

Laurent Series and z-Transform Examples case 0.A

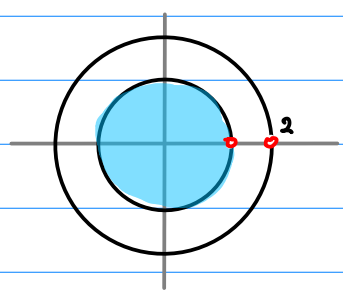
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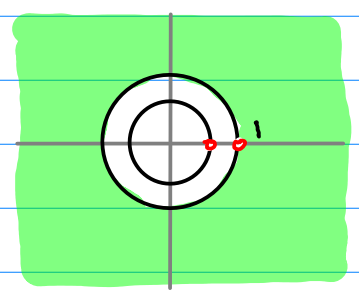
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1.A

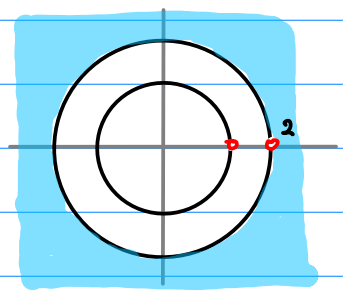
$$f(z) = \frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} X(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$



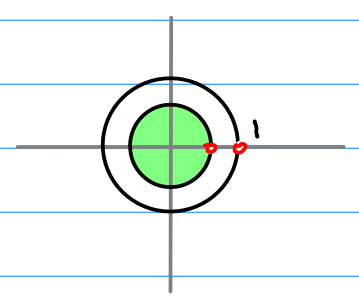
$$\sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^n$$



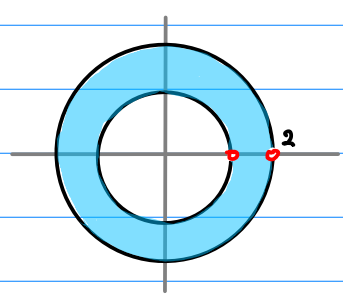
$$\sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^n$$



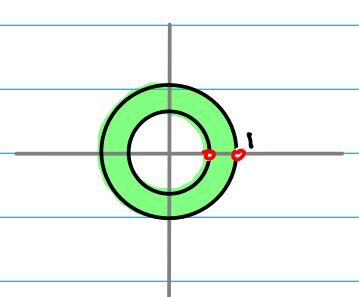
$$\sum_{n=-1}^{\infty} \left(1 - \left(\frac{1}{2}\right)^{n+1} \right) z^n$$



$$\sum_{n=-1}^{\infty} \left(1 - \left(\frac{1}{2}\right)^{n+1} \right) z^n$$



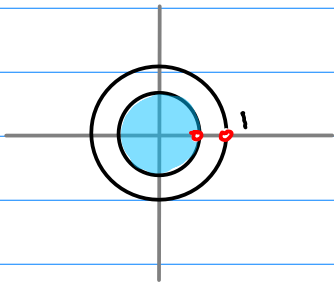
$$\sum_{n=-1}^{\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} \cdot z^n$$



$$\sum_{n=-1}^{\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} \cdot z^n$$

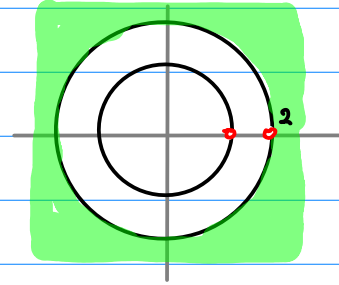
2.A

$$f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} X(z) = \frac{-1}{(z-1)(z-2)}$$

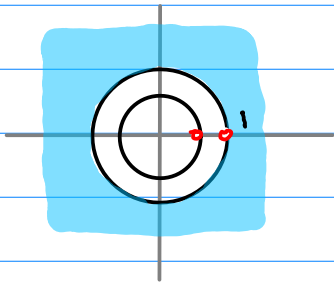


$$\sum_{n=-\infty}^{\infty} [1 - 2^{n-1}] z^n$$

≡

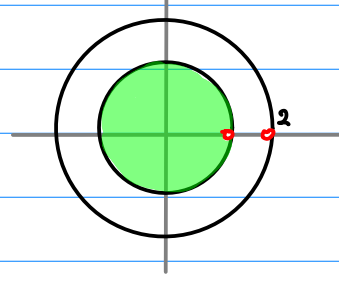


$$\sum_{n=-\infty}^{\infty} [1 - 2^{n-1}] z^n$$

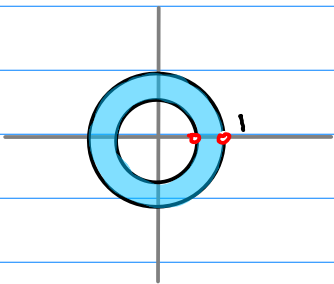


$$\sum_{n=0}^{-\infty} [2^{n-1} - 1] z^n$$

≡

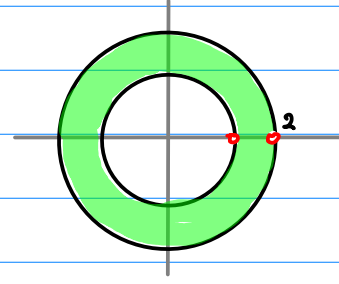


$$\sum_{n=0}^{-\infty} [2^{n-1} - 1] z^n$$



$$\sum_{n=-\infty}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

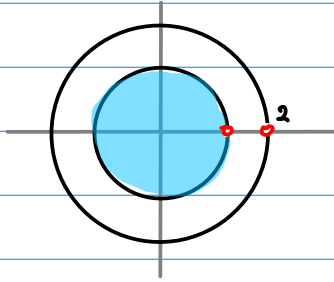
≡



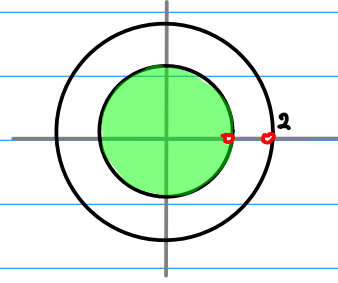
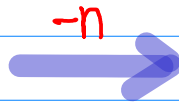
$$\sum_{n=-\infty}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

3. A

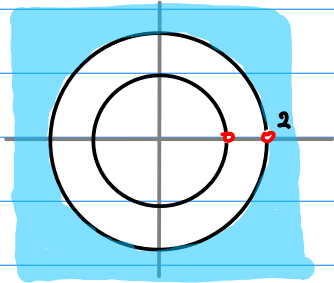
$$f(z) = \frac{-1}{(z-1)(z-2)} = X(z) = \frac{-1}{(z-1)(z-2)}$$



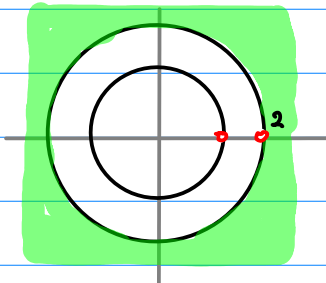
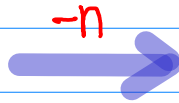
$$\sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^n$$



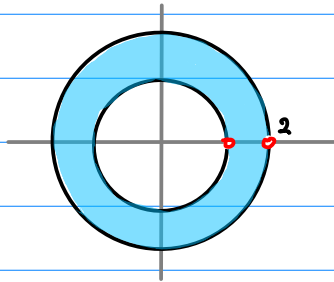
$$\sum_{n=0}^{-\infty} [2^{n-1} - 1] z^{-n}$$



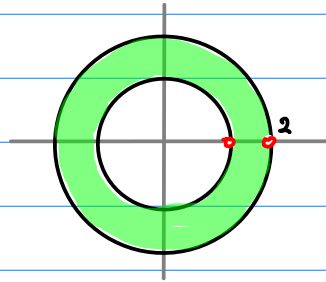
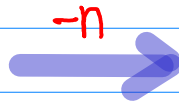
$$\sum_{n=-1}^{\infty} \left[1 - \left(\frac{1}{2}\right)^{n+1} \right] z^n$$



$$\sum_{n=-1}^{\infty} [1 - 2^{n-1}] z^{-n}$$



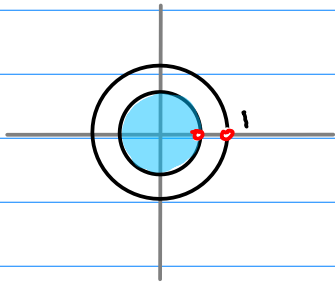
$$\sum_{n=-1}^{\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$



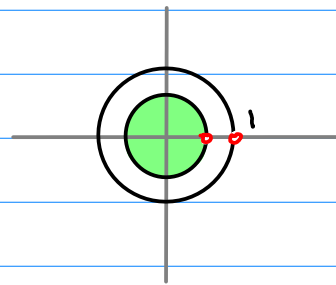
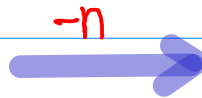
$$+ \sum_{n=-1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

4.A

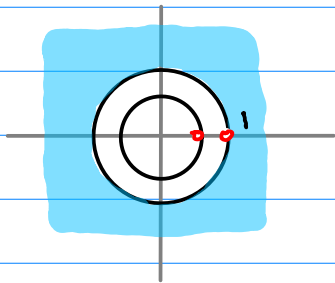
$$f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} = X(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)}$$



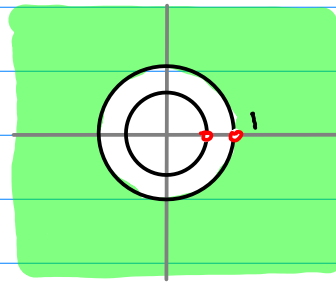
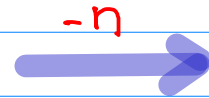
$$\sum_{n=-\infty}^{\infty} [1 - 2^{n-1}] z^n$$



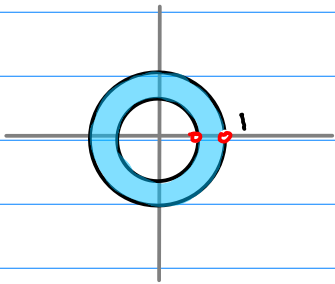
$$\sum_{n=-\infty}^{\infty} [1 - (\frac{1}{2})^{n+1}] z^{-n}$$



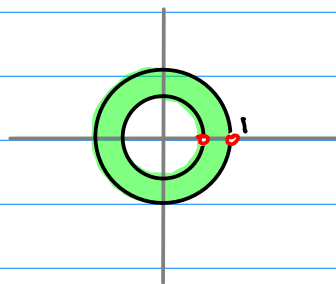
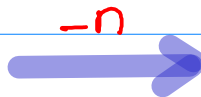
$$\sum_{n=-\infty}^{\infty} [2^{n-1} - 1] z^n$$



$$\sum_{n=-\infty}^{\infty} [(\frac{1}{2})^{n+1} - 1] z^{-n}$$



$$+\sum_{n=-\infty}^{\infty} z^n + \sum_{n=0}^{\infty} 2^{n-1} z^n$$

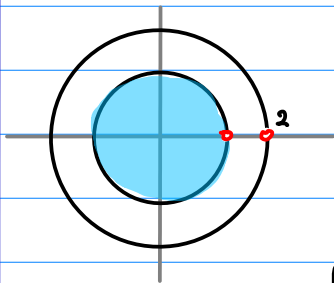


$$+\sum_{n=-\infty}^{\infty} z^{-n} + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n}$$

1. A

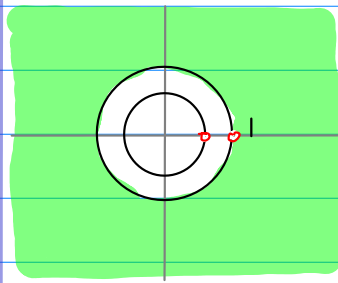
$$f(z) = \frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} X(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$

I



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{|n|} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

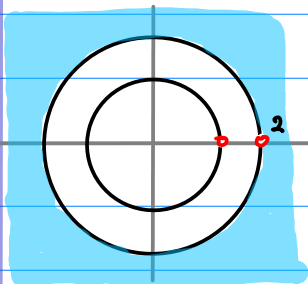
$$f(z) = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{|n|} - 1 \right] z^n$$



$$x_n = \begin{cases} 0 & (n > 0) \\ \left(\frac{1}{2}\right)^{|n|} - 1 & (n \leq 0) \end{cases}$$

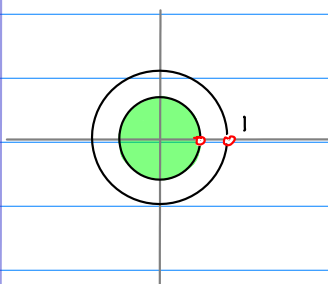
$$X(z) = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{|n|} - 1 \right] z^n$$

II



$$a_n = \begin{cases} 0 & (n \geq 0) \\ \left(1 - \left(\frac{1}{2}\right)^{|n+1}\right) & (n < 0) \end{cases}$$

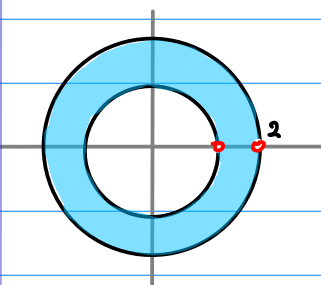
$$f(z) = \sum_{n=-1}^{\infty} \left(1 - \left(\frac{1}{2}\right)^{|n+1}\right) z^n$$



$$x_n = \begin{cases} \left(1 - \left(\frac{1}{2}\right)^{|n+1}\right) & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

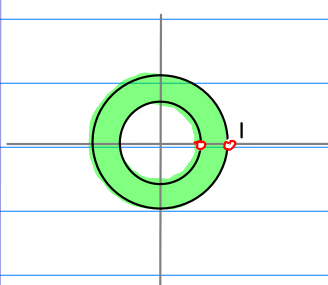
$$X(z) = \sum_{n=-1}^{\infty} \left(1 - \left(\frac{1}{2}\right)^{|n+1}\right) z^n$$

III



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{|n|} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{|n|} z^n$$



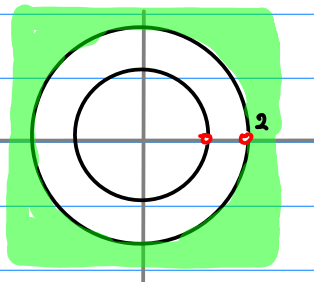
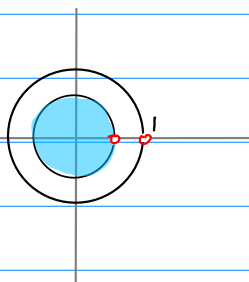
$$x_n = \begin{cases} 1 & (n > 0) \\ \left(\frac{1}{2}\right)^{|n|} & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=-1}^{\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{|n|} z^n$$

2.A

$$f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} X(z) = \frac{-1}{(z-1)(z-2)}$$

I



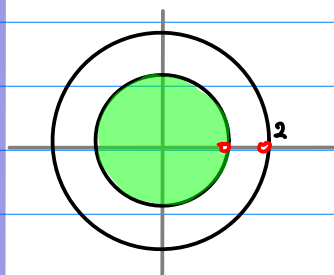
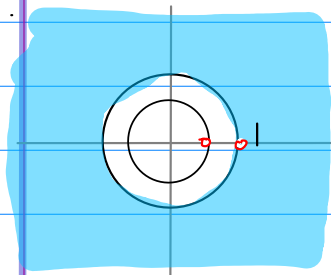
$$a_n = \begin{cases} [1 - 2^{n-1}] & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (n \geq 0) \\ [1 - 2^{n-1}] & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} [1 - 2^{n-1}] z^n$$

$$X(z) = \sum_{n=1}^{\infty} [1 - 2^{n-1}] z^n$$

II



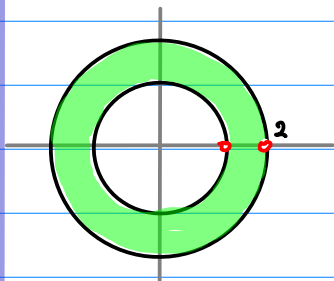
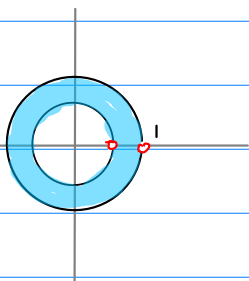
$$a_n = \begin{cases} 0 & (n > 0) \\ [2^{n-1} - 1] & (n \leq 0) \end{cases}$$

$$x_n = \begin{cases} [2^{n-1} - 1] & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{-\infty} [2^{n-1} - 1] z^n$$

$$X(z) = \sum_{n=0}^{-\infty} [2^{n-1} - 1] z^n$$

III



$$a_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

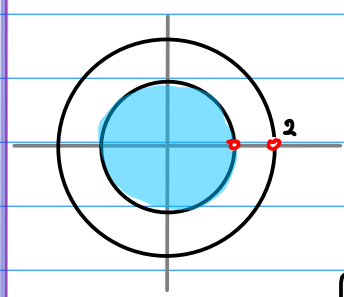
$$x_n = \begin{cases} 2^{n-1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

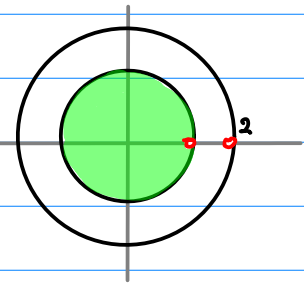
3.A $f(z) = \frac{-1}{(z-1)(z-2)} = X(z) = \frac{-1}{(z-1)(z-2)}$

I



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

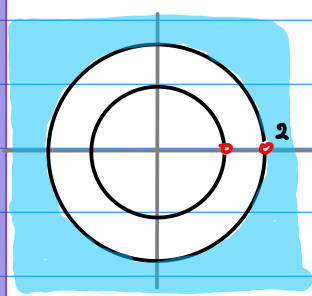
$$f(z) = \sum_{n=0}^{\infty} [(\frac{1}{2})^{n+1} - 1] z^n$$



$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

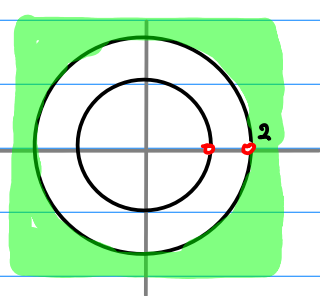
$$X(z) = \sum_{n=-\infty}^{\infty} [2^{n-1} - 1] z^{-n}$$

II



$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

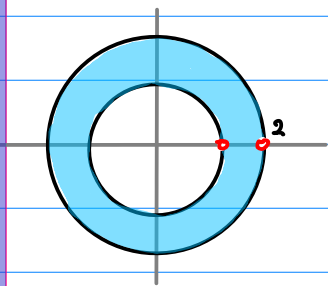
$$f(z) = \sum_{n=-1}^{\infty} [1 - (\frac{1}{2})^{n+1}] z^n$$



$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

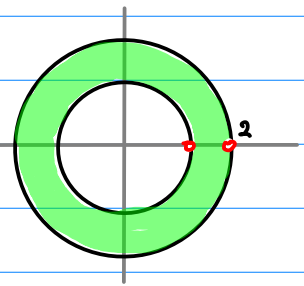
$$X(z) = \sum_{n=1}^{\infty} [1 - 2^{n-1}] z^{-n}$$

III



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{\infty} z^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n$$



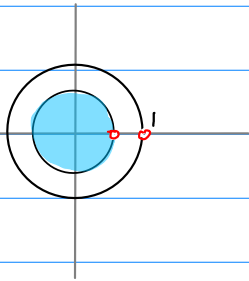
$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

4.A

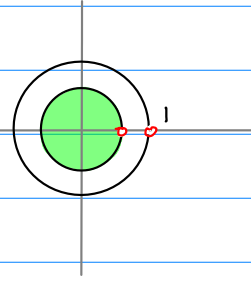
$$f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} = X(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)}$$

I



$$a_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

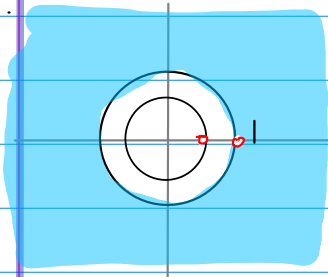
$$f(z) = \sum_{n=-\infty}^{\infty} [1 - 2^{n-1}] z^n$$



$$x_n = \begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

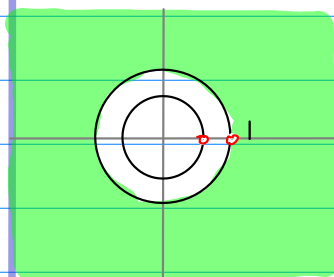
$$X(z) = \sum_{n=-\infty}^{\infty} [1 - (\frac{1}{2})^{n+1}] z^{-n}$$

II



$$a_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

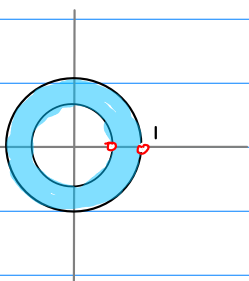
$$f(z) = \sum_{n=-\infty}^{\infty} [2^{n-1} - 1] z^n$$



$$x_n = \begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

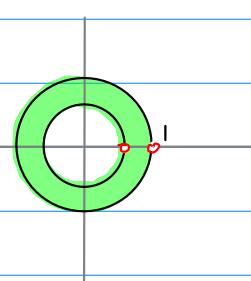
$$X(z) = \sum_{n=-\infty}^{\infty} [(\frac{1}{2})^{n+1} - 1] z^{-n}$$

III



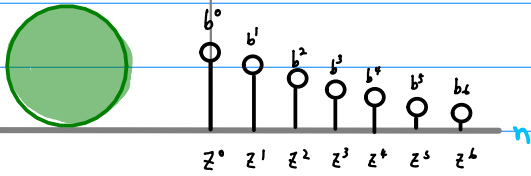
$$a_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$f(z) = + \sum_{n=-\infty}^{\infty} z^n + \sum_{n=0}^{\infty} 2^{n-1} z^n$$



$$x_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$X(z) = + \sum_{n=-\infty}^{\infty} z^{-n} + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n}$$

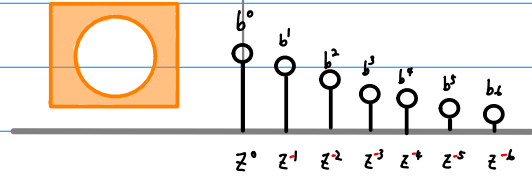
b^n $(n \geq 0)$ 

$$X(z^{-1}) = \frac{z^{-1}}{z^{-1} - 0.5} \quad |z| < 2$$

$$f(z) = \frac{z}{z - 0.5} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^n$$

$$a_n = \left(\frac{1}{2}\right)^n$$

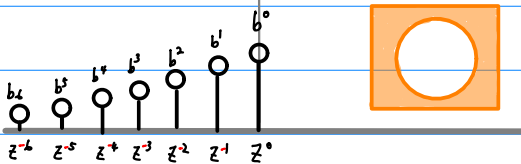
$$= p^{-n} \quad \boxed{p=2}$$

 b^n $(n \geq 0)$ 

$$X(z) = \frac{z}{z - 0.5} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} \quad |z| > \frac{1}{2}$$

$$x_n = \left(\frac{1}{2}\right)^n$$

$$= p^n \quad \boxed{p=\frac{1}{2}}$$

 b^{-n} $(n \leq 0)$ 

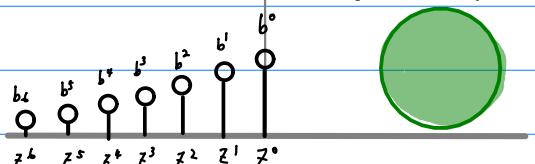
$$X(z^{-1}) = \frac{z}{z - 0.5} \quad |z| > \frac{1}{2}$$

$$f(z) = \frac{z}{z - 0.5} = \sum_{n=-\infty}^0 \left(\frac{1}{2}\right)^{-n} z^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$$

$$a_n = \left(\frac{1}{2}\right)^{-n}$$

$$= p^{-n} \quad \boxed{p=\frac{1}{2}}$$

 b^{-n} $(n \leq 0)$ 

$$X(z) = \frac{z}{z - 0.5} = \sum_{n=-\infty}^0 \left(\frac{1}{2}\right)^{-n} z^{-n}$$

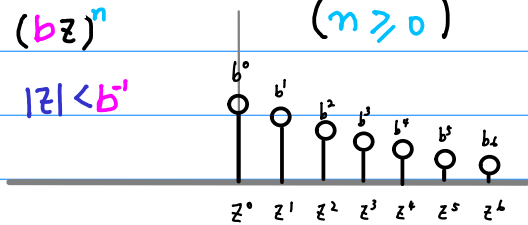
$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^n \quad |z| < 2$$

$$x_n = \left(\frac{1}{2}\right)^{-n}$$

$$= p^n \quad \boxed{p=2}$$

$$(bz)^n \quad (n \geq 0)$$

$$|z| < b^{-1}$$

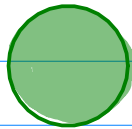


$$f(z) = \frac{1}{1-bz} = \frac{b^{-1}}{b^{-1}-z}$$

$$a_n = b^n$$

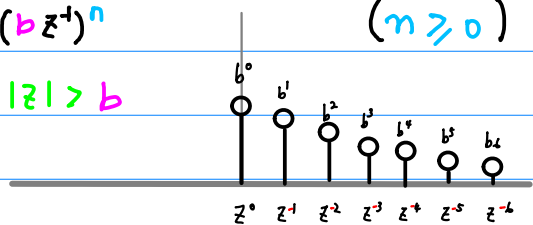
$$= p^{-n}$$

$$p = b^{-1}$$



$$(bz^{-1})^n \quad (n \geq 0)$$

$$|z| > b$$

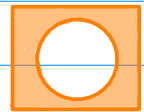


$$X(z) = \frac{1}{1-b/z} = \frac{z}{z-b}$$

$$x_n = b^n$$

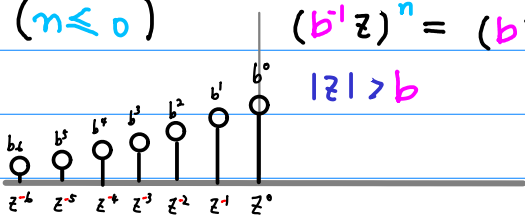
$$= p^n$$

$$p = b$$



$$(n \leq 0) \quad (b^{-1}z)^n = (bz^{-1})^{-n}$$

$$|z| > b$$

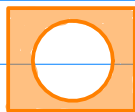


$$f(z) = \frac{1}{1-(b^{-1}z)} = \frac{z}{z-b}$$

$$a_n = b^{-n}$$

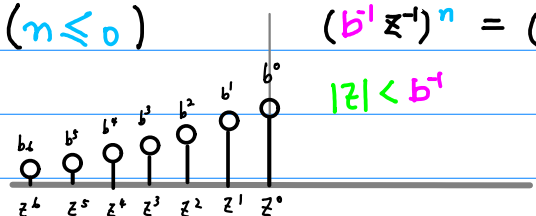
$$= p^{-n}$$

$$p = b$$



$$(n \leq 0) \quad (b^{-1}z^{-1})^n = (bz)^{-n}$$

$$|z| < b^{-1}$$

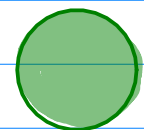


$$X(z) = \frac{1}{1-(bz)} = \frac{b^{-1}}{b^{-1}-z}$$

$$x_n = b^{-n}$$

$$= p^n$$

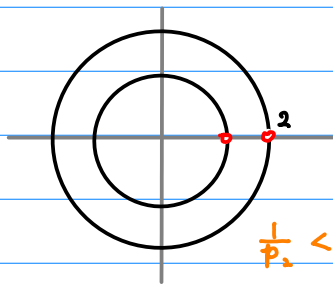
$$p = b^{-1}$$



$$f(z) \xrightarrow{z^{-1}} X(z)$$

1.A

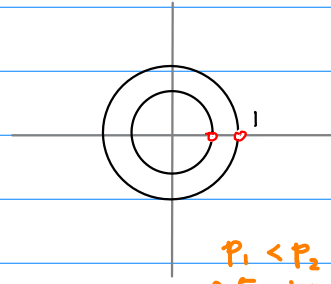
$$\frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} \frac{-0.5z^2}{(z-1)(z-0.5)}$$



$$\frac{1}{p_2} < \frac{1}{p_1}$$

$$\frac{1}{2} < \frac{1}{1}$$

$$\begin{matrix} p_1 = 1 \\ p_2 = 2 \end{matrix}$$



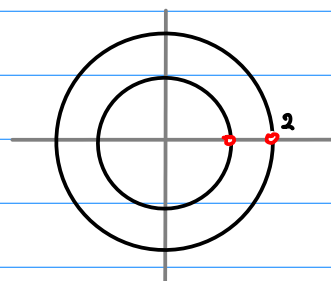
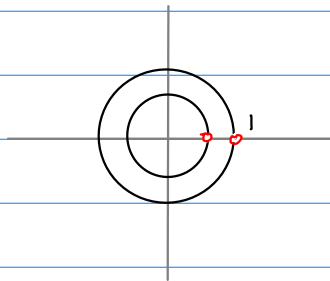
$$p_1 < p_2$$

$$0.5 < 1.0$$

$$\begin{matrix} p_1 = 0.5 \\ p_2 = 1.0 \end{matrix}$$

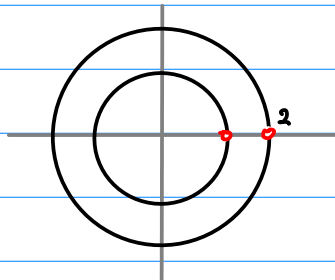
1.B

$$\frac{-0.5z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} \frac{-1}{(z-1)(z-2)}$$

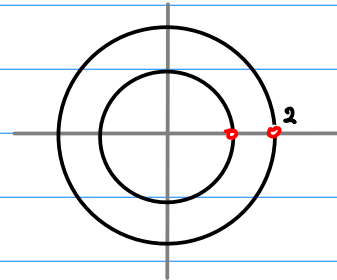


$$f(z) = X(z)$$

3.A $\frac{-1}{(z-1)(z-2)} = \frac{-1}{(z-1)(z-2)}$

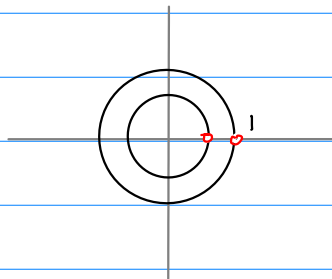


$$p_1 < p_2$$

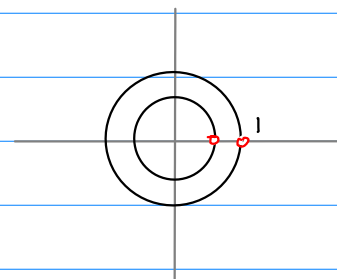


$$p_1 < p_2$$

4.B $\frac{-0.5z^2}{(z-1)(z-0.5)} = \frac{-0.5z^2}{(z-1)(z-0.5)}$



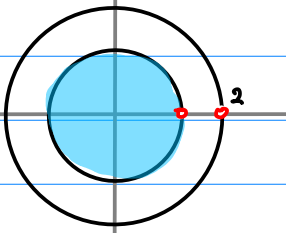
$$p_1 < p_2$$



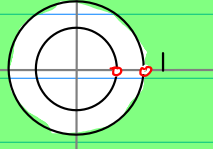
$$p_1 < p_2$$

$$f(z) \xrightarrow{z^{-1}} X(z)$$

I

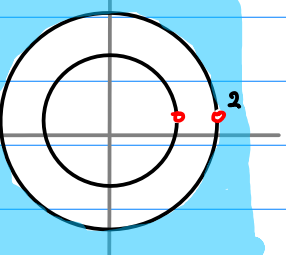


$(\frac{1}{2})^{n+1} - 1$	$(n \geq 0)$
0	$(n < 0)$

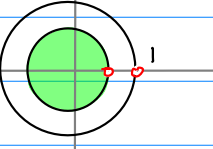


0	$(n > 0)$
$(\frac{1}{2})^{n+1} - 1$	$(n \leq 0)$

II

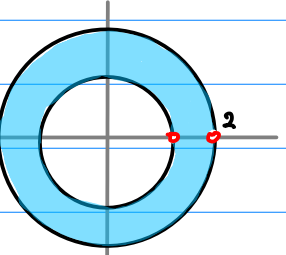


0	$(n \geq 0)$
$1 - (\frac{1}{2})^{n+1}$	$(n < 0)$

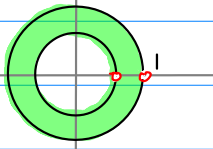


$1 - (\frac{1}{2})^{n+1}$	$(n > 0)$
0	$(n \leq 0)$

III

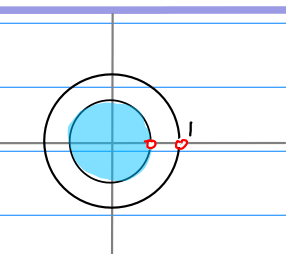


$(\frac{1}{2})^{n+1}$	$(n \geq 0)$
1	$(n < 0)$

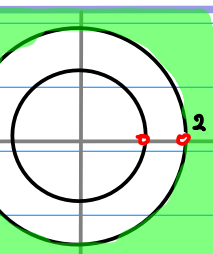


1	$(n > 0)$
$(\frac{1}{2})^{n+1}$	$(n \leq 0)$

I

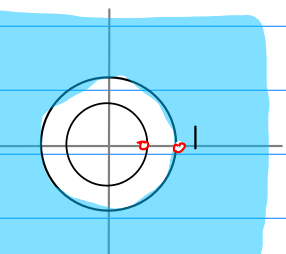


$1 - 2^{n-1}$	$(n > 0)$
0	$(n \leq 0)$

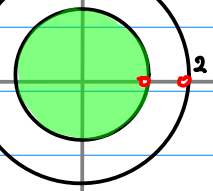


0	$(n \geq 0)$
$1 - 2^{n-1}$	$(n < 0)$

II

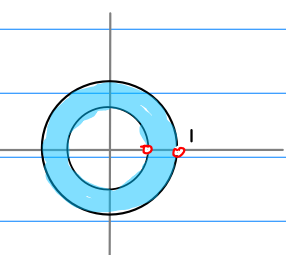


0	$(n > 0)$
$2^{n-1} - 1$	$(n \leq 0)$

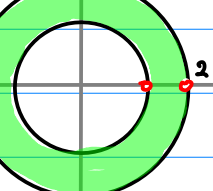


$2^{n-1} - 1$	$(n \geq 0)$
0	$(n < 0)$

III



1	$(n > 0)$
2^{n-1}	$(n \leq 0)$



2^{n-1}	$(n \geq 0)$
1	$(n < 0)$

$$f(z) = X(z)$$

I		$\begin{matrix} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{matrix}$		$\begin{matrix} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{matrix}$
II		$\begin{matrix} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{matrix}$		$\begin{matrix} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{matrix}$
III		$\begin{matrix} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{matrix}$		$\begin{matrix} 1 & (n > 0) \\ 2^n & (n \leq 0) \end{matrix}$
I		$\begin{matrix} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{matrix}$		$\begin{matrix} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{matrix}$
II		$\begin{matrix} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{matrix}$		$\begin{matrix} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{matrix}$
III		$\begin{matrix} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{matrix}$		$\begin{matrix} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{matrix}$

$$f(z) \xrightarrow{z^{-1}} X(z)$$

Ⓘ

$$\begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$\begin{cases} 0 & (n > 0) \\ (\frac{1}{2})^{n+1} - 1 & (n \leq 0) \end{cases}$$

* -1

Ⓙ

$$\begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

$$\begin{cases} 1 - (\frac{1}{2})^{n+1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

Ⓚ

$$\begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$\begin{cases} 1 & (n > 0) \\ (\frac{1}{2})^{n+1} & (n \leq 0) \end{cases}$$

Ⓛ

$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$\begin{cases} 0 & (n \geq 0) \\ 1 - 2^{n-1} & (n < 0) \end{cases}$$

* -1

Ⓜ

$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

$$\begin{cases} 2^{n-1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

Ⓝ

$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$\begin{cases} 2^{n-1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = X(z)$$

Ⓘ

$$\begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

*-1

Ⓜ

$$\begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

*-1

Ⓝ

$$\begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

Ⓘ

$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$\begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

*-1

Ⓜ

$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

$$\begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

*-1

Ⓝ

$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$\begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

n

	-3	-2	-1	0	1	2	3	4
	$\left(\frac{1}{2}\right)^{-3+1}$	$\left(\frac{1}{2}\right)^{-2+1}$	$\left(\frac{1}{2}\right)^{-1+1}$	$\left(\frac{1}{2}\right)^{0+1}$	$\left(\frac{1}{2}\right)^{1+1}$	$\left(\frac{1}{2}\right)^{2+1}$	$\left(\frac{1}{2}\right)^{3+1}$	$\left(\frac{1}{2}\right)^{4+1}$
	$\left(\frac{1}{2}\right)^{-2}$	$\left(\frac{1}{2}\right)^{-1}$	$\left(\frac{1}{2}\right)^0$	$\left(\frac{1}{2}\right)^1$	$\left(\frac{1}{2}\right)^2$	$\left(\frac{1}{2}\right)^3$	$\left(\frac{1}{2}\right)^4$	$\left(\frac{1}{2}\right)^5$
	2^2	2^1	2^0					
	$2^{-(-3)-1}$	$2^{-(-2)-1}$	$2^{-(-1)-1}$					
	2^{-n-1}	2^{-n-1}	2^{-n-1}					

$f(z)$

$p_1 < p_2$

$|z| < p_1$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} \\ 0 \end{array} \quad \begin{array}{l} (n \geq 0) \\ (n < 0) \end{array}$$

$p_1 = 1/2 \quad p_2 = 1$

* -

$|z| > p_2$

$$\begin{array}{l} 0 \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} \end{array} \quad \begin{array}{l} (n \geq 0) \\ (n < 0) \end{array}$$

$p_1 = 1 \quad p_2 = 2$

$p_1 < |z| < p_2$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n+1} \\ \left(\frac{1}{p_1}\right)^{n+1} \end{array} \quad \begin{array}{l} (n \geq 0) \\ (n < 0) \end{array}$$

$p_1 = 1 \quad p_2 = 2$

$X(z)$ $p_1 < p_2$ $|z| < p_1$

$$\begin{array}{l} 0 \\ (p_1)^{n-1} - (p_2)^{n-1} \end{array} \quad \begin{array}{l} (n > 0) \\ (n \leq 0) \end{array}$$

$p_1 = 1/2 \quad p_2 = 1$

 $*-1$ $|z| > p_2$

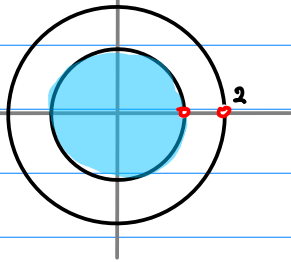
$$\begin{array}{l} (p_2)^{n-1} - (p_1)^{n-1} \\ 0 \end{array} \quad \begin{array}{l} (n > 0) \\ (n \leq 0) \end{array}$$

$p_1 = 1/2 \quad p_2 = 1$

 $p_1 < |z| < p_2$

$$\begin{array}{l} (p_2)^{n-1} \\ (p_1)^{n-1} \end{array} \quad \begin{array}{l} (n > 0) \\ (n \leq 0) \end{array}$$

I

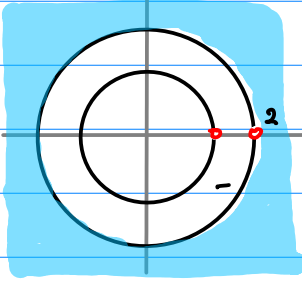


$$\begin{array}{ll} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{array}$$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} \\ 0 \end{array}$$

$$p_1 = 1 \quad p_2 = 2$$

II

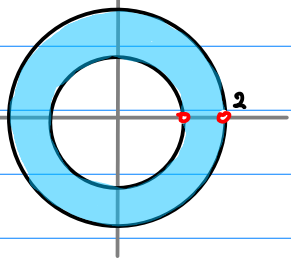


$$\begin{array}{ll} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{array}$$

$$\begin{array}{l} 0 \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} \end{array}$$

$$p_1 = 1 \quad p_2 = 2$$

III

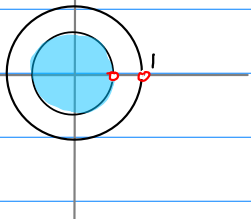


$$\begin{array}{ll} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{array}$$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n+1} \\ \left(\frac{1}{p_1}\right)^{n+1} \end{array}$$

$$p_1 = 1 \quad p_2 = 2$$

I

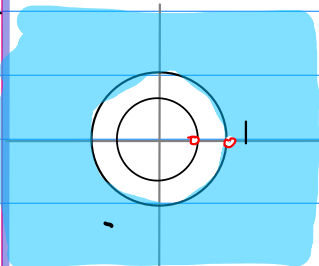


$$\begin{array}{ll} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{array}$$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n-1} - \left(\frac{1}{p_1}\right)^{n-1} \\ 0 \end{array}$$

$$p_1 = \frac{1}{2} \quad p_2 = 1$$

II

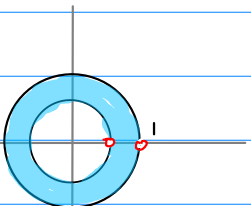


$$\begin{array}{ll} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{array}$$

$$\begin{array}{l} 0 \\ \left(\frac{1}{p_1}\right)^{n-1} - \left(\frac{1}{p_2}\right)^{n-1} \end{array}$$

$$p_1 = \frac{1}{2} \quad p_2 = 1$$

III



$$\begin{array}{ll} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{array}$$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n-1} \\ \left(\frac{1}{p_1}\right)^{n-1} \end{array}$$

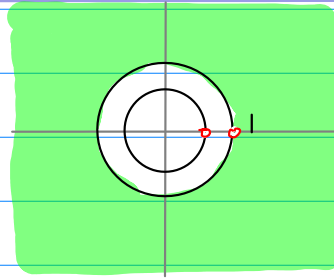
$$p_1 = \frac{1}{2} \quad p_2 = 1$$

I

$$0$$

$$(p_1)^{n+1} - (p_2)^{n+1}$$

$$p_1 = 1/2 \quad p_2 = 1$$



$$0 \quad (n > 0)$$

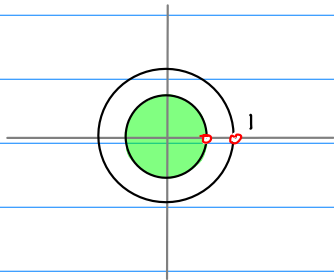
$$\left(\frac{1}{2}\right)^{n+1} - 1 \quad (n \leq 0)$$

II

$$(p_2)^{n+1} - (p_1)^{n+1}$$

$$0$$

$$p_1 = 1/2 \quad p_2 = 1$$



$$1 - \left(\frac{1}{2}\right)^{n+1} \quad (n > 0)$$

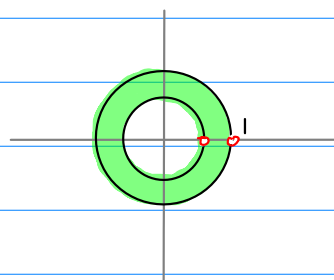
$$0 \quad (n \leq 0)$$

III

$$(p_2)^{n+1}$$

$$(p_1)^{n+1}$$

$$p_1 = 1/2 \quad p_2 = 1$$



$$1 \quad (n > 0)$$

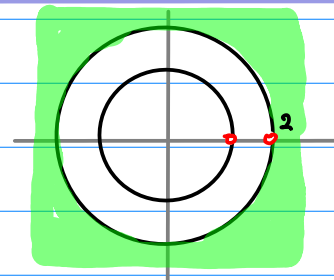
$$\left(\frac{1}{2}\right)^{n+1} \quad (n \leq 0)$$

I

$$0$$

$$(p_1)^{n-1} - (p_2)^{n-1}$$

$$p_1 = 1 \quad p_2 = 2$$



$$0 \quad (n \geq 0)$$

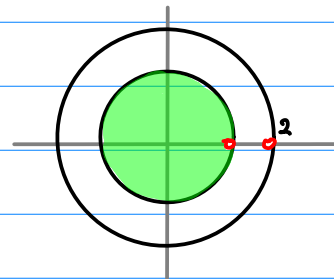
$$1 - 2^{n-1} \quad (n < 0)$$

II

$$(p_2)^{n-1} - (p_1)^{n-1}$$

$$0$$

$$p_1 = 1 \quad p_2 = 2$$



$$2^{n-1} - 1 \quad (n \geq 0)$$

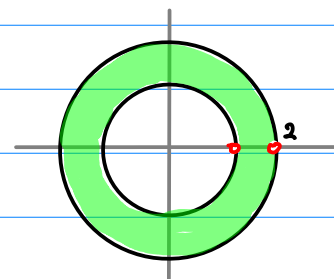
$$0 \quad (n < 0)$$

III

$$(p_2)^{n-1}$$

$$(p_1)^{n-1}$$

$$p_1 = 1 \quad p_2 = 2$$

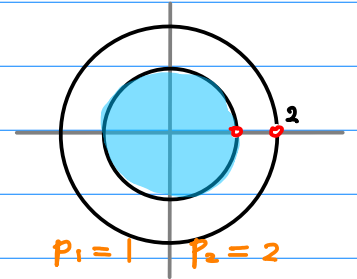
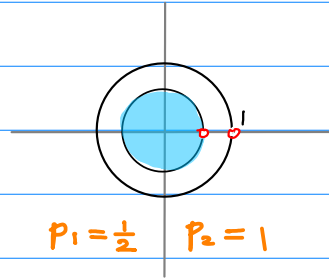


$$2^{n-1} \quad (n \geq 0)$$

$$1 \quad (n < 0)$$

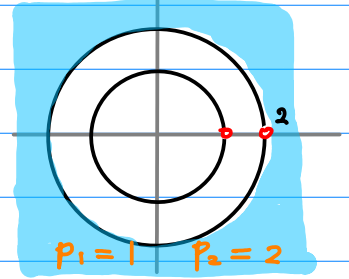
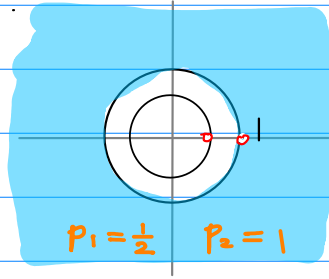
$$\left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1}$$

$$0$$



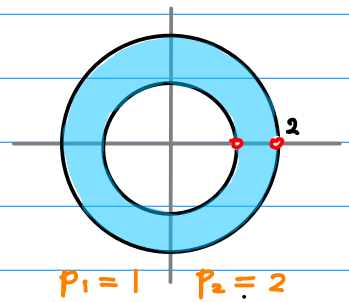
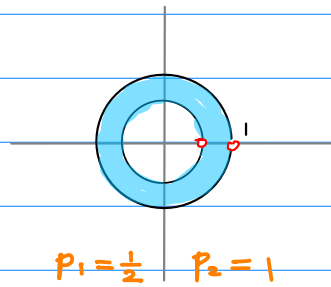
$$0$$

$$\left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1}$$



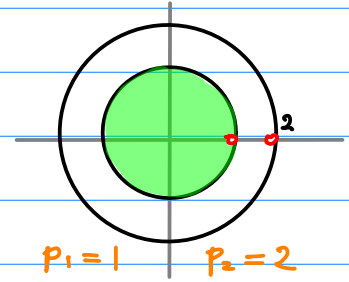
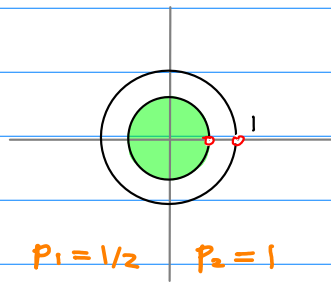
$$\left(\frac{1}{p_2}\right)^{n+1}$$

$$\left(\frac{1}{p_1}\right)^{n+1}$$



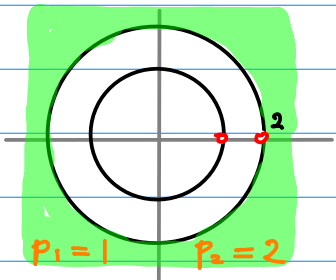
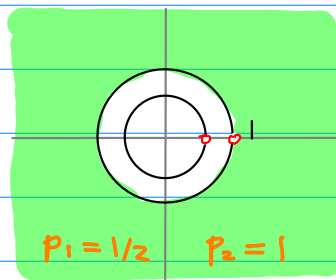
$$(p_2)^{n+1} - (p_1)^{n+1}$$

$$0$$



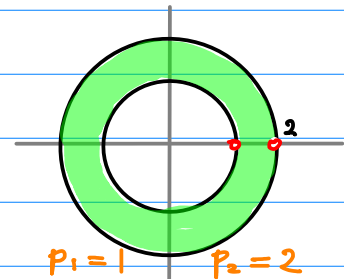
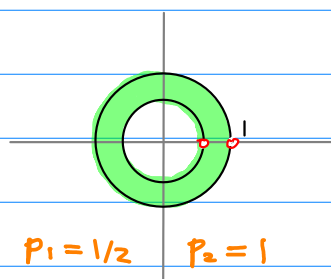
$$0$$

$$(p_1)^{n+1} - (p_2)^{n+1}$$

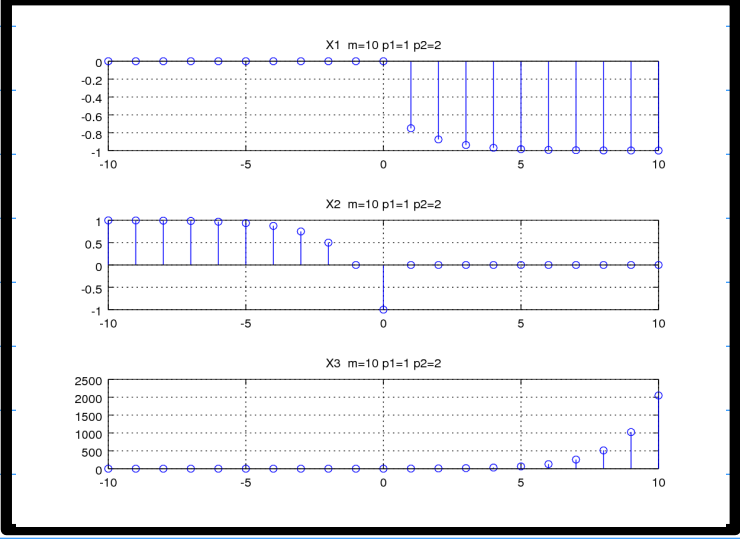
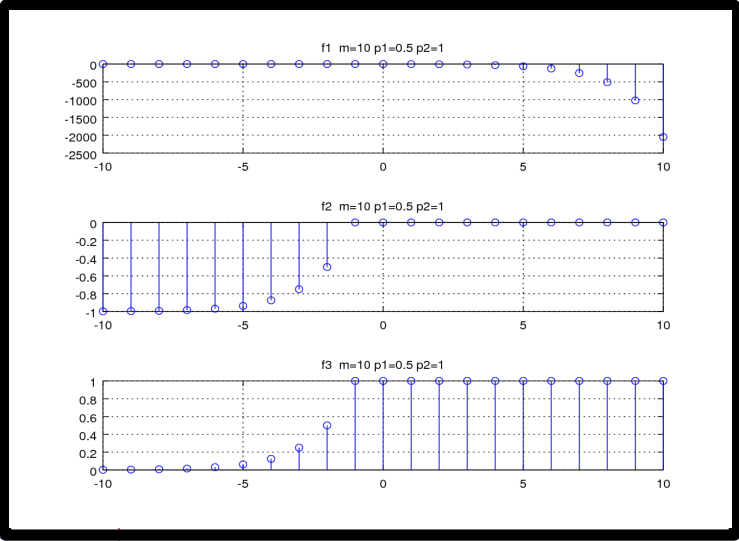
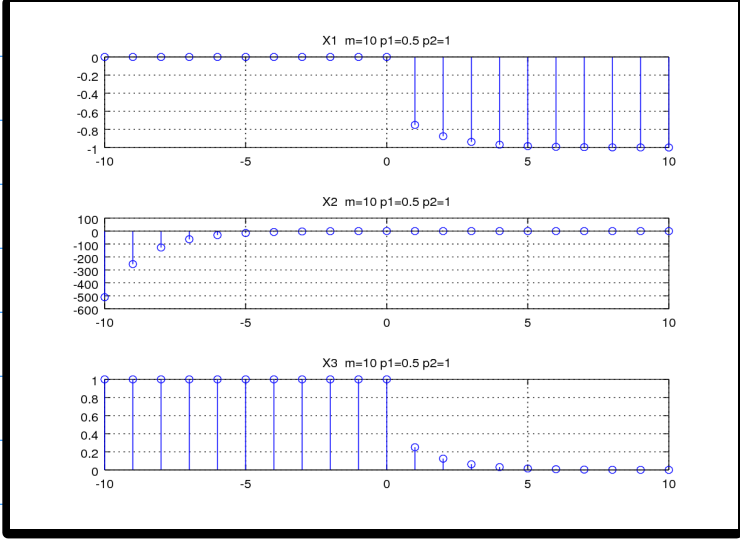
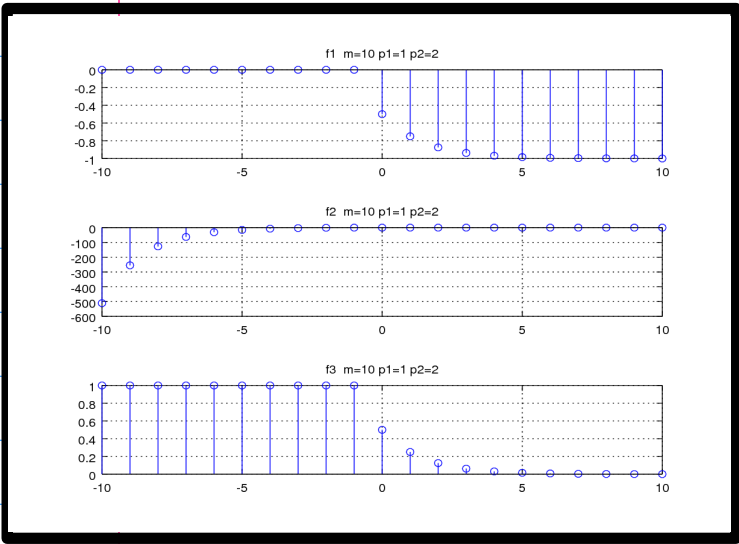


$$(p_2)^{n+1}$$

$$(p_1)^{n+1}$$







P_1
 P_2
 $m+1$
 $p-1$
 check.

```
% Laurent Series and sequences
```

```
function plotseq1(m=1, p1=2, p2=2.1)
```

```
t1p = 0 : m;
```

```
t1n = -m: -1;
```

```
t1 = [t1n, t1p];
```

```
f1 = [zeros(1,m), ((1/p2).^(t1p+1) - (1/p1).^(t1p+1))];
```

```
f2 = [((1/p1).^(t1n+1) - (1/p2).^(t1n+1)), zeros(1,m+1)];
```

```
f3 = [(1/p1).^(t1n+1), (1/p2).^(t1p+1)];
```

```
subplot(3, 1, 1);
```

```
stem(t1, f1);
```

```
grid on
```

```
%axis([0, m])
```

```
title(sprintf("f1 m=%d p1=%g p2=%g", m, p1, p2))
```

```
subplot(3, 1, 2);
```

```
stem(t1, f2);
```

```
grid on
```

```
%axis([0, m])
```

```
title(sprintf("f2 m=%d p1=%g p2=%g", m, p1, p2))
```

```
subplot(3, 1, 3);
```

```
stem(t1, f3);
```

```
grid on
```

```
%axis([0, m])
```

```
title(sprintf("f3 m=%d p1=%g p2=%g", m, p1, p2))
```

```
endfunction
```

```

% z-Transform and sequences
function plotseq2(m=1, p1=2, p2=2.1)

cla;

t1p = 1 : m;
t1n = -m: 0;
t1 = [t1n, t1p];
X1 = [zeros(1,m+1), ((p1).^(t1p-1) - (p2).^(t1p-1))];
X2 = [((p2).^(t1n-1) -(p1).^(t1n-1)), zeros(1,m)];
X3 = [(p2).^(t1n-1), (p1).^(t1p-1)];

subplot(3, 1, 1);
stem(t1, X1);
grid on
%axis([0, m])
title(sprintf("X1 m=%d p1=%g p2=%g", m, p1, p2))

subplot(3, 1, 2);
stem(t1, X2);
grid on
%axis([0, m])
title(sprintf("X2 m=%d p1=%g p2=%g", m, p1, p2))

subplot(3, 1, 3);
stem(t1, X3);
grid on
%axis([0, m])
title(sprintf("X3 m=%d p1=%g p2=%g", m, p1, p2))

endfunction

```