

Complex Random Processes

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles, Jr. and B. Shi

Definition

$$Z(t) = X(t) + jY(t)$$

$$E[Z(t)] = E[X(t)] + jE[Y(t)]$$

$$R_{ZZ}(t, t + \tau) = E[Z(t)Z^*(t + \tau)]$$

$$C_{ZZ}(t, t + \tau) = E[Z(t) - E[Z(t)]] \{E[Z(t + \tau) - E[Z(t + \tau)]]\}^*$$

Pseudo-correlation and covariance functions

N Gaussian random variables

Definition

$$R_{ZZ}(t, t + \tau) = E[Z(t)Z^*(t + \tau)]$$

$$C_{ZZ}(t, t + \tau) = E[Z(t) - E[Z(t)]] \{E[Z(t + \tau) - E[Z(t + \tau)]]\}^*$$

$$\tilde{R}_{ZZ}(t, t + \tau) = E[Z(t)Z(t + \tau)]$$

$$\tilde{C}_{ZZ}(t, t + \tau) = E[Z(t) - E[Z(t)]] \{E[Z(t + \tau) - E[Z(t + \tau)]]\}$$

Proper Random Processes

N Gaussian random variables

Definition

A complex random process $Z(t)$ is said to be proper if the pseudo-autocovariance function is identically zero.

If $Z(t)$ is at least wide-sense stationary, the mean value becomes a constant

$$\bar{Z} = \bar{X} + j\bar{Y}$$

the correlation and pseudo-correlation functions are independent of absolute time

$$R_{ZZ}(t, t + \tau) = R_{ZZ}(\tau) \quad \tilde{R}_{ZZ}(t, t + \tau) = \tilde{R}_{ZZ}(\tau)$$

$$C_{ZZ}(t, t + \tau) = C_{ZZ}(\tau) \quad \tilde{C}_{ZZ}(t, t + \tau) = \tilde{C}_{ZZ}(\tau)$$

Cross / Pseudo-cross, -corelation / -covariance

N Gaussian random variables

Definition

$$R_{Z_i Z_j}(t, t + \tau) = E [Z_i(t) Z_j^*(t + \tau)]$$

$$C_{Z_i Z_j}(t, t + \tau) = E [\{Z_i(t) - E[Z_i(t)]\} \{Z_j(t + \tau) - E[Z_j(t + \tau)]\}^*]$$

$$R_{Z_i Z_j}(t, t + \tau) = E [Z_i(t) Z_j(t + \tau)]$$

$$C_{Z_i Z_j}(t, t + \tau) = E [\{Z_i(t) - E[Z_i(t)]\} \{Z_j(t + \tau) - E[Z_j(t + \tau)]\}]$$

