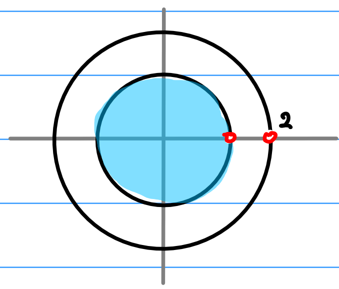
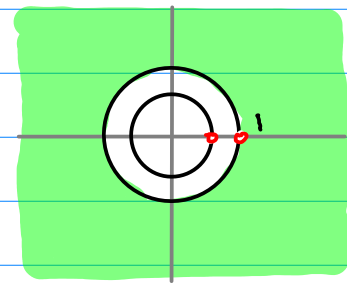
Laurent Series and z-Transform Examples case 0.A

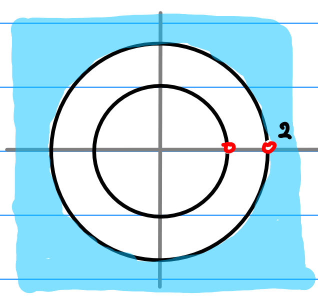
20171106

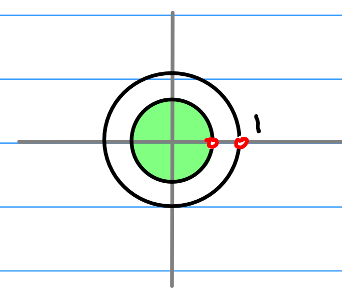
1. A $f(z) = \dfrac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} X(z) = \dfrac{-0.5\, z^2}{(z-1)(z-0.5)}$



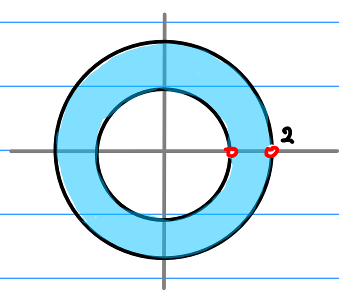


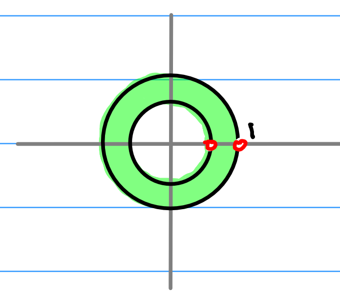
$$\sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1 \right] z^n = \sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1 \right] z^n$$





$$\sum_{n=-1}^{-\infty} \left(1 - \left(\tfrac{1}{2}\right)^{n+1}\right) z^n = \sum_{n=-1}^{-\infty} \left(1 - \left(\tfrac{1}{2}\right)^{n+1}\right) z^n$$

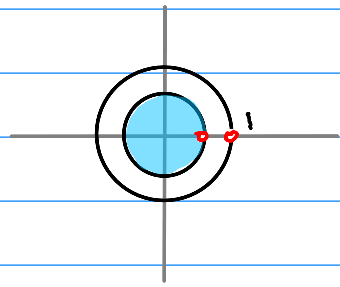


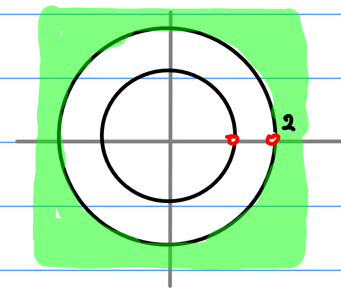


$$\sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^n = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^n$$

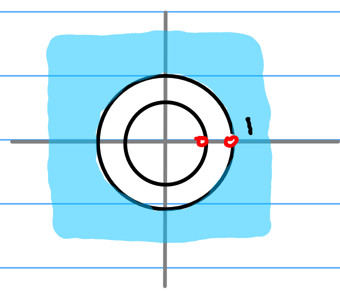
2.A

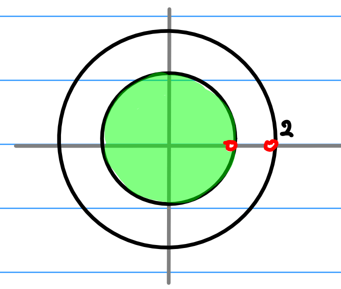
$$f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} X(z) = \frac{-1}{(z-1)(z-2)}$$



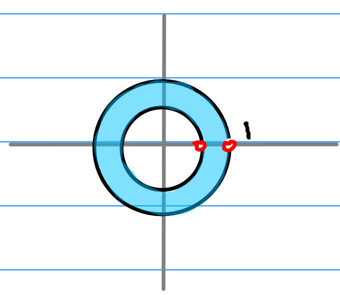


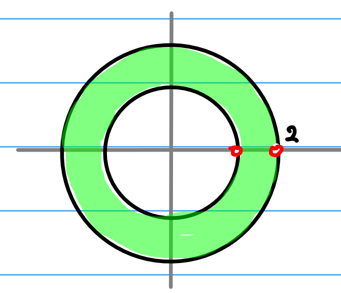
$$\sum_{n=1}^{\infty}\left[1-2^{n-1}\right]z^n = \sum_{n=1}^{\infty}\left[1-2^{n-1}\right]z^n$$





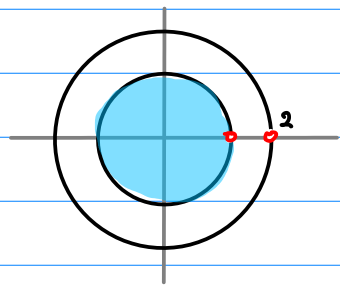
$$\sum_{n=0}^{-\infty}\left[2^{n-1}-1\right]z^n = \sum_{n=0}^{-\infty}\left[2^{n-1}-1\right]z^n$$

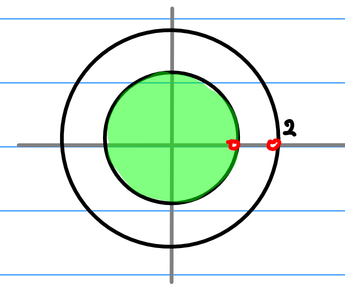




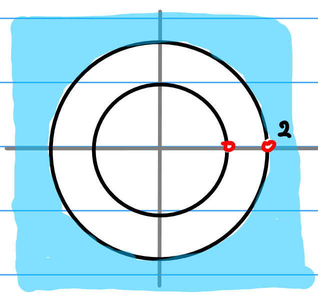
$$\sum_{n=1}^{\infty}1\cdot z^n + \sum_{n=0}^{-\infty}2^{n-1}\cdot z^n = \sum_{n=1}^{\infty}1\cdot z^n + \sum_{n=0}^{-\infty}2^{n-1}\cdot z^n$$

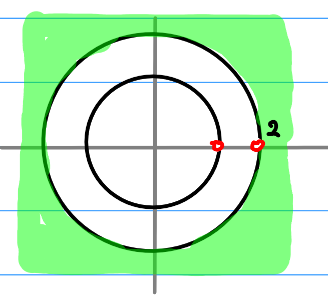
3. A $f(z) = \dfrac{-1}{(z-1)(z-2)} = X(z) = \dfrac{-1}{(z-1)(z-2)}$



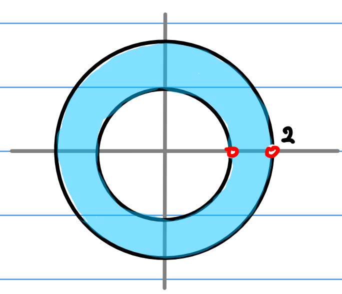


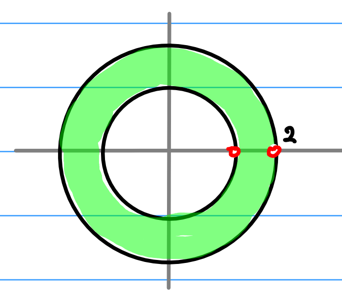
$$\sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1 \right] z^n \xrightarrow{-n} \sum_{n=0}^{-\infty} \left[2^{n-1} - 1 \right] z^{-n}$$





$$\sum_{n=-1}^{-\infty} \left[1 - \left(\tfrac{1}{2}\right)^{n+1} \right] z^n \xrightarrow{-n} \sum_{n=1}^{\infty} \left[1 - 2^{n-1} \right] z^{-n}$$

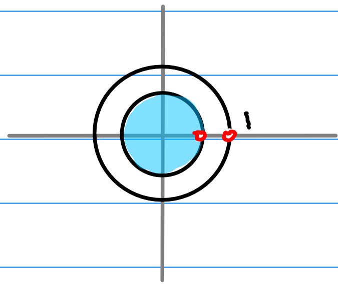


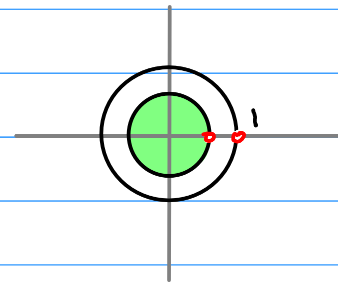


$$\sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^n \xrightarrow{-n} + \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

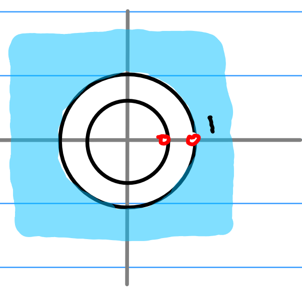
4.A

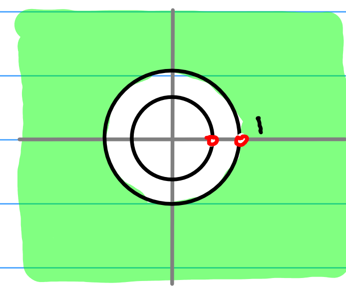
$$f(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)} = X(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$



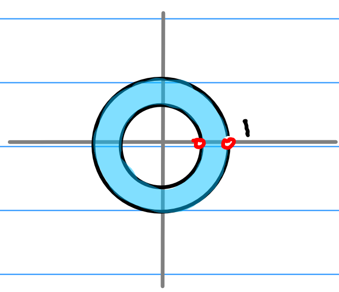


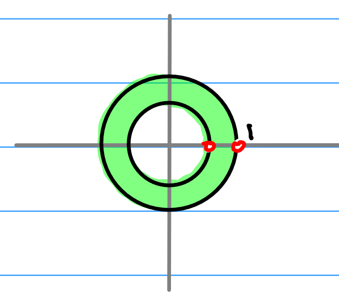
$$\sum_{n=1}^{\infty} \left[1 - 2^{n-1}\right] z^n \xrightarrow{-n} \sum_{n=-1}^{\infty} \left[1 - \left(\tfrac{1}{2}\right)^{n+1}\right] z^{-n}$$





$$\sum_{n=-1}^{-\infty} \left[2^{n-1} - 1\right] z^n \xrightarrow{-n} \sum_{n=1}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1\right] z^{-n}$$





$$+\sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1} z^n \xrightarrow{-n} +\sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^{-n}$$

1. A

$$f(z) = \frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} X(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$

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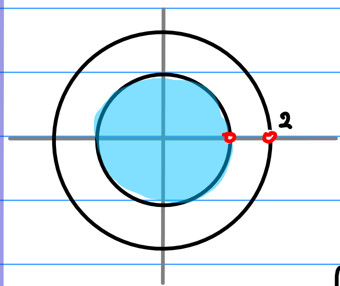

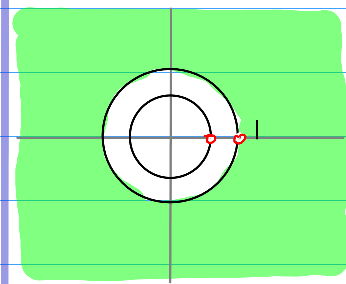

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} - 1 & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^n$$

$$X(z) = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^n$$

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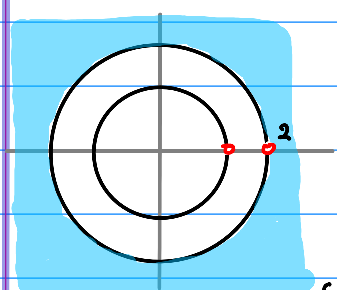

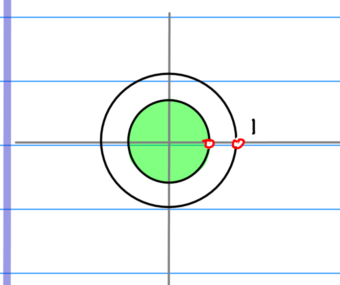

$$a_n = \begin{cases} 0 & (n \geq 0) \\ \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) & (n < 0) \end{cases}$$

$$x_n = \begin{cases} \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) z^n$$

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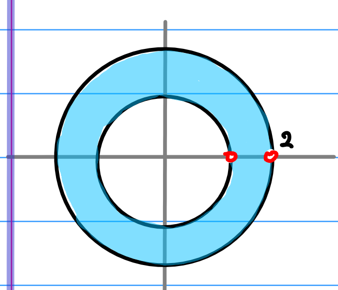

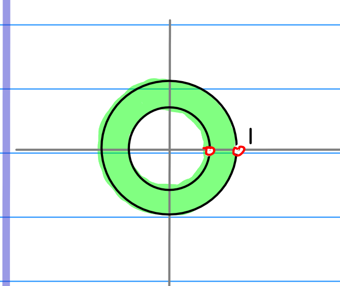

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

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$$f(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

$$X(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

2.A

$$f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} X(z) = \frac{-1}{(z-1)(z-2)}$$

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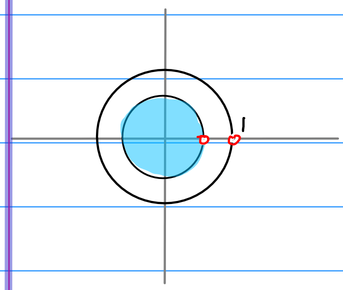

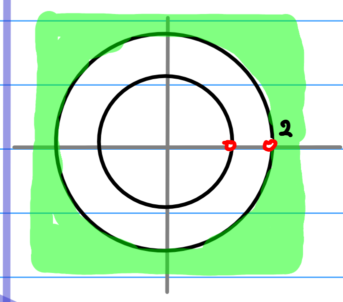

$$a_n = \begin{cases} [1-2^{n-1}] & (n>0) \\ 0 & (n\le 0) \end{cases} \qquad x_n = \begin{cases} 0 & (n\ge 0) \\ [1-2^{n-1}] & (n<0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} [1-2^{n-1}] z^n \qquad X(z) = \sum_{n=1}^{\infty} [1-2^{n-1}] z^n$$

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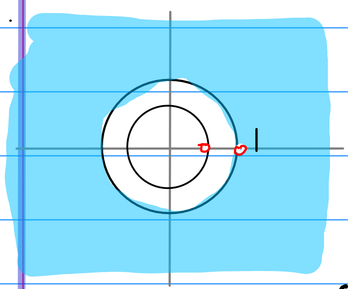

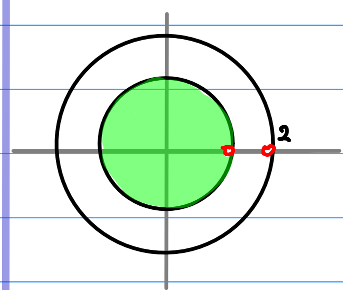

$$a_n = \begin{cases} 0 & (n>0) \\ [2^{n-1}-1] & (n\le 0) \end{cases} \qquad x_n = \begin{cases} [2^{n-1}-1] & (n\ge 0) \\ 0 & (n<0) \end{cases}$$

$$f(z) = \sum_{n=0}^{-\infty} [2^{n-1}-1] z^n \qquad X(z) = \sum_{n=0}^{-\infty} [2^{n-1}-1] z^n$$

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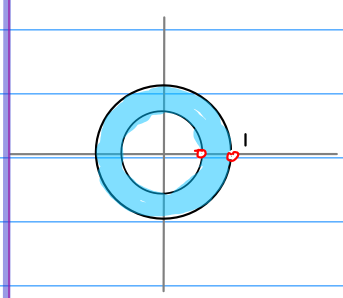

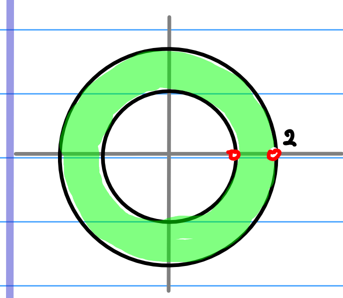

$$a_n = \begin{cases} 1 & (n>0) \\ 2^{n-1} & (n\le 0) \end{cases} \qquad x_n = \begin{cases} 2^{n-1} & (n\ge 0) \\ 1 & (n<0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} 1\cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^n \qquad X(z) = \sum_{n=1}^{\infty} 1\cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^n$$

3 . A $f(z) = \dfrac{-1}{(z-1)(z-2)}$ $=$ $X(z) = \dfrac{-1}{(z-1)(z-2)}$

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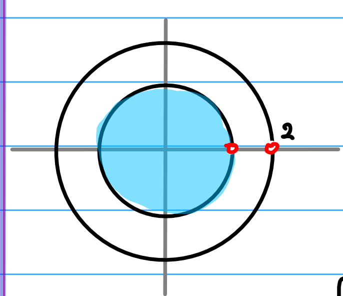

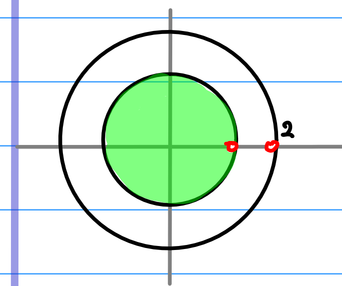

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases} \quad \Rightarrow \quad x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^n \qquad X(z) = \sum_{n=0}^{-\infty} \left[2^{n-1} - 1 \right] z^{-n}$$

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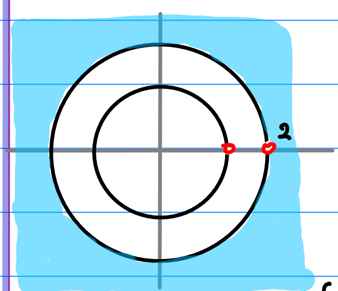

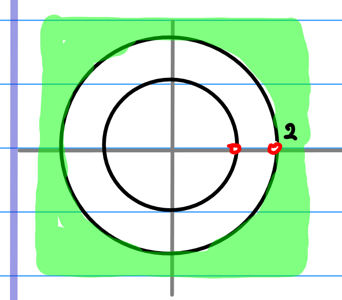

$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases} \quad \Rightarrow \quad x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} \left[1 - \left(\frac{1}{2}\right)^{n+1} \right] z^n \qquad X(z) = \sum_{n=1}^{\infty} \left[1 - 2^{n-1} \right] z^{-n}$$

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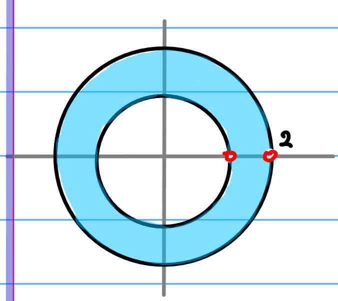

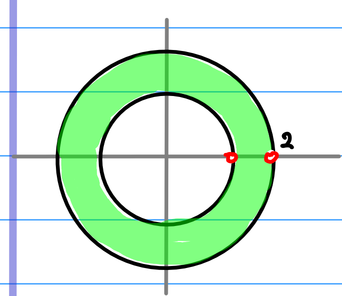

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases} \quad \Rightarrow \quad x_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n \qquad X(z) = + \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

4.A $\quad f(z) = \dfrac{-0.5z^2}{(z-1)(z-0.5)} = X(z) = \dfrac{-0.5z^2}{(z-1)(z-0.5)}$

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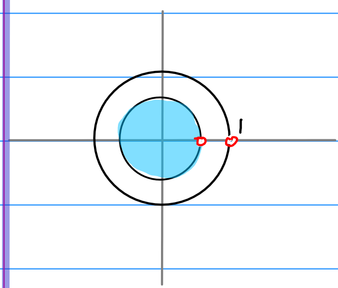

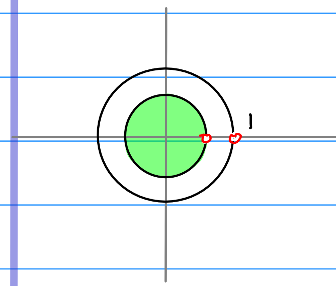

$$a_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \le 0) \end{cases} \qquad x_n = \begin{cases} 0 & (n \ge 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} [1 - 2^{n-1}] z^n \qquad X(z) = \sum_{n=-1}^{-\infty} [1 - \left(\tfrac{1}{2}\right)^{n+1}] z^{-n}$$

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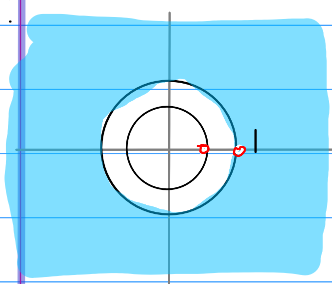

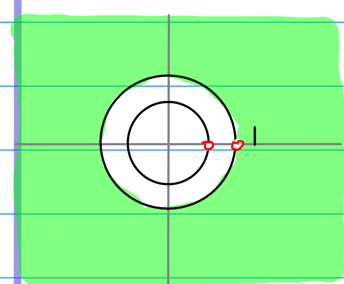

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$$f(z) = \sum_{n=-1}^{-\infty} [2^{n-1} - 1] z^n \qquad X(z) = \sum_{n=1}^{\infty} [\left(\tfrac{1}{2}\right)^{n+1} - 1] z^{-n}$$

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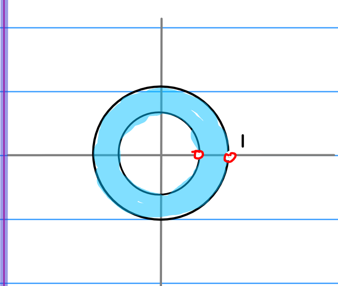
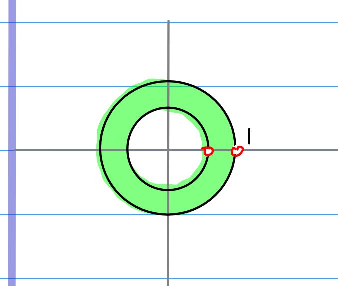
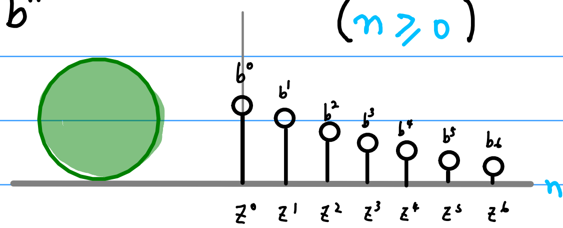




$$a_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \le 0) \end{cases} \qquad x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \ge 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = +\sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1} z^n \qquad X(z) = +\sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^{-n}$$

b^n $(n \geq 0)$



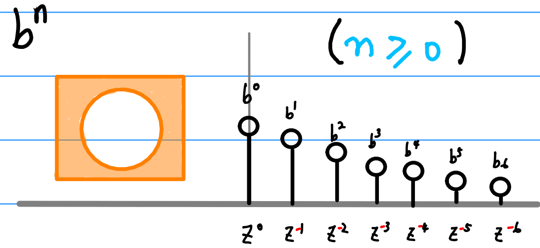
$$X(z^{-1}) = \frac{z^{-1}}{z^{-1} - 0.5} \qquad |z| < 2$$

$$f(z) = \frac{2}{2-z} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^n$$

$$a_n = \left(\frac{1}{2}\right)^n$$

$$= p^{-n} \qquad \boxed{p = 2}$$

b^n $(n \geq 0)$



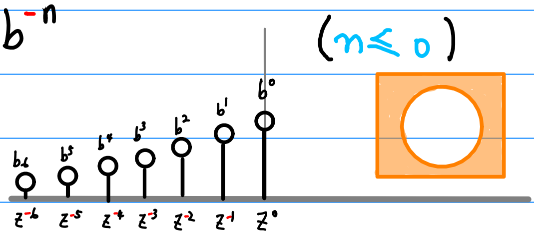
$$|z| > \frac{1}{2}$$

$$X(z) = \frac{z}{z - 0.5} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$$

$$x_n = \left(\frac{1}{2}\right)^n$$

$$= p^n \qquad \boxed{p = \frac{1}{2}}$$

b^{-n} $(n \leq 0)$



$$X(z^{-1}) = \frac{2}{2 - z^{-1}} \qquad |z| > \frac{1}{2}$$

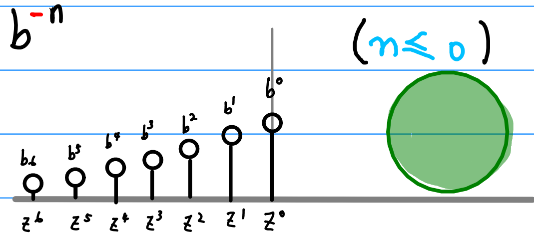
$$f(z) = \frac{z}{z - 0.5} = \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{-n} z^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$$

$$a_n = \left(\frac{1}{2}\right)^{-n}$$

$$= p^{-n} \qquad \boxed{p = \frac{1}{2}}$$

b^{-n} $(n \leq 0)$



$$|z| < 2$$

$$X(z) = \frac{2}{2-z} = \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{-n} z^{-n}$$

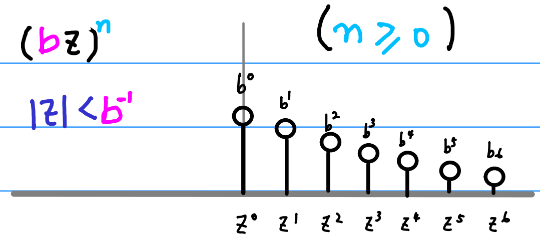
$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^n$$

$$x_n = \left(\frac{1}{2}\right)^{-n}$$

$$= p^n \qquad \boxed{p = 2}$$

$(bz)^n$ $\quad (n \geq 0)$

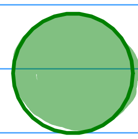
$|z| < b^{-1}$



$$f(z) = \frac{1}{1-bz} = \frac{b^{-1}}{b^{-1}-z}$$

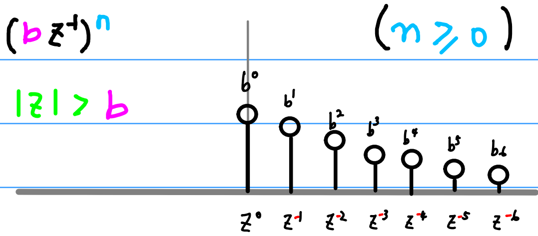
$$a_n = b^n$$
$$= p^{-n}$$

$\boxed{p = b^{-1}}$



$(bz^{-1})^n$ $\quad (n \geq 0)$

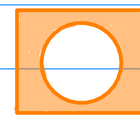
$|z| > b$



$$X(z) = \frac{1}{1-b/z} = \frac{z}{z-b}$$

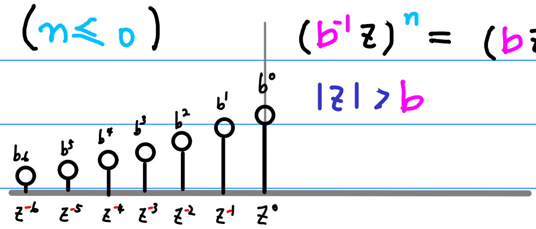
$$x_n = b^n$$
$$= p^n$$

$\boxed{p = b}$



$(n \leq 0)$ $\quad (b^{-1}z)^n = (bz^{-1})^{-n}$

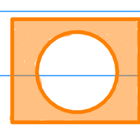
$|z| > b$



$$f(z) = \frac{1}{1-(bz^{-1})} = \frac{z}{z-b}$$

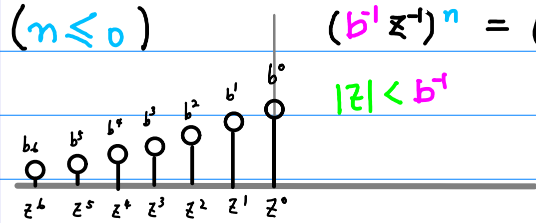
$$a_n = b^{-n}$$
$$= p^{-n}$$

$\boxed{p = b}$



$(n \leq 0)$ $\quad (b^{-1}z^{-1})^n = (bz)^{-n}$

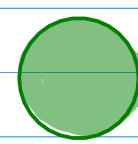
$|z| < b^{-1}$



$$X(z) = \frac{1}{1-(bz)} = \frac{b^{-1}}{b^{-1}-z}$$

$$x_n = b^{-n}$$
$$= p^n$$

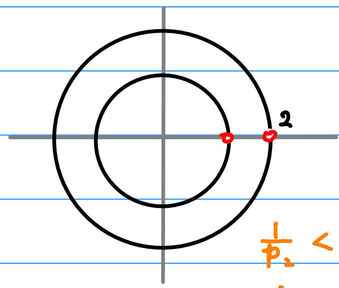
$\boxed{p = b^{-1}}$



$$f(z) \xrightarrow{z^{-1}} X(z)$$

1.A

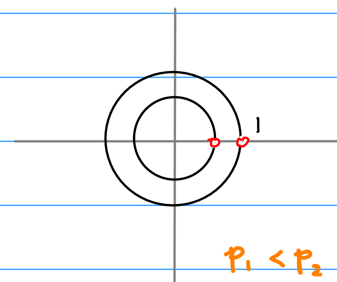
$$\frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} \frac{-0.5z^2}{(z-1)(z-0.5)}$$



$\frac{1}{p_2} < \frac{1}{p_1}$

1 2

$\boxed{p_1 = 1,\ p_2 = 2}$



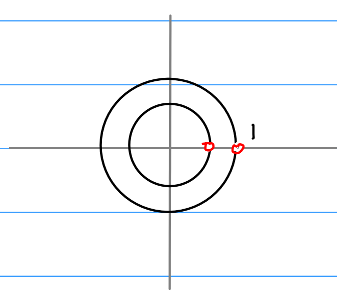
$p_1 < p_2$

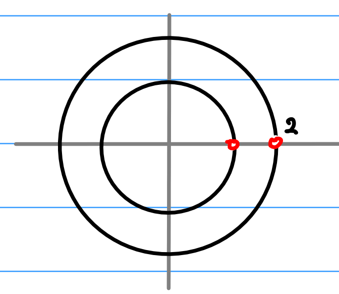
0.5 1.0

$\boxed{p_1 = 0.5,\ p_2 = 1.0}$

1.B

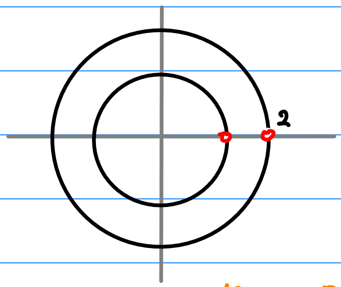
$$\frac{-0.5z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} \frac{-1}{(z-1)(z-2)}$$

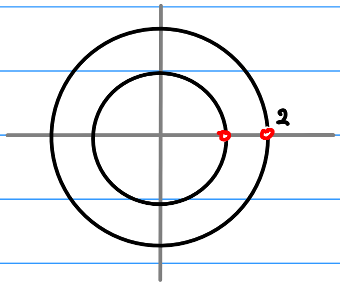




$$f(z) = X(z)$$

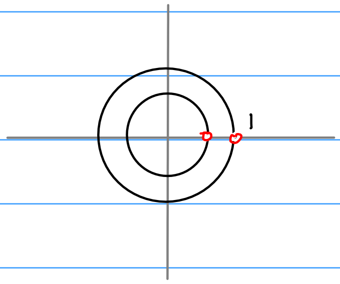
3.A $\quad \dfrac{-1}{(z-1)(z-2)} = \dfrac{-1}{(z-1)(z-2)}$

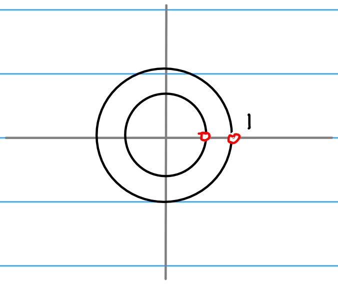




$p_1 < p_2$ $\quad$ $p_1 < p_2$

4.B $\quad \dfrac{-0.5\,z^2}{(z-1)(z-0.5)} = \dfrac{-0.5\,z^2}{(z-1)(z-0.5)}$

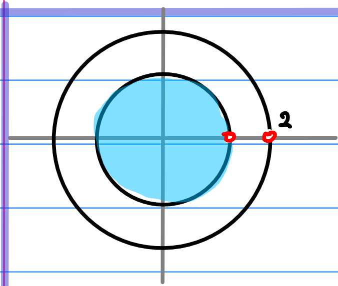




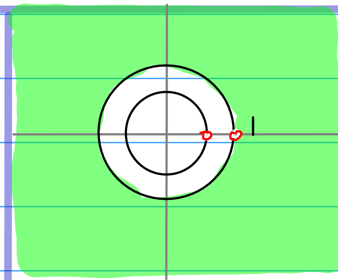
$p_1 < p_2$ $\quad$ $p_1 < p_2$

$$f(z) \xrightarrow{Z^{-1}} X(z)$$

Ⅰ

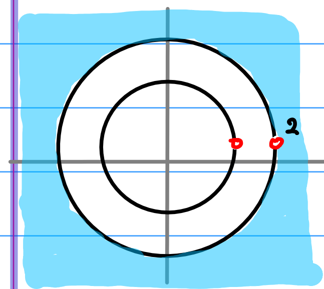


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

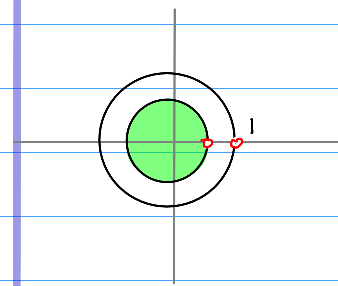


$$\begin{cases} 0 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} - 1 & (n \leq 0) \end{cases}$$

Ⅱ

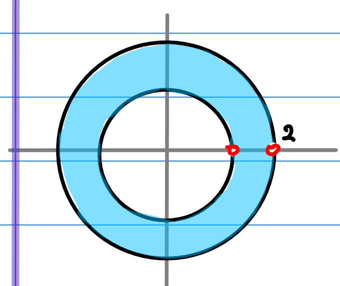


$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

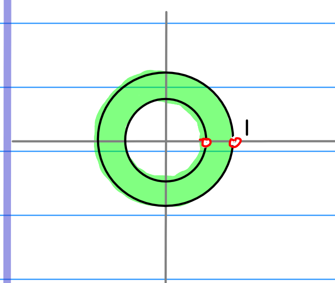


$$\begin{cases} 1 - \left(\frac{1}{2}\right)^{n+1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

Ⅲ

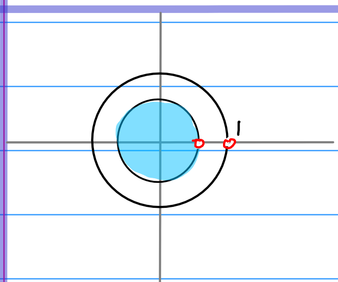


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

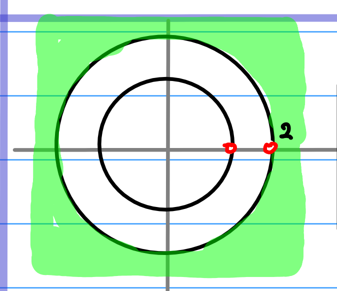


$$\begin{cases} 1 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} & (n \leq 0) \end{cases}$$

Ⅰ

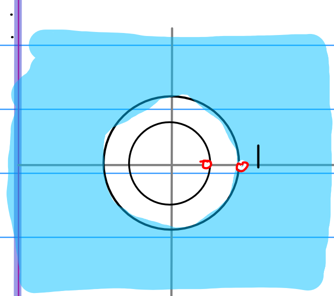


$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

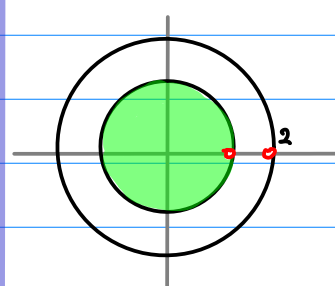


$$\begin{cases} 0 & (n \geq 0) \\ 1 - 2^{n-1} & (n < 0) \end{cases}$$

Ⅱ

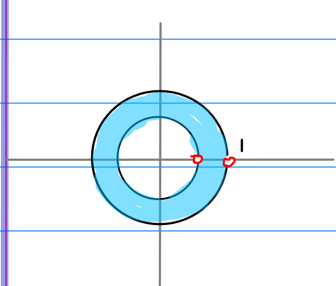


$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

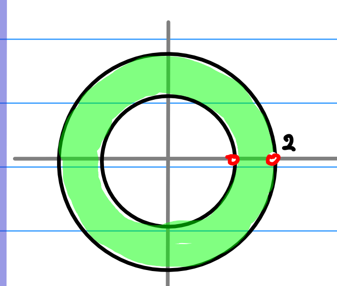


$$\begin{cases} 2^{n-1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

Ⅲ



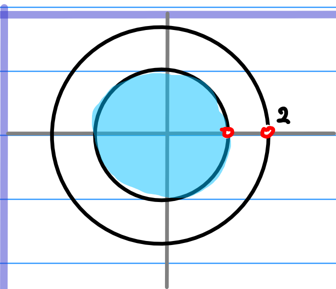
$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$



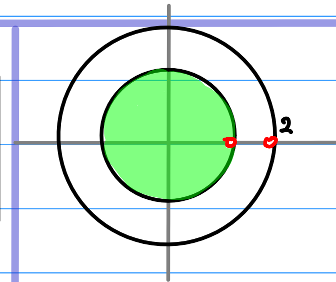
$$\begin{cases} 2^{n-1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$f(z) = X(z)$

Ⅰ

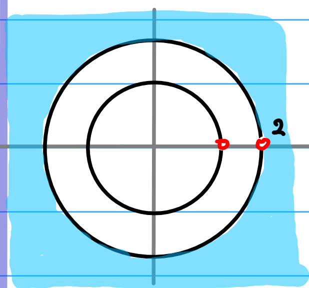


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

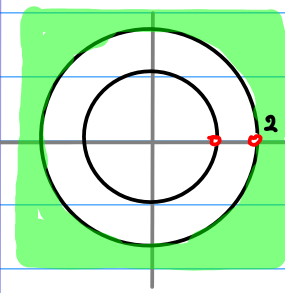


$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

Ⅱ

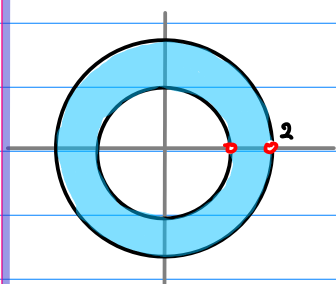


$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

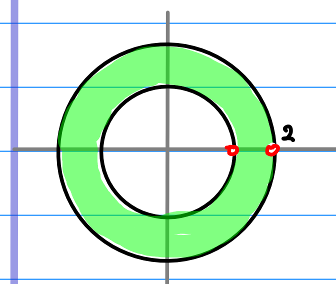


$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

Ⅲ

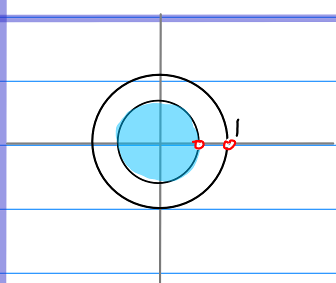


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

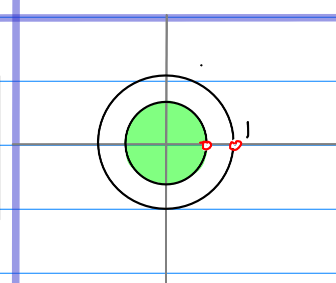


$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

Ⅰ

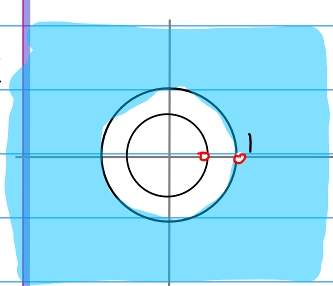


$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

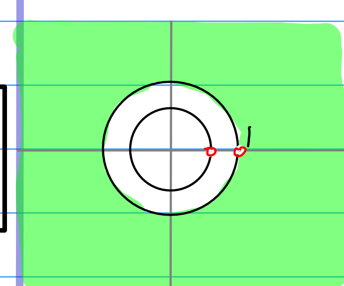


$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

Ⅱ

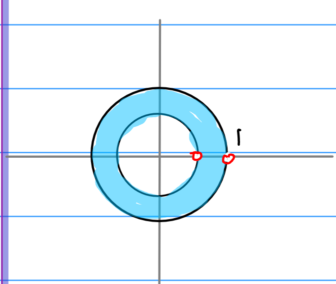


$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

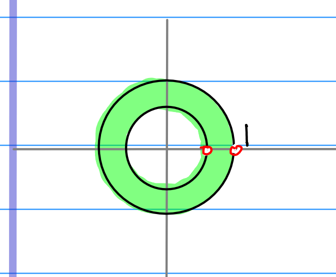


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

Ⅲ



$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$



$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) \xrightarrow{Z^{-1}} X(z)$$

	$f(z)$	$X(z)$
Ⅰ	$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$	$\begin{cases} 0 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} - 1 & (n \leq 0) \end{cases}$
	$* -1$	$* -1$
Ⅱ	$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$	$\begin{cases} 1 - \left(\frac{1}{2}\right)^{n+1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$
Ⅲ	$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$	$\begin{cases} 1 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} & (n \leq 0) \end{cases}$

	$f(z)$	$X(z)$
Ⅰ	$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$	$\begin{cases} 0 & (n \geq 0) \\ 1 - 2^{n-1} & (n < 0) \end{cases}$
	$* -1$	$* -1$
Ⅱ	$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$	$\begin{cases} 2^{n-1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$
Ⅲ	$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$	$\begin{cases} 2^{n-1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$

$$f(z) = X(z)$$

	$f(z)$	$X(z)$
Ⅰ	$\begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$	$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$
	$*-1$	$*-1$
Ⅱ	$\begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$	$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$
Ⅲ	$\begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$	$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$

	$f(z)$	$X(z)$
Ⅰ	$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$	$\begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$
	$*-1$	$*-1$
Ⅱ	$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$	$\begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$
Ⅲ	$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$	$\begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$

$\left(\frac{1}{2}\right)^{n+1} - 1$	$(n \geq 0)$
0	$(n < 0)$

$1 - 2^{n-1}$	$(n > 0)$
0	$(n \leq 0)$

n	-3	-2	-1	0	1	2	3	4
	$\left(\frac{1}{2}\right)^{-3+1}$	$\left(\frac{1}{2}\right)^{-2+1}$	$\left(\frac{1}{2}\right)^{-1+1}$	$\left(\frac{1}{2}\right)^{0+1}$	$\left(\frac{1}{2}\right)^{1+1}$	$\left(\frac{1}{2}\right)^{2+1}$	$\left(\frac{1}{2}\right)^{3+1}$	$\left(\frac{1}{2}\right)^{4+1}$
	$\left(\frac{1}{2}\right)^{-2}$	$\left(\frac{1}{2}\right)^{-1}$	$\left(\frac{1}{2}\right)^{0}$	$\left(\frac{1}{2}\right)^{1}$	$\left(\frac{1}{2}\right)^{2}$	$\left(\frac{1}{2}\right)^{3}$	$\left(\frac{1}{2}\right)^{4}$	$\left(\frac{1}{2}\right)^{5}$
	2^2	2^1	2^0					
	$2^{-(-3)-1}$	$2^{-(-2)-1}$	$2^{-(-1)-1}$					
	2^{-n-1}	2^{-n-1}	2^{-n-1}					

$f(z)$ $\qquad p_1 < p_2$

$|z| < p_1$

$$\boxed{\begin{matrix} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} & (n \geq 0) \\ 0 & (n < 0) \end{matrix}}$$

$p_1 = 1/2 \quad p_2 = 1$

$* -1$

$|z| > p_2$

$$\boxed{\begin{matrix} 0 & (n \geq 0) \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} & (n < 0) \end{matrix}}$$

$p_1 = 1 \quad p_2 = 2$

$p_1 < |z| < p_2$

$$\boxed{\begin{matrix} \left(\frac{1}{p_2}\right)^{n+1} & (n \geq 0) \\ \left(\frac{1}{p_1}\right)^{n+1} & (n < 0) \end{matrix}}$$

$p_1 = 1 \quad p_2 = 2$

$X(z)$ $\qquad p_1 < p_2$

$|z| < p_1$

$$\boxed{\begin{array}{l} 0 \\ (p_1)^{n-1} - (p_2)^{n-1} \end{array}} \quad \begin{array}{l} (n > 0) \\ (n \leq 0) \end{array}$$

$p_1 = 1/2 \qquad p_2 = 1$

$* -1$

$|z| > p_2$

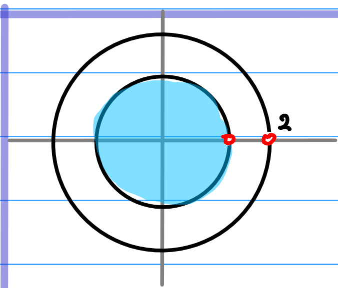
$$\boxed{\begin{array}{l} (p_2)^{n-1} - (p_1)^{n-1} \\ 0 \end{array}} \quad \begin{array}{l} n > 0) \\ n \leq 0) \end{array}$$

$p_1 = 1/2 \qquad p_2 = 1$

$p_1 < |z| < p_2$

$$\boxed{\begin{array}{l} (p_2)^{n-1} \\ (p_1)^{n-1} \end{array}} \quad \begin{array}{l} (n > 0) \\ (n \leq 0) \end{array}$$

Ⅰ

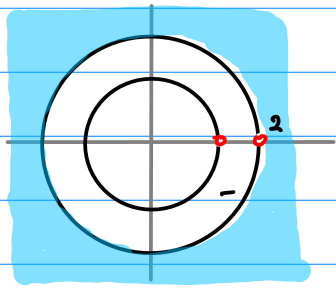


$$\begin{array}{ll} \left(\frac{1}{2}\right)^{n+1}-1 & (n \geq 0) \\ 0 & (n < 0) \end{array}$$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n+1}-\left(\frac{1}{p_1}\right)^{n+1} \\ 0 \end{array}$$

$p_1 = 1 \quad p_2 = 2$

Ⅱ

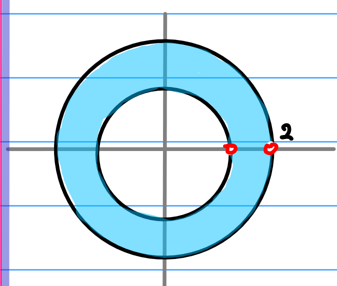


$$\begin{array}{ll} 0 & (n \geq 0) \\ 1-\left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{array}$$

$$\begin{array}{l} 0 \\ \left(\frac{1}{p_1}\right)^{n+1}-\left(\frac{1}{p_2}\right)^{n+1} \end{array}$$

$p_1 = 1 \quad p_2 = 2$

Ⅲ

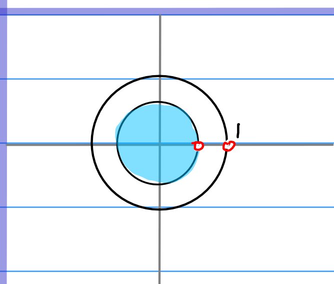


$$\begin{array}{ll} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{array}$$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n+1} \\ \left(\frac{1}{p_1}\right)^{n+1} \end{array}$$

$p_1 = 1 \quad p_2 = 2$

Ⅰ

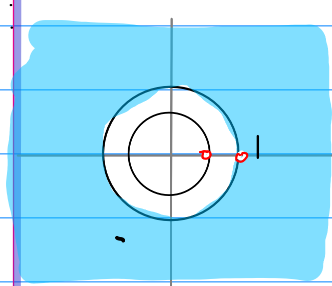


$$\begin{array}{ll} 1-2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{array}$$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n-1}-\left(\frac{1}{p_1}\right)^{n-1} \\ 0 \end{array}$$

$p_1 = \frac{1}{2} \quad p_2 = 1$

Ⅱ

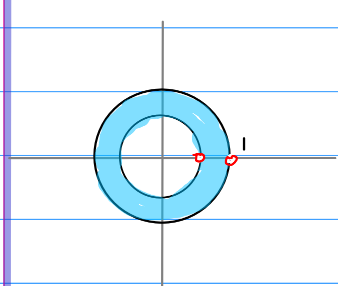


$$\begin{array}{ll} 0 & (n > 0) \\ 2^{n-1}-1 & (n \leq 0) \end{array}$$

$$\begin{array}{l} 0 \\ \left(\frac{1}{p_1}\right)^{n-1}-\left(\frac{1}{p_2}\right)^{n-1} \end{array}$$

$p_1 = \frac{1}{2} \quad p_2 = 1$

Ⅲ



$$\begin{array}{ll} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{array}$$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n-1} \\ \left(\frac{1}{p_1}\right)^{n-1} \end{array}$$

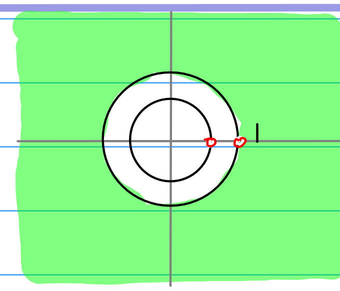
$p_1 = \frac{1}{2} \quad p_2 = 1$

I

0

$(p_1)^{n+1} - (p_2)^{n+1}$

$p_1 = 1/2 \quad p_2 = 1$



$0 \quad (n > 0)$

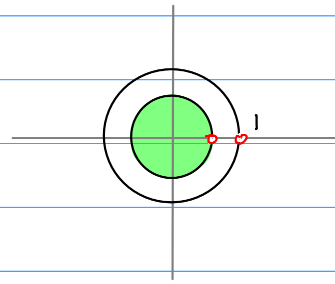
$\left(\frac{1}{2}\right)^{n+1} - 1 \quad (n \leq 0)$

II

$(p_2)^{n+1} - (p_1)^{n+1}$

0

$p_1 = 1/2 \quad p_2 = 1$



$1 - \left(\frac{1}{2}\right)^{n+1} \quad (n > 0)$

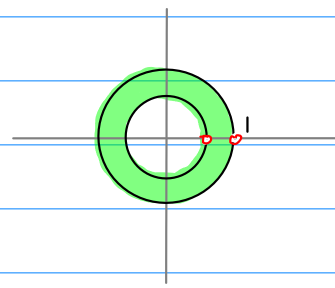
$0 \quad (n \leq 0)$

III

$(p_2)^{n+1}$

$(p_1)^{n+1}$

$p_1 = 1/2 \quad p_2 = 1$



$1 \quad (n > 0)$

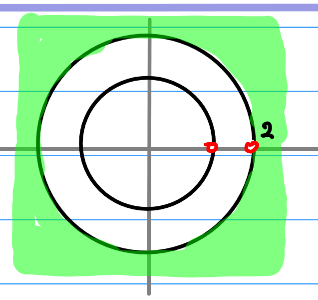
$\left(\frac{1}{2}\right)^{n+1} \quad (n \leq 0)$

I

0

$(p_1)^{n-1} - (p_2)^{n-1}$

$p_1 = 1 \quad p_2 = 2$



$0 \quad (n \geq 0)$

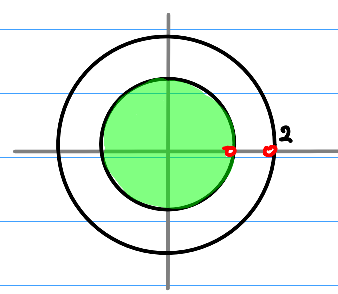
$1 - 2^{n-1} \quad (n < 0)$

II

$(p_2)^{n-1} - (p_1)^{n-1}$

0

$p_1 = 1 \quad p_2 = 2$



$2^{n-1} - 1 \quad (n \geq 0)$

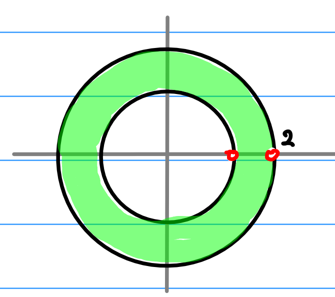
$0 \quad (n < 0)$

III

$(p_2)^{n-1}$

$(p_1)^{n-1}$

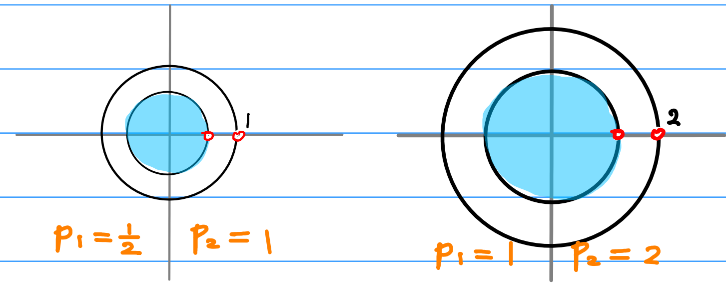
$p_1 = 1 \quad p_2 = 2$



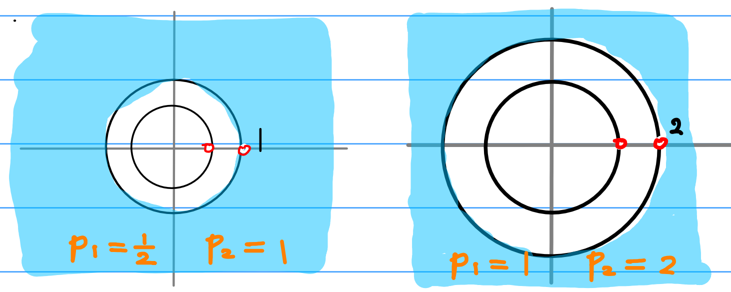
$2^{n-1} \quad (n \geq 0)$

$1 \quad (n < 0)$

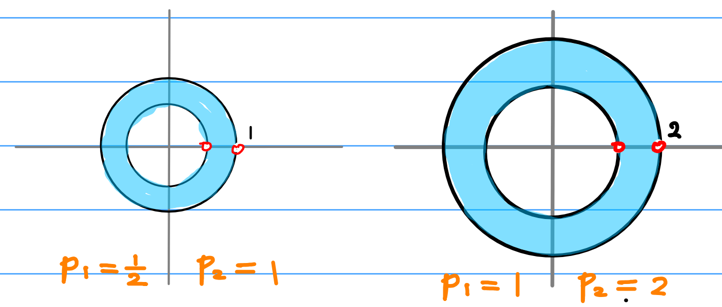
$$\boxed{\begin{array}{l}\left(\frac{1}{p_2}\right)^{n+1}-\left(\frac{1}{p_1}\right)^{n+1}\\ 0\end{array}}$$



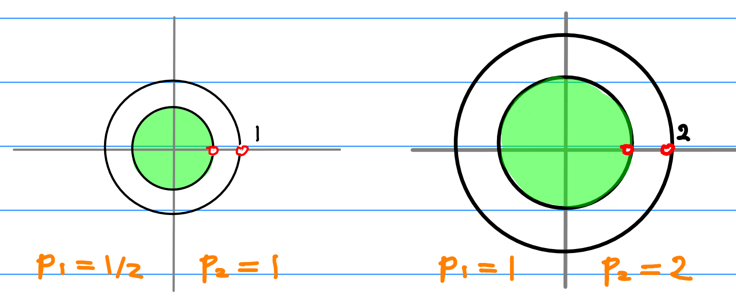
$$\boxed{\begin{array}{l}0\\ \left(\frac{1}{p_1}\right)^{n+1}-\left(\frac{1}{p_2}\right)^{n+1}\end{array}}$$



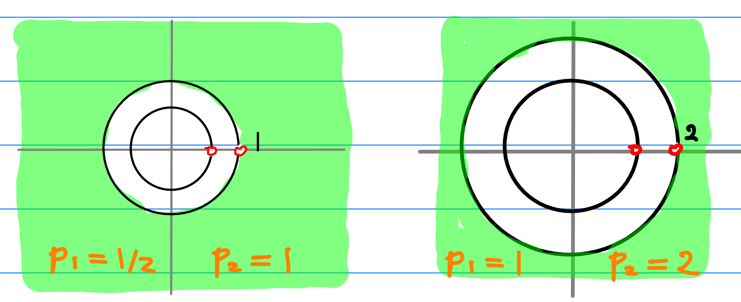
$$\boxed{\begin{array}{l}\left(\frac{1}{p_2}\right)^{n+1}\\ \left(\frac{1}{p_1}\right)^{n+1}\end{array}}$$



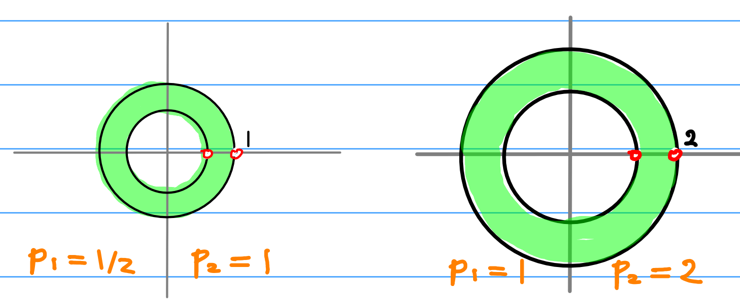
$$\boxed{\begin{array}{l}(p_2)^{n+1}-(p_1)^{n+1}\\ 0\end{array}}$$

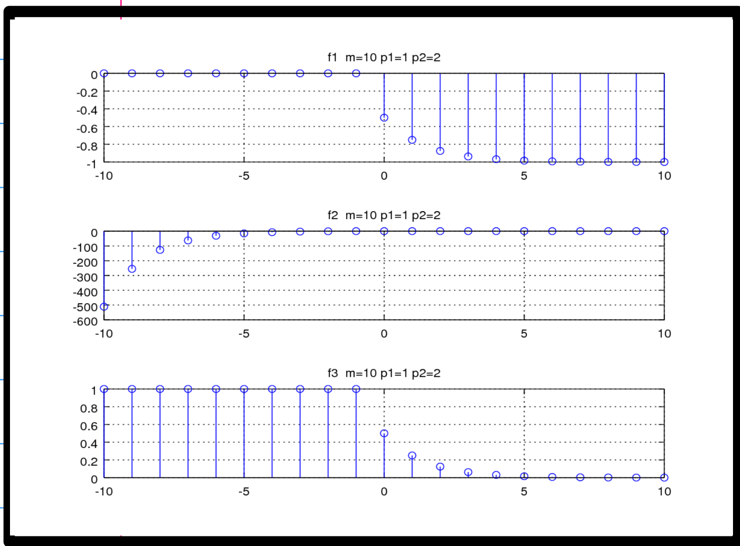


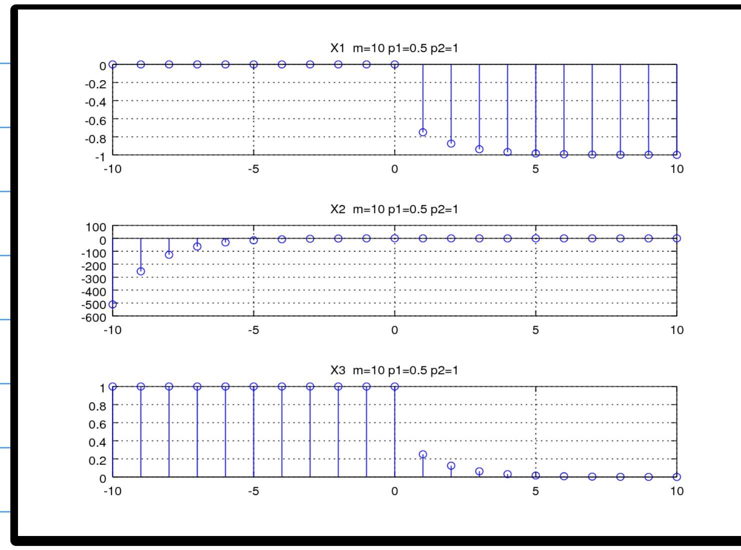
$$\boxed{\begin{array}{l}0\\ (p_1)^{n+1}-(p_2)^{n+1}\end{array}}$$

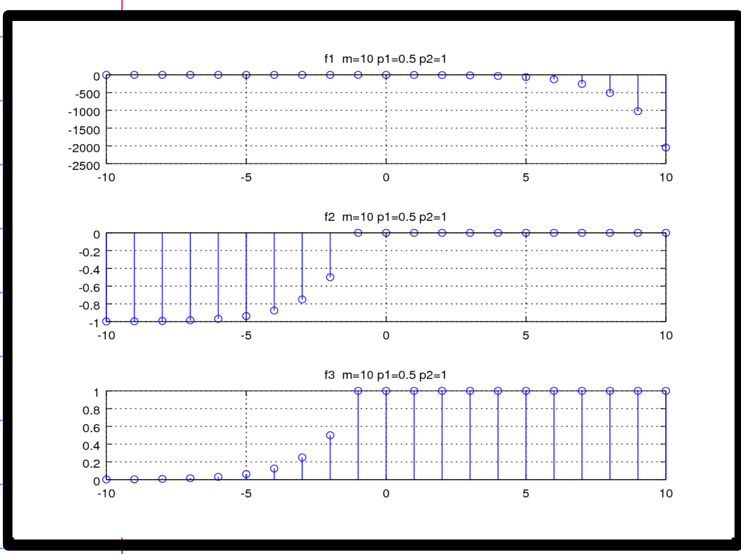


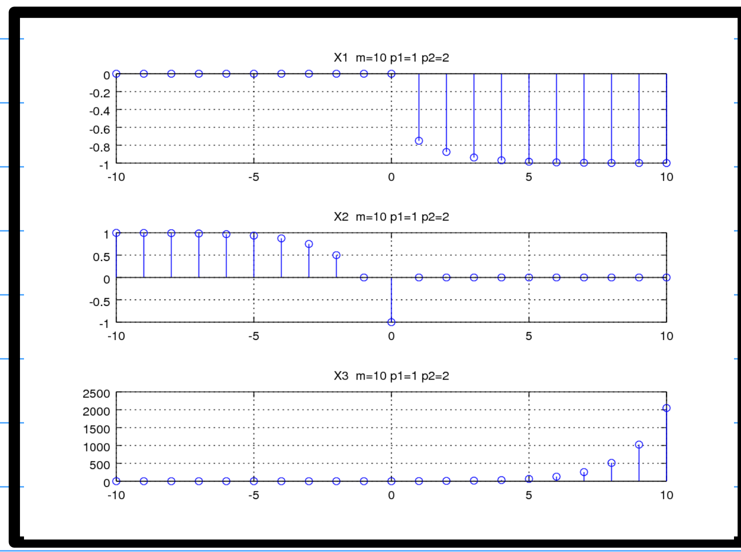
$$\boxed{\begin{array}{l}(p_2)^{n+1}\\ (p_1)^{n+1}\end{array}}$$











P_1 $\quad n+1$ $\quad$ check.

P_2 $\quad n-1$

```
% Laurent Series and sequences
function plotseq1(m=1, p1=2, p2=2.1)

t1p = 0 : m;
t1n = -m: -1;
t1 = [t1n, t1p];
f1 = [zeros(1,m), ((1/p2).^(t1p+1) - (1/p1).^(t1p+1))];
f2 = [((1/p1).^(t1n+1) -(1/p2).^(t1n+1)), zeros(1,m+1)];
f3 = [(1/p1).^(t1n+1), (1/p2).^(t1p+1)];


subplot(3, 1, 1);
stem(t1, f1);
grid on
%axis([0, m])
title(sprintf("f1  m=%d p1=%g p2=%g", m, p1, p2))

subplot(3, 1, 2);
stem(t1, f2);
grid on
%axis([0, m])
title(sprintf("f2  m=%d p1=%g p2=%g", m, p1, p2))

subplot(3, 1, 3);
stem(t1, f3);
grid on
%axis([0, m])
title(sprintf("f3  m=%d p1=%g p2=%g", m, p1, p2))

endfunction
```

```
% z-Transform and sequences
function plotseq2(m=1, p1=2, p2=2.1)

cla;

t1p = 1 : m;
t1n = -m: 0;
t1 = [t1n, t1p];
X1 = [zeros(1,m+1), ((p1).^(t1p-1) - (p2).^(t1p-1))];
X2 = [((p2).^(t1n-1) -(p1).^(t1n-1)), zeros(1,m)];
X3 = [(p2).^(t1n-1), (p1).^(t1p-1)];


subplot(3, 1, 1);
stem(t1, X1);
grid on
%axis([0, m])
title(sprintf("X1  m=%d p1=%g p2=%g", m, p1, p2))

subplot(3, 1, 2);
stem(t1, X2);
grid on
%axis([0, m])
title(sprintf("X2  m=%d p1=%g p2=%g", m, p1, p2))

subplot(3, 1, 3);
stem(t1, X3);
grid on
%axis([0, m])
title(sprintf("X3  m=%d p1=%g p2=%g", m, p1, p2))

endfunction
```