Mtg 37: Fri, 21 Nov 08, EAS 4200C

p. 36-3 (Cont'd)

3) Back to superposition:

\[ q_{ij} = q + q_{ij} \]

- true shear flow
- closed cell
- const. shear flow
- open cell
- piecewise const. shear flow

M) Multi-cell:

M.1) w/o stringers:

\[ R = \sum_{i=1}^{n} R_i \]

n cells = numb. of cells

p. 12-2 \( R_i = 0 \)

p. 36-1 \( R = 0 \)
M.2) w/ strings

\[ R^2 = \sum_i R^2_i = 0 \]

\[ \Rightarrow R^Z = R^Z + V = 0 \]
Solving Eq. 2: Equil of each stringer (Euler cut princ.)

\[ \sum F_x = 0 = \]

\[ \int \left[ \Gamma_{xx} (x + dx) - \Gamma_{xx} (x) \right] dA_3 \]

\[ A_3 \rightarrow \left[ - \hat{N}_{23} - \hat{N}_{43} + \hat{N}_{31} \right] dx \]

Taylor Series: \( \frac{d\Gamma_{xx}}{dx} dx + \text{h.o.t.} \)

\[ \frac{\hat{N}_{31}}{dx} = \hat{N}_{23} + \hat{N}_{43} + \hat{N}^{(3)} \]
\[ q^{(3)} = \int_{A_3} \frac{dM_{z}}{dx} \ dA_3 \]

**Contrib. to Shear flow by Stringer 3.**

Recall \[ V_y = \frac{dM_{z}}{dx} \], \[ V_z = \frac{dM_{y}}{dx} \]

\[ q^{(3)} = - (k_y V_y - k_y z V_z) \ Q_z^{(3)} \]
\[ - (k_z V_z - k_y z V_y) \ Q_y^{(3)} \]
\[ Q_z^{(3)} = \int q \ dA_3 \], \[ Q_y^{(3)} = \text{HW} \]

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**Stringer 2:**

\[ q_{123} = q_{212} - q_{124} + q^{(2)} \]

*because of cut*\[ \Rightarrow \text{Flow in} \]
\[ \text{flow out} \]

\[ q^{(2)} \text{ comp. as in (1)} \]
\[ Q_{2}^{(2)} = \gamma_{2} A_{2} \to \text{area of stringer 2} \]

\[ z \text{ chord of stringer 2} \]

\[ Q_{g}^{(2)} = HW_{7} \]

**Stringer 4:**

\[ \mathbf{q}_{43} = \mathbf{q}_{24} - \mathbf{q}_{41} + q \quad (4) \]

\(q_{(4)}\) comp. as above for \(Q_{g}^{(2)}\).

Hence deduce \(\mathbf{q}_{31}\) (p. 38-1).

**Superposition (again)**

\[ q_{ij} = \mathbf{q}_{ij} + q_{k} \]

\[ q_{12} = \frac{q_{12}}{\text{known}} + \frac{q_{1}}{\text{unknown}} \]

\[ q_{23} = \mathbf{q}_{23} + q_{1} - q_{2} \]

\[ q_{31} = \mathbf{q}_{31} + q_{1} - q_{3} \]
\[ q_{24} = \frac{q_{24}}{\text{known}} + \frac{q_2}{\text{unknown}}. \]

\[ q_{43} = H_W \]

\[ q_{41} = H_W \]

3 unknowns: \( q_1, q_{21}, q_3 \)

Need 3 eqs:

1) Mom. eq.: Take mom of \( V_y, V_z \) and \( \{ q_{12}, \ldots, q_{41} \} \) about any convenient pt. (usually where lines of action of \( V_y \) and \( V_z \) intersect).

2) \( \theta_1 = \theta_2 \) \text{ Comp. eqs}

3) \( \theta_2 = \theta_3 \)
NACA airfoil:
Meeting 39: Mon, 1 Dec 08, EAS 4200 C (39-1)

Solving P2: For each cell,
- Follow path $x_i$
- equil. of each stringer on path

2 ways:
1) Complete method: FBD p. 38-1
2) Consequence of 1st method:

\[
\begin{align*}
\vec{q}_{16} &= \vec{q}_{12} - \vec{q}_{15} \\
\vec{q}_{26} &= \vec{q}_{21} - \vec{q}_{25} \\
\vec{q}_{76} &= \vec{q}_{71} - \vec{q}_{75} \\
\vec{q}_{36} &= \vec{q}_{31} - \vec{q}_{35} \\
\vec{q}_{18} &= \vec{q}_{14} - \vec{q}_{15} \\
\vec{q}_{28} &= \vec{q}_{24} - \vec{q}_{25} \\
\vec{q}_{78} &= \vec{q}_{74} - \vec{q}_{75} \\
\vec{q}_{38} &= \vec{q}_{34} - \vec{q}_{35} \\
\vec{q}_{48} &= \vec{q}_{45} - \vec{q}_{45} \\
\vec{q}_{58} &= \vec{q}_{56} - \vec{q}_{55} \\
\vec{q}_{68} &= \vec{q}_{67} - \vec{q}_{65} \\
\end{align*}
\]
Note: (Jared)

Q: What if we cut cell walls at one stringer was isolated.

\[ q_{12} \neq 0 \]
\[ q_{23} = q_{34} = 0 \]

\[ q_{41} \neq 0 \]

\[ q_{31} = q_{13} - q_{14} \]

\[ q^{(3)} = 0 \]

\[ q^{(3)} = 0 \] not true. (p.38-2)
Another cut

isolated

HWY:
MTG 40: Fri, 5 Dec 08. EAS 4200C 450.1

- Shear buckling (HW7)

Plotting buckling shape under shear

Expr. \{ C_{22}, C_{13}, C_{31}, C_{33} \} in terms of \( C_{11} \), and for \( \nu = 1.5 \)

1) Find \( \lambda \) for \( \nu = 1.5 \)

Eq. (30) in wiki

2) Eval. num. \( K = 5 \times 5 \) in Eq (26)

3) \( K = [ K_{ij} ] \)

\[
\begin{bmatrix}
K_{22} & K_{23} & \cdots & K_{25} \\
\vdots & \ddots & \vdots & \vdots \\
K_{52} & \cdots & K_{55}
\end{bmatrix}
\begin{bmatrix}
C_{22} \\
\vdots \\
C_{33}
\end{bmatrix}
= \begin{bmatrix}
-\frac{4}{9} C_{11} \\
0 \\
0
\end{bmatrix}
\]

Solve \( \{ C_{22}, C_{13}, C_{31}, C_{33} \} \) in terms of \( C_{11} \)
\[ K_{1}^{-1} \text{ inverse of } K \]

\[
\begin{pmatrix}
C_{11} \\
C_{13} \\
C_{31} \\
C_{33}
\end{pmatrix}
= K_{1}^{-1}
\begin{pmatrix}
-4 & C_{11} \\
0 & 0
\end{pmatrix}
\]

\[ u_{z} = C_{11} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \]

\[ + C_{22} \sin \frac{2\pi x}{a} \sin \frac{2\pi y}{b} \]

\[ + C_{13} \sin \frac{\pi x}{a} \sin \frac{3\pi y}{b} \]

\[ + C_{21} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} \]

\[ + C_{33} \sin \frac{3\pi x}{a} \sin \frac{3\pi y}{b} \]

Set \( C_{11} = 1 \), plot \( u_{z}(x, y) \).
Note: Cont. from p. 39-2

Ans. in 2 parts.

Part 1: Equil. of isolated stringer (done)

Part 2: Closed cell, equil. of stringers

**Stringer 1:**

\[ \tilde{q}_{12} = \tilde{q}_{31} + \tilde{q}_{41} + q \]

**Stringer 2:**

\[ q_{24} = q_{12} - q_{23} + q \]

**Stringer 4:**

\[ q_{41} = q_{24} + q_{34} + q \]

\[ = (q_{41} + q (1)) + q (2) + q_{34} + q (4) \]
p. 40-3 cont'd

\[ 0 = q^{(1)} + q^{(2)} + q^{(4)} = -q \]

Not true (or possible) since \( \neq 0 \)

\[ 0 = q^{(1)} + q^{(2)} + q^{(3)} + q^{(4)} \]

\[ 0 = \sum_{e=1}^{4} q^{(e)} \]

p. 38-2:

\[ q^{(e)} = n_{x} Q_{x}^{(e)} + n_{y} Q_{y}^{(e)} \]

\[ n_{x} := - (k_{x} \nabla x - k_{y} x \nabla z) \]

\[ n_{y} := \text{HW} \]

\[ \sum_{e=1}^{4} q^{(e)} = n_{x} \sum_{e=1}^{4} Q_{x}^{(e)} + n_{y} \sum_{e=1}^{4} Q_{y}^{(e)} \]

\[ \boxed{0} \]

\[ \boxed{0} \]
HW7: NACA airfoil - "back of the envelope" verification

\[ \overrightarrow{BE} = \overrightarrow{FH} \]
\[ = \frac{1}{2} (\overrightarrow{BE} + \overrightarrow{FH}) \]

Verify magnitudes of: \( I_y, I_z, I_{yz} \)

(C-section pb) \( \theta \) angle of N.A.

\( \frac{\tau_{xx}}{\theta} \) @ each stringer
Bi-dir. bending (cont'd) p. 34-2 L41-3

R& R Sec 4.2 (4.22a) \rightarrow (4.28c)

\[ M_y = -EI_y \frac{d^2 v_0}{dx^2} - EI_y \frac{d^2 w_0}{dx^2} \] (4.26)

\[ M_z = -EI_z \frac{d^2 v_0}{dx^2} - EI_z \frac{d^2 w_0}{dx^2} \]

Mat. form:

\[
\begin{bmatrix}
M_y \\
M_z
\end{bmatrix} =
\begin{bmatrix}
I_Y & I_y z \\
I_y z & I_z
\end{bmatrix}
\begin{bmatrix}
-E \chi_{xz} \\
-E \chi_{yz}
\end{bmatrix}
\]

(4.25)

\[ \varepsilon_{xx} = -\chi_{xy} - \chi_{xz} \]

= 1 \times \chi \begin{bmatrix}
-\chi_{xz} \\
-\chi_{yz}
\end{bmatrix}

\text{curvatures } J
\[ \sigma_{xx} = \frac{E}{L} \left[ \begin{array}{c} 1 \quad -1 \\ -1 \quad 1 \end{array} \right] \left[ \begin{array}{c} M_y \\ M_z \end{array} \right] \]

\[ D = I_y I_z - (I_{yz})^2 \]

Shear flow: \[ q = - \int_A \frac{d\sigma_{xx}}{dx} dA \]

\[ \frac{d \sigma_{xx}}{dx} = L z \quad \forall \quad I^{-1} \left[ \begin{array}{c} \frac{dM_y}{dx} \\ \frac{dM_z}{dx} \end{array} \right] \]

\[ V_y = \frac{dM_y}{dx}, \quad V_z = \frac{dM_z}{dx} \]

\[ Q_y = \int_A z dA, \quad Q_z = \int_A y dA \]