

Laurent Series and z-Transform - Geometric Series Applications

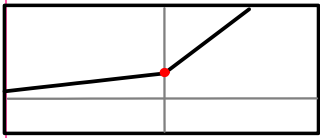


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a^n



*a
/a

Shifting a sequence

$$\frac{1}{1-az} \quad |z| < a^{-1}$$

$a^n u(n)$

$$\frac{a}{1-az} \quad |z| < a^{-1}$$

$a^{n+1} u(n)$

$$\frac{az}{1-az} \quad |z| < a^{-1}$$

$a^n u(n-1)$

$$\frac{z}{1-az} \quad |z| < a^{-1}$$

$a^{n-1} u(n-1)$

$$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad |z| > a^{-1}$$

$a^n u(-n-1)$

$$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad |z| > a^{-1}$$

$a^{n+1} u(-n-1)$

$$-\frac{1}{1-a^{-1}z^{-1}} \quad |z| > a^{-1}$$

$a^n u(-n)$

$$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad |z| > a^{-1}$$

$a^{n-1} u(-n)$

*a

/z

*z

/a

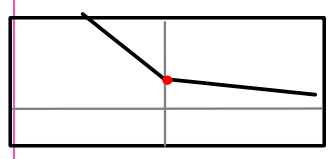
*a

/z

*z

/a

a^{-n}



/a
*a

Shifting a sequence

$\frac{1}{1-a^1z} \quad z < a$	$\xrightarrow{\quad /a \quad}$ $\xrightarrow{\quad /z \quad}$	$\frac{a^1}{1-a^1z} \quad z < a$
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$a^{-n} u(n)$

$a^{-n-1} u(n)$

$\frac{a^1z}{1-a^1z} \quad z < a$	$\xrightarrow{\quad *a \quad}$ $\xrightarrow{\quad *z \quad}$	$\frac{z}{1-a^1z} \quad z < a$
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$a^{-n} u(n-1)$

$a^{-n+1} u(n-1)$

$-\frac{a^1z^{-1}}{1-a^1z^{-1}} \quad z > a$	$\xrightarrow{\quad /a \quad}$ $\xrightarrow{\quad /z \quad}$	$-\frac{z^{-1}}{1-a^1z^{-1}} \quad z > a$
--	--	---

$a^{-n} u(-n-1)$

$a^{-n-1} u(-n-1)$

$-\frac{1}{1-a^1z^{-1}} \quad z > a$	$\xrightarrow{\quad *a \quad}$ $\xrightarrow{\quad *z \quad}$	$-\frac{a}{1-a^1z^{-1}} \quad z > a$
--	--	--

$a^{-n} u(-n)$

$a^{-n+1} u(-n)$

2 formulas

Simple Pole Form

$$\frac{1}{z - p}$$

$$\frac{1}{z^{-1} - p}$$

2 representations each

Geometric Series Form

$$\begin{array}{l} \frac{1}{z - p} \\ \swarrow \searrow \\ \frac{p^{-1}}{1 - p^{-1}z} \triangleq \begin{array}{l} \text{causal} \\ f(z) = \chi(z^{-1}) \\ \text{anti-causal} \end{array} \\ \frac{z^{-1}}{1 - pz^{-1}} \triangleq \begin{array}{l} \text{causal} \\ \gamma(z) = g(z^{-1}) \\ \text{anti-causal} \end{array} \end{array}$$

$$\begin{array}{l} \frac{1}{z^{-1} - p} \\ \swarrow \searrow \\ -\frac{p^{-1}}{1 - p^{-1}z^{-1}} \triangleq \begin{array}{l} \text{causal} \\ \chi(z) = f(z^{-1}) \\ \text{anti-causal} \end{array} \\ \frac{z}{1 - pz} \triangleq \begin{array}{l} \text{causal} \\ g(z) = \gamma(z^{-1}) \\ \text{anti-causal} \end{array} \end{array}$$

Simple Pole Form

Geometric Series Form

Geometric Series (1)

2 formulas

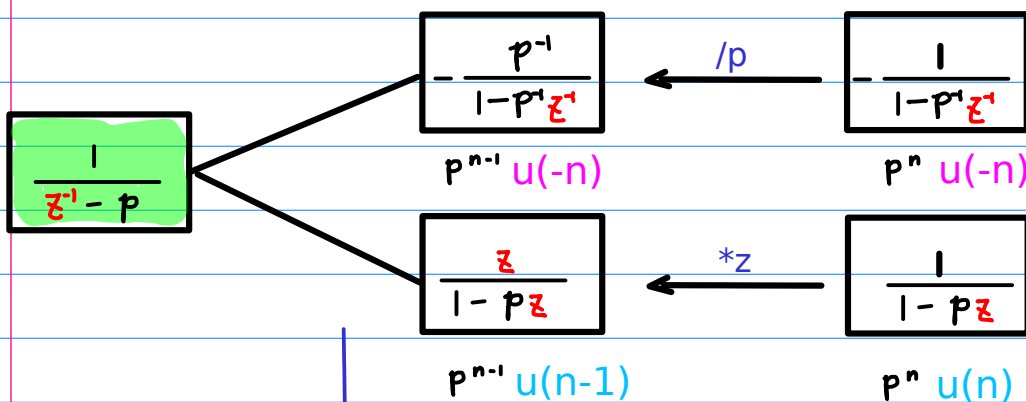
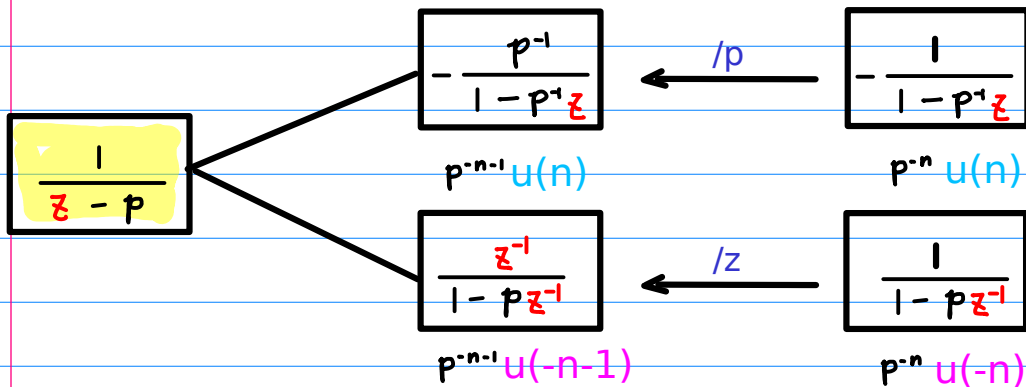
Simple Pole Form

$$\frac{1}{z - p}$$

$$\frac{1}{z^{-1} - p}$$

2 representations each

Geometric Series Form



Simple Pole Form

Geometric Series Form

Geometric Series (2)

2 formulas

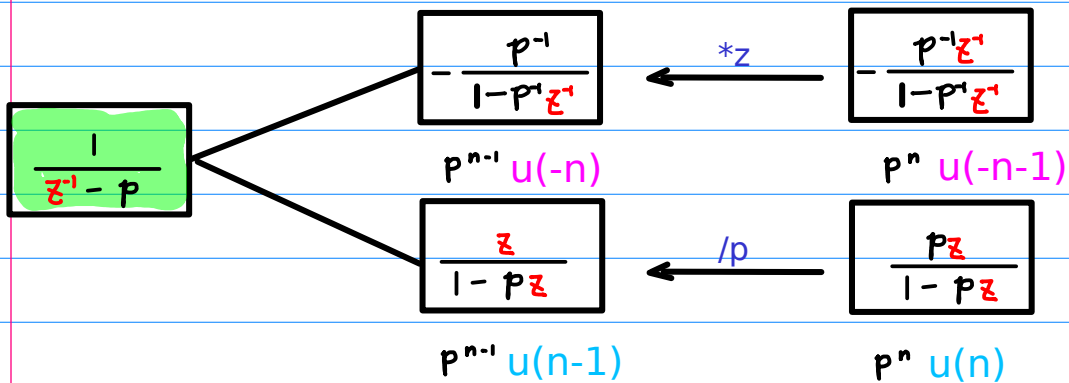
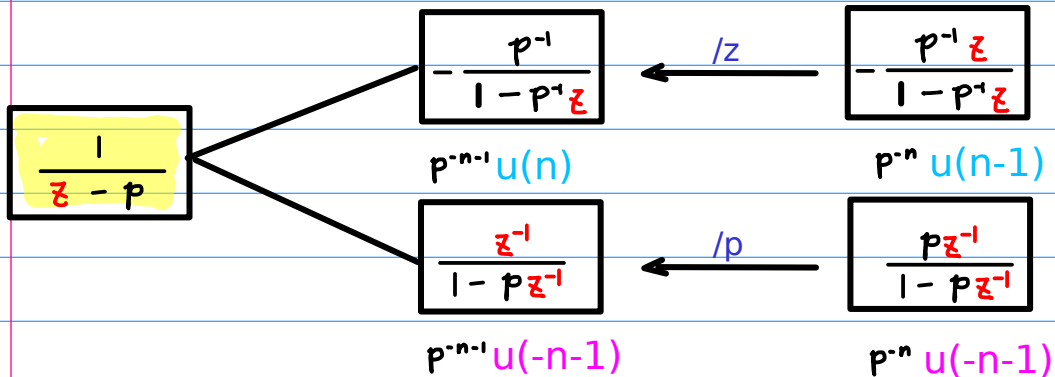
Simple Pole Form

$$\frac{1}{z - p}$$

$$\frac{1}{z^{-1} - p}$$

2 representations each

Geometric Series Form



Simple Pole Form

Geometric Series Form

Geometric Series Form Combinations with a unit start term

$$-\frac{1}{1 - az} \quad -a^n u(n)$$

$$-\frac{1}{1 - az^{-1}} \quad -a^{-n} u(-n)$$

$$+\frac{1}{1 - a^{-1}z^{-1}} \quad a^n u(-n)$$

$$+\frac{1}{1 - a^{-1}z} \quad a^{-n} u(n)$$

$$-\frac{1}{1 - a^{-1}z} \quad -a^{-n} u(n)$$

$$-\frac{1}{1 - a^{-1}z^{-1}} \quad -a^n u(-n)$$

$$+\frac{1}{1 - az^{-1}} \quad a^{-n} u(-n)$$

$$+\frac{1}{1 - az} \quad a^n u(n)$$

Geometric Series Form Combinations with a common-ratio start term

$$+ \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}}$$

$a^n u(-n-1)$

$$+ \frac{a^{-1}z}{1 - a^{-1}z}$$

$a^{-n} u(n-1)$

$$- \frac{az}{1 - az}$$

$a^n u(n-1)$

$$- \frac{az^{-1}}{1 - az^{-1}}$$

$a^{-n} u(-n-1)$

$$+ \frac{az^{-1}}{1 - az^{-1}}$$

$a^{-n} u(-n-1)$

$$+ \frac{az}{1 - az}$$

$a^n u(n-1)$

$$- \frac{a^{-1}z}{1 - a^{-1}z}$$

$a^{-n} u(n-1)$

$$- \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}}$$

$a^n u(-n-1)$

Geometric Series - a unit start term

Laurent Series

Geometric Series - a unit start term

z-Transform

Geometric Series - a unit start term

Laurent Series vs. z-Transform

Geometric Series - a unit start term

Laurent Series

$$-\frac{1}{1-az}$$

$$|z| < a^{-1}$$

$$-(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$a_n = -a^n \quad (n \geq 0)$$

$$-\frac{1}{1-az^{-1}}$$

$$|z| > a$$

$$-(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$-((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

$$a_n = -(\frac{1}{a})^n \quad (n < 0)$$

$$+\frac{1}{1-a^{-1}z}$$

$$|z| > a^{-1}$$

$$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$a_n = a^n \quad (n < 0)$$

$$+\frac{1}{1-az}$$

$$|z| < a$$

$$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

$$a_n = (\frac{1}{a})^n \quad (n \geq 0)$$

$$-\frac{1}{1-a^{-1}z}$$

$$|z| < a$$

$$-(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$-((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

$$a_n = -(\frac{1}{a})^n \quad (n \geq 0)$$

$$-\frac{1}{1-az^{-1}}$$

$$|z| > a^{-1}$$

$$-(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$a_n = -a^n \quad (n < 0)$$

$$+\frac{1}{1-az^{-1}}$$

$$|z| > a$$

$$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

$$a_n = (\frac{1}{a})^n \quad (n < 0)$$

$$+\frac{1}{1-az}$$

$$|z| < a^{-1}$$

$$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$a_n = a^n \quad (n \geq 0)$$

Geometric Series - a unit start term

z-Transform

$$-\frac{1}{1 - az}$$

$$|z| < a^{-1}$$

$$- (a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$- ((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$$

$$a_n = -a^{-n} \quad (n \geq 0)$$

$$a_n = -(\frac{1}{a})^n \quad (n < 0)$$

$$-\frac{1}{1 - az^{-1}}$$

$$|z| > a$$

$$- (a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$- ((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

$$a_n = -(\frac{1}{a})^n \quad (n < 0)$$

$$a_n = -a^n \quad (n \geq 0)$$

$$+\frac{1}{1 - a^{-1}z}$$

$$|z| > a^{-1}$$

$$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$$

$$a_n = a^{-n} \quad (n < 0)$$

$$a_n = (\frac{1}{a})^n \quad (n \geq 0)$$

$$+\frac{1}{1 - a^{-1}z}$$

$$|z| < a$$

$$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$$

$$a_n = (\frac{1}{a})^{-n} \quad (n \geq 0)$$

$$a_n = a^n \quad (n < 0)$$

$$-\frac{1}{1 - a^{-1}z}$$

$$|z| < a$$

$$- (a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$- ((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$$

$$a_n = -(\frac{1}{a})^{-n} \quad (n \geq 0)$$

$$a_n = -a^n \quad (n < 0)$$

$$-\frac{1}{1 - a^{-1}z^{-1}}$$

$$|z| > a^{-1}$$

$$- (a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$- ((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

$$a_n = -a^{-n} \quad (n < 0)$$

$$a_n = -(\frac{1}{a})^n \quad (n \geq 0)$$

$$+\frac{1}{1 - az^{-1}}$$

$$|z| > a$$

$$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$$

$$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$$

$$a_n = (\frac{1}{a})^{-n} \quad (n < 0)$$

$$a_n = a^n \quad (n \geq 0)$$

$$+\frac{1}{1 - az}$$

$$|z| < a^{-1}$$

$$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$$

$$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$$

$$a_n = a^{-n} \quad (n \geq 0)$$

$$a_n = (\frac{1}{a})^n \quad (n < 0)$$

Geometric Series - a unit start term

Laurent Series vs. z-Transform

$-\frac{1}{1-\underline{az}}$	$ z < a$	$-\frac{1}{1-\underline{az^{-1}}}$	$ z > a$
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- $(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$

- $((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$

- $(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$

- $((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$

Laurent

$a_n = -a^n$	$(n \geq 0)$
--------------	--------------

$(n < 1)$

$a_n = -(\frac{1}{a})^n$	$(n < 1)$
--------------------------	-----------

$(n \geq 0)$

z-Trans

$a_n = -(\frac{1}{a})^n$	$(n < 1)$
--------------------------	-----------

$(n \geq 0)$

$a_n = -a^n$	$(n \geq 0)$
--------------	--------------

$(n < 1)$

$+\frac{1}{1-\underline{a^{-1}z^{-1}}}$	$ z > a^{-1}$	$+\frac{1}{1-\underline{a^{-1}z}}$	$ z < a$
---	----------------	------------------------------------	-----------

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$

$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$

Laurent

$a_n = a^n$	$(n < 1)$
-------------	-----------

$(n \geq 0)$

$a_n = (\frac{1}{a})^n$	$(n \geq 0)$
-------------------------	--------------

$(n < 1)$

z-Trans

$a_n = (\frac{1}{a})^n$	$(n \geq 0)$
-------------------------	--------------

$(n < 1)$

$a_n = a^n$	$(n < 1)$
-------------	-----------

$(n \geq 0)$

$-\frac{1}{1-\underline{a^{-1}z}}$	$ z < a$	$-\frac{1}{1-\underline{a^{-1}z^{-1}}}$	$ z > a^{-1}$
------------------------------------	-----------	---	----------------

- $(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$

- $((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$

- $(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$

- $((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$

Laurent

$a_n = -(\frac{1}{a})^n$	$(n \geq 0)$
--------------------------	--------------

$(n < 1)$

$a_n = -a^n$	$(n < 1)$
--------------	-----------

$(n \geq 0)$

z-Trans

$a_n = -a^n$	$(n < 1)$
--------------	-----------

$(n \geq 0)$

$a_n = -(\frac{1}{a})^n$	$(n \geq 0)$
--------------------------	--------------

$(n < 1)$

$+\frac{1}{1-\underline{az^{-1}}}$	$ z > a$	$+\frac{1}{1-\underline{az}}$	$ z < a^{-1}$
------------------------------------	-----------	-------------------------------	----------------

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \dots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \dots)$

$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \dots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \dots)$

Laurent

$a_n = (\frac{1}{a})^n$	$(n < 1)$
-------------------------	-----------

$(n \geq 0)$

$a_n = a^n$	$(n \geq 0)$
-------------	--------------

$(n < 1)$

z-Trans

$a_n = a^n$	$(n \geq 0)$
-------------	--------------

$(n < 1)$

$a_n = (\frac{1}{a})^n$	$(n < 1)$
-------------------------	-----------

$(n \geq 0)$

Geometric Series - a non-unit start term

Laurent Series

Geometric Series - a non-unit start term

z-Transform

Geometric Series - a non-unit start term

Laurent Series vs. z-Transform

Geometric Series - a non-unit start term

Laurent Series

$$+ \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}}$$

$$|z| > a^{-1}$$

$$(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \dots)$$

$$a_n = a^n \quad (n < 0)$$

$$+ \frac{a^{-1}z}{1 - a^{-1}z}$$

$$|z| < a$$

$$(a^{-1}z^1 + a^{-2}z^2 + a^{-3}z^3 + \dots)$$

$$((\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + (\frac{1}{a})^3 z^3 + \dots)$$

$$a_n = (\frac{1}{a})^n \quad (n \geq 1)$$

$$- \frac{az}{1 - az}$$

$$|z| < a^{-1}$$

$$-(a^1 z^1 + a^2 z^2 + a^3 z^3 + \dots)$$

$$a_n = -a^n \quad (n \geq 1)$$

$$- \frac{az^{-1}}{1 - az^{-1}}$$

$$|z| > a$$

$$-(a^1 z^{-1} + a^2 z^{-2} + a^3 z^{-3} + \dots)$$

$$-((\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + (\frac{1}{a})^3 z^{-3} + \dots)$$

$$a_n = -(\frac{1}{a})^n \quad (n < 0)$$

$$+ \frac{az^{-1}}{1 - az^{-1}}$$

$$|z| > a$$

$$(a^1 z^{-1} + a^2 z^{-2} + a^3 z^{-3} + \dots)$$

$$((\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + (\frac{1}{a})^3 z^{-3} + \dots)$$

$$a_n = (\frac{1}{a})^n \quad (n < 0)$$

$$+ \frac{az}{1 - az}$$

$$|z| < a^{-1}$$

$$(a^1 z^1 + a^2 z^2 + a^3 z^3 + \dots)$$

$$a_n = a^n \quad (n \geq 1)$$

$$- \frac{a^{-1}z}{1 - a^{-1}z}$$

$$|z| < a$$

$$-(a^{-1}z^1 + a^{-2}z^2 + a^{-3}z^3 + \dots)$$

$$-((\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + (\frac{1}{a})^3 z^3 + \dots)$$

$$a_n = -(\frac{1}{a})^n \quad (n \geq 1)$$

$$- \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}}$$

$$|z| > a^{-1}$$

$$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \dots)$$

$$a_n = -a^n \quad (n < 0)$$

Geometric Series - a non-unit start term

z-Transform

$$+ \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}} \quad |z| > a^{-1}$$

$$(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \dots)$$

$$((\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + (\frac{1}{a})^{-3}z^{-3} + \dots)$$

$a_n = a^{-n}$	$(-n < 0)$
$a_n = (\frac{1}{a})^n$	$(n \geq 1)$

$$+ \frac{a^{-1}z}{1 - a^{-1}z} \quad |z| < a$$

$$(a^{-1}z + a^{-2}z^2 + a^{-3}z^3 + \dots)$$

$$((\frac{1}{a})z + (\frac{1}{a})^2z^2 + (\frac{1}{a})^3z^3 + \dots)$$

$a_n = (\frac{1}{a})^n$	$(n \geq 1)$
$a_n = a^n$	$(n < 0)$

$$- \frac{az}{1 - az} \quad |z| < a^{-1}$$

$$-(a^1z^1 + a^2z^2 + a^3z^3 + \dots)$$

$$-((\frac{1}{a})^1z^1 + (\frac{1}{a})^2z^2 + (\frac{1}{a})^3z^3 + \dots)$$

$a_n = -a^n$	$(-n \geq 1)$
$a_n = -(\frac{1}{a})^n$	$(n < 0)$

$$- \frac{az^{-1}}{1 - az^{-1}} \quad |z| > a$$

$$-(a^1z^{-1} + a^2z^{-2} + a^3z^{-3} + \dots)$$

$$-((\frac{1}{a})^1z^{-1} + (\frac{1}{a})^2z^{-2} + (\frac{1}{a})^3z^{-3} + \dots)$$

$a_n = -(\frac{1}{a})^{-n}$	$(-n < 0)$
$a_n = -a^n$	$(n \geq 1)$

$$+ \frac{az^{-1}}{1 - az^{-1}} \quad |z| > a$$

$$(a^1z^{-1} + a^2z^{-2} + a^3z^{-3} + \dots)$$

$$((\frac{1}{a})^1z^{-1} + (\frac{1}{a})^2z^{-2} + (\frac{1}{a})^3z^{-3} + \dots)$$

$a_n = (\frac{1}{a})^{-n}$	$(-n < 0)$
$a_n = a^n$	$(n \geq 1)$

$$+ \frac{az}{1 - az} \quad |z| < a^{-1}$$

$$(a^1z + a^2z^2 + a^3z^3 + \dots)$$

$$((\frac{1}{a})z + (\frac{1}{a})^2z^2 + (\frac{1}{a})^3z^3 + \dots)$$

$a_n = a^n$	$(-n \geq 1)$
$a_n = (\frac{1}{a})^n$	$(n < 0)$

$$- \frac{a^{-1}z}{1 - a^{-1}z} \quad |z| < a$$

$$-(a^{-1}z + a^{-2}z^2 + a^{-3}z^3 + \dots)$$

$$-((\frac{1}{a})z + (\frac{1}{a})^2z^2 + (\frac{1}{a})^3z^3 + \dots)$$

$a_n = -(\frac{1}{a})^n$	$(-n \geq 1)$
$a_n = -a^n$	$(n < 0)$

$$- \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}} \quad |z| > a^{-1}$$

$$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \dots)$$

$$-((\frac{1}{a})z^{-1} + (\frac{1}{a})^2z^{-2} + (\frac{1}{a})^3z^{-3} + \dots)$$

$a_n = -a^{-n}$	$(-n < 0)$
$a_n = -(\frac{1}{a})^n$	$(n \geq 1)$

Geometric Series - a non-unit start term

Laurent Series vs. z-Transform

$$+ \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}}$$

$$|z| > a^{-1}$$

$$+ \frac{a^{-1}z}{1 - a^{-1}z}$$

$$|z| < a$$

$$(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \dots)$$

$$((\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + (\frac{1}{a})^{-3}z^{-3} + \dots)$$

$$(a^{-1}z^1 + a^{-2}z^2 + a^{-3}z^3 + \dots)$$

$$((\frac{1}{a})^1z^1 + (\frac{1}{a})^2z^2 + (\frac{1}{a})^3z^3 + \dots)$$

Laurent

$$a_n = a^n \quad (n < 0)$$

$$a_n = (\frac{1}{a})^n \quad (n \geq 1)$$

z-Trans

$$a_n = (\frac{1}{a})^n \quad (n \geq 1)$$

$$a_n = a^n \quad (n < 0)$$

$$- \frac{az}{1 - az}$$

$$|z| < a^{-1}$$

$$- \frac{az^{-1}}{1 - az^{-1}}$$

$$|z| > a$$

$$- (a^1z^1 + a^2z^2 + a^3z^3 + \dots)$$

$$- ((\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + (\frac{1}{a})^{-3}z^{-3} + \dots)$$

$$- (a^1z^{-1} + a^2z^{-2} + a^3z^{-3} + \dots)$$

$$- ((\frac{1}{a})^1z^1 + (\frac{1}{a})^2z^2 + (\frac{1}{a})^3z^3 + \dots)$$

Laurent

$$a_n = -a^n \quad (n \geq 1)$$

$$a_n = -(\frac{1}{a})^n \quad (n < 0)$$

z-Trans

$$a_n = -(\frac{1}{a})^n \quad (n < 0)$$

$$a_n = -a^n \quad (n \geq 1)$$

$$+ \frac{az^{-1}}{1 - az^{-1}}$$

$$|z| > a$$

$$+ \frac{az}{1 - az}$$

$$|z| < a^{-1}$$

$$(a^1z^{-1} + a^2z^{-2} + a^3z^{-3} + \dots)$$

$$((\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + (\frac{1}{a})^{-3}z^{-3} + \dots)$$

$$(a^1z^1 + a^2z^2 + a^3z^3 + \dots)$$

$$((\frac{1}{a})^1z^1 + (\frac{1}{a})^2z^2 + (\frac{1}{a})^3z^3 + \dots)$$

Laurent

$$a_n = (\frac{1}{a})^n \quad (n < 0)$$

$$a_n = a^n \quad (n \geq 1)$$

z-Trans

$$a_n = a^n \quad (n \geq 1)$$

$$a_n = (\frac{1}{a})^n \quad (n < 0)$$

$$- \frac{a^{-1}z}{1 - a^{-1}z}$$

$$|z| < a$$

$$- \frac{a^{-1}z^{-1}}{1 - a^{-1}z^{-1}}$$

$$|z| > a^{-1}$$

$$- (a^{-1}z^1 + a^{-2}z^2 + a^{-3}z^3 + \dots)$$

$$- ((\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + (\frac{1}{a})^{-3}z^{-3} + \dots)$$

$$- (a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \dots)$$

$$- ((\frac{1}{a})^1z^1 + (\frac{1}{a})^2z^2 + (\frac{1}{a})^3z^3 + \dots)$$

Laurent

$$a_n = -(\frac{1}{a})^n \quad (n \geq 1)$$

$$a_n = -a^n \quad (n < 0)$$

z-Trans

$$a_n = -a^n \quad (n < 0)$$

$$a_n = -(\frac{1}{a})^n \quad (n \geq 1)$$

Complement ROC Pairs - Original Geometric Series Form Combinations

unit	$-\frac{1}{1-az} \quad z < a^{-1}$	$-a^n \quad (n \geq 0)$	$-\frac{1}{1-az^{-1}} \quad z > a$	$-\left(\frac{1}{a}\right)^n \quad (n < 1)$
non-unit	$\frac{a^nz^{-1}}{1-a^nz^{-1}} \quad z > a^{-1}$	$a^n \quad (n < 0)$	$\frac{a^nz}{1-a^nz} \quad z < a$	$\left(\frac{1}{a}\right)^n \quad (n \geq 1)$

unit	$\frac{1}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$a^n \quad (n < 1)$	$\frac{1}{1-a^{-1}z} \quad z < a$	$\left(\frac{1}{a}\right)^n \quad (n \geq 0)$
non-unit	$-\frac{az}{1-az} \quad z < a^{-1}$	$-a^n \quad (n \geq 1)$	$\frac{az^{-1}}{1-az^{-1}} \quad z > a$	$-\left(\frac{1}{a}\right)^n \quad (n < 0)$

unit	$-\frac{1}{1-a^{-1}z} \quad z < a$	$-\left(\frac{1}{a}\right)^n \quad (n \geq 0)$	$-\frac{1}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-a^n \quad (n < 1)$
non-unit	$\frac{az^{-1}}{1-az^{-1}} \quad z > a$	$\left(\frac{1}{a}\right)^n \quad (n < 0)$	$\frac{az}{1-az} \quad z < a^{-1}$	$a^n \quad (n \geq 1)$

unit	$\frac{1}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\left(\frac{1}{a}\right)^n \quad (n < 1)$	$\frac{1}{1-a^{-1}z} \quad z < a$	$a^n \quad (n \geq 0)$
non-unit	$-\frac{a^nz}{1-a^nz} \quad z < a$	$-\left(\frac{1}{a}\right)^n \quad (n \geq 1)$	$-\frac{a^nz^{-1}}{1-a^nz^{-1}} \quad z > a^{-1}$	$-a^n \quad (n < 0)$

start term

Complement ROC Pairs - Shifted Geometric Series Form Combinations

$-\frac{a}{1-az} \quad z < a^{-1}$	$-a^{n+1} \quad (n \geq 0)$	$-\frac{a}{1-az^{-1}} \quad z > a$	$-\left(\frac{1}{a}\right)^{n-1} \quad (n < 1)$
$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$a^{n+1} \quad (n < 0)$	$\frac{z}{1-a^{-1}z} \quad z < a$	$\left(\frac{1}{a}\right)^{n-1} \quad (n \geq 1)$

$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$a^{n+1} \quad (n < 0)$	$\frac{z}{1-a^{-1}z} \quad z < a$	$\left(\frac{1}{a}\right)^{n-1} \quad (n \geq 1)$
$-\frac{a}{1-az} \quad z < a^{-1}$	$-a^{n+1} \quad (n \geq 0)$	$\frac{a}{1-az^{-1}} \quad z > a$	$-\left(\frac{1}{a}\right)^{n-1} \quad (n < 1)$

$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-\left(\frac{1}{a}\right)^{n+1} \quad (n \geq 0)$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-a^{n-1} \quad (n < 1)$
$\frac{z^{-1}}{1-az^{-1}} \quad z > a$	$\left(\frac{1}{a}\right)^{n+1} \quad (n < 0)$	$\frac{z}{1-az} \quad z < a^{-1}$	$a^{n-1} \quad (n \geq 1)$

$\frac{z^{-1}}{1-az^{-1}} \quad z > a$	$\left(\frac{1}{a}\right)^{n+1} \quad (n < 0)$	$\frac{z}{1-az} \quad z < a^{-1}$	$a^{n-1} \quad (n \geq 1)$
$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-\left(\frac{1}{a}\right)^{n+1} \quad (n \geq 0)$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-a^{n-1} \quad (n < 1)$

Complement ROC Pairs - Reduced Shifted Geometric Series Form Combinations

$-\frac{a}{1-az}$ $ z < a^{-1}$	$-a^{n+1}$ ($n \geq 0$)	$-\frac{a}{1-az^{-1}}$ $ z > a$	$-(\frac{1}{a})^{n-1}$ ($n < 1$)
$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $ z > a^{-1}$	a^{n+1} ($n < 0$)	$\frac{z}{1-a^{-1}z}$ $ z < a$	$(\frac{1}{a})^{n-1}$ ($n \geq 1$)

$-\frac{a^{-1}}{1-a^{-1}z}$ $ z < a$	$-(\frac{1}{a})^{n+1}$ ($n \geq 0$)	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $ z > a^{-1}$	$-a^{n-1}$ ($n < 1$)
$\frac{z^{-1}}{1-az^{-1}}$ $ z > a$	$(\frac{1}{a})^{n+1}$ ($n < 0$)	$\frac{z}{1-az}$ $ z < a^{-1}$	a^{n-1} ($n \geq 1$)

$$2z$$

$$2z^{-1}$$

$$2^{-1}z^{-1}$$

$$2^{-1}z$$

$$|z| < 0.5$$

$$|z| > 2$$

$$|z| > 0.5$$

$$|z| < 2$$

$$- \frac{2}{1-2z} \xleftrightarrow{z^{-1}} - \frac{2}{1-2z^{-1}}$$

$$\cdot \frac{(2z)^{-1}}{(2z)^{-1}} \cdot \frac{(2z)}{(2z)}$$

$$\cdot \frac{(2z^{-1})^{-1}}{(2z^{-1})^{-1}} \cdot \frac{(2z^{-1})}{(2z^{-1})}$$

$$+ \frac{z^{-1}}{1-0.5z^{-1}} \xleftrightarrow{z^{-1}} + \frac{z}{1-0.5z}$$

scale(a)

scale(a)

	$-\frac{1}{1-a^2z} \quad z < a^{-1}$	$-\frac{a}{1-a^2z} \quad z < a^{-1}$	$-\frac{1}{1-a^2z^{-1}} \quad z > a$	$-\frac{a}{1-a^2z^{-1}} \quad z > a$
Comp.ROC	$\frac{a^2z^{-1}}{1-a^2z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^2z^{-1}} \quad z > a^{-1}$	$\frac{a^2z}{1-a^2z} \quad z < a$	$\frac{z}{1-a^2z} \quad z < a$

scale(1/z)

scale(z)

	$\frac{1}{1-a^2z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^2z^{-1}} \quad z > a^{-1}$	$\frac{1}{1-a^2z} \quad z < a$	$\frac{z}{1-a^2z} \quad z < a$
Comp.ROC	$-\frac{az}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$\frac{az^{-1}}{1-az^{-1}} \quad z > a$	$\frac{a}{1-az^{-1}} \quad z > a$

scale(1/a)

scale(1/a)

	$-\frac{1}{1-a^2z} \quad z < a$	$-\frac{a^{-1}}{1-a^2z} \quad z < a$	$-\frac{1}{1-a^2z^{-1}} \quad z > a^{-1}$	$-\frac{a^{-1}}{1-a^2z^{-1}} \quad z > a^{-1}$
Comp.ROC	$\frac{az^{-1}}{1-az^{-1}} \quad z > a$	$\frac{z^{-1}}{1-az^{-1}} \quad z > a$	$\frac{az}{1-az} \quad z < a^{-1}$	$\frac{z}{1-az} \quad z < a^{-1}$

scale(1/z)

scale(z)

	$\frac{1}{1-a^2z^{-1}} \quad z > a$	$\frac{z^{-1}}{1-a^2z^{-1}} \quad z > a$	$\frac{1}{1-a^2z} \quad z < a^{-1}$	$\frac{z}{1-a^2z} \quad z < a^{-1}$
Comp.ROC	$-\frac{a^2z}{1-a^2z} \quad z < a$	$-\frac{a^2}{1-a^2z} \quad z < a$	$-\frac{a^2z^{-1}}{1-a^2z^{-1}} \quad z > a^{-1}$	$-\frac{a^2}{1-a^2z^{-1}} \quad z > a^{-1}$

scale(a)

scale(a)

Comp.ROC

$-\frac{1}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$-\frac{1}{1-a^{-1}z^{-1}} \quad z > a$	$-\frac{a}{1-a^{-1}z^{-1}} \quad z > a$
$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{a^{-1}z}{1-a^{-1}z} \quad z < a$	$\frac{z}{1-a^{-1}z} \quad z < a$

scale(1/z)

scale(z)

Comp.ROC

$\frac{1}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{1}{1-a^{-1}z} \quad z < a$	$\frac{z}{1-a^{-1}z} \quad z < a$
$-\frac{az}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{a}{1-a^{-1}z^{-1}} \quad z > a$

scale(1/a)

scale(1/a)

Comp.ROC

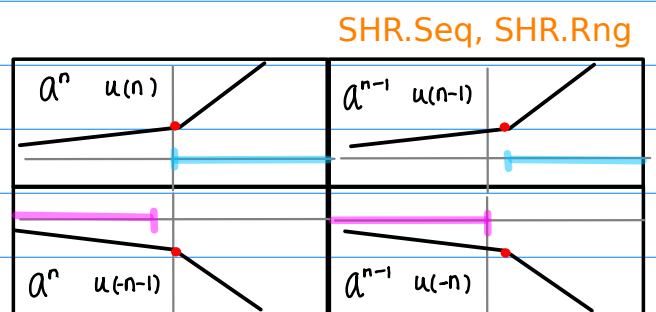
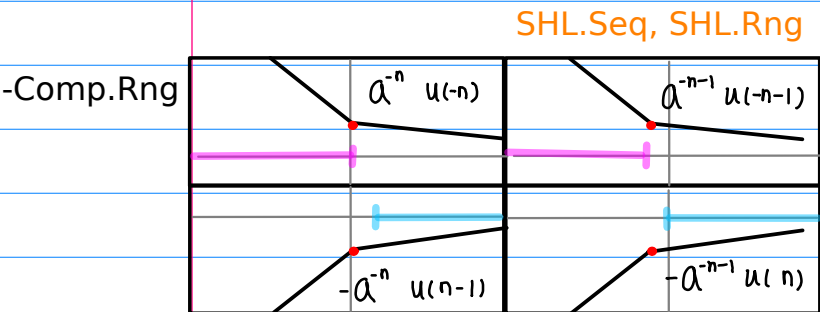
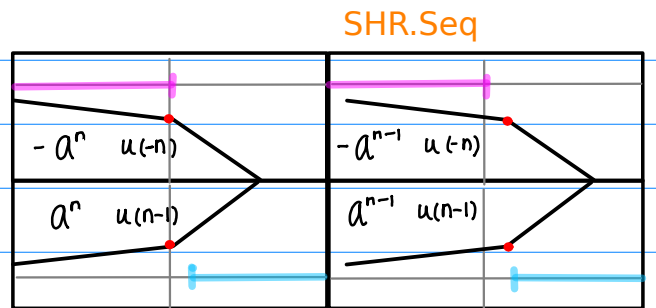
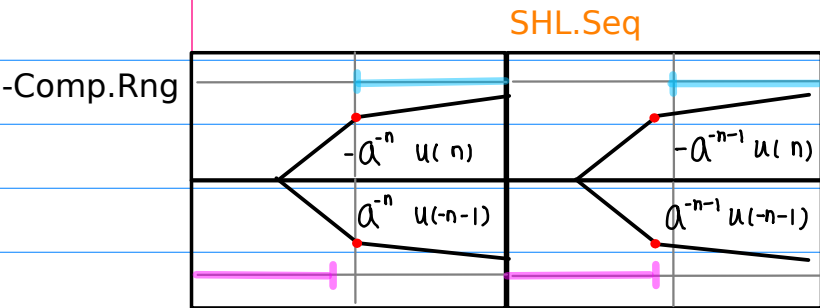
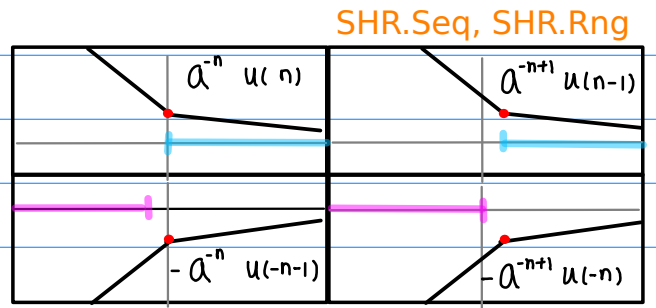
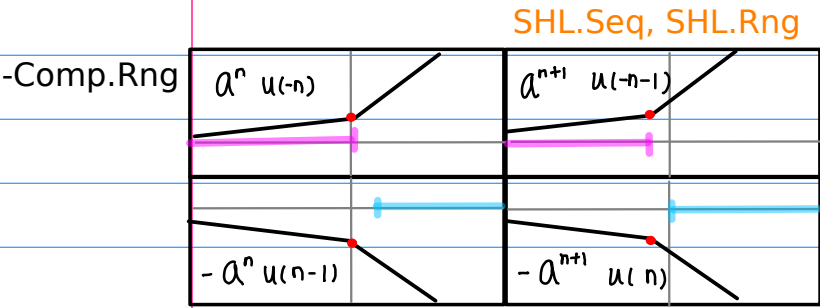
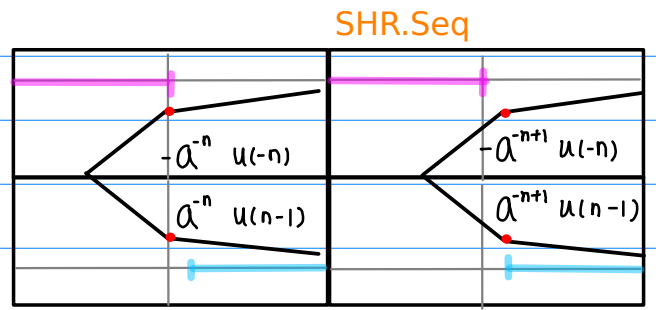
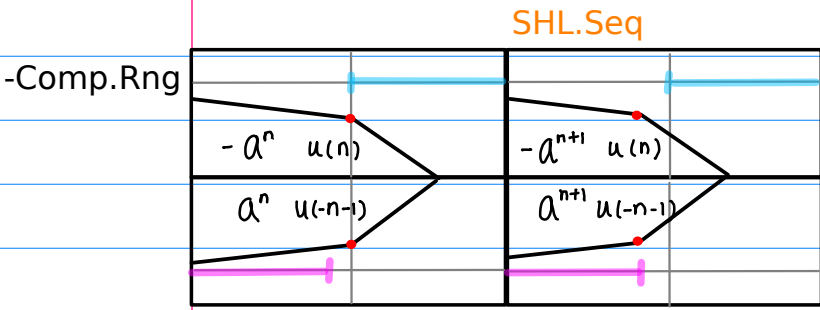
$-\frac{1}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-\frac{1}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$
$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{az}{1-az} \quad z < a^{-1}$	$\frac{z}{1-az} \quad z < a^{-1}$

scale(1/z)

scale(z)

Comp.ROC

$\frac{1}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{1}{1-az} \quad z < a^{-1}$	$\frac{z}{1-az} \quad z < a^{-1}$
$-\frac{a^{-1}z}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$



Left Shifted
Sequence

Right Shifted
Sequence

scale(a)

scale(a)

	$-\frac{1}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$-\frac{1}{1-az^{-1}} \quad z > a$	$-\frac{a}{1-az^{-1}} \quad z > a$
Comp.ROC	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{a^{-1}z}{1-a^{-1}z} \quad z < a$	$\frac{z}{1-a^{-1}z} \quad z < a$

scale(1/z)

scale(z)

	$\frac{1}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$\frac{1}{1-a^{-1}z} \quad z < a$	$\frac{z}{1-a^{-1}z} \quad z < a$
Comp.ROC	$-\frac{az}{1-az} \quad z < a^{-1}$	$-\frac{a}{1-az} \quad z < a^{-1}$	$\frac{az^{-1}}{1-az^{-1}} \quad z > a$	$\frac{a}{1-az^{-1}} \quad z > a$

scale(1/a)

scale(1/a)

	$-\frac{1}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-\frac{1}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$
Comp.ROC	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{z^{-1}}{1-a^{-1}z^{-1}} \quad z > a$	$\frac{az}{1-az} \quad z < a^{-1}$	$\frac{z}{1-az} \quad z < a^{-1}$

scale(1/z)

scale(z)

	$\frac{1}{1-az^{-1}} \quad z > a$	$\frac{z^{-1}}{1-az^{-1}} \quad z > a$	$\frac{1}{1-az} \quad z < a^{-1}$	$\frac{z}{1-az} \quad z < a^{-1}$
Comp.ROC	$-\frac{a^{-1}z}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}}{1-a^{-1}z} \quad z < a$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \quad z > a^{-1}$

SHL.Seq

$-a^n$ ($n \geq 0$) $-(a^0, a^1, a^2, \dots)$	$-a^{n+1}$ ($n \geq 0$) $-(a^1, a^2, a^3, \dots)$
a^n ($n < 0$) $(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$	a^{n+1} ($n < 0$) $(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$

SHR.Seq

$-\left(\frac{1}{a}\right)^n$ ($n < 1$) $-(\dots, a^0, a^1, a^2)$	$-\left(\frac{1}{a}\right)^{n-1}$ ($n < 1$) $-(\dots, a^3, a^2, a^1)$
$\left(\frac{1}{a}\right)^n$ ($n \geq 1$) $(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \dots)$	$\left(\frac{1}{a}\right)^{n-1}$ ($n \geq 1$) $(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \dots)$

-Comp.Rng

SHL.Seq, SHL.Rng

a^n ($n < 1$) $(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$	a^{n+1} ($n < 0$) $(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$
$-a^n$ ($n \geq 1$) $-(a^1, a^2, a^3, \dots)$	$-a^{n+1}$ ($n \geq 0$) $-(a^1, a^2, a^3, \dots)$

SHR.Seq, SHR.Rng

$\left(\frac{1}{a}\right)^n$ ($n \geq 0$) $(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \dots)$	$\left(\frac{1}{a}\right)^{n-1}$ ($n \geq 1$) $(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \dots)$
$-\left(\frac{1}{a}\right)^n$ ($n < 0$) (\dots, a^3, a^2, a^1)	$-\left(\frac{1}{a}\right)^{n-1}$ ($n < 1$) (\dots, a^3, a^2, a^1)

-Comp.Rng

SHL.Seq

$-\left(\frac{1}{a}\right)^n$ ($n \geq 0$) $-(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \dots)$	$-\left(\frac{1}{a}\right)^{n+1}$ ($n \geq 0$) $-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \dots)$
$\left(\frac{1}{a}\right)^n$ ($n < 0$) (\dots, a^3, a^2, a^1)	$\left(\frac{1}{a}\right)^{n+1}$ ($n < 0$) (\dots, a^2, a^1, a^0)

SHR.Seq

$-a^n$ ($n < 1$) $-(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$	$-a^{n-1}$ ($n < 1$) $-(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$
a^n ($n \geq 1$) (a^1, a^2, a^3, \dots)	a^{n-1} ($n \geq 1$) (a^0, a^1, a^2, \dots)

-Comp.Rng

SHL.Seq, SHL.Rng

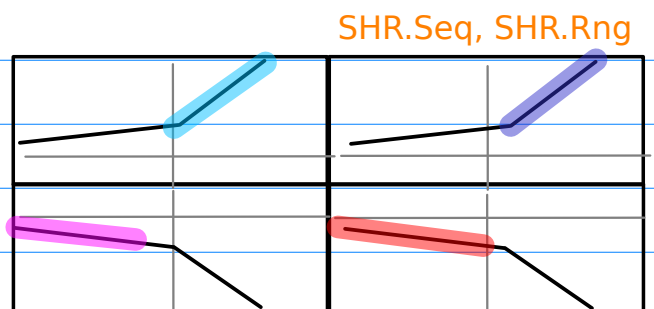
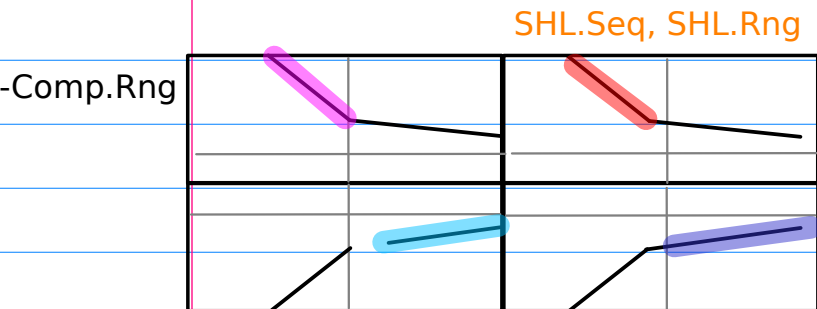
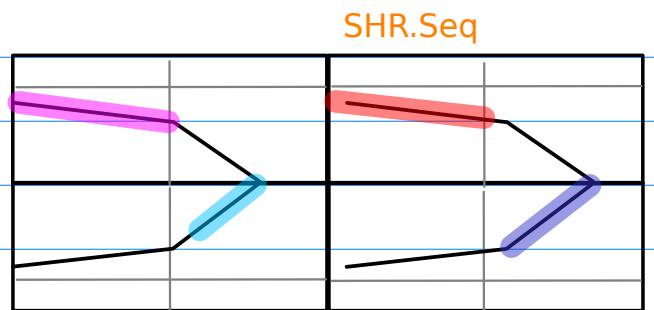
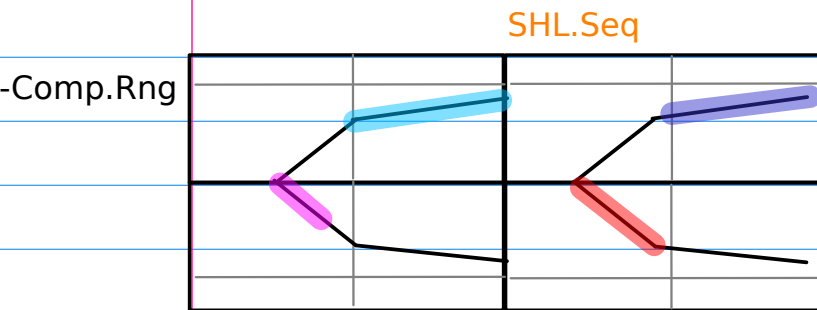
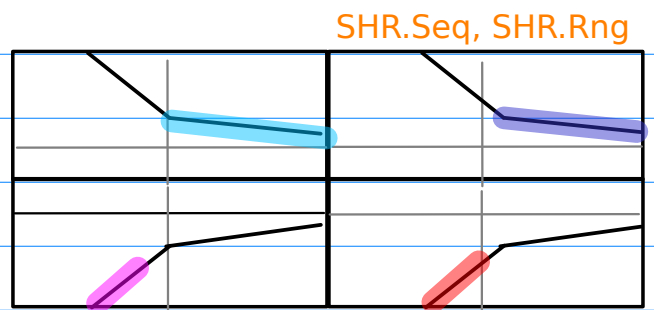
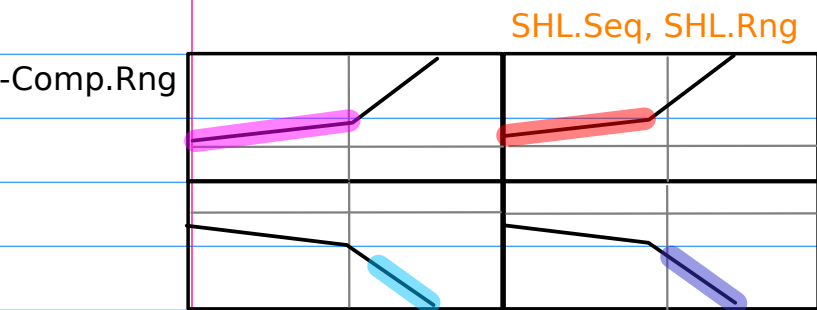
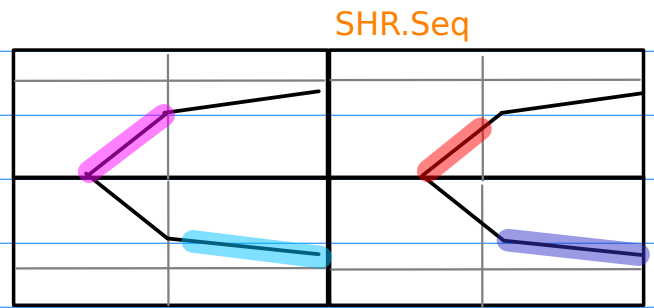
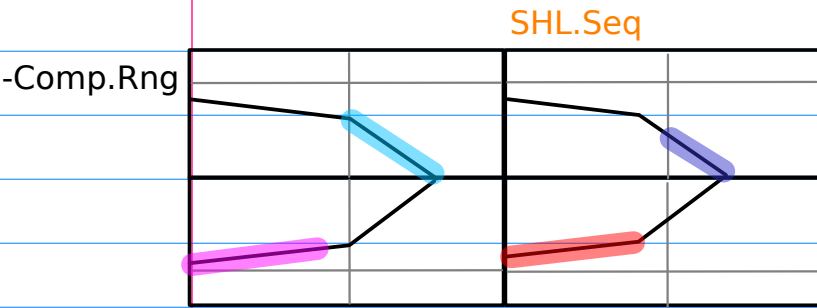
$\left(\frac{1}{a}\right)^n$ ($n < 1$) (\dots, a^2, a^1, a^0)	$\left(\frac{1}{a}\right)^{n+1}$ ($n < 0$) (\dots, a^2, a^1, a^0)
$-\left(\frac{1}{a}\right)^n$ ($n \geq 1$) $-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \dots)$	$-\left(\frac{1}{a}\right)^{n+1}$ ($n \geq 0$) $-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \dots)$

SHR.Seq, SHR.Rng

a^n ($n \geq 0$) (a^0, a^1, a^2, \dots)	a^{n-1} ($n \geq 1$) (a^0, a^1, a^2, \dots)
$-a^n$ ($n < 0$) $-(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$	$-a^{n-1}$ ($n < 1$) $-(\dots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^4})$

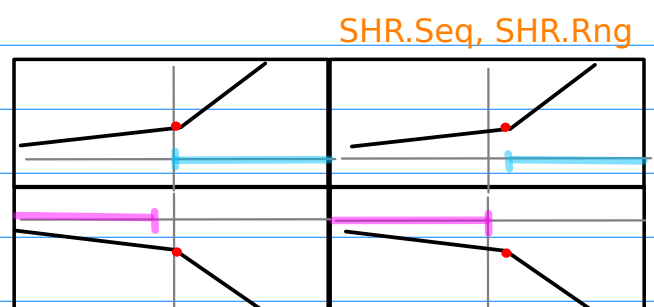
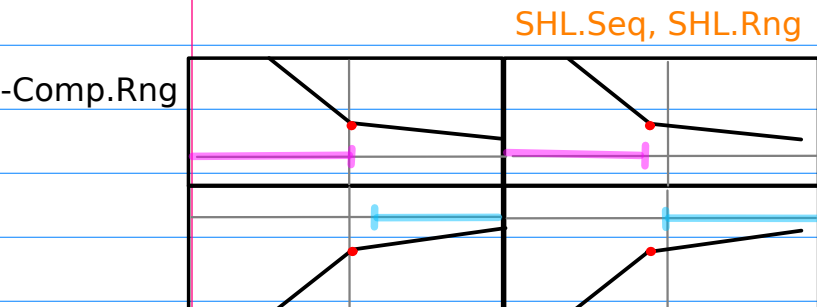
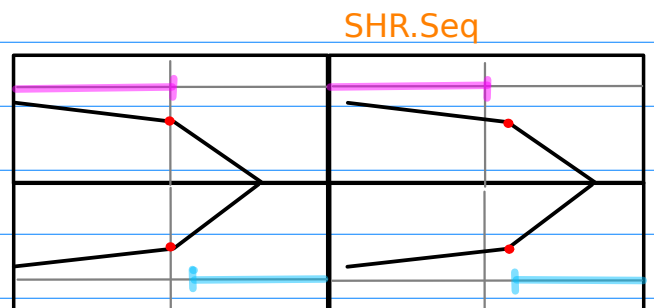
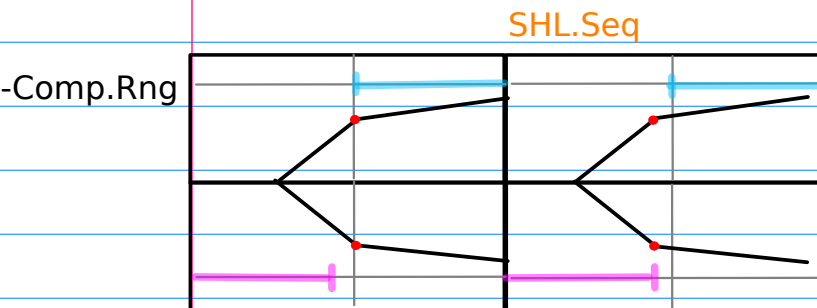
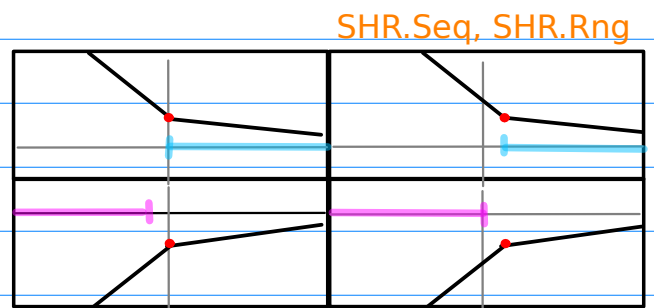
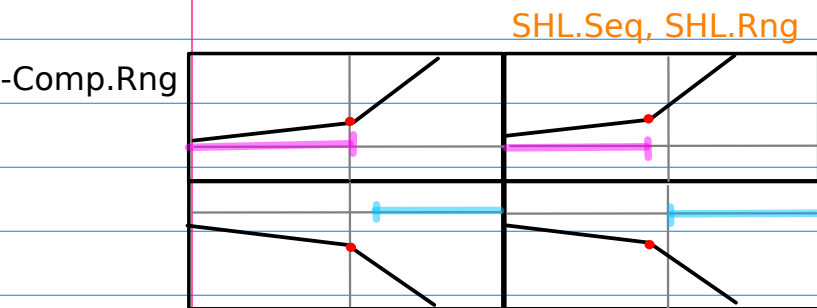
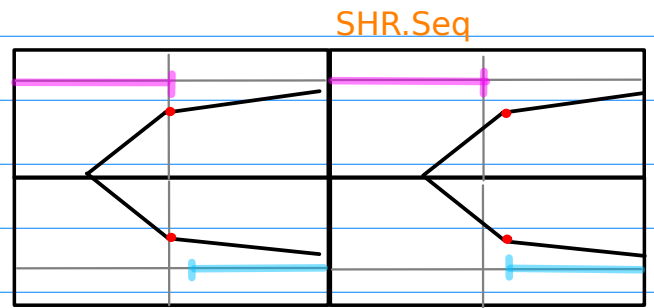
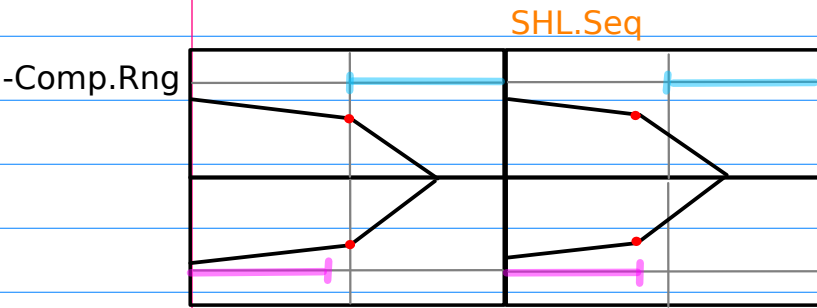
-Comp.Rng

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SequenceRight Shifted
Sequence



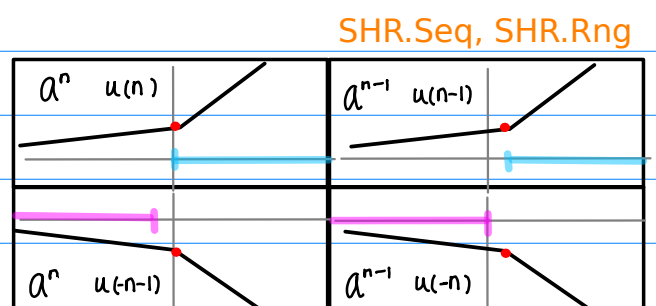
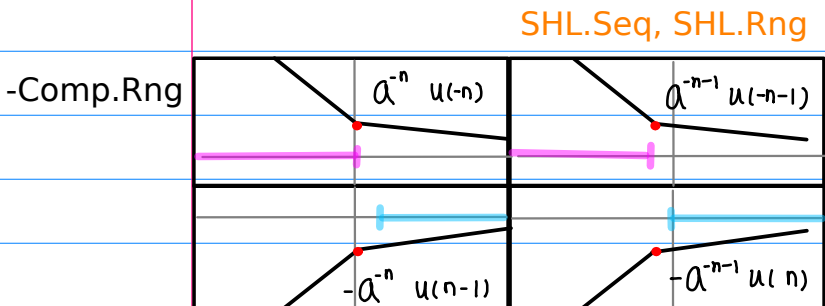
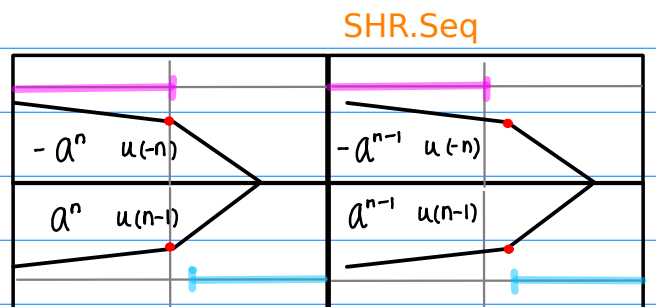
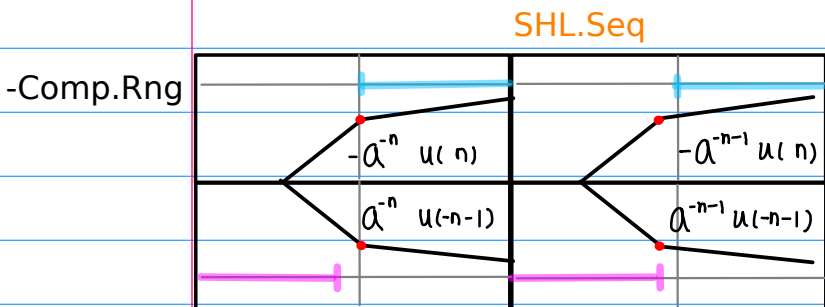
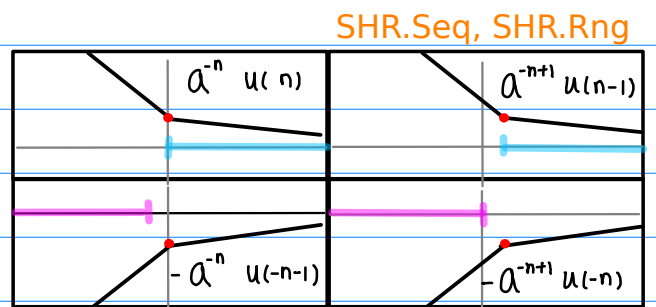
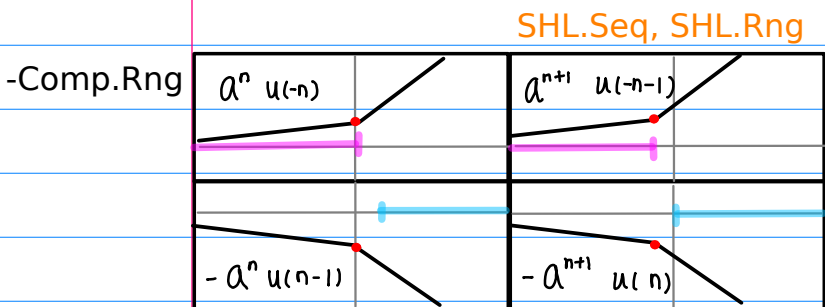
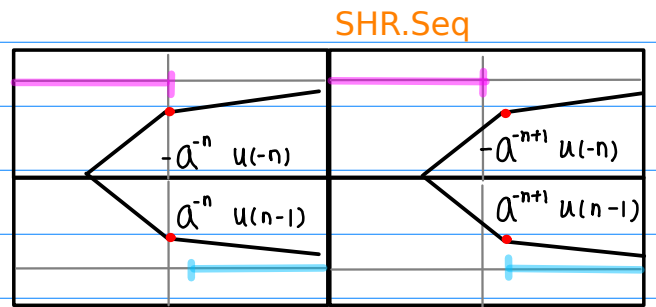
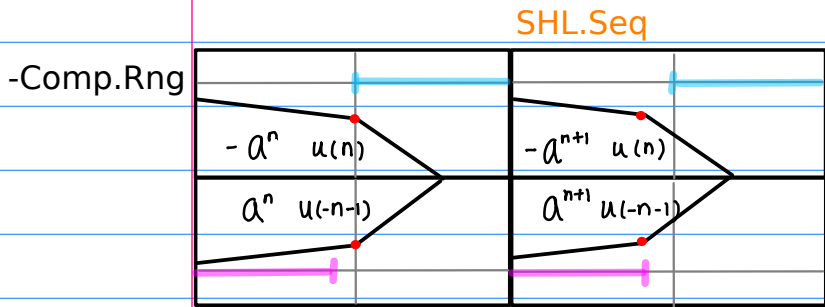
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Sequence

Right Shifted
Sequence



Left Shifted
Sequence

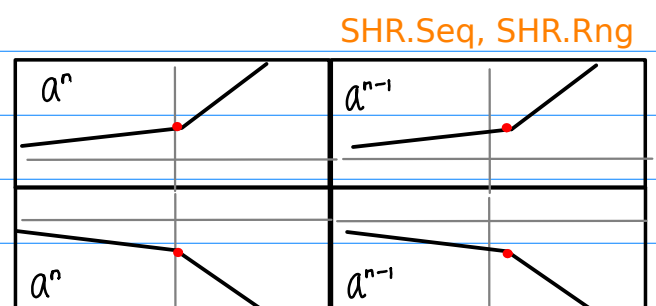
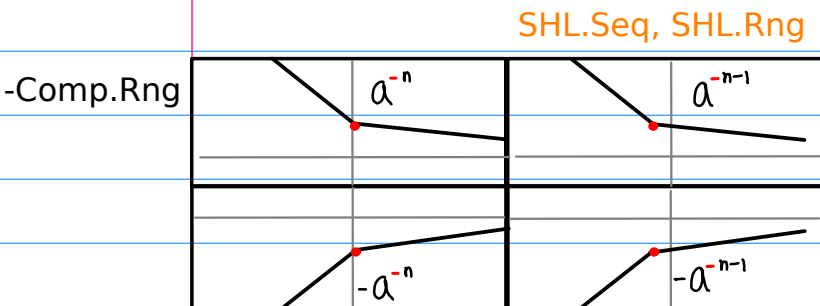
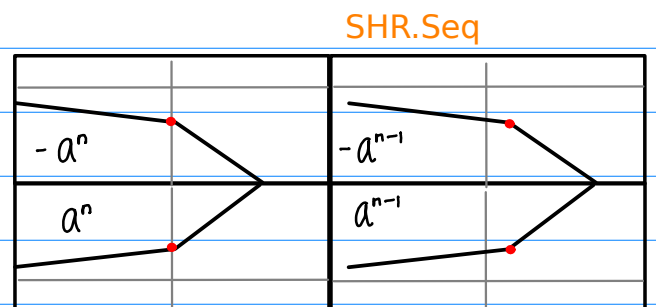
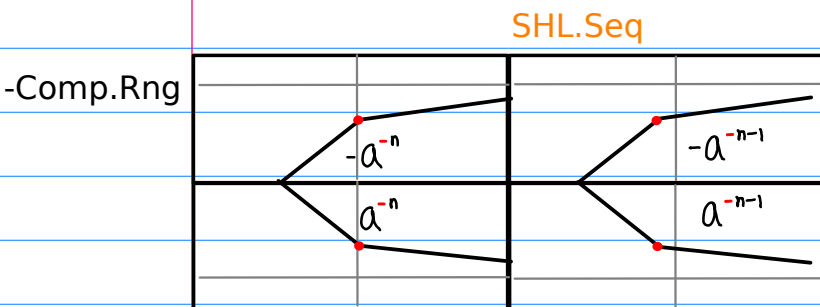
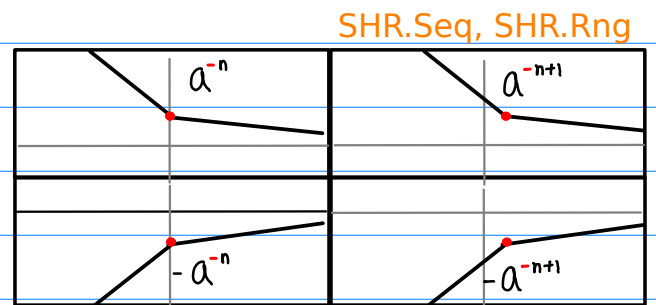
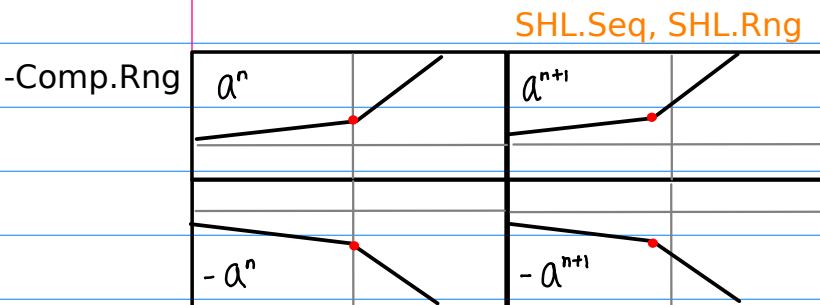
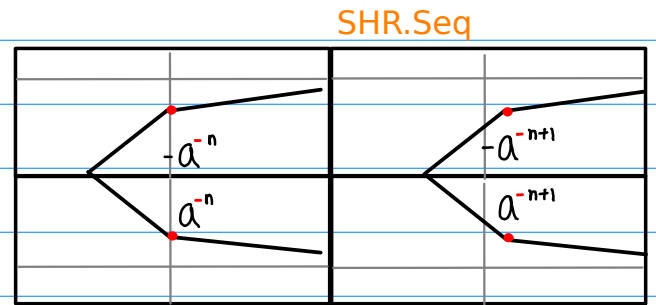
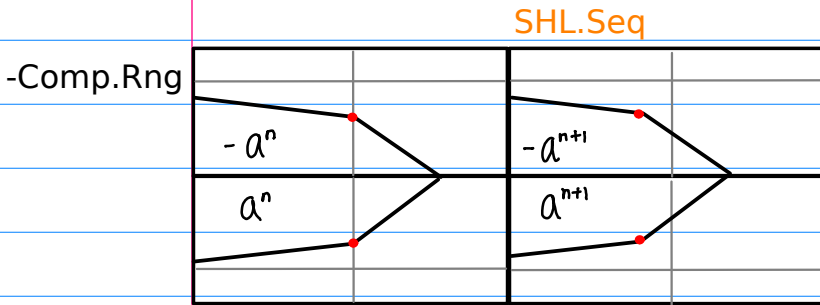
Right Shifted
Sequence



Left Shifted Sequence

Right Shifted Sequence

a Sequence Function



Left Shifted Sequence

Right Shifted Sequence

Range of a Sequence

