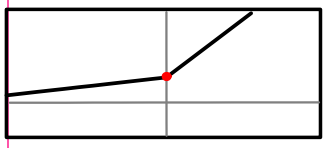
Laurent Series and z-Transform - Geometric Series Applications

A

20200709 Thr

a^n



*a

/a

Shifting a sequence

$\frac{1}{1-az}$ $|z|<a^{-1}$ — $a^n\ u(n)$

*a → $\frac{a}{1-az}$ $|z|<a^{-1}$ — $a^{n+1}\ u(n)$

/z → $\frac{az}{1-az}$ $|z|<a^{-1}$ — $a^n\ u(n-1)$

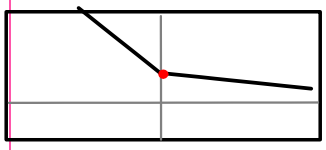
*z → $\frac{z}{1-az}$ $|z|<a^{-1}$ — $a^{n-1}\ u(n-1)$

/a → $-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$ — $a^n\ u(-n-1)$

*a → $-\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$ — $a^{n+1}\ u(-n-1)$

/z → $-\frac{1}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$ — $a^n\ u(-n)$

*z → $-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$ — $a^{n-1}\ u(-n)$

a^{-n} 

/a

*a

Shifting a sequence

$\frac{1}{1-a^{-1}z}$ $|z|<a$ — $a^{-n}\,u(n)$

/a

/z

$\frac{a^{-1}}{1-a^{-1}z}$ $|z|<a$ — $a^{-n-1}\,u(n)$

*z

$\frac{a^{-1}z}{1-a^{-1}z}$ $|z|<a$ — $a^{-n}\,u(n-1)$

*a

$\frac{z}{1-a^{-1}z}$ $|z|<a$ — $a^{-n+1}\,u(n-1)$

$-\frac{az^{-1}}{1-az^{-1}}$ $|z|>a$ — $a^{-n}\,u(-n-1)$

/a

/z

$-\frac{z^{-1}}{1-az^{-1}}$ $|z|>a$ — $a^{-n-1}\,u(-n-1)$

*z

$-\frac{1}{1-az^{-1}}$ $|z|>a$ — $a^{-n}\,u(-n)$

*a

$-\frac{a}{1-az^{-1}}$ $|z|>a$ — $a^{-n+1}\,u(-n)$

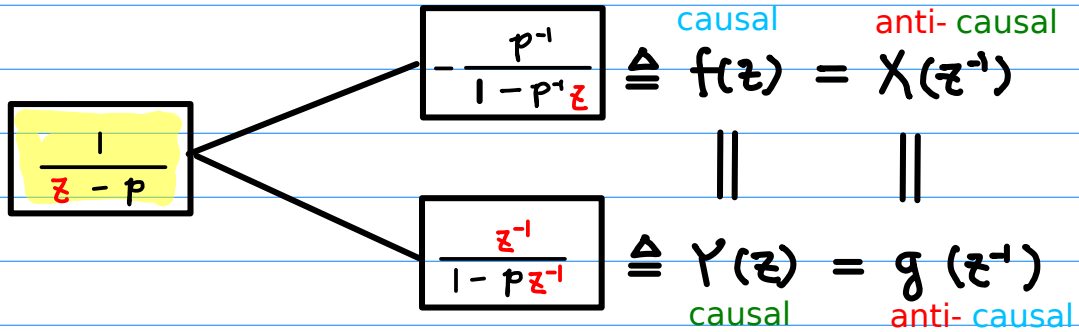
2 formulas

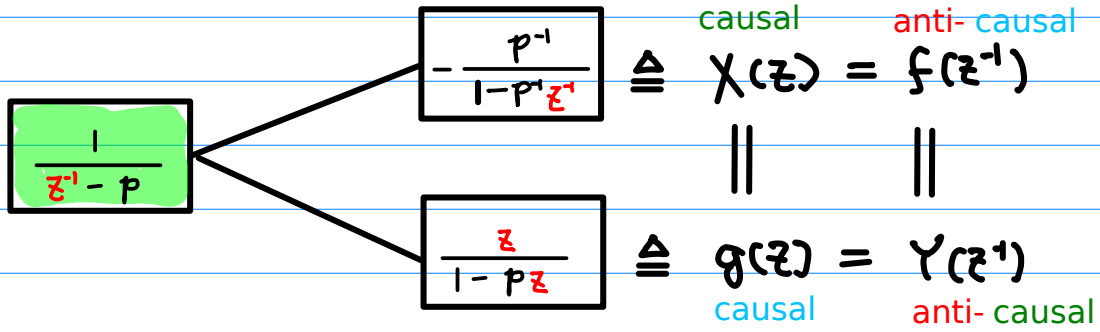
Simple Pole Form

$$\frac{1}{z - p} \qquad \frac{1}{z^{-1} - p}$$

2 representations each

Geometric Series Form





Simple Pole Form	Geometric Series Form

Geometric Series (1)

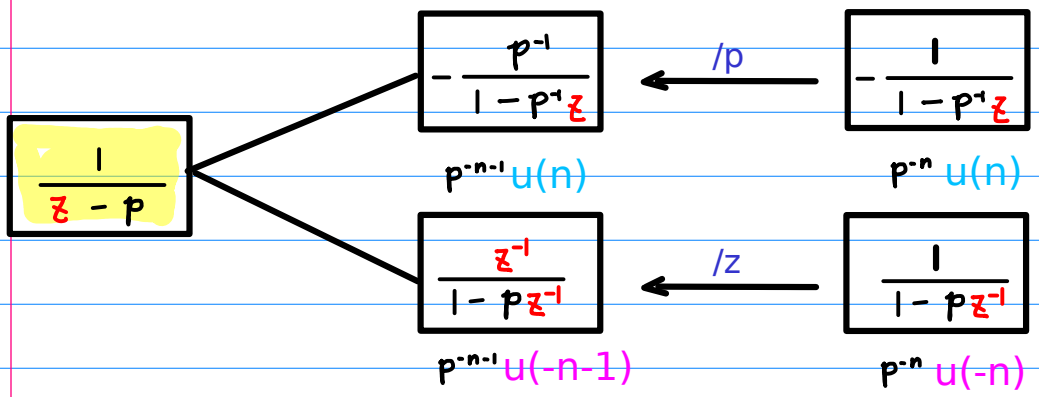
2 formulas

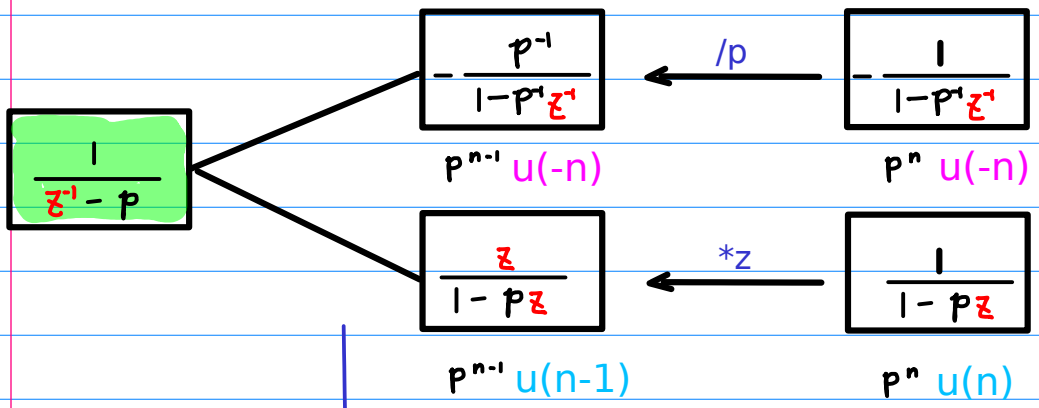
Simple Pole Form

$\frac{1}{z - p}$ $\frac{1}{z^{-1} - p}$

2 representations each

Geometric Series Form





Simple Pole Form | Geometric Series Form

Geometric Series (2)

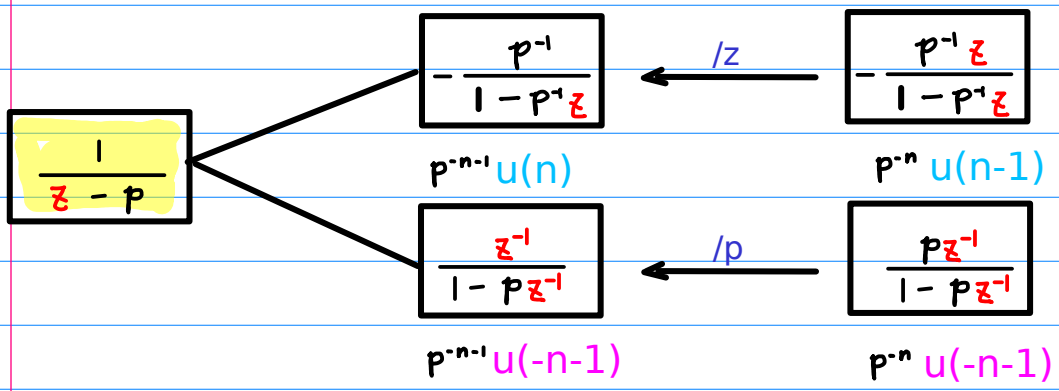
2 formulas

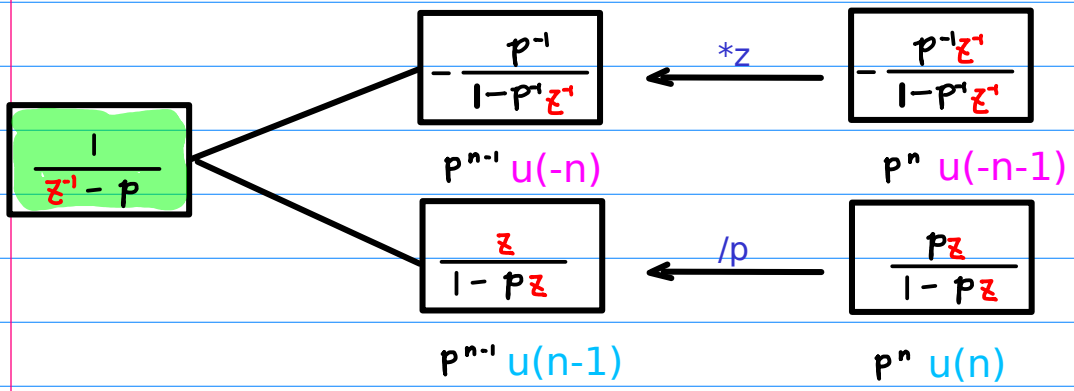
Simple Pole Form

$$\frac{1}{z - p} \qquad \frac{1}{z^{-1} - p}$$

2 representations each

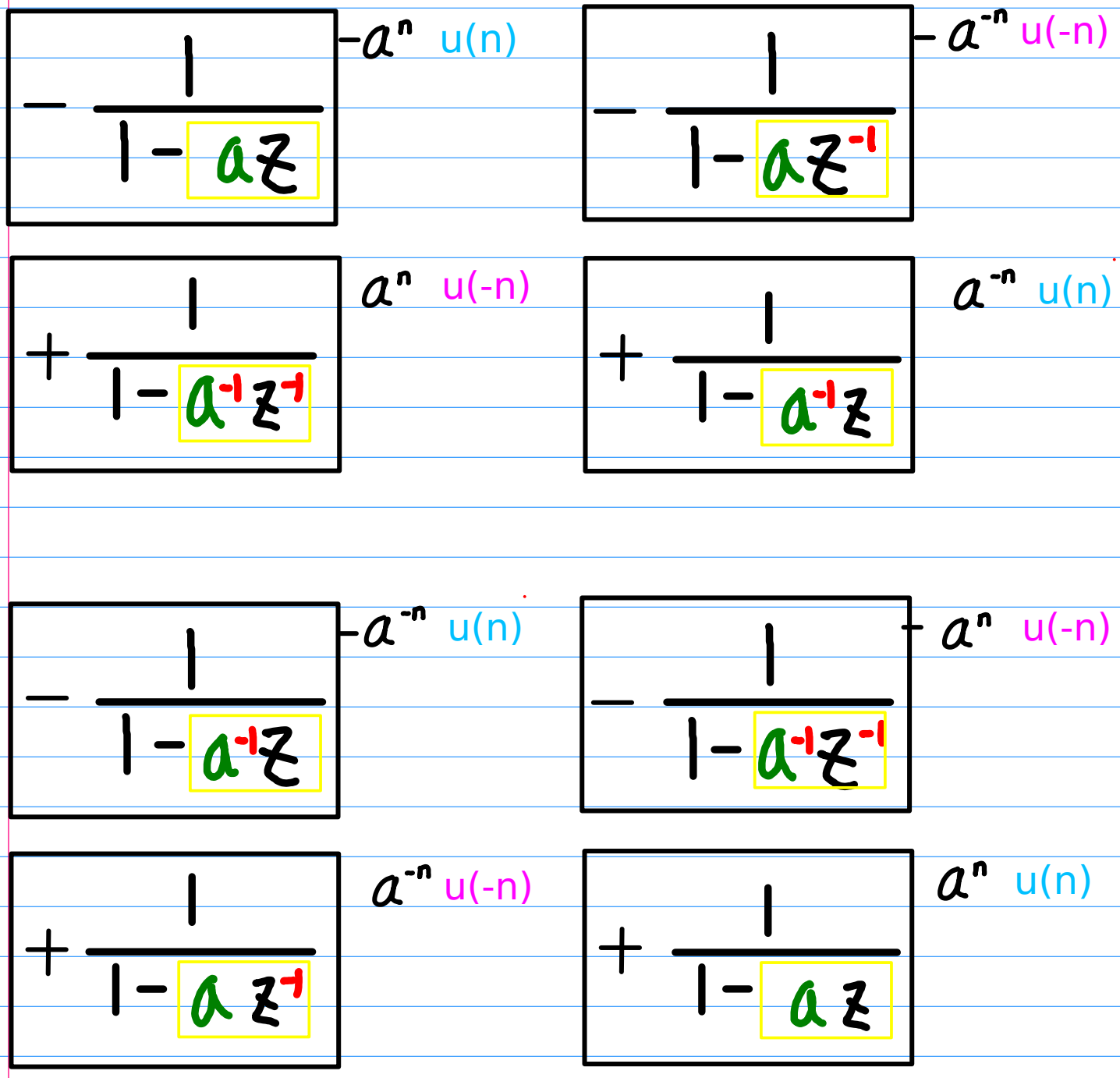
Geometric Series Form



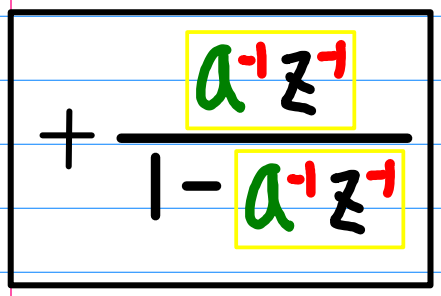


Simple Pole Form | Geometric Series Form

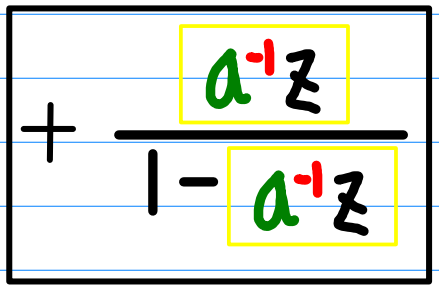
Geometric Series Form Combinations with a unit start term



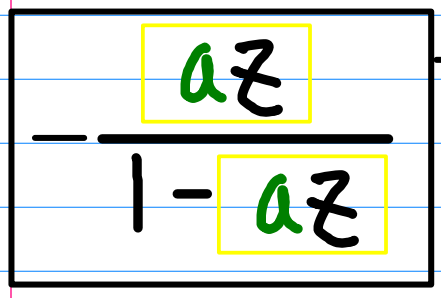
Geometric Series Form Combinations with a common-ratio start term



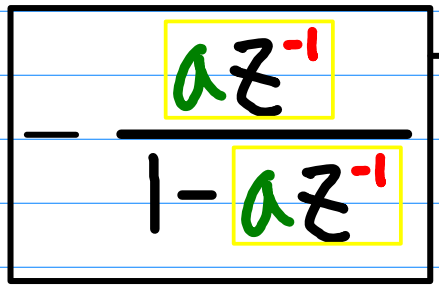
$a^{n}\ u(-n-1)$



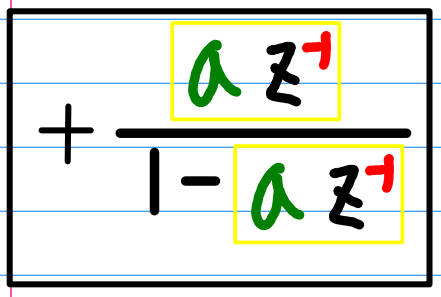
$a^{-n}\ u(n-1)$



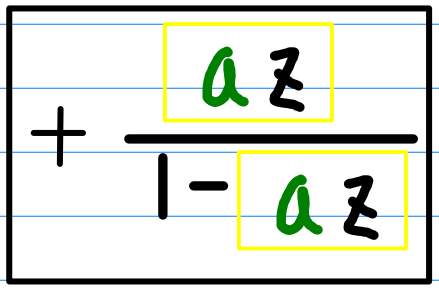
$-a^{n}\ u(n-1)$



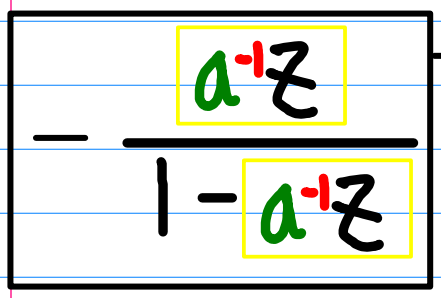
$-a^{-n}\ u(-n-1)$



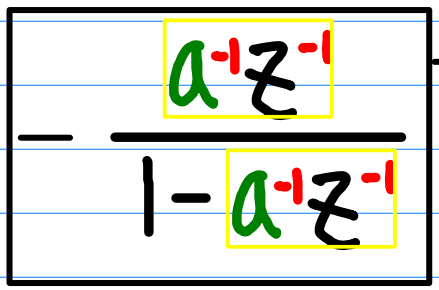
$a^{-n}\ u(-n-1)$



$a^{n}\ u(n-1)$



$-a^{-n}\ u(n-1)$



$-a^{n}\ u(-n-1)$

Geometric Series - a unit start term

Laurent Series

Geometric Series - a unit start term

z-Transform

Geometric Series - a unit start term

Laurent Series vs. z-Transform

Geometric Series - a unit start term

Laurent Series

$$-\frac{1}{1-az} \qquad |z| < a^{-1}$$

$$-\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$$

$$a_n = -a^n \qquad (n \geq 0)$$

$$-\frac{1}{1-az^{-1}} \qquad |z| > a$$

$$-\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right)$$

$$-\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^1 z^{-1} + \left(\tfrac{1}{a}\right)^2 z^{-2} + \cdots\right)$$

$$a_n = -\left(\tfrac{1}{a}\right)^n \qquad (n < 1)$$

$$+\frac{1}{1-a^{-1}z^{-1}} \qquad |z| > a^{-1}$$

$$\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right)$$

$$a_n = a^n \qquad (n < 1)$$

$$+\frac{1}{1-a^{-1}z} \qquad |z| < a$$

$$\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$$

$$\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^1 z^1 + \left(\tfrac{1}{a}\right)^2 z^2 + \cdots\right)$$

$$a_n = \left(\tfrac{1}{a}\right)^n \qquad (n \geq 0)$$

$$-\frac{1}{1-a^{-1}z} \qquad |z| < a$$

$$-\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^1 z^1 + \left(\tfrac{1}{a}\right)^2 z^2 + \cdots\right)$$

$$a_n = -\left(\tfrac{1}{a}\right)^n \qquad (n \geq 0)$$

$$-\frac{1}{1-a^{-1}z^{-1}} \qquad |z| > a^{-1}$$

$$-\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right)$$

$$a_n = -a^n \qquad (n < 1)$$

$$+\frac{1}{1-az^{-1}} \qquad |z| > a$$

$$\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right)$$

$$\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^1 z^{-1} + \left(\tfrac{1}{a}\right)^2 z^{-2} + \cdots\right)$$

$$a_n = \left(\tfrac{1}{a}\right)^n \qquad (n < 1)$$

$$+\frac{1}{1-az} \qquad |z| < a^{-1}$$

$$\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$$

$$a_n = a^n \qquad (n \geq 0)$$

Geometric Series - a unit start term

z-Transform

$$-\frac{1}{1-az} \qquad |z|<a^{-1}$$

$-\left(a^0z^0+a^1z^{-1}+a^2z^{-2}+\cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0z^0+\left(\frac{1}{a}\right)^1z^{-1}+\left(\frac{1}{a}\right)^2z^{-2}+\cdots\right)$

$a_n = -a^n$	$(-n \geq 0)$
$a_n = -\left(\frac{1}{a}\right)^n$	$(n<1)$

$$-\frac{1}{1-az^{-1}} \qquad |z|>a$$

$-\left(a^0z^0+a^1z^{-1}+a^2z^{-2}+\cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0z^0+\left(\frac{1}{a}\right)^1z^{-1}+\left(\frac{1}{a}\right)^2z^{-2}+\cdots\right)$

$a_n = -\left(\frac{1}{a}\right)^{-n}$	$(-n<1)$
$a_n = -a^n$	$(n \geq 0)$

$$+\frac{1}{1-a^{-1}z^{-1}} \qquad |z|>a^{-1}$$

$\left(a^0z^0+a^{-1}z^{-1}+a^{-2}z^{-2}+\cdots\right)$

$\left(\left(\frac{1}{a}\right)^0z^0+\left(\frac{1}{a}\right)^1z^{-1}+\left(\frac{1}{a}\right)^2z^{-2}+\cdots\right)$

$a_n = a^{-n}$	$(-n<1)$
$a_n = \left(\frac{1}{a}\right)^n$	$(n \geq 0)$

$$+\frac{1}{1-a^{-1}z} \qquad |z|<a$$

$\left(a^0z^0+a^{-1}z^1+a^{-2}z^2+\cdots\right)$

$\left(\left(\frac{1}{a}\right)^0z^0+\left(\frac{1}{a}\right)^1z^1+\left(\frac{1}{a}\right)^2z^2+\cdots\right)$

$a_n = \left(\frac{1}{a}\right)^{-n}$	$(-n \geq 0)$
$a_n = a^n$	$(n<1)$

$$-\frac{1}{1-a^{-1}z} \qquad |z|<a$$

$-\left(a^0z^0+a^{-1}z^1+a^{-2}z^2+\cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0z^0+\left(\frac{1}{a}\right)^1z^1+\left(\frac{1}{a}\right)^2z^2+\cdots\right)$

$a_n = -\left(\frac{1}{a}\right)^{-n}$	$(-n \geq 0)$
$a_n = -a^n$	$(n<1)$

$$-\frac{1}{1-a^{-1}z^{-1}} \qquad |z|>a^{-1}$$

$-\left(a^0z^0+a^{-1}z^{-1}+a^{-2}z^{-2}+\cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0z^0+\left(\frac{1}{a}\right)^1z^{-1}+\left(\frac{1}{a}\right)^2z^{-2}+\cdots\right)$

$a_n = -a^{-n}$	$(-n<1)$
$a_n = -\left(\frac{1}{a}\right)^n$	$(n \geq 0)$

$$+\frac{1}{1-az^{-1}} \qquad |z|>a$$

$\left(a^0z^0+a^1z^{-1}+a^2z^{-2}+\cdots\right)$

$\left(\left(\frac{1}{a}\right)^0z^0+\left(\frac{1}{a}\right)^1z^{-1}+\left(\frac{1}{a}\right)^2z^{-2}+\cdots\right)$

$a_n = \left(\frac{1}{a}\right)^{-n}$	$(-n<1)$
$a_n = a^n$	$(n \geq 0)$

$$+\frac{1}{1-az} \qquad |z|<a^{-1}$$

$\left(a^0z^0+a^1z^1+a^2z^2+\cdots\right)$

$\left(\left(\frac{1}{a}\right)^0z^0+\left(\frac{1}{a}\right)^1z^1+\left(\frac{1}{a}\right)^2z^2+\cdots\right)$

$a_n = a^{-n}$	$(-n \geq 0)$
$a_n = \left(\frac{1}{a}\right)^n$	$(n<1)$

Geometric Series - a unit start term

Laurent Series vs. z-Transform

$$-\frac{1}{1-az} \qquad |z|<a^{-1}$$

$-\left(a^0z^0+a^1z^1+a^2z^2+\cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0z^0+\left(\frac{1}{a}\right)^1z^1+\left(\frac{1}{a}\right)^2z^2+\cdots\right)$

Laurent	$a_n = -a^n$	$(n\geq 0)$
z-Trans	$a_n = -\left(\frac{1}{a}\right)^n$	$(n<1)$

$$-\frac{1}{1-az^{-1}} \qquad |z|>a$$

$-\left(a^0z^0+a^1z^{-1}+a^2z^{-2}+\cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0z^0+\left(\frac{1}{a}\right)^1z^{-1}+\left(\frac{1}{a}\right)^2z^{-2}+\cdots\right)$

Laurent	$a_n = -\left(\frac{1}{a}\right)^n$	$(n<1)$
z-Trans	$a_n = -a^n$	$(n\geq 0)$

$$+\frac{1}{1-a^{-1}z^{-1}} \qquad |z|>a^{-1}$$

$\left(a^0z^0+a^{-1}z^{-1}+a^{-2}z^{-2}+\cdots\right)$

$\left(\left(\frac{1}{a}\right)^0z^0+\left(\frac{1}{a}\right)^1z^{-1}+\left(\frac{1}{a}\right)^2z^{-2}+\cdots\right)$

Laurent	$a_n = a^n$	$(n<1)$
z-Trans	$a_n = \left(\frac{1}{a}\right)^n$	$(n\geq 0)$

$$+\frac{1}{1-a^{-1}z} \qquad |z|<a$$

$\left(a^0z^0+a^{-1}z^1+a^{-2}z^2+\cdots\right)$

$\left(\left(\frac{1}{a}\right)^0z^0+\left(\frac{1}{a}\right)^1z^1+\left(\frac{1}{a}\right)^2z^2+\cdots\right)$

Laurent	$a_n = \left(\frac{1}{a}\right)^n$	$(n\geq 0)$
z-Trans	$a_n = a^n$	$(n<1)$

$$-\frac{1}{1-a^{-1}z} \qquad |z|<a$$

$-\left(a^0z^0+a^{-1}z^1+a^{-2}z^2+\cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0z^0+\left(\frac{1}{a}\right)^1z^1+\left(\frac{1}{a}\right)^2z^2+\cdots\right)$

Laurent	$a_n = -\left(\frac{1}{a}\right)^n$	$(n\geq 0)$
z-Trans	$a_n = -a^n$	$(n<1)$

$$-\frac{1}{1-a^{-1}z^{-1}} \qquad |z|>a^{-1}$$

$-\left(a^0z^0+a^{-1}z^{-1}+a^{-2}z^{-2}+\cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0z^0+\left(\frac{1}{a}\right)^1z^{-1}+\left(\frac{1}{a}\right)^2z^{-2}+\cdots\right)$

Laurent	$a_n = -a^n$	$(n<1)$
z-Trans	$a_n = -\left(\frac{1}{a}\right)^n$	$(n\geq 0)$

$$+\frac{1}{1-az^{-1}} \qquad |z|>a$$

$\left(a^0z^0+a^1z^{-1}+a^2z^{-2}+\cdots\right)$

$\left(\left(\frac{1}{a}\right)^0z^0+\left(\frac{1}{a}\right)^1z^{-1}+\left(\frac{1}{a}\right)^2z^{-2}+\cdots\right)$

Laurent	$a_n = \left(\frac{1}{a}\right)^n$	$(n<1)$
z-Trans	$a_n = a^n$	$(n\geq 0)$

$$+\frac{1}{1-az} \qquad |z|<a^{-1}$$

$\left(a^0z^0+a^1z^1+a^2z^2+\cdots\right)$

$\left(\left(\frac{1}{a}\right)^0z^0+\left(\frac{1}{a}\right)^1z^1+\left(\frac{1}{a}\right)^2z^2+\cdots\right)$

Laurent	$a_n = a^n$	$(n\geq 0)$
z-Trans	$a_n = \left(\frac{1}{a}\right)^n$	$(n<1)$

Geometric Series - a non-unit start term

Laurent Series

Geometric Series - a non-unit start term

z-Transform

Geometric Series - a non-unit start term

Laurent Series vs. z-Transform

Geometric Series - a non-unit start term

Laurent Series

$$+\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \qquad |z|>a^{-1}$$

$$\left(a^{-1}z^{-1}+a^{-2}z^{-2}+a^{-3}z^{-3}+\cdots\right)$$

$$a_n = a^n \qquad (n<0)$$

$$+\frac{a^{-1}z}{1-a^{-1}z} \qquad |z|<a$$

$$\left(a^{-1}z^{1}+a^{-2}z^{2}+a^{-3}z^{3}+\cdots\right)$$

$$\left(\left(\tfrac{1}{a}\right)^{1}z^{1}+\left(\tfrac{1}{a}\right)^{2}z^{2}+\left(\tfrac{1}{a}\right)^{3}z^{3}+\cdots\right)$$

$$a_n = \left(\tfrac{1}{a}\right)^n \qquad (n\geq 1)$$

$$-\frac{az}{1-az} \qquad |z|<a^{-1}$$

$$-\left(a^{1}z^{1}+a^{2}z^{2}+a^{3}z^{3}+\cdots\right)$$

$$a_n = -a^n \qquad (n\geq 1)$$

$$-\frac{az^{-1}}{1-az^{-1}} \qquad |z|>a$$

$$-\left(a^{1}z^{-1}+a^{2}z^{-2}+a^{3}z^{-3}+\cdots\right)$$

$$-\left(\left(\tfrac{1}{a}\right)^{1}z^{-1}+\left(\tfrac{1}{a}\right)^{2}z^{-2}+\left(\tfrac{1}{a}\right)^{3}z^{-3}+\cdots\right)$$

$$a_n = -\left(\tfrac{1}{a}\right)^n \qquad (n<0)$$

$$+\frac{az^{-1}}{1-az^{-1}} \qquad |z|>a$$

$$\left(a^{1}z^{-1}+a^{2}z^{-2}+a^{3}z^{-3}+\cdots\right)$$

$$\left(\left(\tfrac{1}{a}\right)^{1}z^{-1}+\left(\tfrac{1}{a}\right)^{2}z^{-2}+\left(\tfrac{1}{a}\right)^{3}z^{-3}+\cdots\right)$$

$$a_n = \left(\tfrac{1}{a}\right)^n \qquad (n<0)$$

$$+\frac{az}{1-az} \qquad |z|<a^{-1}$$

$$\left(a^{1}z^{1}+a^{2}z^{2}+a^{3}z^{3}+\cdots\right)$$

$$a_n = a^n \qquad (n\geq 1)$$

$$-\frac{a^{-1}z}{1-a^{-1}z} \qquad |z|<a$$

$$-\left(a^{-1}z^{1}+a^{-2}z^{2}+a^{-3}z^{3}+\cdots\right)$$

$$-\left(\left(\tfrac{1}{a}\right)^{1}z^{1}+\left(\tfrac{1}{a}\right)^{2}z^{2}+\left(\tfrac{1}{a}\right)^{3}z^{3}+\cdots\right)$$

$$a_n = -\left(\tfrac{1}{a}\right)^n \qquad (n\geq 1)$$

$$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \qquad |z|>a^{-1}$$

$$-\left(a^{-1}z^{-1}+a^{-2}z^{-2}+a^{-3}z^{-3}+\cdots\right)$$

$$a_n = -a^n \qquad (n<0)$$

Geometric Series - a non-unit start term

z-Transform

$$+\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \qquad |z|>a^{-1}$$

$$(a^{-1}z^{-1}+a^{-2}z^{-2}+a^{-3}z^{-3}+\cdots)$$

$$((\tfrac{1}{a})^{1}z^{-1}+(\tfrac{1}{a})^{2}z^{-2}+(\tfrac{1}{a})^{3}z^{-3}+\cdots)$$

$a_n = a^{-n}$	$(-n<0)$
$a_n = (\frac{1}{a})^{n}$	$(n\geq 1)$

$$+\frac{a^{-1}z}{1-a^{-1}z} \qquad |z|<a$$

$$(a^{-1}z^{1}+a^{-2}z^{2}+a^{-3}z^{3}+\cdots)$$

$$((\tfrac{1}{a})^{1}z^{1}+(\tfrac{1}{a})^{2}z^{2}+(\tfrac{1}{a})^{3}z^{3}+\cdots)$$

$a_n = (\frac{1}{a})^{-n}$	$(-n\geq 1)$
$a_n = a^{n}$	$(n<0)$

$$-\frac{az}{1-az} \qquad |z|<a^{-1}$$

$$-(a^{1}z^{1}+a^{2}z^{2}+a^{3}z^{3}+\cdots)$$

$$-((\tfrac{1}{a})^{1}z^{1}+(\tfrac{1}{a})^{2}z^{2}+(\tfrac{1}{a})^{3}z^{3}+\cdots)$$

$a_n = -a^{n}$	$(-n\geq 1)$
$a_n = -(\frac{1}{a})^{n}$	$(n<0)$

$$-\frac{az^{-1}}{1-az^{-1}} \qquad |z|>a$$

$$-(a^{1}z^{-1}+a^{2}z^{-2}+a^{3}z^{-3}+\cdots)$$

$$-((\tfrac{1}{a})^{1}z^{-1}+(\tfrac{1}{a})^{2}z^{-2}+(\tfrac{1}{a})^{3}z^{-3}+\cdots)$$

$a_n = -(\frac{1}{a})^{-n}$	$(-n<0)$
$a_n = -a^{n}$	$(n\geq 1)$

$$+\frac{az^{-1}}{1-az^{-1}} \qquad |z|>a$$

$$(a^{1}z^{-1}+a^{2}z^{-2}+a^{3}z^{-3}+\cdots)$$

$$((\tfrac{1}{a})^{1}z^{-1}+(\tfrac{1}{a})^{2}z^{-2}+(\tfrac{1}{a})^{3}z^{-3}+\cdots)$$

$a_n = (\frac{1}{a})^{-n}$	$(-n<0)$
$a_n = a^{n}$	$(n\geq 1)$

$$+\frac{az}{1-az} \qquad |z|<a^{-1}$$

$$(a^{1}z^{1}+a^{2}z^{2}+a^{3}z^{3}+\cdots)$$

$$((\tfrac{1}{a})^{1}z^{1}+(\tfrac{1}{a})^{2}z^{2}+(\tfrac{1}{a})^{3}z^{3}+\cdots)$$

$a_n = a^{-n}$	$(-n\geq 1)$
$a_n = (\frac{1}{a})^{n}$	$(n<0)$

$$-\frac{a^{-1}z}{1-a^{-1}z} \qquad |z|<a$$

$$-(a^{-1}z^{1}+a^{-2}z^{2}+a^{-3}z^{3}+\cdots)$$

$$-((\tfrac{1}{a})^{1}z^{1}+(\tfrac{1}{a})^{2}z^{2}+(\tfrac{1}{a})^{3}z^{3}+\cdots)$$

$a_n = -(\frac{1}{a})^{-n}$	$(-n\geq 1)$
$a_n = -a^{n}$	$(n<0)$

$$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \qquad |z|>a^{-1}$$

$$-(a^{-1}z^{-1}+a^{-2}z^{-2}+a^{-3}z^{-3}+\cdots)$$

$$-((\tfrac{1}{a})^{1}z^{-1}+(\tfrac{1}{a})^{2}z^{-2}+(\tfrac{1}{a})^{3}z^{-3}+\cdots)$$

$a_n = -a^{-n}$	$(-n<0)$
$a_n = -(\frac{1}{a})^{n}$	$(n\geq 1)$

Geometric Series - a non-unit start term

Laurent Series vs. z-Transform

$+\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$, $|z|>a^{-1}$

$(a^{-1}z^{-1}+a^{-2}z^{-2}+a^{-3}z^{-3}+\cdots)$

$((\frac{1}{a})^{1}z^{-1}+(\frac{1}{a})^{2}z^{-2}+(\frac{1}{a})^{3}z^{-3}+\cdots)$

Laurent	$a_n = a^n$	$(n<0)$
z-Trans	$a_n = (\frac{1}{a})^n$	$(n\geq 1)$

$+\dfrac{a^{-1}z}{1-a^{-1}z}$, $|z|<a$

$(a^{-1}z^{1}+a^{-2}z^{2}+a^{-3}z^{3}+\cdots)$

$((\frac{1}{a})^{1}z^{1}+(\frac{1}{a})^{2}z^{2}+(\frac{1}{a})^{3}z^{3}+\cdots)$

Laurent	$a_n = (\frac{1}{a})^n$	$(n\geq 1)$
z-Trans	$a_n = a^n$	$(n<0)$

$-\dfrac{az}{1-az}$, $|z|<a^{-1}$

$-(a^{1}z^{1}+a^{2}z^{2}+a^{3}z^{3}+\cdots)$

$-((\frac{1}{a})^{-1}z^{1}+(\frac{1}{a})^{-2}z^{2}+(\frac{1}{a})^{-3}z^{3}+\cdots)$

Laurent	$a_n = -a^n$	$(n\geq 1)$
z-Trans	$a_n = -(\frac{1}{a})^n$	$(n<0)$

$-\dfrac{az^{-1}}{1-az^{-1}}$, $|z|>a$

$-(a^{1}z^{-1}+a^{2}z^{-2}+a^{3}z^{-3}+\cdots)$

$-((\frac{1}{a})^{-1}z^{-1}+(\frac{1}{a})^{-2}z^{-2}+(\frac{1}{a})^{-3}z^{-3}+\cdots)$

Laurent	$a_n = -(\frac{1}{a})^n$	$(n<0)$
z-Trans	$a_n = -a^n$	$(n\geq 1)$

$+\dfrac{az^{-1}}{1-az^{-1}}$, $|z|>a$

$(a^{1}z^{-1}+a^{2}z^{-2}+a^{3}z^{-3}+\cdots)$

$((\frac{1}{a})^{-1}z^{-1}+(\frac{1}{a})^{-2}z^{-2}+(\frac{1}{a})^{-3}z^{-3}+\cdots)$

Laurent	$a_n = (\frac{1}{a})^n$	$(n<0)$
z-Trans	$a_n = a^n$	$(n\geq 1)$

$+\dfrac{az}{1-az}$, $|z|<a^{-1}$

$(a^{1}z^{1}+a^{2}z^{2}+a^{3}z^{3}+\cdots)$

$((\frac{1}{a})^{-1}z^{1}+(\frac{1}{a})^{-2}z^{2}+(\frac{1}{a})^{-3}z^{3}+\cdots)$

Laurent	$a_n = a^n$	$(n\geq 1)$
z-Trans	$a_n = (\frac{1}{a})^n$	$(n<0)$

$-\dfrac{a^{-1}z}{1-a^{-1}z}$, $|z|<a$

$-(a^{-1}z^{1}+a^{-2}z^{2}+a^{-3}z^{3}+\cdots)$

$-((\frac{1}{a})^{1}z^{1}+(\frac{1}{a})^{2}z^{2}+(\frac{1}{a})^{3}z^{3}+\cdots)$

Laurent	$a_n = -(\frac{1}{a})^n$	$(n\geq 1)$
z-Trans	$a_n = -a^n$	$(n<0)$

$-\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$, $|z|>a^{-1}$

$-(a^{-1}z^{-1}+a^{-2}z^{-2}+a^{-3}z^{-3}+\cdots)$

$-((\frac{1}{a})^{1}z^{-1}+(\frac{1}{a})^{2}z^{-2}+(\frac{1}{a})^{3}z^{-3}+\cdots)$

Laurent	$a_n = -a^n$	$(n<0)$
z-Trans	$a_n = -(\frac{1}{a})^n$	$(n\geq 1)$

Complemnt ROC Pairs -
Original Geometric Series Form Combinations

unit	$-\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$	$-a^n \quad (n \geq 0)$	$-\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$	$-\left(\frac{1}{a}\right)^n \quad (n < 1)$
non-unit	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$a^n \quad (n < 0)$	$\frac{a^{-1}z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\left(\frac{1}{a}\right)^n \quad (n \geq 1)$

unit	$\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$a^n \quad (n < 1)$	$\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\left(\frac{1}{a}\right)^n \quad (n \geq 0)$
non-unit	$-\frac{az}{1-az}$	$\lvert z\rvert < a^{-1}$	$-a^n \quad (n \geq 1)$	$\frac{az^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$-\left(\frac{1}{a}\right)^n \quad (n < 0)$

unit	$-\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\left(\frac{1}{a}\right)^n \quad (n \geq 0)$	$-\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$-a^n \quad (n < 1)$
non-unit	$\frac{az^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\left(\frac{1}{a}\right)^n \quad (n < 0)$	$\frac{az}{1-az}$	$\lvert z\rvert < a^{-1}$	$a^n \quad (n \geq 1)$

unit	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$	$\left(\frac{1}{a}\right)^n \quad (n < 1)$	$\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$	$a^n \quad (n \geq 0)$
non-unit	$-\frac{a^{-1}z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\left(\frac{1}{a}\right)^n \quad (n \geq 1)$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$-a^n \quad (n < 0)$

start term

Complemnt ROC Pairs - Shifted Geometric Series Form Combinations

$-\frac{a}{1-az}$	$\lvert z\rvert < a^{-1}$	$-a^{n+1}$ $(n \geq 0)$	$-\frac{a}{1-az^{-1}}$	$\lvert z\rvert > a$	$-\left(\frac{1}{a}\right)^{n-1}$ $(n \leq 1)$
$\frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	a^{n+1} $(n < 0)$	$\frac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\left(\frac{1}{a}\right)^{n-1}$ $(n \geq 1)$

$\frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	a^{n+1} $(n < 0)$	$\frac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\left(\frac{1}{a}\right)^{n-1}$ $(n \geq 1)$
$-\frac{a}{1-az}$	$\lvert z\rvert < a^{-1}$	$-a^{n+1}$ $(n \geq 0)$	$\frac{a}{1-az^{-1}}$	$\lvert z\rvert > a$	$-\left(\frac{1}{a}\right)^{n-1}$ $(n \leq 1)$

$-\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\left(\frac{1}{a}\right)^{n+1}$ $(n \geq 0)$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$-a^{n-1}$ $(n \leq 1)$
$\frac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\left(\frac{1}{a}\right)^{n+1}$ $(n < 0)$	$\frac{z}{1-az}$	$\lvert z\rvert < a^{-1}$	a^{n-1} $(n \geq 1)$

$\frac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\left(\frac{1}{a}\right)^{n+1}$ $(n < 0)$	$\frac{z}{1-az}$	$\lvert z\rvert < a^{-1}$	a^{n-1} $(n \geq 1)$
$-\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\left(\frac{1}{a}\right)^{n+1}$ $(n \geq 0)$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$-a^{n-1}$ $(n \leq 1)$

Complemnt ROC Pairs - Reduced Shifted Geometric Series Form Combinations

$-\frac{a}{1-az}$	$\lvert z\rvert < a^{-1}$	$-a^{n+1}$ $(n \geq 0)$	$-\frac{a}{1-az^{-1}}$	$\lvert z\rvert > a$	$-\left(\frac{1}{a}\right)^{n-1}$ $(n \leq 1)$
$\frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	a^{n+1} $(n < 0)$	$\frac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\left(\frac{1}{a}\right)^{n-1}$ $(n \geq 1)$

$-\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\left(\frac{1}{a}\right)^{n+1}$ $(n \geq 0)$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$-a^{n-1}$ $(n \leq 1)$
$\frac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\left(\frac{1}{a}\right)^{n+1}$ $(n < 0)$	$\frac{z}{1-az}$	$\lvert z\rvert < a^{-1}$	a^{n-1} $(n \geq 1)$

$2z$	$2z^{-1}$	$2^{-1}z^{-1}$	$2^{-1}z$
$\lvert z\rvert < 0.5$	$\lvert z\rvert > 2$	$\lvert z\rvert > 0.5$	$\lvert z\rvert < 2$

$$-\frac{2}{1-2z} \xleftrightarrow{z^{-1}} -\frac{2}{1-2z^{-1}}$$

$$\cdot\frac{(2z)^{-1}}{(2z)^{-1}} \cdot\frac{(2z)}{(2z)} \qquad \cdot\frac{(2z^{-1})^{-1}}{(2z^{-1})^{-1}} \cdot\frac{(2z^{-1})}{(2z^{-1})}$$

$$+\frac{z^{-1}}{1-0.5z^{-1}} \xleftrightarrow{z^{-1}} +\frac{z}{1-0.5z}$$

	scale(a)		scale(a)	
	$-\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	$-\frac{a}{1-az}$ $\lvert z\rvert<a^{-1}$	$-\frac{1}{1-az^{-1}}$ $\lvert z\rvert>a$	$-\frac{a}{1-az^{-1}}$ $\lvert z\rvert>a$
Comp.ROC	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert<a$

	scale(1/z)		scale(z)	
	$\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{1}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert<a$
Comp.ROC	$-\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	$-\frac{a}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{a}{1-az^{-1}}$ $\lvert z\rvert>a$

	scale(1/a)		scale(1/a)	
	$-\frac{1}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$
Comp.ROC	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert<a^{-1}$

	scale(1/z)		scale(z)	
	$\frac{1}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert<a^{-1}$
Comp.ROC	$-\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$

	scale(a)		scale(a)	
	$-\frac{1}{1-az}$ $\lvert z\rvert < a^{-1}$	$-\frac{a}{1-az}$ $\lvert z\rvert < a^{-1}$	$-\frac{1}{1-az^{-1}}$ $\lvert z\rvert > a$	$-\frac{a}{1-az^{-1}}$ $\lvert z\rvert > a$
Comp.ROC	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert < a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert < a$

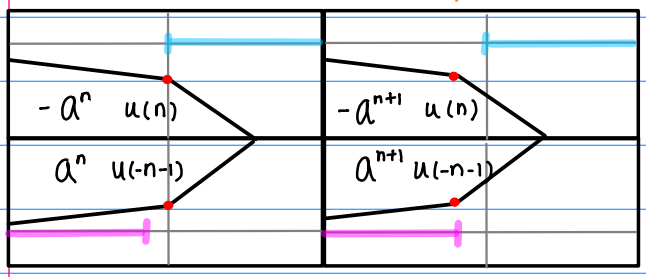
	scale(1/z)		scale(z)	
	$\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$\frac{1}{1-a^{-1}z}$ $\lvert z\rvert < a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert < a$
Comp.ROC	$-\frac{az}{1-az}$ $\lvert z\rvert < a^{-1}$	$-\frac{a}{1-az}$ $\lvert z\rvert < a^{-1}$	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$\frac{a}{1-az^{-1}}$ $\lvert z\rvert > a$

	scale(1/a)		scale(1/a)	
	$-\frac{1}{1-a^{-1}z}$ $\lvert z\rvert < a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert < a$	$-\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$
Comp.ROC	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$\frac{az}{1-az}$ $\lvert z\rvert < a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert < a^{-1}$

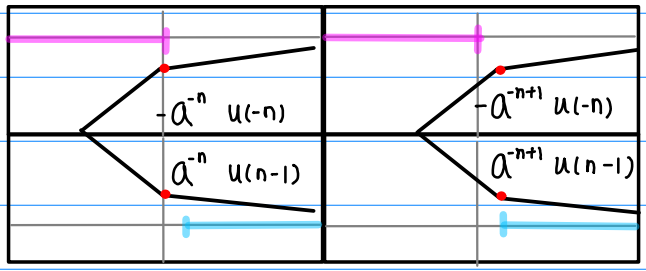
	scale(1/z)		scale(z)	
	$\frac{1}{1-az^{-1}}$ $\lvert z\rvert > a$	$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$\frac{1}{1-az}$ $\lvert z\rvert < a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert < a^{-1}$
Comp.ROC	$-\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert < a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert < a$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$

SHL.Seq

-Comp.Rng

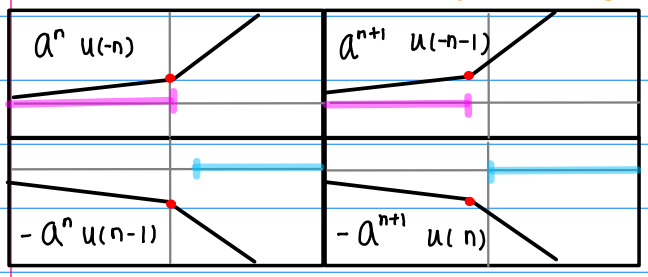


SHR.Seq

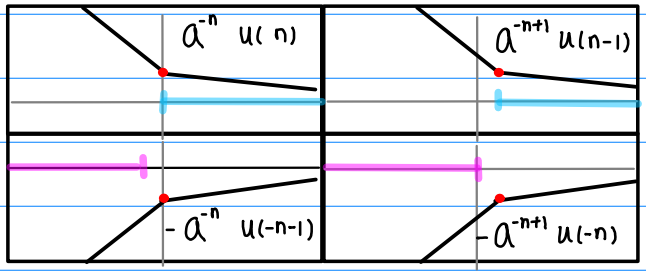


SHL.Seq, SHL.Rng

-Comp.Rng

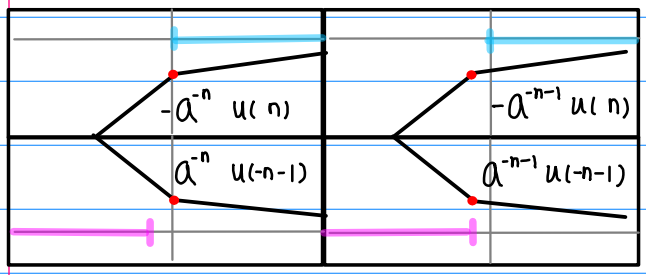


SHR.Seq, SHR.Rng

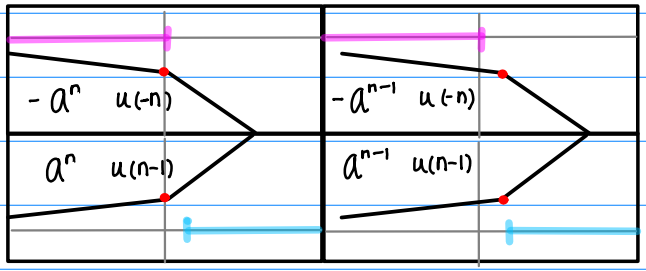


SHL.Seq

-Comp.Rng

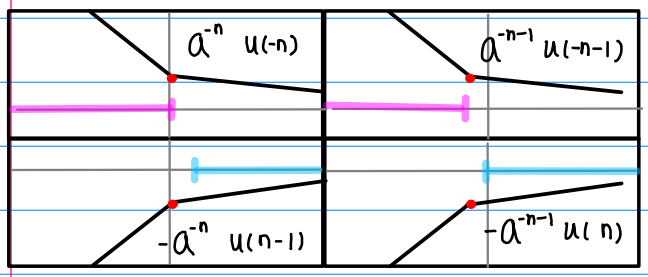


SHR.Seq

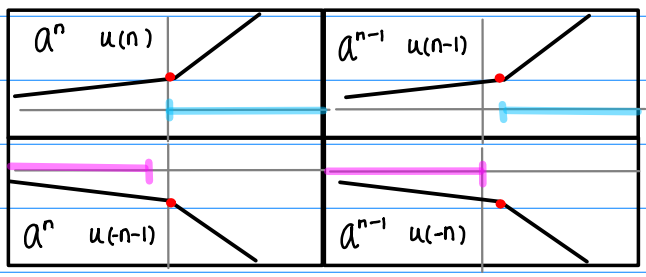


SHL.Seq, SHL.Rng

-Comp.Rng



SHR.Seq, SHR.Rng



Left Shifted Sequence

Right Shifted Sequence

	scale(a)		scale(a)	
	$-\frac{1}{1-az}$ $\lvert z\rvert < a^{-1}$	$-\frac{a}{1-az}$ $\lvert z\rvert < a^{-1}$	$-\frac{1}{1-az^{-1}}$ $\lvert z\rvert > a$	$-\frac{a}{1-az^{-1}}$ $\lvert z\rvert > a$
Comp.ROC	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert < a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert < a$

	scale(1/z)		scale(z)	
	$\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$\frac{1}{1-a^{-1}z}$ $\lvert z\rvert < a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert < a$
Comp.ROC	$-\frac{az}{1-az}$ $\lvert z\rvert < a^{-1}$	$-\frac{a}{1-az}$ $\lvert z\rvert < a^{-1}$	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$\frac{a}{1-az^{-1}}$ $\lvert z\rvert > a$

	scale(1/a)		scale(1/a)	
	$-\frac{1}{1-a^{-1}z}$ $\lvert z\rvert < a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert < a$	$-\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$
Comp.ROC	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$\frac{az}{1-az}$ $\lvert z\rvert < a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert < a^{-1}$

	scale(1/z)		scale(z)	
	$\frac{1}{1-az^{-1}}$ $\lvert z\rvert > a$	$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$\frac{1}{1-az}$ $\lvert z\rvert < a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert < a^{-1}$
Comp.ROC	$-\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert < a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert < a$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$

SHL.Seq

$-a^n \ (n \geq 0)$	$-a^{n+1} \ (n \geq 0)$
$-(a^0, a^1, a^2, \cdots)$	$-(a^1, a^2, a^3, \cdots)$
$a^n \ (n < 0)$	$a^{n+1} \ (n < 0)$
$(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$	$(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$

-Comp.Rng

SHR.Seq

$-(\frac{1}{a})^n \ (n < 1)$	$-(\frac{1}{a})^{n-1} \ (n < 1)$
$-(\cdots, a^2, a^1, a^0)$	$-(\cdots, a^3, a^2, a^1)$
$(\frac{1}{a})^n \ (n \geq 1)$	$(\frac{1}{a})^{n-1} \ (n \geq 1)$
$(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \cdots)$	$(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \cdots)$

SHL.Seq, SHL.Rng

$a^n \ (n < 1)$	$a^{n+1} \ (n < 0)$
$(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$	$(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$
$-a^n \ (n \geq 1)$	$-a^{n+1} \ (n \geq 0)$
$-(a^1, a^2, a^3, \cdots)$	$-(a^1, a^2, a^3, \cdots)$

-Comp.Rng

SHR.Seq, SHR.Rng

$(\frac{1}{a})^n \ (n \geq 0)$	$(\frac{1}{a})^{n-1} \ (n \geq 1)$
$(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \cdots)$	$(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \cdots)$
$-(\frac{1}{a})^n \ (n < 0)$	$-(\frac{1}{a})^{n-1} \ (n < 1)$
$(\cdots, a^3, a^2, a^1)$	$(\cdots, a^3, a^2, a^1)$

SHL.Seq

$-(\frac{1}{a})^n \ (n \geq 0)$	$-(\frac{1}{a})^{n+1} \ (n \geq 0)$
$-(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2}, \cdots)$	$-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \cdots)$
$(\frac{1}{a})^n \ (n < 0)$	$(\frac{1}{a})^{n+1} \ (n < 0)$
$(\cdots, a^3, a^2, a^1)$	$(\cdots, a^2, a^1, a^0)$

-Comp.Rng

SHR.Seq

$-a^n \ (n < 1)$	$-a^{n-1} \ (n < 1)$
$-(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$	$-(\cdots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^1})$
$a^n \ (n \geq 1)$	$a^{n-1} \ (n \geq 1)$
$(a^1, a^2, a^3, \cdots)$	$(a^0, a^1, a^2, \cdots)$

SHL.Seq, SHL.Rng

$(\frac{1}{a})^n \ (n < 1)$	$(\frac{1}{a})^{n+1} \ (n < 0)$
$(\cdots, a^2, a^1, a^0)$	$(\cdots, a^2, a^1, a^0)$
$-(\frac{1}{a})^n \ (n \geq 1)$	$-(\frac{1}{a})^{n+1} \ (n \geq 0)$
$-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \cdots)$	$-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3}, \cdots)$

-Comp.Rng

SHR.Seq, SHR.Rng

$a^n \ (n \geq 0)$	$a^{n-1} \ (n \geq 1)$
$(a^0, a^1, a^2, \cdots)$	$(a^0, a^1, a^2, \cdots)$
$-a^n \ (n < 0)$	$-a^{n-1} \ (n < 1)$
$-(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$	$-(\cdots, \frac{1}{a^3}, \frac{1}{a^2}, \frac{1}{a^1})$

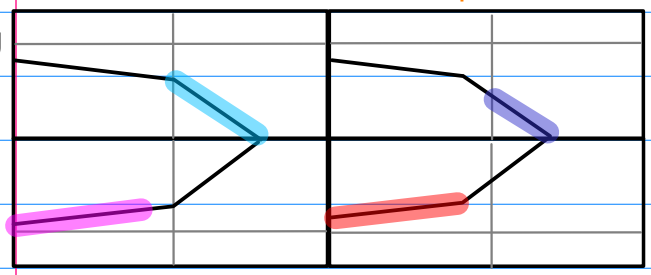
Left Shifted Sequence

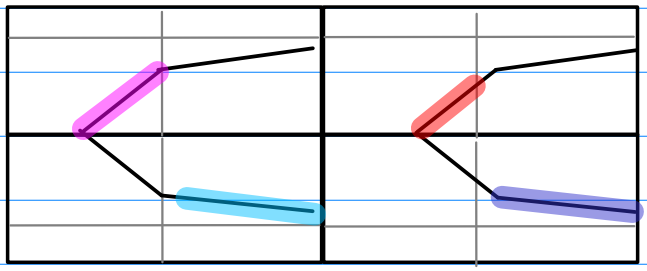
Right Shifted Sequence

SHL.Seq

SHR.Seq

-Comp.Rng

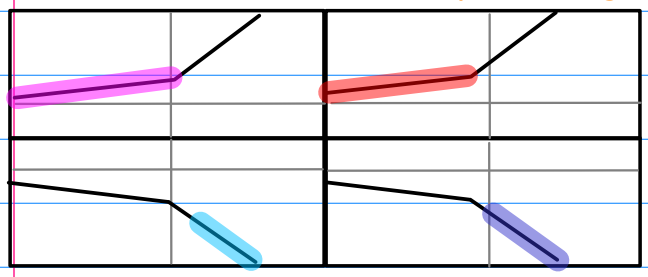


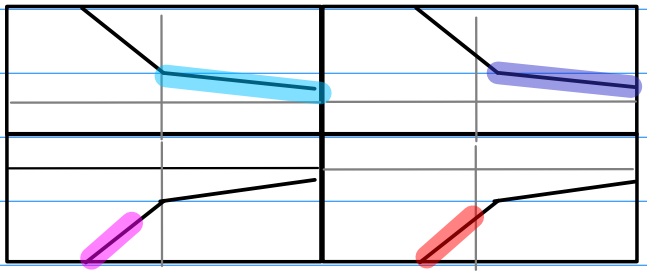


SHL.Seq, SHL.Rng

SHR.Seq, SHR.Rng

-Comp.Rng

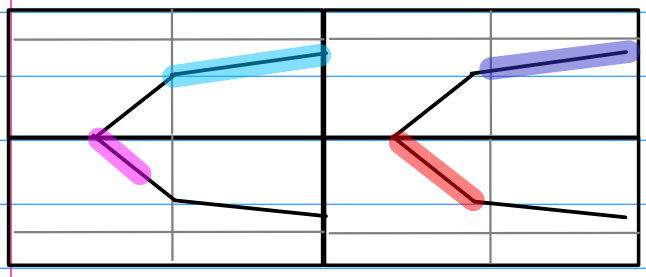


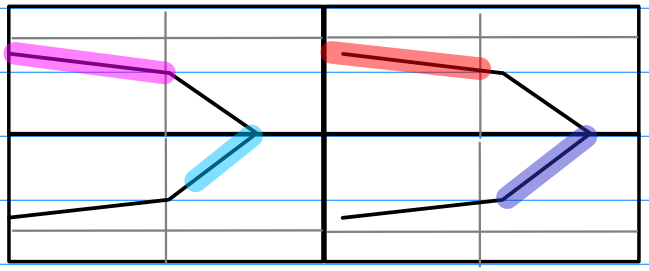


SHL.Seq

SHR.Seq

-Comp.Rng

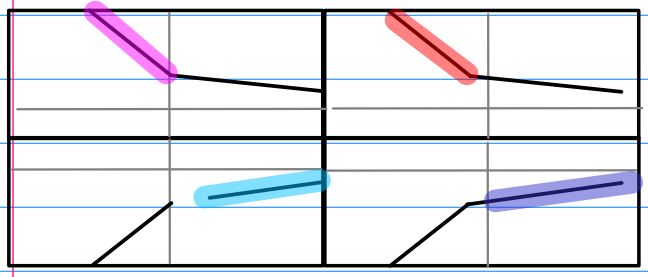


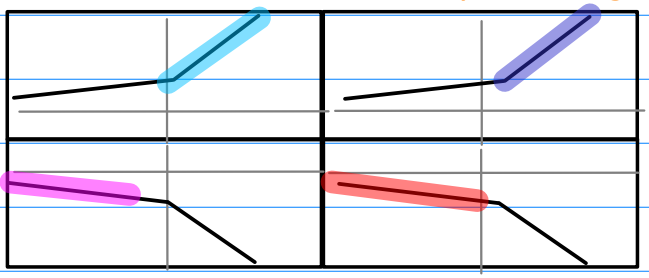


SHL.Seq, SHL.Rng

SHR.Seq, SHR.Rng

-Comp.Rng



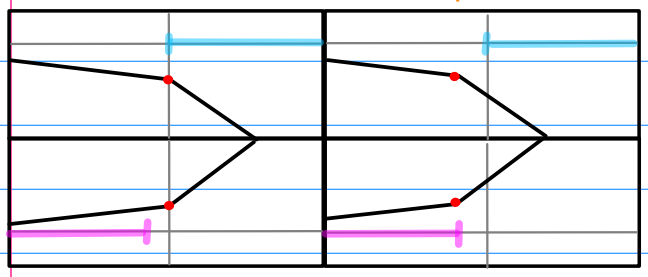


Left Shifted Sequence

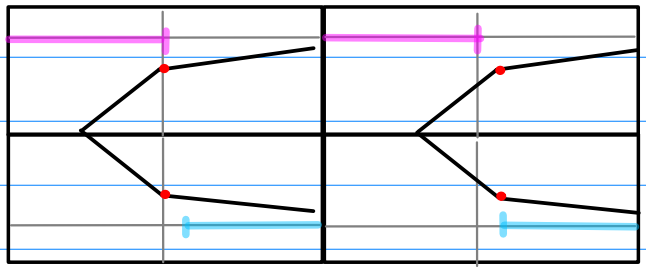
Right Shifted Sequence

SHL.Seq

-Comp.Rng

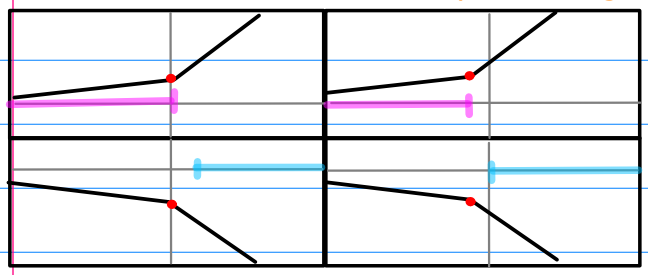


SHR.Seq

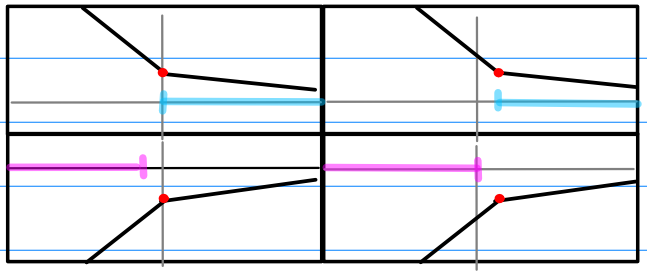


SHL.Seq, SHL.Rng

-Comp.Rng

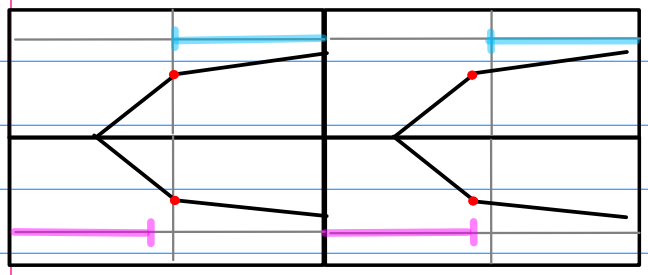


SHR.Seq, SHR.Rng

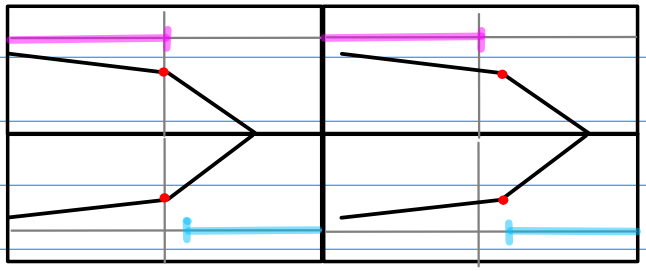


SHL.Seq

-Comp.Rng

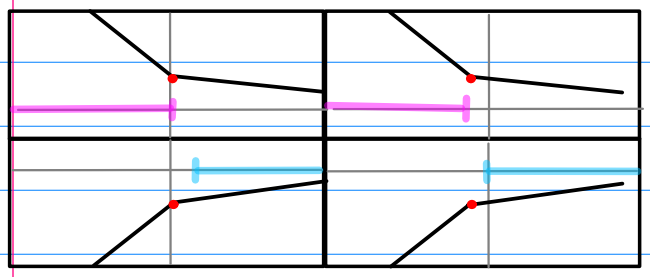


SHR.Seq

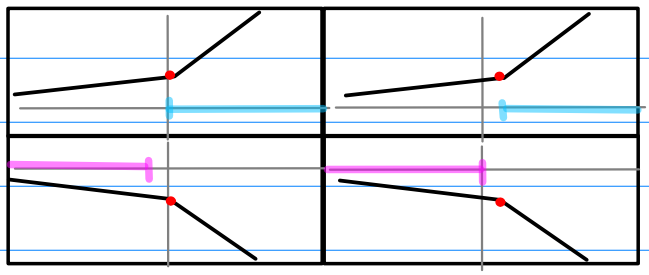


SHL.Seq, SHL.Rng

-Comp.Rng



SHR.Seq, SHR.Rng

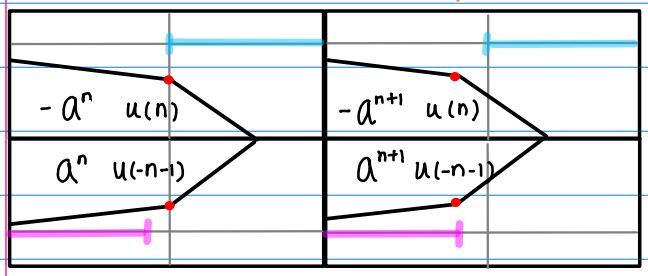


Left Shifted Sequence

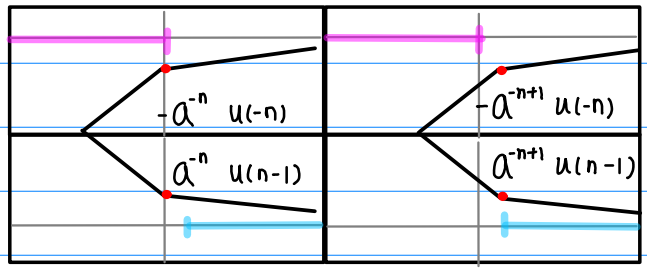
Right Shifted Sequence

SHL.Seq

-Comp.Rng

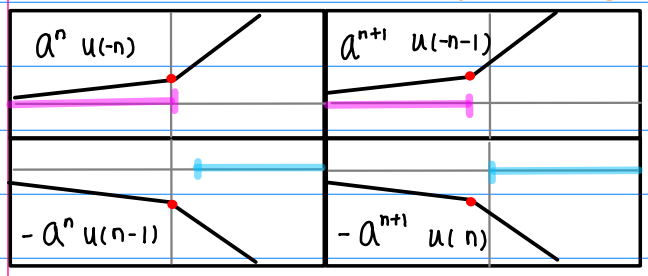


SHR.Seq

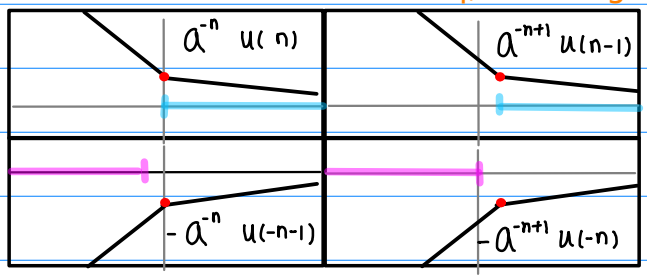


SHL.Seq, SHL.Rng

-Comp.Rng

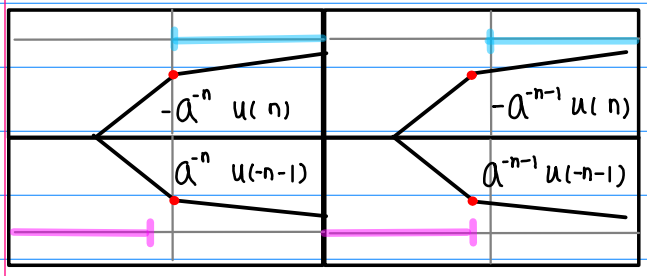


SHR.Seq, SHR.Rng

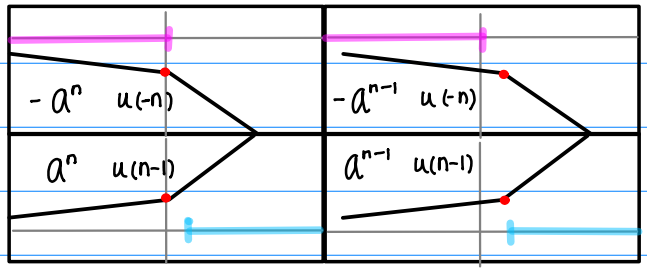


SHL.Seq

-Comp.Rng

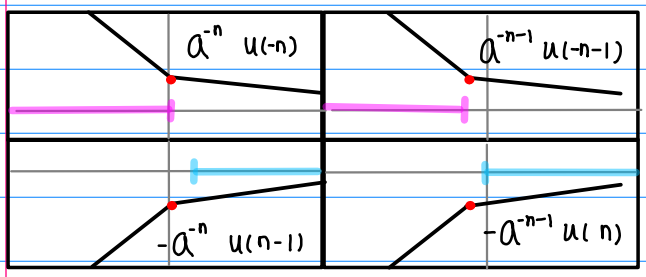


SHR.Seq

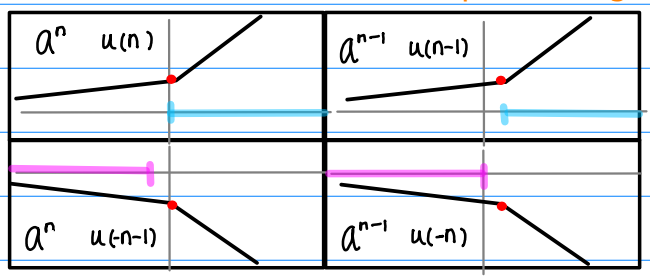


SHL.Seq, SHL.Rng

-Comp.Rng



SHR.Seq, SHR.Rng



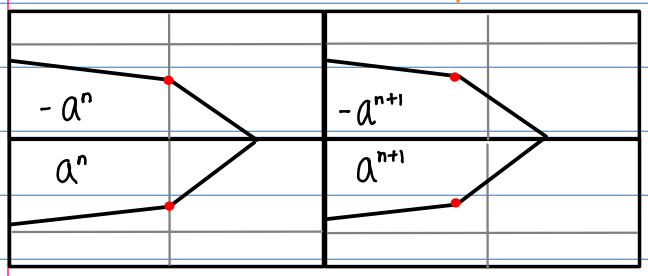
Left Shifted Sequence

Right Shifted Sequence

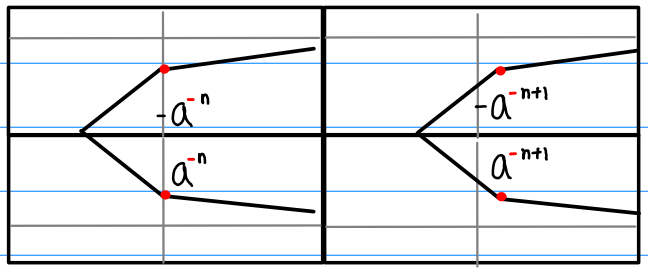
a Sequence Function

SHL.Seq

-Comp.Rng

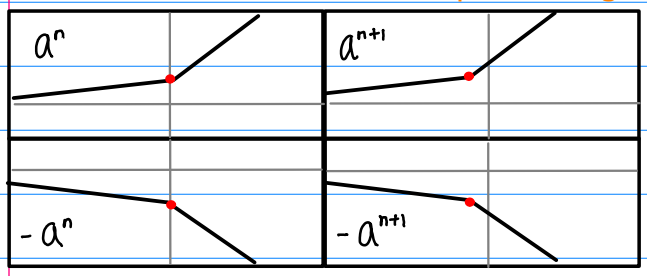


SHR.Seq

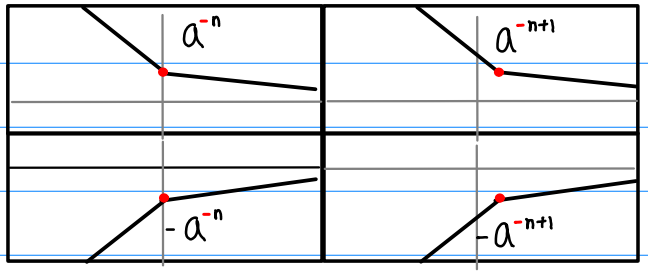


SHL.Seq, SHL.Rng

-Comp.Rng

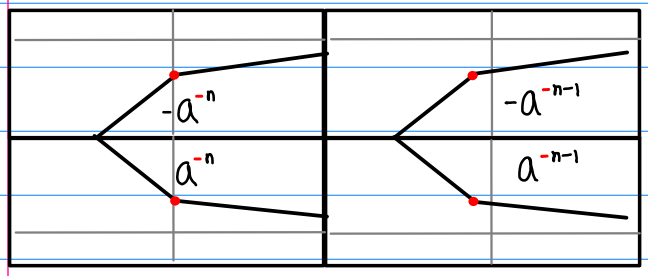


SHR.Seq, SHR.Rng

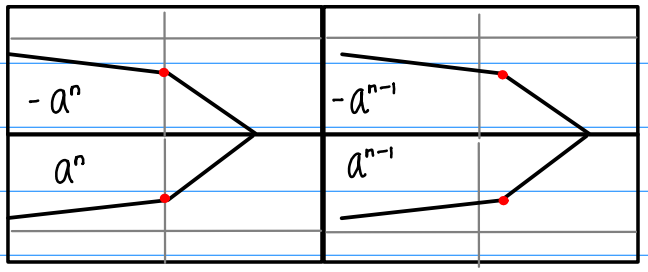


SHL.Seq

-Comp.Rng

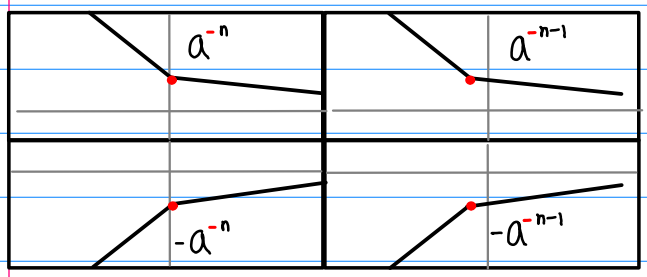


SHR.Seq

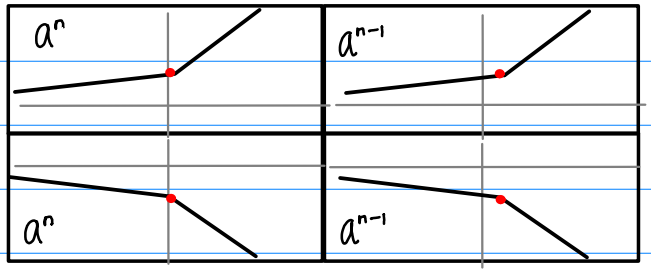


SHL.Seq, SHL.Rng

-Comp.Rng



SHR.Seq, SHR.Rng



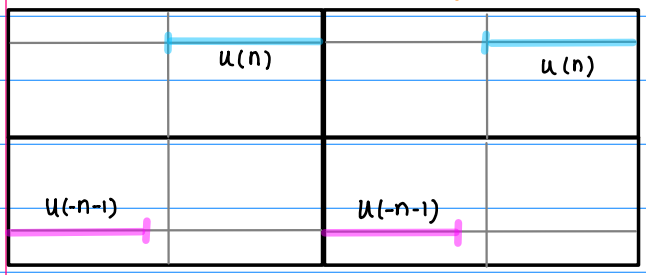
Left Shifted Sequence

Right Shifted Sequence

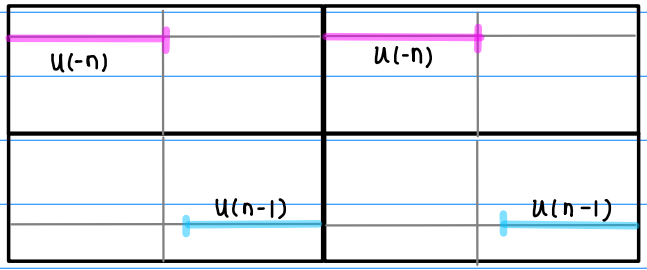
Range of a Sequence

SHL.Seq

-Comp.Rng

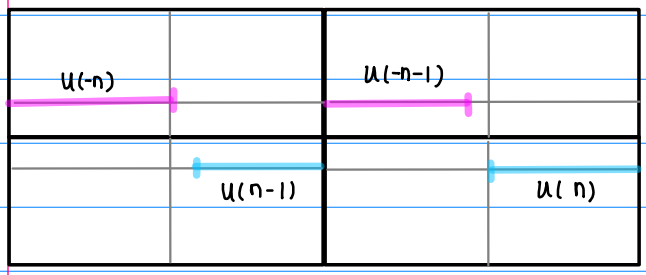


SHR.Seq

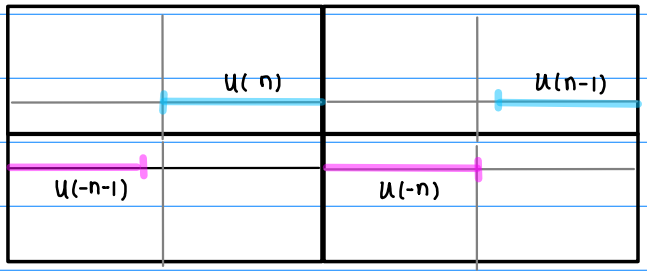


SHL.Seq, SHL.Rng

-Comp.Rng

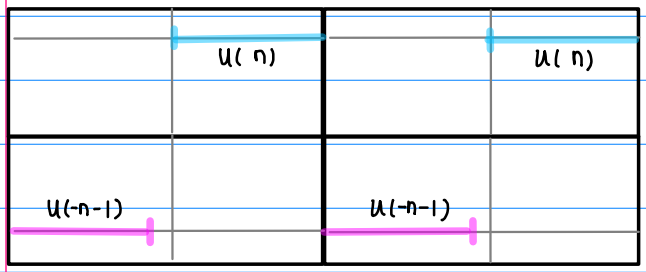


SHR.Seq, SHR.Rng

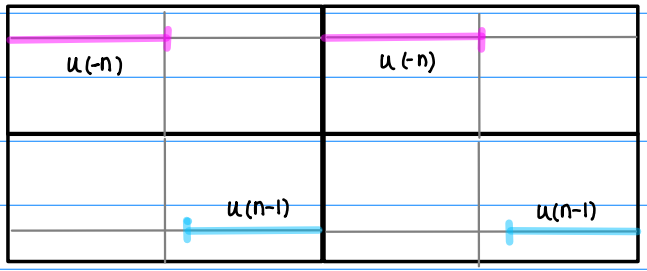


SHL.Seq

-Comp.Rng

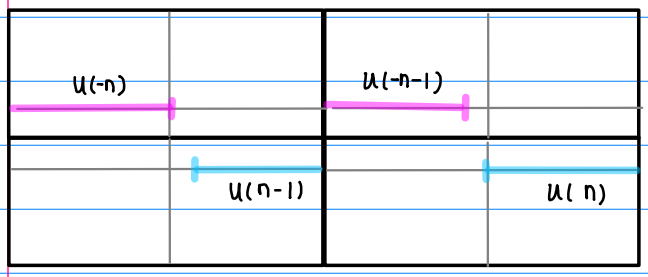


SHR.Seq



SHL.Seq, SHL.Rng

-Comp.Rng



SHR.Seq, SHR.Rng