First Order Logic – Semantics (3A)

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Based on

Contemporary Artificial Intelligence,
R.E. Neapolitan & X. Jiang

Logic and Its Applications,
Burkey & Foxley

Model

First specify a signature
- Constant Symbols
- Predicate Symbols
- Function Symbols

Determines the language

Given a language
- A model is specified
  - A domain of discourse
  - An interpretation

Model

1. a nonempty set D of **entities** called a **domain of discourse**
   - this domain is a set
   - each element in the set : entity
   - each constant symbol : one entity in the domain

2. an **interpretation**
   (a) an entity in D is assigned to each of the constant symbols.
   Normally, every entity is assigned to a constant symbol.
   (b) for each **function**,
   an entity is assigned to each possible input of entities to the **function**
   (c) the predicate '**True**' is always assigned the value T
   The predicate '**False**' is always assigned the value F
   (d) for every other **predicate**,
   the value T or F is assigned
   to each possible input of entities to the **predicate**

Model

Constant assignment

(a) an entity → the constant symbols.

Function assignments

(b) an entity → each possible input of entities to the **function**

Truth value assignments

(c) the value T →the predicate '**True**'
the value F → the predicate '**False**'

(d) for every other **predicate**,
the value T or F is assigned → every other predicate
to each possible input of entities to the **predicate**

Signature Model Examples

Signature

1. constant symbols = { Mary, Fred, Sam }
2. predicate symbols = { married, young }
   married(x, y) : arity two
   young(x) : arity one

Model

1. domain of discourse D : the set of three particular individuals
2. interpretation
   (a) a different individual is assigned to each of the constant symbols
   (b) the truth value assignments
   young(Mary) = F, young(Fred) = F, young(Sam) = T
   married(Mary, Mary) = F, married(Mary, Fred) = T, married(Mary, Sam) = F
   married(Fred, Mary) = T, married(Fred, Fred) = F, married(Fred, Sam) = F
   married(Sam, Mary) = F, married(Sam, Fred) = F, married(Sam, Sam) = F

Signature Model Examples

Signature

1. constant symbols = { Fred, Mary, Sam }
2. predicate symbols = { love } love(x, y) : arity two
3. function symbols = { mother } mother(x) : arity one

Model

1. domain of discourse D : the set of three particular individuals
2. interpretation
   (a) a different individual is assigned to each of the constant symbols
   (b) the truth value assignments
   love(Fred, Fred) = F, love(Fred, Mary) = F, love(Fred, Ann) = F
   love(Mary, Fred) = T, love(Mary, Mary) = F, love(Mary, Ann) = T
   love(Ann, Fred) = T, love(Ann, Mary) = T, love(Ann, Ann) = F
   (c) the function assignments
   mother(Fred) = Mary, mother(Mary) = Ann, mother(Ann) = - (no assignment)

The truth value and sentences

The truth values of **all sentences** are assigned :

1. the truth values for **sentences** developed with the symbols ¬, ∧, ∨, ⇒, ⇔ are assigned as in propositional logic.

2. the truth values for two terms connected by the = symbol is T if both terms refer to the same entity; otherwise it is F

3. the truth values for ∀x p(x) has value T if p(x) has value T for **every assignment** to x of an entity in the domain D; otherwise it has value F

4. the truth values for ∃x p(x) has value T if p(x) has value T for **at least one assignment** to x of an entity in the domain D; otherwise it has value F

5. the operator **precedence** is as follows ¬, =, ∧, ∨, ⇒, ⇔

6. the **quantifiers** have precedence over the operators

7. **parentheses** change the order of the precedence

Formulas and Sentences

An **formula**
- A **atomic formula**
- The operator ¬ followed by a **formula**
- Two formulas separated by $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$
- A quantifier following by a variable followed by a formula

A **sentence**
- A **formula** with no free variables

∀x love(x,y)	: free variable y	: not a sentence
∀x tall(x)	: no free variable	: a sentence

Finding the truth value

Find the truth values of **all sentences**

1. $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$

2. $=$ symbol

3. $\forall x$ p(x)

4. $\exists x$ p(x)

5. the **operator precedence** is as follows $\neg$, $=$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$

6. the **quantifiers** $\forall$ $\exists$ have precedence over the **operators**

7. **parentheses** change the order of the precedence

Formal Language

Signature

∀ ∃

Constant Symbols = {Socrates, Plato, Zeus, Fido}
Predicate Symbols = {human, mortal, legs} all arity one

Model
D: the set of these four particular individuals
Interpretation
(a) a different individual is assigned to each of the constant symbols
(b) the truth value assignment
human(Socrates)=T, human(Plato)=T, human(Zeus)=F, human(Fido)=F
mortal(Socrates)=T, mortal(Plato)=T, mortal(Zeus)=F, mortal(Fido)=T
legs(Socrates)=T, legs(Plato)=T, legs(Zeus)=T, legs(Fido)=T

Formal Language

Sentence 1: human(Zeus) $\wedge$ human(Fido) $\vee$ human(Socrates) T
Sentence 2: human(Zeus) $\wedge$ (human(Fido) $\vee$ haman(Socrates)) F
Sentence 3: $\forall x$ human(x)
Sentence 4: $\forall x$ mortal(x)
Sentence 5: $\forall x$ legs(x)
Sentence 6: $\exists x$ human(x)
Sentence 7: $\forall x$ (human(x) $\Rightarrow$ mortal(x))

References

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