Applications of Pointers (1A)

Please send corrections (or suggestions) to youngwlim@hotmail.com.
This document was produced by using LibreOffice.

Address-of operator and dereferencing operator

the address of a variable :
address-of operator ***&***

& variable *:*
returns the address of a variable

variable *has memory locations*
whose value can be changed
by an assignment

(***variable*** *must be an lvalue*)

the content at an address :
dereferencing operator *******

**** address*** *:*
returns the value at the address

**** address*** *has memory locations*
whose value can be changed
by an assignment

(**** address*** *is an lvalue*)

lvalue and rvalue in assignments

Left Hand Side		Right Hand Side
LHS	=	***RHS***

```
int    a,  b = 10 ;
int  * p, q = &a ;
```

LHS		RHS		Example LHS		Example RHS
lvalue	=	*lvalue*		p	=	q ;
lvalue	=	*rvalue*		p	=	&a ;
~~*rvalue*~~	=	*lvalue*		~~&a~~	=	p ;
~~*rvalue*~~	=	*rvalue*		~~&a~~	=	&b ;

*in the **LHS,** only lvalue can exist*
*rvalue can exist only in the **RHS***

a, b, p, q	: lvalues	… variables	… RW
*p, *q	: lvalues	… variables	… RW
&a, &b	: rvalues	… constants	… RO

lvalue and rvalue with * and & operators

lvalue	⇐	* lvalue	lvalue	⇐	* p
~~rvalue~~	⇐	* lvalue	~~rvalue~~	⇐	* p
lvalue	⇐	* rvalue	lvalue	⇐	* (&a)
~~rvalue~~	⇐	* rvalue	~~rvalue~~	⇐	* (&a)

~~lvalue~~	⇐	& lvalue	~~lvalue~~	⇐	& p
rvalue	⇐	& lvalue	rvalue	⇐	& p
~~lvalue~~	⇐	& ~~rvalue~~	lvalue	⇐	~~& (&a)~~
~~rvalue~~	⇐	& ~~rvalue~~	rvalue	⇐	~~& (&a)~~

```
int   a = 10 ;
int * p = &a ;
```

* can be applied
to either an **lvalue** variable
or a **rvalue** address

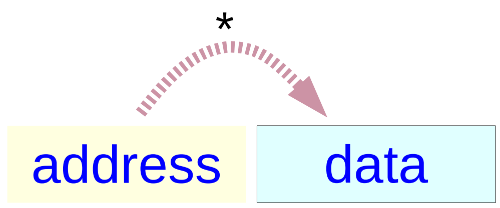
* **operand** becomes
an **lvalue** variable
thus can be applied
successively.

& can be applied
to only an **lvalue** variable and
returns only an **rvalue** address

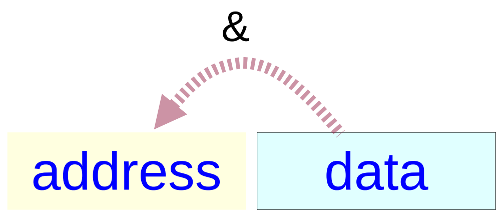
a, **p**	: **l**values	… variables	… RW
*__p__	: **l**values	… variables	… RW
&__a__	: **r**values	… constants	… RO

Address-of and dereference operators

Primitive Data Type

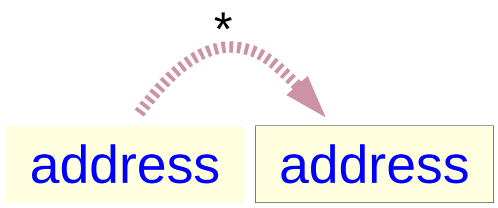


pointer	*lvalue*	**p**	*lvalue*	***p**
constant	*rvalue*	**&a**	*lvalue*	**a**

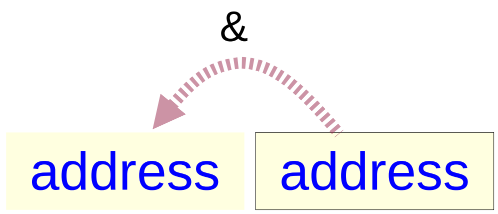


constant	*rvalue*	**&a**	*lvalue*	**a**

Pointer Data Type

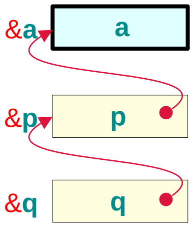


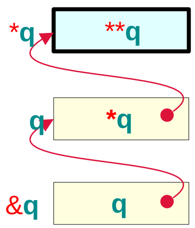
pointer	*lvalue*	**q**	*lvalue*	***q**
constant	*rvalue*	**&q**	*lvalue*	**q**



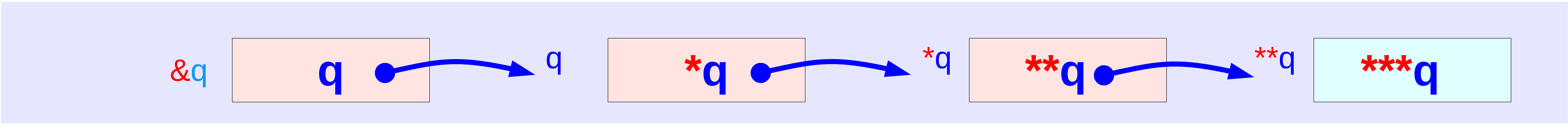
constant	*rvalue*	**&q**	*lvalue*	**q**

```
int    a ;
int *  p ;
int ** q ;
```

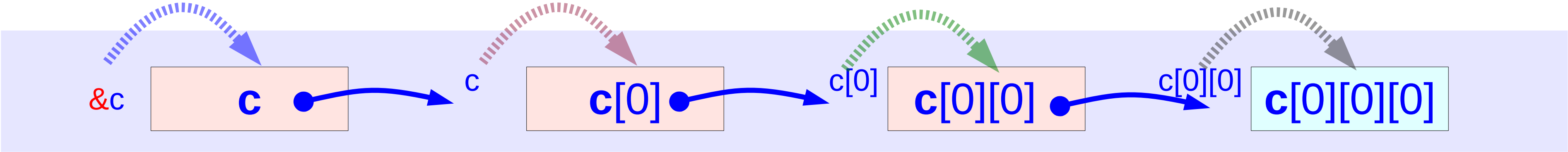




Pointer Chain Types

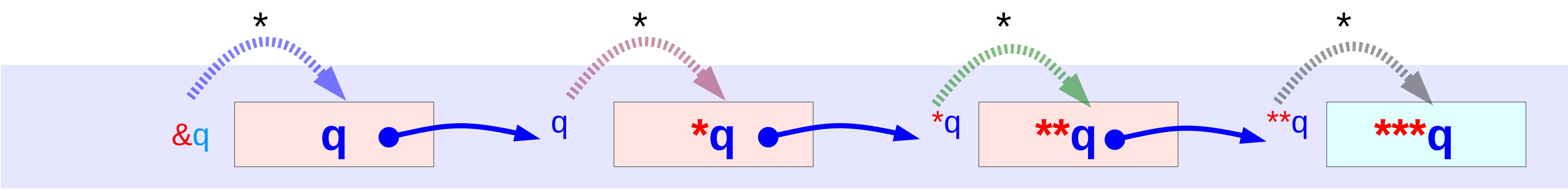


- **dynamically allocated multi-dimensional arrays**



- **statically allocated multi-dimensional arrays**

Pointer Chain Type 1



*(&q) = q

C expression returns data value(q) which is an address

*(q) = *q

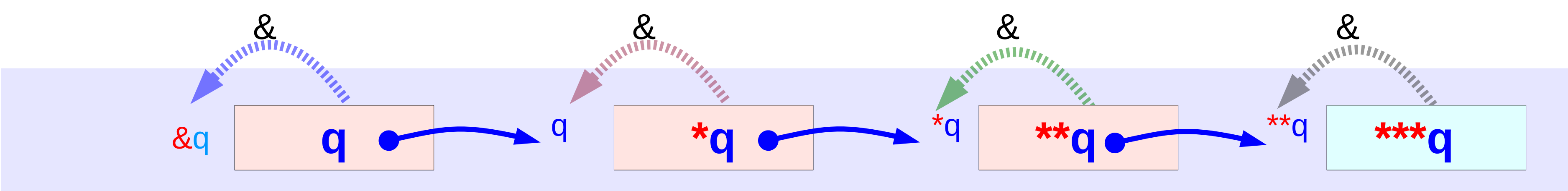
C expression returns data value(*q) which is an address

*(*q) = **q

C expression returns data value(**q) which is an integer

*(**q) = ***q

C expression returns data value(**q) which is an integer



&q

C expression returns address value(&q) which is the address of a variable q

&(*q) = q

C expression returns address value(q) which is an address of a variable *q

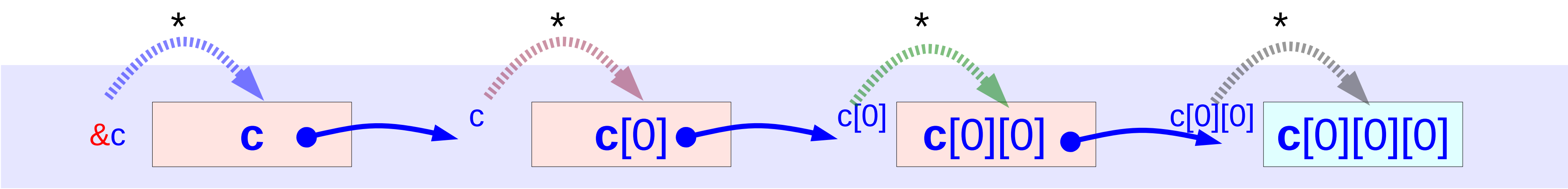
&(**q) = *q

C expression returns address value(*q) which is an address of a variable **q

&(***q) = **q

C expression returns address value(**q) which is an address of a variable **q

Pointer Chain Type 2 (1)



*(&c) = c

*(c) = c[0]

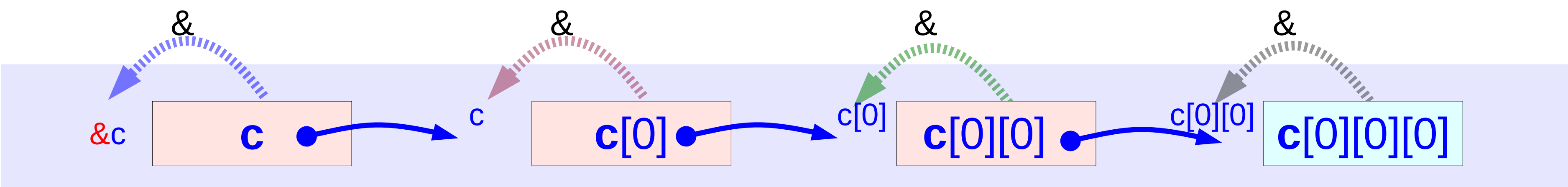
*(c[0]) = c[0][0]

*(c[0][0]) = c[0][0][0]

(int (*)[3][4]) c can be viewed as a pointer to (int [3][4]) c[0]

(int (*)[4]) c[0] can be viewed as a pointer to (int [4]) c[0][0]

(int (*)) c[0][0] can be viewed as a pointer to (int) c[0][0][0]



&c

&(c[0]) = c

&(c[0][0]) = c[0]

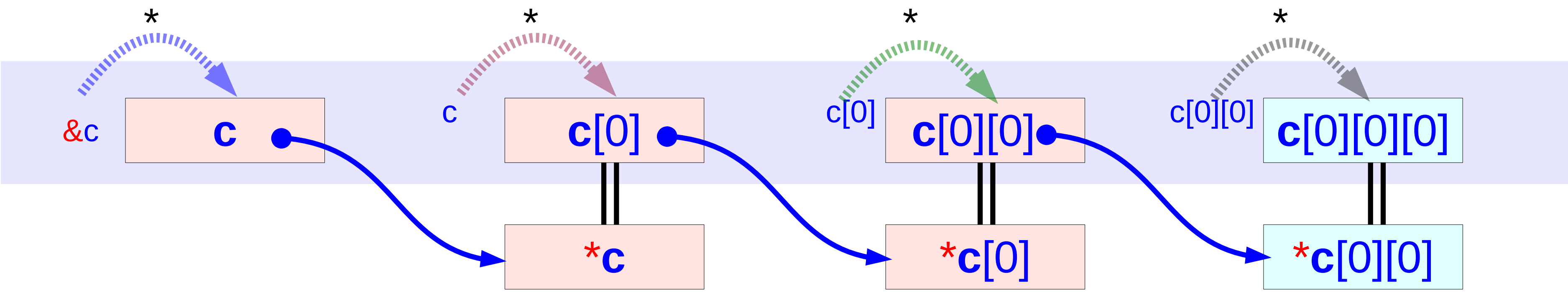
&(c[0][0][0]) = c[0][0]

(int (*)[3][4]) c has the address value of (int [3][4]) c[0]

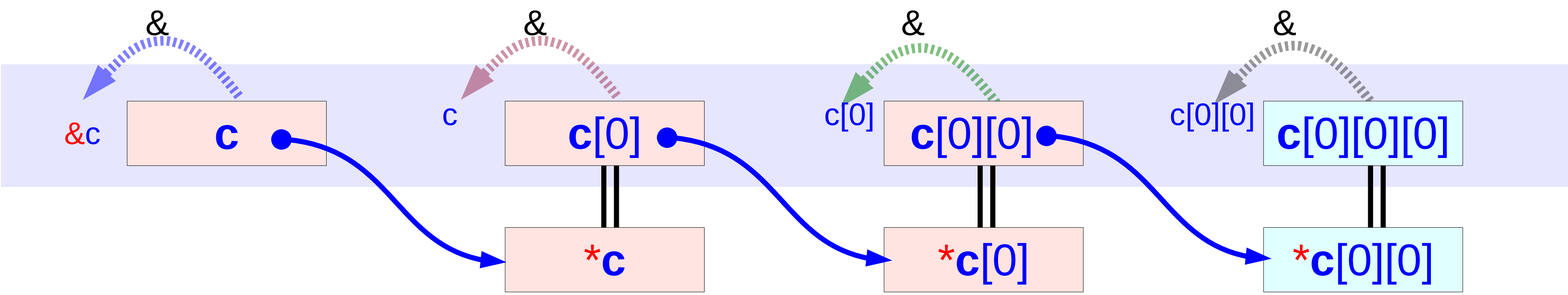
(int (*)[4]) c[0] has the address value of (int [4] c[0][0]

(int (*)) c[0][0] has the address value of (int) c[0][0][0]

Pointer Chain Type 2 (2)



*(&c) = c   *(c) = c[0]   *(c[0]) = c[0][0]   *(c[0][0]) = c[0][0][0]



&c   &(c[0]) = c   &(c[0][0]) = c[0]   &(c[0][0][0]) = c[0][0]

2-d array access

Array of Pointers

```
int      a [4] ;
int *    b [3] ;
```

int | a | [4]

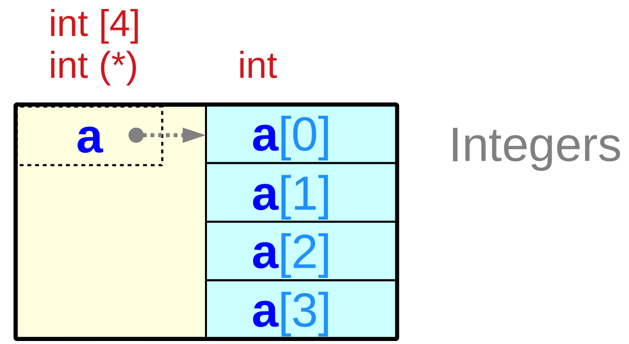
the type of each element: an integer

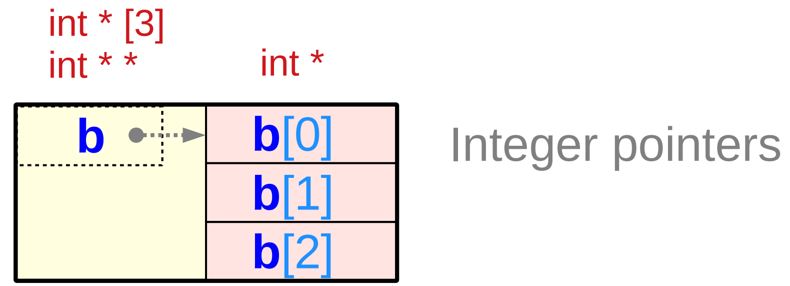
there are 4 elements

int * | b | [3]

the type of each element: a pointer an integer

there are 3 elements

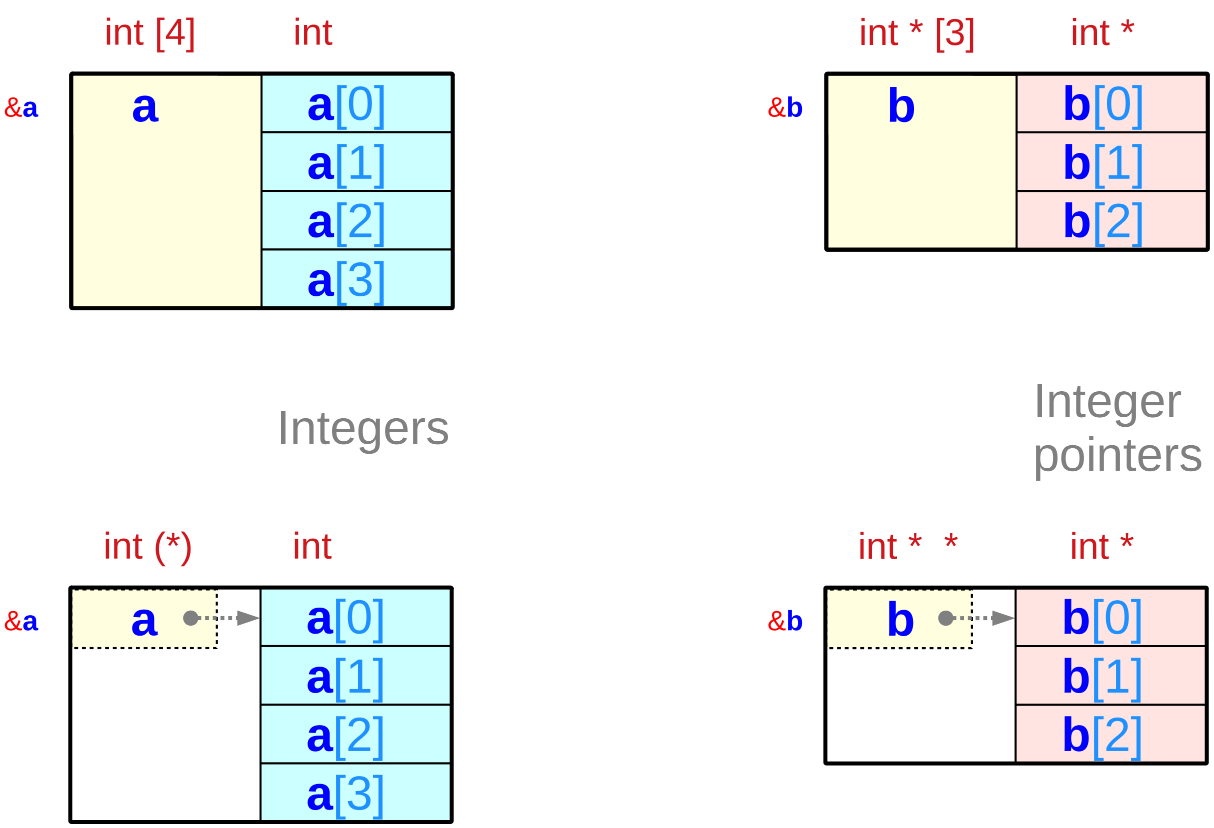




Array of Pointers – a type view

```
int      a [4] ;
```

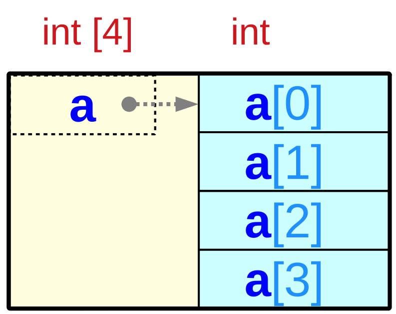
```
int *    b [3] ;
```



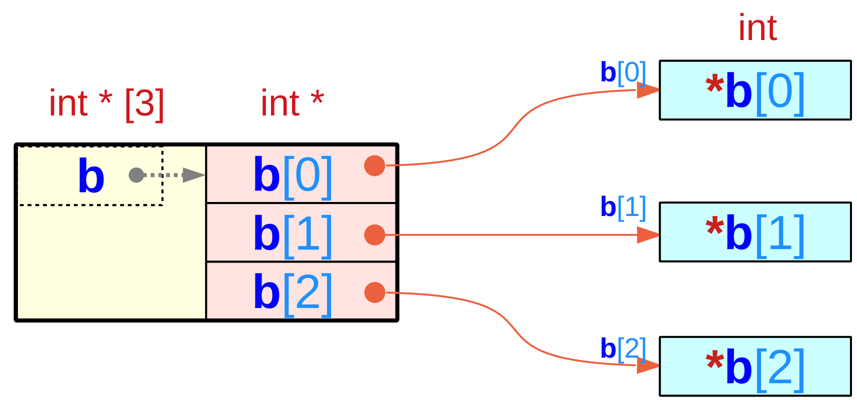
Array of Pointers – a variable view

int a [4] ;

int * b [3] ;



Integers



Integer pointers

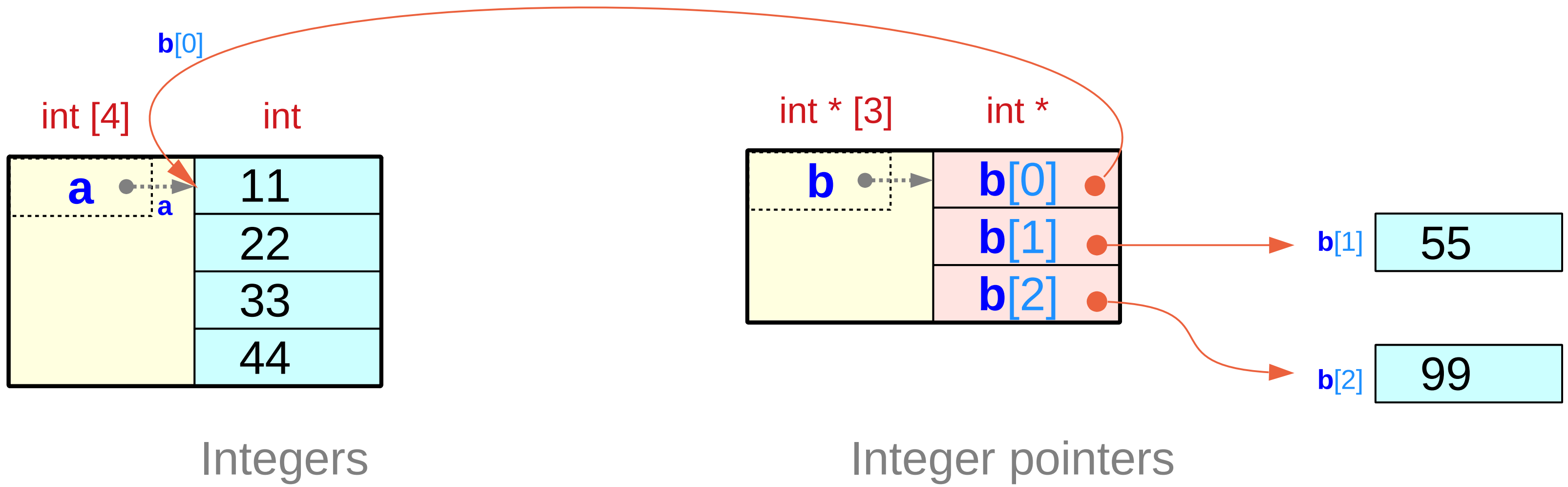
Assigning a **1-d** array name

int * b [3] ;

int a [4] ;

assignment

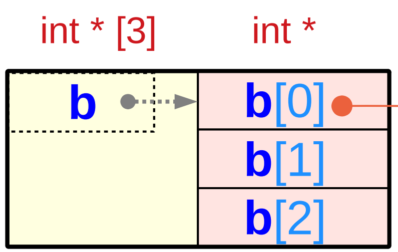
b[0] = &**a**[0] (= **a**)



Assigning a **1-d** array name – equivalence

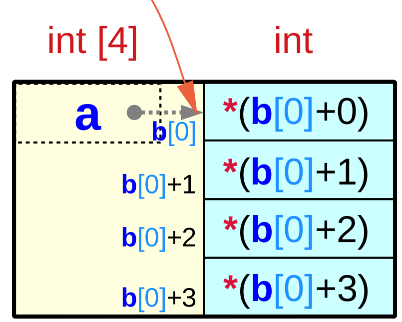
```
int *     b [3] ;
```

```
int       a [4] ;
```



assignment

b[0] = &a[0] (= a)

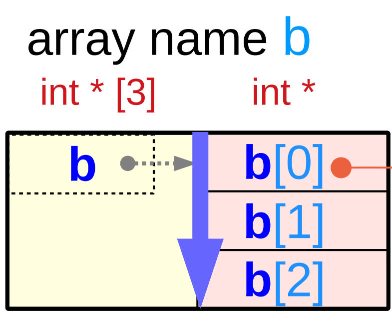


Array of Pointers – extended dimension

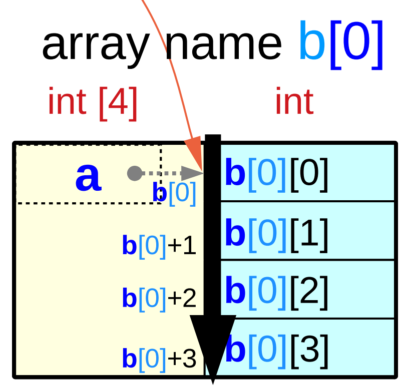
int * b [3] ;

int a [4] ;

a[0]	≡ b[0][0]	≡ *(*(b+0)+0)
a[1]	≡ b[0][1]	≡ *(*(b+0)+1)
a[2]	≡ b[0][2]	≡ *(*(b+0)+2)
a[3]	≡ b[0][3]	≡ *(*(b+0)+3)



1st dim

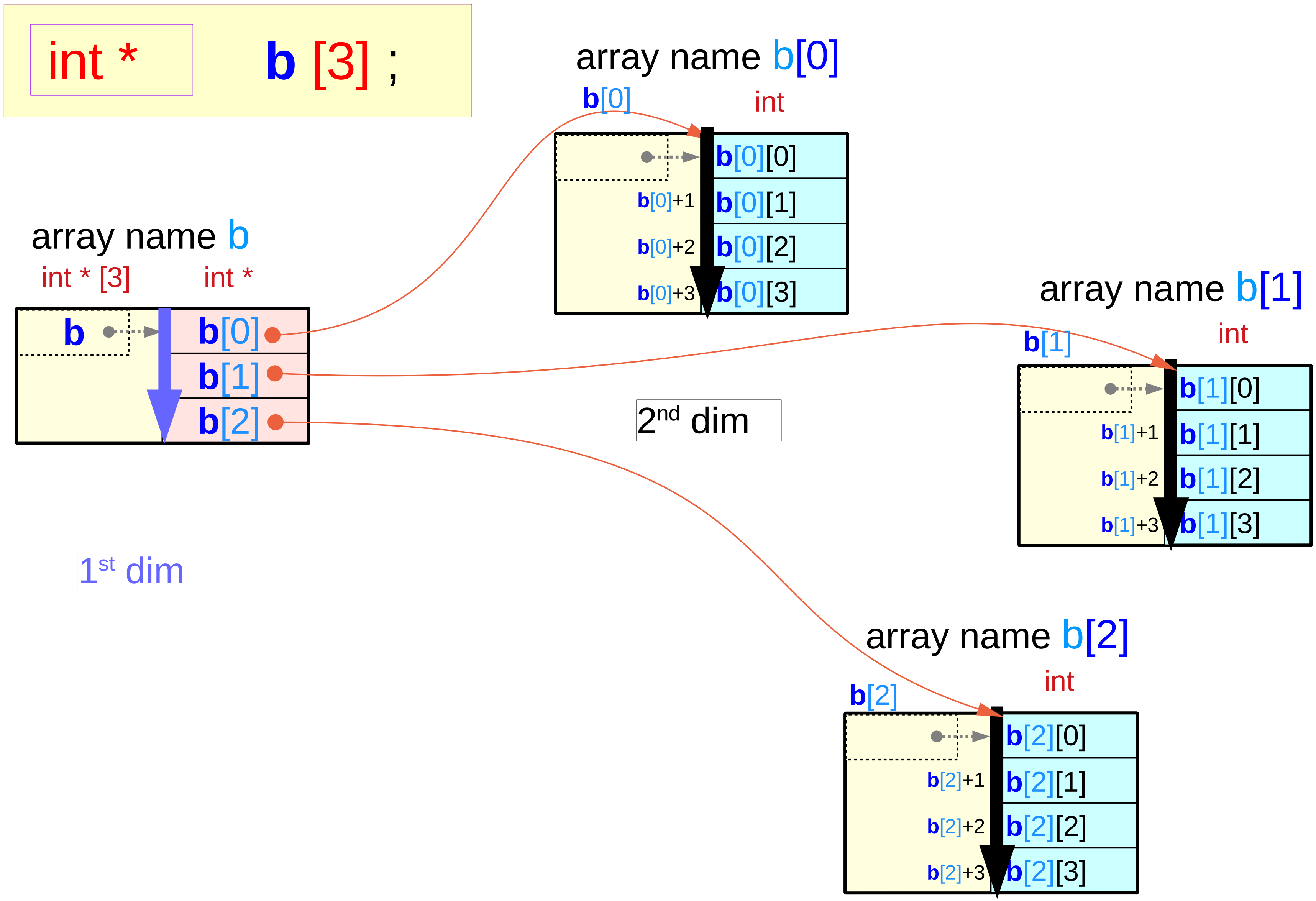


2nd dim

assignment

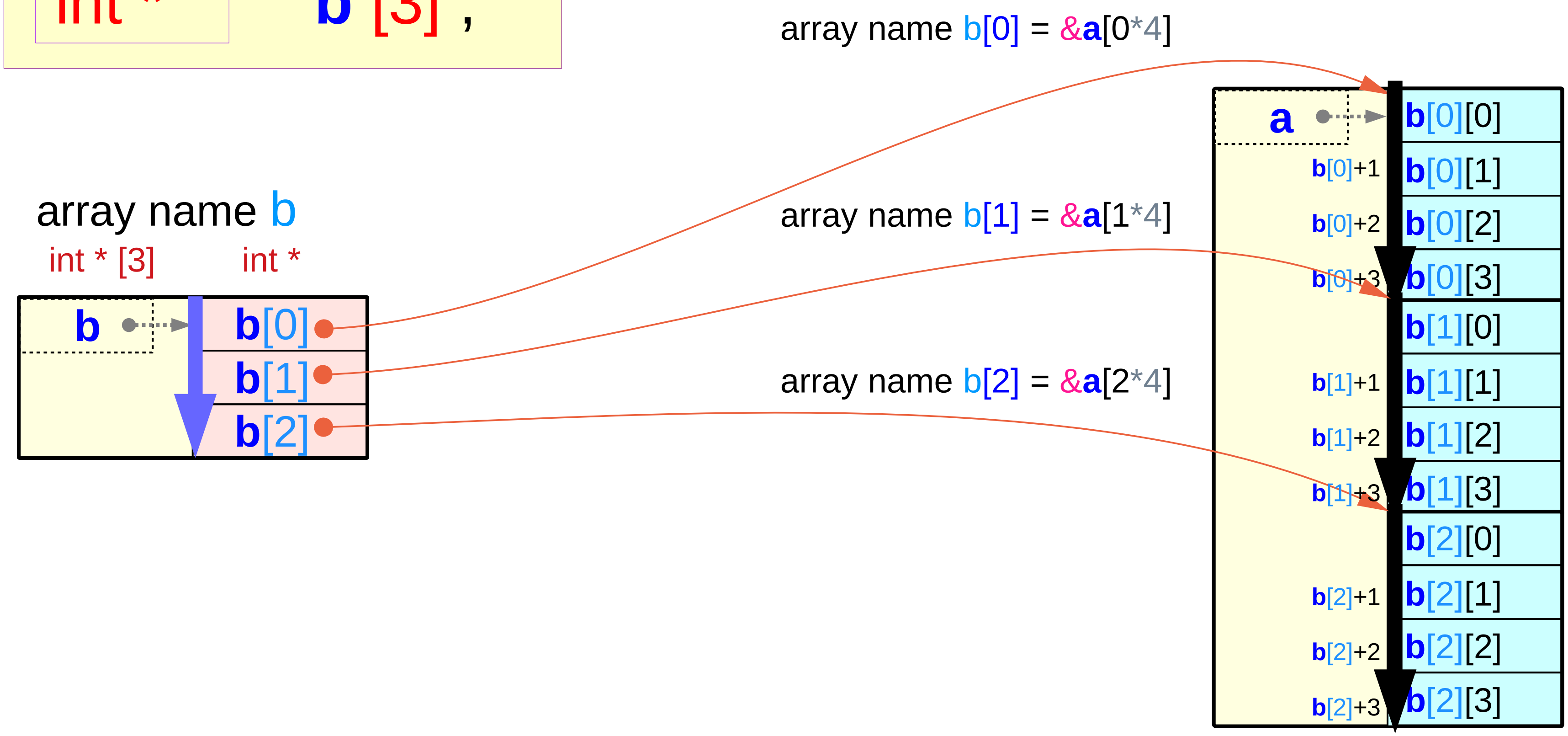
b[0] = &a[0] (= a)

2-d access of 1-d arrays



2-d access of a 1-d array

int * b [3] ;



int * a [3*4] ;

2-d access of a **1-d** array – pointer array assignments

int * **b** [3] ;

int **a** [3*4] ;

constraint : contiguous b[i][j] over j

Assignments

b[0] = &**a**[0*4] (= **a** +0*4)

b[1] = &**a**[1*4] (= **a** +1*4)

b[2] = &**a**[2*4] (= **a** +2*4)

2-d access of a **1-d** array

b[i][j] ≡ *(**b**[i] +j)

a[i*4+j] ≡ *(**a**+i*4 +j)

1-d access of a **1-d** array

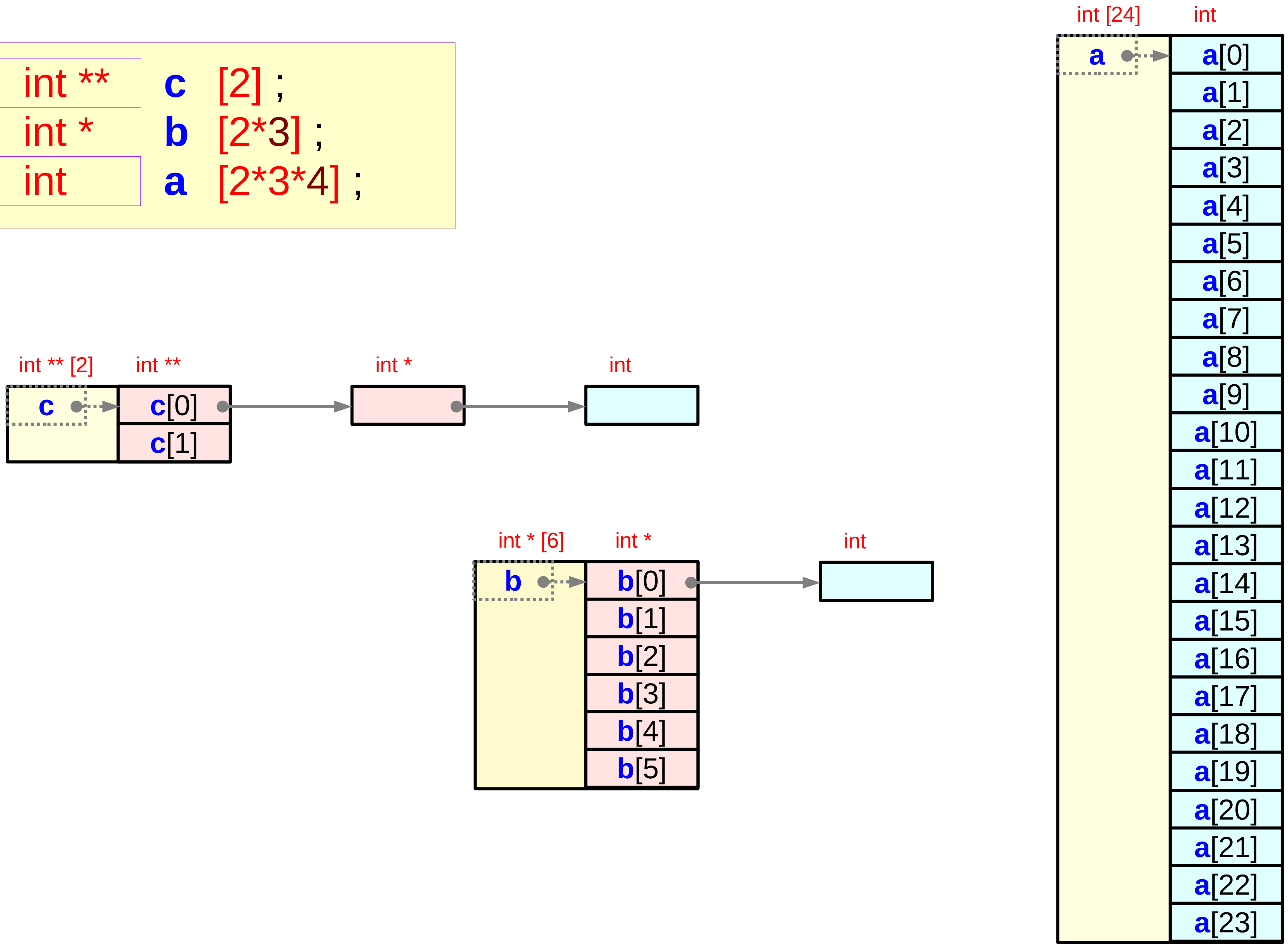
constraint : contiguous a[i*4+j] over j

*(b+i) = a+*f*(*i*)

3-d array access of a **1-d** array

Using pointer arrays **b**, **c**

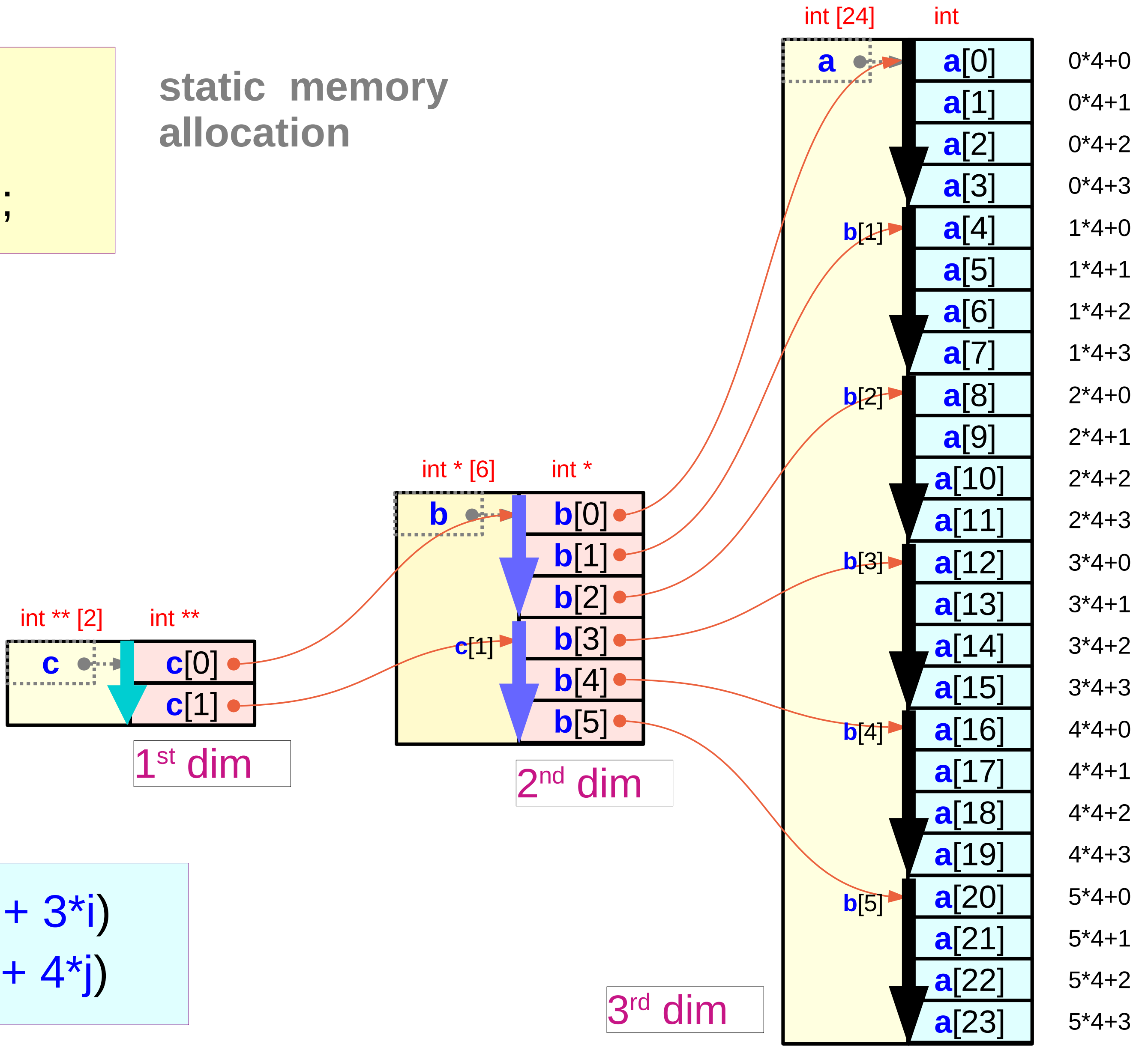
```
int **  c  [2] ;
int *   b  [2*3] ;
int     a  [2*3*4] ;
```



Using static memory allocation

int **	c	[2] ;
int *	b	[2*3] ;
int	a	[2*3*4] ;

static memory allocation

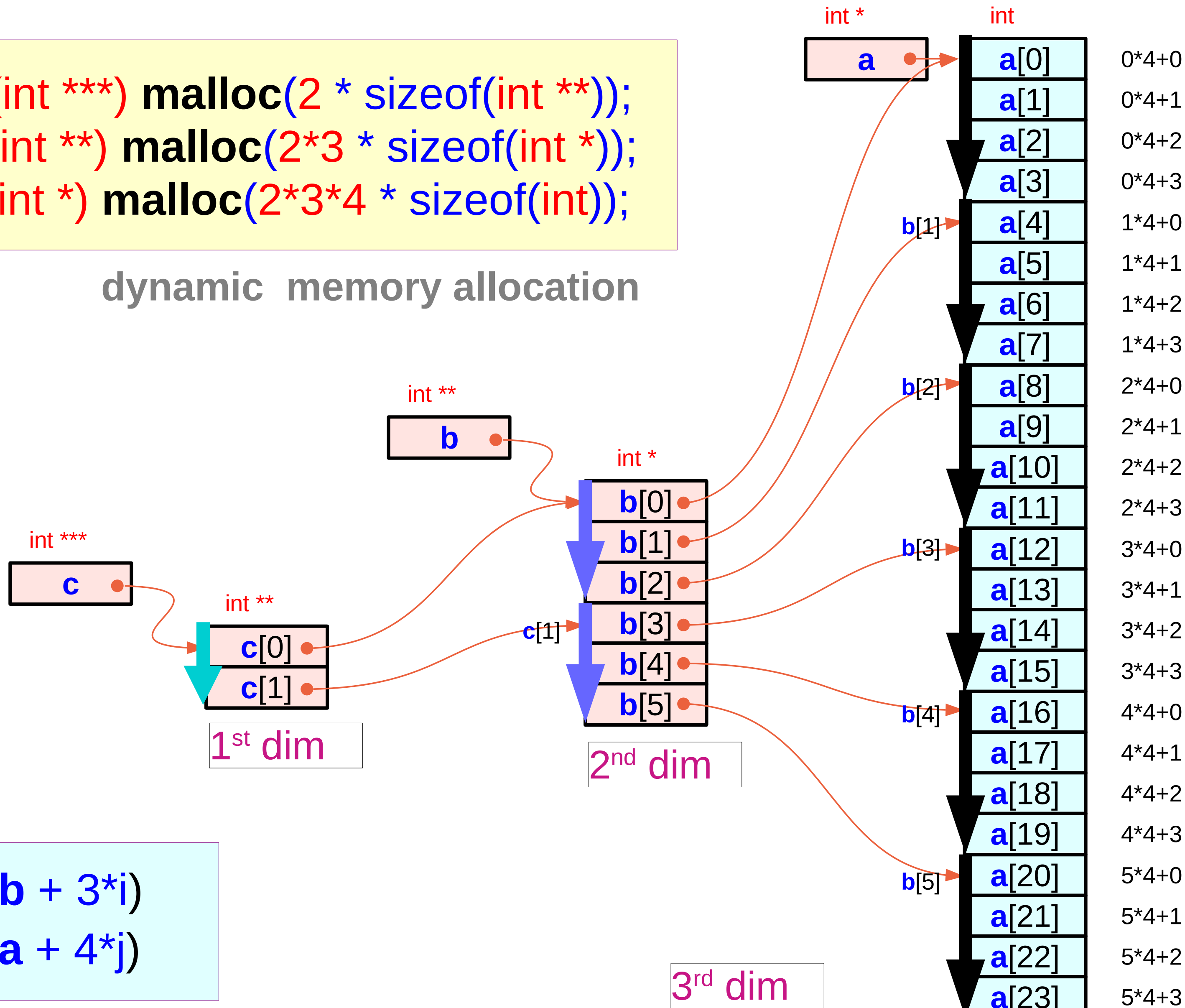


c[i] = &b[3*i] (= b + 3*i)
b[j] = &a[4*j] (= a + 4*j)

Using dynamic memory allocation

```
int ***   c   = (int ***) malloc(2 * sizeof(int **));
int **    b   = (int **) malloc(2*3 * sizeof(int *));
int *     a   = (int *) malloc(2*3*4 * sizeof(int));
```

dynamic memory allocation

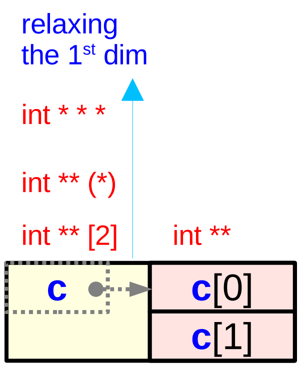


c[i] = &b[3*i] (= b + 3*i)

b[j] = &a[4*j] (= a + 4*j)

Static v.s. dynamic memory allocation (1)

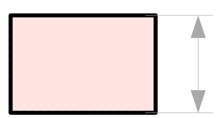
int *** c = (int ***) malloc(2 * sizeof(int **));

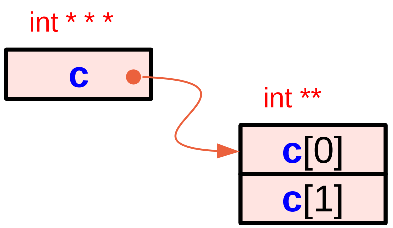


int ** c [2] ;

static memory allocation

malloc(2 * sizeof(int **));



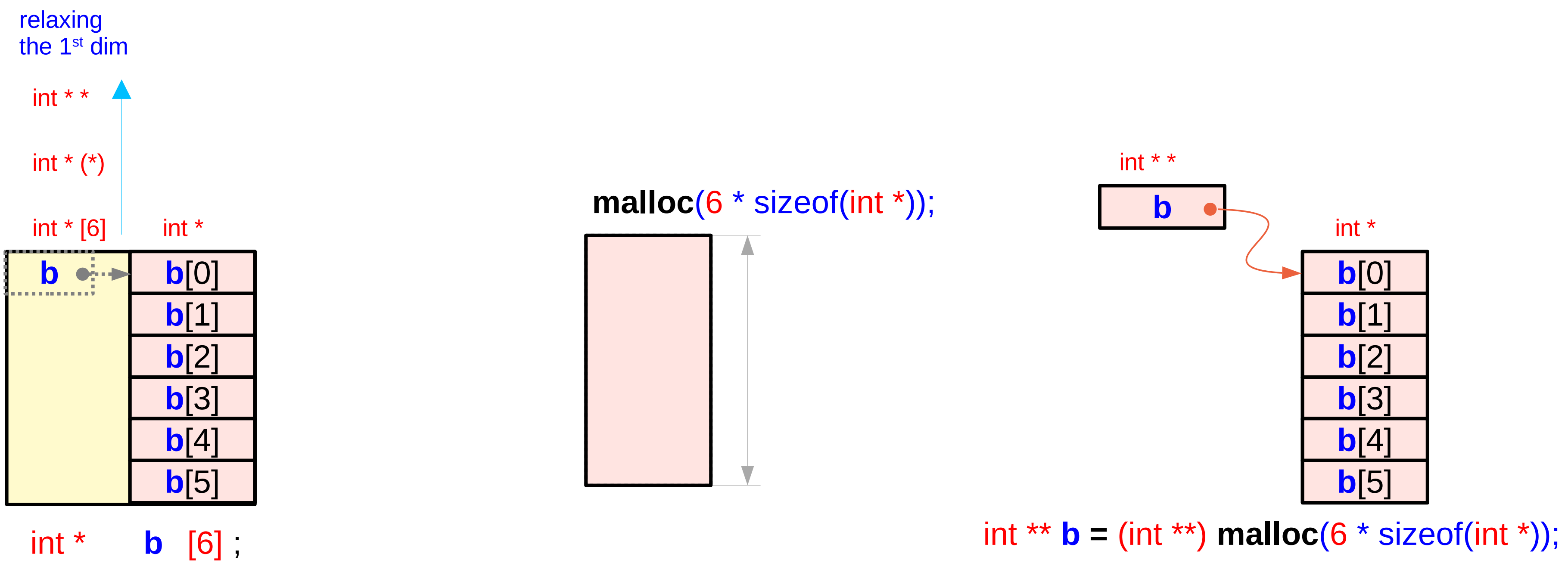


int *** c = (int ***) malloc(2 * sizeof(int **));

dynamic memory allocation

Static v.s. dynamic memory allocation (2)

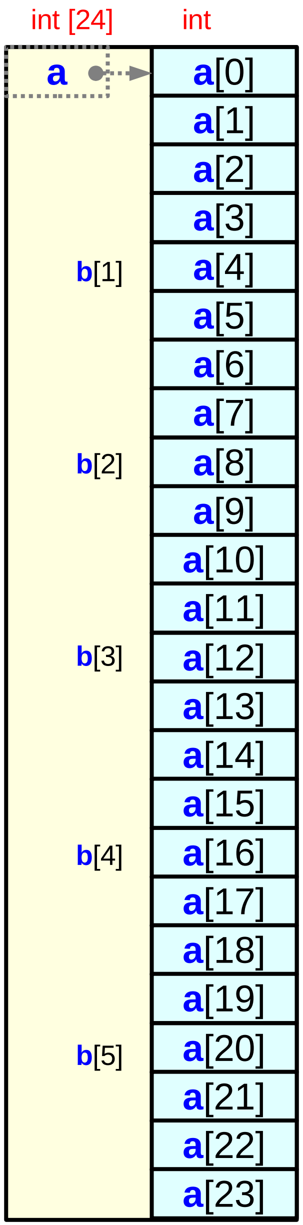
int ** b = (int ***) malloc(6 * sizeof(int *));



static memory allocation

dynamic memory allocation

Static v.s. dynamic memory allocation (3)




```
int * a = (int *)  malloc(24 * sizeof(int));
```

relaxing
the 1st dim

int *

int (*)

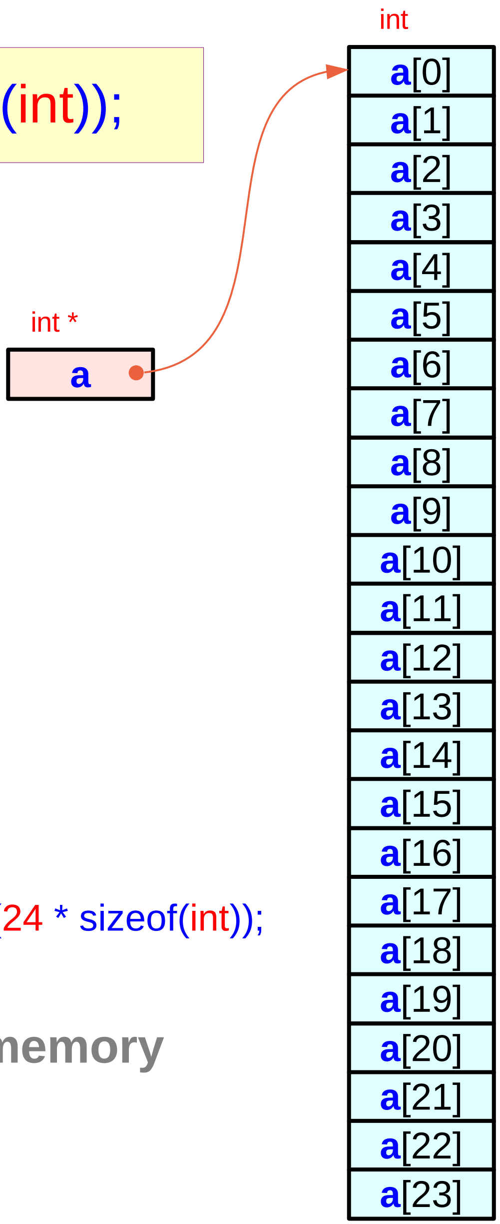
int [24]

`int * a [24] ;`

static memory allocation

`int * a = (int *) malloc(24 * sizeof(int));`

dynamic memory allocation



Static v.s. dynamic memory allocation (4)

int **	**c**	[2] ;
int *	**b**	[2*3] ;
int	**a**	[2*3*4] ;

static memory allocation

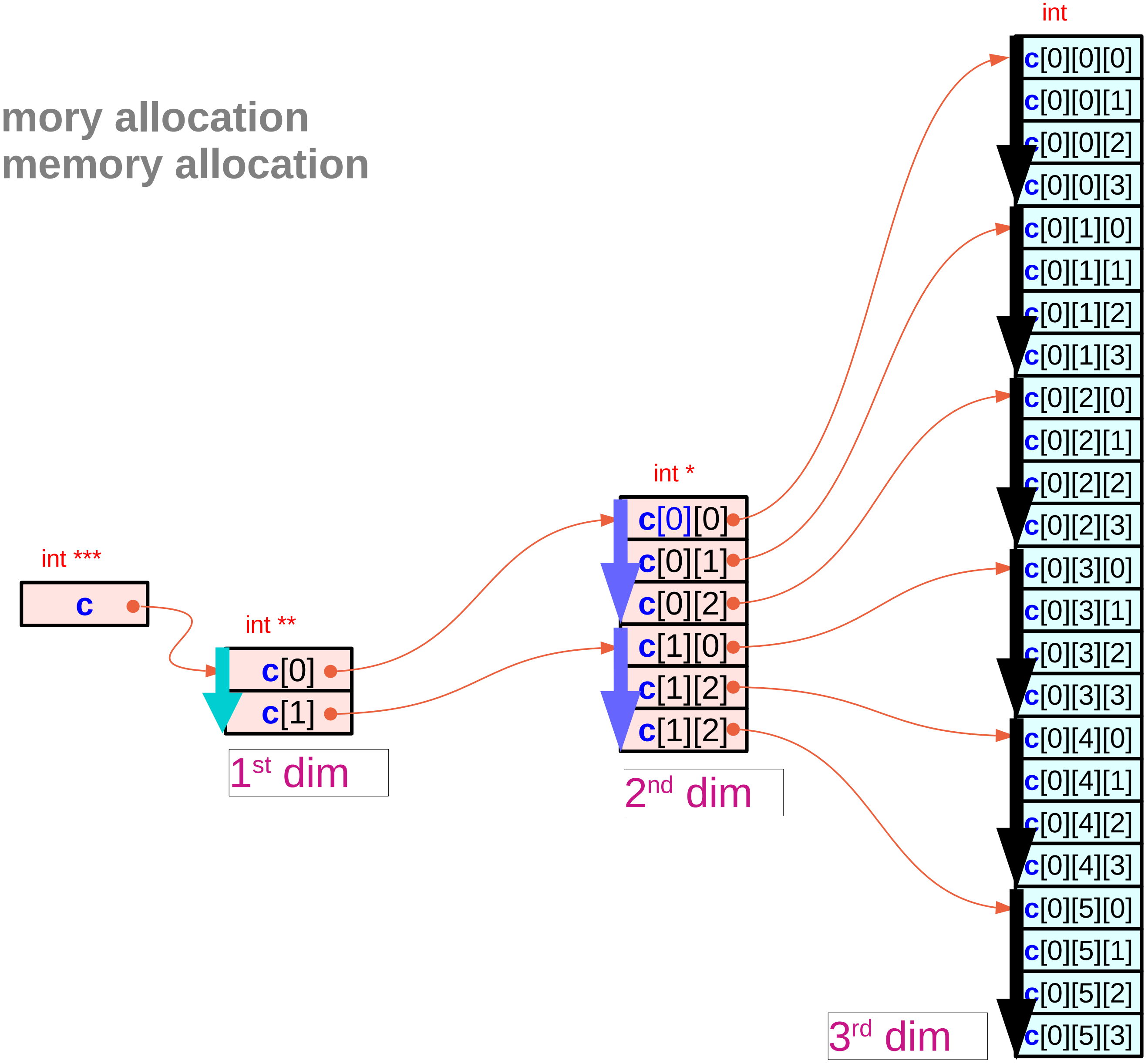
int ***	**c**	= (int ***) **malloc**(2 * sizeof(int **));
int **	**b**	= (int **) **malloc**(2*3 * sizeof(int *));
int *	**a**	= (int *) **malloc**(2*3*4 * sizeof(int));

dynamic memory allocation

c[i] = &**b**[3*i] (= **b** + 3*i)
b[j] = &**a**[4*j] (= **a** + 4*j)

Static v.s. dynamic memory allocation (5)

- **static memory allocation**
- **dynamic memory allocation**



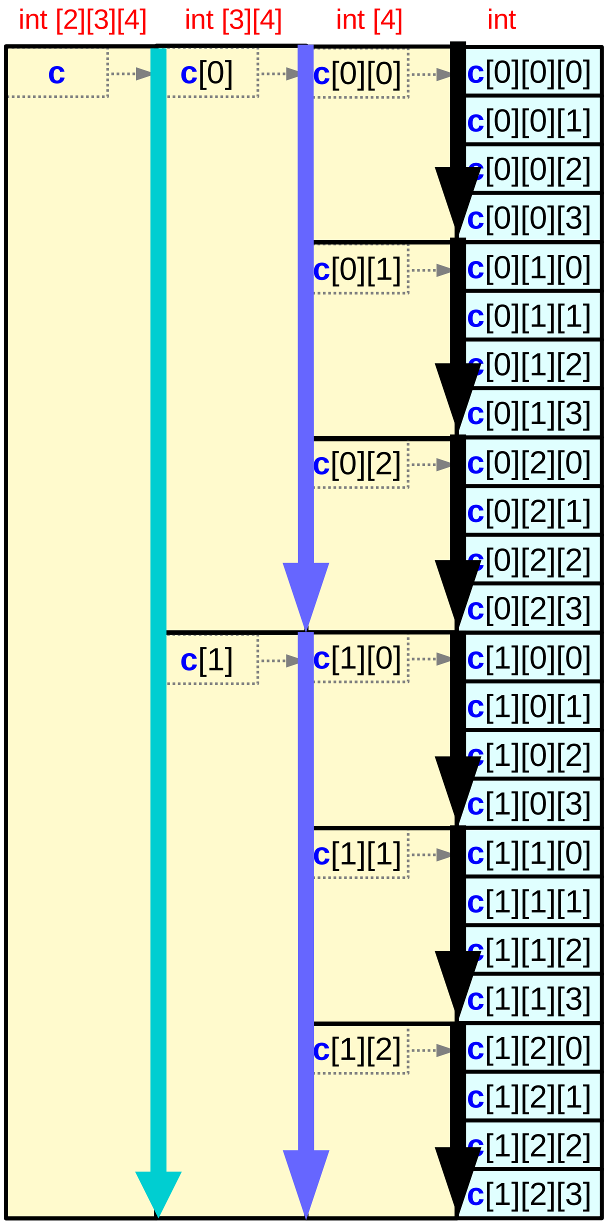
Static memory allocation of an **3-d** array

int **c** [2][3][4] ;

static memory allocation

value(**c**) = value(**c**[0]) = value(**c**[0][0]) = &**c**[0][0][0]
value(**c**[0][1]) = &**c**[0][1][0]
value(**c**[0][2]) = &**c**[0][1][0]
value(**c**[1]) = value(**c**[1][0]) = &**c**[1][0][0]
value(**c**[1][1]) = &**c**[1][1][0]
value(**c**[1][2]) = &**c**[1][1][0]

sizeof(**c**) = 2*3*4 * sizeof(int)
sizeof(**c**[i]) = 3*4 * sizeof(int)
sizeof(**c**[i][j]) = 4 * sizeof(int)



Finding sub-array sizes

int c [2][3][4] ;

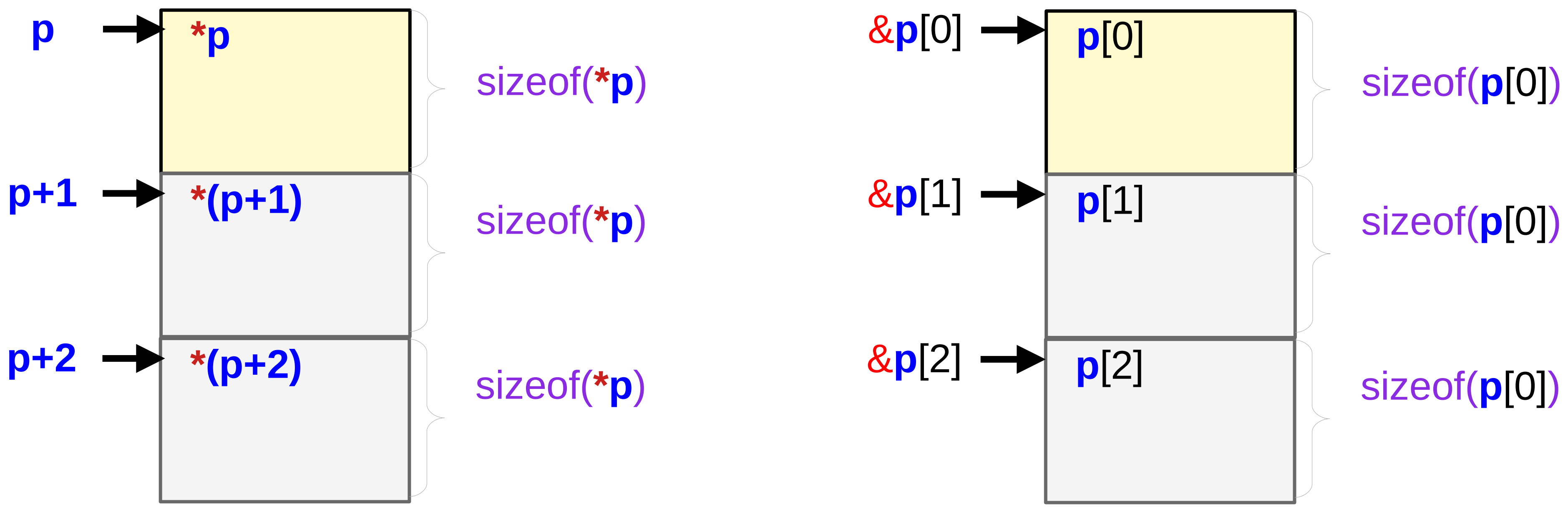
```
[2] [3] [4]
sizeof(c[i][j][0])   = sizeof(int)

[2] [3] [4]
sizeof(c[i][0])      = 4 * sizeof(int)

[2] [3] [4]
sizeof(c[i])         = 3 * 4 * sizeof(int)

[2] [3] [4]
sizeof(c)            = 2 * 3 * 4 * sizeof(int))
```

Byte addresses in an array



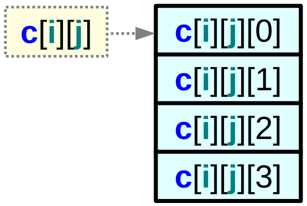
value(p+1) = value(p) + 1 * sizeof(*p)
value(p+2) = value(p) + 2 * sizeof(*p)

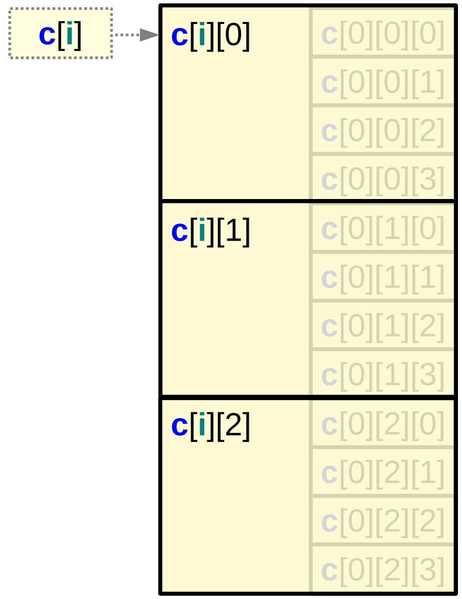
byte address　　byte address　　byte size

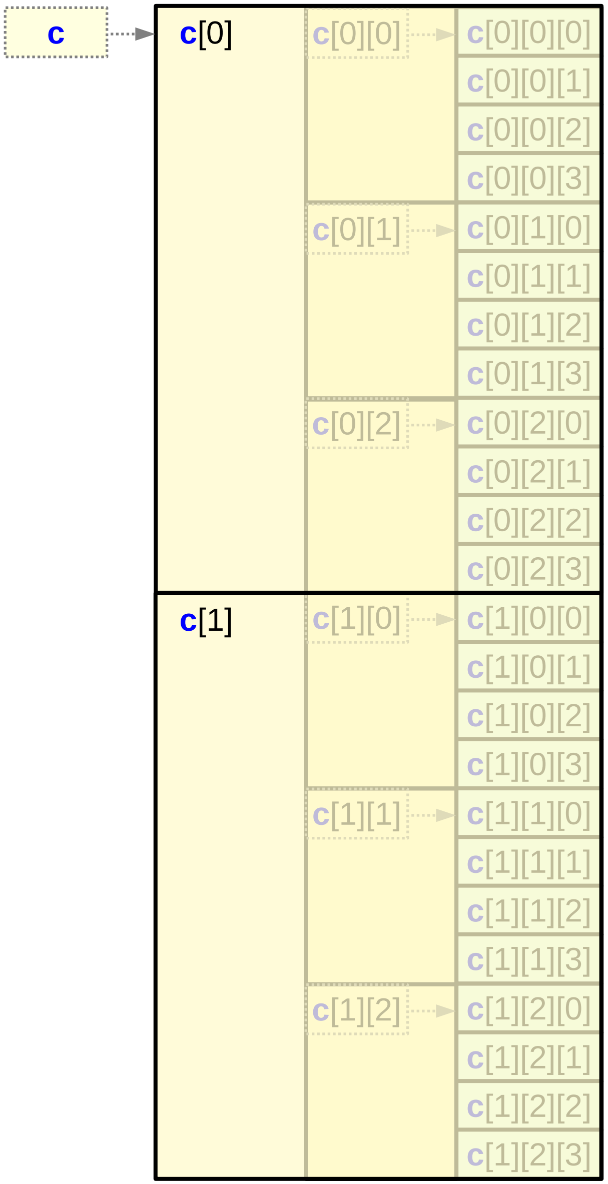
value(&p[1]) = value(p) + 1 * sizeof(p[0])
value(&p[2]) = value(p) + 2 * sizeof(p[0])

byte address　　byte address　　byte size

Byte addresses of &c[i], &c[i][j], &c[i][j][k]







value(&c[i][j][k]) **k** = 0:3
= value(c[i][j]) + **k** * sizeof(*c[i][j])
= value(c[i][j]) + **k** * sizeof(c[i][j][0])
= value(c[i][j]) + **k** * sizeof(int)

value(&c[i][j]) **j** = 0:2
= value(c[i]) + **j** * sizeof(*c[i])
= value(c[i]) + **j** * sizeof(c[i][0])
= value(c[i]) + **j** * sizeof(int) * 4

value(&c[i]) **i** = 0 : 1
= value(c) + **i** * sizeof(*c)
= value(c) + **i** * sizeof(c[0])
= value(c) + **i** *sizeof(int) * 3 * 4

Abstract and byte addresses of sub-arrays

int c [2][3][4] ;

c[i][j][k]	= *(c[i][j] + k)
c[i][j]	= *(c[i] + j)
c[i]	= *(c + i)

abstract address *type independent*

&c[i][j][k]	= c[i][j] + k	*after k sizeof(*c[i][j])*
&c[i][j]	= c[i] + j	*after j sizeof(*c[i])*
&c[i]	= c + i	*after i sizeof(*c)*

byte address *type dependent*

value(&c[i][j][k])	= value(c[i][j]) + k * sizeof(*c[i][j])	= value(c[i][j]) + k * sizeof(int)
value(&c[i][j])	= value(c[i]) + j * sizeof(*c[i])	= value(c[i]) + j * 4 * sizeof(int)
value(&c[i])	= value(c) + i * sizeof(*c)	= value(c) + i * 3 * 4 * sizeof(int)

Values of &c[i][j], c[i][j], and &c[i], c[i]

int c [2][3][4] ;

Byte address

value(&c[i][j][k]) = value(c[i][j]) + k * sizeof(int)

value(&c[i][j]) = value(c[i]) + j * 4 * sizeof(int)

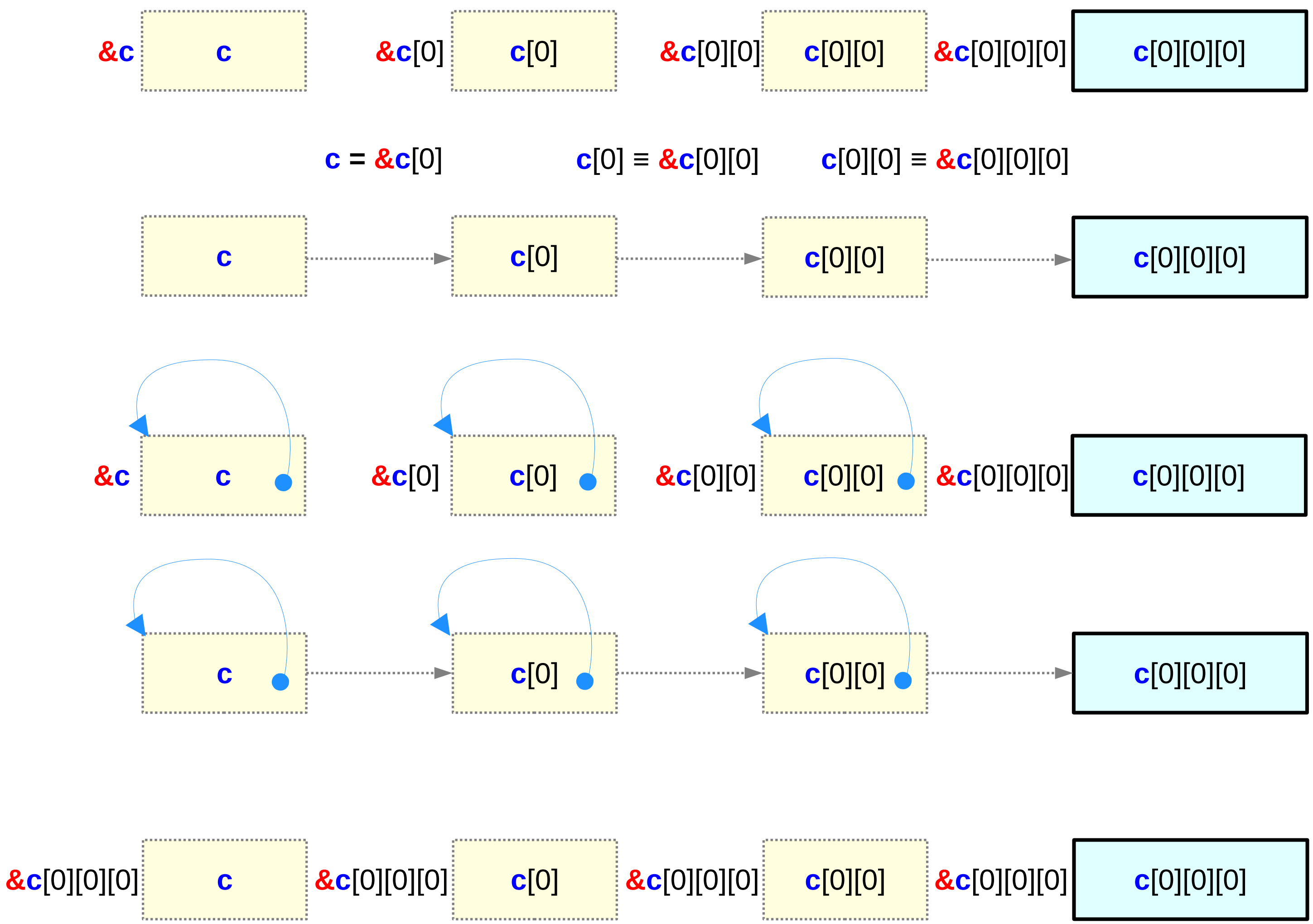
value(&c[i]) = value(c) + i * 3 * 4 * sizeof(int)

what if
value(c[i][j]) = value(&c[i][j])
value(c[i]) = value(&c[i])

c[i][j][k] = *(c[i][j]+k)
c[i][j] = *(c[i]+j)
c[i] = *(c+i)

value(&c[i][j][k]) = value(c[i][j]) + k * sizeof(*c[i][j])
value(&c[i][j]) = value(c[i]) + j * sizeof(*c[i])
value(&c[i]) = value(c) + i * sizeof(*c)

Virtual pointers – subarray names **c**, **c**[0], **c**[0][0]



equivalences

new conditions
value(**c**[i][j]) = value(&**c**[i][j])
value(**c**[i]) = value(&**c**[i])
value(**c**) = value(&**c**)

c, **c**[0], **c**[0][0] : virtual pointers
the same address and value

a physical location
has a unique address

Byte addresses of sub-arrays in an array

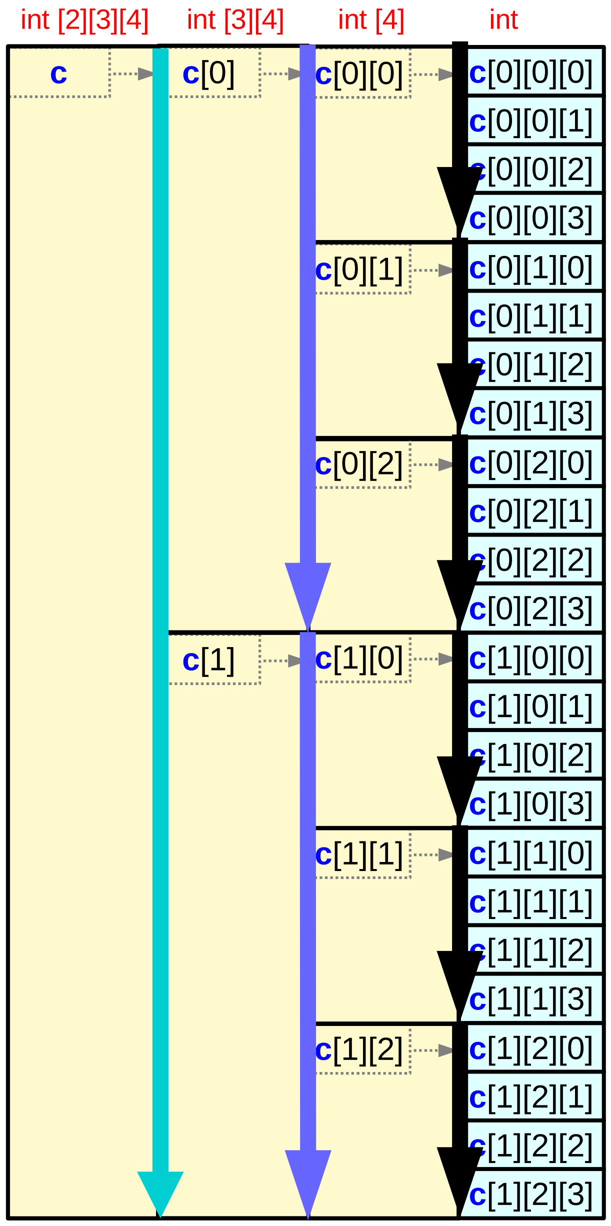
value(c) = value(c[0]) = value(c[0][0]) = value(&c[0][0][0])
value(c[0][1]) = value(&c[0][1][0])
value(c[0][2]) = value(&c[0][1][0])
value(c[1]) = value(c[1][0]) = value(&c[1][0][0])
value(c[1][1]) = value(&c[1][1][0])
value(c[1][2]) = value(&c[1][1][0])

value(c) = value(c[0]) = value(c[0][0]) = value(&c[0][0][0])
value(c[i]) = value(c[i][j]) = value(&c[i][j][0])

new conditions
value(c[i][j]) = value(&c[i][j])
value(c[i]) = value(&c[i])
value(c) = value(&c)

equivalences
value(c[i][j]) = value(&c[i][j][0])
value(c[i]) = value(&c[i][0])
value(c) = value(&c[0])

virtual pointers



Address values of &c[i][j][k] (1)

int c [2][3][4] ;

Byte address

c[i][j][k] = *(c[i][j]+k)   value(&c[i][j][k]) = value(c[i][j]) + k * sizeof(*c[i][j])
c[i][j] = *(c[i]+j)   value(&c[i][j]) = value(c[i]) + j * sizeof(*c[i])
c[i] = *(c+i)   value(&c[i]) = value(c) + i * sizeof(*c)

value(c[i][j]) = value(&c[i][j])
value(c[i]) = value(&c[i])

value(&c[i][j][k]) = value(c[i][j]) + k * sizeof(*c[i][j])
= value(c[i]) + j * sizeof(*c[i]) + k * sizeof(*c[i][j])
= value(c) + i * sizeof(*c) + j * sizeof(*c[i]) + k * sizeof(*c[i][j])

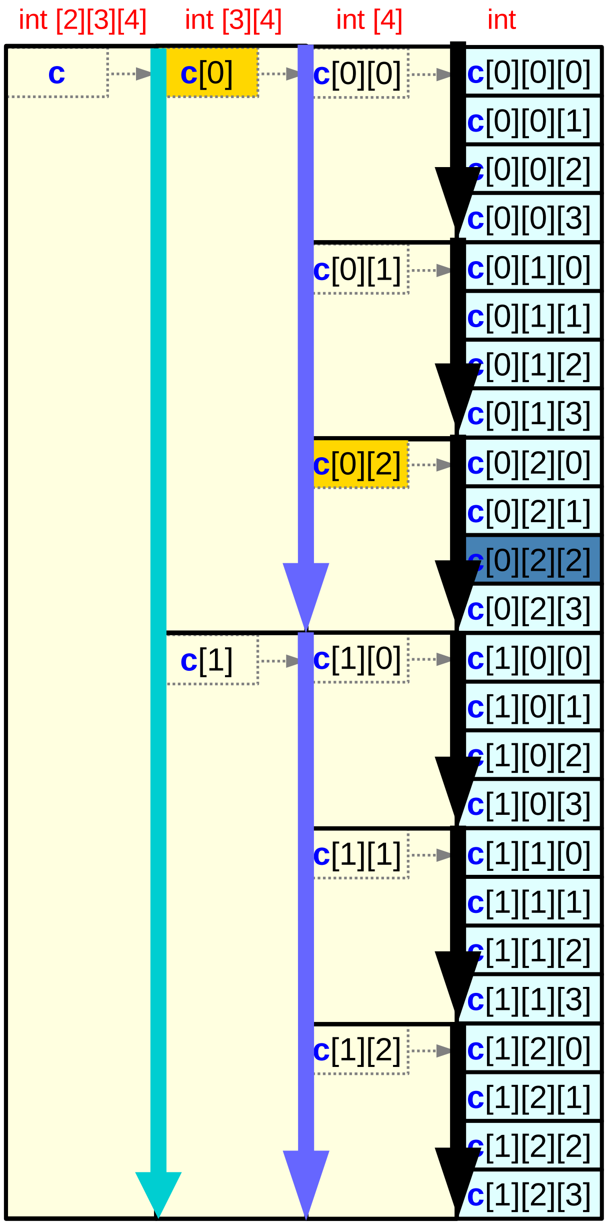
Byte addresses of sub-arrays in an array

c[i][j][k] = *(c[i][j]+k)
(c[i][j])[k] = *(*(c[i]+j)+k)
((c[i])[j])[k] = *(*(*(c+i)+j)+k)

&(c[i][j][k]) = (c[i][j]+k)
&(&(c[i][j])[k]) = ((c[i]+j)+k)
&(&(&(c[i])[j])[k]) = (((c+i)+j)+k)

c[i][j][k] = *(c[i][j]+k)
c[i][j] = *(c[i]+j)
c[i] = *(c+i)

&c[i][j][k] = (c[i][j]+k)
&c[i][j] = (c[i]+j)
&c[i] = (c+i)



Byte addresses of sub-arrays in an array

$$\&(\&(\&(c[i])[j])[k]) \;=\; (((c+i)+j)+k)$$

sizeof(c[0])
3 * 4 * sizeof(int)

sizeof(c[0][0])
4 * sizeof(int)

sizeof(c[0][0][0])
sizeof(int)

Though they are equivalent mathematically,
in the respect of pointer arithmetic,
they are very different and
parentheses shall be used to distinguish them
As another way, value() expression is used,
Which returns the address value.

$$\neq\; c + i + j + k$$

sizeof(c[0])

Byte addresses of sub-arrays in an array

$$\&(\&(\&(c[i])[j])[k]) \;=\; (((c+i)+j)+k)$$

Ideal & operator

C & operator

can be applied to only **lvalue** variable
returns address value
thus, the above expression is not possible
Successive application of & is not possible

In constrast, ***p** becomes a lvalue variable
* operator can be applied successively.

Address values of &c[i][j][k] (1)

c[i][j][k]	=	*(c[i][j]+k)	&(c[i][j][k])	=	(c[i][j]+k)
(c[i][j])[k]	=	*(*(c[i]+j)+k)	&(&(c[i][j])[k])	=	((c[i]+j)+k)
((c[i])[j])[k]	=	*(*(*(c+i)+j)+k)	&(&(&(c[i])[j])[k])	=	(((c+i)+j)+k)

value(&(c[i][j][k])) = value(c[i][j]+k)
value(&(&(c[i][j])[k])) = value(value(c[i]+j)+k)
value(&(&(&(c[i])[j])[k])) = value(value(value(c+i)+j)+k)

value(&c[i][j][k]) = value(c[i][j]) + k * sizeof(*c[i][j])
= value(c[i]) + j * sizeof(*c[i]) + k * sizeof(*c[i][j])
= value(c) + i * sizeof(*c) + j * sizeof(*c[i]) + k * sizeof(*c[i][j])

Address values of &c[i][j][k] (2)

int c [L][M][N] ;

```
value(&c[i][j][k])  = value(c[i][j]) + k * sizeof(*c[i][j])
                    = value(c[i][j]) + k * sizeof(c[i][j][0])
                    = value(c[i][j]) + k * sizeof(int)

                    = value(c[i]) + j * sizeof(*c[i]) + k * sizeof(*c[i][j])
                    = value(c[i]) + j * sizeof(c[i][0]) + k * sizeof(c[i][j][0])
                    = value(c[i]) + (j * N + k) * sizeof(int)

                    = value(c) + i * sizeof(*c) + j * sizeof(*c[i]) + k * sizeof(*c[i][j])
                    = value(c) + i * sizeof(c[0]) + j * sizeof(c[i][0]) + k * sizeof(c[i][j][0])
                    = value(c) + (i * M * N  + j * N  + k) * sizeof(int)
                    = value(c) + ((i * M + j ) * N  + k) * sizeof(int)
```

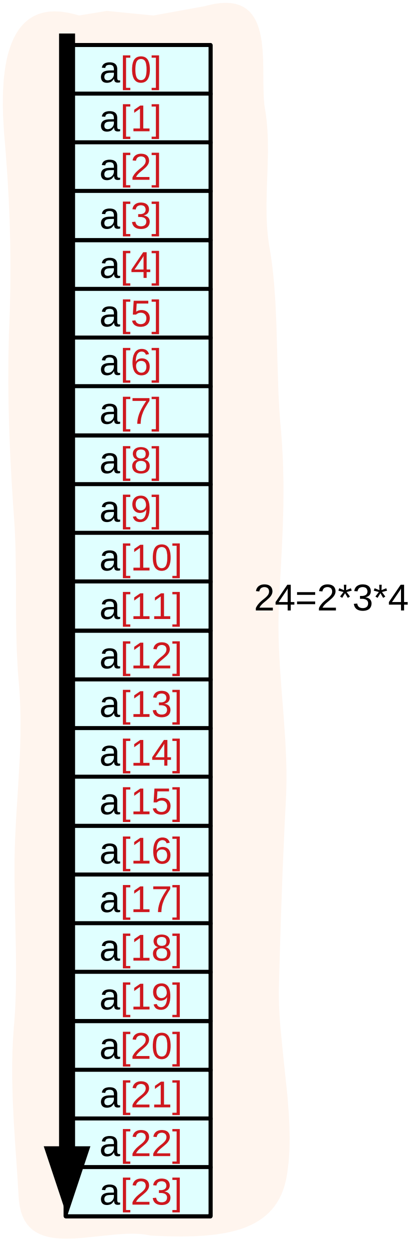
- **1-d** array access
- **2-d** array access
- **3-d** array access

Accessing an int array **a** as a **1-d** array

int **a** [2*3*4] ;

a [**k**]

k = 0,1, …,23



c[i][j][k]	≡ *(*(*(c+i)+j)+k)	int c[2][3][4] ;
b[i][j]	≡ *(*(b+i)+j)	int b[2*3][4] ;
a[i]	≡ *(a+i)	int a[2*3*4] ;

Accessing an int array **a** as a **2-d** array using **b**

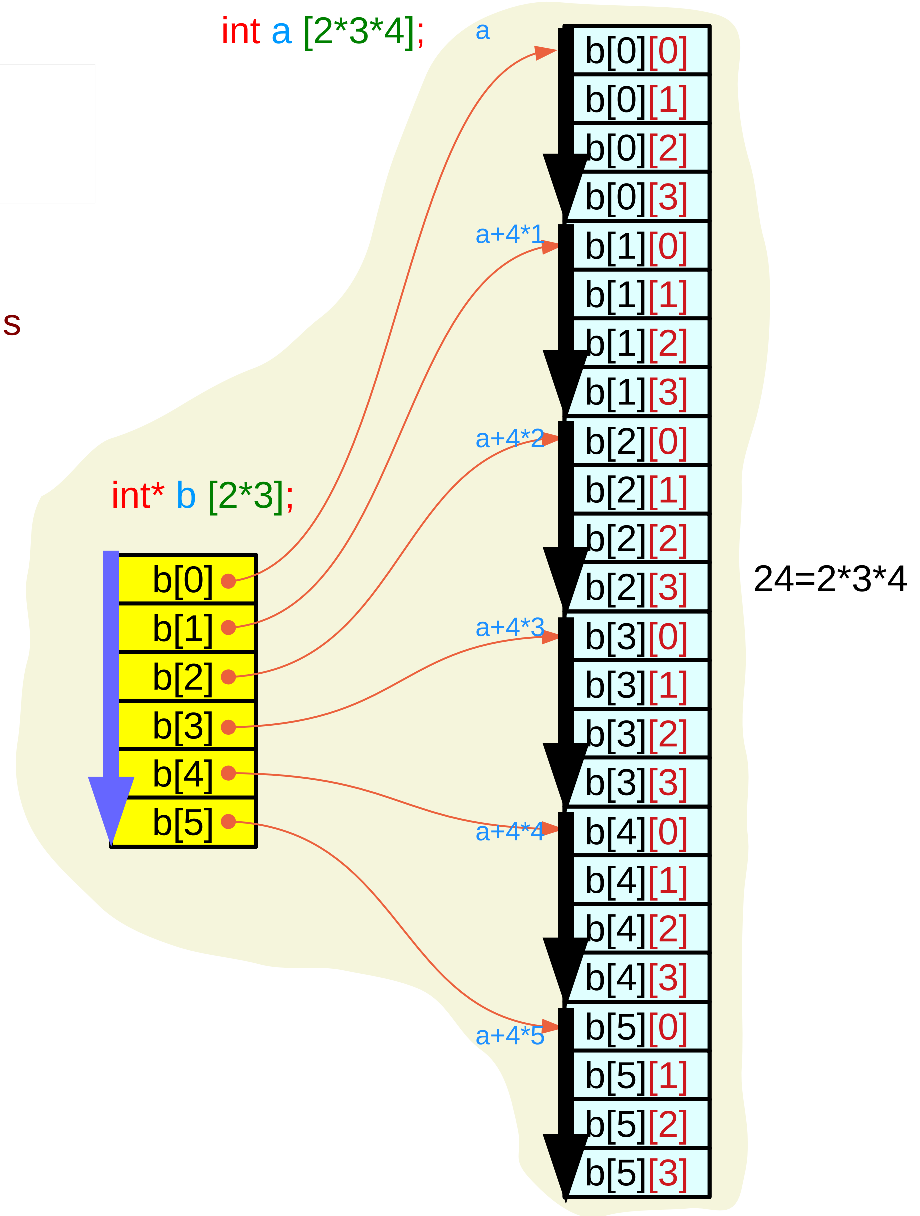
int a [2*3*4] ;
int * b [2*3] ;

b [j][k] ≡ a [j*4 +k]

j = 0:5
k = 0:4

b[j] = &a[j*4];

b take actual memory locations



c[i][j][k]	≡ *(*(*(c+i)+j)+k)	int c[2][3][4] ;
b[i][j]	≡ *(*(b+i)+j)	int b[2*3][4] ;
a[i]	≡ *(a+i)	int a[2*3*4] ;

Accessing an int array **a** as a **3-d** array

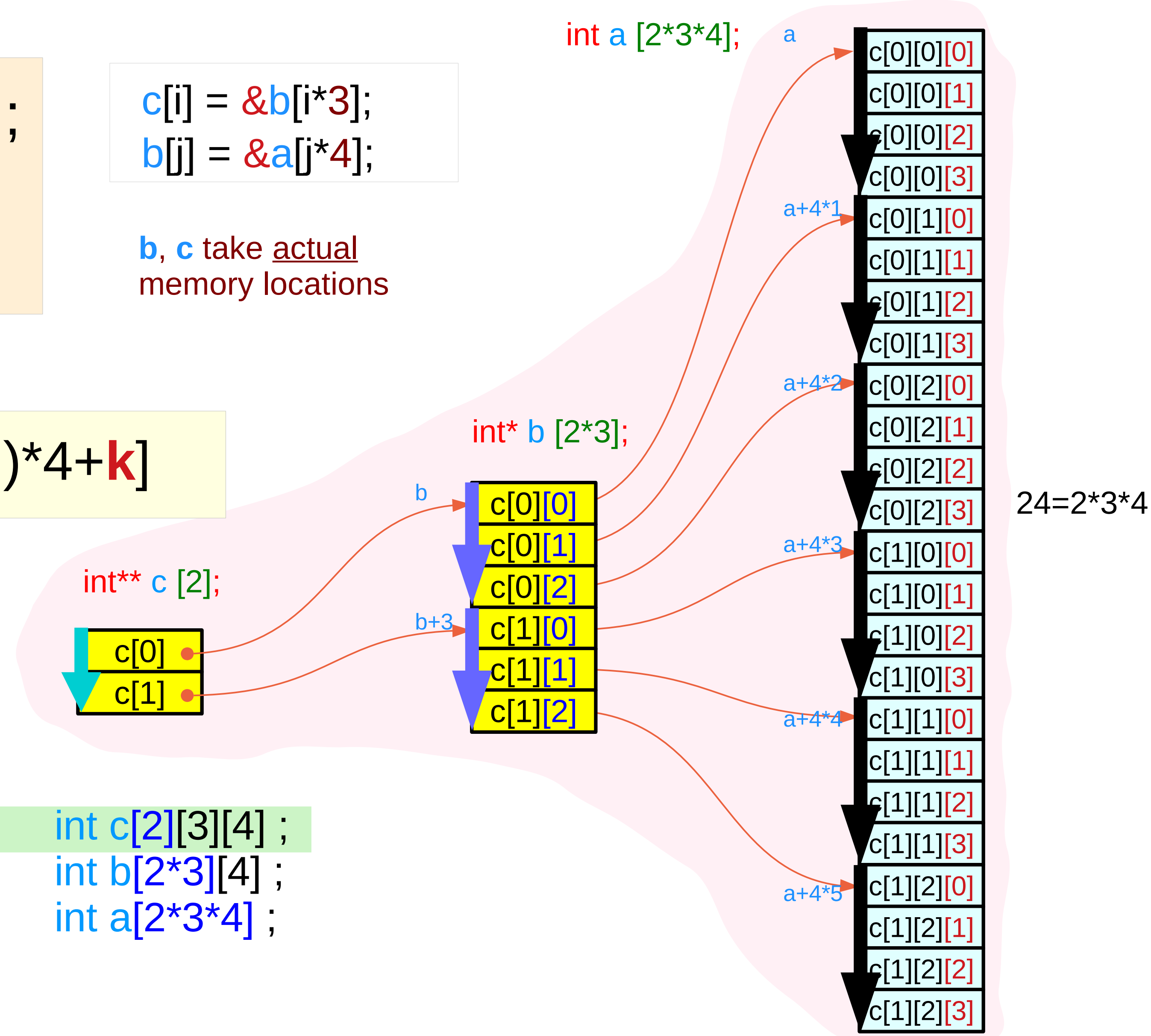
int a [2*3*4] ;
int * b [2*3] ;
int ** c [2] ;

c[i] = &b[i*3];
b[j] = &a[j*4];

b, **c** take actual memory locations

c [i][j][k] ≡ a [(i*3+j)*4+k]

i = 0, 1
j = 0, 1, 2
k = 0, 1, 2, 3



c[i][j][k]	≡ *(*(*(c+i)+j)+k)	int c[2][3][4] ;
b[i][j]	≡ *(*(b+i)+j)	int b[2*3][4] ;
a[i]	≡ *(a+i)	int a[2*3*4] ;

Accessing **non-contiguous 1-d** arrays as a **3-d** array (1)

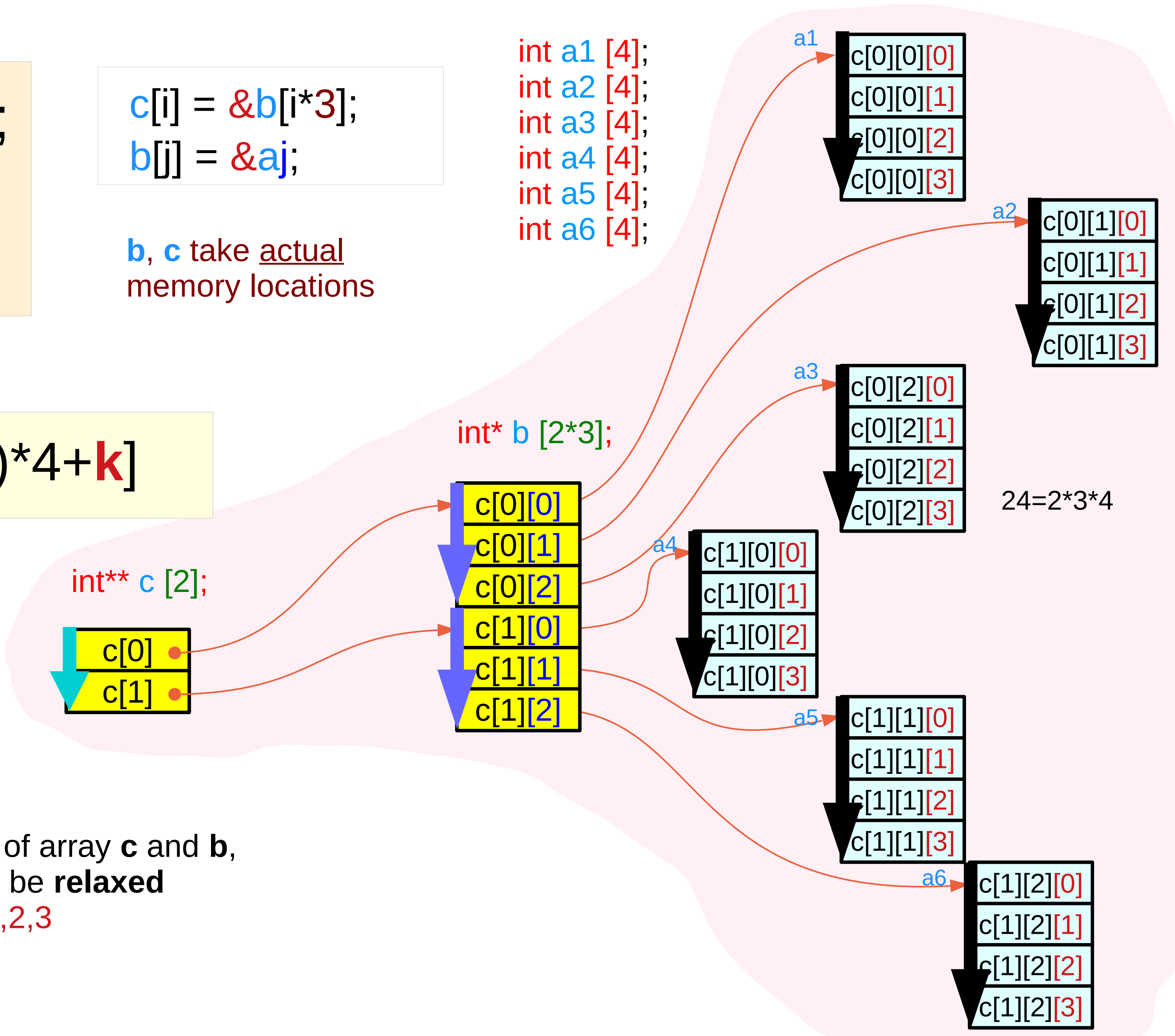
```
int      a [2*3*4] ;
int *    b [2*3] ;
int **   c [2] ;
```

c[i] = &b[i*3];
b[j] = &aj;

b, **c** take actual memory locations

c [i][j][k] ≡ a [(i*3+j)*4+k]

i = 0, 1
j = 0, 1, 2
k = 0, 1, 2, 3



Because the physical **allocation** of array **c** and **b**, the **contiguous constraints** can be **relaxed**
contiguous c[i][j][k] only for k=0,1,2,3

Accessing **non-contiguous 1-d** arrays as a **3-d** array (2)

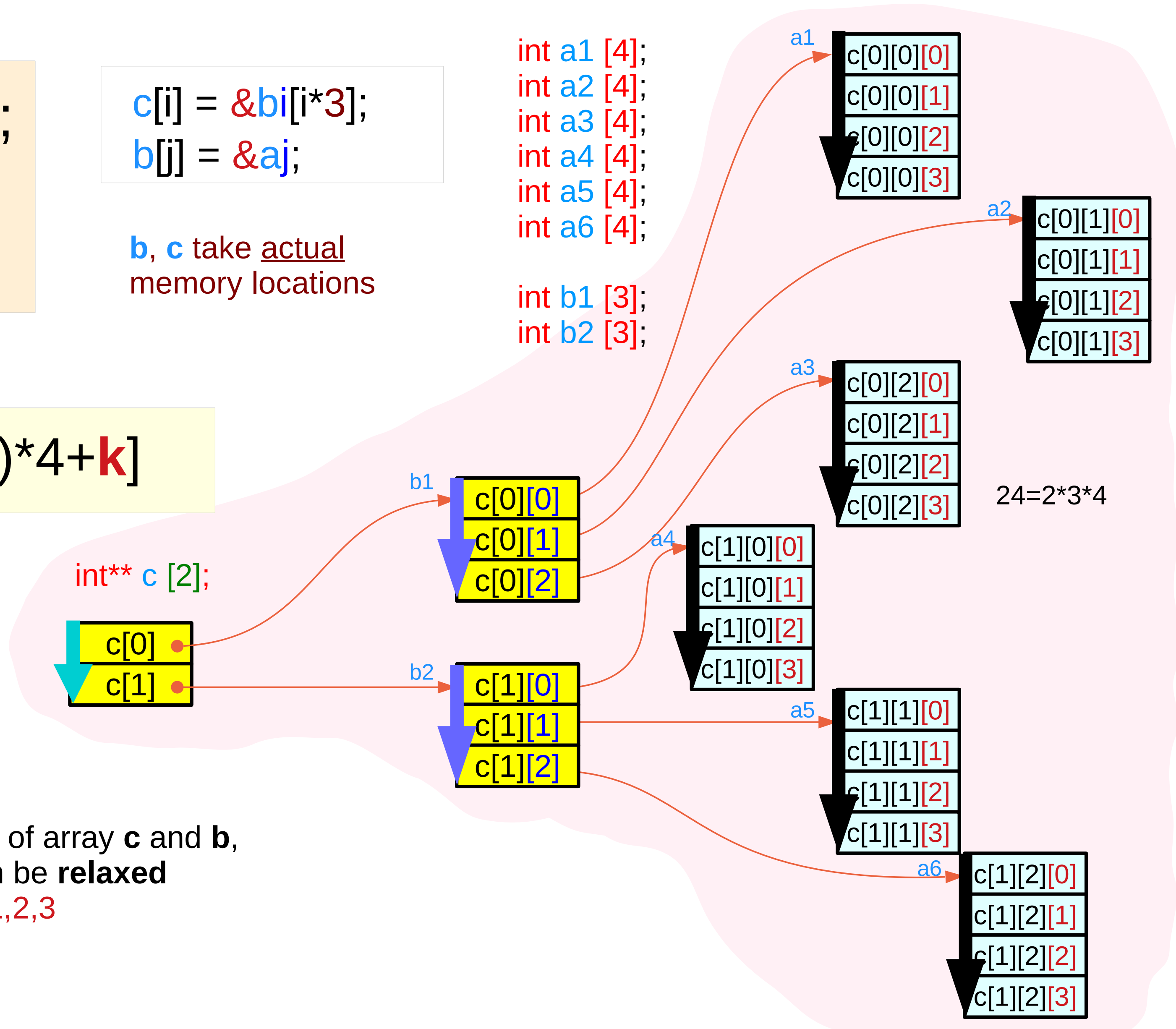
```
int     a [2*3*4] ;
int *   b [2*3] ;
int **  c [2] ;
```

c[i] = &bi[i*3];
b[j] = &aj;

b, **c** take actual memory locations

c [i][j][k] ≡ a [(i*3+j)*4+k]

i = 0, 1
j = 0, 1, 2
k = 0, 1, 2, 3



Because the physical **allocation** of array **c** and **b**,
the **contiguous constraints** can be **relaxed**
contiguous c[i][j][k] only for k=0,1,2,3

3-d access of a 1-d array – pointer array assignment

int	a	[2*3*4] ;
int *	b	[2*3] ;
int **	c	[2] ;

int a [2*3*4] ;
int * b [2*3] ;

b [j][k] ≡ a [j*4 +k]

j = [0:5] k = [0:3]

j*4+k = [0:23]

int * b [2*3] ;
int ** c [2] ;

c [i][j][k] ≡ a [(i*3+j)*4+k]

i = [0:1] j = [0:2] k = [0:3]

(i*3+j)*4+k = [0:23]

Assignments

b[j] = &a[j*4] (= a+j*4)
c[i] = &b[i*3] (= b+i*3)

Initialization of
pointer arrays **b** and **c**

3-d access of a 1-d array – pointer array assignment

int	a	[2*3*4] ;
int *	b	[2*3] ;
int **	c	[2] ;

a[k] ≡ *(a+k)
b[j][k] ≡ *(*(b+j)+k)
c[i][j][k] ≡ *(*(*(c+i)+j)+k)

constraint : contiguous a[i], b[i], c[i]

Assignments

c[i] = &b[i*3] (= b+i*3)
b[j] = &a[j*4] (= a+j*4)

Initialization of
pointer arrays b and c

3-d access of a **1-d** array

c[i][j][k] ≡
a[(i*3+j)*4 +k] ≡ a[i*3*4+j*4+k]

1-d access of a **1-d** array

*(c+i) = b+*g*(i)
*(b+j) = a+*f*(j)

3-d access of a 1-d array – pointer array assignment

int	a [2*3*4] ;
int *	b [2*3] ;
int **	c [2] ;

a[k] ≡ *(a+k)

contiguous over k = 0:23

b[j][k] ≡ *(*(b+j)+k)

➡ *(b[j]+k) = *(a+j*4+k) = a[j*4+k]

contiguous over j = 0:5 & k = 0:3

⇦ b[j] = &a[j*4] (= a+j*4)

partition 24 into 6 * 4

c[i][j][k] ≡ *(*(*(c+i)+j)+k)

➡ *(*(c[i]+j)+k) = *(*(b+i*3+j)+k)

➡ *(b[i*3+j]+k) = *(a+(i*3+j)*4+k)

➡ a[(i*3+j)*4+k]

contiguous over i =0:1 & j= 0:2 & k = 0:3

partition 6 into 2 * 3

⇦ c[i] = &b[i*3] (= b+i*3)

⇦ b[j] = &a[j*4] (= a+j*4)

partition 24 into 2 * 3 * 4

3-d access of a 1-d array – pointer array assignment

int	a [2*3*4] ;
int *	b [2*3] ;
int **	c [2] ;

b[j] = &a[j*4] (= a+j*4)

partition 24 into 6 * 4
partition size = 4

contiguous a over k = 0:3
contiguous b over j = 0:5

b [j][k] ≡ a [j*4 +k]

b[j] = &a[j*4] (= a+j*4)
c[i] = &b[i*3] (= b+i*3)

(1) partition 24 into 6 * 4
1st partition size = 4

(2) partition 6 into 2 * 3
2nd partition size = 3

contiguous a over k = 0:3
contiguous b over j = 0:2
contiguous c over i = 0:1

c [i][j][k] ≡ a [(i*3+j)*4+k]

3-d access of a 1-d array – pointer array assignment

int	a	[2*3*4] ;
int *	b	[2*3] ;
int **	c	[2] ;

(1) partition 24 into six 4's (6 * 4)
1st partition size = 4

(2) partition 6 into two 3's (2 * 3)
2nd partition size = 3

b[j] = &a[j*4] (= a+j*4)

b [j][k] ≡ a [j*4 +k]

contiguous a over k = 0:3 (=4-1)
contiguous b over j = 0:5 (=6-1)

b[j] = &a[j*4] (= a+j*4)
c[i] = &b[i*3] (= b+i*3)

c [i][j][k] ≡ a [(i*3+j)*4+k]

contiguous a over k = 0:3 (=4-1)

c [i][j][k] ≡ a [(i*3+j)*4+k]

contiguous b over j = 0:2 (=3-1)
contiguous c over i = 0:1 (=2-1)

3-d access of a **1-d** array – pointer array sizes

int **	**c** [2] ;
int *	**b** [2*3] ;

int	**a** [2*3*4] ;

sizeof(int **) = 4 or 8 bytes
sizeof(int *) = 4 or 8 bytes

on a 32-bit machine

on a 64-bit machine

sizeof(int) = 4 bytes

sizeof(**c**) = 2*sizeof(int **)
sizeof(**b**) = 2*3*sizeof(int *)

sizeof(**a**) = 2*3*4*sizeof(int)

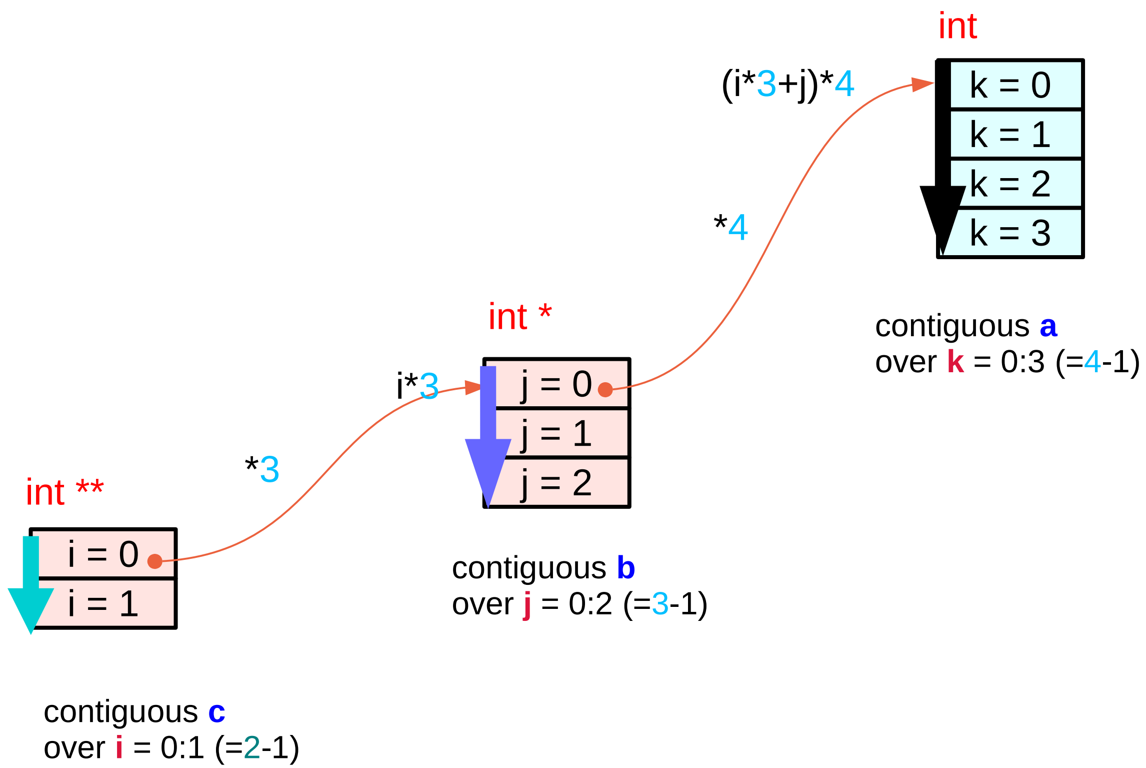
Using pointer arrays

int	a [2*3*4] ;
int *	b [2*3] ;
int **	c [2] ;

c [i][j][k]

conditions

b[j] = &a[j*4] (= a+j*4)
c[i] = &b[i*3] (= b+i*3)



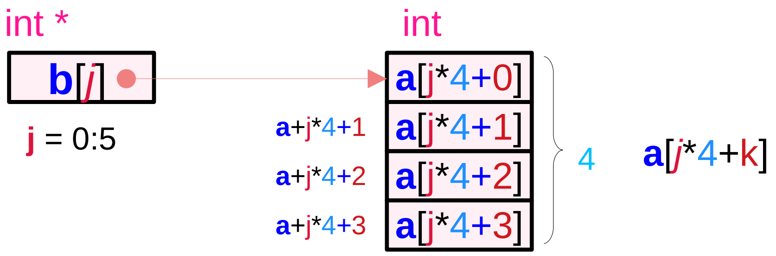
c [i][j][k] ≡ a [(i*3+j)*4+k]

Integer array a and pointer arrays b, c

int	a [2*3*4] ;
int *	b [2*3] ;
int **	c [2] ;

(1) partition 24 into six 4's (6 * 4)
1st partition size = 4

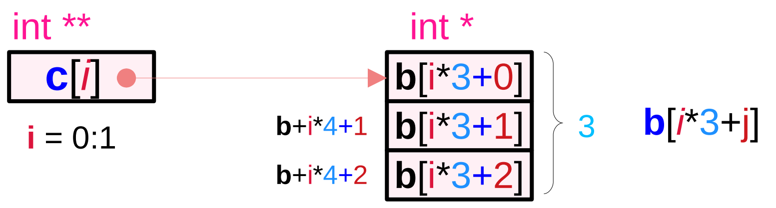
(2) partition 6 into two 3's (2 * 3)
2nd partition size = 3



contiguous a over k = 0:3 (=4-1)

j = 0:5

b[0] = &a[0*4]; (= a + 0*4)
b[1] = &a[1*4]; (= a + 1*4)
b[2] = &a[2*4]; (= a + 2*4)
b[3] = &a[3*4]; (= a + 3*4)
b[4] = &a[4*4]; (= a + 4*4)
b[5] = &a[5*4]; (= a + 5*4)



i = 0:1

c[0] = &b[0*3]; (= b + 0*3)
c[1] = &b[1*3]; (= b + 1*3)

contiguous b over j = 0:2 (=3-1)
contiguous c over i = 0:1 (=2-1)

c [i][j][k] ≡ a [(i*3+j)*4+k]

Static memory allocation of an **3-d** array

```
int  c [2][3][4] ;          int * p = (int *) c ;
```

```
int *              int *
&c[i][j][k] = (p+(i*3+j)*4+k)
```

```
int               int *
c[i][j][k] = *(c[i][j]+k)
&c[i][j][k] = (c[i][j]+k)

int *         int **
c[i][j] = *(c[i]+j)
&c[i][j] = (c[i]+j)

int **      int ***
c[i] = *(c+i)
&c[i] = (c+i)
```

```
int
c[i][j][0] = *c[i][j]
&c[i][j][0] = c[i][j]

int *
c[i][0] = *c[i]
&c[i][0] = c[i]

int **
c[0] = *c
&c[0] = c
```

```
int *
c[i][j]      = (p+(i*3+i)*4)

int **
c[i]         = (int **) (p+i*3)

c
```

```
c[i][j][k]  = *(c[i][j]+k)
            = *(*(c[i]+j)+k)
            = *(*(*(c+i)+j)+k)
```

Static memory allocation of an **3-d** array

int **c** [2][3][4] ;   int * **p** = (int *) **c** ;

&**c**[**i**][**j**][**k**] = (**p**+(**i***3+**j**)*4+**k**)   (int *, int *)

int	**c**[**i**][**j**][**k**]
int *	**c**[**i**][**j**] + k = c[i][j] + k * sizeof(*c[i][j])
int **	**c**[**i**] + j = c[i] + j * sizeof(*c[i])
int **	**c**

c[**i**][**j**][**k**] = *(**c**[**i**][**j**]+**k**)
= *(*(**c**[**i**]+**j**)+**k**)
= *(*(*(**c**+**i**)+**j**)+**k**)

Static memory allocation of an **3-d** array

int c [2][3][4] ;

value(c) = value(c[0]) = value(c[0][0]) = &c[0][0][0]
value(c[0][1]) = &c[0][1][0]
value(c[0][2]) = &c[0][1][0]
value(c[1]) = value(c[1][0]) = &c[1][0][0]
value(c[1][1]) = &c[1][1][0]
value(c[1][2]) = &c[1][1][0]

c[i][j][0] = *(c+i*3+j*4)
&c[i][j][0] = (c+i*3+j*4)

c[i][j] → c[i][j][0]   if c[i][j] = &c[i][j][0]   c[i][j] = (c+i*3+j*4)

c[i] → c[i][0]   if c[i] = &c[i][0]
= &c[i][0][0]   c[i] = (c+i*3)

c → c[0]   if c = &c[0]
= &c[i][0]
= &c[i][0][0]   c

Static memory allocation of an **3-d** array

int **c** [2][3][4] ;

value(**c**) = value(**c**[0]) = value(**c**[0][0]) = &**c**[0][0][0]
value(**c**[0][1]) = &**c**[0][1][0]
value(**c**[0][2]) = &**c**[0][1][0]
value(**c**[1]) = value(**c**[1][0]) = &**c**[1][0][0]
value(**c**[1][1]) = &**c**[1][1][0]
value(**c**[1][2]) = &**c**[1][1][0]

c[**i**][**j**][0] = *(**c**+**i***3+**j***4)
&**c**[**i**][**j**][0] = (**c**+**i***3+**j***4)

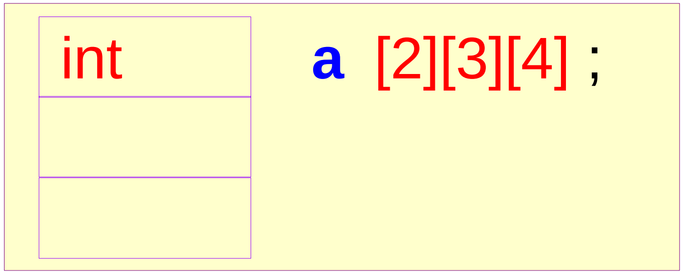
c[**i**][**j**] = (**c**+**i***3+**j***4) if **c**[**i**][**j**] = &**c**[**i**][**j**][0]

c[**i**] = (**c**+**i***3) if **c**[**i**] = &**c**[**i**][0][0]

c[**i**][0] = (**c**+**i***3) if **c**[**i**][0] = &**c**[**i**][0][0]
&**c**[**i**][0] = (**c**+**i***3) if

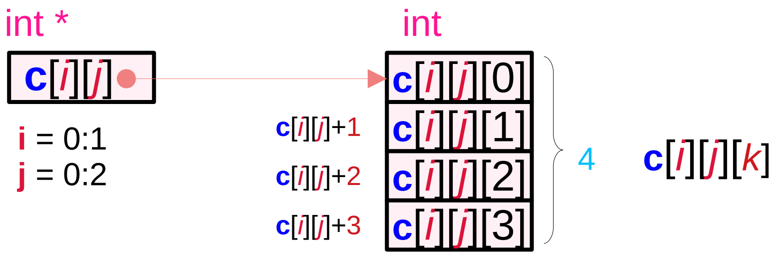
c[**i**] = (**c**+**i***3)

Integer array **a** and pointer arrays **b**, **c**



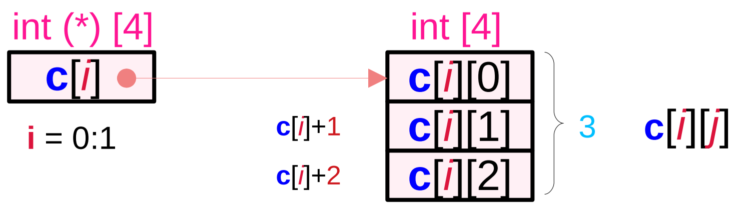
(1) partition 24 into six 4's (6 * 4)
1st partition size = 4

(2) partition 6 into two 3's (2 * 3)
2nd partition size = 3



value(**c**) = value(**c**[0]) = value(**c**[0][0]) = &**c**[0][0][0]
value(**c**[0][1]) = &**c**[0][1][0]
value(**c**[0][2]) = &**c**[0][1][0]
value(**c**[1]) = value(**c**[1][0]) = &**c**[1][0][0]
value(**c**[1][1]) = &**c**[1][1][0]
value(**c**[1][2]) = &**c**[1][1][0]

contiguous **c**[i][j] over **k** = 0:3 (=4-1)



contiguous **c**[i] over **j** = 0:2 (=3-1)

contiguous **c** over **i** = 0:1 (=2-1)

Leading elements : **c[i][0][0]**, **c[i][j][0]**

```
int     a [L*M*N];
int*    b [L*M];
int**   c [L];
```

c [i][j][k]

i = 0, 1
j = 0, 1, 2
k = 0, 1, 2, 3

L=2	i=0 i=1	i*3*4 = 0 i*3*4 = 12
M=3	j=0 j=1 j=2	j*4 = 0 j*4 = 4 j*4 = 8
N=4	k=0 k=1 k=2 k=3	k*1= 0 k*1= 1 k*1= 2 k*1= 3

c[0][0][0] = a[0]   0
c[1][0][0] = a[12]   12

c[0][0][0] = a[0]   0+0
c[0][1][0] = a[4]   0+4
c[0][2][0] = a[8]   0+8
c[1][0][0] = a[12]   12+0
c[1][1][0] = a[16]   12+4
c[1][2][0] = a[20]   12+8

c[0][0][0]	a[0]
c[0][0][1]	a[1]
c[0][0][2]	a[2]
c[0][0][3]	a[3]
c[0][1][0]	a[4]
c[0][1][1]	a[5]
c[0][1][2]	a[6]
c[0][1][3]	a[7]
c[0][2][0]	a[8]
c[0][2][1]	a[9]
c[0][2][2]	a[10]
c[0][2][3]	a[11]
c[1][0][0]	a[12]
c[1][0][1]	a[13]
c[1][0][2]	a[14]
c[1][0][3]	a[15]
c[1][1][0]	a[16]
c[1][1][1]	a[17]
c[1][1][2]	a[18]
c[1][1][3]	a[19]
c[1][2][0]	a[20]
c[1][2][1]	a[21]
c[1][2][2]	a[22]
c[1][2][3]	a[23]

Initialization of pointer arrays – a general case

```
int      a [L*M*N];
```

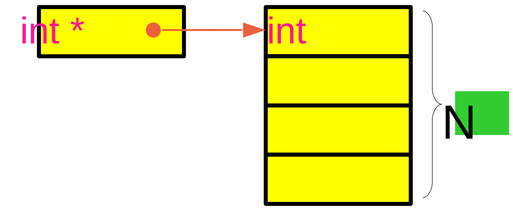
```
int*     b [L*M];
int**    c [L];
```

pointer arrays b, c

```
int      c [L][M][N];
```

```
int *       b[L*M];
int         a[L*M*N];
```

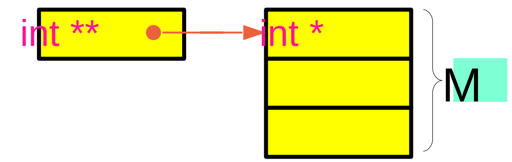
```
b[j] = &a[j*N];
j=0,…, L*M-1
```



b[j] get the address of the every N^th element of a

```
int **      c[L];
int *       b[L*M];
```

```
c[i] = &b[i*M];
i=0, …. L-1
```



c[i] get the address of the every M^th element of b

3-d and 1-d accesses (recursive pointers vs. brackets)

conditions

```
c[i] = &b[i*M];
b[j] = &a[j*N];
```

c[i][j][k] ≡ a[i*M*N + j*N + k]
≡ a[(i*M + j)*N + k]

```
int **   c[L];
int *    b[L*M];

for (i=0; i<L; ++i)
    c[i] = &b[i*M];
```

```
int *    b[L*M];
int      a[L*M*N];

for (j=0; j<L*M; ++j)
    b[j] = &a[j*N];
```

c[i][j][k]
= *(*(*(c+i)+j)+k)
= *(*(c[i]+j)+k) ← c[i] =&b[i*M]
= *(*(&b[i*M]+j)+k) → *(*(b+i*M+j)+k)
= *(b[i*M+j]+k) ← b[m] = &a[m*N]
= *(&a[(i*M+j)*N]+k) → *(a+(i*M+j)*N+k)
= a[(i*M+j)*N+k]

Pointer Arrays for recursive indirections

1-d array of (int **) pointers	int** c [2];
1-d array of (int *) pointers	int* b [2*3];
1-d array of (int)	int a [2*3*4];

→ 3-d access c **[i][j][k]**

Recursive indirections in a 3-d array

```
int     c [L][M][N];

        c [i][j][k]
```

left-to-right associativity

(c) [i][j][k]
(c [i]) [j][k]
((c [i]) [j]) [k]
(((c [i]) [j]) [k])

equivalence relations				multiple indirections
c [i]	≡	*(c+i)	≡	*(c+i)
c [i][j]	≡	*(c[i]+j)	≡	*(*(c+i)+j)
c [i][j][k]	≡	*(c[i][j]+k)	≡	*(*(*(c+i)+j)+k)

&c[i][j][k]	= c[i][j]+k
&c[i][j]	= c[i]+j
&c[i]	= c+i

&c[i][j][0]	= c[i][j]
&c[i][0]	= c[i]
&c[0]	= c

3-d access pattern **c[i][j][k]**

General requirements

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

Pointer array approach

```
int** c[2];
int*  b[2*3];
int   c[2*3*4];
```

```
c[i][j][k]   :: int
c[i][j]      :: int *
c[i]         :: int **
```

```
c[i]  ⇐  &b[i*3]
b[j]  ⇐  &a[j*4]
```

Hierarchical Pointer Array Constraints

Abstract Data Type

Array pointer approach

```
int c[2][3][4];
```

```
c[i][j][k]   :: int
c[i][j]      :: int [4]
c[i]         :: int (*) [4]
```

```
c       = &c[0][0][0]
c[i]    = &c[i][0][0]
c[i][j] = &c[i][j][0]
```

Virtual Array Pointer Constraints

Abstract Data Type

3-d access pattern **c[i][j][k]** – pointer array approach

General requirements

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

Pointer array approach

```
int** c[2];
int*  b[2*3];
int   c[2*3*4];
```

```
c[i][j][k]   :: int
c[i][j]      :: int *
c[i]         :: int **
```

```
c[i]  ⇐  &b[i*3]
b[j]  ⇐  &a[j*4]
```

Types and values of **c[i]** and **c[i][j]** for **int c[2][3][4];**

c [i][j][k];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

int c [2][3][4];

c[i] virtual array pointer of the type int (*) [4] … a narrow sense
can also be viewed as the int** type … a wide sense

&c[0][0][0] = c[0][0]
&c[1][0][0] = c[1][0]

int*

&c[0][0] = c[0]
&c[1][0] = c[1]

int**

c[i][j] virtual int pointer of the type int (*) … a narrow sense
can also be viewed as the int* type … a wide sense

&c[0][0][0] = c[0][0]
&c[0][1][0] = c[0][1]
&c[0][2][0] = c[0][2]
&c[1][0][0] = c[1][0]
&c[1][1][0] = c[1][1]
&c[1][2][0] = c[1][2]

int*

Using **int**** and **int*** pointer arrays for **3-d** accesses

General Requirements

```
int c[2][3][4];
&c[i][0]    = c[i]
```

```
&c[0][0]    = c[0]
&c[1][0]    = c[1]
```

int**

```
int c[2][3][4];
&c[i][j][0]    = c[i][j]
```

```
&c[0][0][0] = c[0][0]
&c[0][1][0] = c[0][1]
&c[0][2][0] = c[0][2]
&c[1][0][0] = c[1][0]
&c[1][1][0] = c[1][1]
&c[1][2][0] = c[1][2]
```

int*

Pointer Array Implementation

```
int** c[2];
c[i]   = &b[i*3]
```

```
c[0] = &b[0*3]
c[1] = &b[1*3]
```

int**

```
int* b[2*3];
b[j]   = &a[j*4]
```

```
b[0] = &a[0*4]
b[1] = &a[1*4]
b[2] = &a[2*4]
b[3] = &a[3*4]
b[4] = &a[4*4]
b[5] = &a[5*4]
```

int*

instead of using **int c[2][3][4]**,
use these 1-d arrays of pointers
int c[2]** and **int* b[2*3]**
with proper initializations:
c[i] = &b[i*3] and **b[j] = &a[j*4]**

then **c[i][j][k]** can be used
to access the 1-d array
int a[2*3*4]

Assignments and their Equivalent Relations

assignments

c[i] ⟸ &b[i*3]
b[j] ⟸ &a[j*4]

[2][3][4]

equivalences

c[i] ≡ (b+i*3);
b[j] ≡ (a+j*4);

extending a dimension
extending a dimension

*(c[i]+j) = *(b+i*3+j);
*(b[j]+k) = *(a+j*4+k);

c[i][j] ≡ b[i*3+j]
b[j][k] ≡ a[j*4+k]

extending a dimension
substitute j ← (i*3+j)

*(c[i][j]+k) = *(b[i*3+j]+k)
*(b[i*3+j]+k) = *(a+(i*3+j)*4+k)

c[i][j][k] ≡ b[i*3+j][k]
≡ a[(i*3+j)*4+k]

The leading elements of pointer arrays

c[i] ⇐ &b[i*3]	
b[j] ⇐ &a[j*4]	

assignments

```
c[i]    ≡ (b+i*3);
b[j]    ≡ (a+j*4);
```

equivalence

```
c[i][j]   ≡ b[i*3+j]
b[j][k]  ≡ a[j*4+k]
```

equivalence

```
c[i][0] ≡ b[i*3];
b[j][0] ≡ a[j*4];
```

The 1st elements of **c[i][j]**, **b[i][j]**

```
c[i][j][k] ≡ b[i*3+j][k]
 ≡ a[(i*3+j)*4+k]
```

equivalence

```
c[i][j][0]   ≡ b[i*3+j];
c[i][0][0]  ≡ a[(i*3)*4];
```

The 1st elements of **c[i][j][k]**

c[i], **c[i][j]**, **c[i][j][k]** in terms of array **a** and **b**

c[i] ⟸ &b[i*3]
b[j] ⟸ &a[j*4]

assignments

c[i] ≡ (b+i*3);
b[j] ≡ (a+j*4);

equivalence

c[i] = &b[i*3]
= ~~&&a[(i*3)*4]~~

&& is not allowed

c[i][j] ≡ b[i*3+j]
b[j][k] ≡ a[j*4+k]

equivalence

c[i][j] ≡ b[i*3+j]
≡ &a[(i*3+j)*4]

c[i][j][k] ≡ b[i*3+j][k]
≡ a[(i*3+j)*4+k]

equivalence

c[i][j][k] ≡ b[i*3+j][k]
≡ a[(i*3+j)*4+k]

Pointer Arrays – **c[i]** reaches **c[i][0][0]** via **c[i][0]**

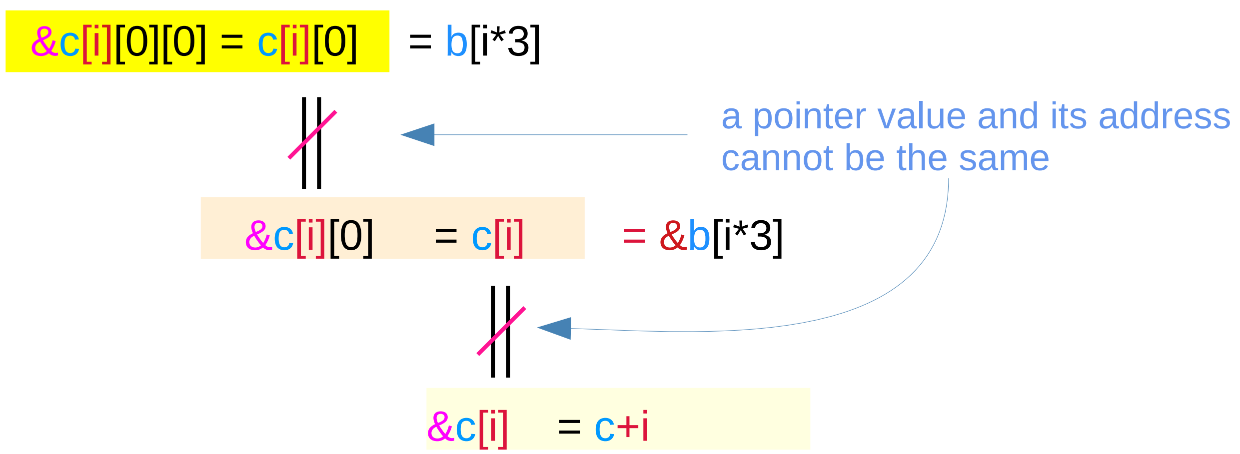
c [i][j][k];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

```
int**   c[2];
int*    b[2*3];
int     a[2*3*4];
```

c[i] ⟸ &b[i*3]
b[j] ⟸ &a[j*4]



Pointer Arrays – **c[i][j]** reaches **c[i][j][0]**

c [i][j][k];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

&c[i][j][k] = c[i][j]+k
&c[i][j] = c[i]+j
&c[i] = c+i

```
int**    c[2];
int*     b[2*3];
int      a[2*3*4];
```

c[i] ⬅ &b[i*3]
b[j] ⬅ &a[j*4]

&c[i][j][0] = c[i][j] = b[i*3+j] = &a[(i*3+j)*4]

Recursive Indirections – thinking pointer substitutions

c[i][j][k]	≡	*(c[i][j] +k)
*(c[i][j] +k)	≡	*(*(c[i] +j) +k)
((c[i] +j) +k)	≡	*(*(*(c +i) +j) +k)

X = c[i][j]	int *
Y = c[i]	int **
Z = c	int ***

for a given **i**, **j**, **k**

X[k]	≡	*(X+k)
Y[j][k]	≡	*(*(Y+j)+k)
Z[i][j][k]	≡	*(*(*(Z+i)+j)+k)

Recursive Indirections – general cases of i, j, k

c[i][j][k]	≡	*(c[i][j] +k)
*(c[i][j] +k)	≡	*(*(c[i] +j) +k)
((c[i] +j) +k)	≡	*(*(*(c +i) +j) +k)

$X_{i,j}$ = c[i][j]		int *
Y_i = c[i]		int **
Z = c = Y		int ***

for general cases of indices i, j, k, **X** and **Y** need to be arrays of pointers

$X_{i,j}$[k]	≡	*($X_{i,j}$+k)
Y_i[j][k]	≡	*(*(Y_i+j)+k)
Z[i][j][k]	≡	*(*(*(Z+i)+j)+k)

Recursive Indirections – Pointer array initialization

c[i][j][k] ≡ *(c[i][j] +k)
*(c[i][j] +k) ≡ *(*(c[i] +j) +k)
((c[i] +j) +k) ≡ *(*(*(c +i) +j) +k)

```
int     c [L][M][N]  ;
```

$X_{i,j}$ = c[i][j]   int *
Y_i = c[i]   int **
Z = c = Y   int ***

X[i*M+j] = c[i][j];
Y[i] = c[i];

$X_{i,j}$[k] ≡ *($X_{i,j}$+k)
Y_i[j][k] ≡ *(*(Y_i+j)+k)
Y[i][j][k] ≡ *(*(*(Y+i)+j)+k)

```
int     W  [L*M*N]  ;
int *   X  [L*M]    ;
int **  Y  [L]      ;
```

Recursive Indirections – Substitution Analysis

X[i*M+j]= c[i][j];

Y[i] = c[i];

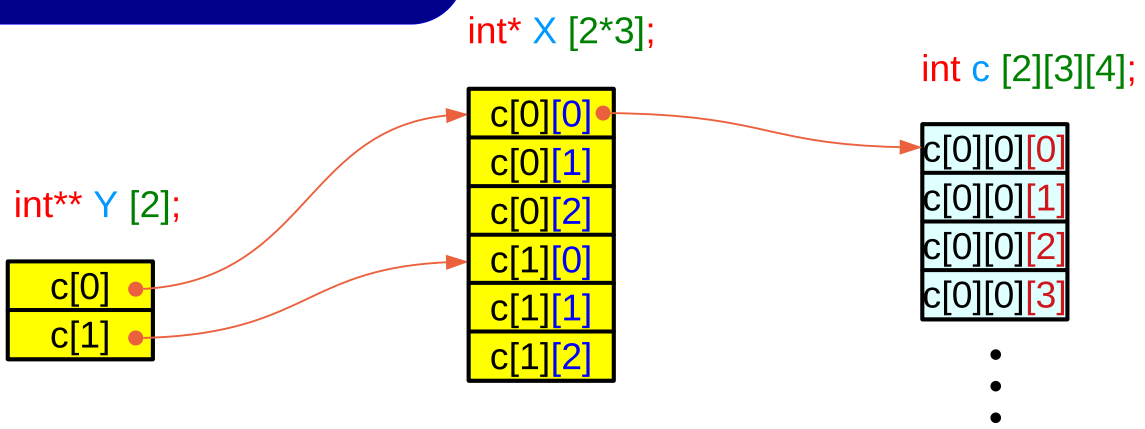
Y[i][j] = *(Y[j]+j)
= *(c[i]+j)
= c[i][j]

Y[i][j][k] = *(Y[i][j]+k)
= *(c[i][j]+k)
= c[i][j][k]

&Y[i][j][0] = &c[i][j][0] = c[i][j] = Y[i][j]

&Y[i][0] = &c[i][0] = c[i] = Y[i]

&Y[i] = Y+i



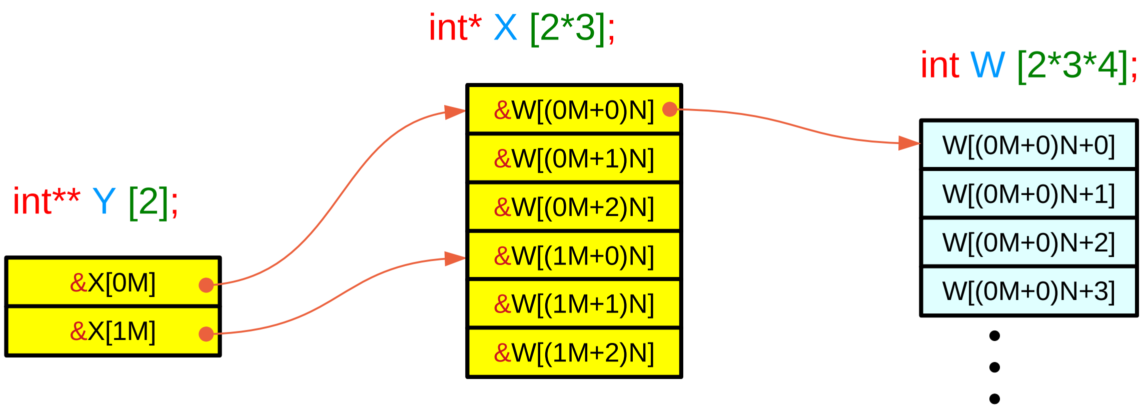
Recursive Indirections – one continuous int array W

X[i*M+j] = &W[(i*M+j)*N];
Y[i] = &X[i*M];

Y[i][j] = *(Y[i]+j)
= *(X+i*M+j)
= X[i*M+j]

Y[i][j][k] = *(Y[i][j]+k)
= *(W+(i*M+j)*N+k)
= W[(i*M+j)*N+k]

&Y[i][j][0]	=	&W[(i*M+j)*N+0]	= Y[i][j]
	=	X[(i*M+j)]	
&Y[i][0]	=	&X[i*M+0]	= Y[i]
&Y[i]			= Y+i



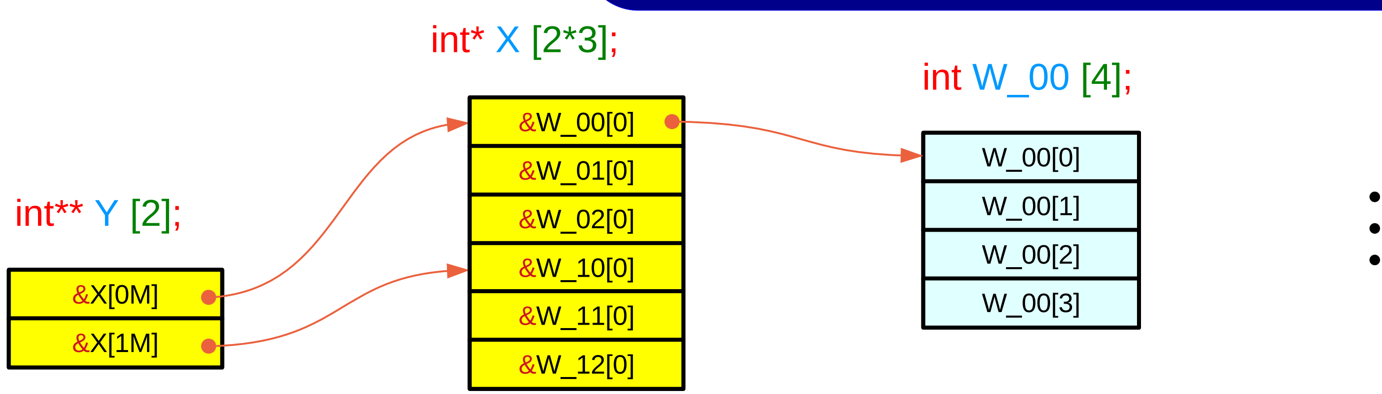
Recursive Indirections – non-contiguous 1-d arrays W_ij

&Y[i][j][0]	=	&W_ij	= Y[i][j]
	=	X[(i*M+j)]	
&Y[i][0]	=	&X[i*M+0]	= Y[i]
&Y[i]			= Y+i

```
X[i*M+j]     = &W_ij[0];
Y[i]         = &X[i*M];
```

```
Y[i][j]      = *(Y[i]+j)
             = *(X+i*M+j)
             = X[i*M+j]
```

```
Y[i][j][k]   = *(Y[i][j]+k)
             = *(W+(i*M+j)*N+k)
             = W[(i*M+j)*N+k]
```



Recursive Indirections – contiguous v.s. non-contiguous

```
int     W  [L*M*N]  ;
int *   X  [L*M]    ;
int **  Y  [L]      ;
```

```
int     W_00  [N]    ;
int     W_01  [N]    ;
        :
int *   X     [L*M]  ;
int **  Y     [L]    ;
```

W[(i*M+j)*N+k];

one contiguous 1-d array
with the size of L*M*N

W_ij[k];

L*M non-contiguous 1-d arrays
with the size of N

Contiguity Constraints

c [i][j][k];

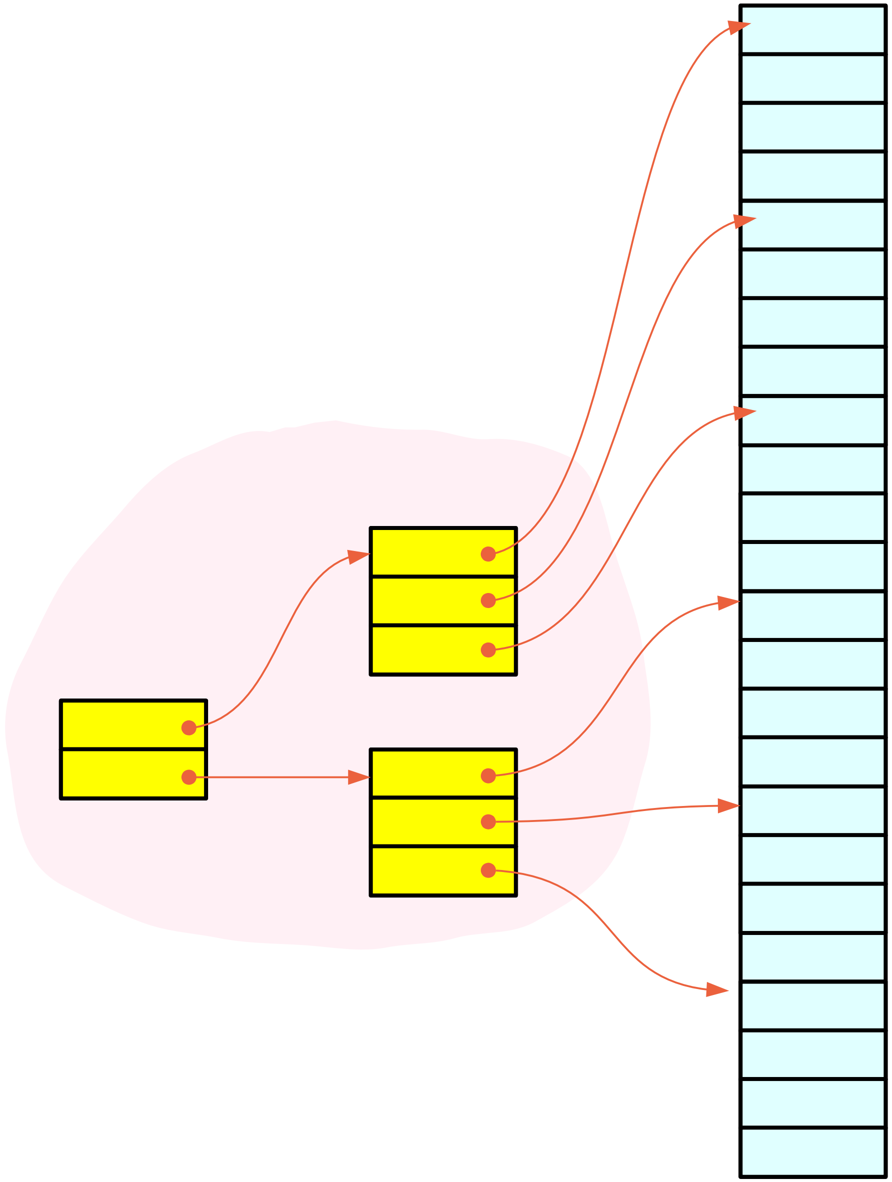
Pointer Arrays and Contiguity

Using pointer arrays

int * [N], int ** [M], int *** [L], …

Pointer array approach for 3-d access patterns

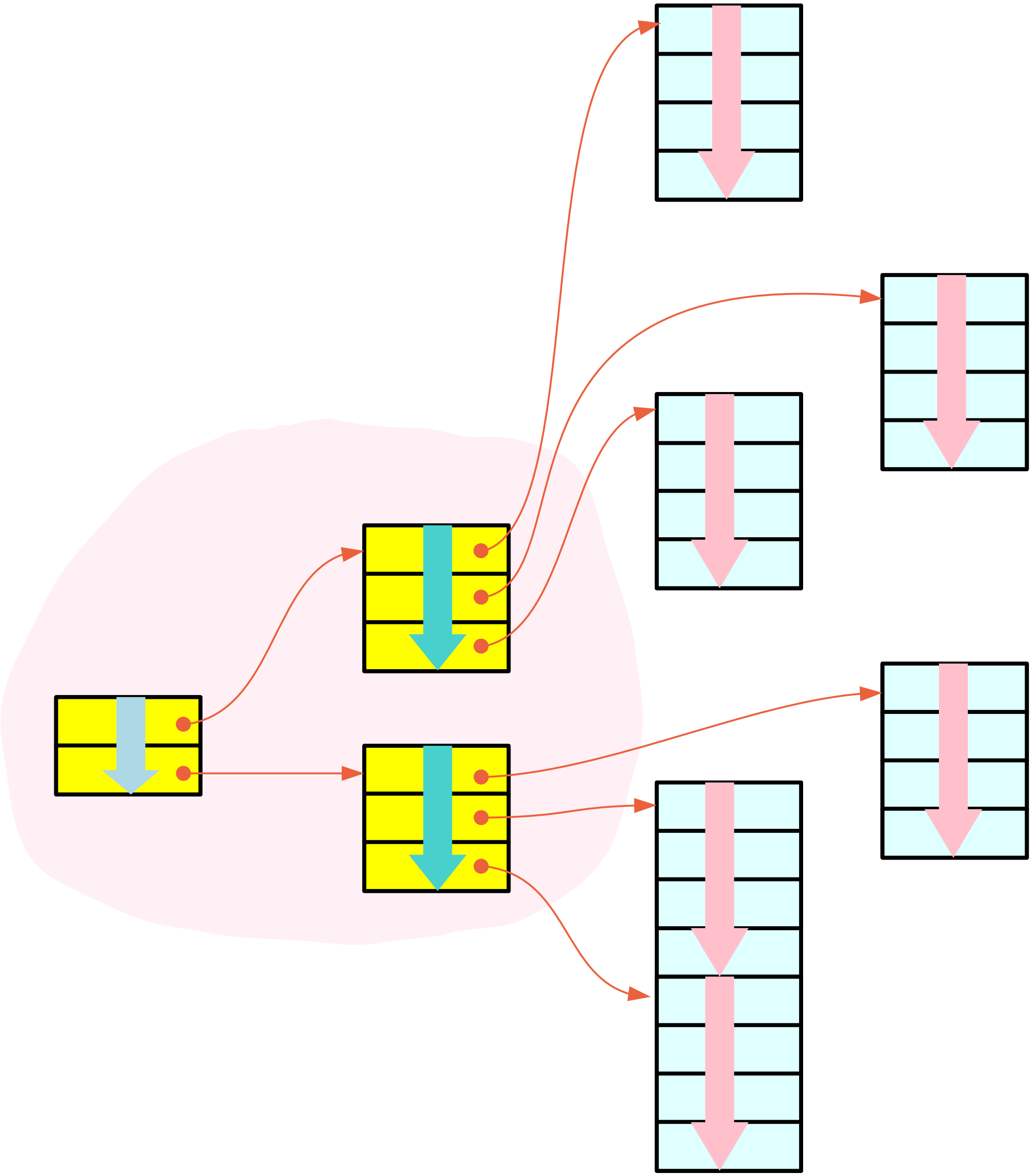
A programmer manually allocates
memory locations for pointer arrays



Pointer Array Approach
(array of pointers)

Pointer array approach – contiguity constraints

contiguity constraints
can be relaxed



Pointer Array Approach
(array of pointers)

Three contiguity constraints

c[i][j][k]	➡	*(c[i][j] +k)
*(c[i][j] +k)	➡	*(*(c[i] +j) +k)
((c[i] +j) +k)	➡	*(*(*(c +i) +j) +k)

c[i][j][k] ⬌ *(*(*(c+i)+j)+k)

sizeof(c[i][j][k]) = 4
sizeof(c[i][j]) = 4*4
sizeof(c[i]) = 3*4*4

c[2][3][4]

c[i][j][k]	*(c[i][j] +k)
c[i][j]	*(c[i] +j)
c[i]	*(c +i)

c[i][j][k] *(*(*(c+i)+j)+k)

sizeof(*c[i][j]) = 4
sizeof(*c[i]]) = 4*4
sizeof(*c]) = 3*4*4

c[2][3][4]

sizeof(c[i][j][0]) = 4
sizeof(c[i][0]) = 4*4
sizeof(c[0]) = 3*4*4

c[2][3][4]

Three contiguity constraints

Pointer Array Approach (array of pointers)

c[i][j][k]	➡	*(c[i][j] +k)
*(c[i][j] +k)	➡	*(*(c[i] +j) +k)
((c[i] +j) +k)	➡	*(*(*(c +i) +j) +k)

contiguous **1-d** array elements	int
contiguous **int** pointers	int *
contiguous **int** double pointers	int **

The contiguity constraints are satisfied by the allocated arrays of pointers

Array Pointer Approach (pointer to arrays)

c[i][j][k]	➡	*(c[i][j] +k)
*(c[i][j] +k)	➡	*(*(c[i] +j) +k)
((c[i] +j) +k)	➡	*(*(*(c +i) +j) +k)

contiguous **1-d** array elements	int
contiguous **1-d** arrays	int [4]
contiguous **1-d** array pointers	int (*) [4]

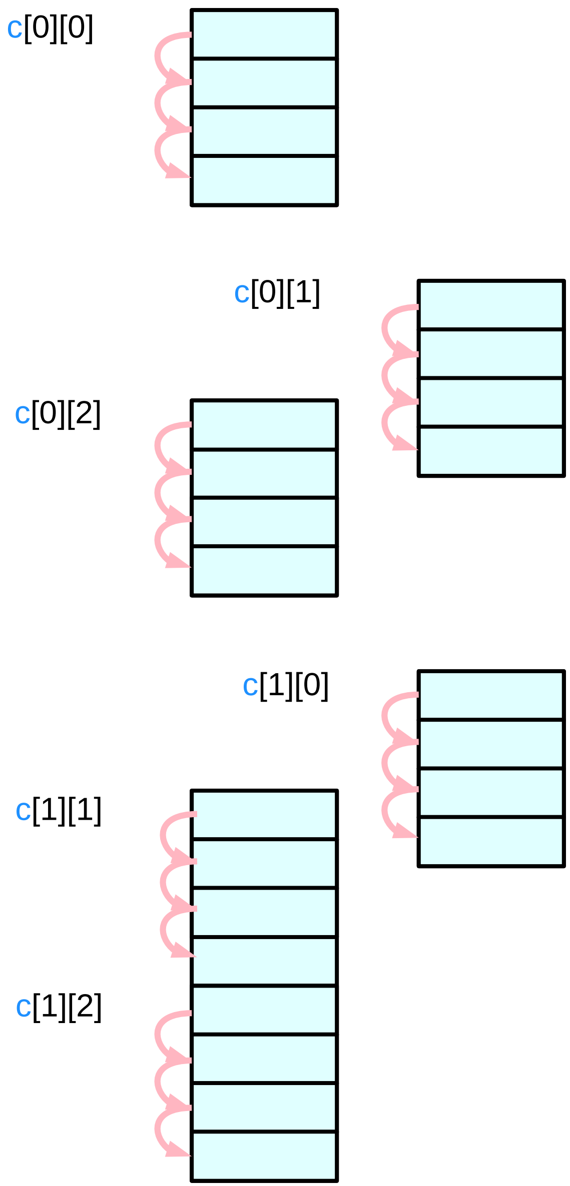
The contiguity constraints are satisfied by row major ordered linear data layout

c[i][j][k] ≡ *(c[i][j] +k)

c[0][0][0] =	*(c[0][0] + 0)
c[0][0][1] =	*(c[0][0] + 1)
c[0][0][2] =	*(c[0][0] + 2)
c[0][0][3] =	*(c[0][0] + 3)
c[0][1][0] =	*(c[0][1] + 0)
c[0][1][1] =	*(c[0][1] + 1)
c[0][1][2] =	*(c[0][1] + 2)
c[0][1][3] =	*(c[0][1] + 3)

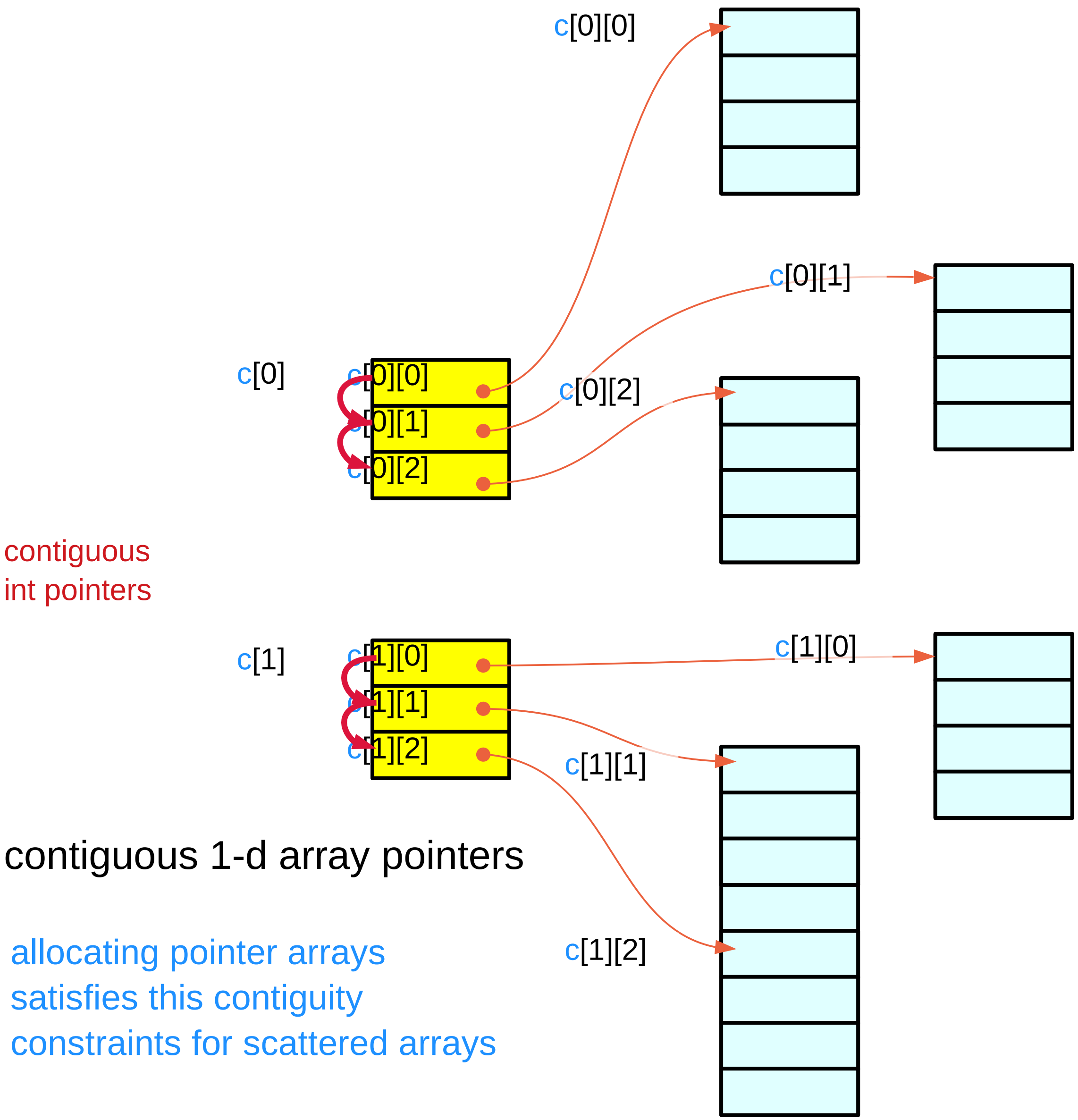
⋮ ⋮

contiguous 1-d
array elements



c[i][j] ≡ *(c[i] +j)

c[0][0] = *(c[0] + 0)
c[0][1] = *(c[0] + 1)
c[0][2] = *(c[0] + 2)
c[1][0] = *(c[1] + 0)
c[1][1] = *(c[2] + 1)
c[1][2] = *(c[3] + 2)



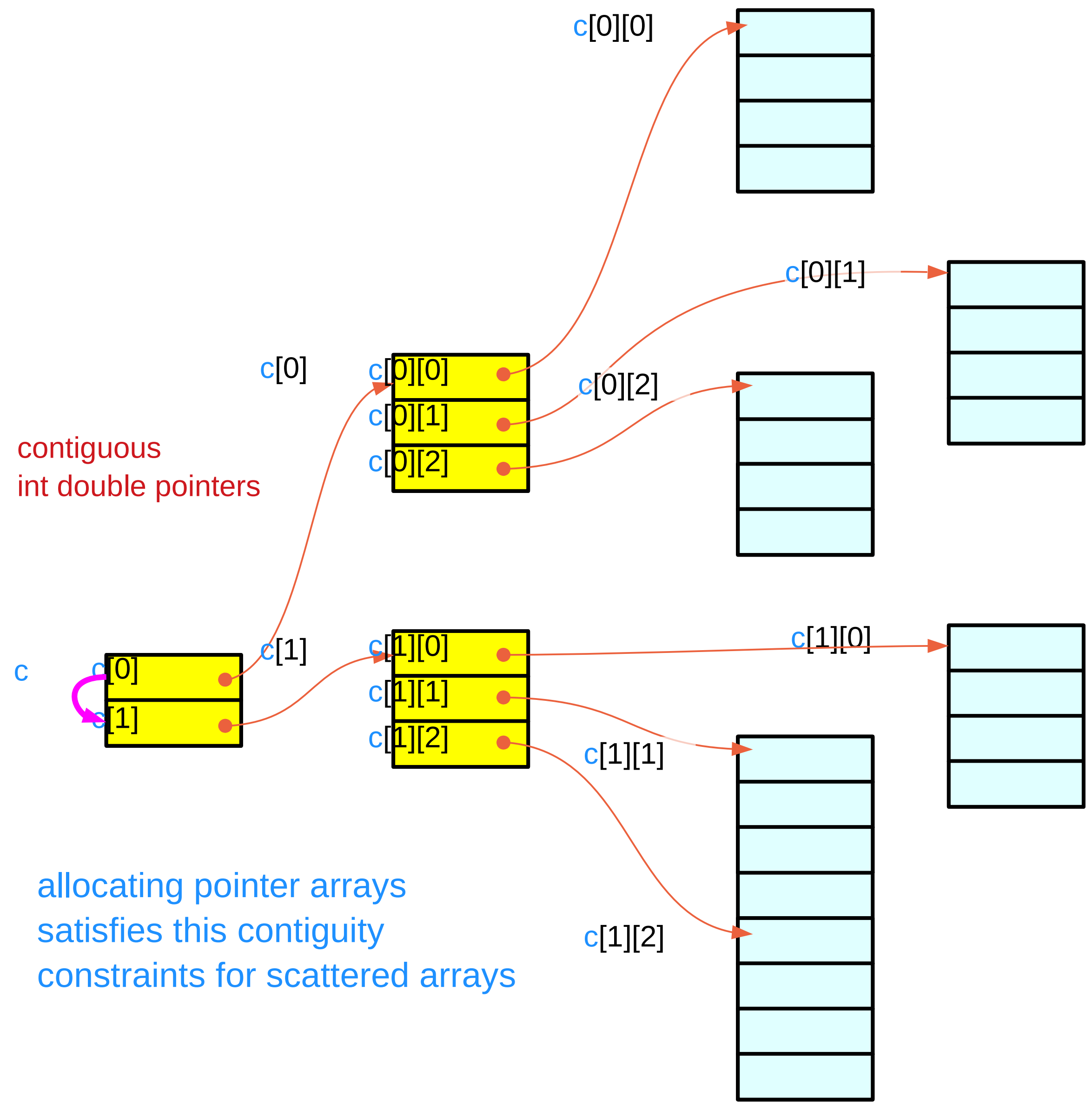
contiguous 1-d array pointers

allocating pointer arrays
satisfies this contiguity
constraints for scattered arrays

c[i] ≡ *(c +i)

c[0] =	*(c + 0)
c[1] =	*(c + 1)

contiguous 1-d array pointers



const pointers

const type, const pointer type (1)

`const int * p;`	⟺	`int * p;`
`int * const q ;`	⟺	`int * q ;`
`const int * const r ;`	⟺	`int * r ;`

constant

must not be changed
must not be updated
must not be written
must not be assigned

const type, const pointer type (2)

```
const int * p ;
```
constant integer

*p : constant integer **value**

```
int * const q ;
```
constant pointer

q : constant (int *) **pointer**

```
const int * const r ;
```
constant integer

*r : constant integer **value**

```
const int * const r ;
```
constant pointer

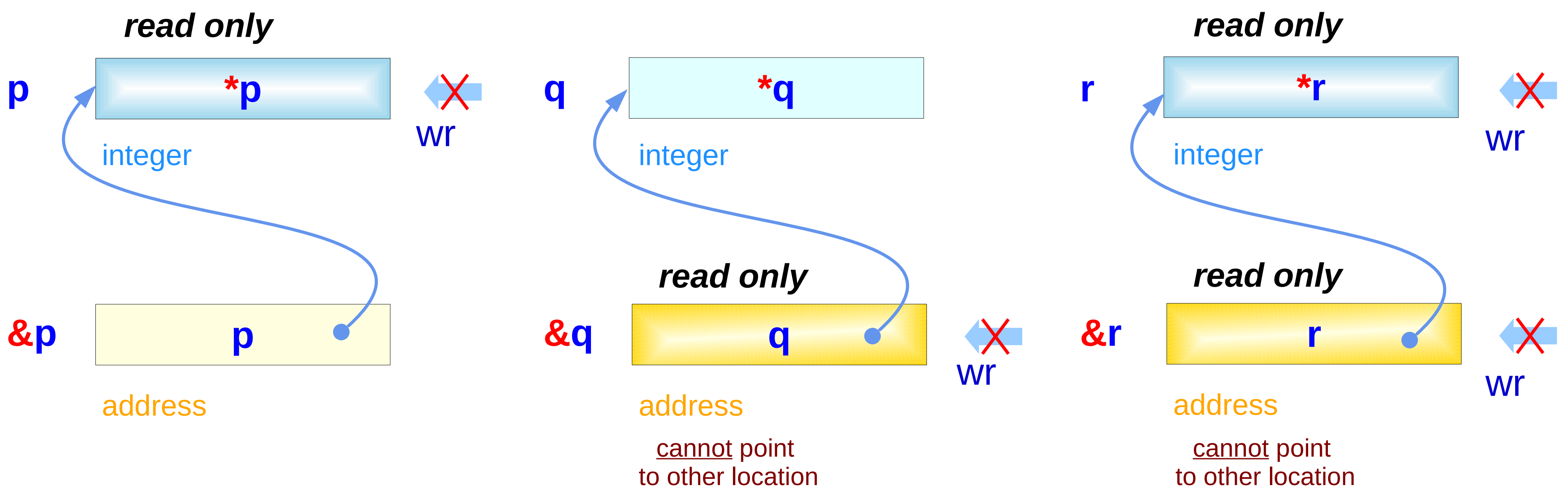
r : constant (int *) **pointer**

const [ ]

group with the following

const type, const pointer type (3)

```
const int *p;          int * const q ;          const int * const r ;
```



const examples (1)

```
const int * pc;
      int * p, i;
const int   ic;
```

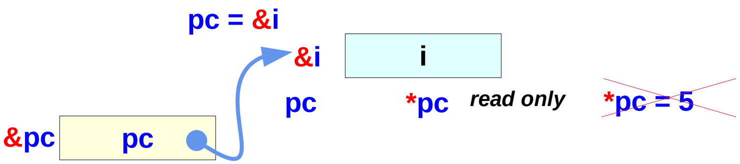
```
 pc = &i;     // (const int *) ← (int *)
*pc  = 5;     // (const int) error
```

Writing to the writable memory location (**i**)
is forbidden via **pc** … (no harm, OK)

```
 p   = &ic;   // (int *) ← (const int *) warning
*p   = 5;     // (int)
```

Writing to the read only memory location (**ic**)
is not forbidden via **p** … (hazardous, not OK)

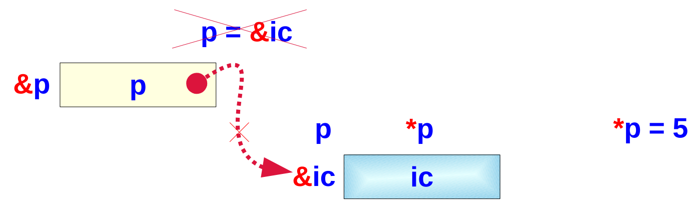
C A Reference Manual, Harbison & Steele Jr.



pc can point to **i**
***pc** must be **const**

the same memory location
that can be written via **i**
cannot be written via ***pc**

***pc** should not write
the writable memory location



Assume **p** points to **const ic**

the same memory location
that cannot be written via **ic,**
can be written via ***p**

thus ***p** can write
the **const** memory location

therefore, **p** should not point to **const ic**

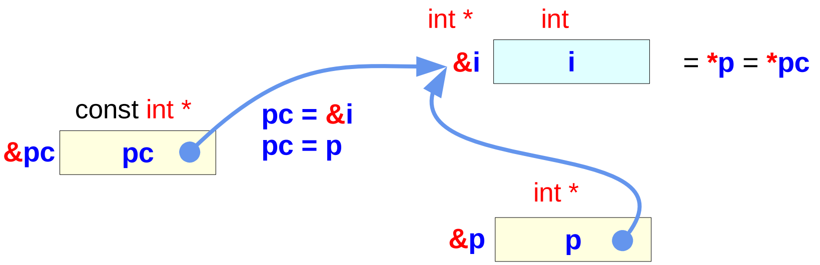
const examples (2)

```
const int * pc;
      int * p, i = 5;
const int   ic = 7;
```

```
p   = &i;
pc  = &ic
```

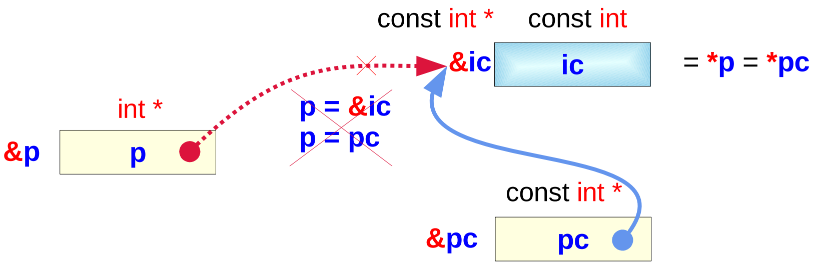
// more constrained type ← general type (O)

```
pc  = &i;   // (const int * ←  int *)
pc  =  p;   // (const int * ←  int *)
```



// general type ← more constrained type (X)

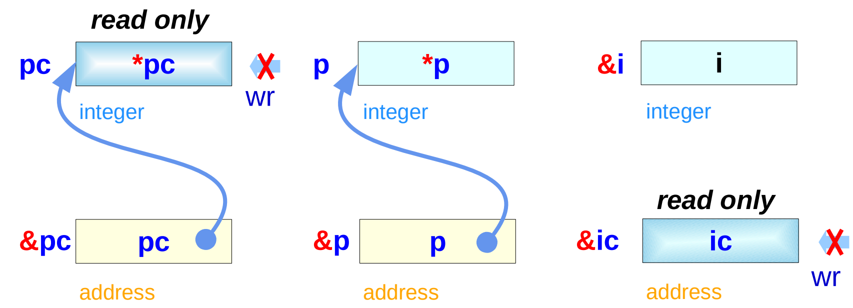
```
p   = &ic;  // (int * ← const int *)  warning
p   =  pc;  // (int * ← const int *)  warning
```



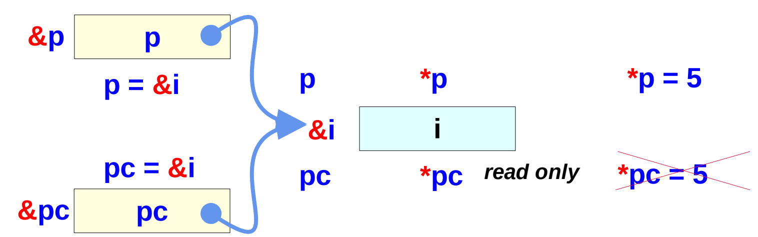
C A Reference Manual, Harbison & Steele Jr.

const examples (3)

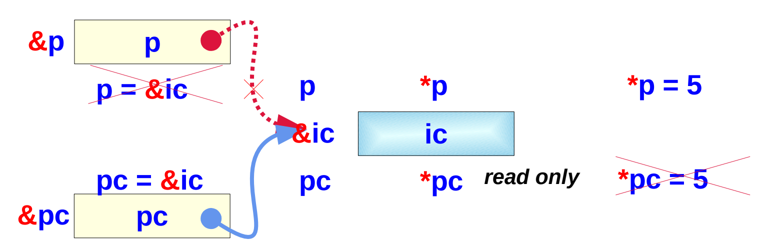
```
const int * pc;
      int * p, i;
const int   ic;
```




```
 p   = &i;     // (int *) ← (int *)
*p   = 5;      // (int)
 pc  = &i;     // (const int *) ← (int *)
*pc  = 5;      // (const int) error
```



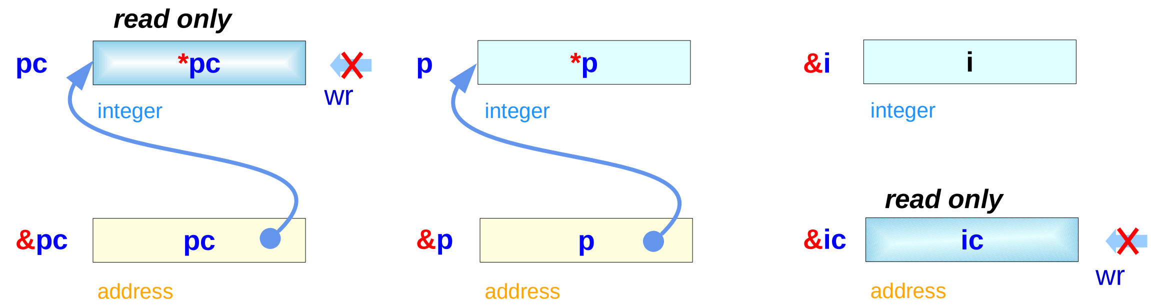

```
 p   = &ic;    // (int *) ← (const int *) warning
*p   = 5;      // (int)
 pc  = &ic;    // (const int *) ← (const int *)
*pc  = 5;      // (const int) error
```



C A Reference Manual, Harbison & Steele Jr.

const examples (4)

```
const int * pc;
      int * p, i;
const int   ic;
```




```
 pc  =   p = &i;
 pc  = &ic
*p   =   5;
*pc  =   5;          // invalid      *pc :: cons int

pc   = &i;           //              (const int * ←  int *)
pc   =   p;          //              (const int * ←  int *)
p    = &ic;          // invalid      (int * ← const int *)
p    =   pc;         // invalid      (int * ← const int *)
p    = (int *) &ic;  // type cast
p    = (int *)  pc;  // type cast
```

C A Reference Manual, Harbison & Steele Jr.

References

[1] Essential C, Nick Parlante
[2] Efficient C Programming, Mark A. Weiss
[3] C A Reference Manual, Samuel P. Harbison & Guy L. Steele Jr.
[4] C Language Express, I. K. Chun