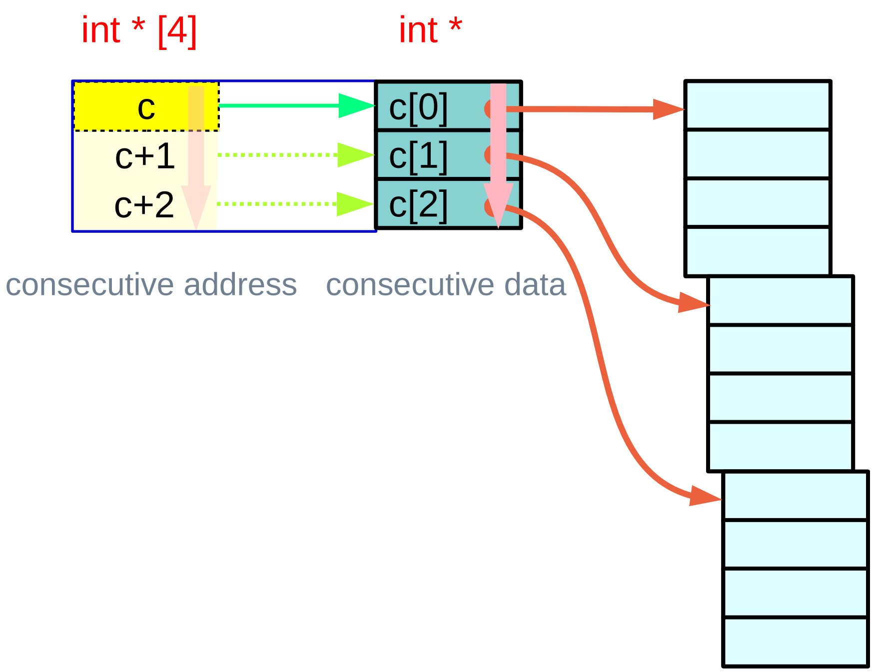
int * c[M] ; with proper assignments

*(b+m)	⟺	b[m]
*(b[m]+n)	⟺	b[m][n]

*(c+m)	⟺	c[m] or
*(c[m]+n)	⟺	c[m][n]

Pointer Arrays vs Array Pointers





int (*b)[N] ;

int * c[M] ; with proper assignments

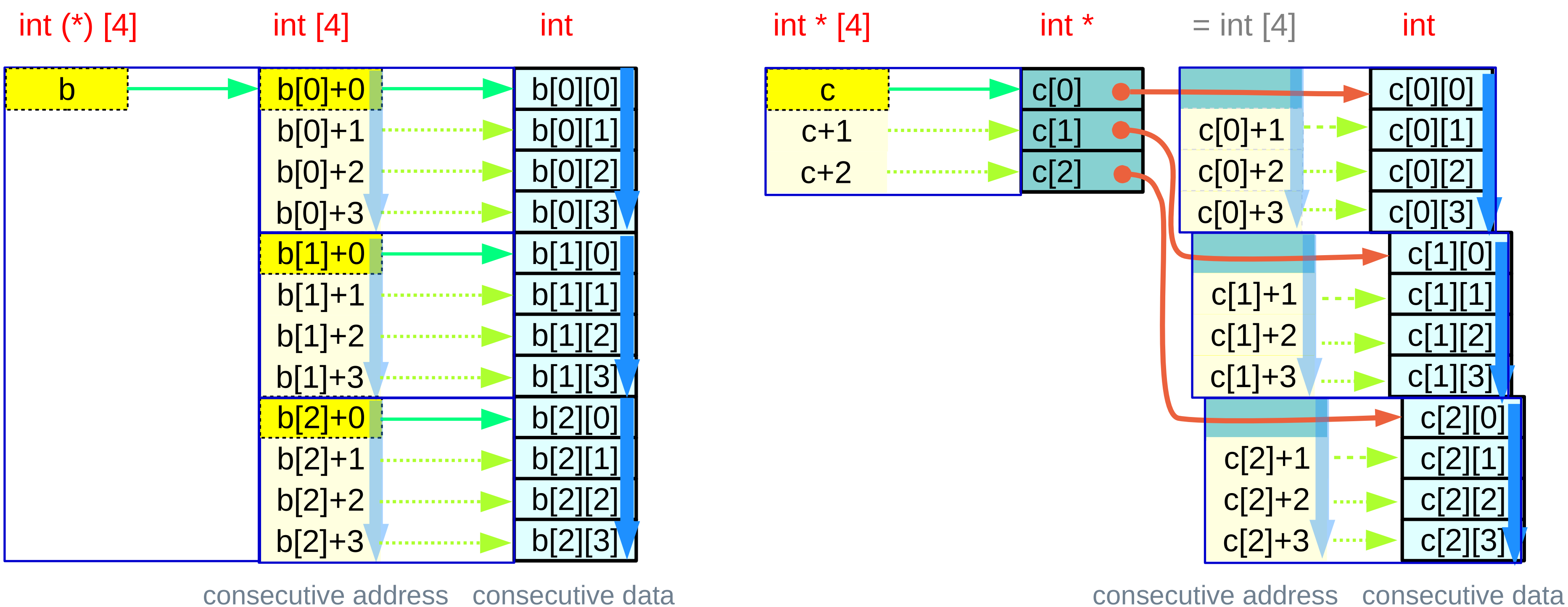
*(b+m) ⟺ b[m]

*(b[m]+n) ⟺ b[m][n]

*(c+m) ⟺ c[m] or

*(c[m]+n) ⟺ c[m][n]

Pointer Arrays vs Array Pointers



int (*b)[N] ;

int * c[M] ; with proper assignments

*(b+m) ⟺ b[m]	*(c+m) ⟺ c[m]
*(b[m]+n) ⟺ b[m][n]	*(c[m]+n) ⟺ c[m][n]

Three contiguity constraints for 3-d arrays

Pointer Array Approach (array of pointers)

c[i][j][k]	➡	*(c[i][j] +k)
*(c[i][j] +k)	➡	*(*(c[i] +j) +k)
((c[i] +j) +k)	➡	*(*(*(c +i) +j) +k)

contiguous **int**	int
contiguous pointers to **int**	int *
contiguous double pointers to **int**	int **

the contiguity constraints are satisfied by allocating arrays of pointers

Array Pointer Approach (pointer to arrays)

c[i][j][k]	➡	*(c[i][j] +k)
*(c[i][j] +k)	➡	*(*(c[i] +j) +k)
((c[i] +j) +k)	➡	*(*(*(c +i) +j) +k)

contiguous **0-d** arrays	int	int
contiguous **1-d** arrays	int [4]	int *
contiguous **2-d** arrays	int [3][4]	int (*) [4]

The contiguity constraints are satisfied by row major ordered linear data layout

Contiguous array pointers c[i][j][k] ≡ *(c[i][j] +k)

c[0][0][0] = *(c[0][0] + 0)
c[0][0][1] = *(c[0][0] + 1)
c[0][0][2] = *(c[0][0] + 2)
c[0][0][3] = *(c[0][0] + 3)
c[0][1][0] = *(c[0][1] + 0)
c[0][1][1] = *(c[0][1] + 1)
c[0][1][2] = *(c[0][1] + 2)
c[0][1][3] = *(c[0][1] + 3)

⋮ ⋮

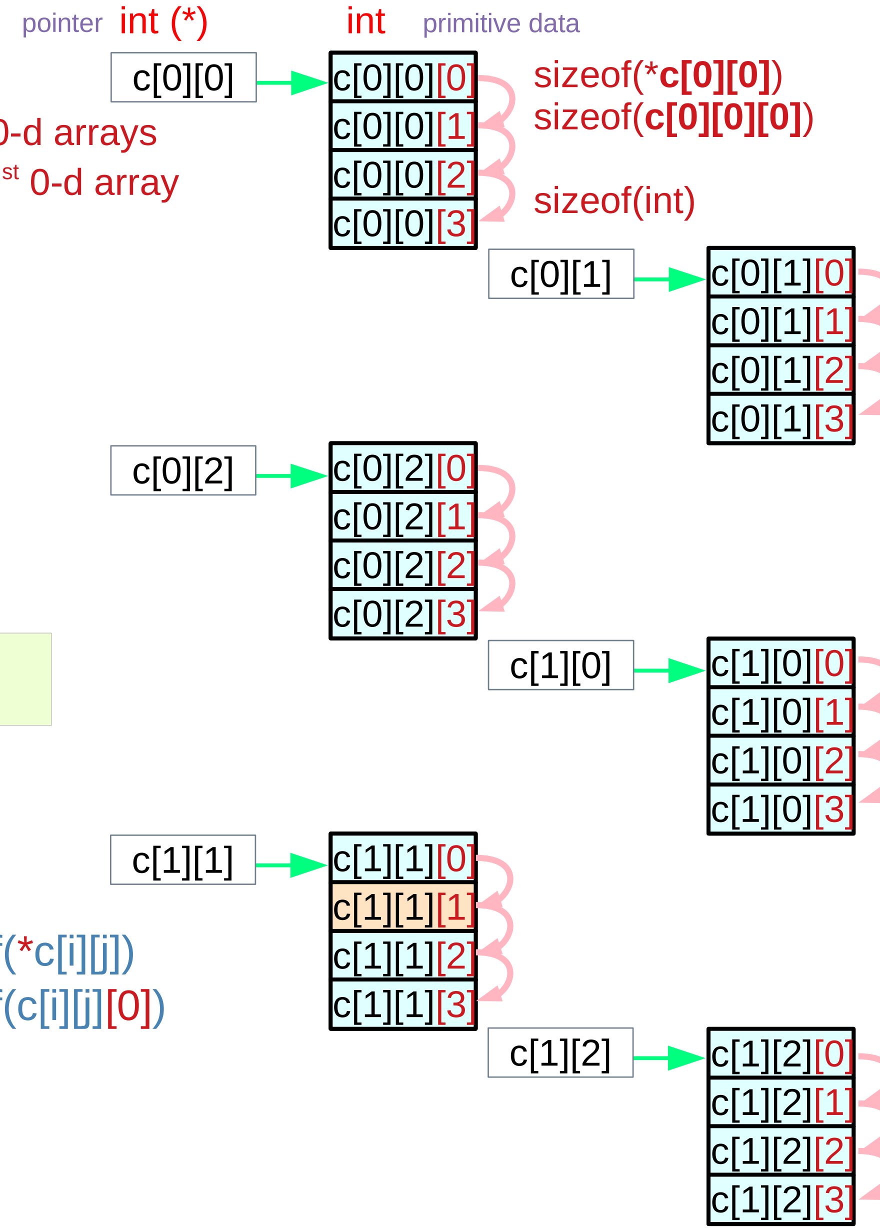
contiguous 1-d array elements

c[i][j]	
int [4]	**4** contiguous 0-d arrays
int *	points to the 1st 0-d array
int	0-d array

sizeof(**c[i][j]**) [k]
sizeof(**c[i][j][k]**) * 4
sizeof(**int**) * 4

int c[2][3][4];

Address Value
c[i][j] + k
&c[i][j][0] + k * sizeof(*c[i][j])
&c[i][j][0] + k * sizeof(c[i][j][0])
&c[i][j][0] + k * 4



Contiguous array pointers c[i][j] ≡ *(c[i] +j)

c[0][0] = *(c[0] + 0)
c[0][1] = *(c[0] + 1)
c[0][2] = *(c[0] + 2)
c[1][0] = *(c[1] + 0)
c[1][1] = *(c[2] + 1)
c[1][2] = *(c[3] + 2)

c[i]

int [3][4] **3** contiguous 1-d arrays

int (*) [4] points to the 1st 1-d array

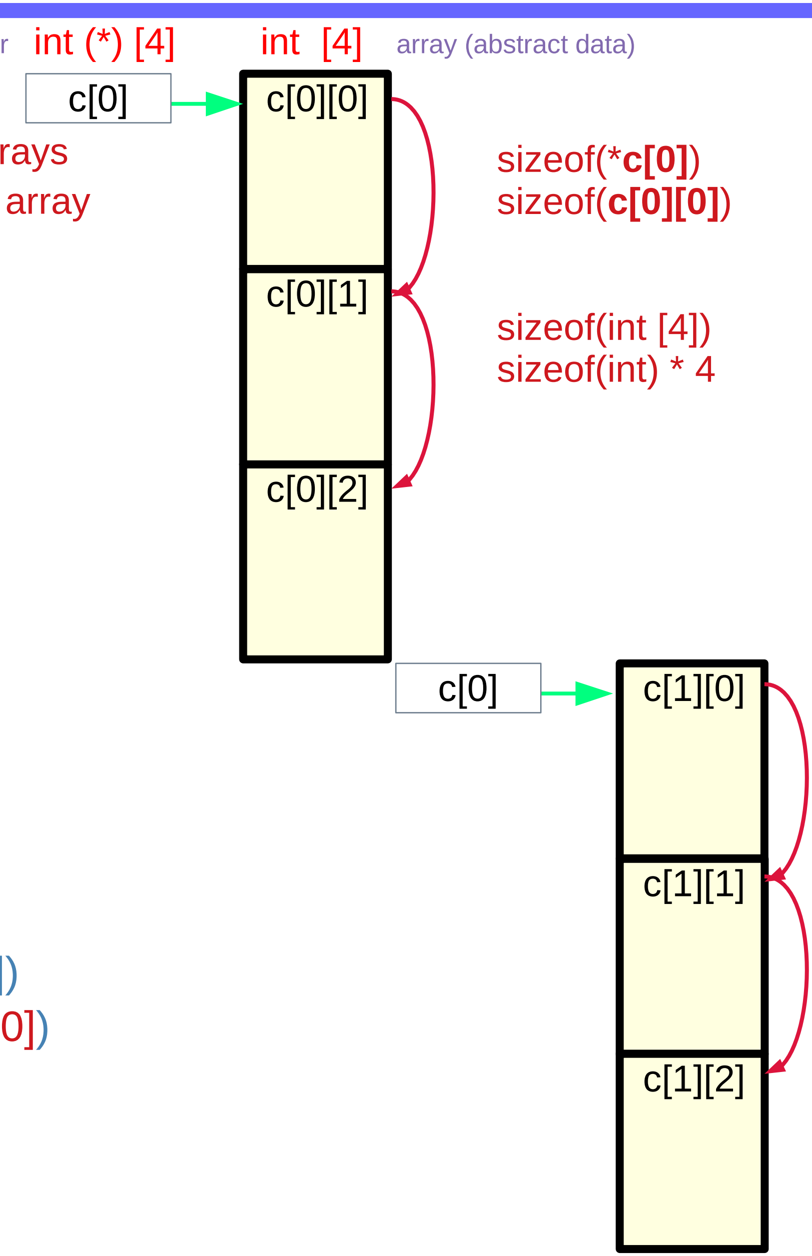
int [4] 1-d array

sizeof(**c[i]**) [j] [k]

sizeof(**c[i][j][k]**) * 3 * 4

sizeof(**int**) * 3 * 4

array pointer



int c[2][3][4];

Address Value

c[i] + j

&c[i][0][0] + j * sizeof(*c[i])

&c[i][0][0] + j * sizeof(c[i][0])

&c[i][0][0] + j * 4 * 4

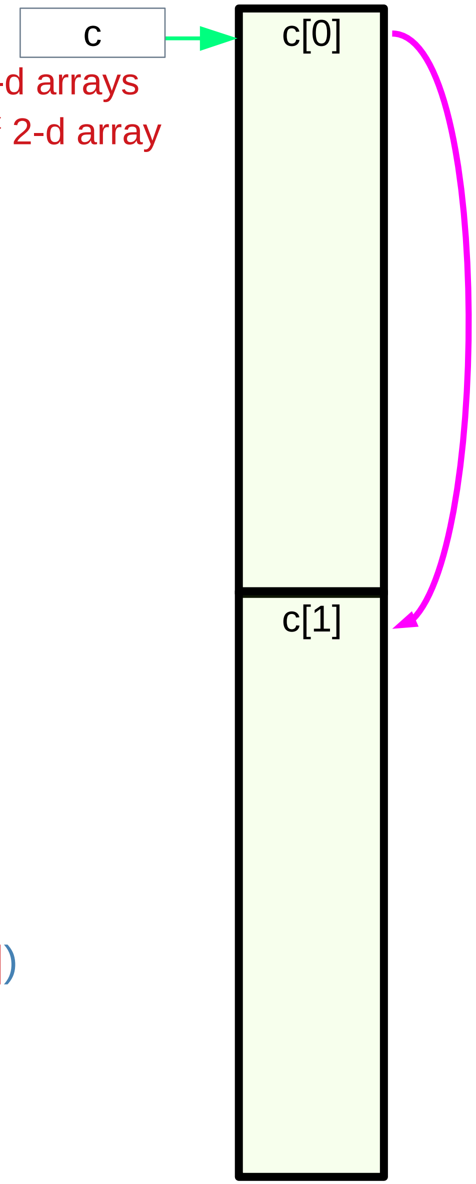
Contiguous array pointers c[i] ≡ *(c +i)

c[0] = *(c + 0)
c[1] = *(c + 1)

c
int [2][3][4] — **2** contiguous 2-d arrays
Int (*) [3][4] — points to the 1st 2-d array
int [3][4] — 2-d array

sizeof(**c**)
sizeof(**c[i][j][k]**) * 2 * 3 * 4 ([i] [j] [k])
sizeof(**int**) * 2 * 3 * 4

array pointer int (*) [3][4]   int [3][4] array (abstract data)



sizeof(*c)
sizeof(**c[0]**)

sizeof(int [3][4])
sizeof(int) * 3 * 4

```
int c[2][3][4];
```

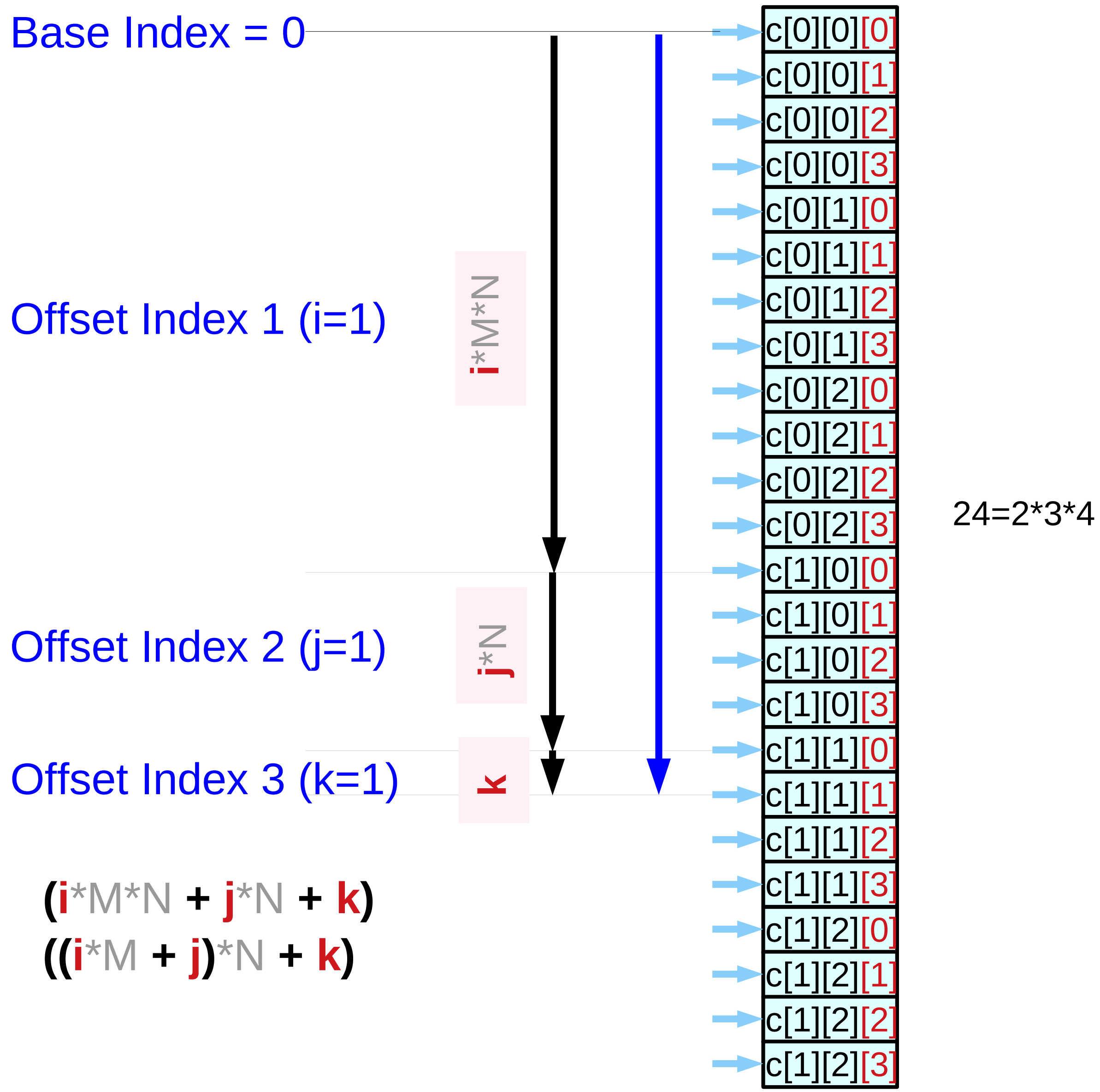
Address Value
c + i
&c[0][0][0] + i * sizeof(*c)
&c[0][0][0] + i * sizeof(c[0])
&c[0][0][0] + i * 4 * 3 * 4

Contiguous linear layout

```
int     c [L][M][N];

        c [i][j][k];
```

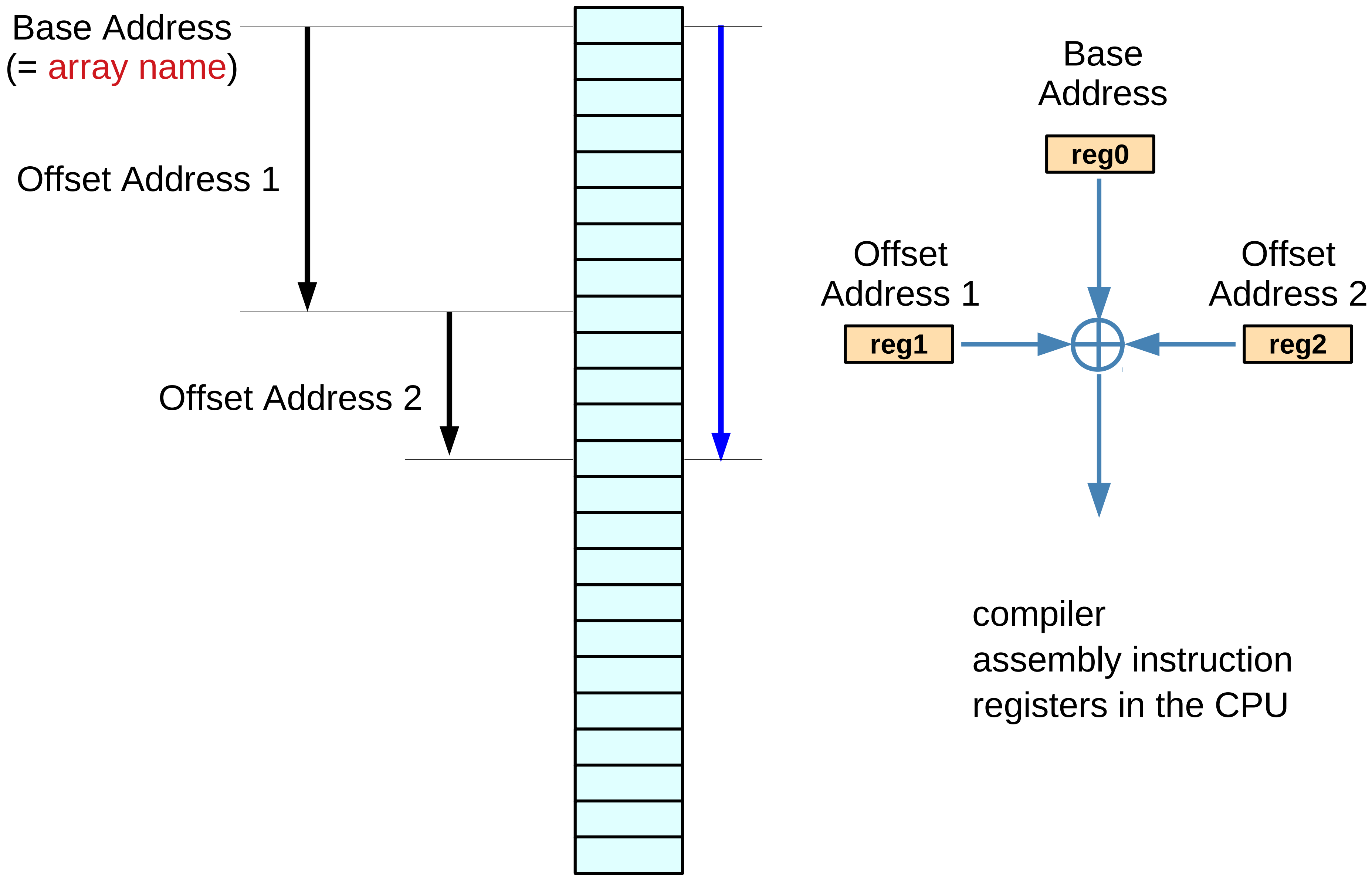
L	M	N
i	j	k
i*M*N	j*N	k



(i*M*N + j*N + k)

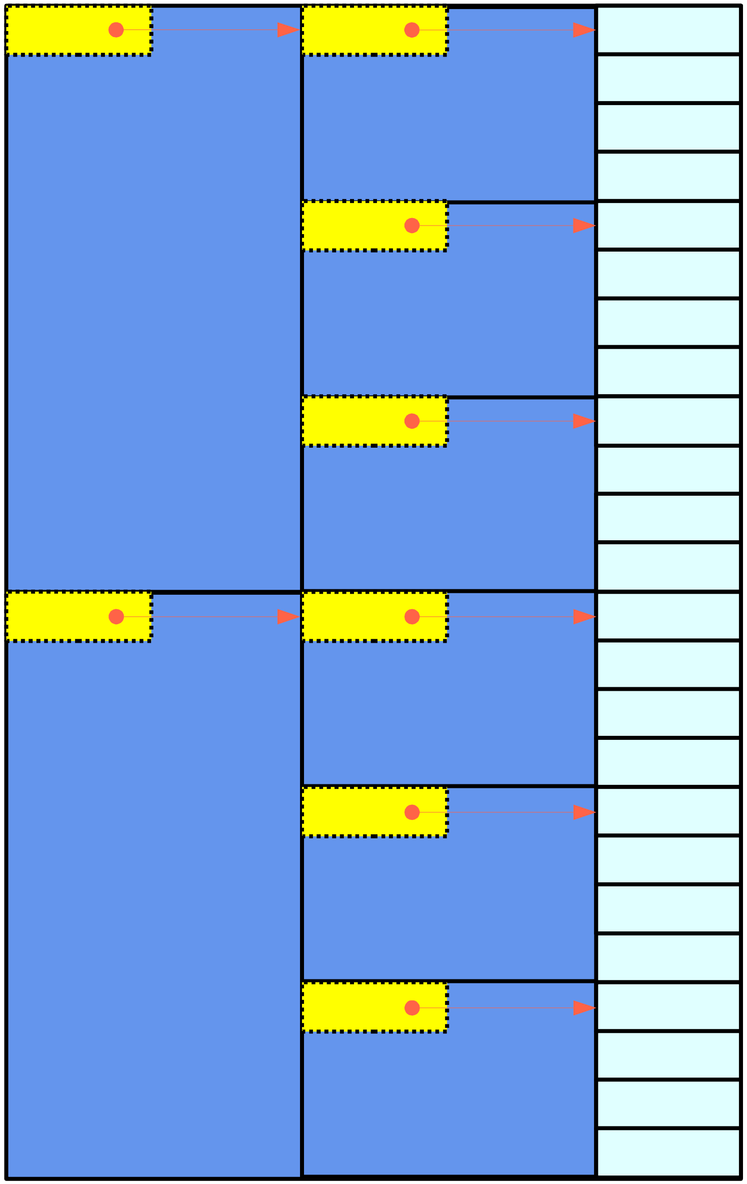
((i*M + j)*N + k)

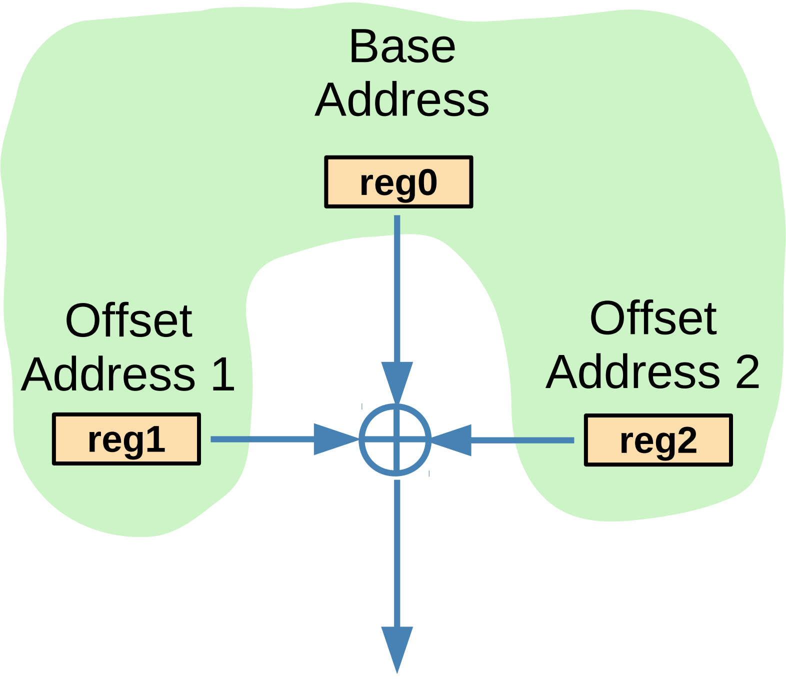
Base and Offset Addressing



compiler
assembly instruction
registers in the CPU

Array Pointer Approach





register based address **computations**
eliminate the pointer arrays – by a compiler

Array Pointer Approach
(pointer to arrays)

References

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[2] Efficient C Programming, Mark A. Weiss
[3] C A Reference Manual, Samuel P. Harbison & Guy L. Steele Jr.
[4] C Language Express, I. K. Chun