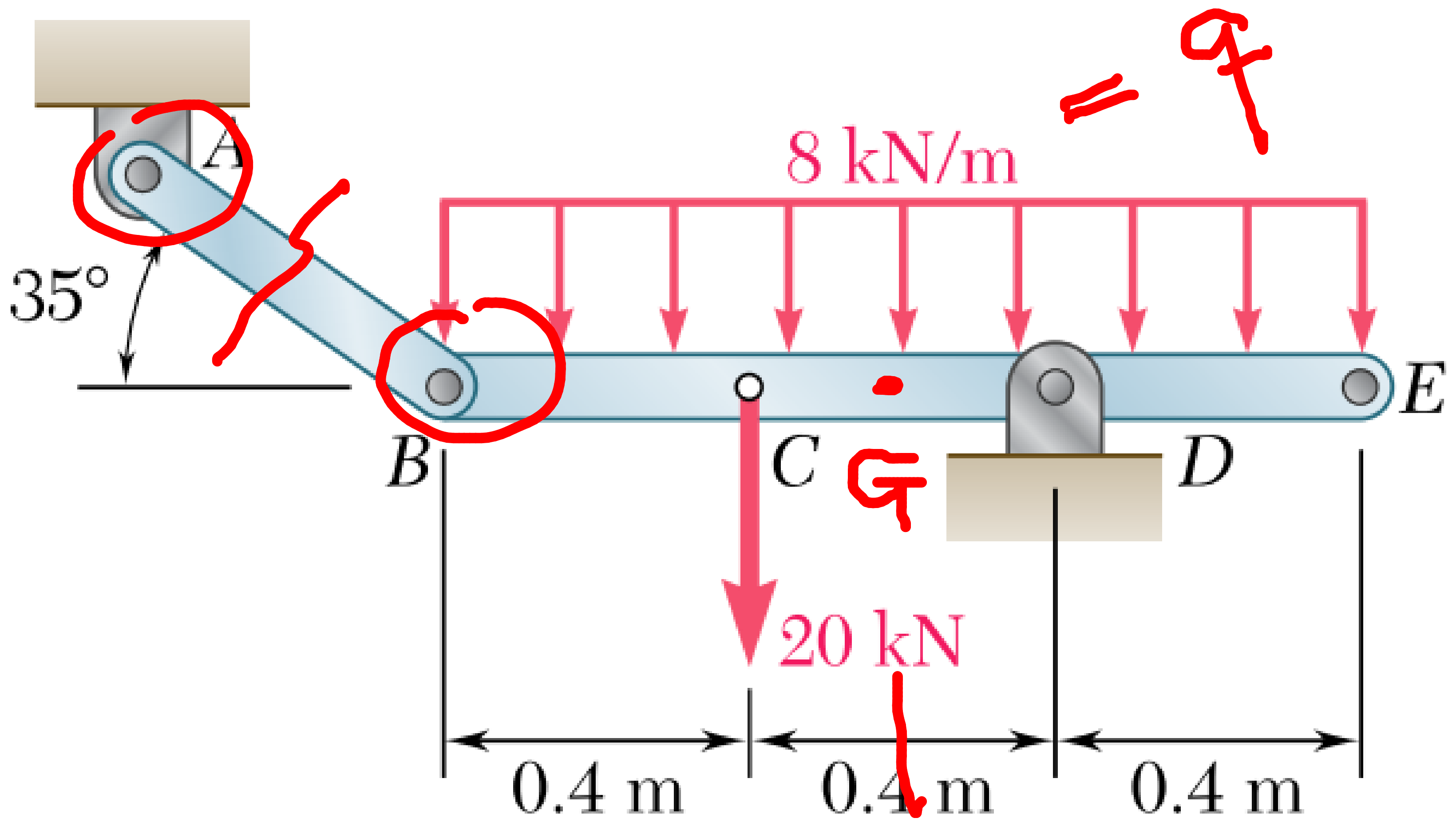
Sec.5

EGM 3520 Mechanics of Materials (MoM)

Beer et al. 2012, Mechanics of Materials, McGraw-Hill.

P1.41, p.37

P1.41, p.37



Link *AB* is to be made of a steel for which the ultimate normal stress is 450 MPa. Determine the cross-sectional area of *AB* for which the factor of safety will be 3.50. Assume that the link will be adequately reinforced around the pins at *A* and *B*.

Pause video NOW !

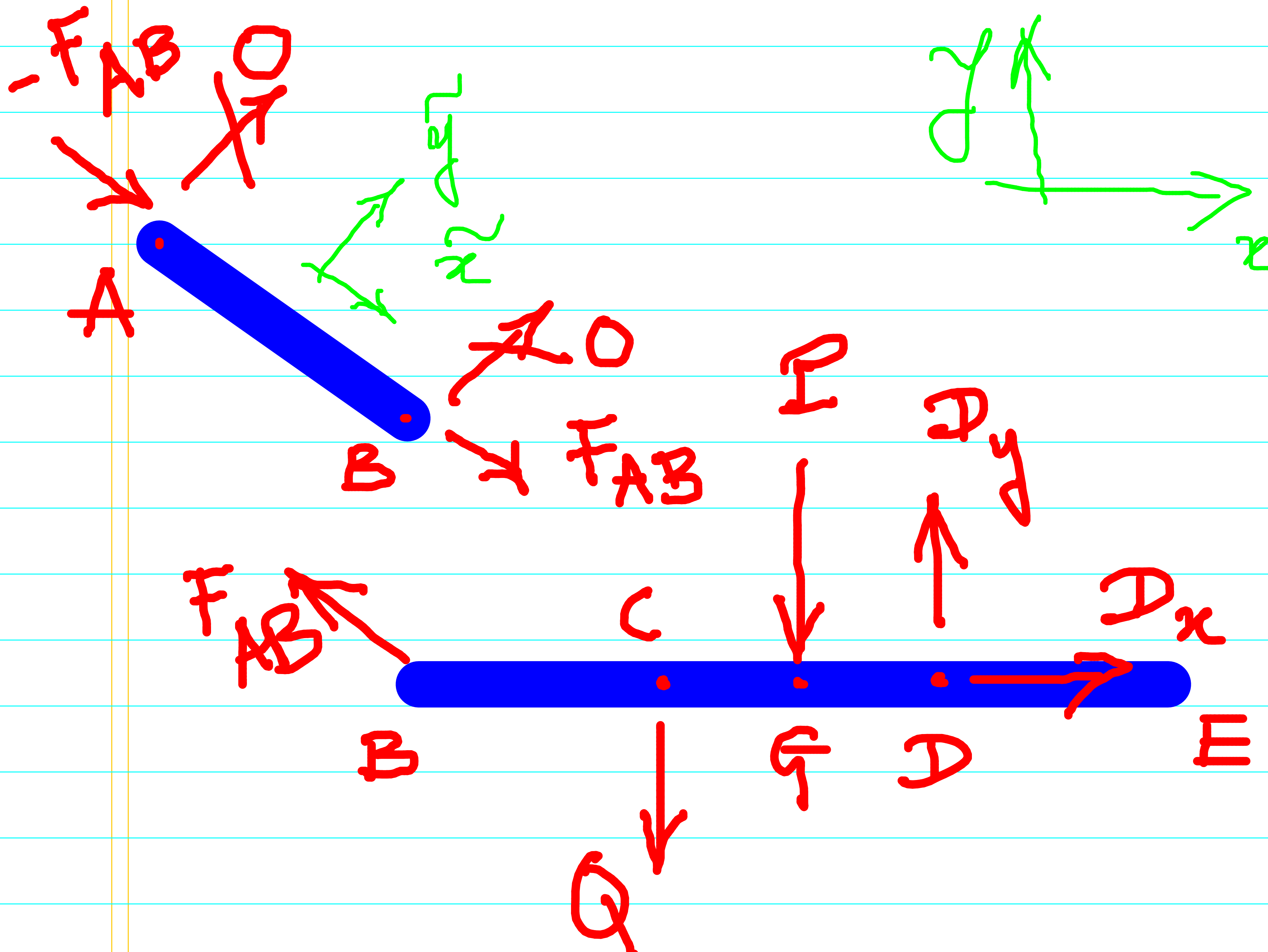
Work out the next step

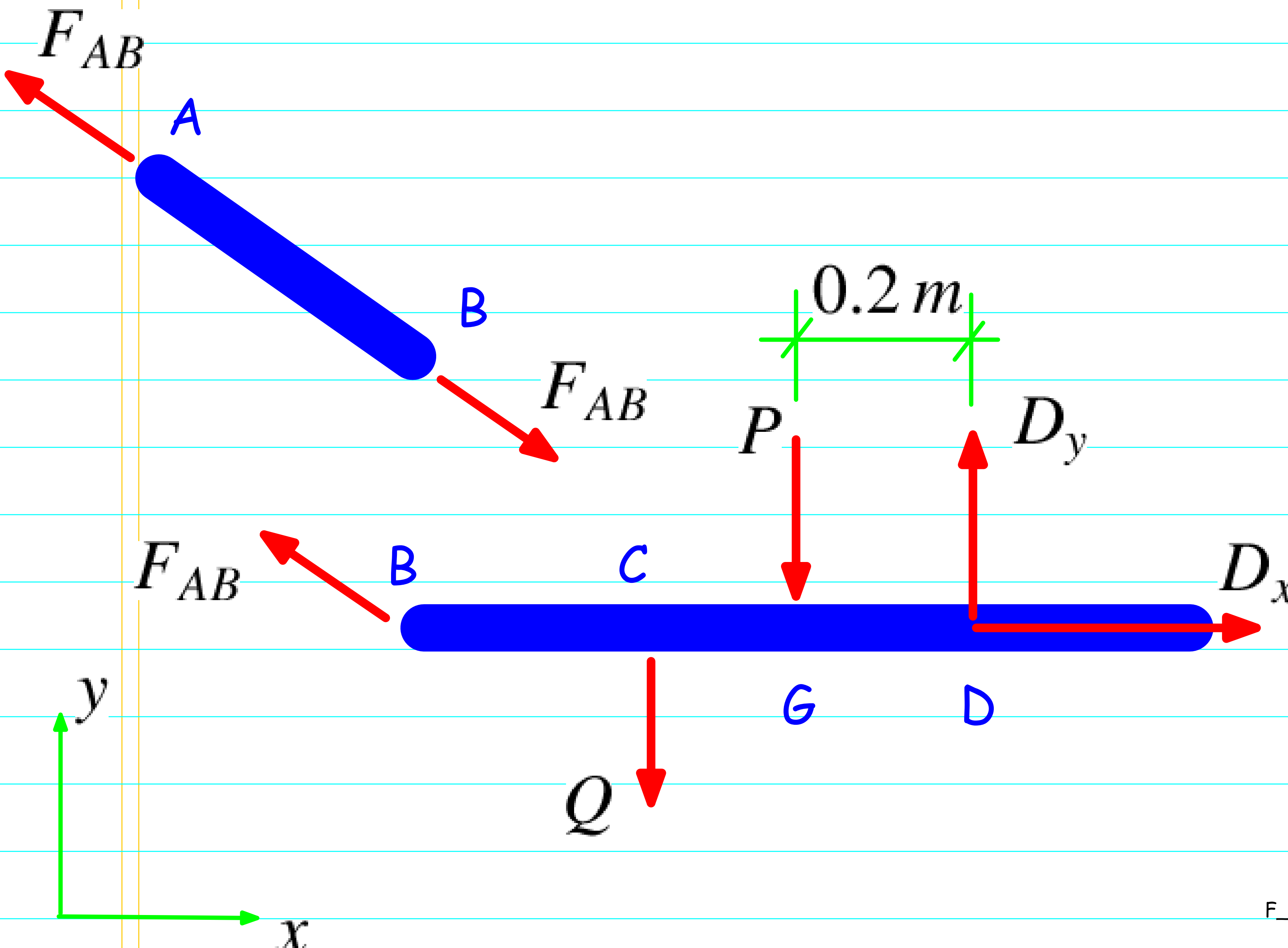
→ individually first

→ discuss with teammates

if you get stuck

then continue to watch the video


-FAB
O
A
B
FAB
O
y
x
I
Dy
FAB
C
Dx
B
G
D
E
Q
y
x



F_{AB}

Pause video NOW !

Work out the next step

→ individually first

→ discuss with teammates

if you get stuck

then continue to watch the video

Method

$$Q = 20\,kN \qquad (1)$$

Q = 20 \, kN

$$q = 8\,kN/m \qquad (2)$$

q = 8 \, kN / m

Resultant force due to distributed load

$$P = q \cdot \overline{BE} \qquad (3)$$

P = q \cdot \overline{BE}

3 unknowns: F_{AB} D_x D_y

3 eqs for 2-D static equilibrium

Find F_{AB} using sum of moments about point D:

$$\sum_i \left. M_i \right|_D = 0 \qquad (4)$$

\sum_i \left. M_i \right|_D = 0

Link AB adequately reinforced around the pin, so it will not fail at the pin, but would fail in the link itself.

So need to design the **smallest** cross section in link AB that can withstand the ultimate normal stress with a F.S. = 3.50, i.e., the **maximum** normal stress in link AB should be:

$$\sigma_{AB,max} = \frac{\sigma_{ult}}{\text{F.S.}} \quad (1)$$

\sigma_{AB, max} = \frac{\sigma_{ult}}{\text{F.S.}}

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} \Rightarrow \sigma_{AB,max} = \frac{F_{AB}}{A_{AB,min}} \quad (2)$$

\sigma_{AB} = \frac{F_{AB}}{A_{AB}} \Rightarrow \sigma_{AB, max} = \frac{F_{AB}}{A_{AB, min}}

Hence the **smallest** cross section area in link AB is

$$A_{AB,min} = \frac{F_{AB}}{\sigma_{AB,max}} = \frac{F_{AB} \cdot \text{F.S.}}{\sigma_{ult}} \quad (3)$$

A_{AB, min} = \frac{F_{AB}}{\sigma_{AB, max}} = \frac{F_{AB} \cdot \text{F.S.}}{\sigma_{ult}}

Pause video NOW !

Work out the next step

→ individually first

→ discuss with teammates

if you get stuck

then continue to watch the video

Computation

(3) p.5-4:

$$P = q \cdot \overline{BE} = 8\,kN/m \cdot 1.2\,m \tag{1}$$

P = q \cdot \overline{BE} = 8 \, kN/m \cdot 1.2 \, m

(4) p.5-4:

$$\sum_i \left. M_i \right|_D = 0 \tag{2}$$

$$= \overline{DG} \cdot P + \overline{DC} \cdot Q - \overline{DB} \cdot F_{AB} \sin 35^\circ$$

\sum_i \left. M_i \right|_D = 0 = \overline{DG} \cdot P + \overline{DC} \cdot Q - \overline{DB} \cdot F_{AB} \sin 35^\circ

(3) p.5-5:

$$A_{AB,min} = \frac{F_{AB} \cdot \text{F.S.}}{\sigma_{ult}} \tag{3}$$

A_{AB, min} = \frac{F_{AB} \cdot \text{F.S.}}{\sigma_{ult}}

$$\sigma_{ult} = 450\,MPa \tag{4}$$

\sigma_{ult} = 450 \, MPa

Additional questions

Review the example with a simple-shear connection in Beer 2012 p.14 fig.1.22, or in the lecture notes Oler.1 p.3-15, as shown below:

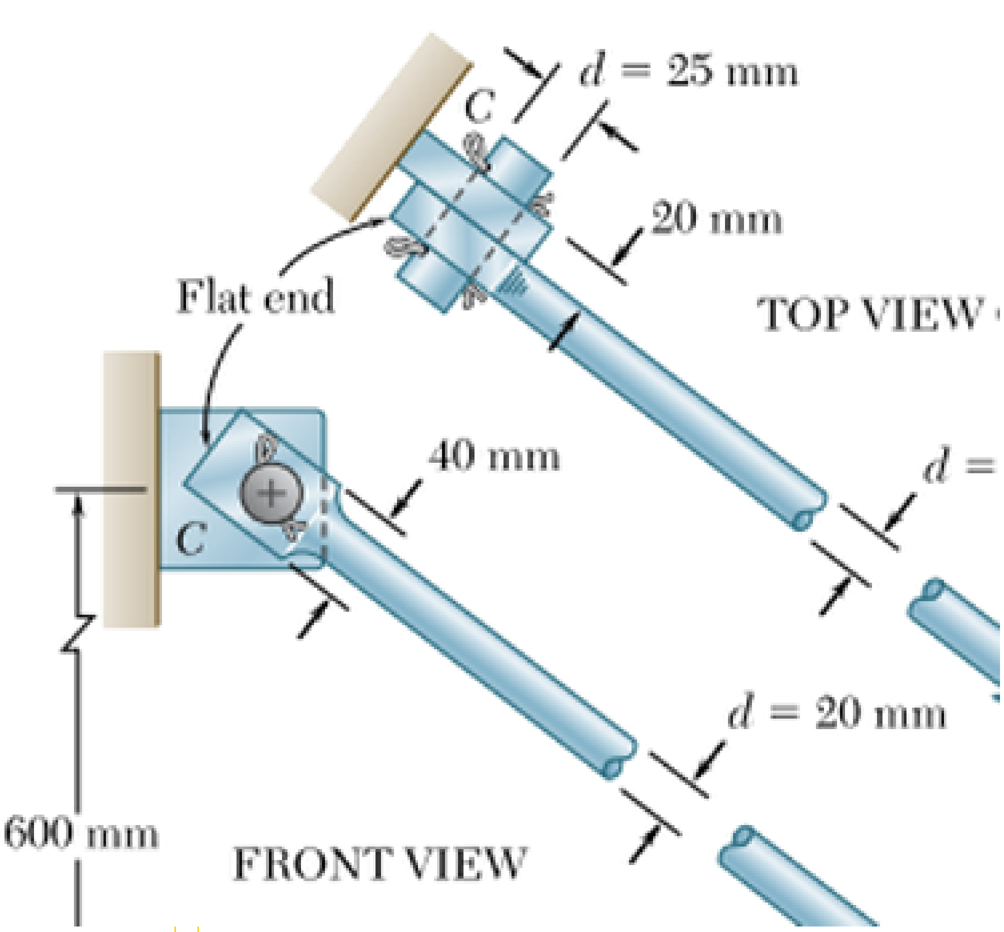


Fig.1.22

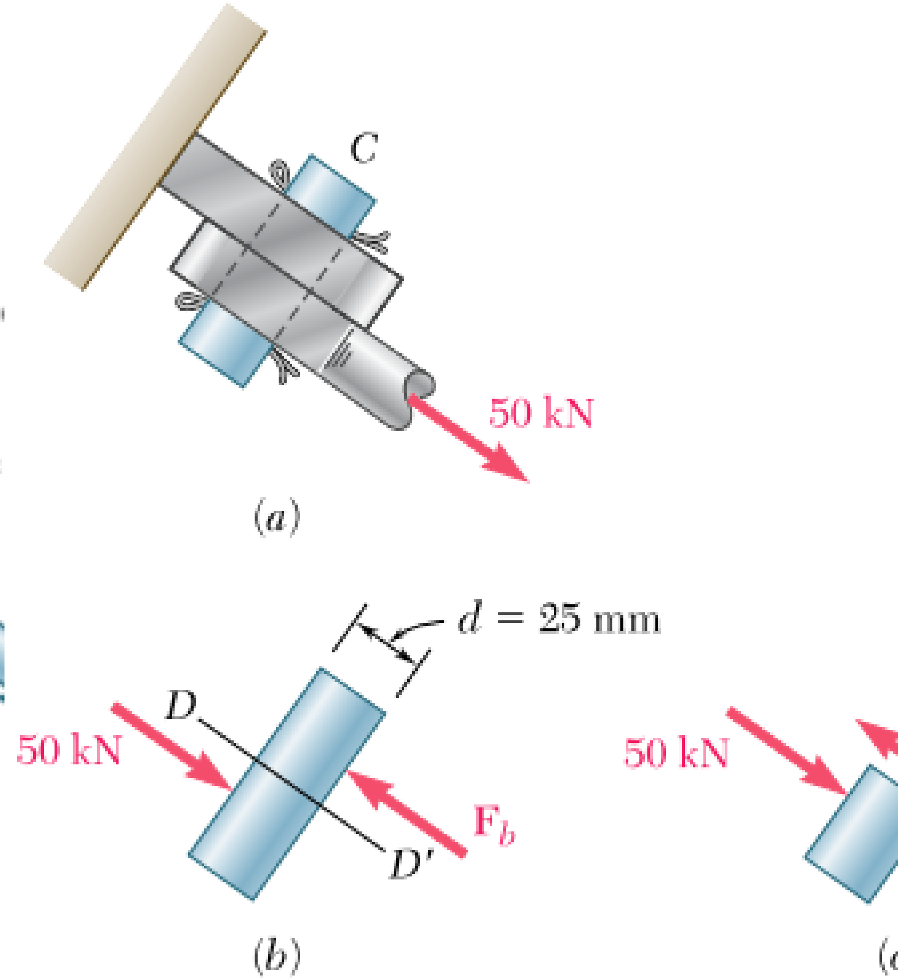


Fig. 1.23

The rod has a flat end at point C, with the rectangular cross section 20 mm (base) x 40 mm (height); the pin at point C has a diameter of 25 mm.

Assume that the pins at A and B in P1.41 p.37 is of the same type as shown above. Bar AB has a rectangular cross section with a base (thickness) of b = 20 mm and a heigth (depth) h to be determined.

1. The pin diameter is d = 25 mm.

   Find the pin bearing stress $\sigma_{b,AB}$ in bar AB at point A, and determine the F.S. of this pin bearing stress.

   If this F.S. is less than 3.50, find a new pin diameter for which the F.S. is 3.50.

\sigma_{b,AB}

2. Find the height h of the cross section such that the normal stress in bar AB has a F.S. of 3.50.

   The normal stress at the cross section of bar AB that passes through the pin is $(\sigma_{AB})_{end}$.

(\sigma_{AB})_{end}

Pause video NOW !

Work out the next step

→ individually first

→ discuss with teammates

if you get stuck

then continue to watch the video

Pin bearing stress at point A

$$\sigma_{b,AB} = \frac{F_{AB}}{d\,b} = \frac{F_{AB}}{(25\ mm)\,(20\ mm)} \qquad (1)$$

\sigma_{b,AB} \sigma_{ult} = \frac{F_{AB}}{(25 \, mm) \, (20 \, mm)}

$$\text{F.S.} = \frac{\sigma_{ult}}{\sigma_{b,AB}} \qquad (2)$$

\text{F.S.} = \frac{\sigma_{ult}}{\sigma_{b,AB}}

If this F.S. is less than 3.50, then calculate the new pin diameter with the maximum allowable pin bearing stress:

$$(\sigma_{b,AB})_{max} = \frac{\sigma_{ult}}{3.50} = \frac{F_{AB}}{d_{min}\,b} \Rightarrow d_{min} = \frac{F_{AB} \times 3.50}{\sigma_{ult}\,b} \qquad (3)$$

(\sigma_{b,AB})_{max} = \frac{\sigma_{ult}}{3.50} = \frac{F_{AB}}{d_{min} \, b} \Rightarrow d_{min}
= \frac{F_{AB} \times 3.50}{\sigma_{ult} \, b}

Another way to find the minimum pin diameter is to use the minimum area in bar AB for a F.S. of 3.50 in (3) p.5-6, i.e.,

$$A_{AB,min} \le d_{min} b \Rightarrow d_{min} = \frac{A_{AB,min}}{b} \qquad (4)$$

A_{AB, min} \le d_{min} b \Rightarrow d_{min} = \frac{A_{AB, min}}{b}

Clearly, (3) and (4) are the same due to (3) p.5-6.

The smallest area in the bar AB is where the pin is located at point A, and is equal to:

$$A_{AB,pin} = (h - d_{min})\, b \quad (1)$$

A_{AB,pin} = (h - d_{min}) \, b

$$\sigma_{AB,pin} = \frac{F_{AB}}{A_{AB,pin}} = \frac{\sigma_{ult}}{3.50} \quad (2)$$

\sigma_{AB,pin} = \frac{F_{AB}}{A_{AB,pin}} = \frac{\sigma_{ult}}{3.50}

The minimum height h can be found from (1) and (2).

Another way is to use (3) p.5-6:

$$A_{AB,min} \le A_{AB,pin} \quad (3)$$

A_{AB, min} \le A_{AB,pin}