StarCore : Computing Correlation Function

Please send corrections (or suggestions) to i.

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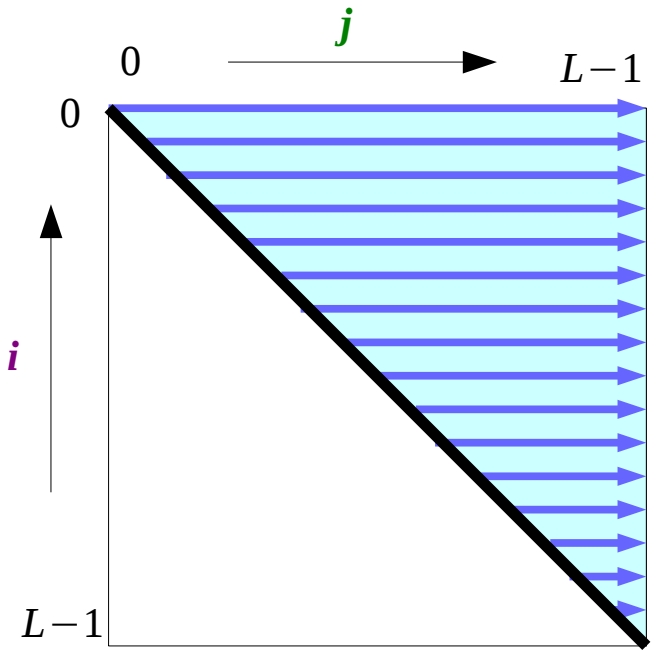
Based on

Cross Correlation
http://cache.freescale.com/files/dsp/doc/app_note/AN2266.pdf

Correlation Code

```
L_max = 0;
for (i = L-1; i >= 0; i--) {
        Acc = 0;
        for (j = i; j < L; j++)
                Acc = L_mac(Acc, x[j], h[j-i]);
        y[i] = Acc;
        Acc = L_abs(Acc);
        if ( Acc > L_max ) {
                L_max = Acc;
        }
}
```

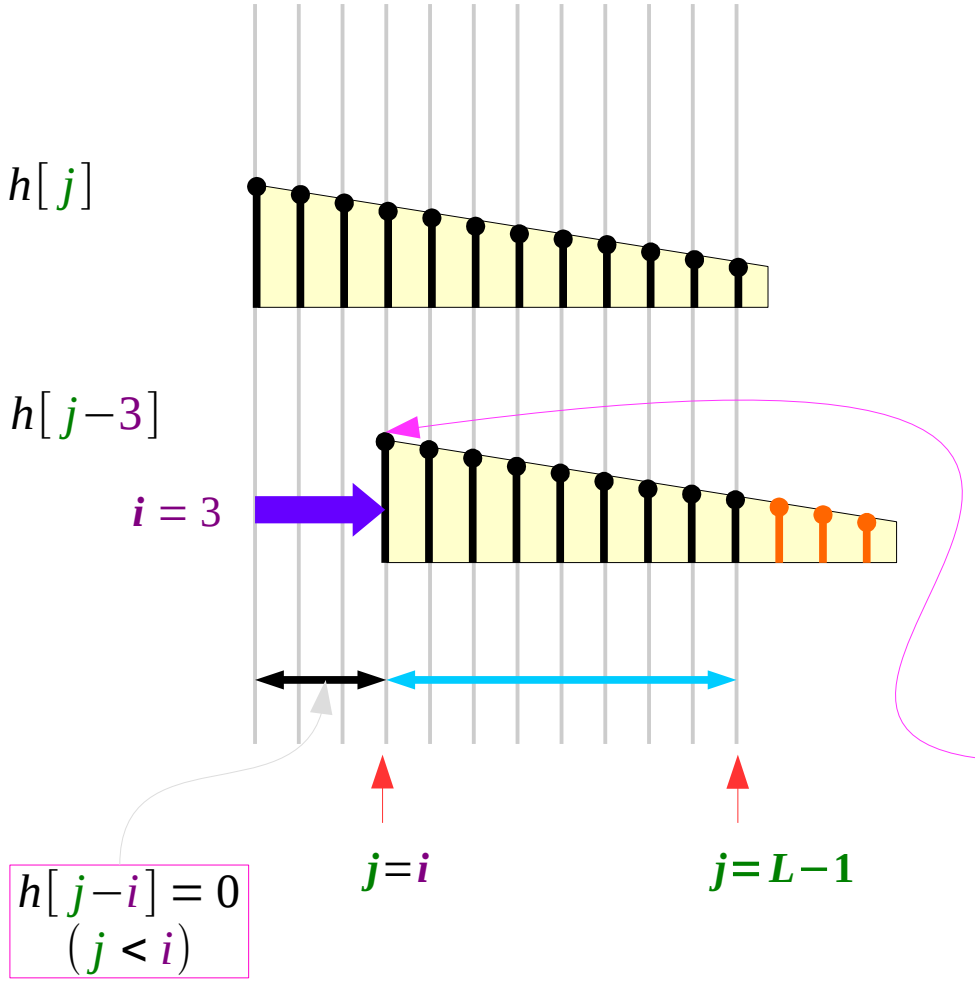


$$y[i] = \sum_{j=i}^{L-1} x[j]\,h[j-i]$$

L_max
L_mac
L_abs

h[j] and h[0]

$$y[i]=\sum_{j=i}^{L-1} x[j]h[j-1]$$



$h[j]$

$h[j-3]$ $\quad$ $h[j]$

$h[j-i]$ shifted version of $h[j]$

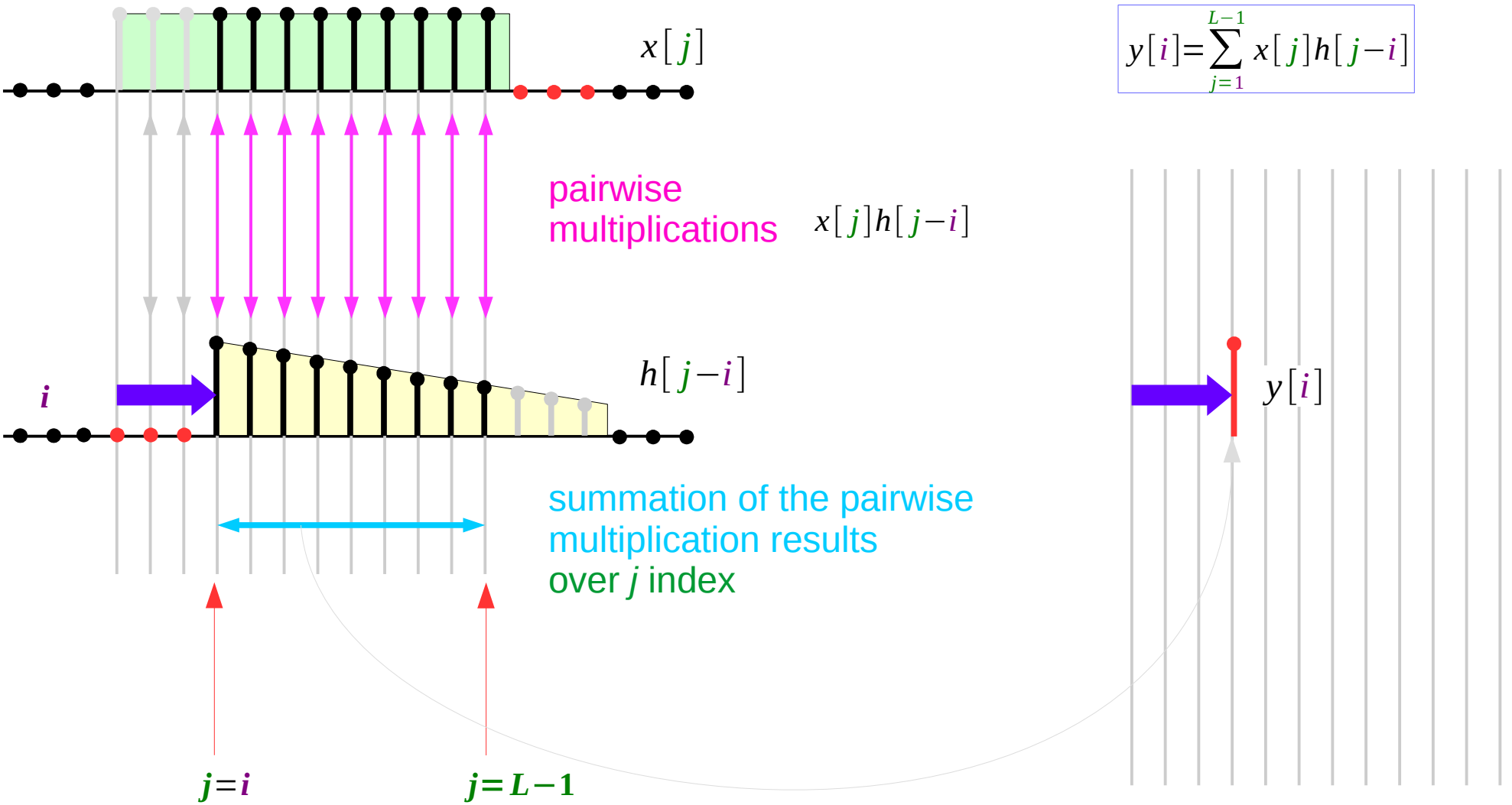
$h[n-a]$ $\quad$ $h[n]$

$h(t-a)$ $\quad$ $h(t)$

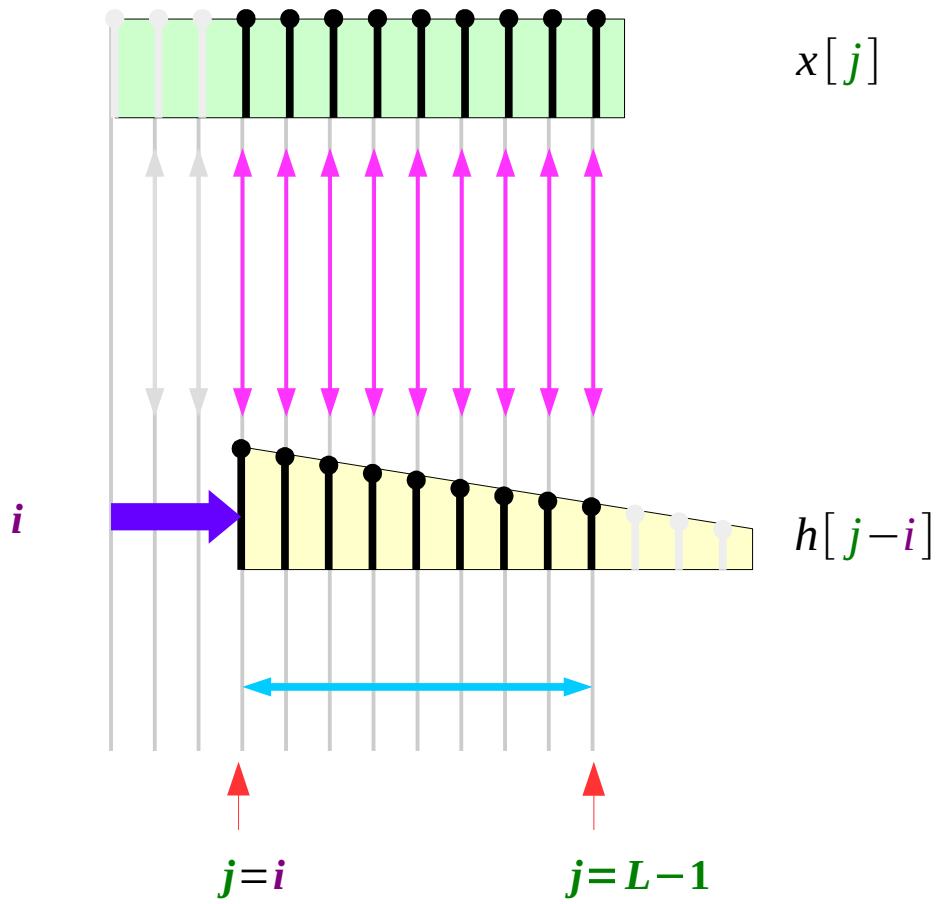
$h[j-i]=h[0]$ *when* $j=i$

Correlation y[*i*] – for a given *i*

$$y[i]=\sum_{j=1}^{L-1} x[j]h[j-i]$$



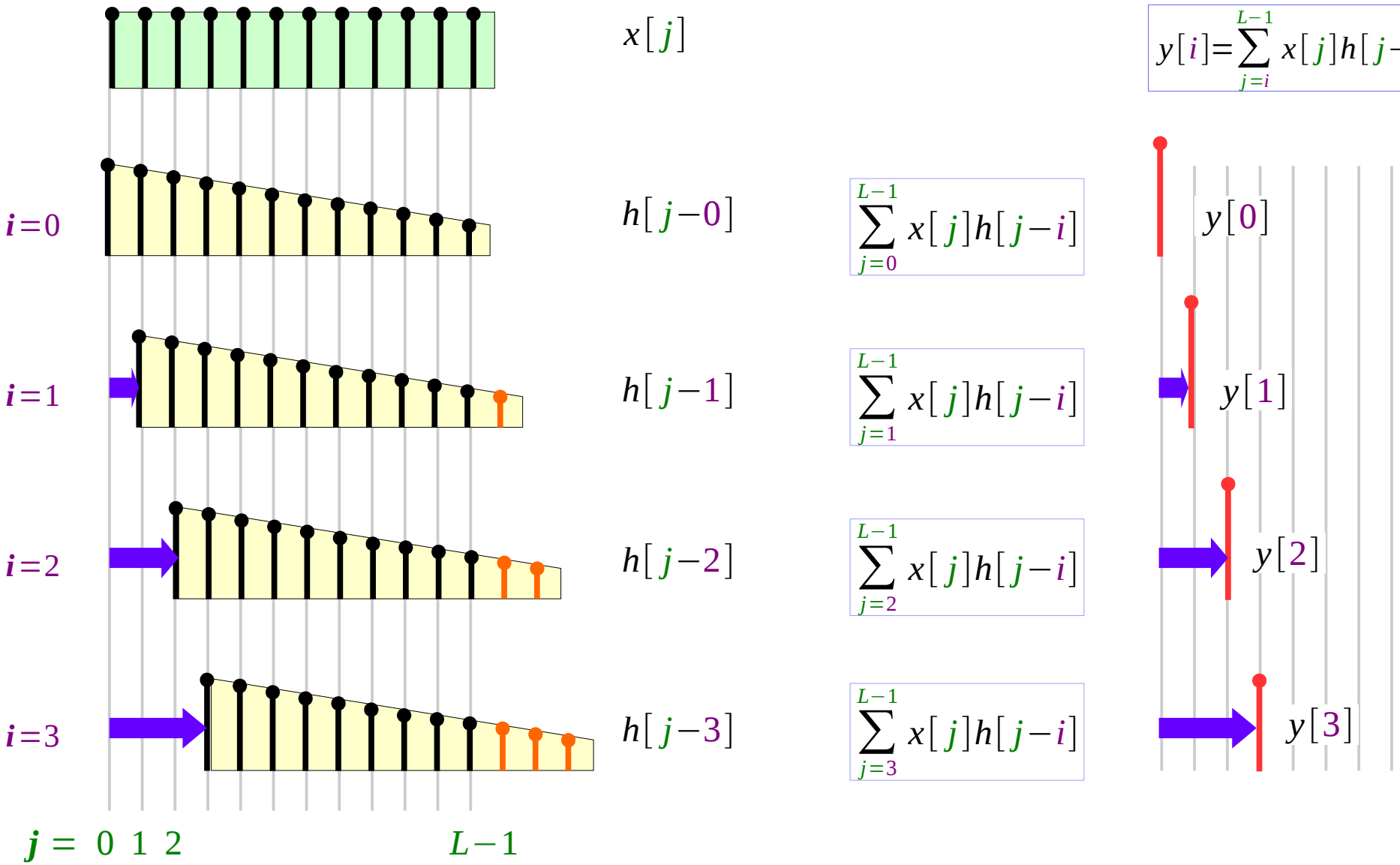
Summing pairwise multiplications



$$y[1]=\sum_{j=1}^{L-1} x[j]h[j-1]$$

$$\begin{array}{rr}
x[0] & 0 \\
x[1] & 0 \\
\vdots & \vdots \\
x[i] & h[0] \\
x[i+1] & h[1] \\
 & \\
 & \\
x[L-1] & h[L-1-i] \\
\vdots & \vdots \\
0 & h[L-2] \\
0 & h[L-1]
\end{array}$$

Cross Correlation Function y[*i*]



Loop Unroll to OL with IL0, IL1, IL2, IL3

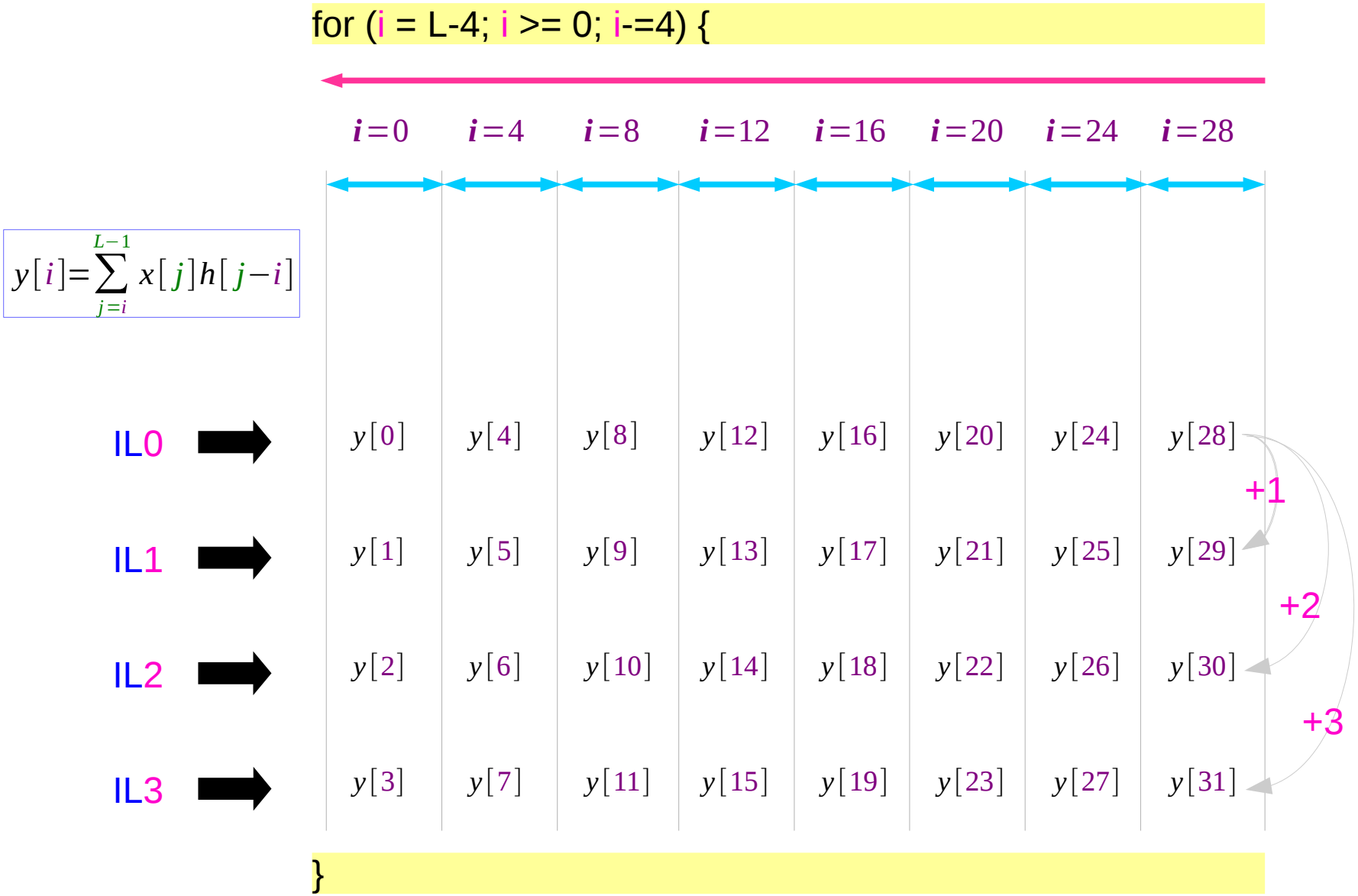
```
Lx = 0;
for (i = L-1; i >= 0; i--) {
      Acc = comp_y(i);
      Acc = L_abs(Acc);
      if ( Acc > Lx ) {
            Lx = Acc;
      }
}
```

```
Acc = 0;
for (j = i; j < L; j++)
      Acc = L_mac(Acc, x[j], h[j-i]);
y[i] = Acc;
```

```
Lx0 = Lx1 = Lx2 = Lx3= 0;
for (i = L-4; i >= 0; i-=4) {
  IL0 ➡ Lx0 = find_max_y(i, 0);
  IL1 ➡ Lx1 = find_max_y(i, 1);
  IL2 ➡ Lx2 = find_max_y(i, 2);
  IL3 ➡ Lx3 = find_max_y(i, 3);
}

Lx = calc_max(Lx0,Lx1,Lx2,Lx3););
```

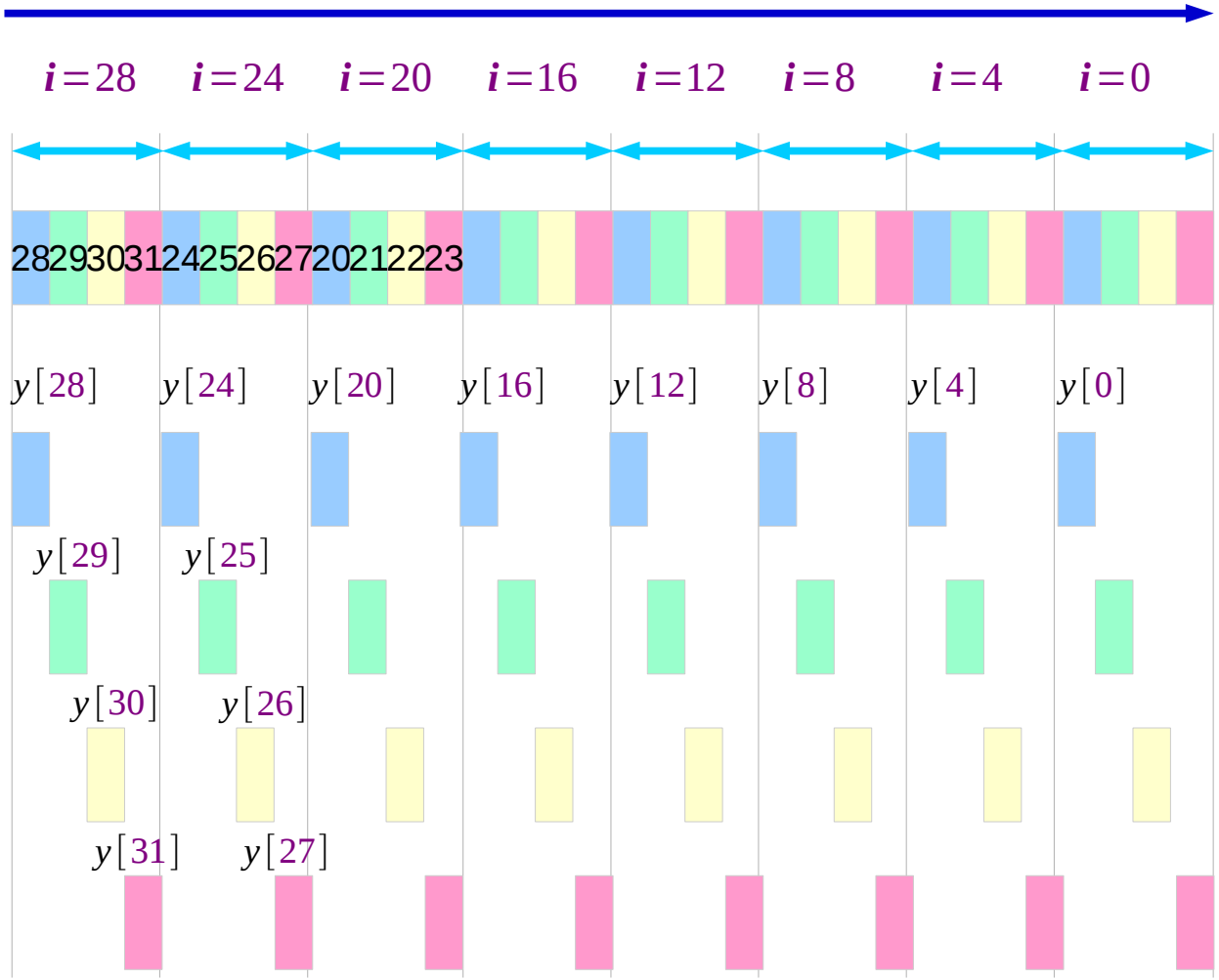
```
Lxi = 0;

Acck = comp_y(i+k);
Acck = L_abs(Acc);
if ( Acck > Lxk ) {
      Lxk = Acc;
}
```

New Inner Loops : IL0, IL1, IL2, IL3



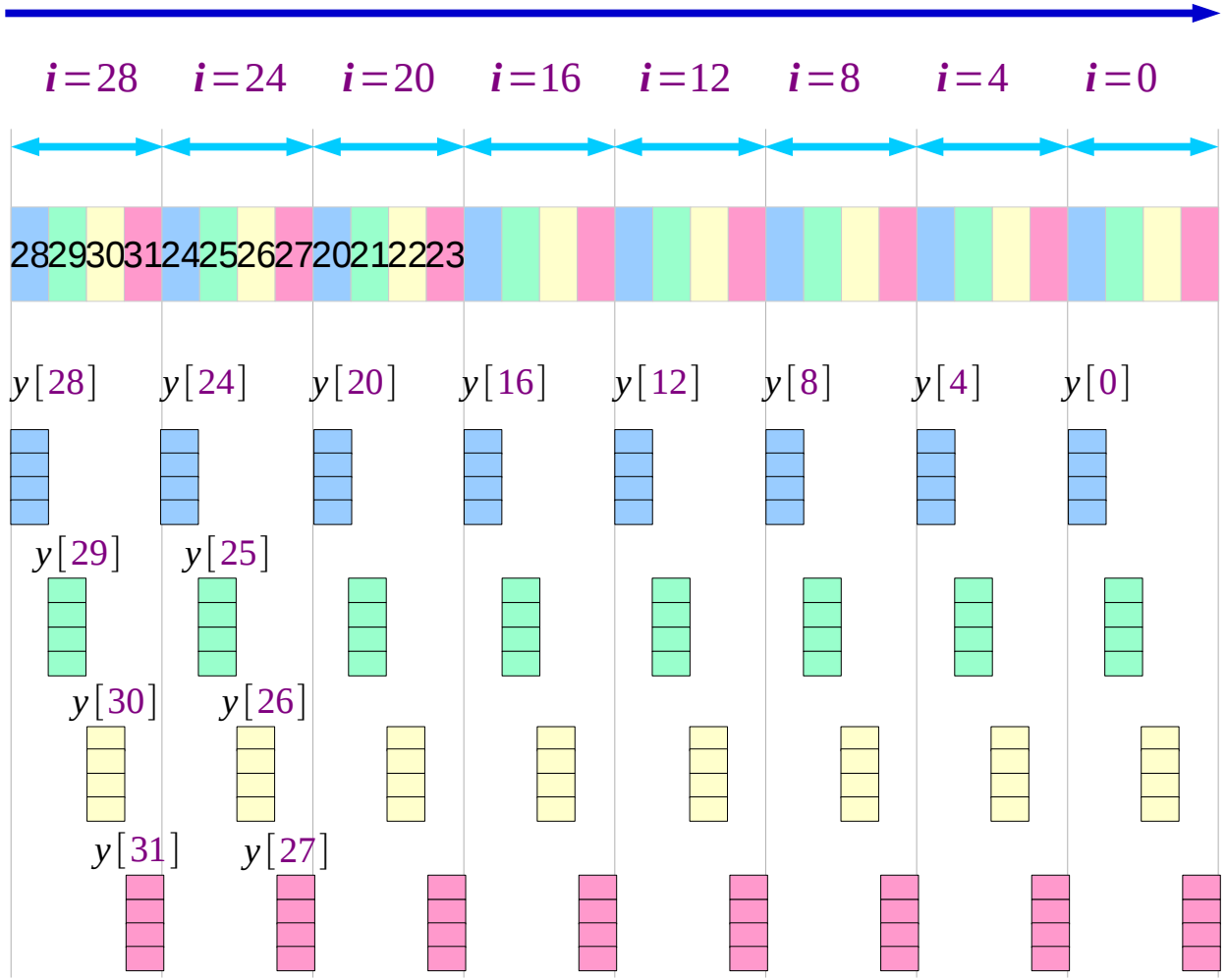
Partitioning y[*i*], *i* = 0, … , L-1

$$y[i]=\sum_{j=i}^{L-1} x[j]h[j-i]$$



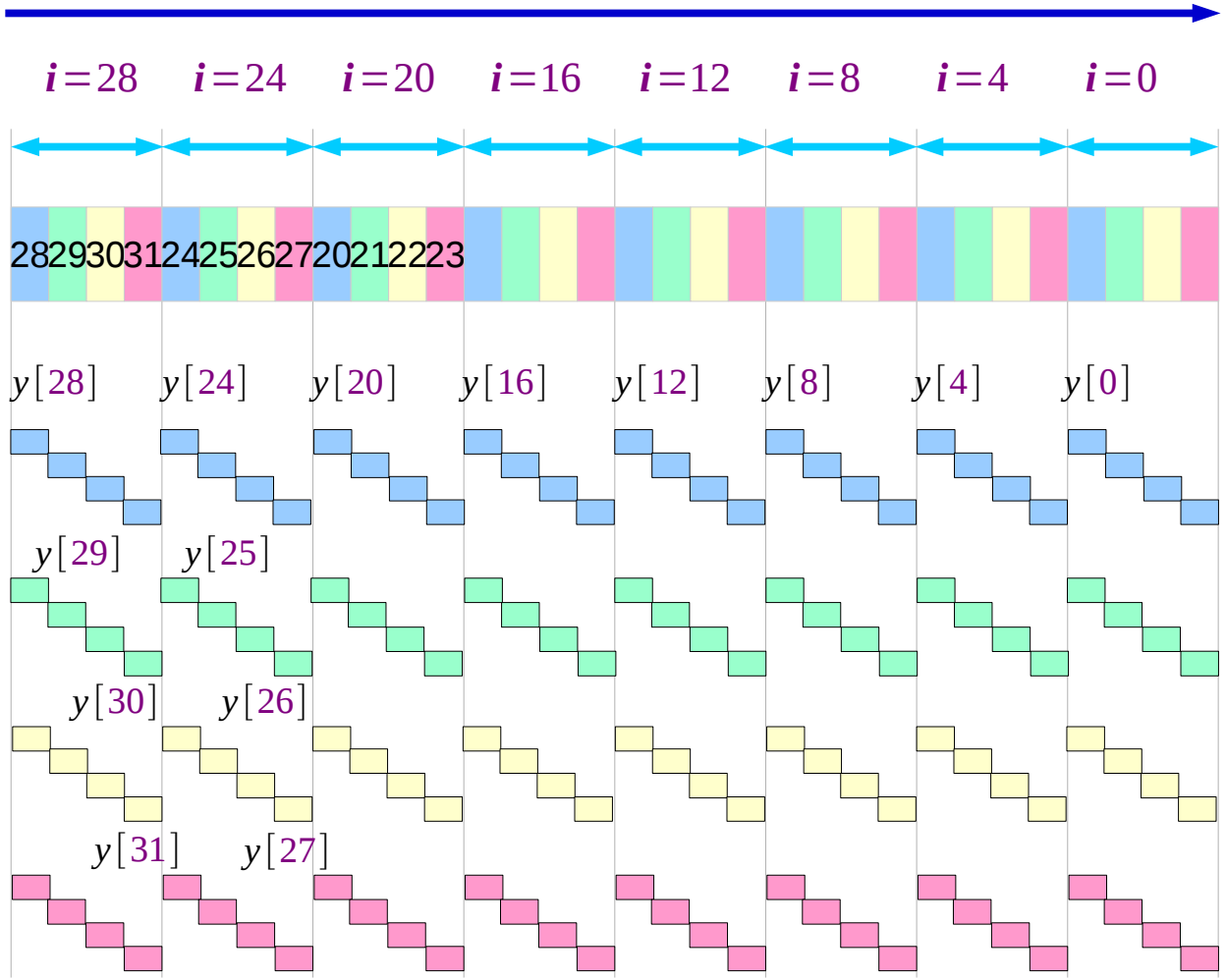
Partitioning each y[*i*]

$$y[i]=\sum_{j=i}^{L-1} x[j]h[j-i]$$



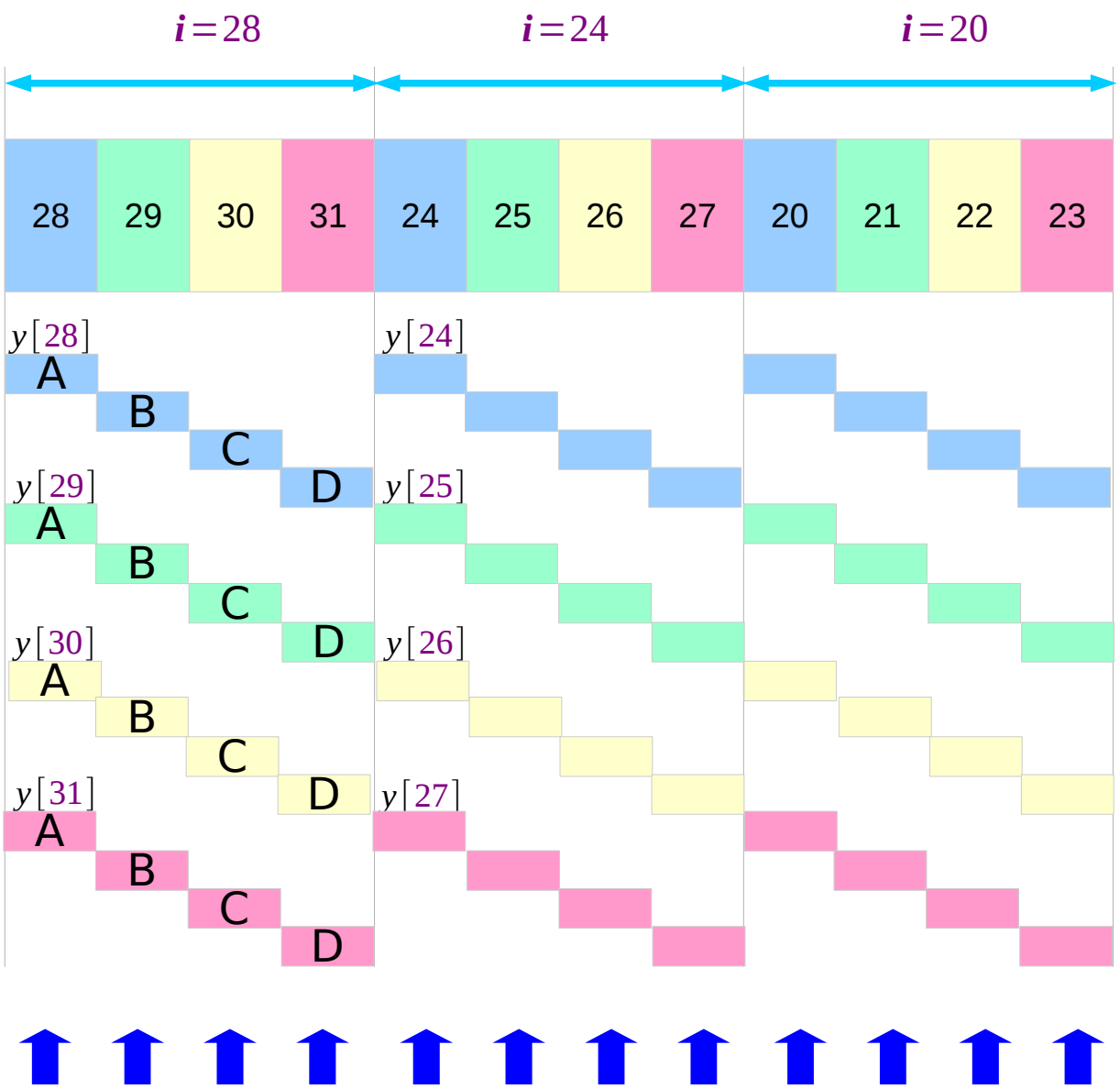
Rearranging partitions

$$y[i]=\sum_{j=i}^{L-1} x[j]h[j-i]$$



Detailed View

$$y[i]=\sum_{j=i}^{L-1} x[j]h[j-i]$$



Detailed Code View

```
for (i = L-4; i >= 0; i-=4) {
```

IL0 ➡ $y[i+0] = \sum_{j=i}^{L-1} X[j]h[j-i-0]$

$j = i+0, i+4, i+8, \cdots, L-4$ ➡ IL0-A

$j = i+1, i+5, i+9, \cdots, L-3$ ➡ IL0-B

$j = i+2, i+6, i+10, \cdots, L-2$ ➡ IL0-C

$j = i+3, i+7, i+11, \cdots, L-1$ ➡ IL0-D

IL1 ➡ $y[i+1] = \sum_{j=i}^{L-1} X[j]h[j-i-1]$

$j = i+1, i+5, i+9, \cdots, L-3$ ➡ IL1-A

$j = i+2, i+6, i+10, \cdots, L-2$ ➡ IL1-B

$j = i+3, i+7, i+11, \cdots, L-1$ ➡ IL1-C

$j = i+4, i+8, i+12, \cdots, L-4$ ➡ IL1-D

IL2 ➡ $y[i+2] = \sum_{j=i}^{L-1} X[j]h[j-i-2]$

$j = i+2, i+6, i+10, \cdots, L-2$ ➡ IL1-A

$j = i+3, i+7, i+11, \cdots, L-1$ ➡ IL1-B

$j = i+4, i+8, i+12, \cdots, L-4$ ➡ IL1-C

$j = i+5, i+9, i+13, \cdots, L-3$ ➡ IL1-D

IL3 ➡ $y[i+3] = \sum_{j=i}^{L-1} X[j]h[j-i-3]$

$j = i+3, i+7, i+11, \cdots, L-1$ ➡ IL1-A

$j = i+4, i+8, i+12, \cdots, L-4$ ➡ IL1-B

$j = i+5, i+9, i+13, \cdots, L-3$ ➡ IL1-C

$j = i+6, i+10, i+14, \cdots, L-2$ ➡ IL1-D

```
}
```

Accessing h[j] array

```
for (i = L-4; i >= 0; i-=4) {
```

$$y[i+0] = \sum_{j=i}^{L-1} X[j]h[j-i-0]$$

$$\sum \quad j = i+0, i+4, i+8, \cdots, L-4 \Rightarrow h[0], h[4], h[8], \cdots, h[L-4]$$
$$\sum \quad j = i+1, i+5, i+9, \cdots, L-3 \Rightarrow h[1], h[5], h[9], \cdots, h[L-3]$$
$$\sum \quad j = i+2, i+6, i+10, \cdots, L-2 \Rightarrow h[2], h[6], h[10], \cdots, h[L-2]$$
$$\sum \quad j = i+3, i+7, i+11, \cdots, L-1 \Rightarrow h[3], h[7], h[11], \cdots, h[L-1]$$

$$y[i+1] = \sum_{j=i}^{L-1} X[j]h[j-i-1]$$

$$\sum \quad j = i+1, i+5, i+9, \cdots, L-3 \Rightarrow h[0], h[4], h[8], \cdots, h[L-4]$$
$$\sum \quad j = i+2, i+6, i+10, \cdots, L-2 \Rightarrow h[1], h[5], h[9], \cdots, h[L-3]$$
$$\sum \quad j = i+3, i+7, i+11, \cdots, L-1 \Rightarrow h[2], h[6], h[10], \cdots, h[L-2]$$
$$\sum \quad j = i+4, i+8, i+12, \cdots, L-4 \Rightarrow h[3], h[7], h[11], \cdots, h[L-1]$$

$$y[i+2] = \sum_{j=i}^{L-1} X[j]h[j-i-2]$$

$$\sum \quad j = i+2, i+6, i+10, \cdots, L-2 \Rightarrow h[0], h[4], h[8], \cdots, h[L-4]$$
$$\sum \quad j = i+3, i+7, i+11, \cdots, L-1 \Rightarrow h[1], h[5], h[9], \cdots, h[L-3]$$
$$\sum \quad j = i+4, i+8, i+12, \cdots, L-4 \Rightarrow h[2], h[6], h[10], \cdots, h[L-2]$$
$$\sum \quad j = i+5, i+9, i+13, \cdots, L-3 \Rightarrow h[3], h[7], h[11], \cdots, h[L-1]$$

$$y[i+3] = \sum_{j=i}^{L-1} X[j]h[j-i-3]$$

$$\sum \quad j = i+3, i+7, i+11, \cdots, L-1 \Rightarrow h[0], h[4], h[8], \cdots, h[L-4]$$
$$\sum \quad j = i+4, i+8, i+12, \cdots, L-4 \Rightarrow h[1], h[5], h[9], \cdots, h[L-3]$$
$$\sum \quad j = i+5, i+9, i+13, \cdots, L-3 \Rightarrow h[2], h[6], h[10], \cdots, h[L-2]$$
$$\sum \quad j = i+6, i+10, i+14, \cdots, L-2 \Rightarrow h[3], h[7], h[11], \cdots, h[L-1]$$

```
}
```

Rearranging for Parallel Execution

for (i = L-4; i >= 0; i-=4) {

$\sum\ j = i+0, i+4, i+8, \cdots, L-4$ ➡ IL0-A

$\sum\sum\ j = i+1, i+5, i+9, \cdots, L-3$ ➡ IL1-A

$\sum\sum\ j = i+2, i+6, i+10, \cdots, L-2$ ➡ IL1-A

$\sum\sum\ j = i+3, i+7, i+11, \cdots, L-1$ ➡ IL1-A

Parallel Execution among 4 ALUs

$\sum\sum\ j = i+1, i+5, i+9, \cdots, L-3$ ➡ IL0-B

$\sum\sum\ j = i+2, i+6, i+10, \cdots, L-2$ ➡ IL1-B

$\sum\sum\ j = i+3, i+7, i+11, \cdots, L-1$ ➡ IL1-B

$\sum\sum\ j = i+4, i+8, i+12, \cdots, L-4$ ➡ IL1-B

Parallel Execution among 4 ALUs

$\sum\sum\ j = i+2, i+6, i+10, \cdots, L-2$ ➡ IL0-C

$\sum\sum\ j = i+3, i+7, i+11, \cdots, L-1$ ➡ IL1-C

$\sum\sum\ j = i+4, i+8, i+12, \cdots, L-4$ ➡ IL1-C

$\sum\sum\ j = i+5, i+9, i+13, \cdots, L-3$ ➡ IL1-C

Parallel Execution among 4 ALUs

$\sum\sum\ j = i+3, i+7, i+11, \cdots, L-1$ ➡ IL0-D

$\sum\sum\ j = i+4, i+8, i+12, \cdots, L-4$ ➡ IL1-D

$\sum\sum\ j = i+5, i+9, i+13, \cdots, L-3$ ➡ IL1-D

$\sum\ j = i+6, i+10, i+14, \cdots, L-2$ ➡ IL1-D

Parallel Execution among 4 ALUs

}

Memory Access Pattern of h[*j*]

```
for (i = L-4; i >= 0; i-=4) {
```

Sequential Execution

$\sum\ j = i+0, i+4, i+8, \cdots, L-4$	IL0-A	$h[0], h[4], h[8], \cdots, h[L-4]$	Parallel Execution
$\sum\ j = i+1, i+5, i+9, \cdots, L-3$	IL1-A	$h[0], h[4], h[8], \cdots, h[L-4]$	
$\sum\ j = i+2, i+6, i+10, \cdots, L-2$	IL1-A	$h[0], h[4], h[8], \cdots, h[L-4]$	
$\sum\ j = i+3, i+7, i+11, \cdots, L-1$	IL1-A	$h[0], h[4], h[8], \cdots, h[L-4]$	
$\sum\ j = i+1, i+5, i+9, \cdots, L-3$	IL0-B	$h[1], h[5], h[9], \cdots, h[L-3]$	Parallel Execution
$\sum\ j = i+2, i+6, i+10, \cdots, L-2$	IL1-B	$h[1], h[5], h[9], \cdots, h[L-3]$	
$\sum\ j = i+3, i+7, i+11, \cdots, L-1$	IL1-B	$h[1], h[5], h[9], \cdots, h[L-3]$	
$\sum\ j = i+4, i+8, i+12, \cdots, L-4$	IL1-B	$h[1], h[5], h[9], \cdots, h[L-3]$	
$\sum\ j = i+2, i+6, i+10, \cdots, L-2$	IL0-C	$h[2], h[6], h[10], \cdots, h[L-2]$	Parallel Execution
$\sum\ j = i+3, i+7, i+11, \cdots, L-1$	IL1-C	$h[2], h[6], h[10], \cdots, h[L-2]$	
$\sum\ j = i+4, i+8, i+12, \cdots, L-4$	IL1-C	$h[2], h[6], h[10], \cdots, h[L-2]$	
$\sum\ j = i+5, i+9, i+13, \cdots, L-3$	IL1-C	$h[2], h[6], h[10], \cdots, h[L-2]$	
$\sum\ j = i+3, i+7, i+11, \cdots, L-1$	IL0-D	$h[3], h[7], h[11], \cdots, h[L-1]$	Parallel Execution
$\sum\ j = i+4, i+8, i+12, \cdots, L-4$	IL1-D	$h[3], h[7], h[11], \cdots, h[L-1]$	
$\sum\ j = i+5, i+9, i+13, \cdots, L-3$	IL1-D	$h[3], h[7], h[11], \cdots, h[L-1]$	
$\sum\ j = i+6, i+10, i+14, \cdots, L-2$	IL1-D	$h[3], h[7], h[11], \cdots, h[L-1]$	

```
}
```

Loop Unroll to IL0, IL1, IL2, IL3

```
x_curr = X[i]
h0 = h[0]; h1 = h2 = h3 = 0;
for (j = i; j < L_SUBFR; j+=4)
{
   L_s0 = L_mac(L_s0, x_curr, h0);
   L_s1 = L_mac(L_s1, x_curr, h1);
   L_s2 = L_mac(L_s2, x_curr, h2);
   L_s3 = L_mac(L_s3, x_curr, h3);
   h3 = h[j+1-i]; x_curr = X[j+1];

   L_s0 = L_mac(L_s0, x_curr, h3);
   L_s1 = L_mac(L_s1, x_curr, h0);
   L_s2 = L_mac(L_s2, x_curr, h1);
   L_s3 = L_mac(L_s3, x_curr, h2);
   h2 = h[j+2-i]; x_curr = X[j+2];

   L_s0 = L_mac(L_s0, x_curr, h2);
   L_s1 = L_mac(L_s1, x_curr, h3);
   L_s2 = L_mac(L_s2, x_curr, h0);
   L_s3 = L_mac(L_s3, x_curr, h1);
   h1 = h[j+3-i]; x_curr = X[j+3];

   L_s0 = L_mac(L_s0, x_curr, h1);
   L_s1 = L_mac(L_s1, x_curr, h2);
   L_s2 = L_mac(L_s2, x_curr, h3);
   L_s3 = L_mac(L_s3, x_curr, h0);
   h0 = h[j+4-i]; x_curr = X[j+4];
}
```

```
x_curr = X[i]
h0 = h[0]; h1 = h2 = h3 = 0;
for (j = i; j < L_SUBFR; j+=4)
{
   L_s0 = L_mac(L_s0, x_curr, h0);   ➡ IL0-A
   L_s1 = L_mac(L_s1, x_curr, h1);   ➡ IL1-A
   L_s2 = L_mac(L_s2, x_curr, h2);   ➡ IL2-A
   L_s3 = L_mac(L_s3, x_curr, h3);   ➡ IL3-A
   h3 = h[j+1-i]; x_curr = X[j+1];   ⬅ Update

   L_s0 = L_mac(L_s0, x_curr, h3);   ➡ IL0-B
   L_s1 = L_mac(L_s1, x_curr, h0);   ➡ IL1-B
   L_s2 = L_mac(L_s2, x_curr, h1);   ➡ IL2-B
   L_s3 = L_mac(L_s3, x_curr, h2);   ➡ IL3-B
   h2 = h[j+2-i]; x_curr = X[j+2];   ⬅ Update

   L_s0 = L_mac(L_s0, x_curr, h2);   ➡ IL0-C
   L_s1 = L_mac(L_s1, x_curr, h3);   ➡ IL1-C
   L_s2 = L_mac(L_s2, x_curr, h0);   ➡ IL2-C
   L_s3 = L_mac(L_s3, x_curr, h1);   ➡ IL3-C
   h1 = h[j+3-i]; x_curr = X[j+3];   ⬅ Update

   L_s0 = L_mac(L_s0, x_curr, h1);   ➡ IL0-D
   L_s1 = L_mac(L_s1, x_curr, h2);   ➡ IL1-D
   L_s2 = L_mac(L_s2, x_curr, h3);   ➡ IL2-D
   L_s3 = L_mac(L_s3, x_curr, h0);   ➡ IL3-D
   h0 = h[j+4-i]; x_curr = X[j+4];   ⬅ Update
}
```

Rearranged for easy understanding

```
{
    L_s0 = L_mac(L_s0, x_curr, h0);
    L_s0 = L_mac(L_s0, x_curr, h3);
    L_s0 = L_mac(L_s0, x_curr, h2);
    L_s0 = L_mac(L_s0, x_curr, h1);
}
```

h3 = h[j+1-i]; x_curr = X[j+1]; ➡ IL0-A ➡ L_s0 += X[j+0] * h[j+0-i];
h2 = h[j+2-i]; x_curr = X[j+2]; ➡ IL0-B ➡ L_s0 += X[j+1] * h[j+1-i];
h1 = h[j+3-i]; x_curr = X[j+3]; ➡ IL0-C ➡ L_s0 += X[j+2] * h[j+2-i];
h0 = h[j+4-i]; x_curr = X[j+4]; ➡ IL0-D ➡ L_s0 += X[j+3] * h[j+3-i];

```
{
    L_s1 = L_mac(L_s1, x_curr, h1);
    L_s1 = L_mac(L_s1, x_curr, h0);
    L_s1 = L_mac(L_s1, x_curr, h3);
    L_s1 = L_mac(L_s1, x_curr, h2);
}
```

h3 = h[j+1-i]; x_curr = X[j+1]; ➡ IL1-A ➡ L_s1 += X[j+0] * h[j+3-i];
h2 = h[j+2-i]; x_curr = X[j+2]; ➡ IL1-B ➡ L_s1 += X[j+1] * h[j+0-i];
h1 = h[j+3-i]; x_curr = X[j+3]; ➡ IL1-C ➡ L_s1 += X[j+2] * h[j+1-i];
h0 = h[j+4-i]; x_curr = X[j+4]; ➡ IL1-D ➡ L_s1 += X[j+3] * h[j+2-i];

Change h3 ↔ h1

```
{
    L_s2 = L_mac(L_s2, x_curr, h2);
    L_s2 = L_mac(L_s2, x_curr, h1);
    L_s2 = L_mac(L_s2, x_curr, h0);
    L_s2 = L_mac(L_s2, x_curr, h3);
}
```

h3 = h[j+1-i]; x_curr = X[j+1]; ➡ IL2-A ➡ L_s2 += X[j+0] * h[j+2-i];
h2 = h[j+2-i]; x_curr = X[j+2]; ➡ IL2-B ➡ L_s2 += X[j+1] * h[j+3-i];
h1 = h[j+3-i]; x_curr = X[j+3]; ➡ IL2-C ➡ L_s2 += X[j+2] * h[j+0-i];
h0 = h[j+4-i]; x_curr = X[j+4]; ➡ IL2-C ➡ L_s2 += X[j+3] * h[j+1-i];

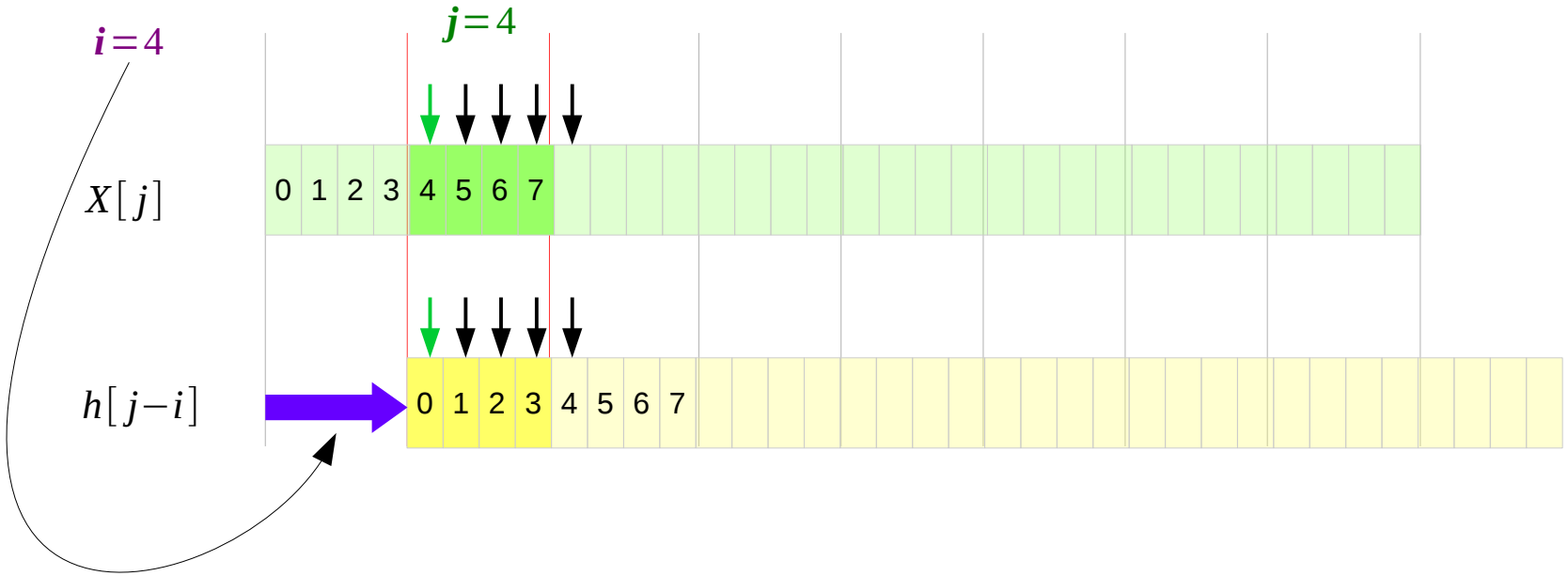
```
{
    L_s3 = L_mac(L_s3, x_curr, h3);
    L_s3 = L_mac(L_s3, x_curr, h2);
    L_s3 = L_mac(L_s3, x_curr, h1);
    L_s3 = L_mac(L_s3, x_curr, h0);
}
```

h3 = h[j+1-i]; x_curr = X[j+1]; ➡ IL3-A ➡ L_s3 += X[j+0] * h[j+1-i];
h2 = h[j+2-i]; x_curr = X[j+2]; ➡ IL3-B ➡ L_s3 += X[j+1] * h[j+2-i];
h1 = h[j+3-i]; x_curr = X[j+3]; ➡ IL3-C ➡ L_s3 += X[j+2] * h[j+3-i];
h0 = h[j+4-i]; x_curr = X[j+4]; ➡ IL3-D ➡ L_s3 += X[j+3] * h[j+0-i];

Updating h0, h1, h2, h3

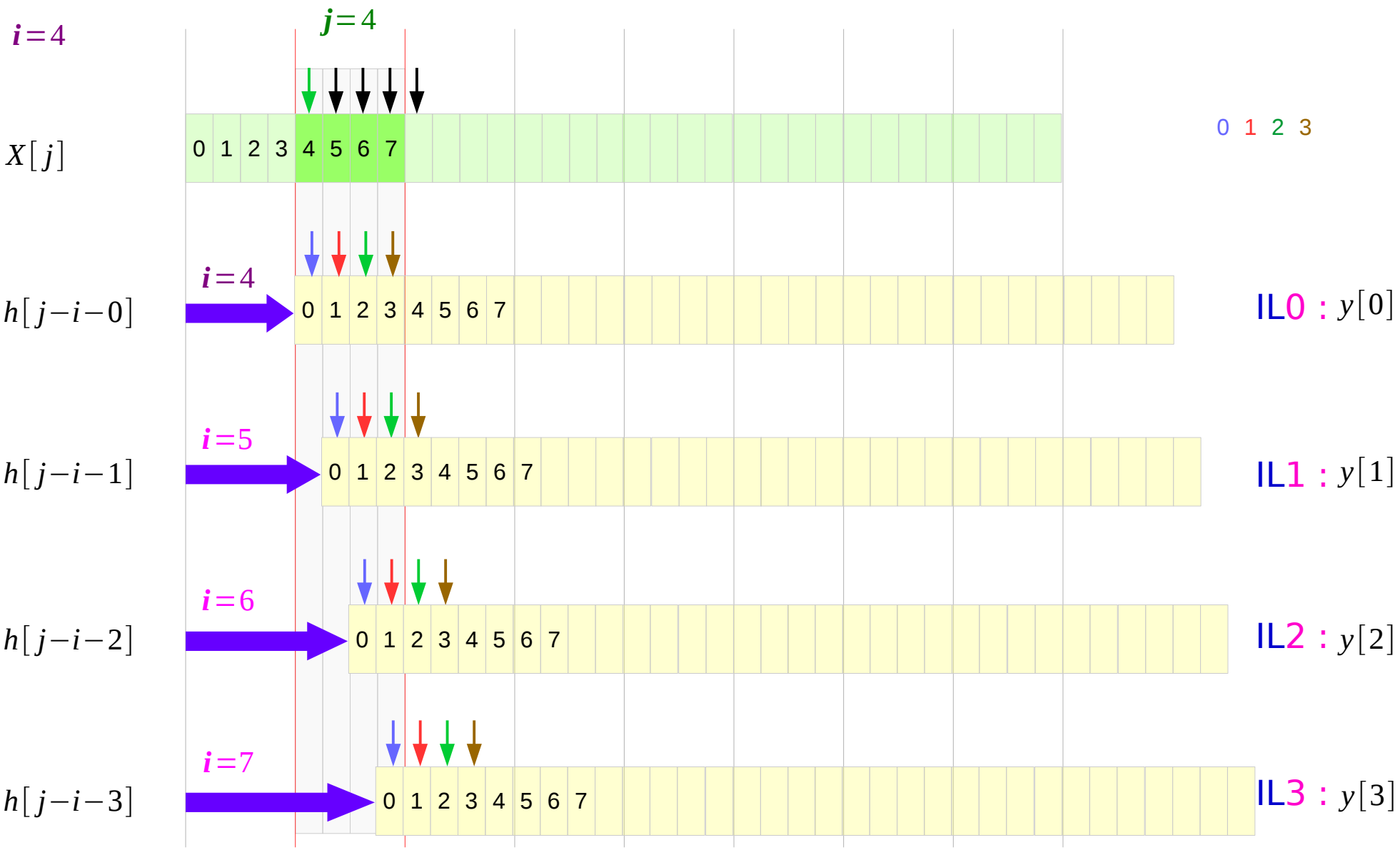
```
{                                   old
    L_s0 = L_mac(L_s0, x_curr, h0);     h1 = h[j+1-i]; x_curr = X[j+1];     L_s0 += X[j+0] * h[j+0-i];   old
    L_s0 = L_mac(L_s0, x_curr, h1);     h2 = h[j+2-i]; x_curr = X[j+2];     L_s0 += X[j+1] * h[j+1-i];
    L_s0 = L_mac(L_s0, x_curr, h2);     h3 = h[j+3-i]; x_curr = X[j+3];     L_s0 += X[j+2] * h[j+2-i];
    L_s0 = L_mac(L_s0, x_curr, h3);     h0 = h[j+4-i]; x_curr = X[j+4];     L_s0 += X[j+3] * h[j+3-i];
}

{                                   old
    L_s1 = L_mac(L_s1, x_curr, h3);     h1 = h[j+1-i]; x_curr = X[j+1];     L_s1 += X[j+0] * h[j+3-i];   old
    L_s1 = L_mac(L_s1, x_curr, h0);     h2 = h[j+2-i]; x_curr = X[j+2];     L_s1 += X[j+1] * h[j+0-i];
    L_s1 = L_mac(L_s1, x_curr, h1);     h3 = h[j+3-i]; x_curr = X[j+3];     L_s1 += X[j+2] * h[j+1-i];
    L_s1 = L_mac(L_s1, x_curr, h2);     h0 = h[j+4-i]; x_curr = X[j+4];     L_s1 += X[j+3] * h[j+2-i];
}

{                                   old
    L_s2 = L_mac(L_s2, x_curr, h2);     h1 = h[j+1-i]; x_curr = X[j+1];     L_s2 += X[j+0] * h[j+2-i];   old
    L_s2 = L_mac(L_s2, x_curr, h3);     h2 = h[j+2-i]; x_curr = X[j+2];     L_s2 += X[j+1] * h[j+3-i];
    L_s2 = L_mac(L_s2, x_curr, h0);     h3 = h[j+3-i]; x_curr = X[j+3];     L_s2 += X[j+2] * h[j+0-i];
    L_s2 = L_mac(L_s2, x_curr, h1);     h0 = h[j+4-i]; x_curr = X[j+4];     L_s2 += X[j+3] * h[j+1-i];
}

{                                   old
    L_s3 = L_mac(L_s3, x_curr, h1);     h1 = h[j+1-i]; x_curr = X[j+1];     L_s3 += X[j+0] * h[j+1-i];   old
    L_s3 = L_mac(L_s3, x_curr, h2);     h2 = h[j+2-i]; x_curr = X[j+2];     L_s3 += X[j+1] * h[j+2-i];
    L_s3 = L_mac(L_s3, x_curr, h3);     h3 = h[j+3-i]; x_curr = X[j+3];     L_s3 += X[j+2] * h[j+3-i];
    L_s3 = L_mac(L_s3, x_curr, h0);     h0 = h[j+4-i]; x_curr = X[j+4];     L_s3 += X[j+3] * h[j+0-i];
}
```

Accessing X[j] & h[j-i]

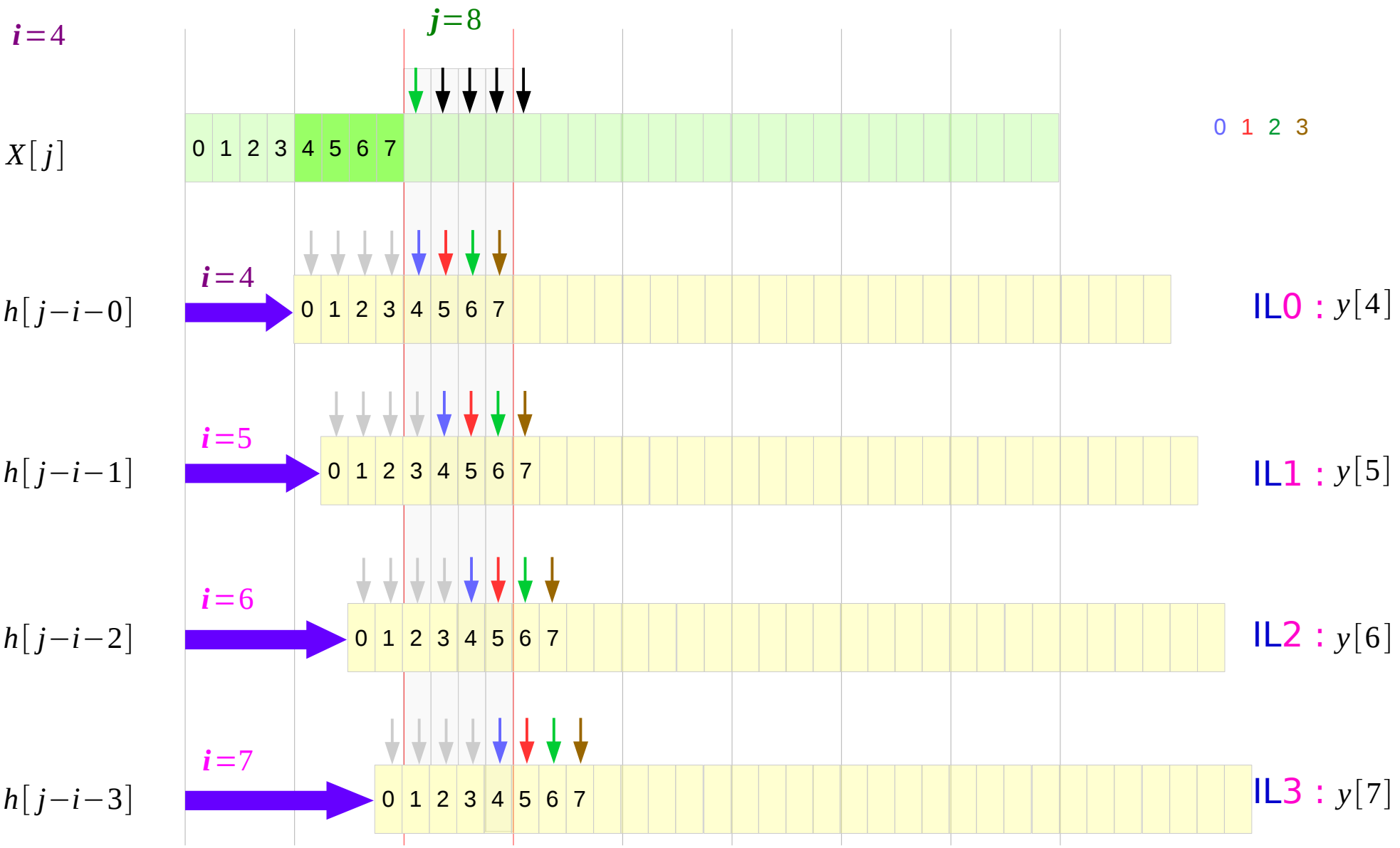



```
h1 = h[j+1-i] = h[j-i+1];      x_curr = X[j+1];
h2 = h[j+2-i] = h[j-i+2];      x_curr = X[j+2];
h3 = h[j+3-i] = h[j-i+3];      x_curr = X[j+3];
h0 = h[j+4-i] = h[j-i+4];      x_curr = X[j+4];
```

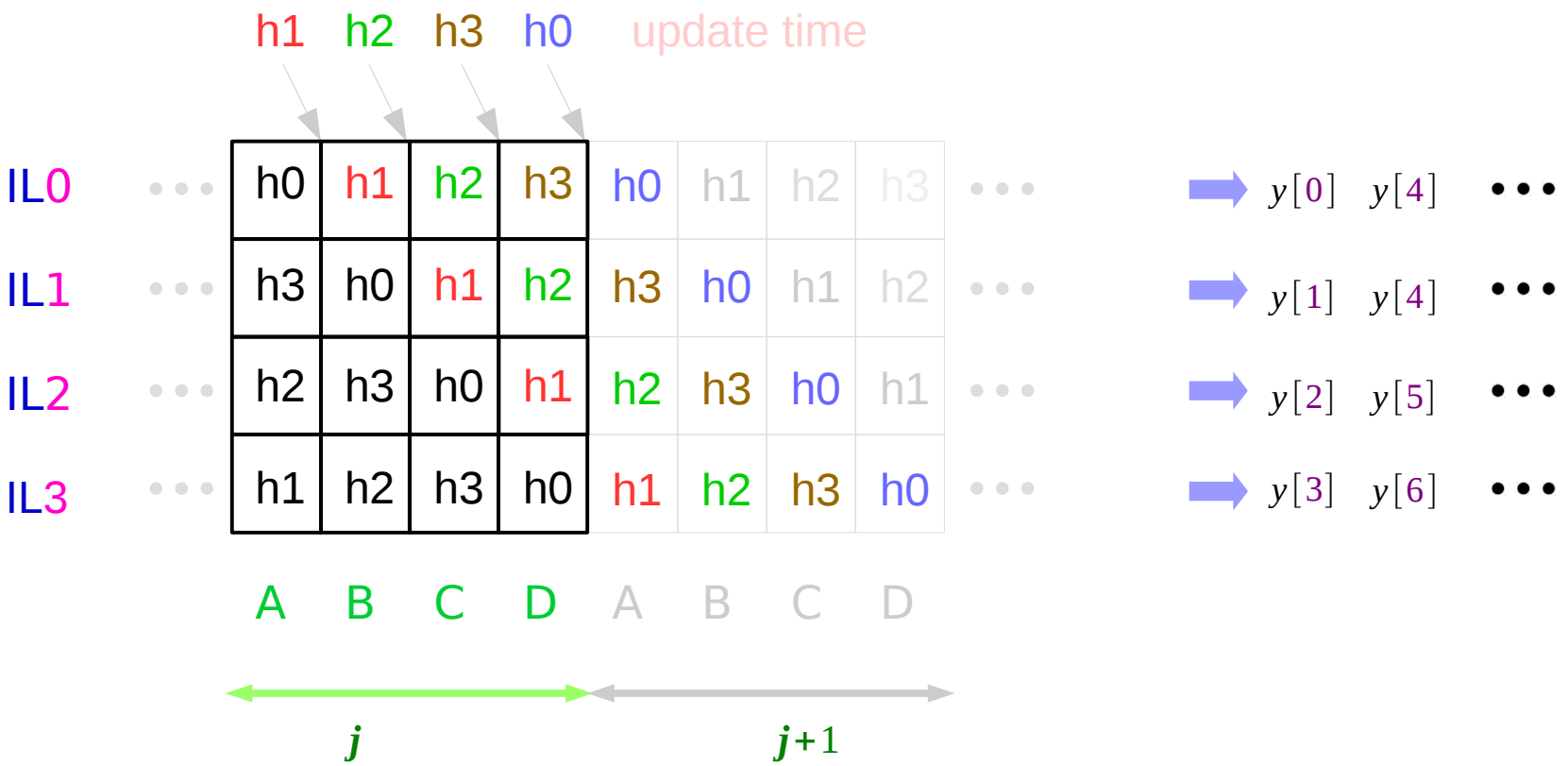
Accessing X[j] & h[j-i]



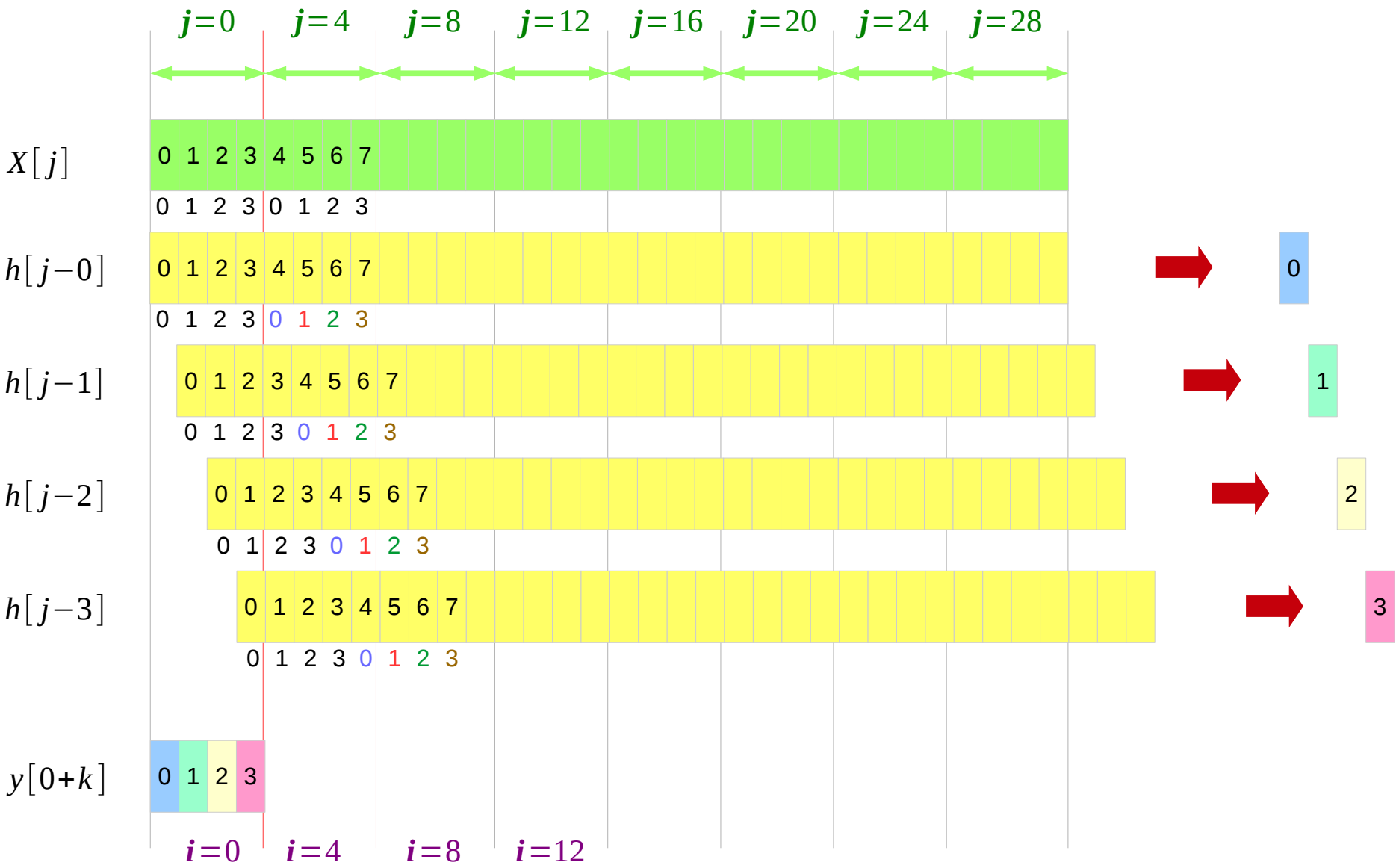
Accessing X[j] & h[j-i]



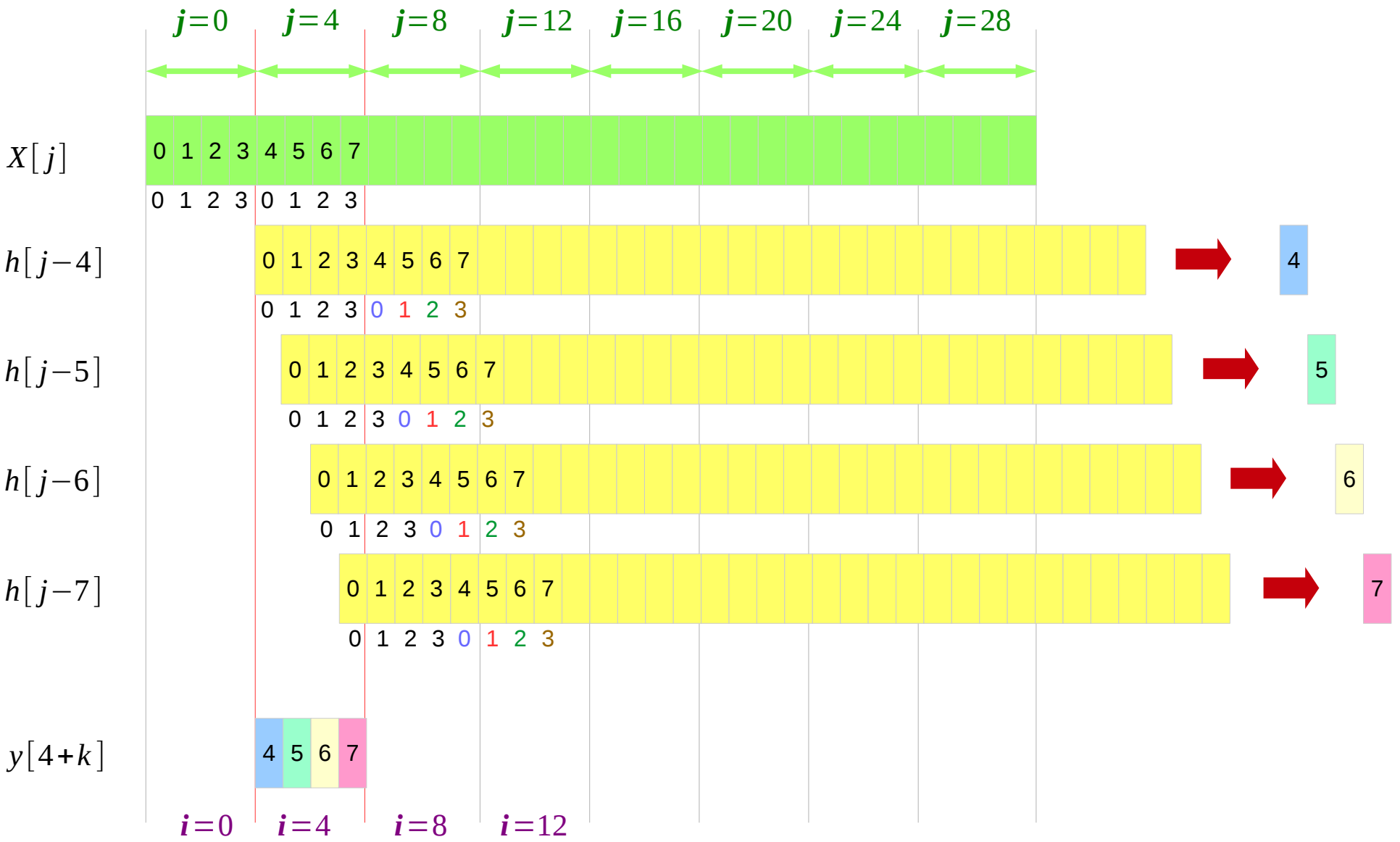
Data Access Patterns: h0, h1, h2, h3



Computing y[0], y[1], y[2], y[3]



Computing y[4], y[5], y[6], y[7]



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