Baseband Demodulation (3B)

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Signal Space

N-dim orthogonal space

Characterized by a set of N linearly independent functions

Basis functions $\Psi_j(t)$

Independent → not interfering in detection

$$\int_0^T \Psi_j(t)\Psi_k(t)\,dt = K_j \delta_{jk} \qquad 0 \le t \le T \qquad j, k = 1, \cdots, N$$

Kronecker delta functions

$$\delta_{jk} = \begin{cases} 1 & for\ j = k \\ 0 & otherwise \end{cases}$$

N-dim orthonormal space $K_j = 1$

$$E_j = \int_0^T \Psi_j^2(t)\,dt = K_j$$

Linear Combination

Any finite set of waveform $\{s_i(t)\}$ $\quad i = 1, \cdots, M$

Characterized by a set of N linearly independent functions

$$s_1(t) = a_{11}\Psi_1(t) + a_{12}\Psi_2(t) + \cdots + a_{1n}\Psi_N(t)$$

$$s_2(t) = a_{21}\Psi_1(t) + a_{22}\Psi_2(t) + \cdots + a_{2n}\Psi_N(t)$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$s_M(t) = a_{M1}\Psi_1(t) + a_{M2}\Psi_2(t) + \cdots + a_{MN}\Psi_N(t)$$

$$s_1(t) = \sum_{j=1}^{N} a_{ij}\Psi_j(t) \qquad i = 1, \cdots, M$$

$$N \le M$$

Linear Combination

Any finite set of waveform $\{s_i(t)\}$ $\quad i = 1, \cdots, M$

Characterized by a set of N linearly independent functions

$$s_i(t) = \sum_{j=1}^{N} a_{ij} \Psi_j(t) \qquad i = 1, \cdots, M$$

$$N \le M$$

$$a_{ij} = \frac{1}{K_j} \int_0^T s_i(t) \Psi_j(t)\, dt \qquad i = 1, \cdots, M \qquad 0 \le t \le T$$

$$j = 1, \cdots, N$$

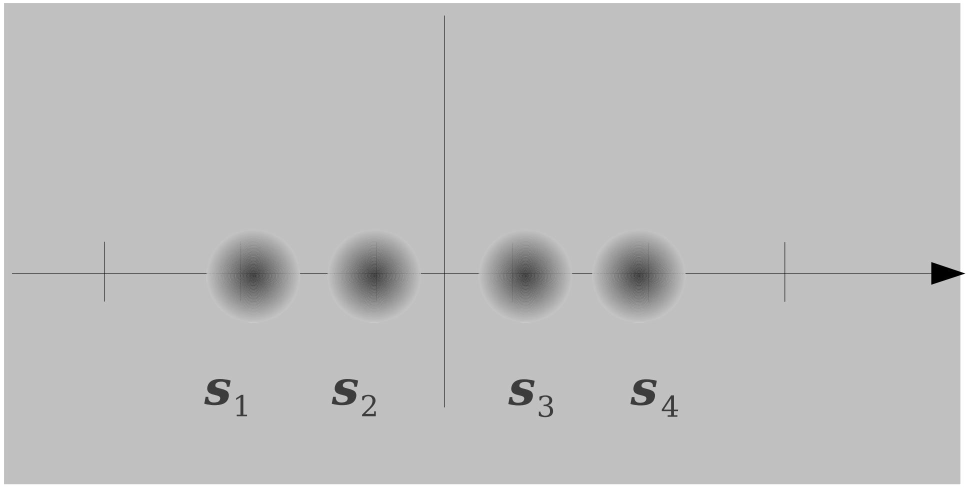
$$\{s_i(t)\} \Longleftrightarrow \{\mathbf{s}_i\} = \{a_{i1}, a_{i2}, \cdots, a_{iN}\} \qquad i = 1, \cdots, M$$

Signals and Noise

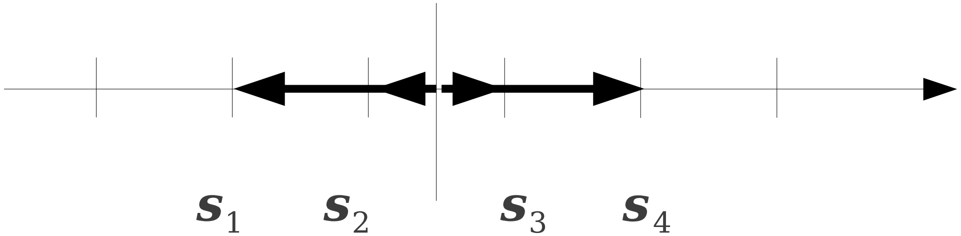
Any finite set of waveform $\{s_i(t)\}$ $i = 1, \cdots, M$

Characterized by a set of N linearly independent functions

$$\{s_i(t)\} \Longleftrightarrow \{\boldsymbol{s}_i\} = \{a_{i1}, a_{i2}, \cdots, a_{iN}\} \quad i = 1, \cdots, M$$



4-ary PAM



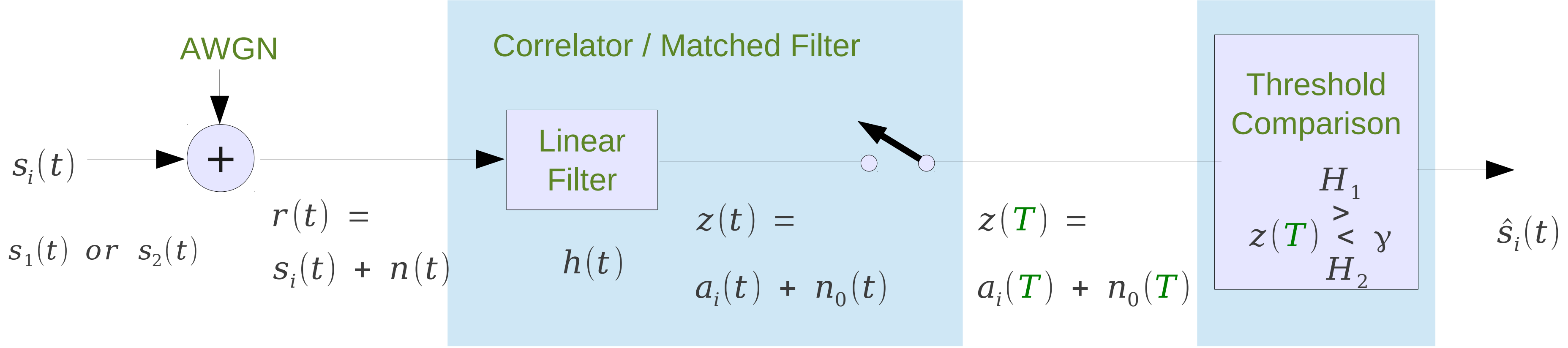
Detection of Binary Signals

Transmitted Signal

$$s_i(t) = \begin{cases} s_1(t) & 0 \le t \le T \quad \text{for a binary } 1 \\ s_2(t) & 0 \le t \le T \quad \text{for a binary } 0 \end{cases}$$

Received Signal

$$r(t) = s_i(t) + n(t) \qquad i = 1,2; \qquad 0 \le t \le T$$



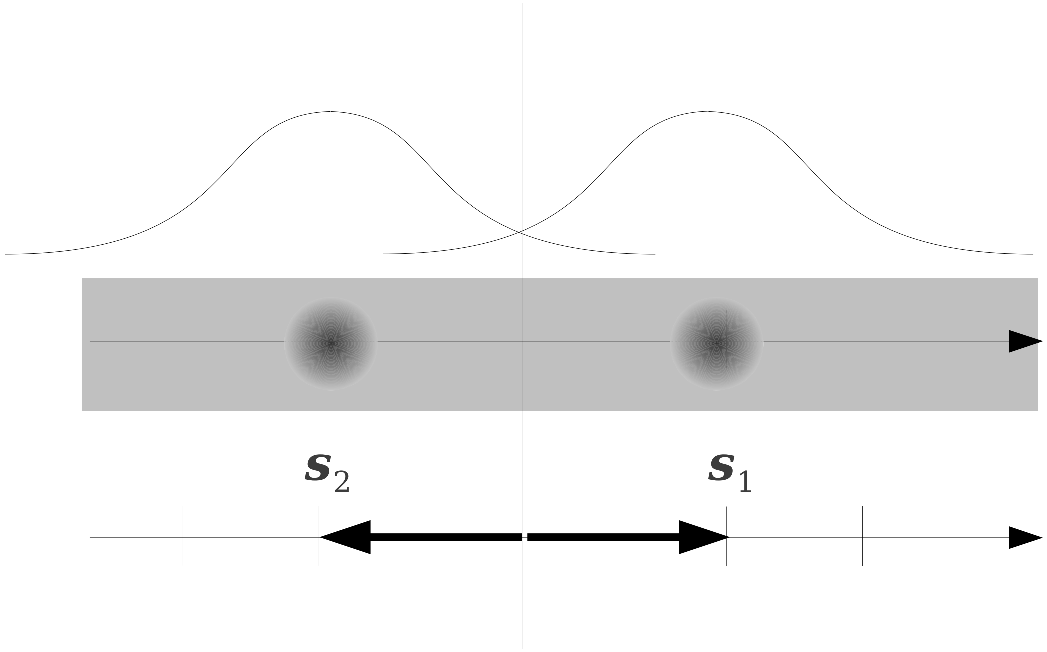
Detection of Binary Signals

$$z(T) = a_i(T) + n_0(T) \quad \Rightarrow \quad z = a_i + n_0$$

$$p(n_0) = \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{n_0}{\sigma_0}\right)^2\right]$$

$$p(z|s_1) = \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z-a_1}{\sigma_0}\right)^2\right]$$

$$p(z|s_2) = \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z-a_2}{\sigma_0}\right)^2\right]$$



$$z(T) \underset{H_2}{\overset{H_1}{\gtrless}} \gamma$$

$$\frac{p(z|s_1)}{p(z|s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{P(s_2)}{P(s_1)}$$

$$\frac{p(z|s_1)}{p(z|s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{a_1+a_2}{2} = \gamma_0$$

Error Probability

error e

$$p(e|s_1) = p(H_2|s_1) = \int_{-\infty}^{\gamma_0} p(z|s_1)\,dz$$

$$p(e|s_2) = p(H_1|s_2) = \int_{\gamma_0}^{-\infty} p(z|s_2)\,dz$$

probability of bit error P_B

$$P_B = P(e|s_1)P(s_1) + P(e|s_2)P(s_2)$$
$$= P(H_2|s_1)P(s_1) + P(H_1|s_2)P(s_2)$$

equal a priori probabilities

$$P_B = \tfrac{1}{2}P(H_2|s_1) + \tfrac{1}{2}P(H_1|s_2)$$
$$= P(H_2|s_1) = P(H_1|s_2)$$

$$P_B = \int_{\gamma_0 = (a_1+a_2)/2}^{+\infty} p(z|s_2)\,dz$$

$$= \int_{\gamma_0 = (a_1+a_2)/2}^{+\infty} \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z-a_2}{\sigma_0}\right)^2\right] dz$$

$u = (z - a_2)/\sigma_0$ $\sigma_0\,du = dz$

$$= \int_{u = (a_1-a_2)/2\sigma_0}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-u^2}{2}\right) du$$

$$= Q\left(\frac{a_1-a_2}{2\sigma_0}\right)$$

complementary error function
(co-error function)

$$Q(x) = \int_{x}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-x^2}{2}\right) dx$$

ISI

Maximum Likelihood Receiver

maximum likelihood detector

$P(s_1) = P(s_2)$ equal a priori probability

$p(z|s_1), \quad p(z|s_2)$ symmetric likelihood

$\gamma_0 = \dfrac{(a_1 + a_2)}{2}$ optimum threshold for minimizing the error probability

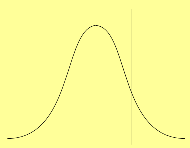
select the hypothesis with the maximum likelihood

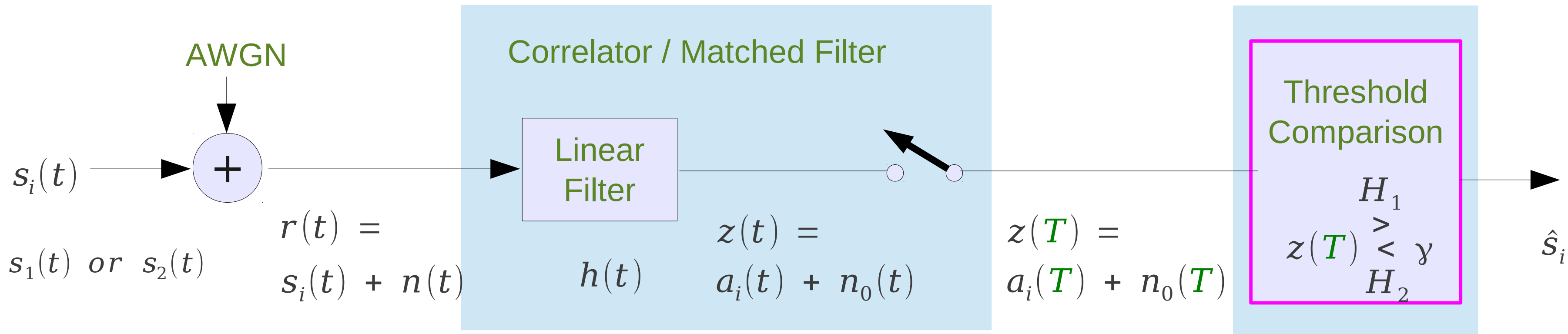
complementary error function

$$Q(x) = \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-u^2}{2}\right) du$$

$$P_B = \int_{\gamma_0 = (a_1 - a_2)/2\sigma_0}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-u^2}{2}\right) du$$

$$= Q\left(\frac{a_1 - a_2}{2\sigma_0}\right)$$





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Matched Filter Minimizes P_B by Maximizing SNR

Matched Filter / Correlator

maximize $\dfrac{(a_1 - a_2)^2}{\sigma_0^2}$ → maximize $\dfrac{(a_1 - a_2)^2}{2\sigma_0}$ →

minimize

$$P_B = Q\left(\frac{a_1 - a_2}{2\sigma_0}\right)$$

maximize $\left(\dfrac{S}{N}\right)_T = \dfrac{a_i^2}{\sigma_0^2}$ $\quad max \left(\dfrac{S}{N}\right)_T = \dfrac{2E}{N_0}$

matched to $s_1 - s_2$

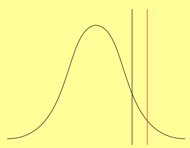
$$\left(\frac{S}{N}\right)_T = \frac{a_1 - a_2{}^2}{\sigma_0{}^2} = \frac{2E_d}{N_0}$$

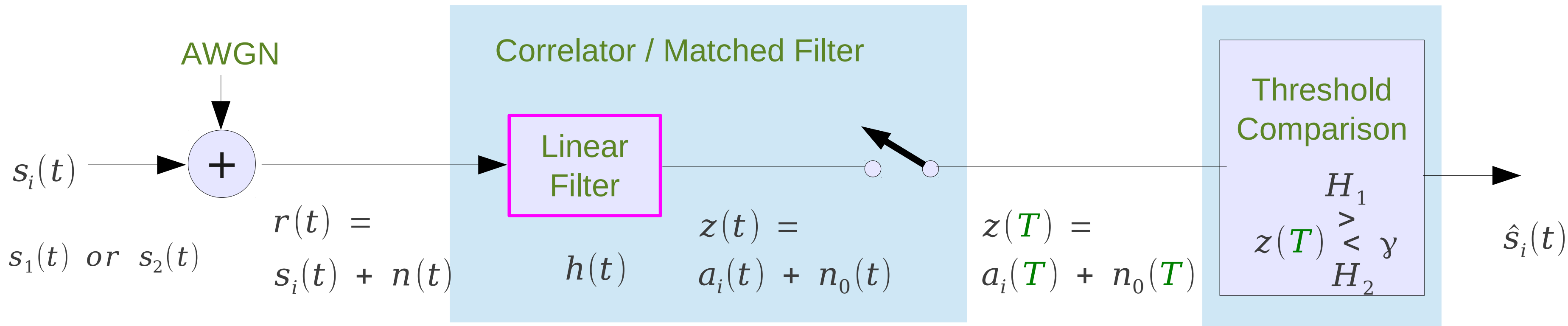
complementary error function

$$Q(x) = \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-u^2}{2}\right) du$$

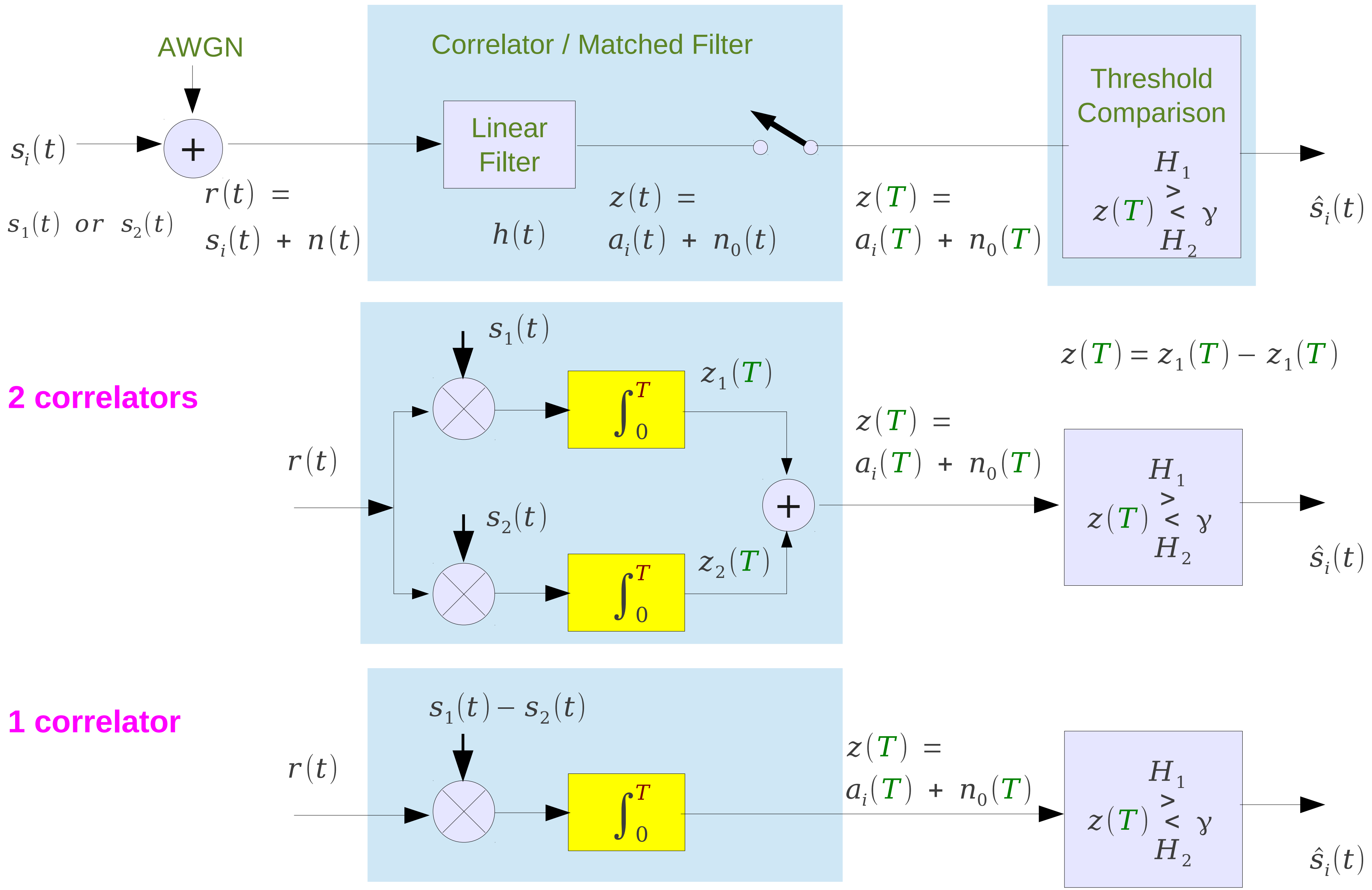
$$P_B = \int_{\gamma_0 = (a_1 - a_2)/2\sigma_0}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-u^2}{2}\right) du$$

$$= Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$$

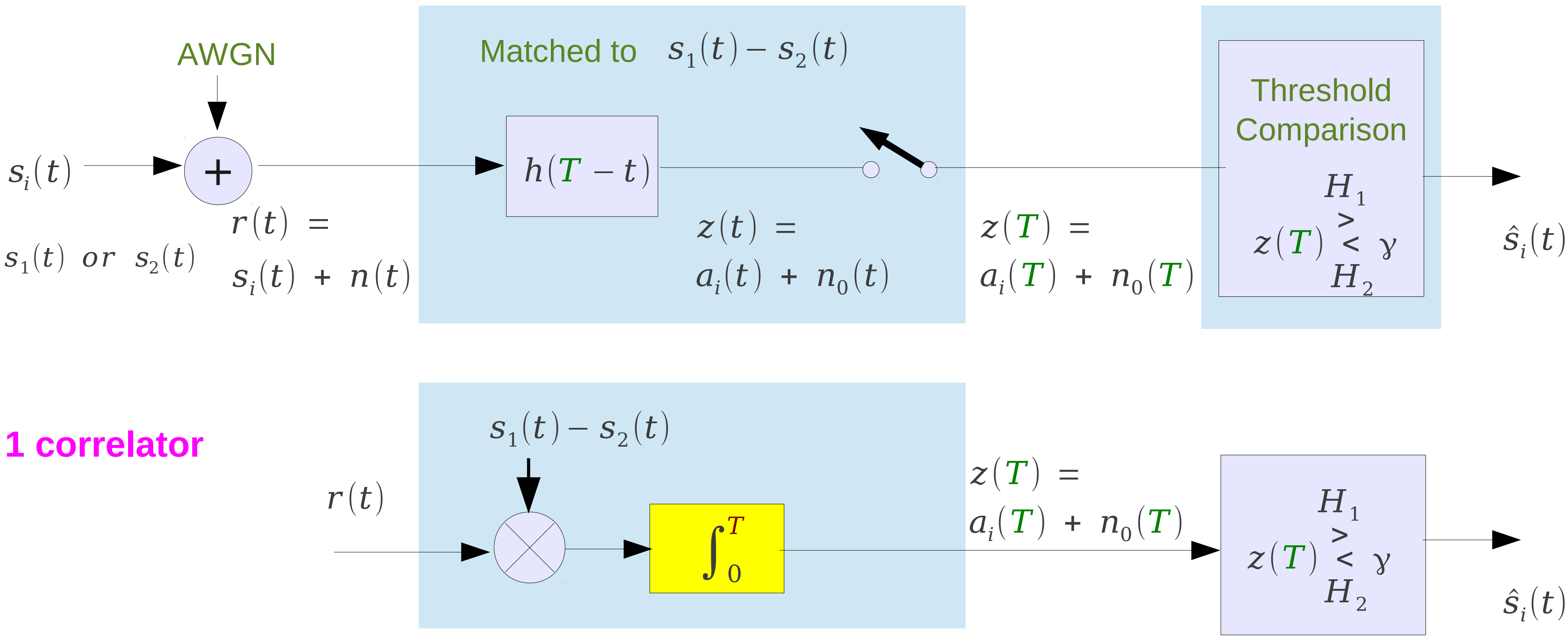




Binary Correlator Receiver



Energy Difference E_b



matched to $s_1 - s_2$

$$\left(\frac{S}{N}\right)_T = \frac{a_1 - a_2{}^2}{\sigma_0{}^2} = \frac{2E_d}{N_0}$$

$$\frac{1}{2}\frac{a_1 - a_2}{\sigma_0} = \sqrt{\frac{2E_d}{N_0}\frac{1}{4}}$$

Energy Difference

$$E_d = \int_0^T [s_1(t) - s_2(t)]^2 dt$$

Bit-Error Probability

$$P_B = Q\left(\frac{E_d}{2N_0}\right)$$

Time Averaging and Ergodicity

References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] B. Sklar, “Digital Communications: Fundamentals and Applications”