State Monad Examples (3E)

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Based on

Haskell in 5 steps
https://wiki.haskell.org/Haskell_in_5_steps

Monad, Monoid

monad (plural monads)

- An ultimate atom, or simple, unextended point; something ultimate and indivisible.
- (mathematics, computing) A monoid in the category of endofunctors.
- (botany) A single individual (such as a pollen grain) that is free from others, not united in a group.

monoid (plural monoids)

- (mathematics) A **set** which is closed under an associative binary operation, and which contains an element which is an identity for the operation.

class Monad **m** where ...

m a

m b

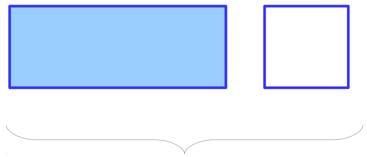
instance Monad **Maybe** where ...

m a

m b

Maybe a

single parameter



Monadic type

https://en.wiktionary.org/wiki/monad, monoid

Maybe Monad

```
class Monad m where
  return :: a -> m a
  (>>=)  :: m a -> (a -> m b) -> m b




instance Monad Maybe where
   -- return      :: a -> Maybe a
   return x       =  Just x

   -- (>>=)       :: Maybe a -> (a -> Maybe b) -> Maybe b
   Nothing   >>= _ =  Nothing
   (Just x)  >>= f =  f x
```

```
   f :: a -> m b
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Maybe Monad

a monad is a parameterized type **m**
that supports **return** and **>>=** functions of the specified types

m must be a parameterized type,
rather than just a type (**Maybe** is not a concrete type)

It is because of this declaration
that the **do** notation can be used to sequence **Maybe** values.

(Just x) **>>=** f = f x

More generally, Haskell supports the use of this notation
with any monadic type.

examples of types that are monadic,
the benefits that result from recognizing and exploiting this fact.

class Monad **m** where ...

m a

m b

instance Monad **Maybe** where ...

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

List Monad

The **Maybe** monad provides a simple model of computations that can fail,

a value of type Maybe a is either **Nothing** (failure)

the form Just x for some x of type a (success)

The **list** monad generalises this notion,

by permitting multiple results in the case of success.

More precisely, a value of [a] is

either the empty list **[ ]** (failure)

or the form of a non-empty list [x1,x2,...,xn] (success)

for some xi of type a

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

List Monad

```
instance Monad [] where
   -- return :: a -> [a]
   return x  =  [x]

   -- (>>=)  :: [a] -> (a -> [b]) -> [b]
   xs >>= f  =  concat (map f xs)
```

return converts a value into a *successful* result containing that value

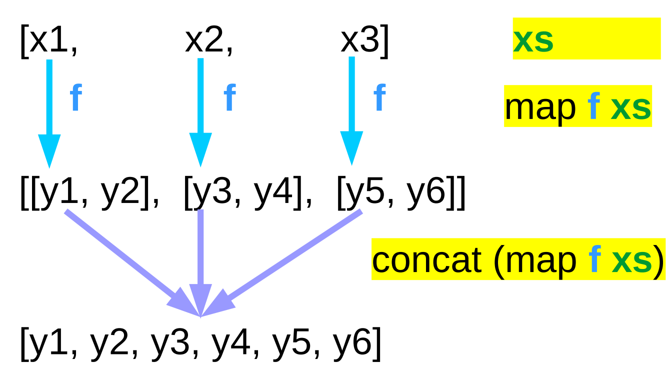
>>= provides a means of *sequencing* computations
that may produce *multiple results*:

xs >>= **f** applies the function **f** to each of the *results* in the list **xs**
to give a *nested list* of *results*,
which is then concatenated to give a *single list* of *results*.

(Aside: in this context, [] denotes the list type [a] without its parameter.)

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

xs :: [a]
f :: a -> [b]
(>>=) :: [a] -> (a -> [b]) -> [b]



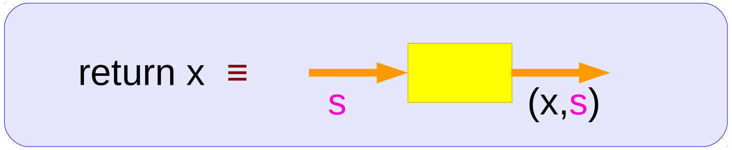
List Monad and ST Monad

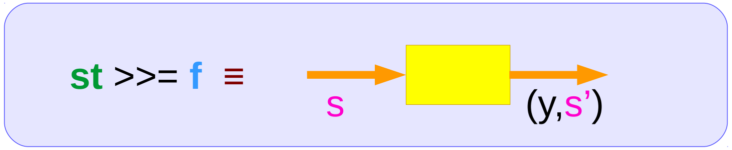
```
instance Monad [] where
  -- return :: a -> [a]
  return x  =  [x]

  -- (>>=)  :: [a] -> (a -> [b]) -> [b]
  xs >>= f  =  concat (map f xs)
```

```
instance Monad ST where
  -- return :: a -> ST a
  return x  =  \s -> (x,s)

  -- (>>=)  :: ST a -> (a -> ST b) -> ST b
  st >>= f  =  \s -> let (x,s') = st s in f x s'
```

(x,s) → (x,s') → (y,s')

(x,s') = st s   f x s'





https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

A State Transformer

```
type State = ...

type ST = State -> State
```

the problem of writing functions that manipulate some kind of state, represented by a type, whose detail is not our concern now.

a state transformer (ST), which takes the current state as its argument, and produces a modified state as its result, which reflects any side effects performed by the function:

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

A Generalized State Transformer

```
type State = ...

type ST = State -> State

type ST a = State -> (a, State)
```

want to return a result value in addition to the modified state
generalized state transformers also return a result value,
as a parameter of the **ST** type

```
State -> (a, State)
   s   ->  (v, s')
```

s: input state, v: the result value, s': output state

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Returning a result value

type **ST** a = State -> (a, State)   generalized ST

st s (x, s')

st :: **ST** a
s :: State
st s :: **ST** a State

x :: a
s' :: State
(x, s') :: (a, State)

type **ST** a State = (a, State)

st s (x, s')

to return a result value in addition to updating the state

return a result through a parameter of the ST type:

(a_result, updated_state) :: (a, State)

st s :: **ST** a State   st s :: (a, State)
(x, s') :: **ST** a State   (x, s') :: (a, State)

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Taking an argument

a state transformer that takes a character
and returns an integer

type ST a = State -> (a, State)   generalized ST

possible further generalization of the state transformer ST
which takes an argument of type b

type STT a **b** = **b** -> State -> (a, State)   further generalized ST
type STT **b** a = **b** -> State -> (a, State)   further generalized ST

no need to use more generalized ST type
can be exploiting currying.

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

A Curried Generalized State Transformer

type ST a = State -> (a, State)	generalized ST
type STT **b** a = **b** -> State -> (a, State)	further generalized ST
b -> ST a = **b** -> State -> (a, State)	think currying

a state transformer that takes a character and returns an integer would have type Char -> ST Int

Char -> State -> (**Int**, State)	**curried form**

* Curried Function

f x y	**f :: a -> b -> c**
(f x) y	**f :: a -> (b -> c)**

f x returns a function of type **b -> c**

g y	**g :: b -> c**

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Two State Transformers

```
instance Monad ST where
  -- return :: a -> ST a
  return x  =  \s -> (x,s)

  -- (>>=)  :: ST a -> (a -> ST b) -> ST b
  st >>= f  =  \s -> let (x,s') = st s in f x s'
```

>>= provides a means of sequencing state transformers:
st >>= f applies the state transformer **st** to an initial state s,
then applies the function **f** to the resulting value x
to give a second state transformer (**f** x),
which is then applied to the modified state s' to give the final result:

```
st >>= f  =  \s -> f x s'
        where (x,s') = st s

st >>= f  =  \s -> (y,s')
        where (x,s') = st s
              (y,s') = f x s'
```

(x,s') = st s

f x s'

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

The type of the sequencer **>>=**

```
instance Monad ST where
  -- return :: a -> ST a
  return x  =  \s -> (x,s)

  -- (>>=)  :: ST a -> (a -> ST b) -> ST b
  st >>= f  =  \s -> let (x,s') = st s in f x s'

st :: ST a
f :: a -> ST b
(>>=) :: ST a -> (a -> ST b) -> ST b

st :: State -> (a, State)
f :: a -> State -> (b, State)
(>>=) :: State -> (a, State) -> (a -> State -> (b, State)) -> State -> (b, State)
```

```
(x,s') = st s          s → (x,s')
(y,s')= f x s'         s' → (y,s')
```

```
type ST a = State -> (a, State)
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

The type of st s and f x s’

```
st :: State -> (a, State)
f :: a -> State -> (b, State)
(>>=) :: State -> (a, State) -> (a -> State -> (b, State)) -> State -> (b, State)
```

```
st :: State -> (a, State)

st s ::  (b, State)
```

(x,s') = st s   s → (x,s')

```
f :: a -> State -> (b, State)

f  x :: State -> (b, State)

f  x s’ :: (b, State)
```

(y,s')= f x s'   s' → (y,s')

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

let … in …

```
cylinder :: (RealFloat a) => a -> a -> a
cylinder r h =
    let sideArea = 2 * pi * r * h
        topArea = pi * r ^2
    in  sideArea + 2 * topArea
```

The form is **let** <bindings> **in** <expression>.

The names that you define in the **let** part
are accessible to the expression after the **in** part.

Notice that the names are also aligned in a single column.

For now it just seems that **let** puts the bindings first
and the expression that uses them later
whereas where is the other way around.

http://learnyouahaskell.com/syntax-in-functions

List, Monad, and ST Monads

```
instance Monad [] where
  -- return :: a -> [a]
  return x  =  [x]

  -- (>>=)  :: [a] -> (a -> [b]) -> [b]
  xs >>= f  =  concat (map f xs)
```

```
instance Monad ST where
  -- return :: a -> ST a
  return x  =  \s -> (x,s)

  -- (>>=)  :: ST a -> (a -> ST b) -> ST b
  st >>= f  =  \s -> let (x,s') = st s in f x s'
```

```
instance Monad Maybe where
  -- return      :: a -> Maybe a
  return x       =  Just x

  -- (>>=)       :: Maybe a -> (a -> Maybe b) -> Maybe b
  Nothing   >>= _ =  Nothing
  (Just x)  >>= f =  f x
```

return x ≡

st >>= f ≡

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Dummy Constructor DC

```
type ST a = State -> (a, State)
```

generalized ST

```
data ST0 a = DC (State -> (a, State))
```

types defined using the **type** mechanism
cannot be made into instances of classes.

types defined using the **data** mechanism
can be made into instances of classes.
but requires a dummy constructor (DC)

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

The application function **apply0**

```
type ST a = State -> (a, State)                TYPE – NO INSTANCE

data ST0 a = DC (State -> (a, State))          DATA – INSTANCE ok
```

In order to strip away the dummy constructor,
define also the application function **apply0**

```
apply0 :: ST0 a -> State -> (a, State)
           input     output
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Removing Data Constructor

```
data ST0 a = DC (State -> (a, State))          Data Constructor

apply0  :: ST0 a -> State -> (a, State)         Application Function

        x :: a
        f :: a -> State -> (b, State)
        f x :: State -> (b, State)              State Transformer

(DC f) x :: ST0 a

apply0 (DC f) x :: State -> (b, State)

apply0 (DC f) x = f x                           Definition to remove DC
```

```
(.)     :: (b->c) -> (a->b) -> (a->c)
f . g   = \x -> f (g x)
f . g  x = f (g x)

DC f
DC (a -> State -> (b, State))
a -> DC (State -> (b, State))

(DC f) x :: DC (State -> (b, State))
(DC f) x :: ST0 a
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

apply0 st s and apply0 f x s’

```
type ST a = State -> (a, State)

data ST0 a = DC (State -> (a, State))

apply0  :: ST0 a -> State -> (a, State)
apply0 (DC f) x = f x
```

```
apply0 st s =  (x,s')          s → (x,s')
apply0 f x s =  (y,s')         s' → (y,s')
```

```
st :: State -> (a, State)
st s ::  (b, State)

f :: a -> State -> (b, State)
f  x :: State -> (b, State)
f  x s’ :: (b, State)
```

```
(x,s') = st s          s → (x,s')
(y,s')= f x s'         s' → (y,s')
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

ST0 Monad

```
instance Monad ST0 where
  -- return :: a -> ST a
 return x   = DC (\s -> (x,s))

  -- (>>=)  :: ST a -> (a -> STT b) -> ST b
  st >>= f   = DC ( \s -> let (x, s') = apply0 st s in apply0 (f x) s' )
```

```
apply0 st s =  (x,s')            s → (x,s')
apply0 f x s =  (y,s')           s' → (y,s')
```

the runtime <u>overhead</u> of manipulating the dummy constructor S
can be <u>eliminated</u> by defining ST using the **newtype** mechanism

```
instance Monad ST where
    -- return :: a -> ST a
    return x  =  \s -> (x,s)

    -- (>>=)  :: ST a -> (a -> ST b) -> ST b
    st >>= f  =  \s -> let (x,s') = st s in f x s'
```

```
(x,s') = st s          s → (x,s')
(y,s')= f x s'         s' → (y,s')
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

ST0 Monad Summary

```
type ST a = State -> (a, State)                    generalized ST
data ST0 a = DC (State -> (a, State))

apply0  :: ST0 a -> State -> (a, State)
apply0 (DC f) x = f x

apply0 st s = (x,s')
apply0 f x s = (y,s')

instance Monad ST0 where
  -- return :: a -> ST a
 return x   = DC (\s -> (x,s))

  -- (>>=)  :: ST a -> (a -> STT b) -> ST b
  st >>= f   = DC (\s -> let (x, s') = apply0 st s in apply0 (f x) s')
```

```
(x,s') = st s          s → (x,s')
(y,s')= f x s'         s' → (y,s')
```

```
instance Monad ST where
   -- return :: a -> ST a
   return x  =  \s -> (x,s)

   -- (>>=)  :: ST a -> (a -> ST b) -> ST b
   st >>= f  =  \s -> let (x,s') = st s in f x s'
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Examples I (1)

```
pairs :: [a] -> [b] -> [(a,b)]
pairs xs ys =  do x <- xs
                  y <- ys
                  return (x, y)
```

do method

this function returns all possible ways of pairing elements from two lists

each possible value x from the list **xs** — x <- **xs**

each possible value y from the list **ys** — y <- **ys**

return the pair (x,y).

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Examples I (2)

```
pairs :: [a] -> [b] -> [(a,b)]
pairs xs ys =  do x <- xs
                  y <- ys
                  return (x, y)
```

do notation

```
pairs xs ys = [(x,y) | x <- xs, y <- ys]
```

comprehension notation

In fact, there is a formal connection
between the **do** notation and the **comprehension** notation.
Both are simply different shorthands
for repeated use of the **>>=** operator for lists.

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Example II (1)

```
(>>) :: Monad m => m a -> m b -> m b;
```

a1 >> a2 takes the actions a1 and a2 and
returns the mega action which is
a1-then-a2-returning-the-value-returned-by-a2.

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Simple Examples (1)

```
type State = Int

fresh :: ST0 Int
fresh =  DC (\n -> (n, n+1))

wtf1 = fresh >>
           fresh >>
           fresh >>
           fresh

ghci> apply0 wtf1 0
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Simple Examples (2)

the >>= sequencer is kind of like >>

only it allows you to “remember” intermediate values
that may have been returned.

return :: a -> ST0 a

takes a value x and yields an action
that doesnt actually transform the state,
but just returns the same value x

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Simple Examples (2)

```
wtf2 = fresh >>= \n1 ->
          fresh >>= \n2 ->
          fresh >>
          fresh >>
          return [n1, n2]

wtf2' = do {n1 <- fresh;
                 n2 <- fresh;
                 fresh ;
                 fresh ;
                 return [n1, n2];
          }

ghci> apply0 wtf2 0
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Simple Examples (3)

```
wtf3 = do  n1 <- fresh
           fresh
           fresh
           fresh
           return n1
```

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Dice Examples

to generate Int dice - result : a number between 1 and 6
throw results from a pseudo-random generator of type StdGen.

the type of the **state processors** will be

State StdGen Int

StdGen -> (Int, StdGen)

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

randomR

the StdGen type : an instance of **RandomGen**

randomR :: (**Random** a, **RandomGen** g) => (a, a) -> g -> (a, g)

assume a is Int and g is StdGen

the type of **randomR**

randomR (1, 6) :: StdGen -> (Int, StdGen)

already have a **state processing function**

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

randomR

```
randomR (1, 6) :: StdGen -> (Int, StdGen)

rollDie :: State StdGen Int
rollDie = state $ randomR (1, 6)
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

Some Examples (1)

```
module StateGame where

import Control.Monad.State

-- Example use of State monad
-- Passes a string of dictionary {a,b,c}
-- Game is to produce a number from the string.
-- By default the game is off, a C toggles the
-- game on and off. A 'a' gives +1 and a b gives -1.
-- E.g
-- 'ab'    = 0
-- 'ca'    = 1
-- 'cabca' = 0
-- State = game is on or off & current score
--       = (Bool, Int)
```

https://wiki.haskell.org/State_Monad

Some Examples (2)

```
type GameValue = Int
type GameState = (Bool, Int)

playGame :: String -> State GameState GameValue
playGame []     = do
    (_, score) <- get
    return score
```

https://wiki.haskell.org/State_Monad

Some Examples (3)

```
playGame (x:xs) = do
    (on, score) <- get
    case x of
        'a' | on -> put (on, score + 1)
        'b' | on -> put (on, score - 1)
        'c'      -> put (not on, score)
        _        -> put (on, score)
    playGame xs

startState = (False, 0)

main = print $ evalState (playGame "abcaaacbbcabbab") startState
```

https://wiki.haskell.org/State_Monad

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf