Fourier Analysis Overview (0B)

- CTFS: Continuous Fourier Series
- CTFT: Continuous Time Fourier Transform
- DTFT: Discrete Time Fourier Transform
- DFT: Discrete Fourier Transform

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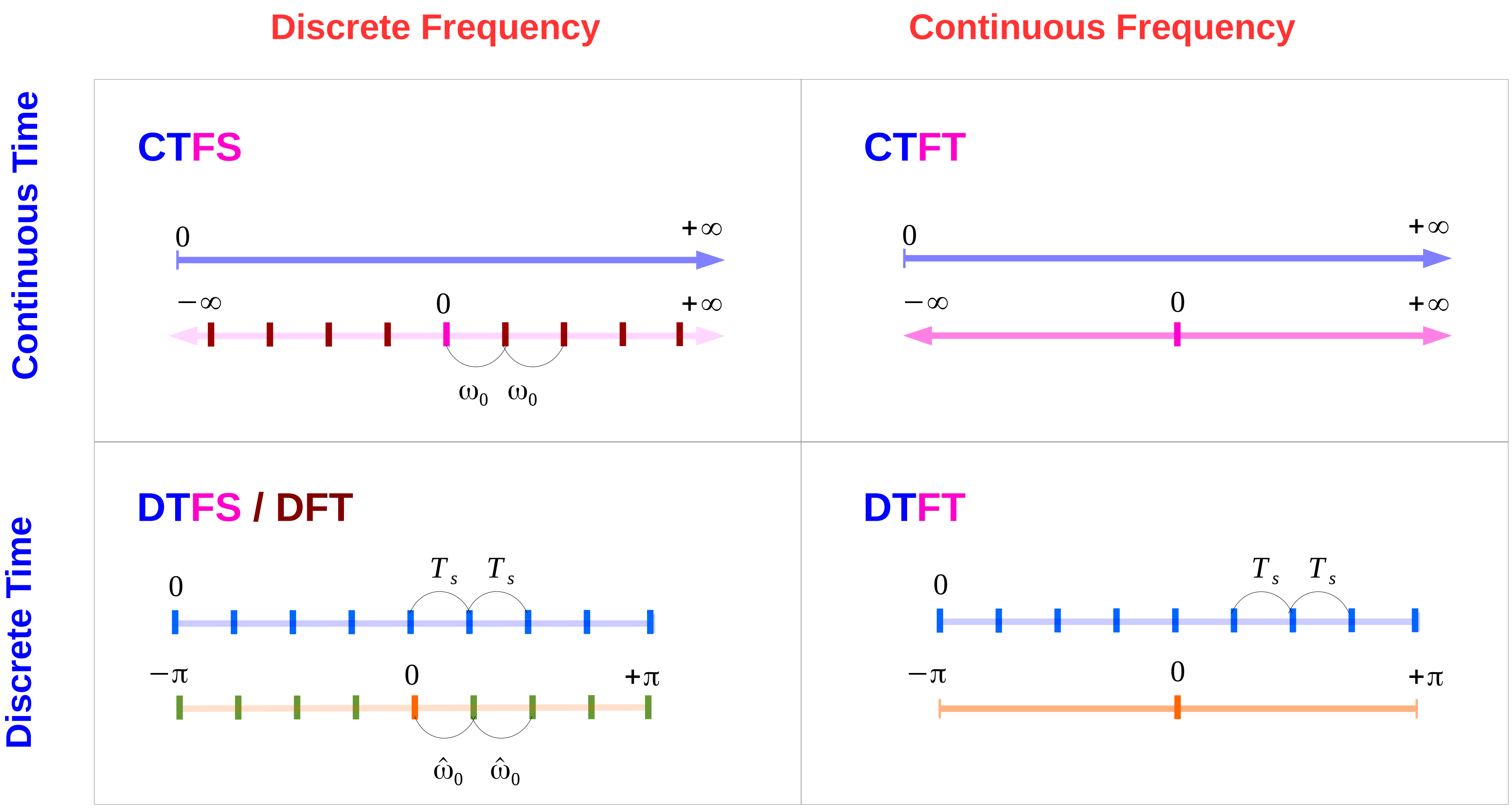
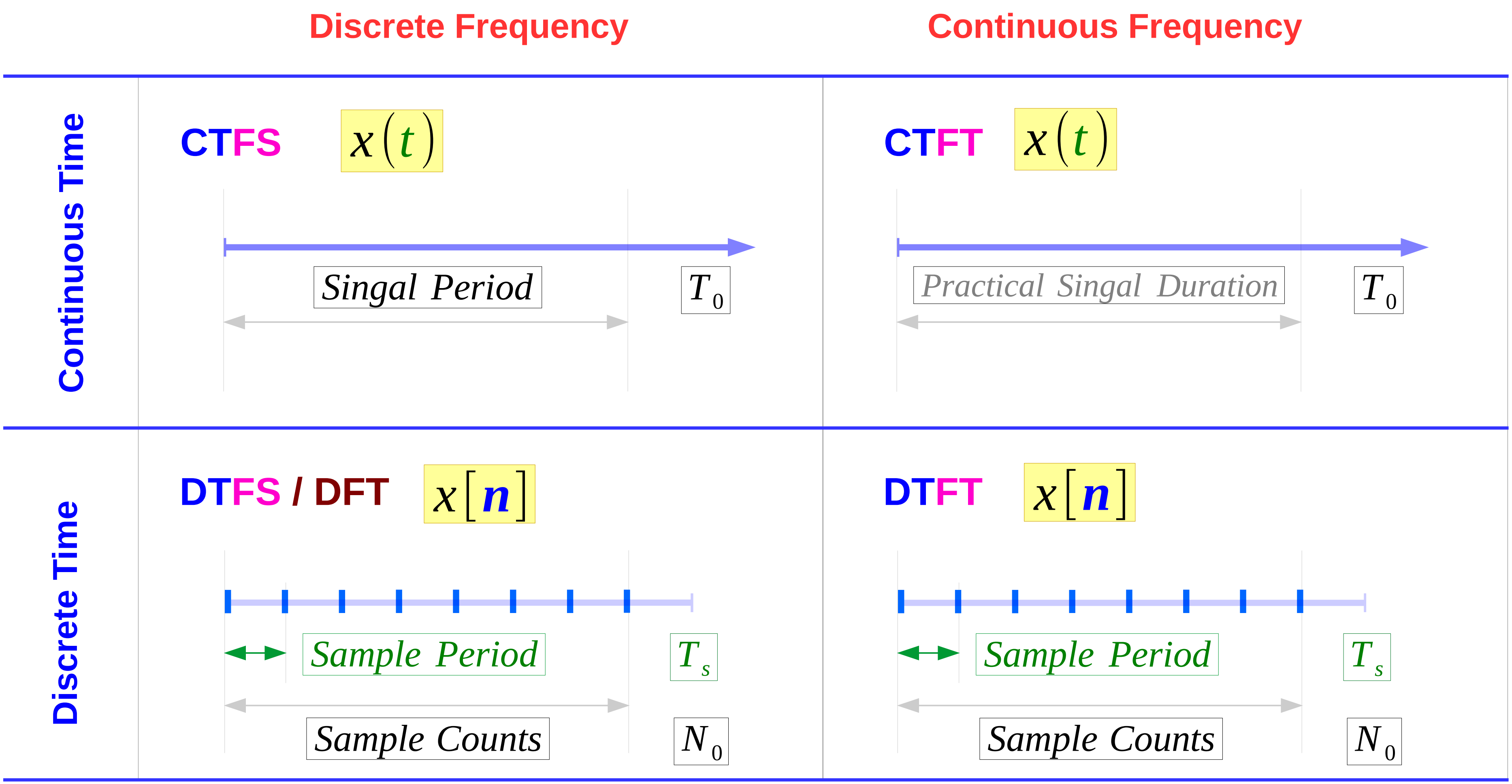
Fourier Analysis Methods

	Discrete Frequency		Continuous Frequency	
Continuous Time	**CTFS** C_k	Periodic in time Aperiodic in freq	**CTFT** $X(j\omega)$	Aperiodic in time Aperiodic in freq
Discrete Time	**DTFS / DFT** γ_k / $X[k]$	Periodic in time Periodic in freq	**DTFT** $X(j\hat{\omega})$	Aperiodic in time Periodic in freq

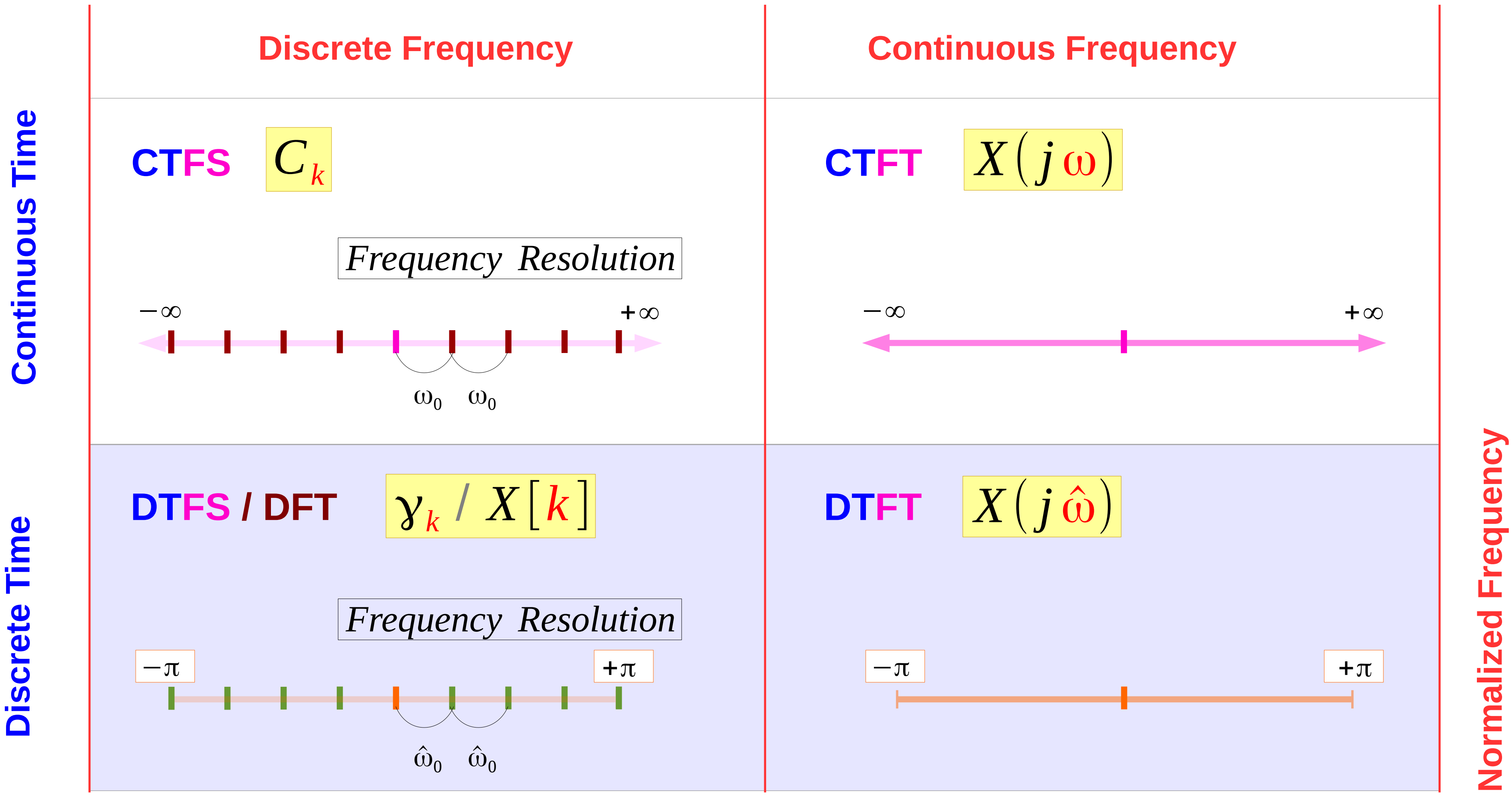
Time and Frequency Domains



Time Domain


Discrete Frequency
Continuous Frequency
Continuous Time
Discrete Time
CTFS
$x(t)$
Singal Period
T_0
CTFT
$x(t)$
Practical Singal Duration
T_0
DTFS / DFT
$x[n]$
Sample Period
T_s
Sample Counts
N_0
DTFT
$x[n]$
Sample Period
T_s
Sample Counts
N_0

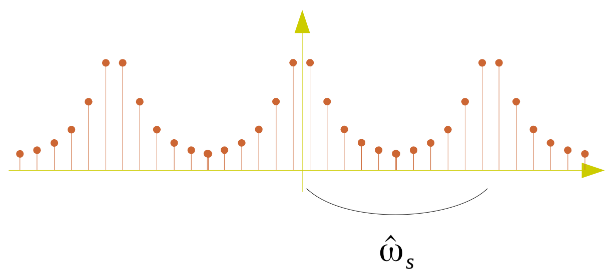
Frequency Domain



Discrete Time and Periodic Frequency

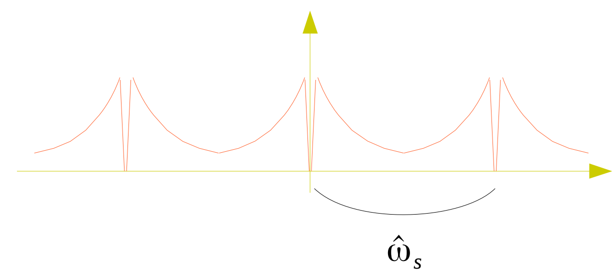
Discrete Frequency

Periodic in *discrete* freq



Continuous Frequency

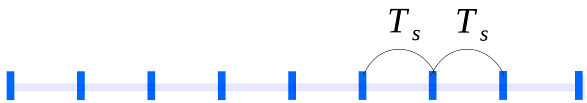
Periodic in *continuous* freq



Discrete Time

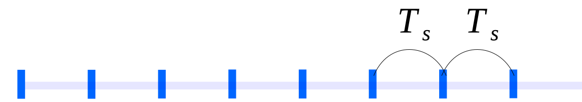
DTFS / DFT

Discrete Time



DTFT

Discrete Time

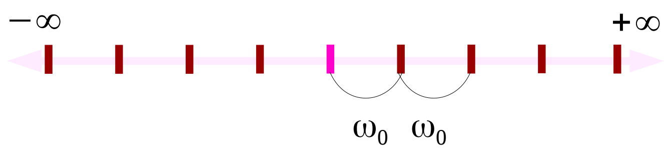


Periodic Time and Discrete Frequency

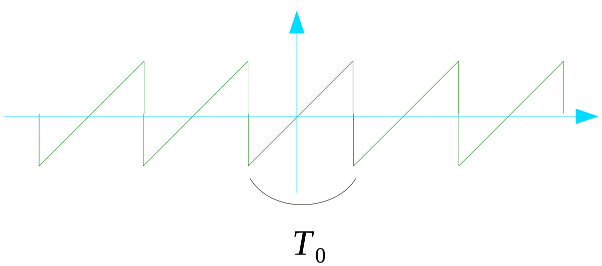
Discrete Frequency

Continuous Time

CTFS — Discrete Frequency

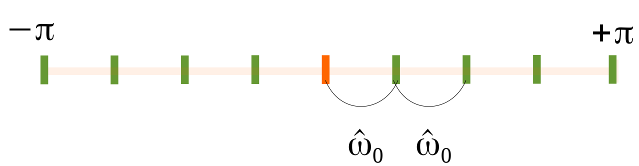


Periodic in *continuous* time

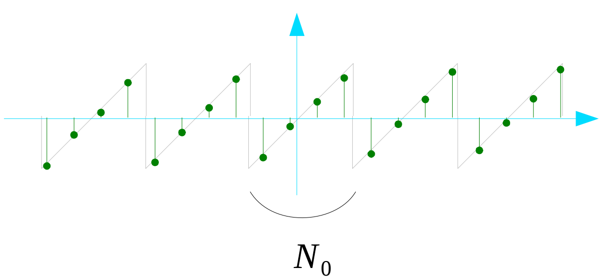


Discrete Time

DTFS / DFT — Discrete Frequency



Periodic in *discrete* time



Discrete Time Resolution

Discrete Frequency

ω_s: *replication frequency*
$\hat{\omega}_s$: *normalized replication frequency*

$$\omega_s = \frac{2\pi}{T_s} \quad \Longleftrightarrow \quad \hat{\omega}_s = \frac{2\pi}{1}$$

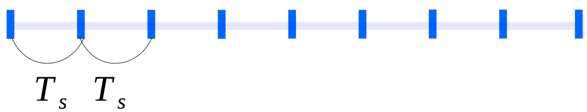
Continuous Frequency

ω_s: *replication frequency*
$\hat{\omega}_s$: *normalized replication frequency*

$$\omega_s = \frac{2\pi}{T_s} \quad \Longleftrightarrow \quad \hat{\omega}_s = \frac{2\pi}{1}$$

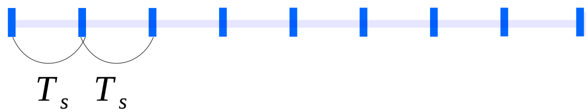
Discrete Time

DTFS / DFT



Discrete Time
Periodic in *discrete* freq

DTFT



Discrete Time
Periodic in *continuous* freq

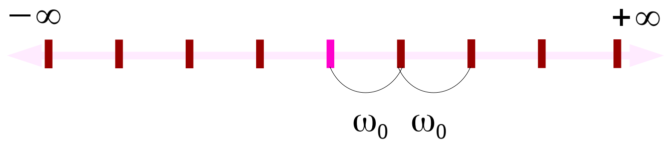
Discrete Frequency Resolution

Discrete Frequency

Continuous Time

CTFS

Periodic in *continuous* time
Discrete Frequency



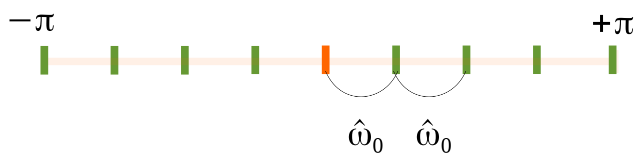
$$\omega_0 = \frac{2\pi}{T_0}$$

$period: T_0 \ seconds$

Discrete Time

DTFS / DFT

Periodic in *discrete* time
Discrete Frequency



$$\hat{\omega}_0 = \frac{2\pi}{N_0}$$

$period: N_0 \ samples$

Normalized Frequency

Normalized Discrete Frequency

$$\hat{\omega}_s = \frac{2\pi}{1} \qquad = \left(\frac{2\pi}{T_s}\right) T_s$$

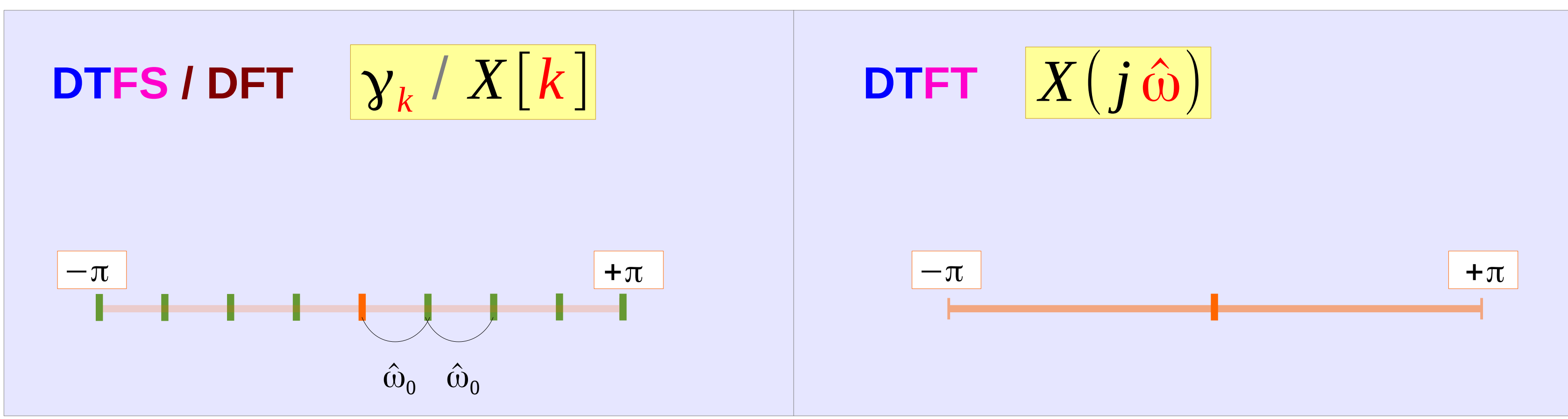
$$\hat{\omega}_0 = \frac{2\pi}{N_0} \qquad = \left(\frac{2\pi}{T_0}\right) T_s$$

Normalized Continuous Frequency

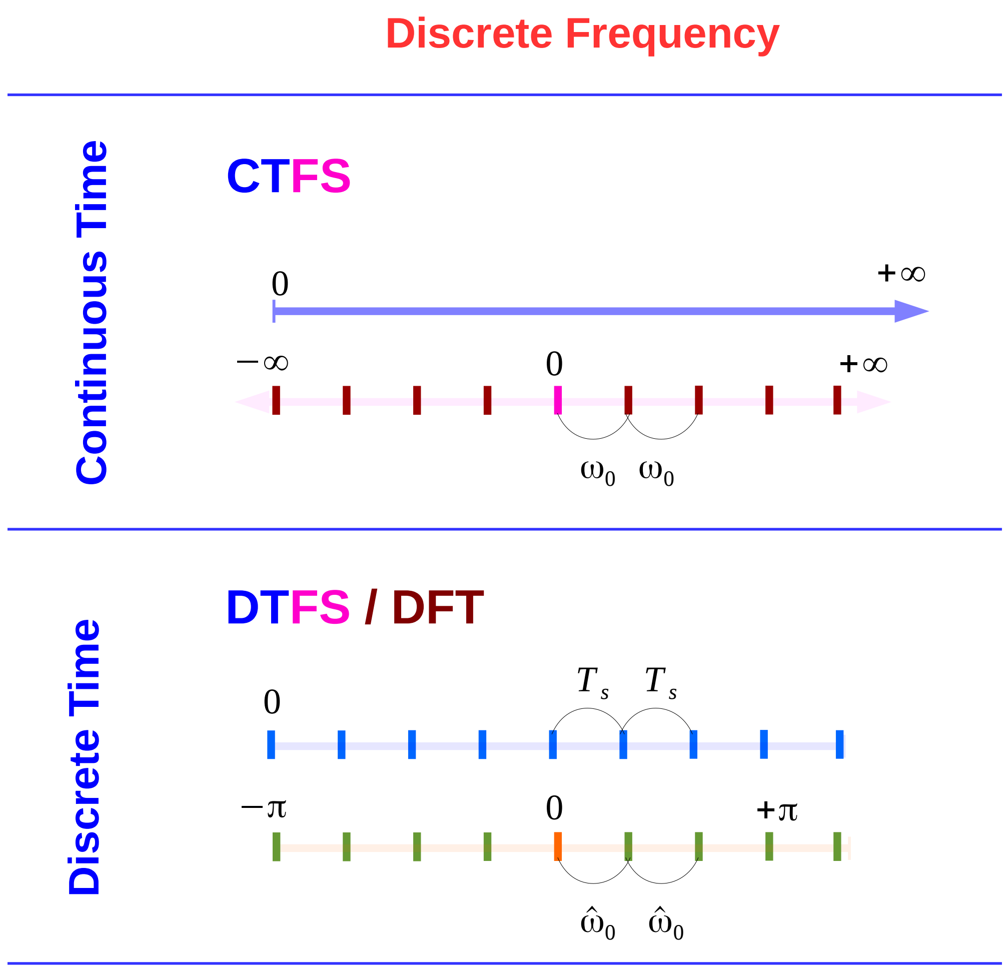
$$\hat{\omega}_s = \frac{2\pi}{1} \qquad = \left(\frac{2\pi}{T_s}\right) T_s$$

continuous variable $\hat{\omega}$

Discrete Time



Normalized by $1/T_s$



frequency resolution

$$\omega_0 = \frac{2\pi}{T_0}$$

$$T_0 = N_0 T_s$$

$$\omega_0 = \frac{2\pi}{N_0 T_s}$$

$\cdot T_s$ $\quad$ $\cdot f_s$

$$\frac{\omega_0}{1/T_s} = \frac{2\pi}{N_0 \cancel{T_s}} \cancel{T_s}$$

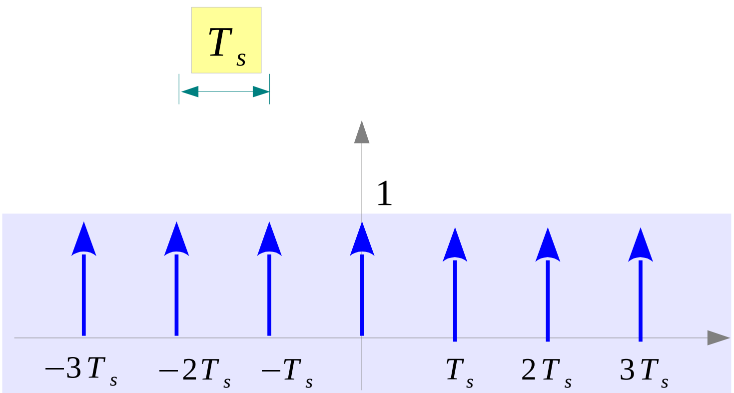
$$\hat{\omega}_0 = \frac{2\pi}{N_0}$$

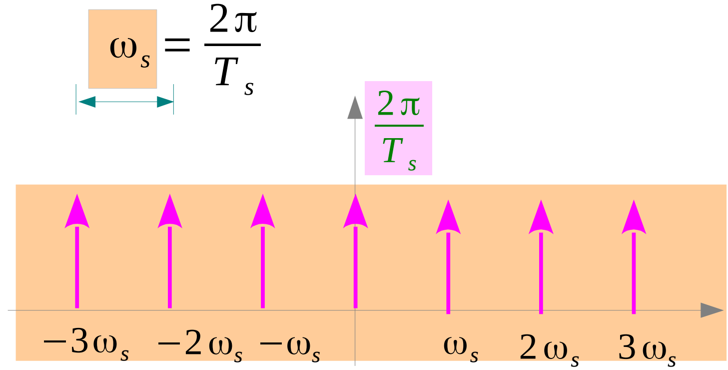
normalized frequency resolution

CTFT pair of an impulse train

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \quad \Longleftrightarrow \quad P(j\omega) = \sum_{k=-\infty}^{\infty} \left(\frac{2\pi}{T_s}\right) \delta(\omega - k\omega_s)$$

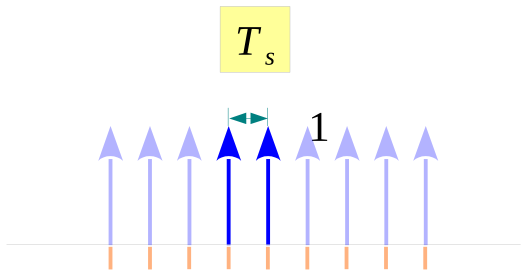
$$\frac{2\pi}{T_s}$$





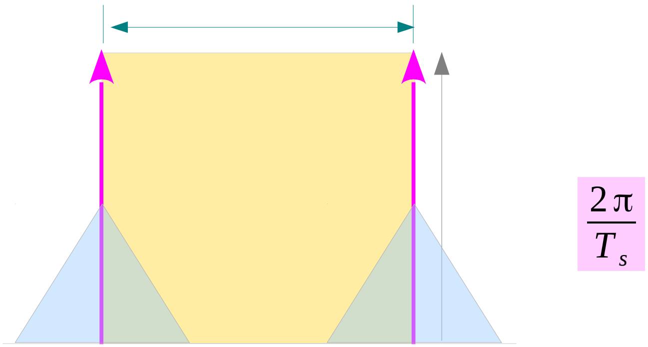
Sampling **Replication**

Sampling and Replicating

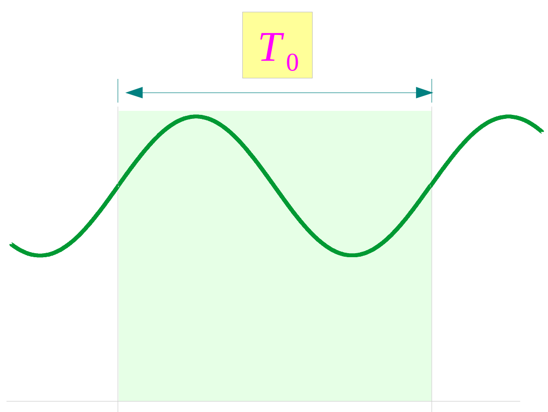


Sampling

$$\omega_s = \frac{2\pi}{T_s}$$

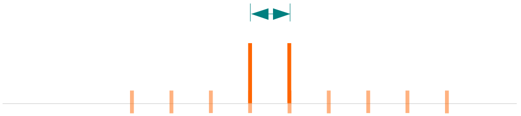


Replication



Period

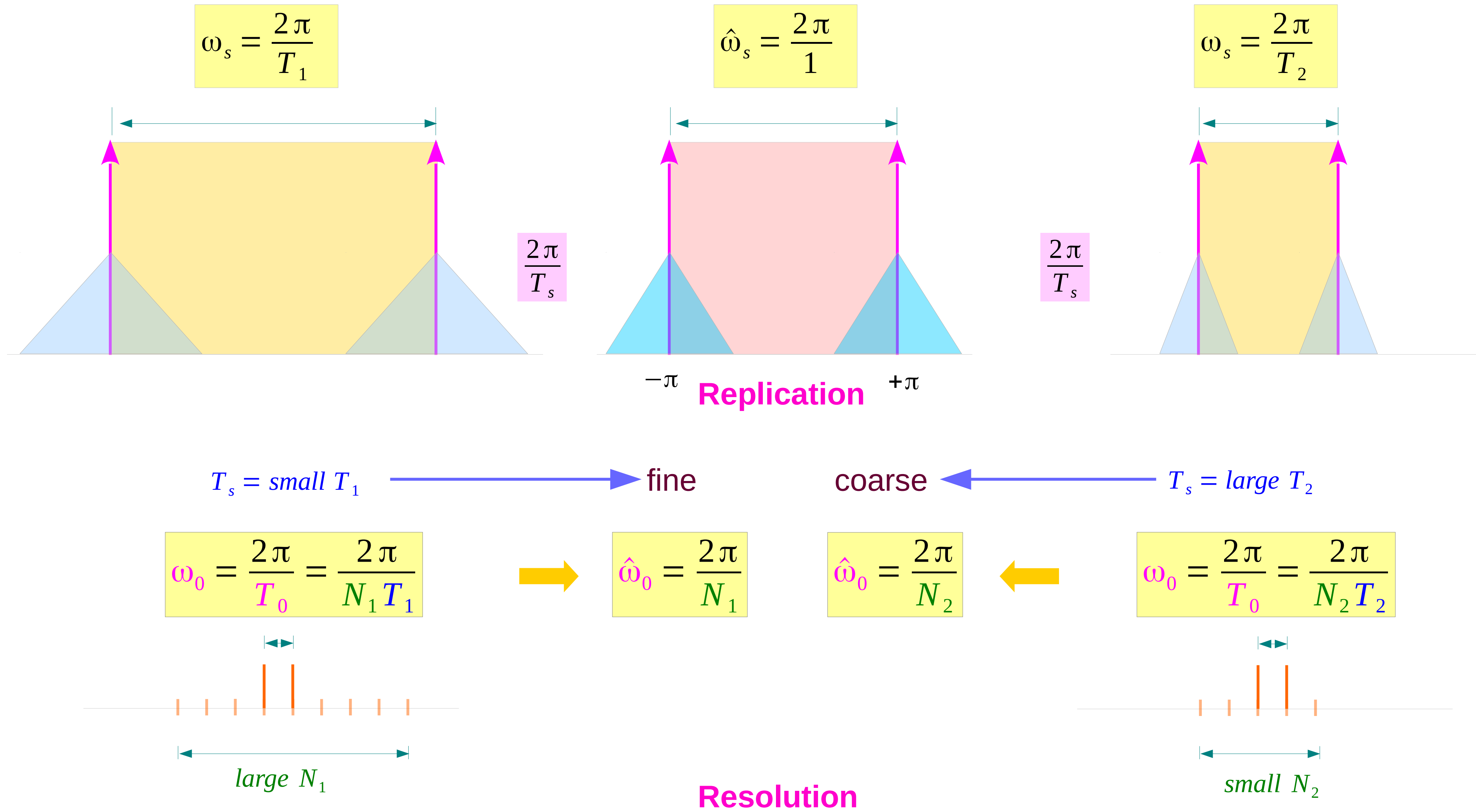
$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{N_1 T_1} = \frac{2\pi}{N_2 T_2}$$



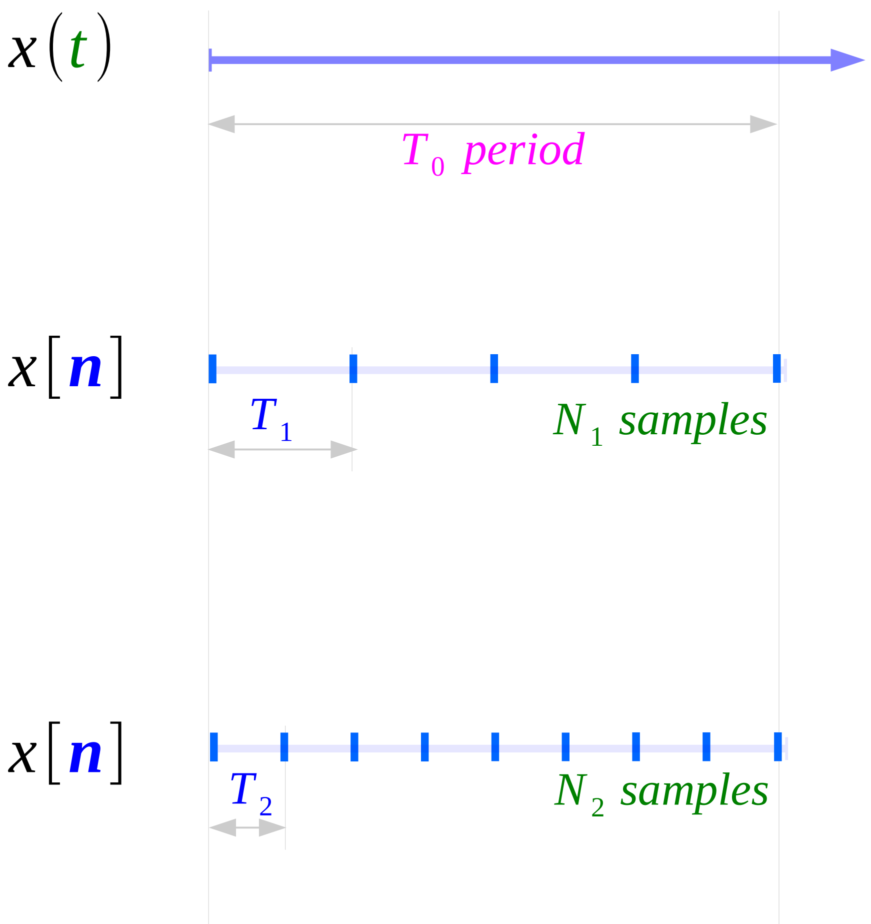
Resolution

Normalization

Relatively scaled figures



Sampling Period and the Number of Samples



$$\textit{fundamental period } T_0 = T_1 N_1 = T_2 N_2$$

$\textit{sampling period}: \quad T_1 \qquad (T_1 > T_2)$

$\textit{number of samples}: \quad N_1 \qquad (N_1 < N_2)$

$\textit{sampling period}: \quad T_2 \qquad (T_1 > T_2)$

$\textit{number of samples}: \quad N_2 \qquad (N_1 < N_2)$

Periodic Relationship

fundamental period T_0 ⟷ *frequency resolution* ω_0

$$T_0 = T_1 N_1 = T_2 N_2$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{N_1 T_1} = \frac{2\pi}{N_2 T_2}$$

sampling period T_s: ⟷ *replication period* ω_1, ω_2:

$$T_1 > T_2$$

$$\omega_1 = \frac{2\pi}{T_1} < \omega_2 = \frac{2\pi}{T_2}$$

no of samples N_0: ⟷ *normalized frequency resolution* $\hat{\omega}_0$

$$N_1 < N_2$$

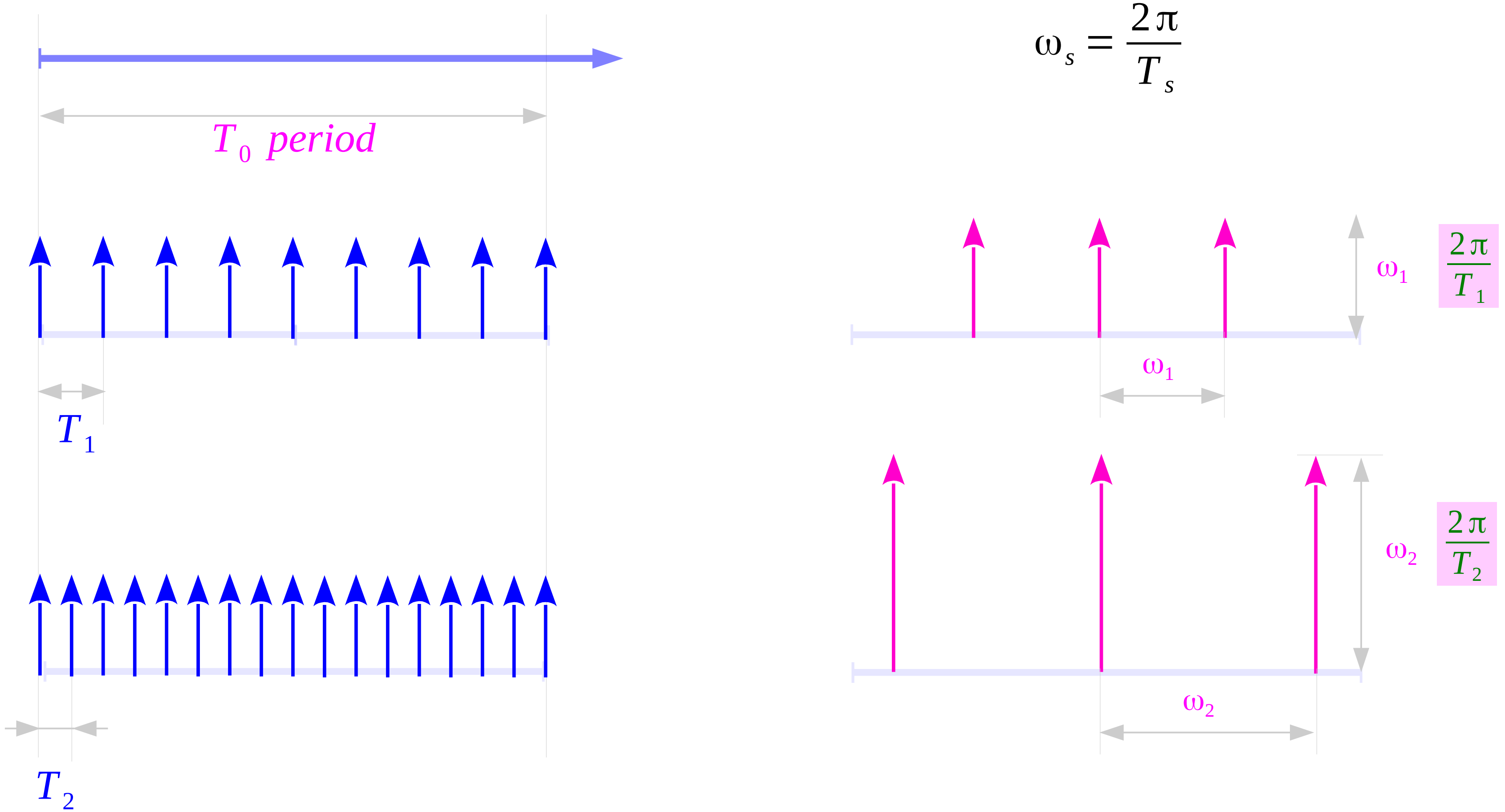
$$\hat{\omega}_0 = \frac{2\pi}{N_1} > \hat{\omega}_0 = \frac{2\pi}{N_2}$$

$$\hat{\omega}_0 = \omega_0 T_1 \qquad \hat{\omega}_0 = \omega_0 T_2$$

coarse fine

Sampling Period and Replication Period

$$\omega_s = \frac{2\pi}{T_s}$$

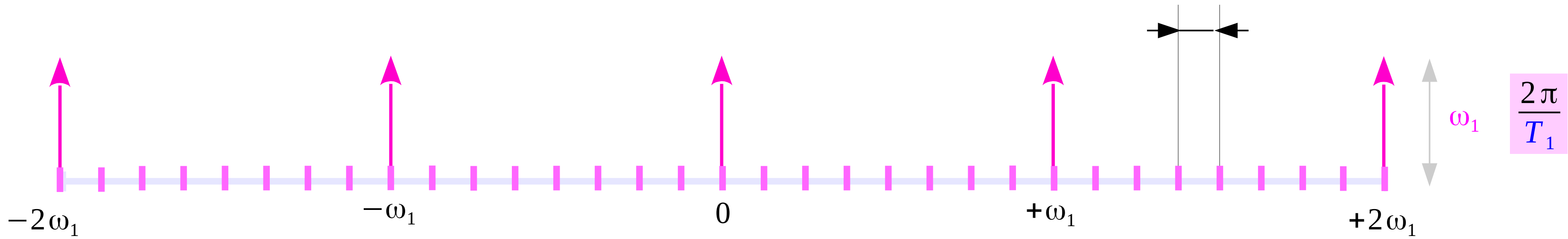


Frequency Resolution

the same signal period T_0

the same frequency resolution ω_0

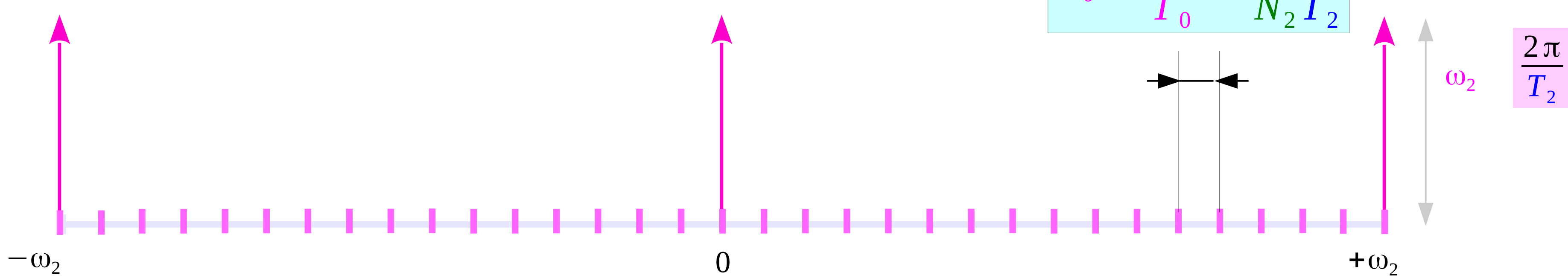
$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{N_1 T_1}$$



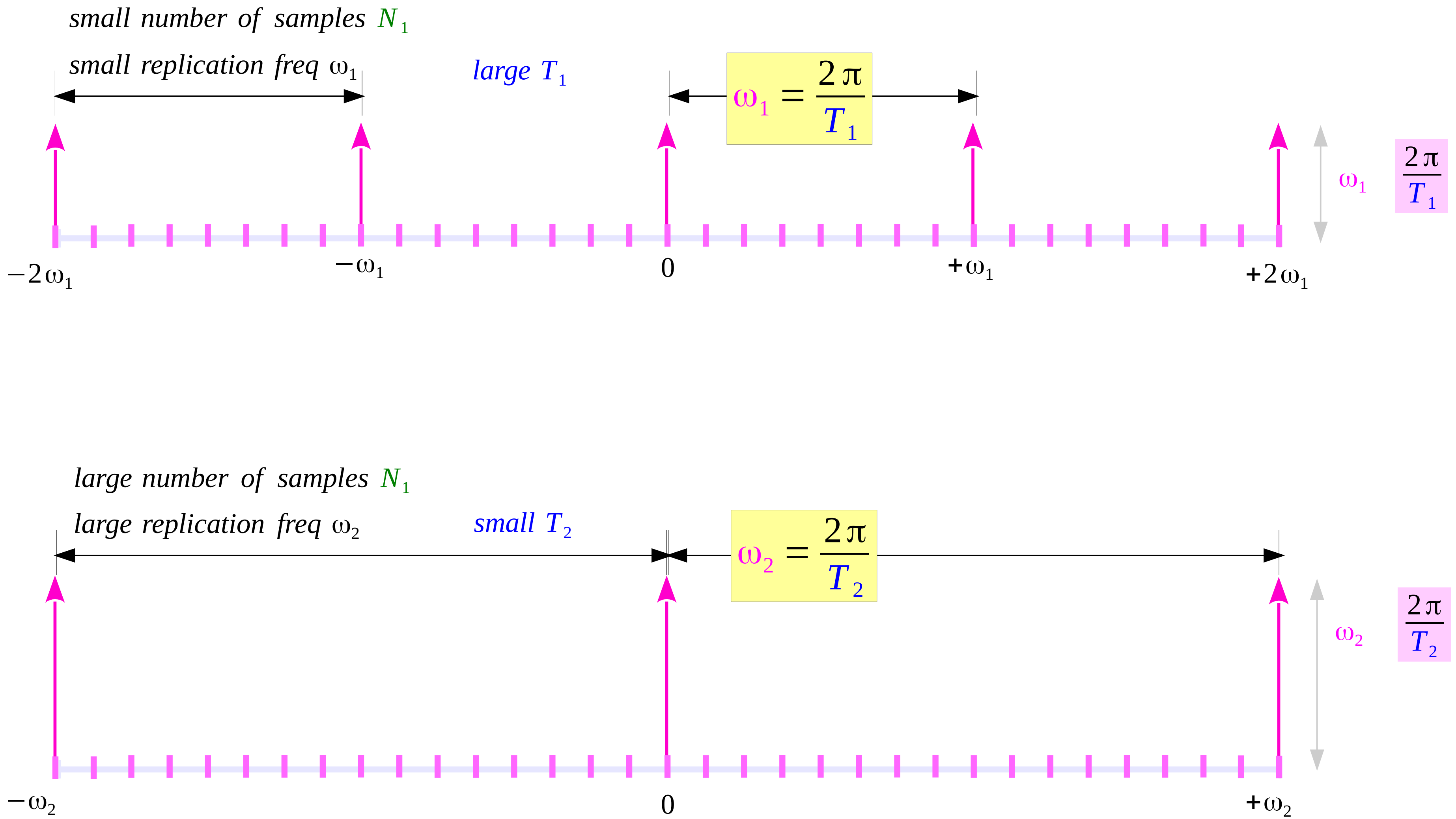
the same signal period T_0

the same frequency resolution ω_0

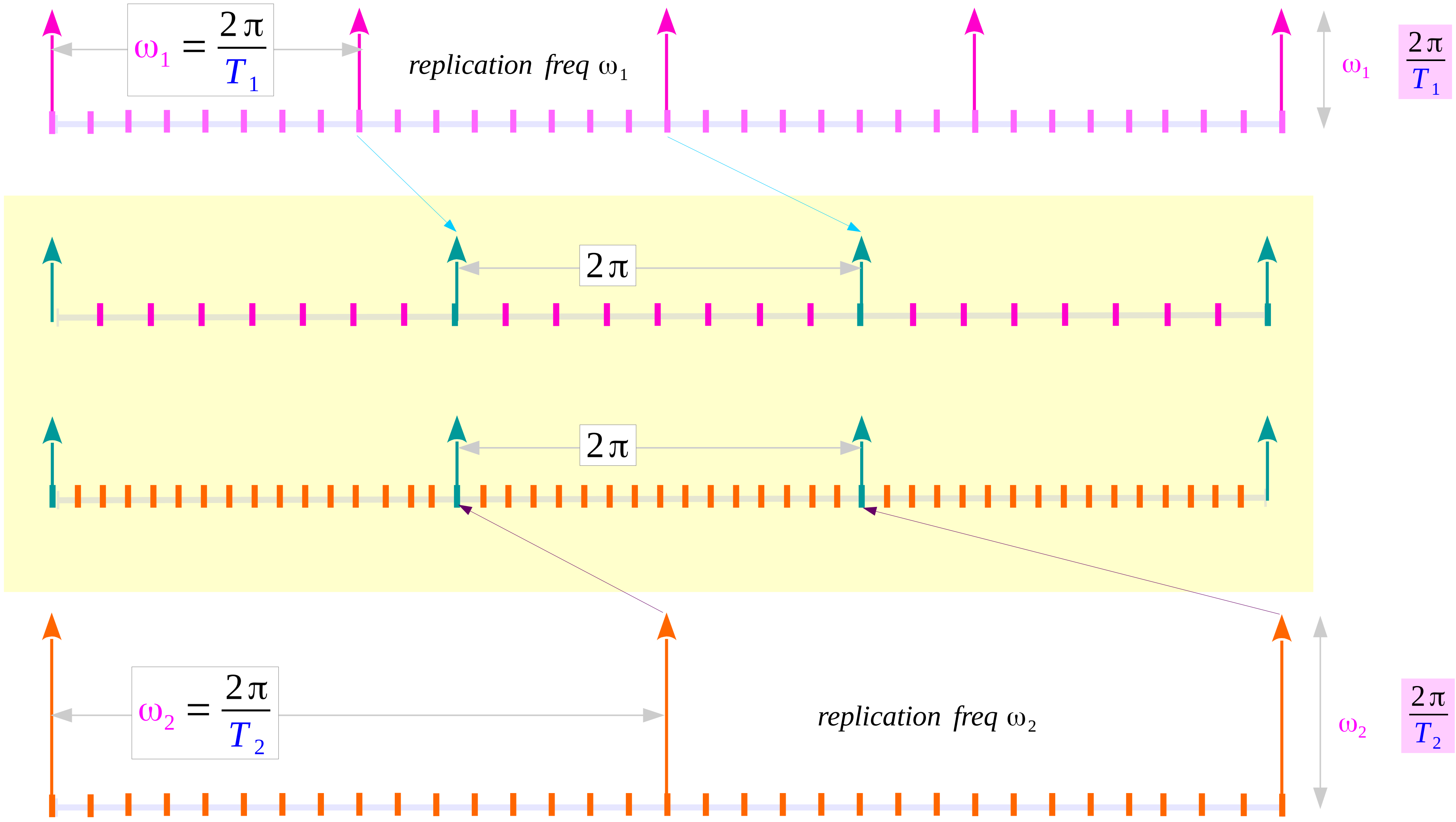
$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{N_2 T_2}$$



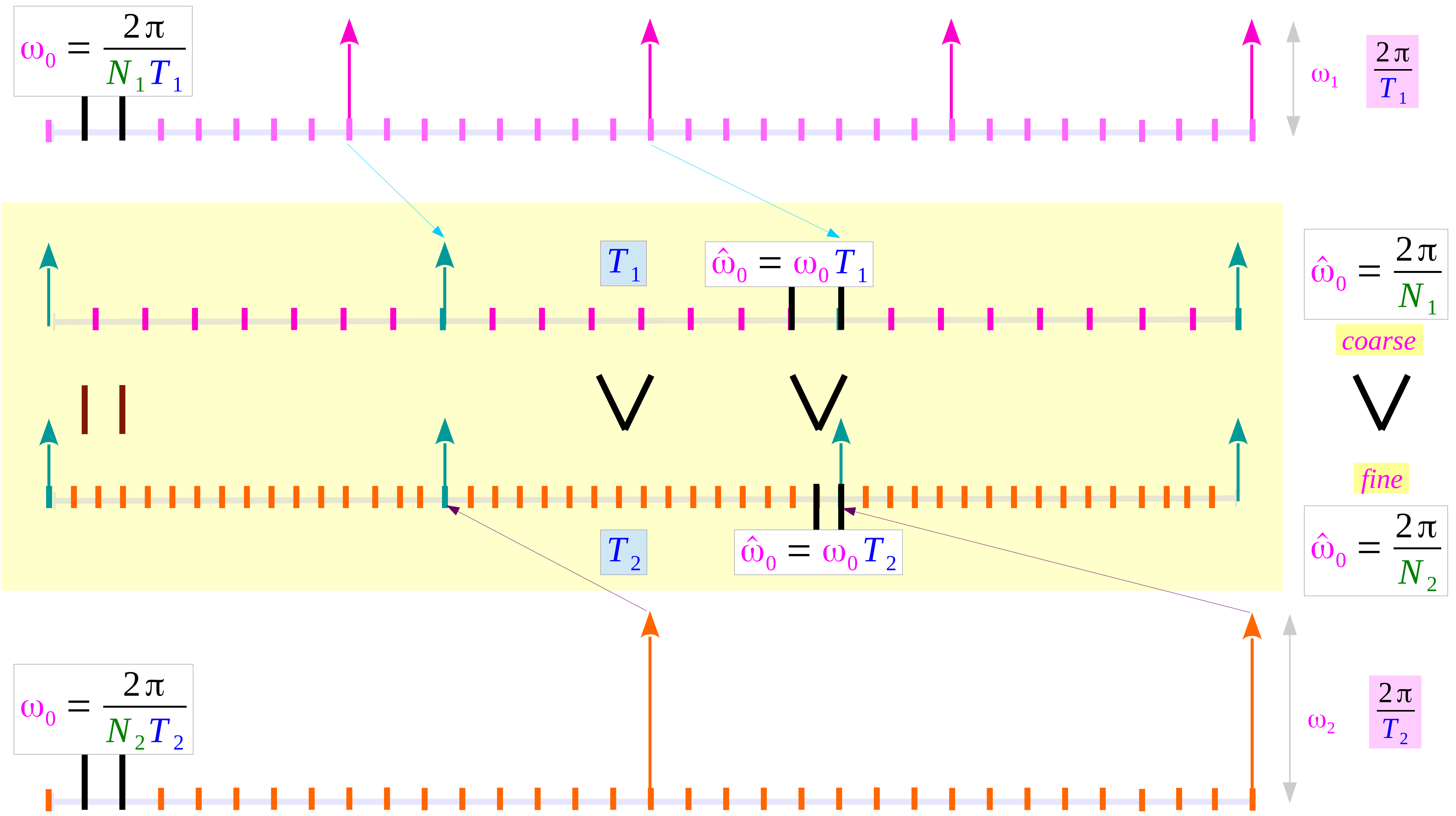
Replication Frequency



Normalized Frequency for Comparison

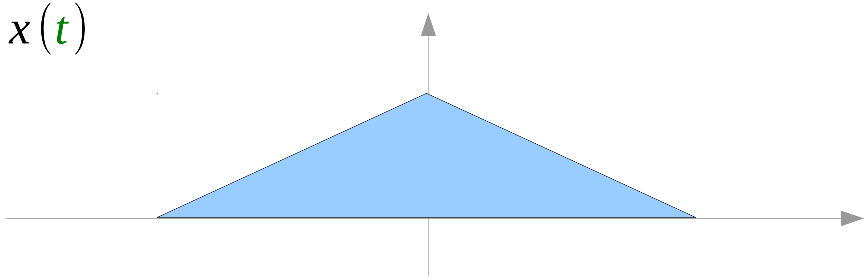


Normalized Frequency Resolution



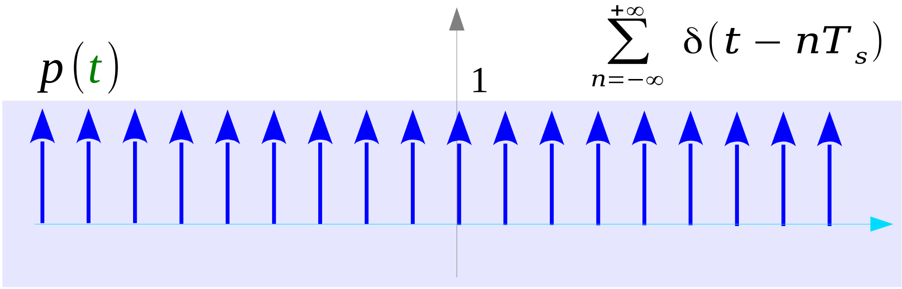
Multiplication with an Impulse Train

$x(t)$



$p(t)$

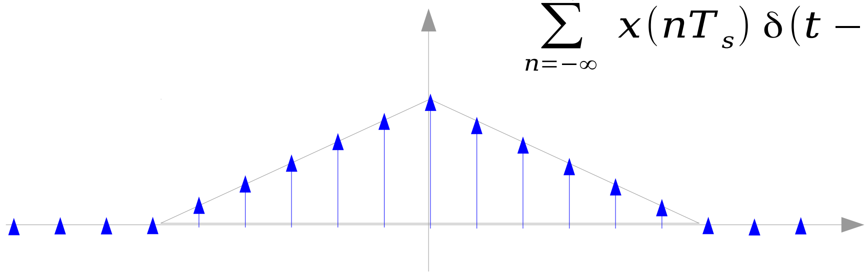
$$\sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$$



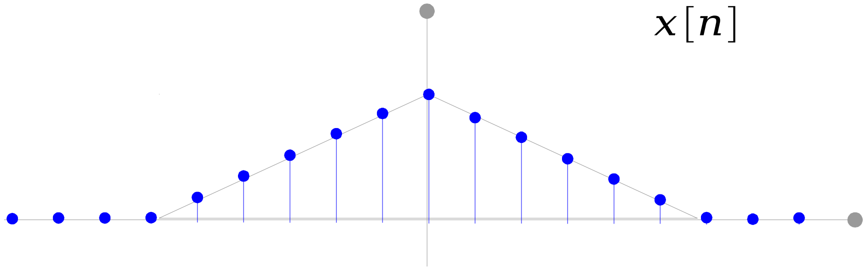
$x(t) \cdot p(t)$

Multiplication with a dense impulse train

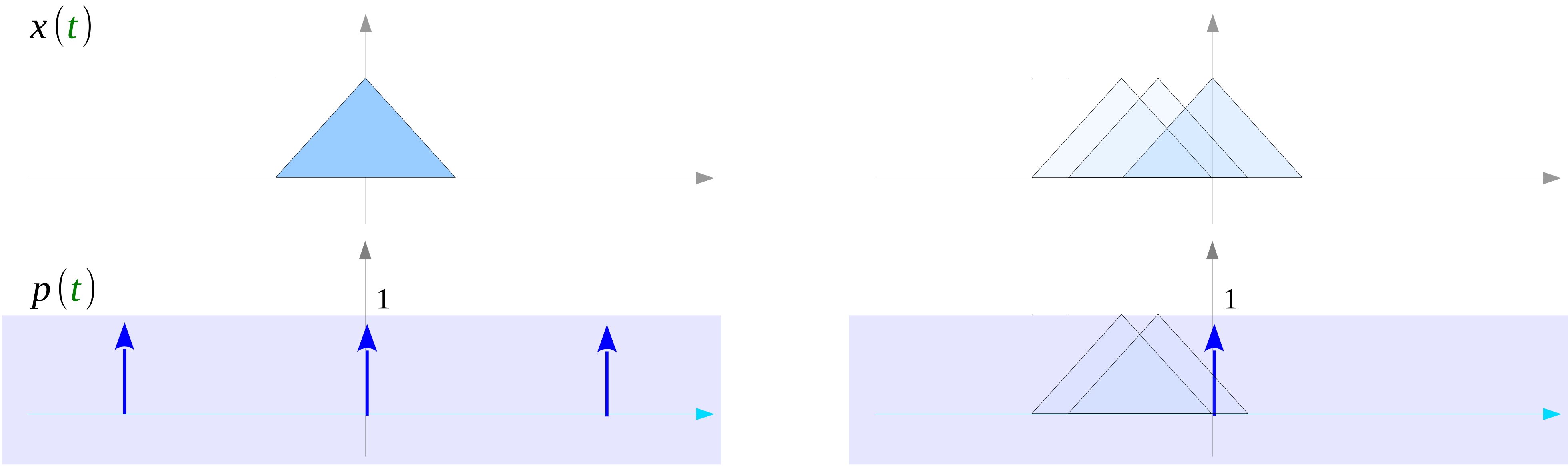
$$\sum_{n=-\infty}^{+\infty} x(nT_s)\,\delta(t - nT_s)$$



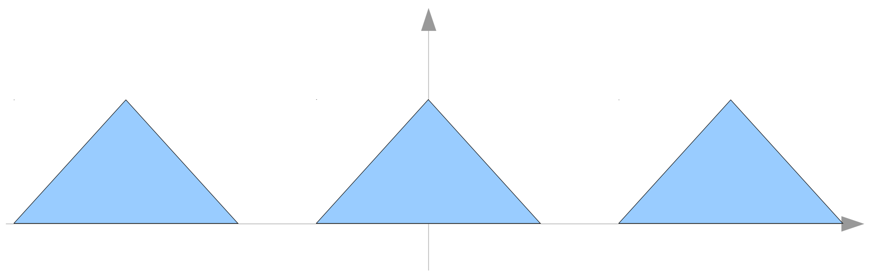
$x[n]$



Convolution with an Impulse Train



$x(t) * p(t)$ **Multiplication with a sparse impulse train**



Convolution & Multiplication Properties

$$x(t) * y(t) \Longleftrightarrow X(j\omega) \cdot Y(j\omega)$$

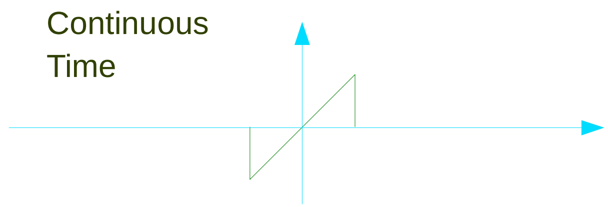
$$x(t) \cdot y(t) \Longleftrightarrow \frac{1}{2\pi} X(j\omega) * Y(j\omega)$$

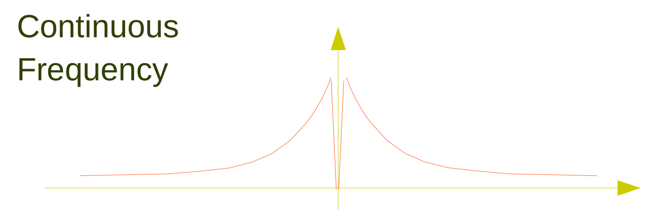
$$x(t) * y(t) \Longleftrightarrow X(f) \cdot Y(f)$$

$$x(t) \cdot y(t) \Longleftrightarrow X(f) * Y(f)$$

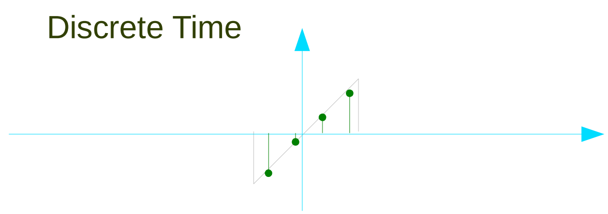
Types of Fourier Transforms

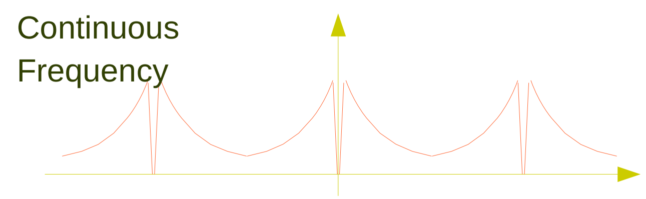
Continuous Time
Fourier Transform
CTFT



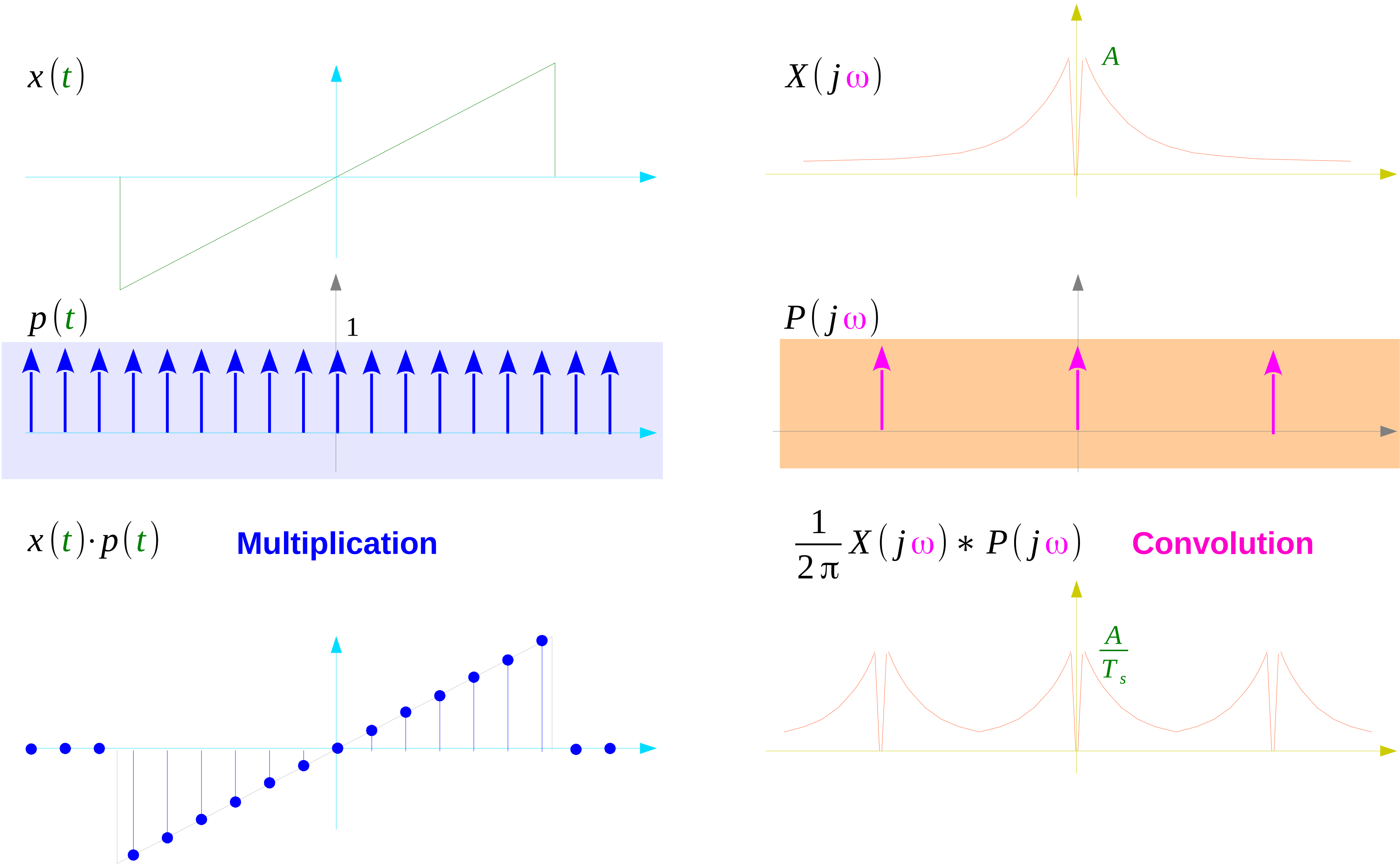


Discrete Time
Fourier Transform
DTFT

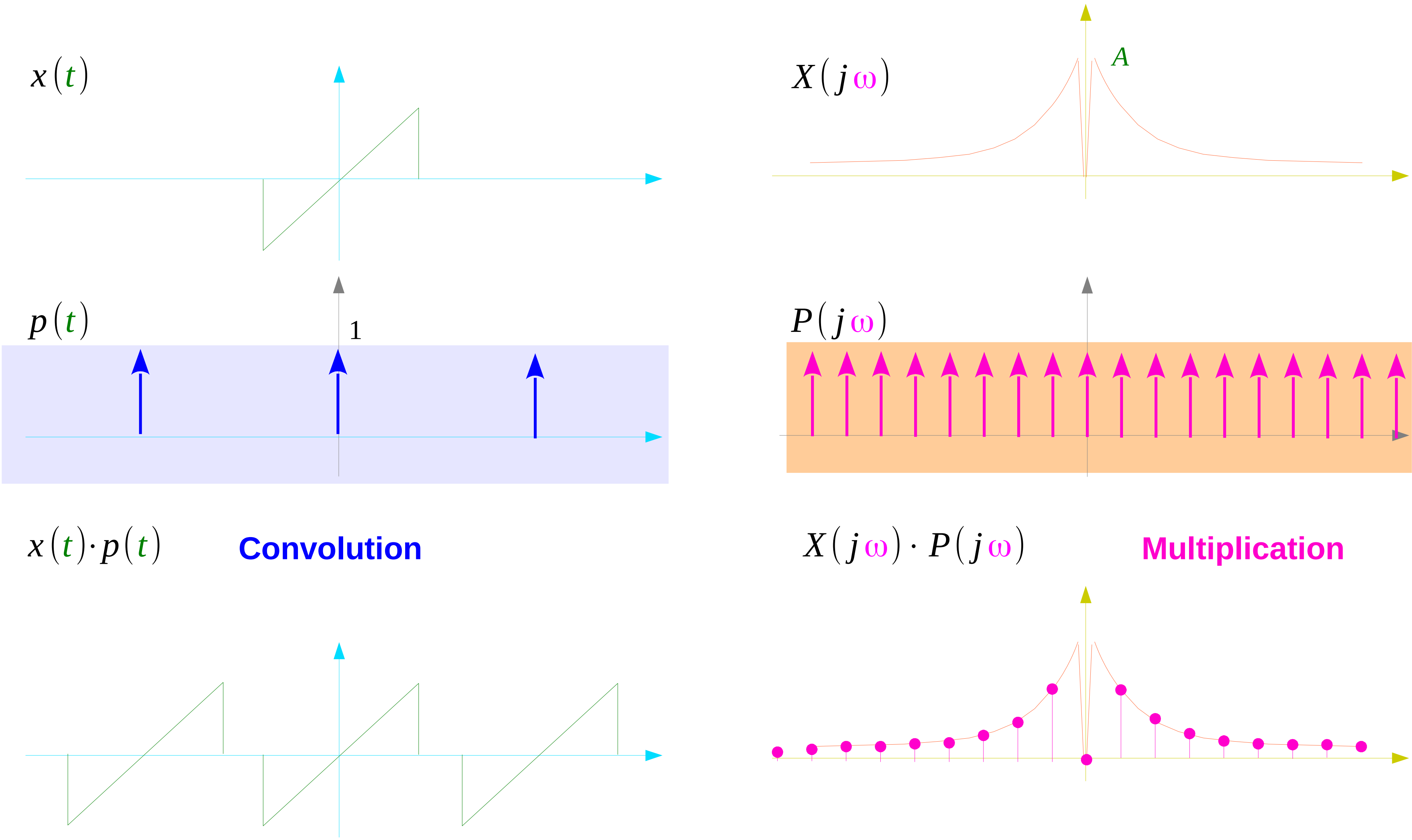




Multiplication & Convolution



Convolution & Multiplication



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] M.J. Roberts, Fundamentals of Signals and Systems
[4] S.J. Orfanidis, Introduction to Signal Processing
[5] K. Shin, et al., Fundamentals of Signal Processing for Sound and Vibration Engineerings