

Capacitor in an AC circuit

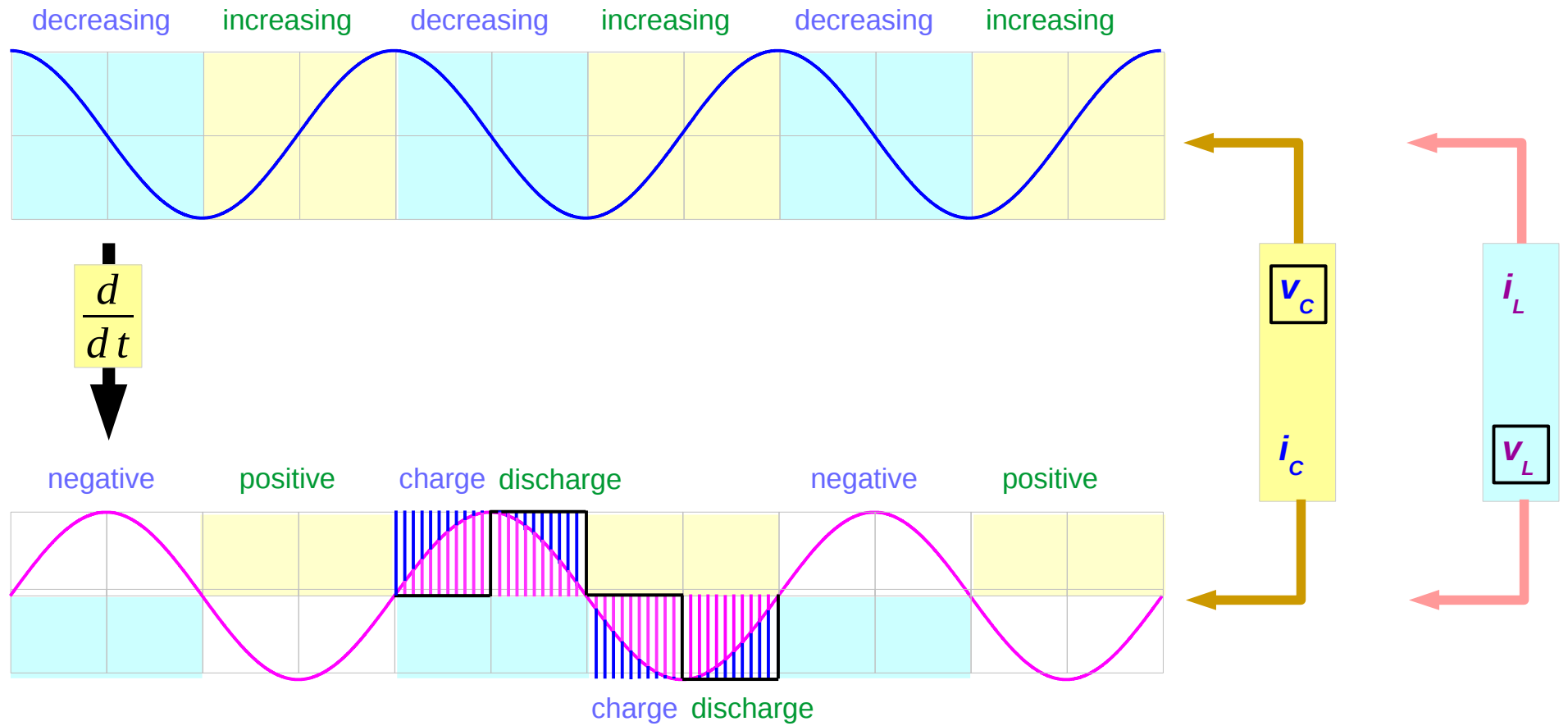
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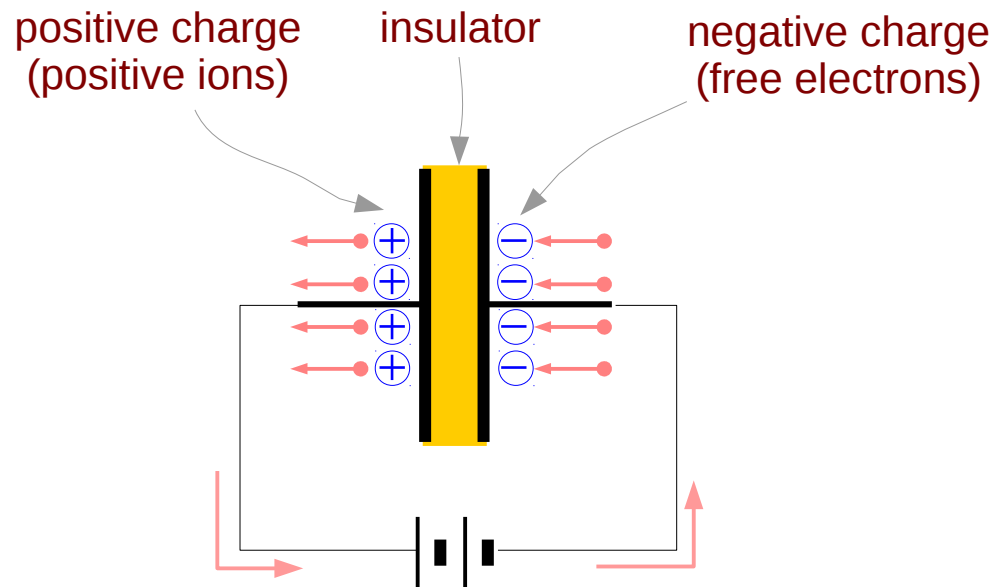
Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

Everchanging signal pairs



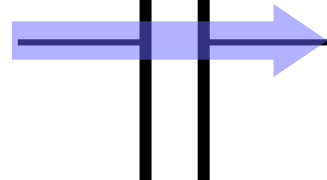
Capacitor Current



No actual electrons movement across insulator materials



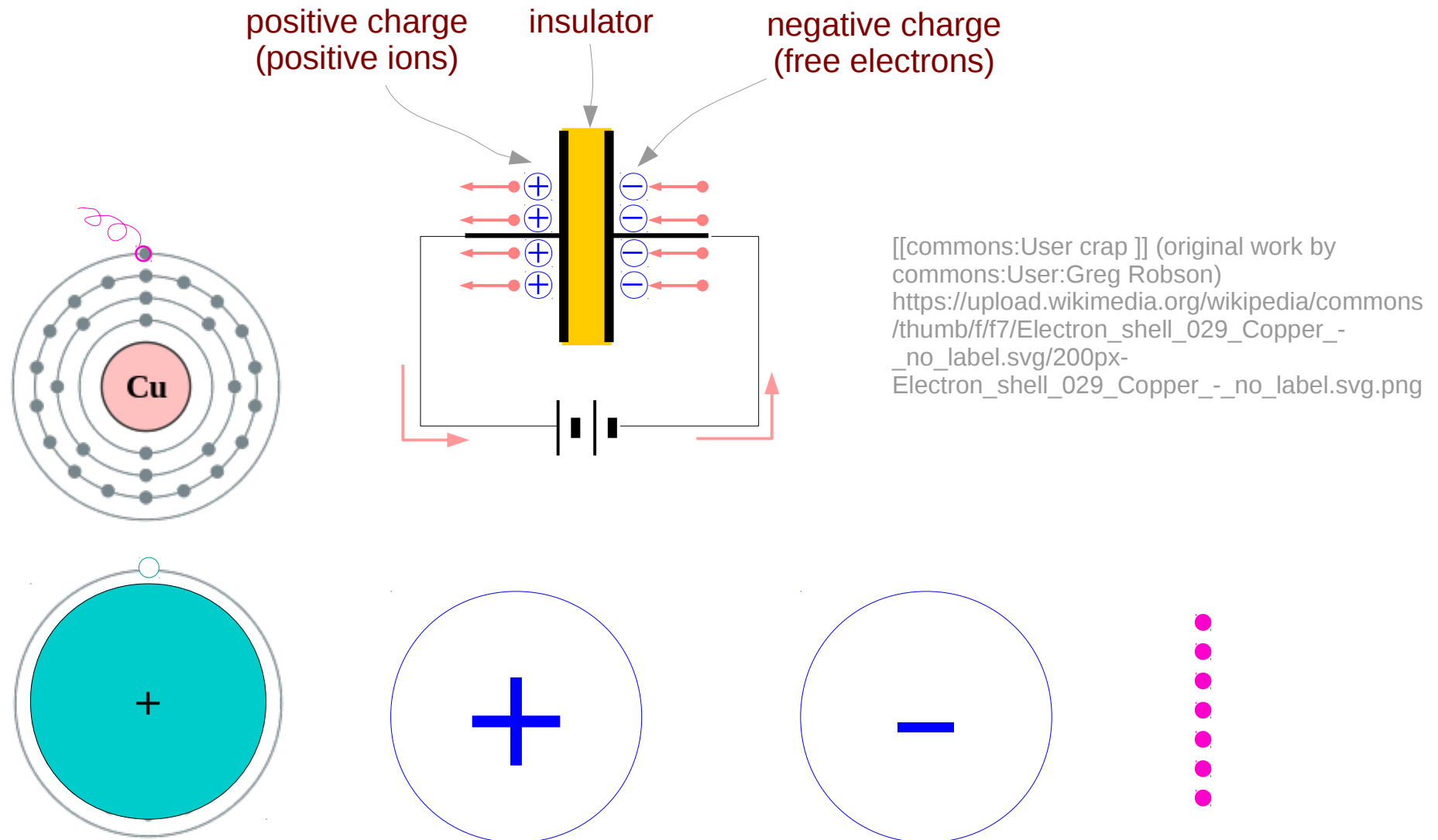
But, think as



Displacement Current

flows through the capacitor

Positive ions and free electrons



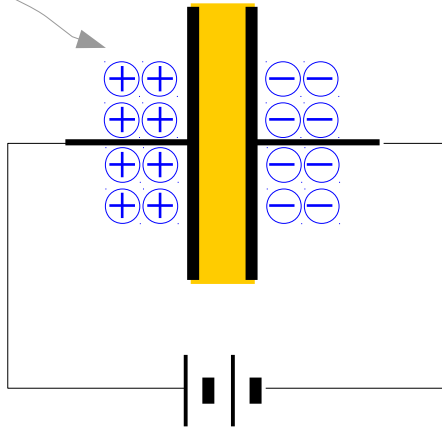
[[commons:User crap]] (original work by commons:User:Greg Robson)
https://upload.wikimedia.org/wikipedia/commons/thumb/f/f7/Electron_shell_029_Copper_-_no_label.svg/200px-Electron_shell_029_Copper_-_no_label.svg.png

Three States

positive charge
(positive ions)

Positively Charged State

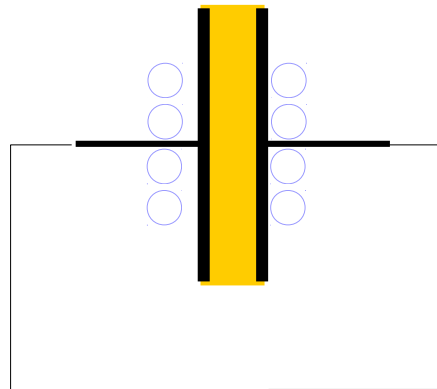
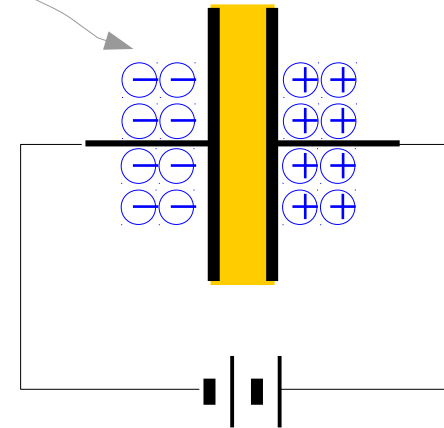
fully charged → no current



negative charge
(free electrons)

Negatively Charged State

fully charged → no current

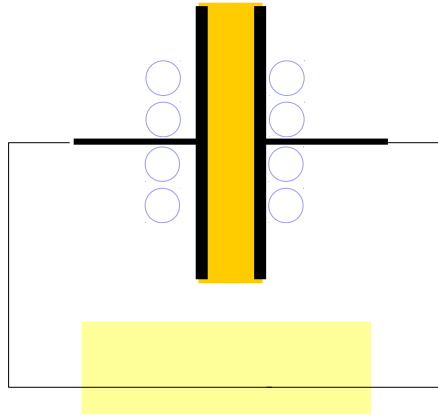


Fully Discharged State

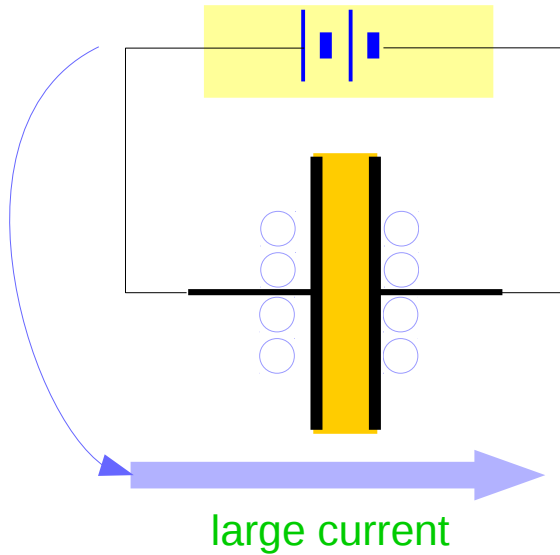
possible large current

Currents in the Fully Discharged State

Initially no current



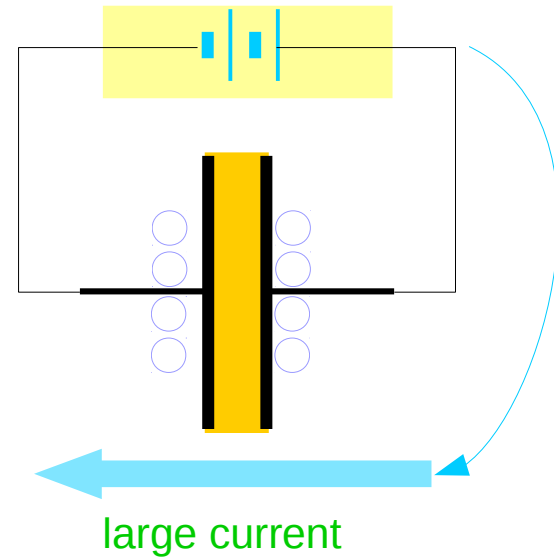
Fully Discharged State



Fully Discharged State

This state can flow large current in either direction

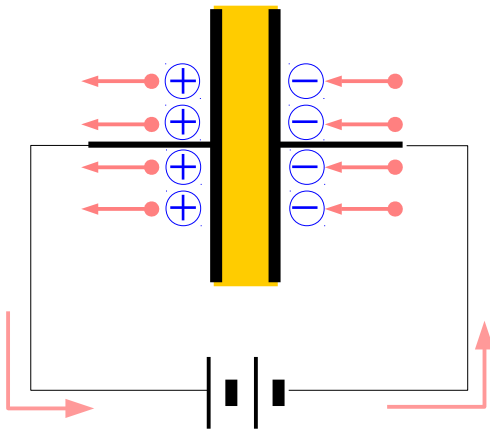
Fully Discharged State



Inter-State Current Flowing

Under Positively Charging

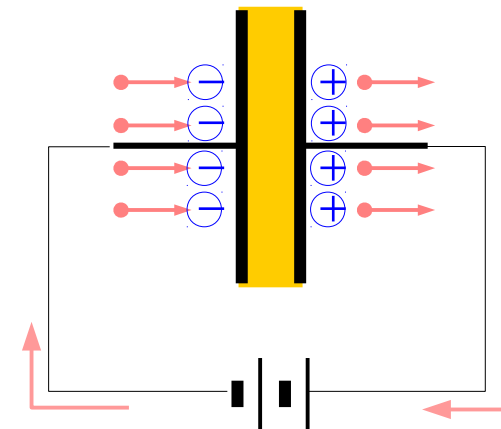
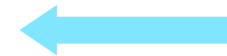
(+) current flow direction



electron flow direction

Under Negatively Charging

(-) current flow direction

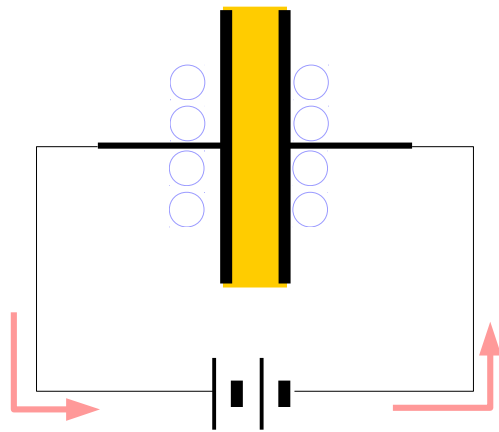


electron flow direction

Inter-State Current Flowing

Fully Discharged State

(+) current flow direction

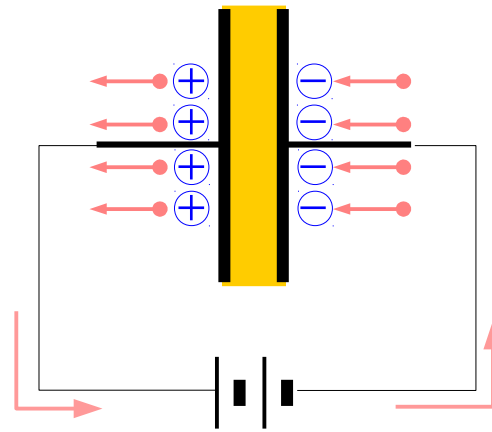


electron flow direction

Initial large current

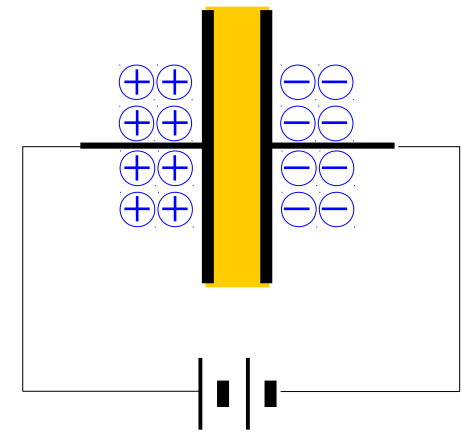
Under Positively Charging

(+) current flow direction



electron flow direction

Positively Charged State



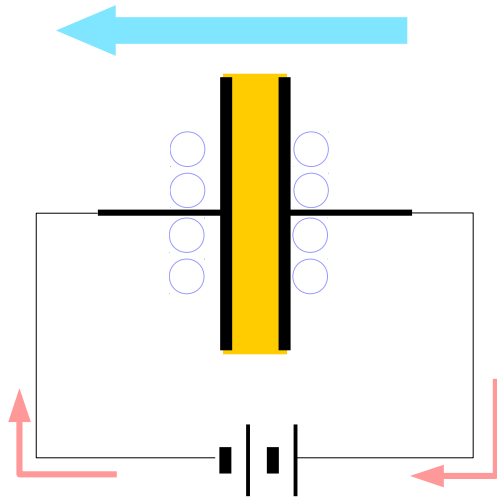
Crowded →
No more space

no current

Inter-State Current Flowing

Fully Discharged State

(-) current flow direction

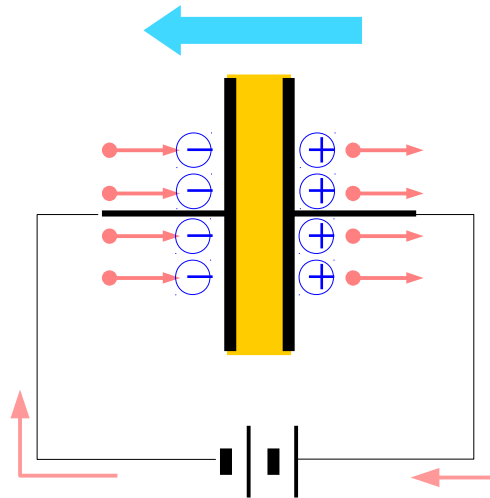


electron flow direction

Initial large current

Under Negatively Charging

(-) current flow direction

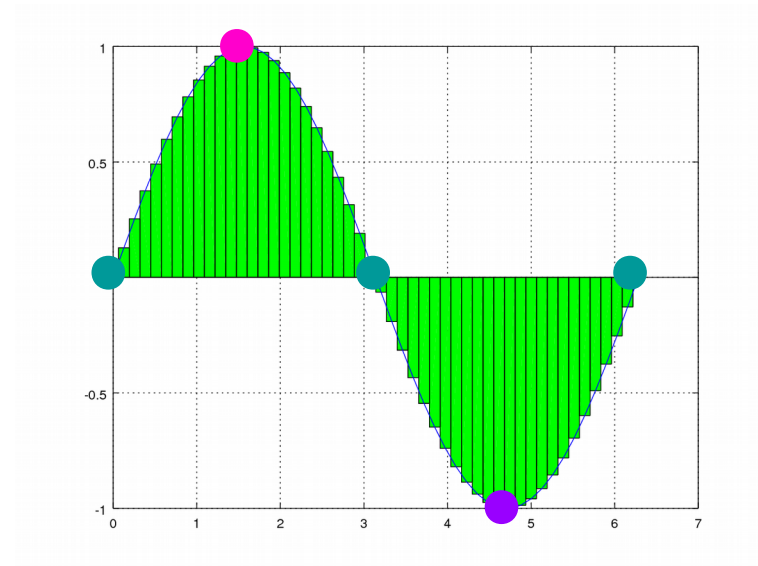
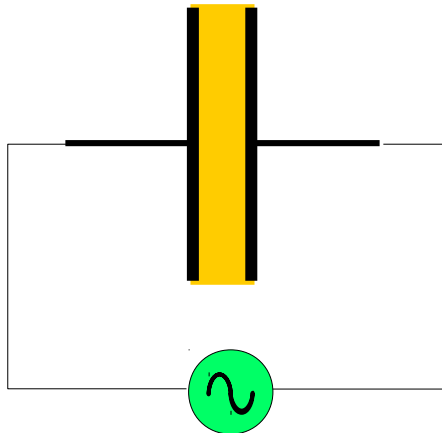


electron flow direction

Crowded →
No more space

no current

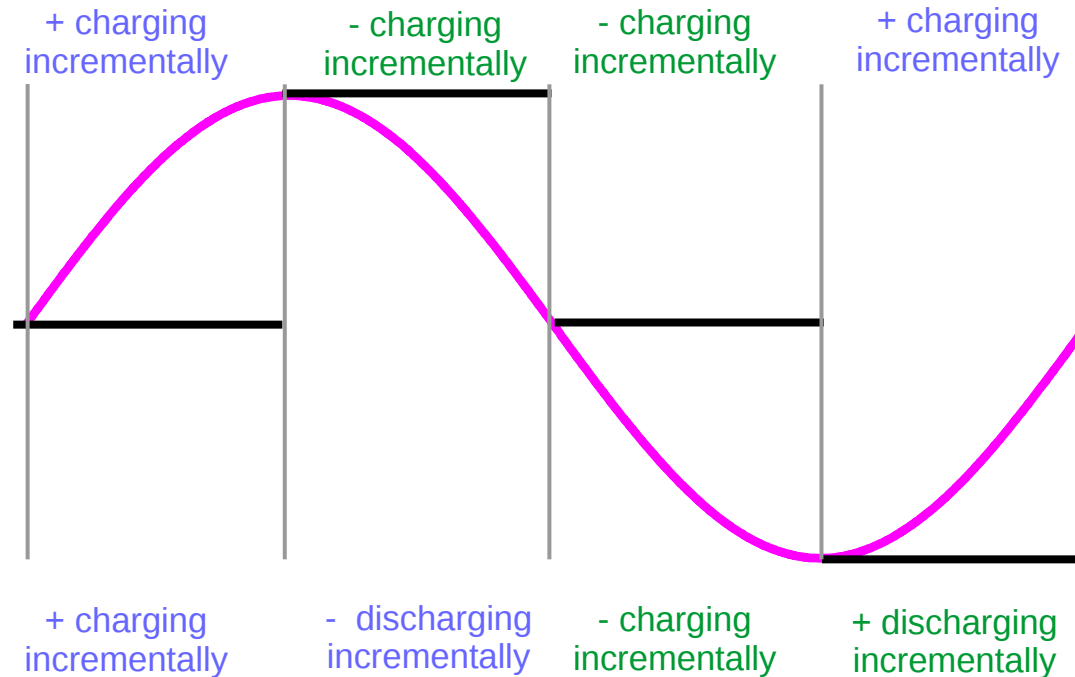
An AC Voltage Source



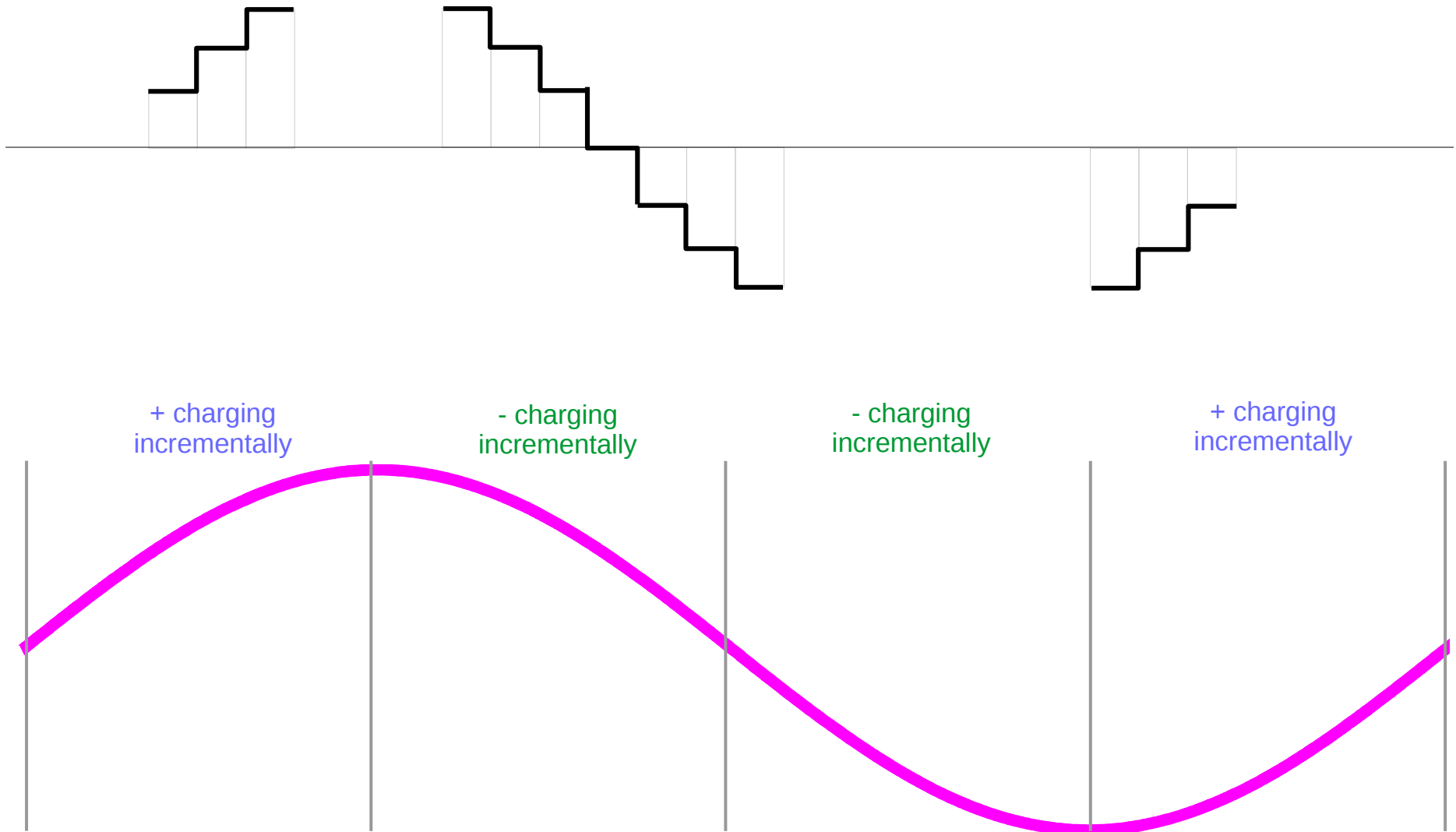
Continuous (Ever-) Charging Operations

Incremental Voltage Increment \Rightarrow + Charging incrementally

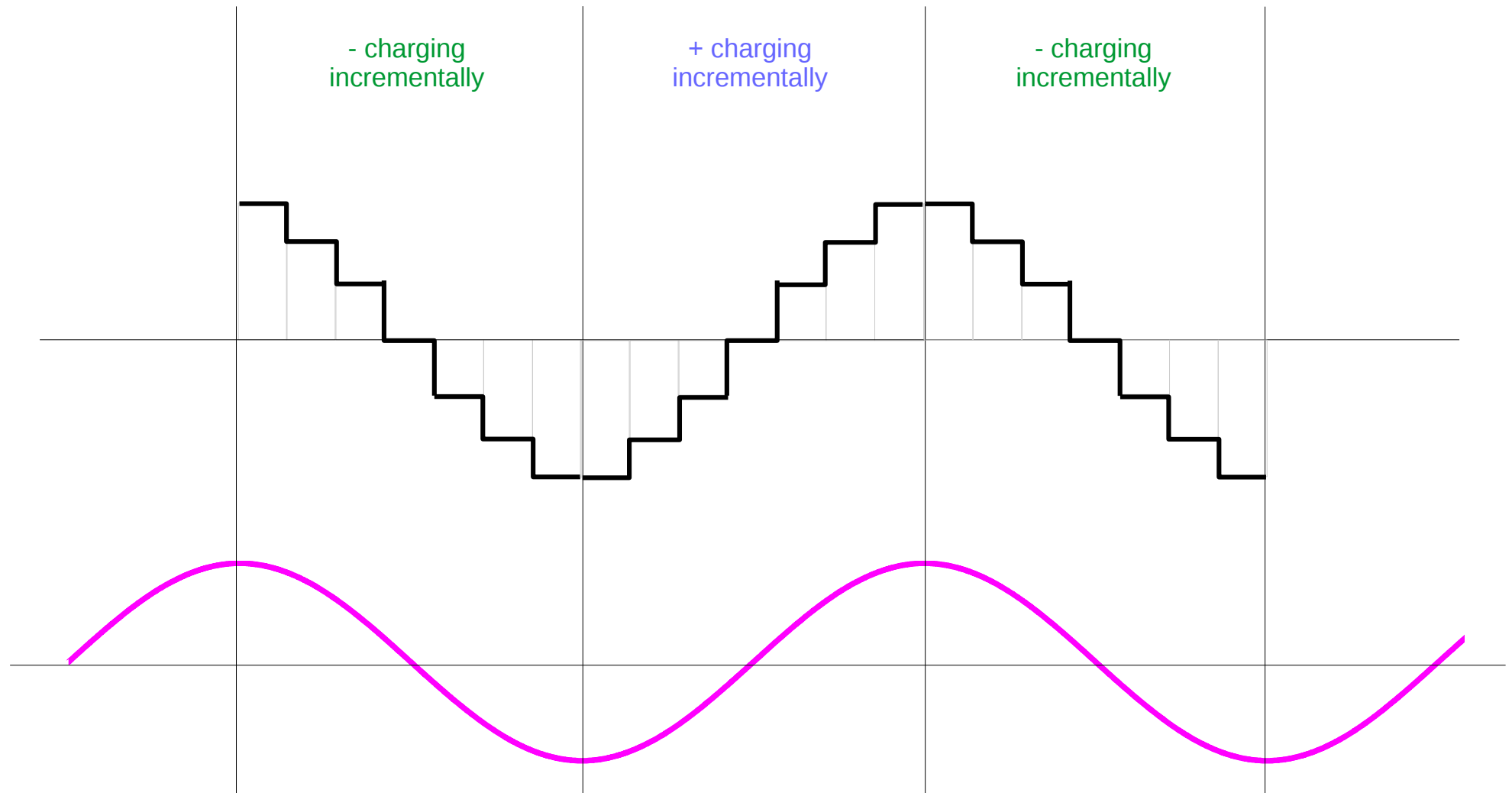
Incremental Voltage Decrement \Rightarrow - Charging incrementally



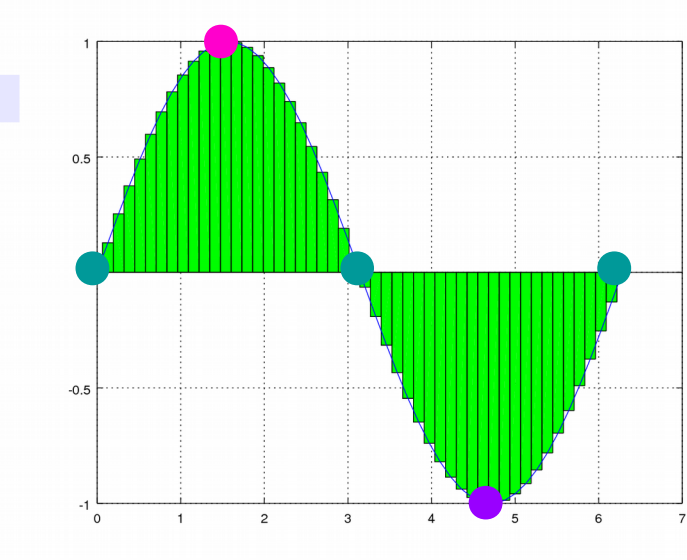
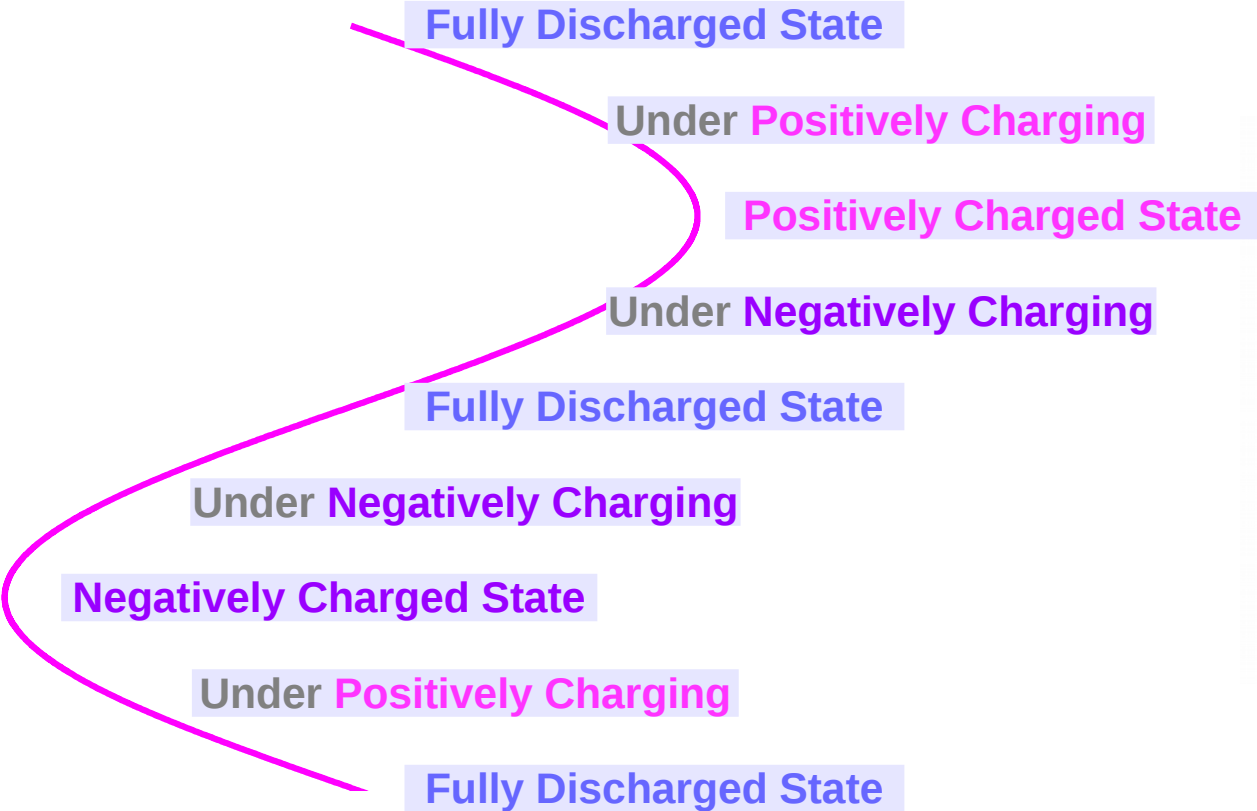
Continuous Charging and Discharging Operations



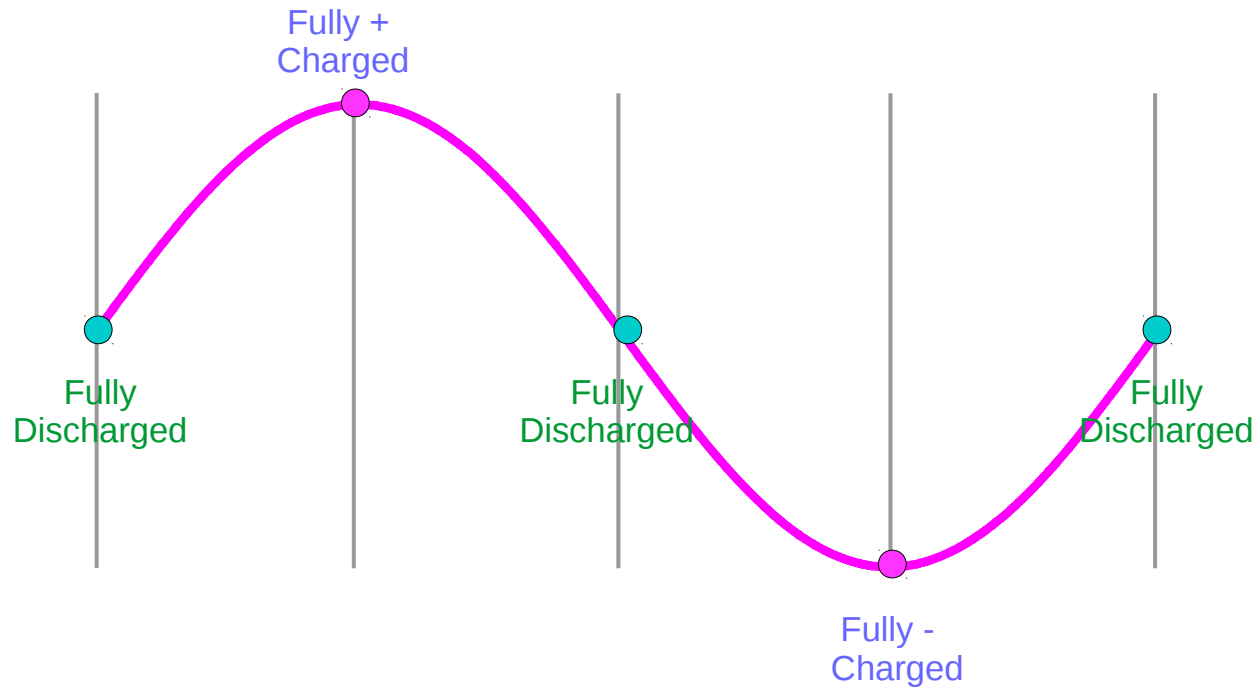
Incrementally Charging



An AC Voltage Source

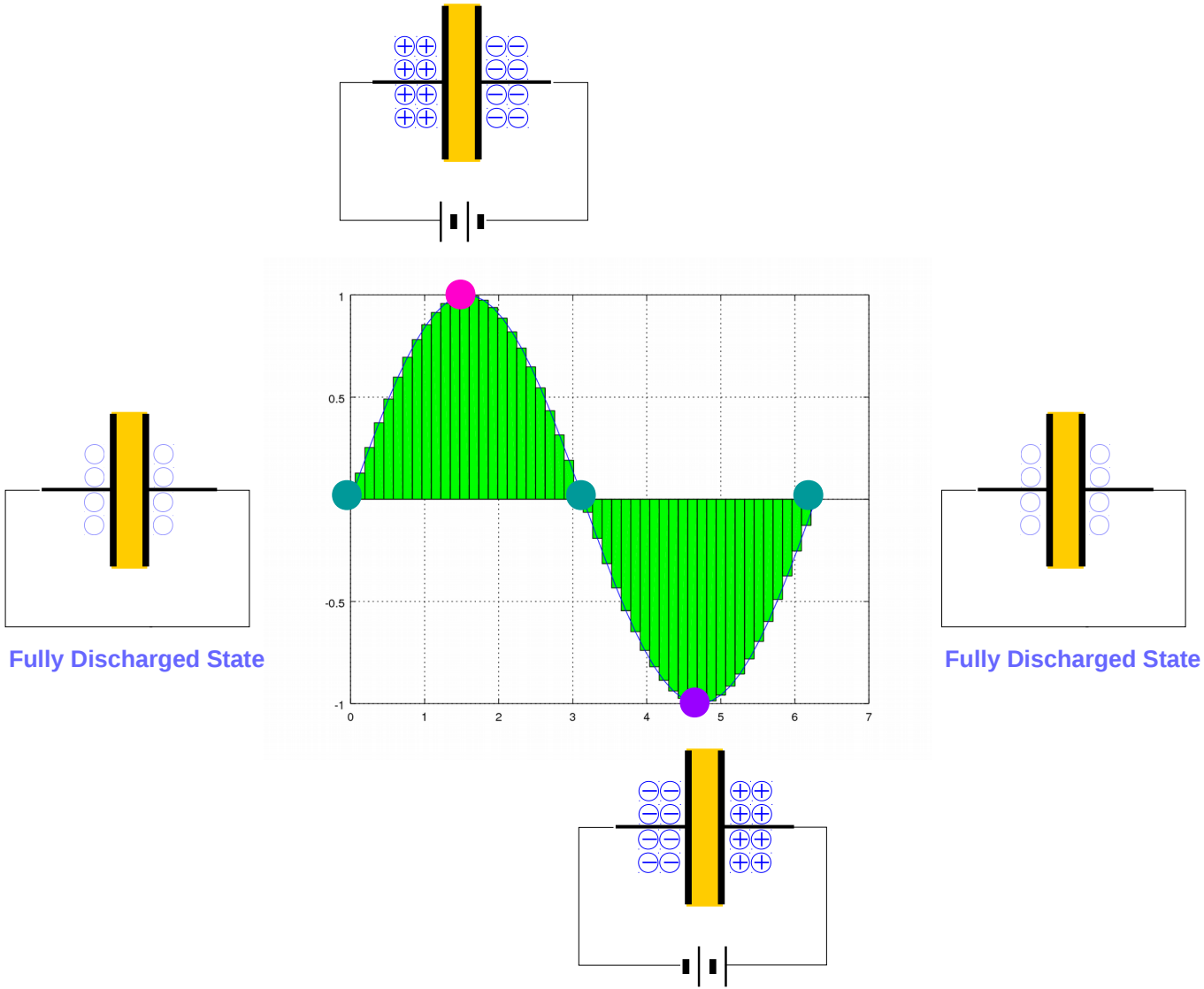


Fully Charged and Fully Discharged

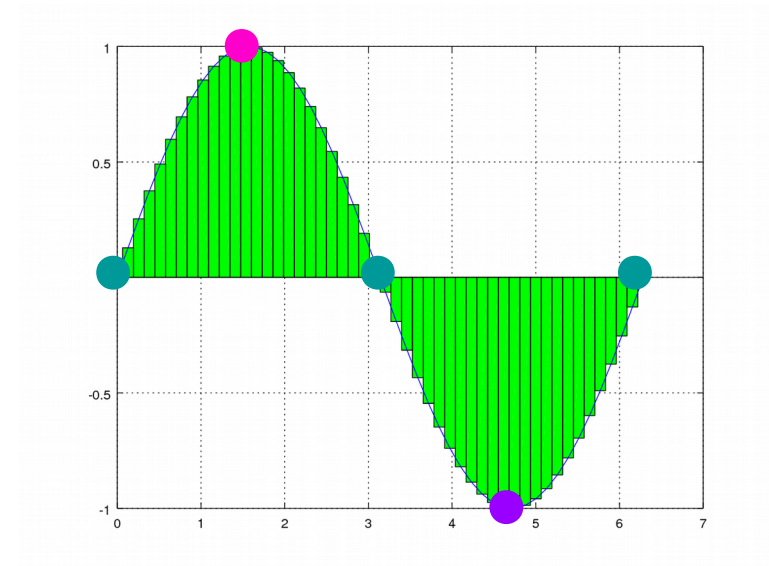
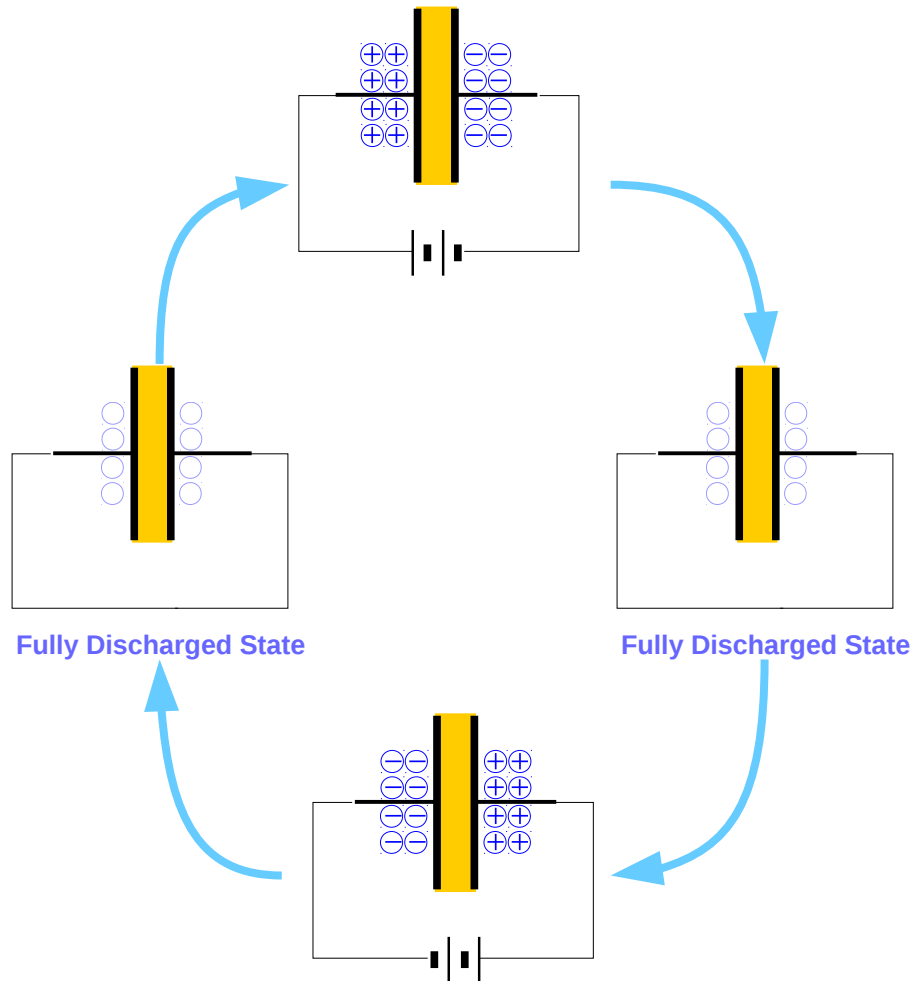


(+) Charging	(-) Charging	(-) Charging	(+) Charging
(+) Current	(-) Current	(-) Current	(+) Current
(+) Charging	(+) Discharging	(-) Charging	(-) Discharging

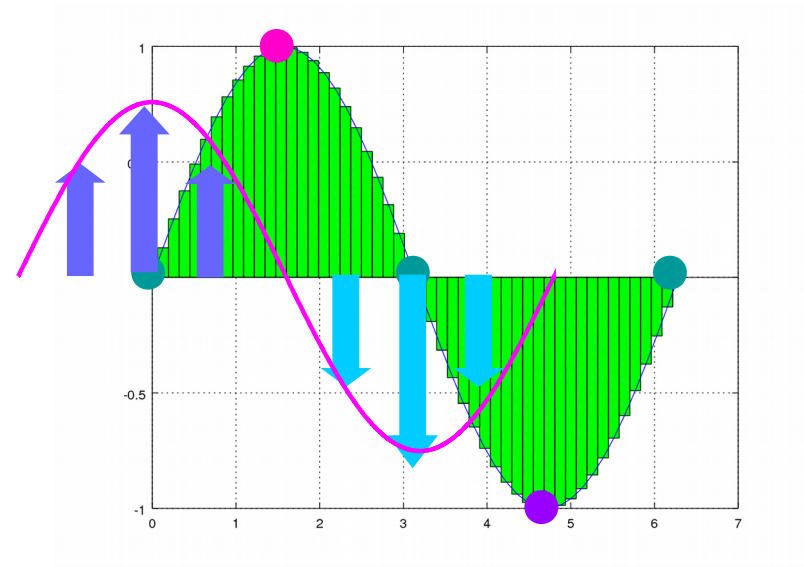
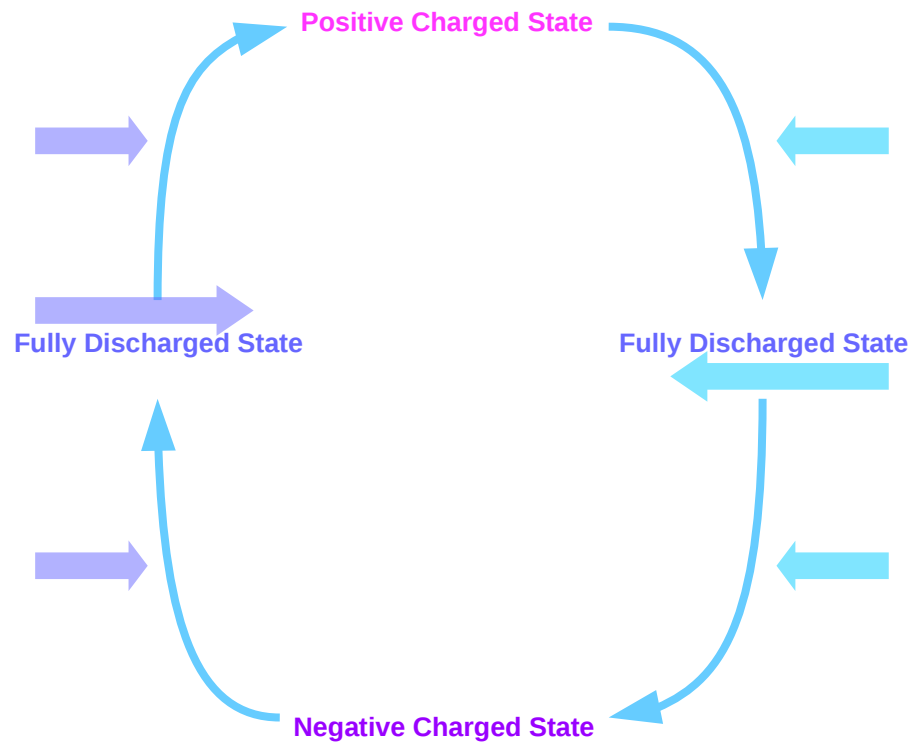
A Cycle



State Transition Diagram



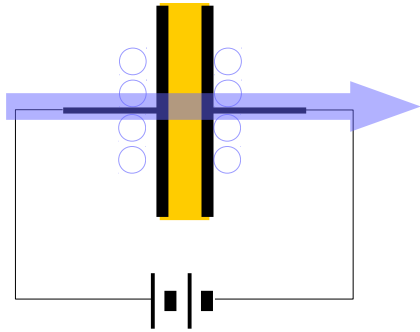
Current Flow



Fully Discharged : Large Current

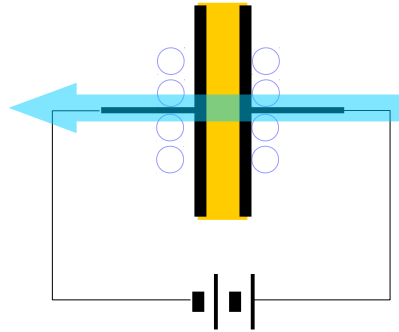
Fully Discharged State

large current



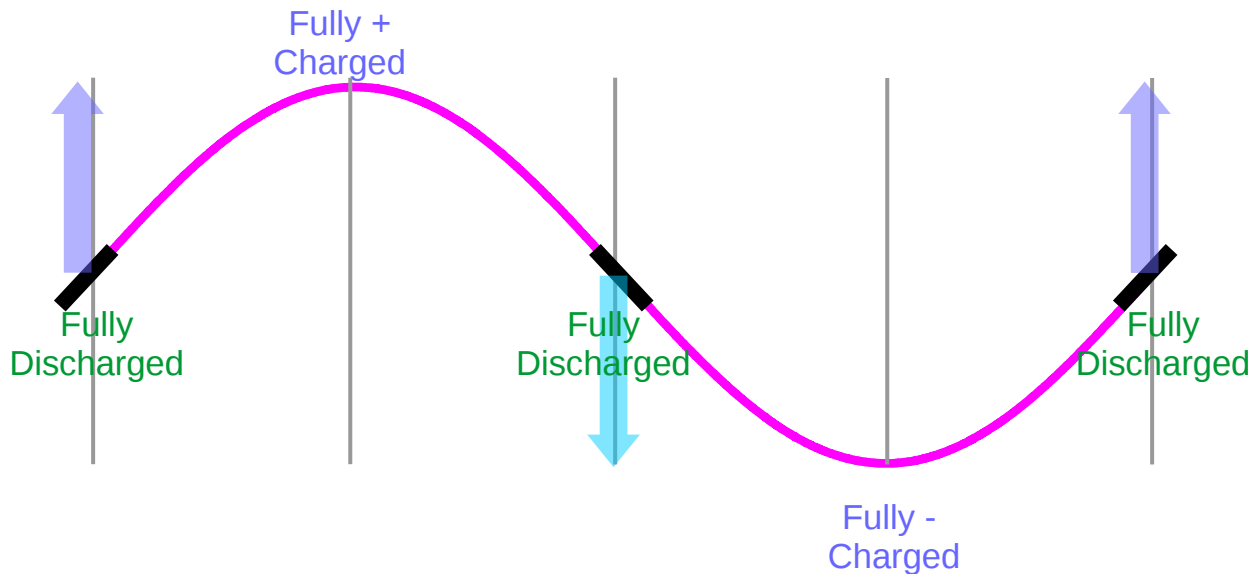
Fully Discharged State

large current



Enough space for large movement of charges

This state can flow large current in either direction

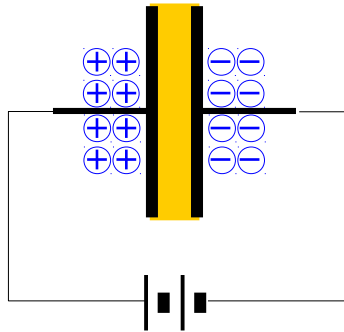


$$\left| \frac{dv_c}{dt} \right| = 1 \quad (\text{max value})$$

Fully Charged : Zero Current

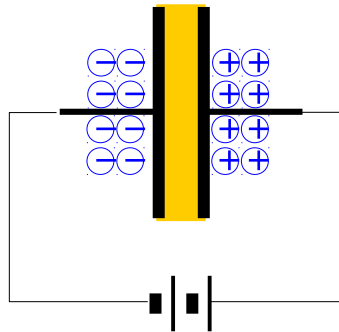
Positively Charged State

fully charged → no current



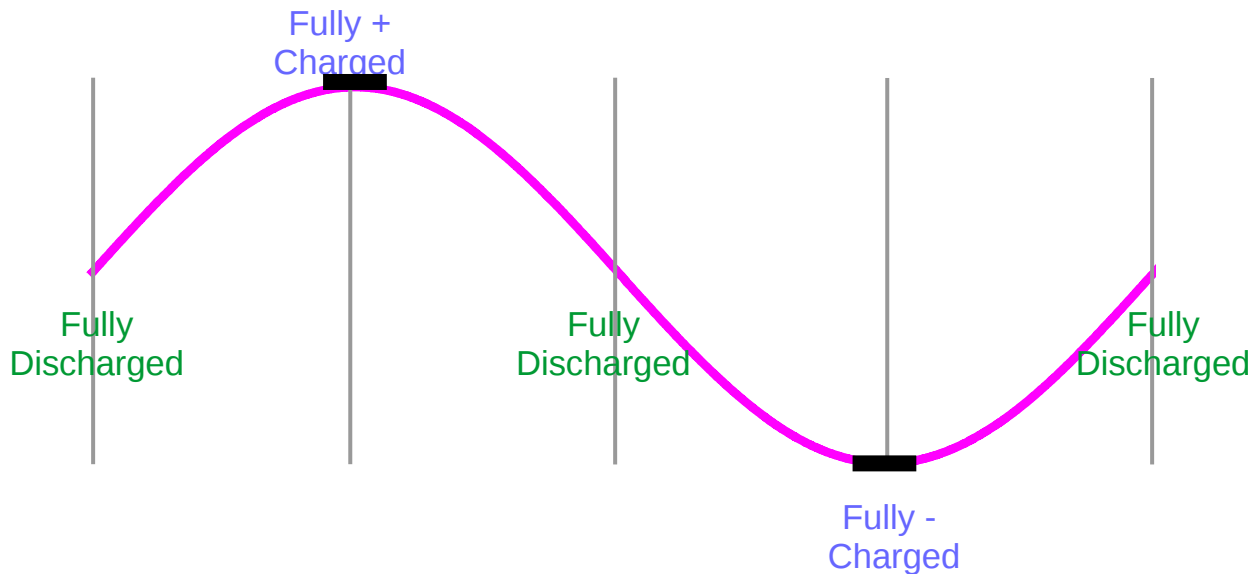
Negatively Charged State

fully charged → no current



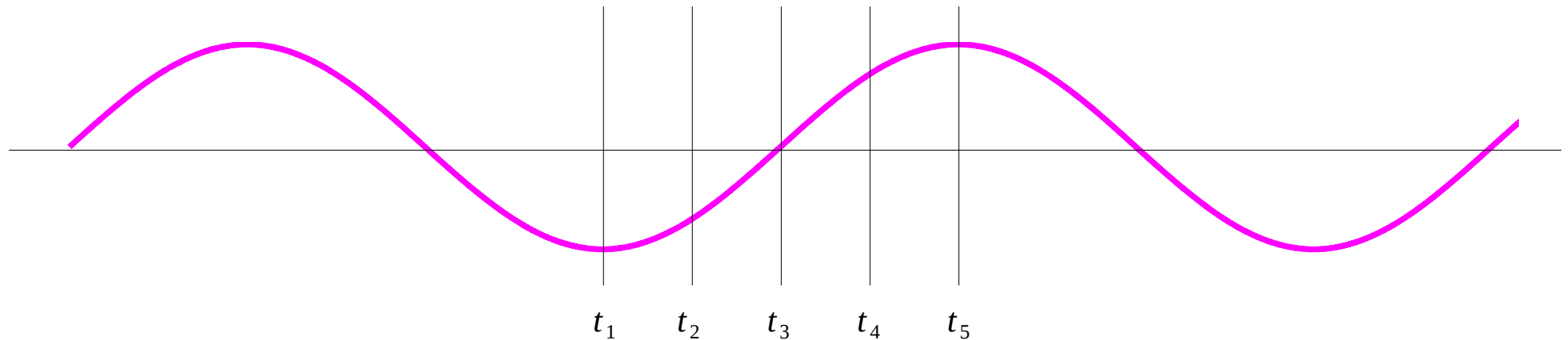
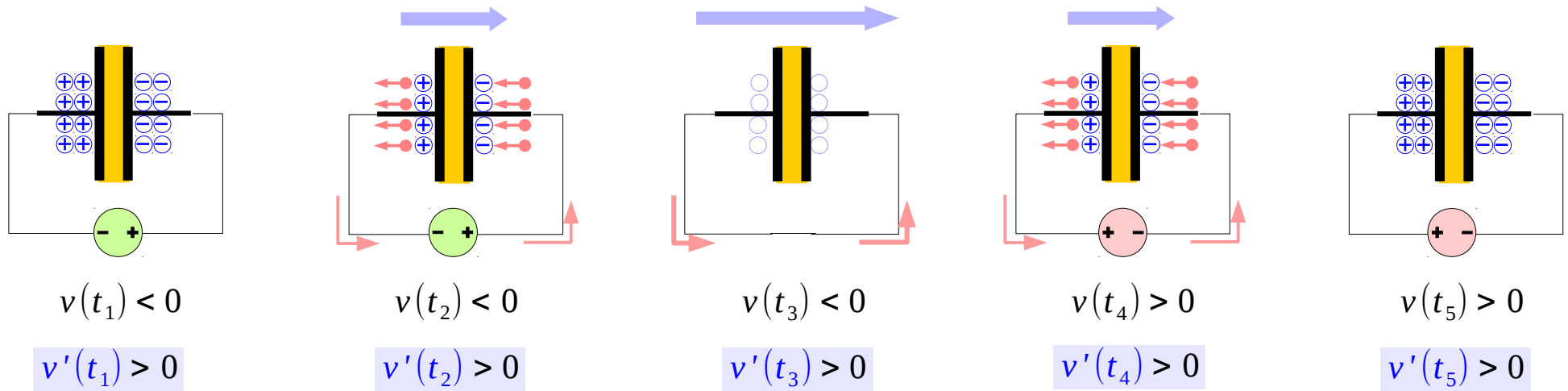
Crowded →
No more space

no current

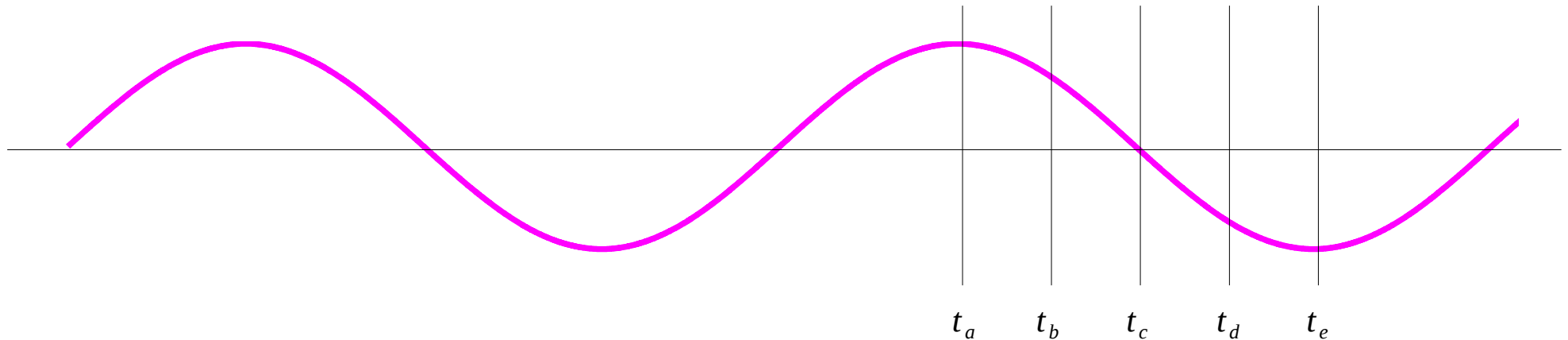
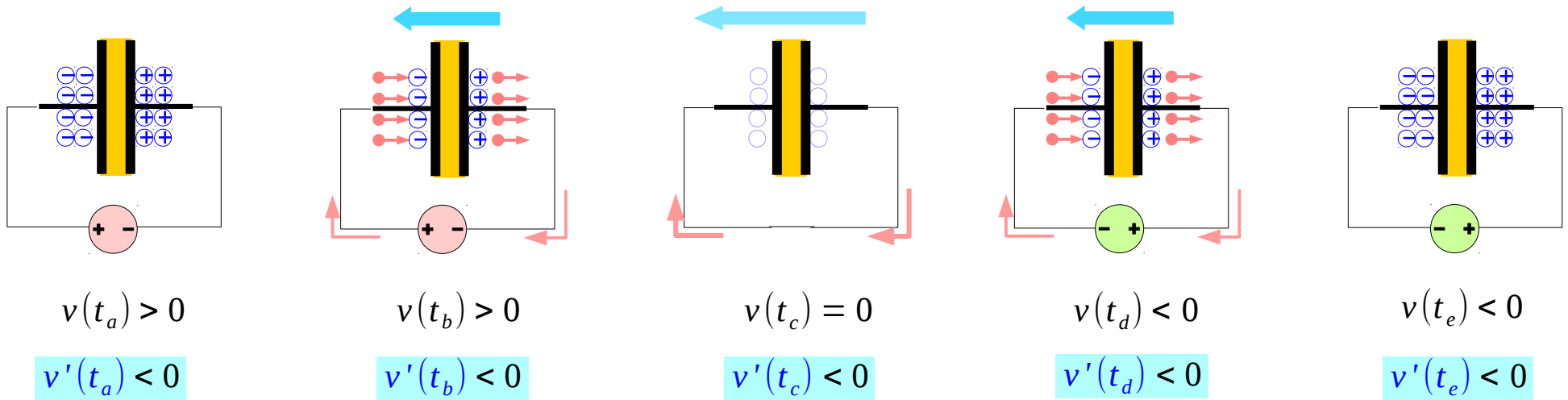


$$\left| \frac{dv_c}{dt} \right| = 0 \quad (\text{min value})$$

Incrementally, Charging Positively



Incrementally, Charging Negatively



Difference of Samples

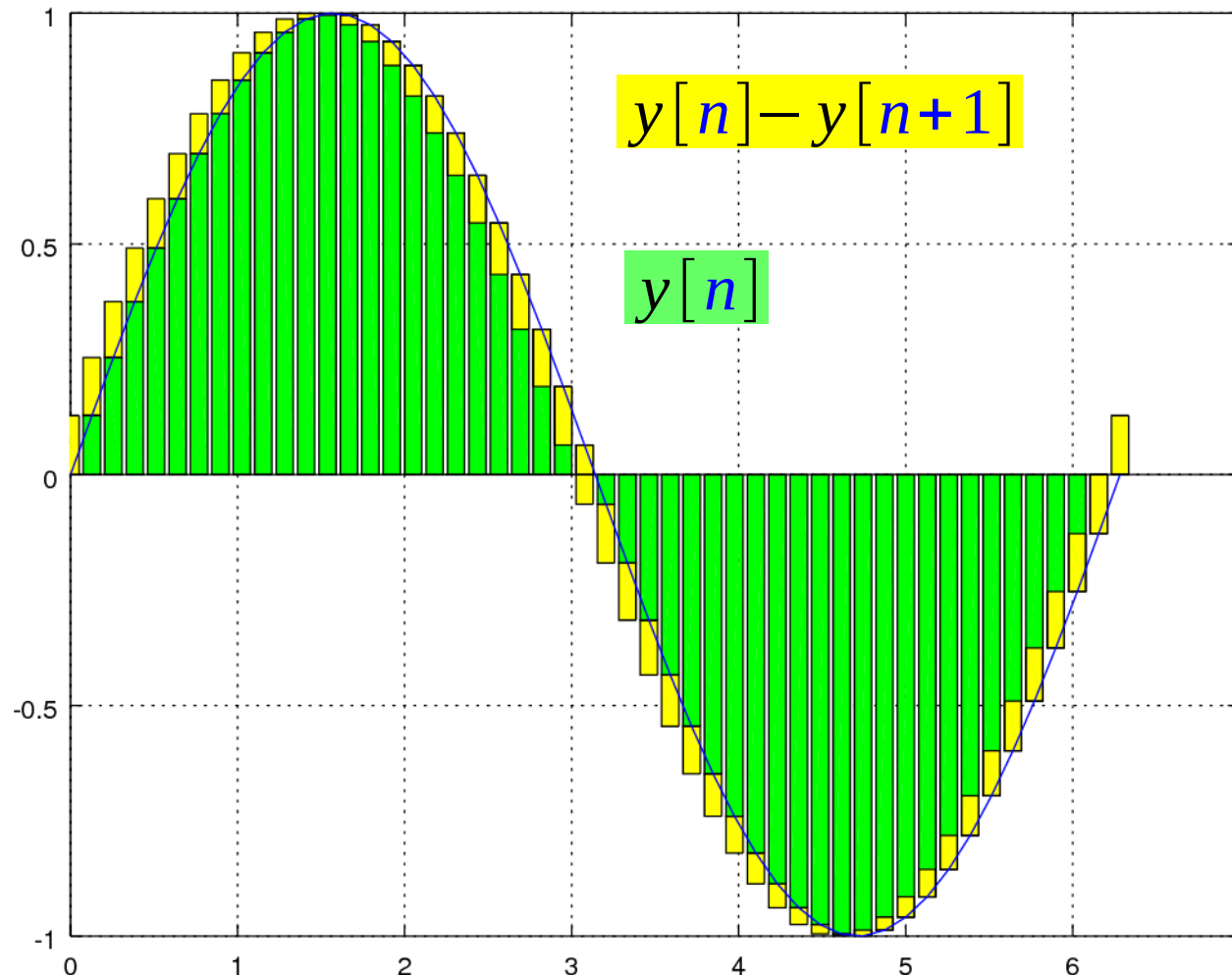
$$y(t) = \sin(t)$$

$$y[n] = \sin(nT)$$

$$y[n] - y[n+1] = \sin(nT) - \sin((n+1)T)$$

$$\frac{y[n] - y[n+1]}{T} \propto \frac{dy}{dt}$$

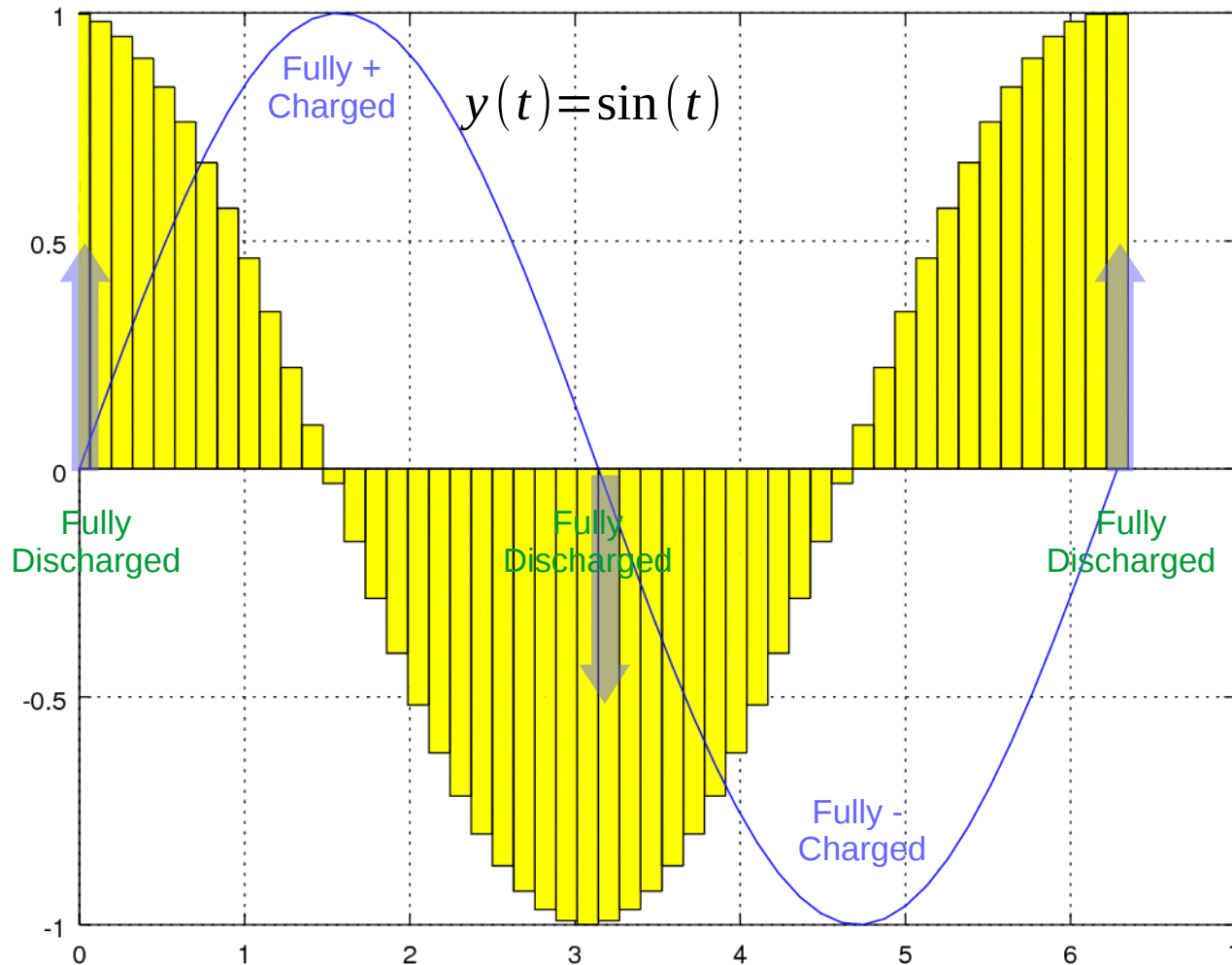
Fully Charged and Fully Discharged



```
h = bar(t1, [y1' y2'],  
"stacked")  
set(h(1), "facecolor", "g");  
set(h(2), "facecolor", "y");  
hold on  
plot(t1, y1)  
axis([0 pi]);
```

$$y[n] - y[n+1] = y(nT) - y((n+1)T) = \sin(nT) - \sin((n+1)T)$$

Fully Charged and Fully Discharged

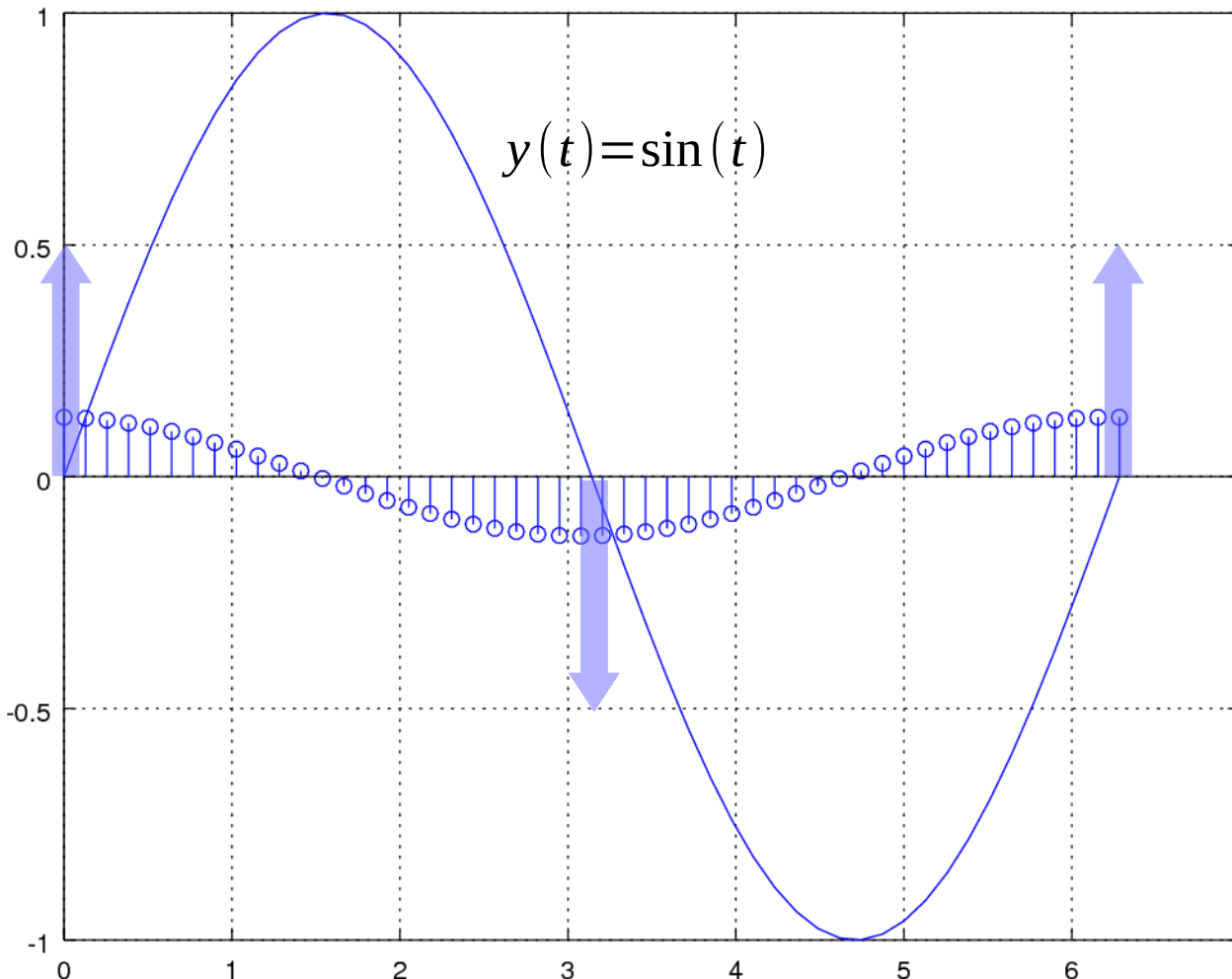


```
h = bar(t1, y2/t(2), "hist")
set(h(1), "facecolor", "y");
hold on
plot(t1, y1)
axis([0 7 -1 1]);
```

$$\frac{y[n] - y[n+1]}{T}$$

$$\propto \frac{dy}{dt}$$

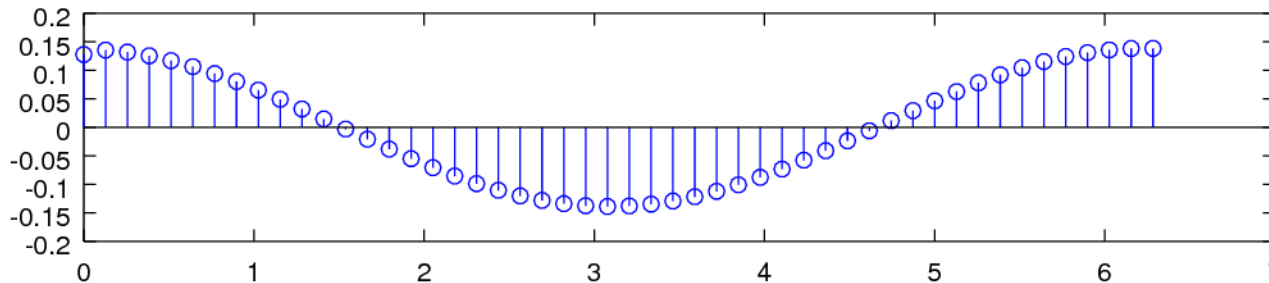
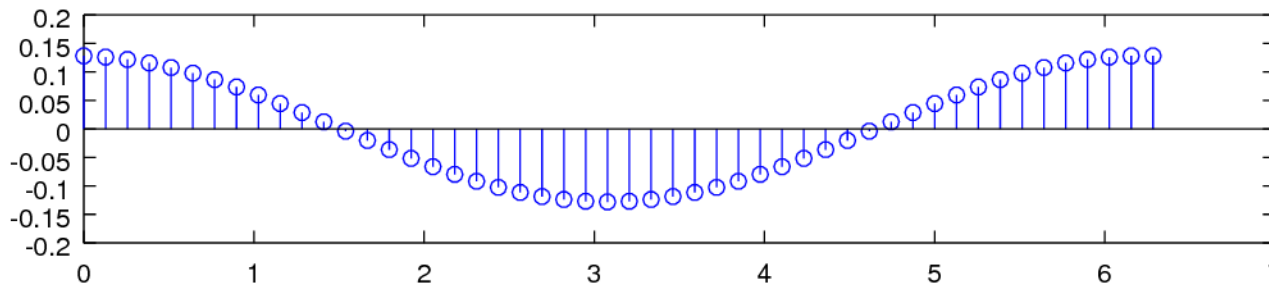
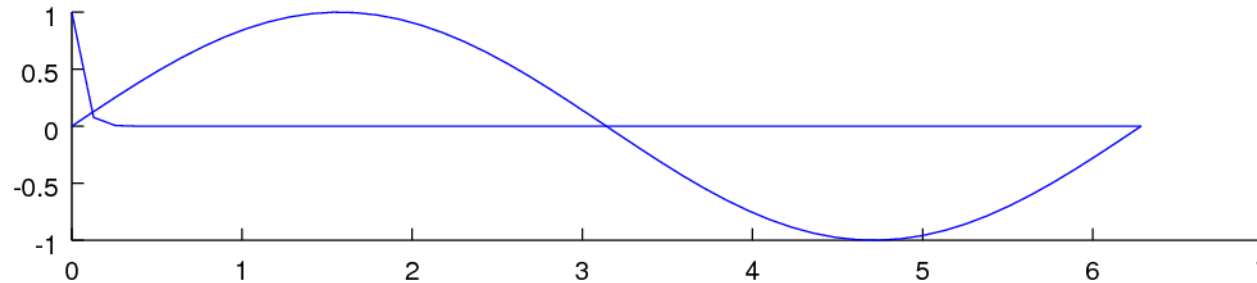
$y[n+1] - y[n]$



```
t = linspace(0, pi*2, 50);  
t1 = t;  
t2 = t + t(2);  
y1 = sin(t1);  
y2 = sin(t2) - sin(t1);  
stem(t1, y2)  
hold on  
plot(t1, y1)
```

$$y[n] - y[n+1] = y(nT) - y((n+1)T) = \sin(nT) - \sin((n+1)T)$$

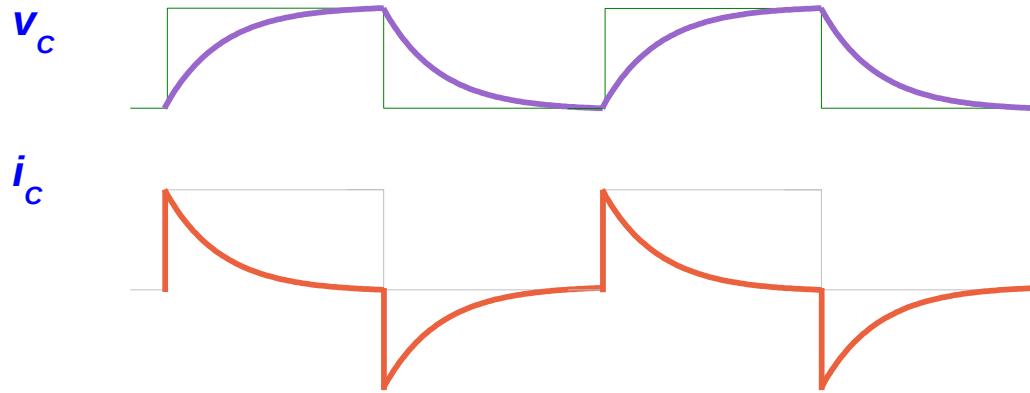
Fully Charged and Fully Discharged



```
clf
t = linspace(0, pi*2, 50);
t1 = t;
t2 = t + t(2);
y1 = sin(t1);
y2 = sin(t2) - sin(t1);
y3 = e.^(-20*t);
y4 = conv(y2, y3);
y5 = y4([1:length(t1)]);
```

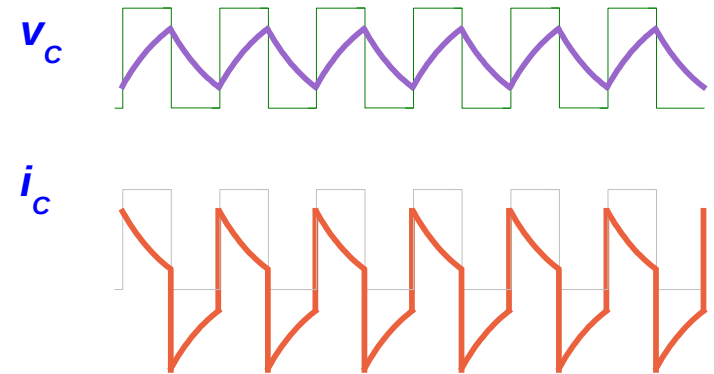
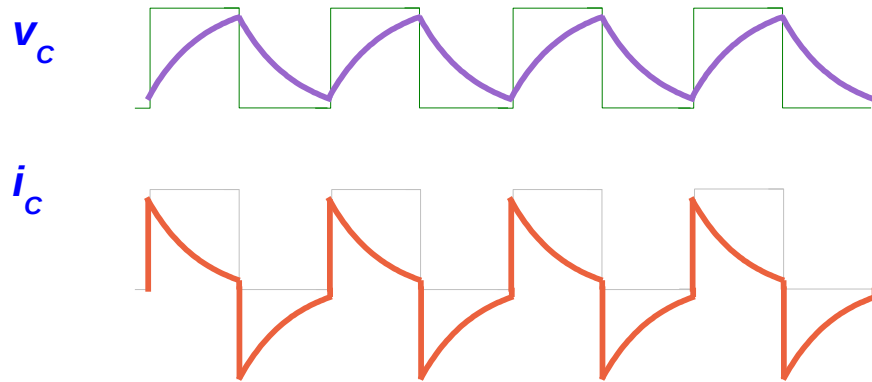
```
subplot(3, 1, 2);
stem(t1, y2)
subplot(3, 1, 1);
hold on
plot(t1, y1);
plot(t1, y3);
subplot(3, 1, 3);
stem(t1, y5);
```

Pulse



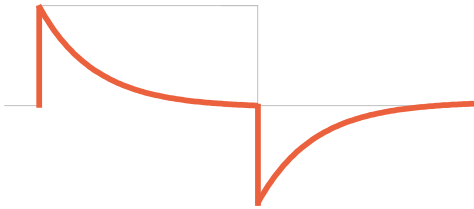
$$i_c = C \frac{dv_c}{dt}$$

ω ↑ i_c ↑ X_c ↓

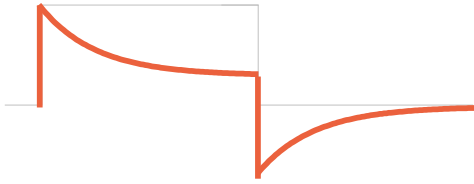


Time Constants

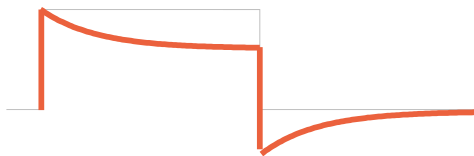
i_c



$\tau = RC$ *small time constant*

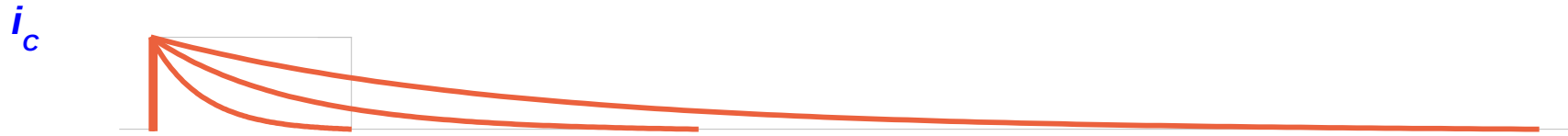


$\tau = RC$ *medium time constant*



$\tau = RC$ *large time constant*

Time Constants



$$\tau_1 < \tau_2 < \tau_3$$

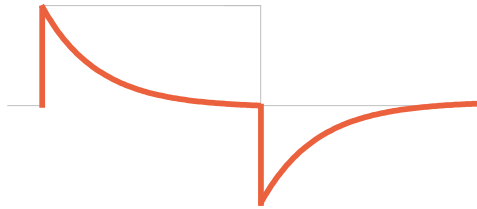
$$a_1 > a_2 > a_3$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}} = e^{-at}$$

$$\tau = RC = \frac{1}{a}$$

Time Constants

i_c



$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

small τ
small C

$$\text{large } \frac{1}{\omega C} \gg R$$

Fully Capacitive



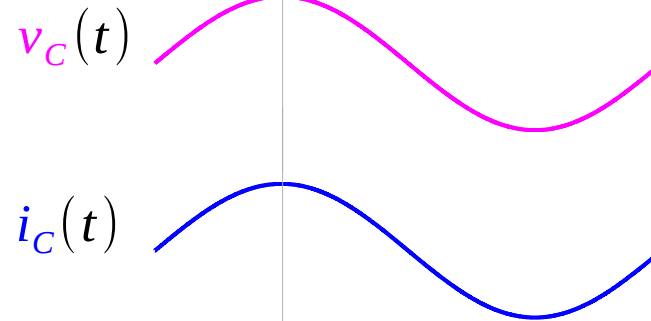
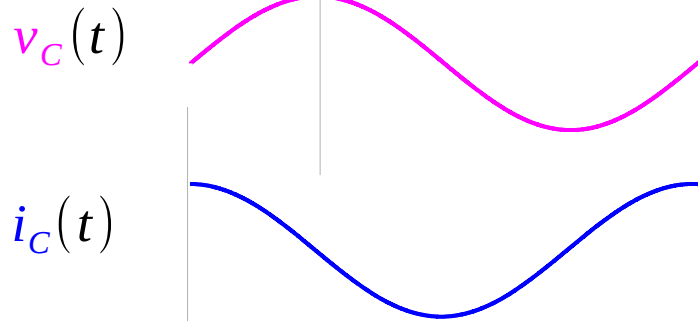
$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

large τ
large C

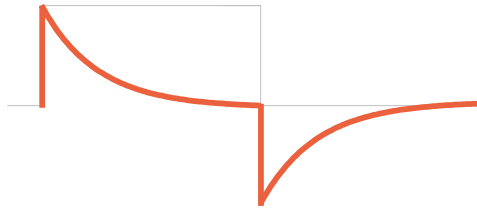
$$\text{small } \frac{1}{\omega C} \ll R$$

Fully Resistive



Time Constants

i_c

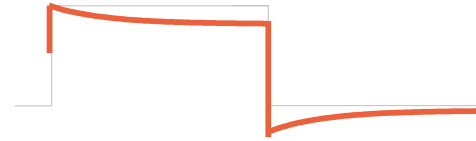


$$\tau = RC$$
$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

small τ
small C

$$\text{large } \frac{1}{\omega C} \gg R$$

Fully Capacitive



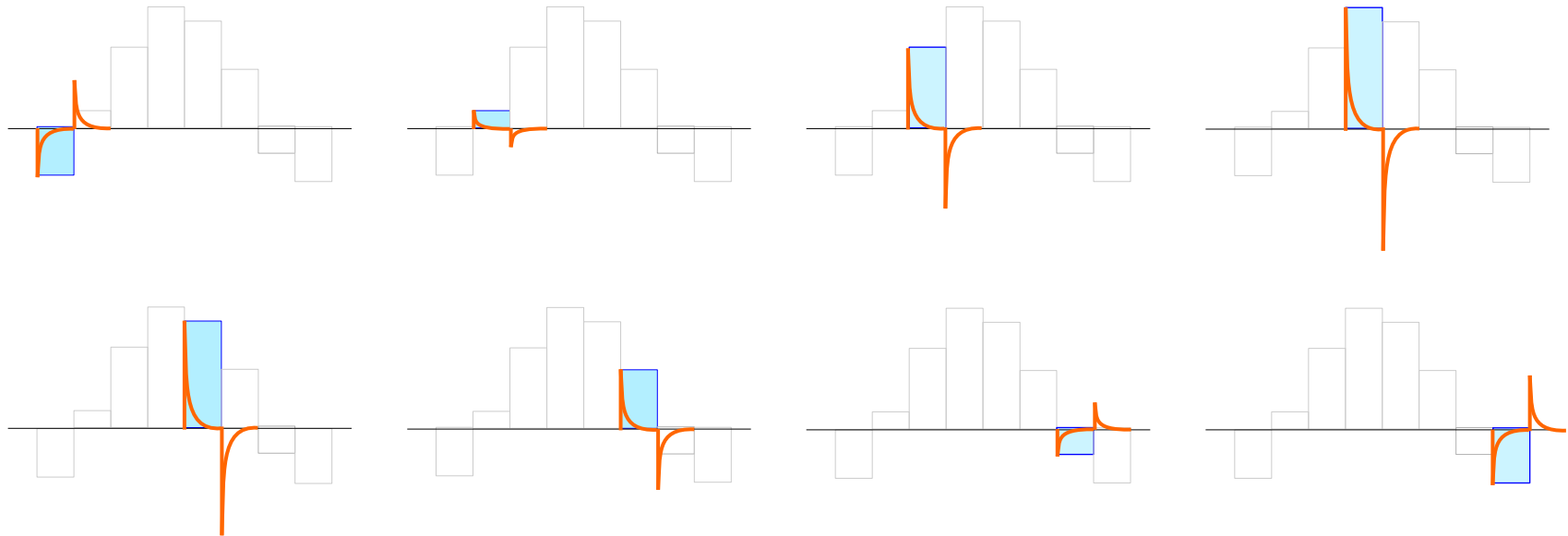
$$\tau = RC$$
$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

large τ
large C

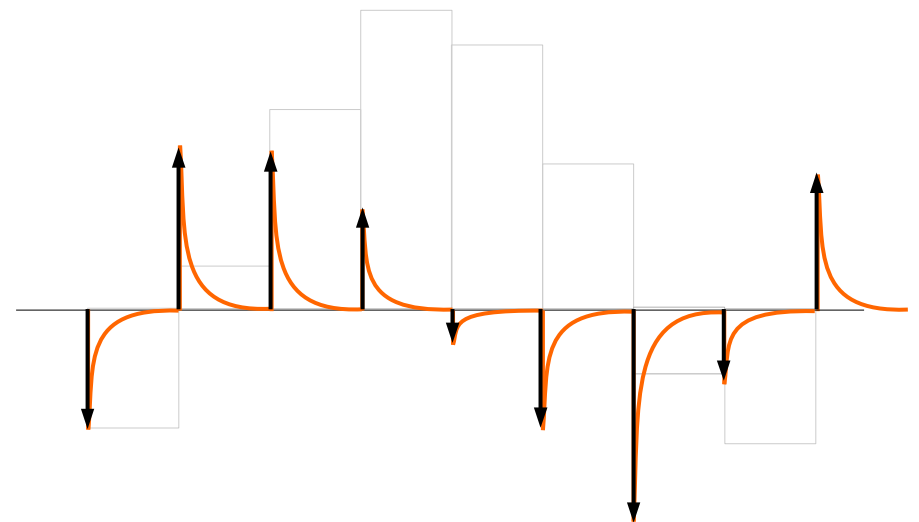
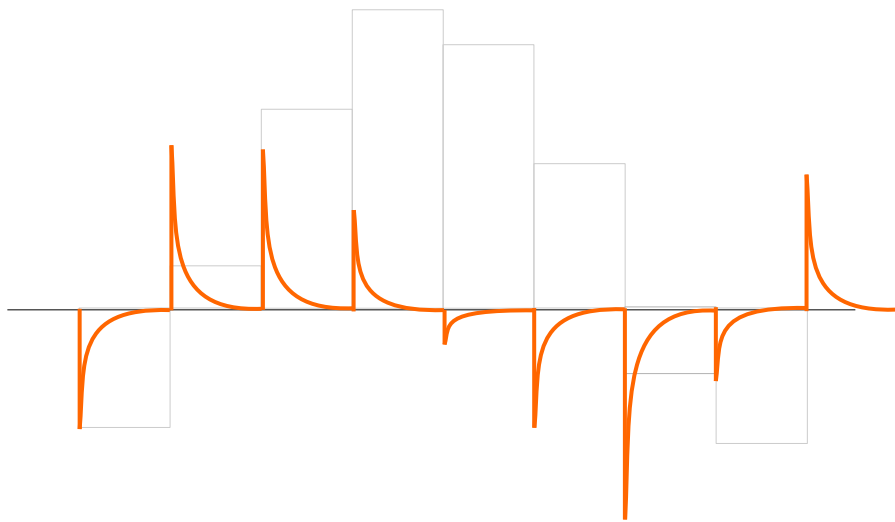
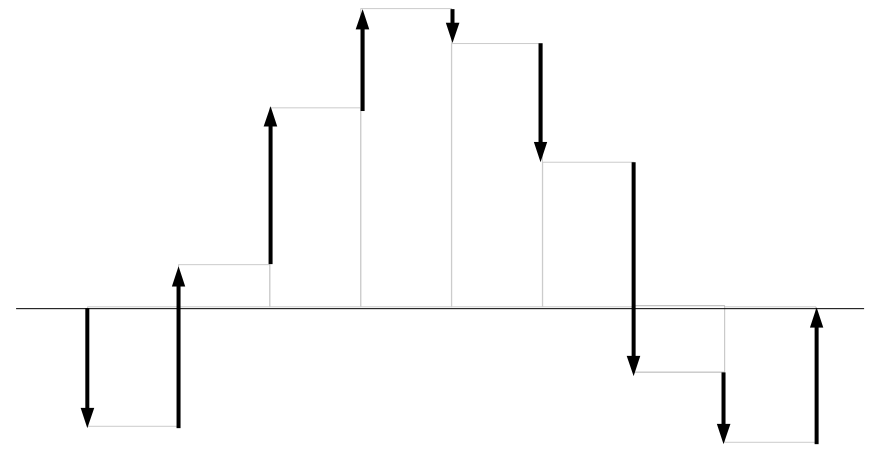
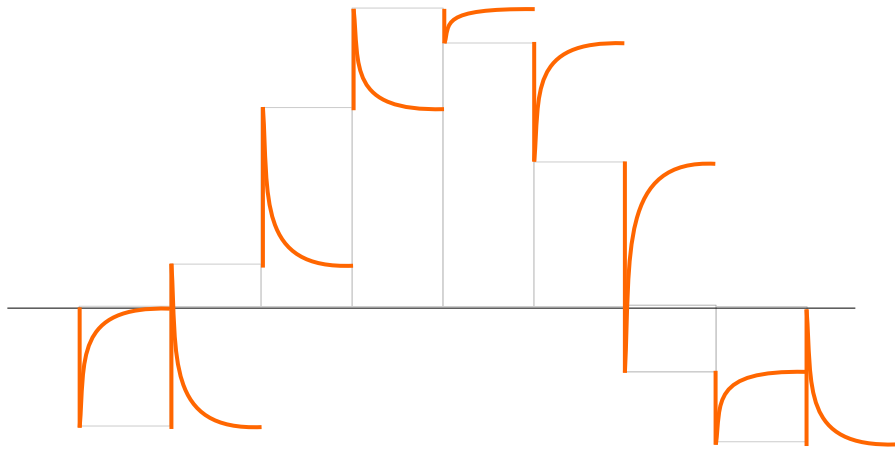
$$\text{small } \frac{1}{\omega C} \ll R$$

Fully Resistive

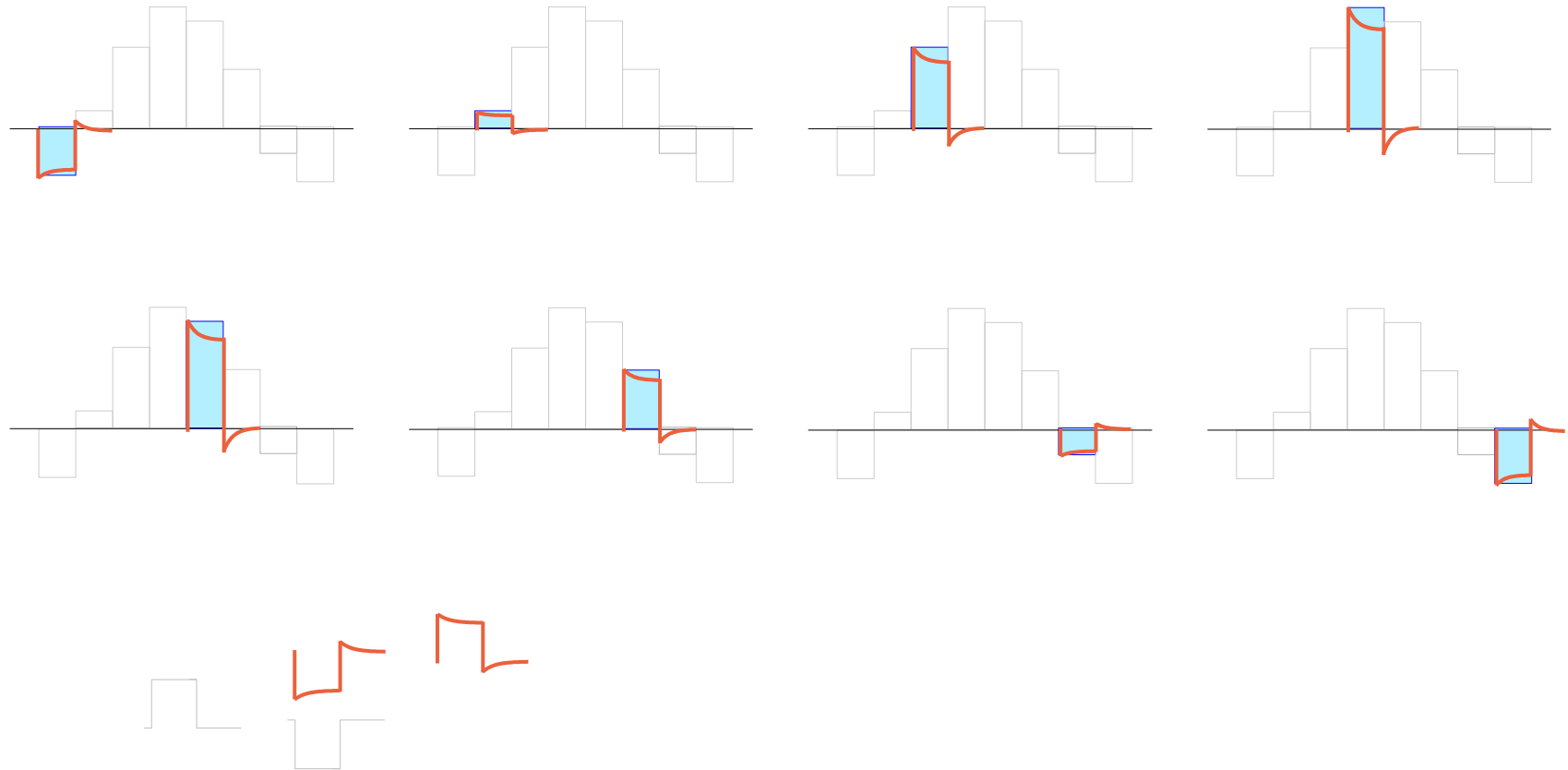
Superposition - Small Time Constant



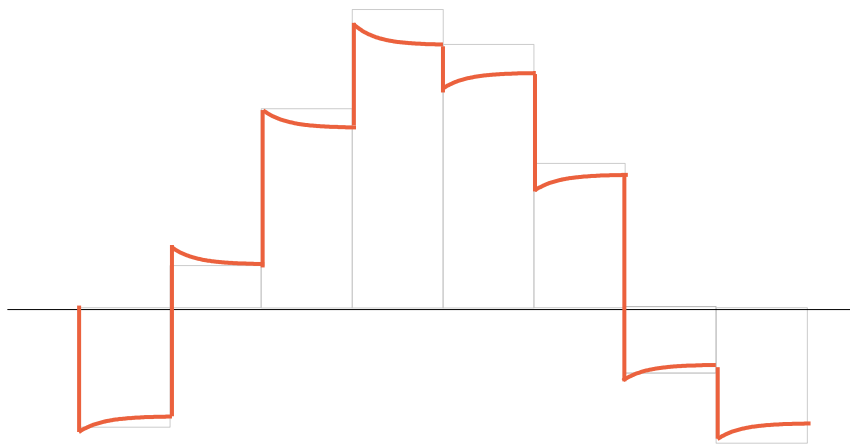
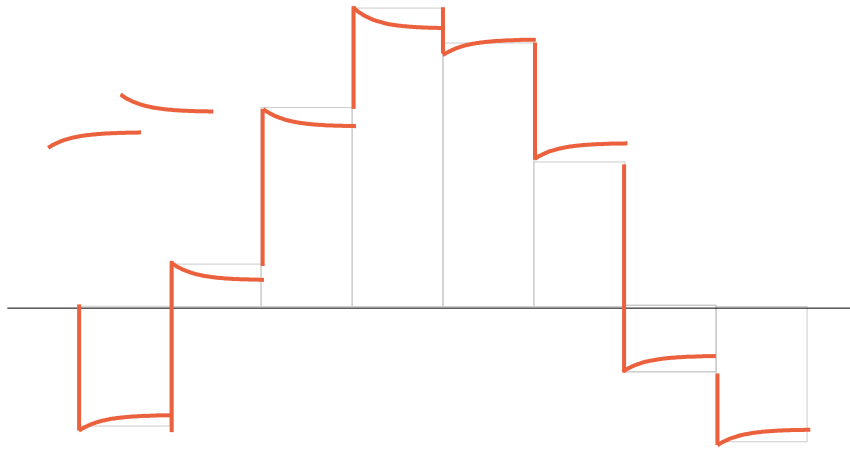
Small Time Constants



Superposition - Large Time Constant

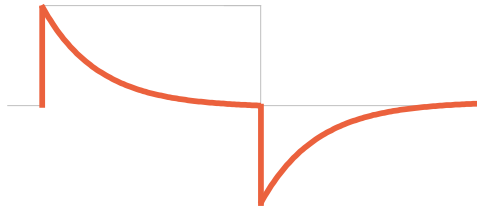


Large Time Constants



Time Constants

i_c



$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

small τ
small C

$$\text{large } \frac{1}{\omega C} \gg R$$

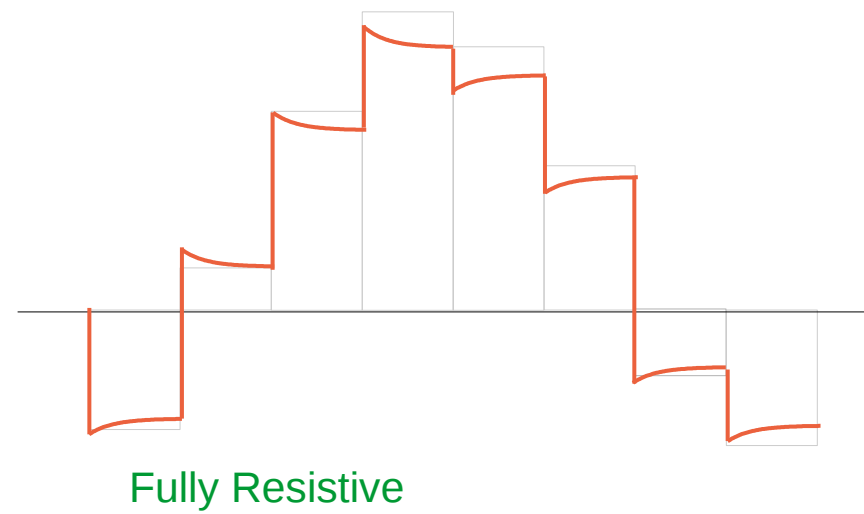
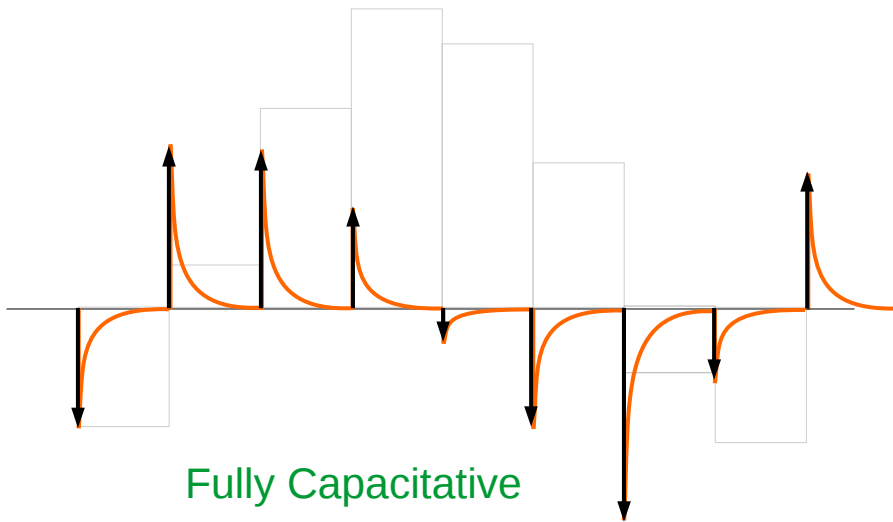


$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

large τ
large C

$$\text{small } \frac{1}{\omega C} \ll R$$



Plotting superposition results

```
clf
t = linspace(0, pi*2, 50);
tt= linspace(0, pi*2, 500);
N = length(t);
NN= length(tt);

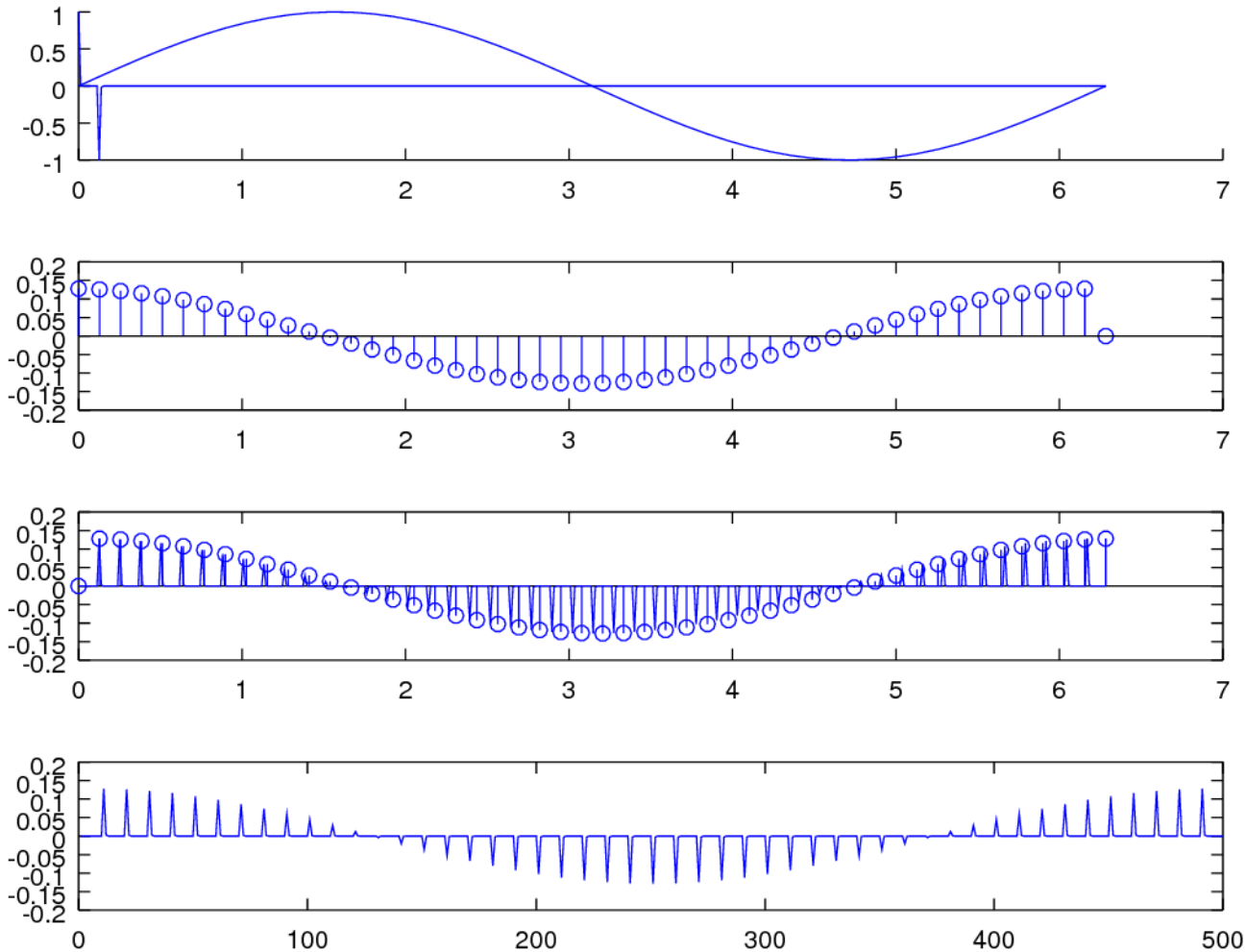
t1 = t;
t2 = [t(2:N), t(N)];
y1 = sin(t1);
y2 = sin(t2) - sin(t1);

yy = [y1; zeros(NN/N-1, N)];
yy2= yy(:)';
a = 1/300;
yy3= e.^(-a*tt);
yy3 =yy3 - [zeros(1, NN/N),
e.^(-a*tt)](1:NN);

svec = zeros(1, NN);
for i = 1:NN;
    tvec = zeros(1, NN);
    tvec = [zeros(1, i-1), yy3];
    tvec = yy2(i) * tvec(1:NN);
    svec = svec + tvec;
endfor
yy4 = svec;
% yy4= conv(yy2, yy3);
y5 = yy4([1:NN/N:NN]);
yy5= yy4([1:NN]);

subplot(4, 1, 2);
stem(t1, y2)
subplot(4, 1, 1);
hold on
plot(t1, y1);
plot(tt, yy3);
subplot(4, 1, 3);
stem(t1, y5); hold on
plot(tt, yy5)
subplot(4, 1, 4);
plot(yy4);
```

Small Time Constant



```
yy = [y1;  
zeros(NN/N-1, N)];  
yy2= yy(:)';  
a = 300;  
yy3= e.^(-a*tt);  
yy3 =yy3 -  
[zeros(1, NN/N),  
e.^(-a*tt)](1:NN);
```

$$\tau = RC$$

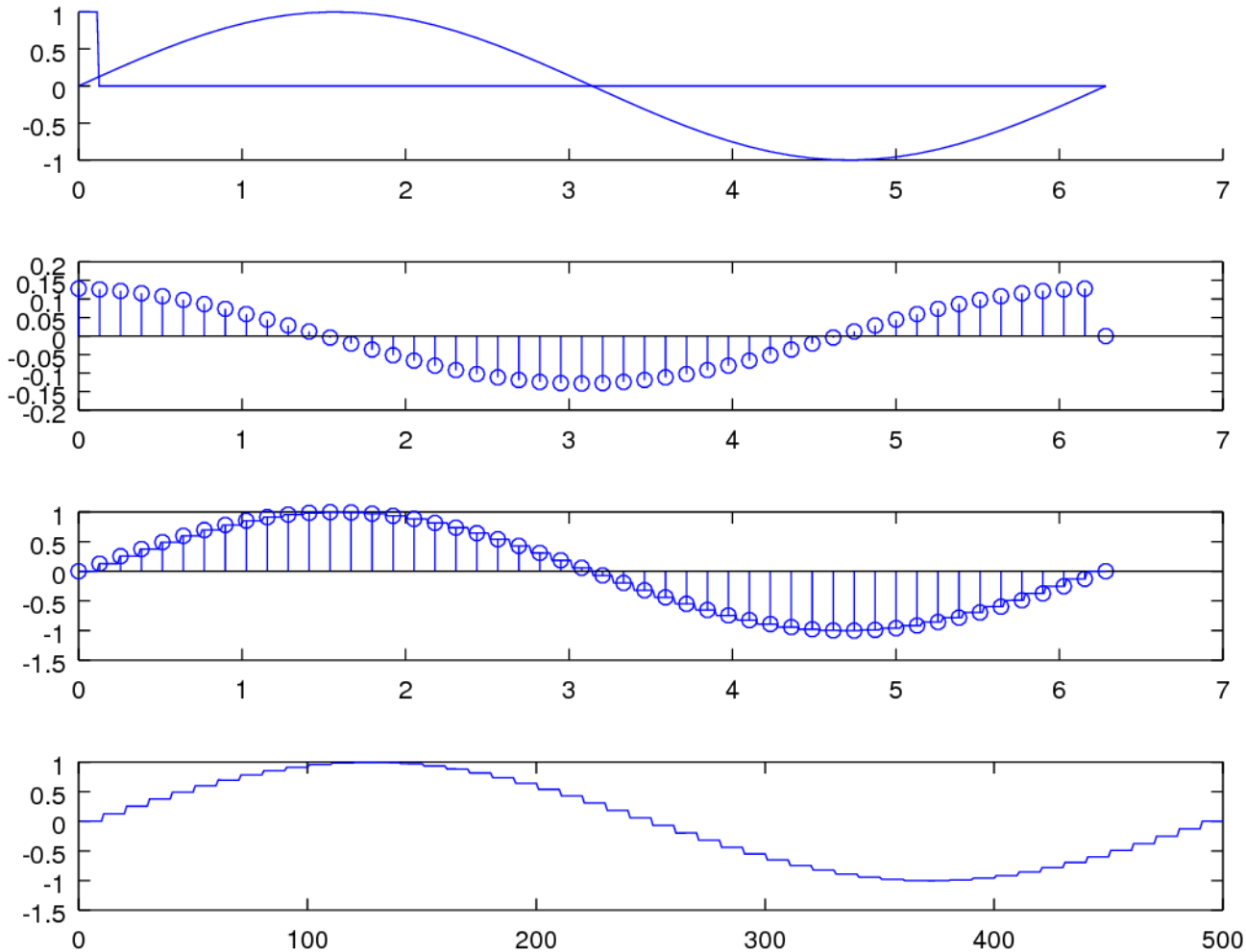
$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

small τ

small C

large $\frac{1}{\omega C}$

Large Time Constant



```
yy = [y1;
zeros(NN/N-1, N)];
yy2= yy(:)';
a = 1/300;
yy3= e.^(-a*tt);
yy3 =yy3 -
[zeros(1, NN/N),
e.^(-a*tt)](1:NN);
```

$$\tau = RC$$

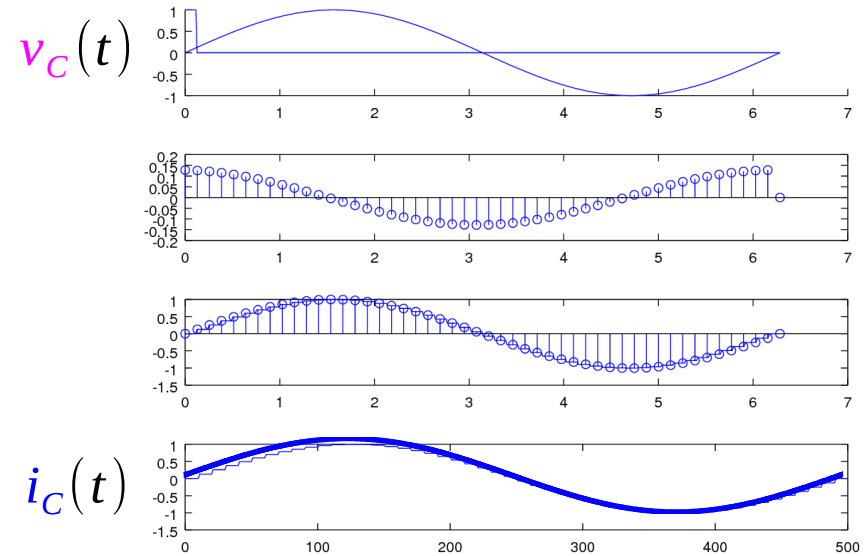
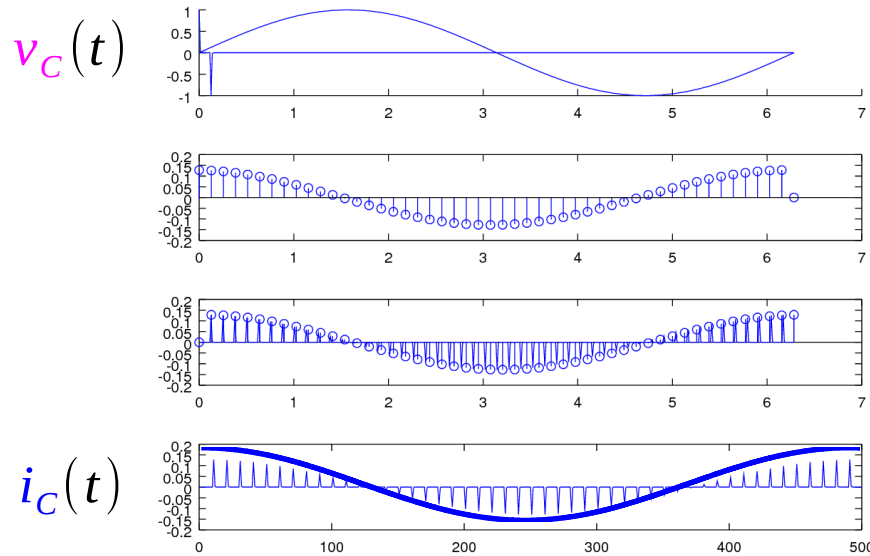
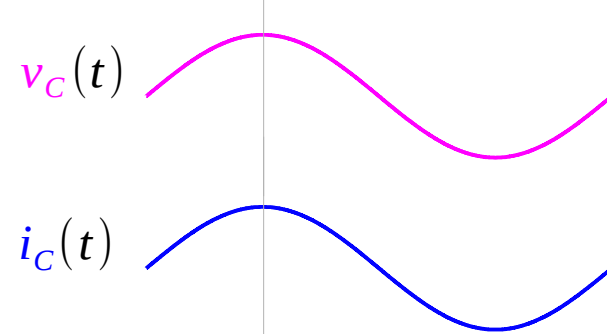
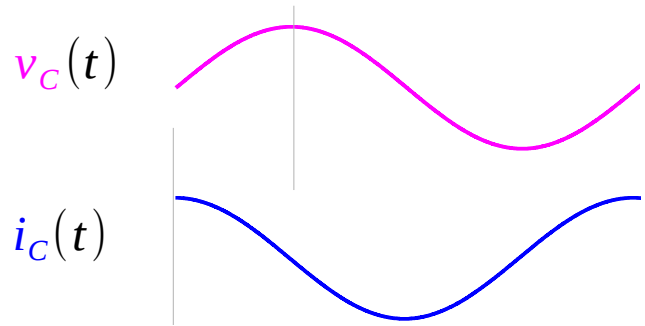
$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

large τ

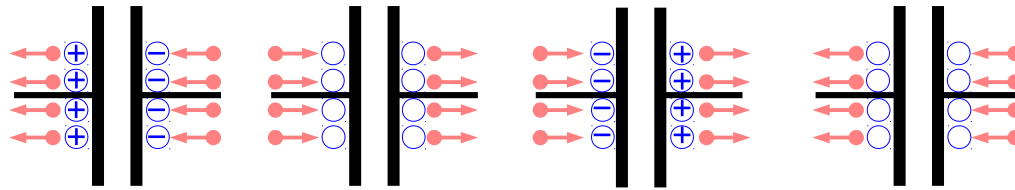
large C

small $\frac{1}{\omega C}$

Envelope of the samples

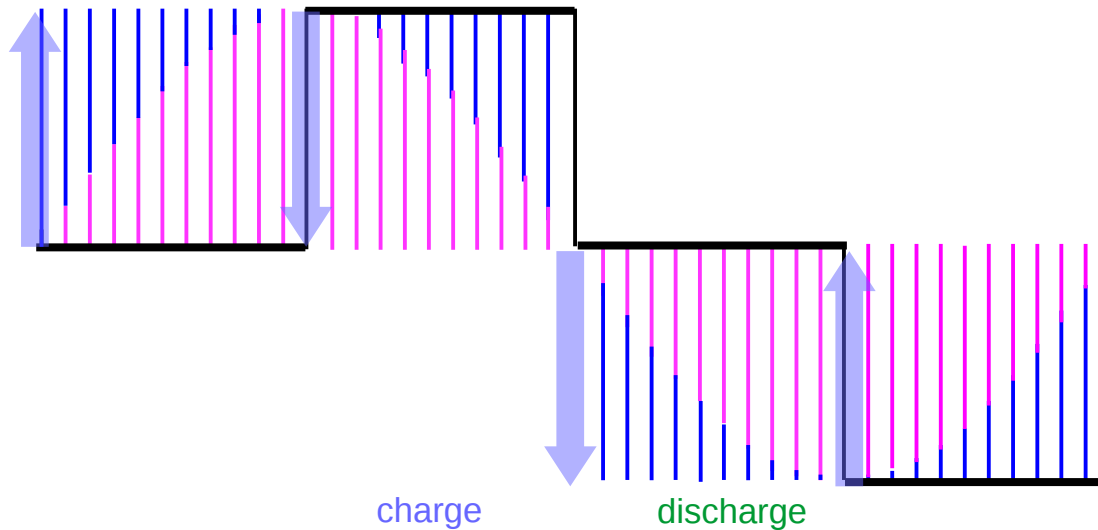


Evercharging signal pairs



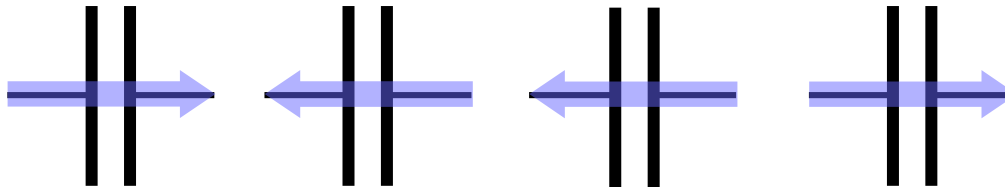
charge

discharge

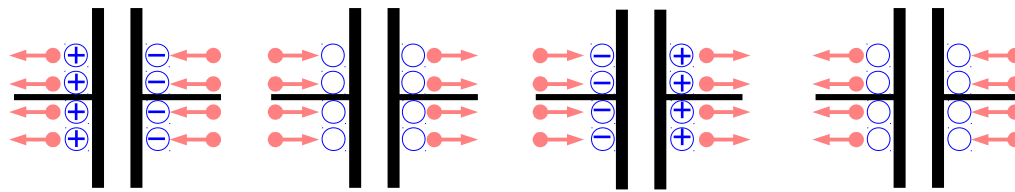


charge

discharge

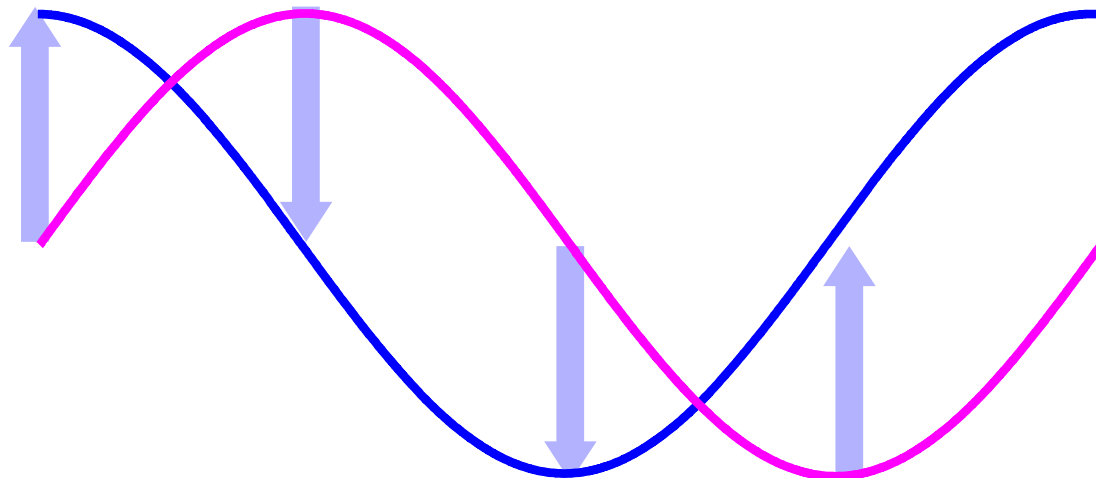


Evercharging signal pairs



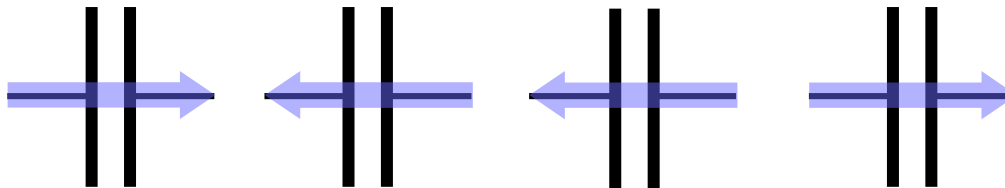
charge

discharge

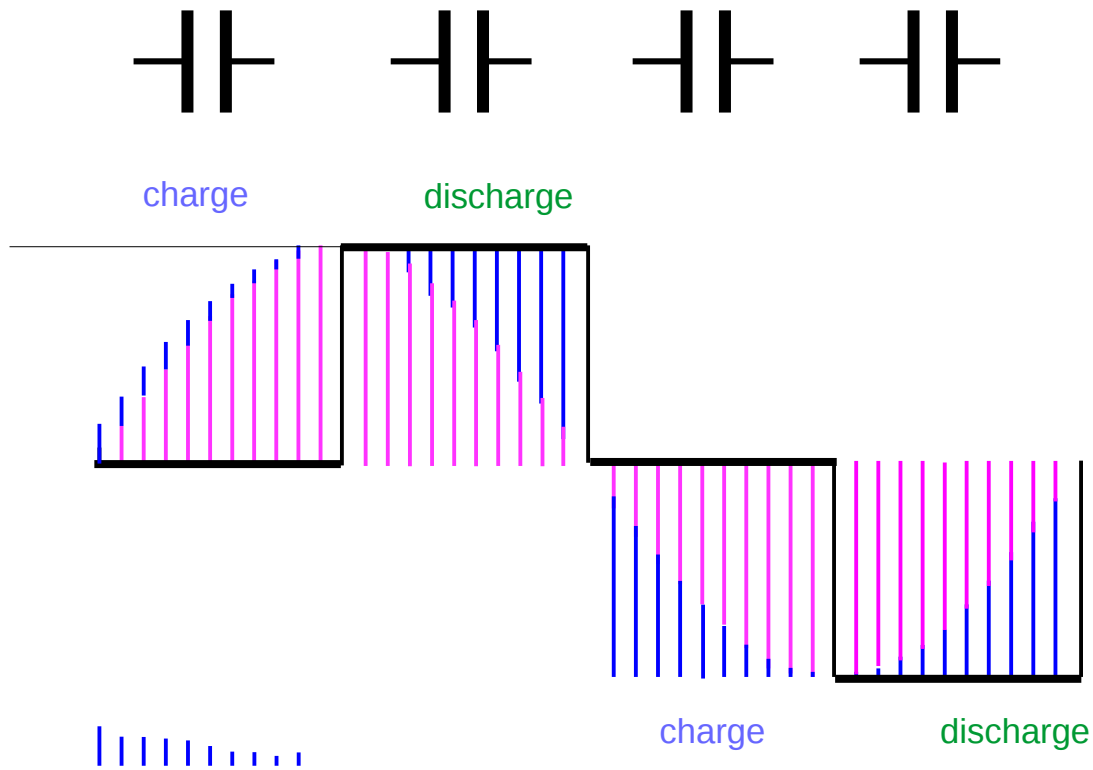


charge

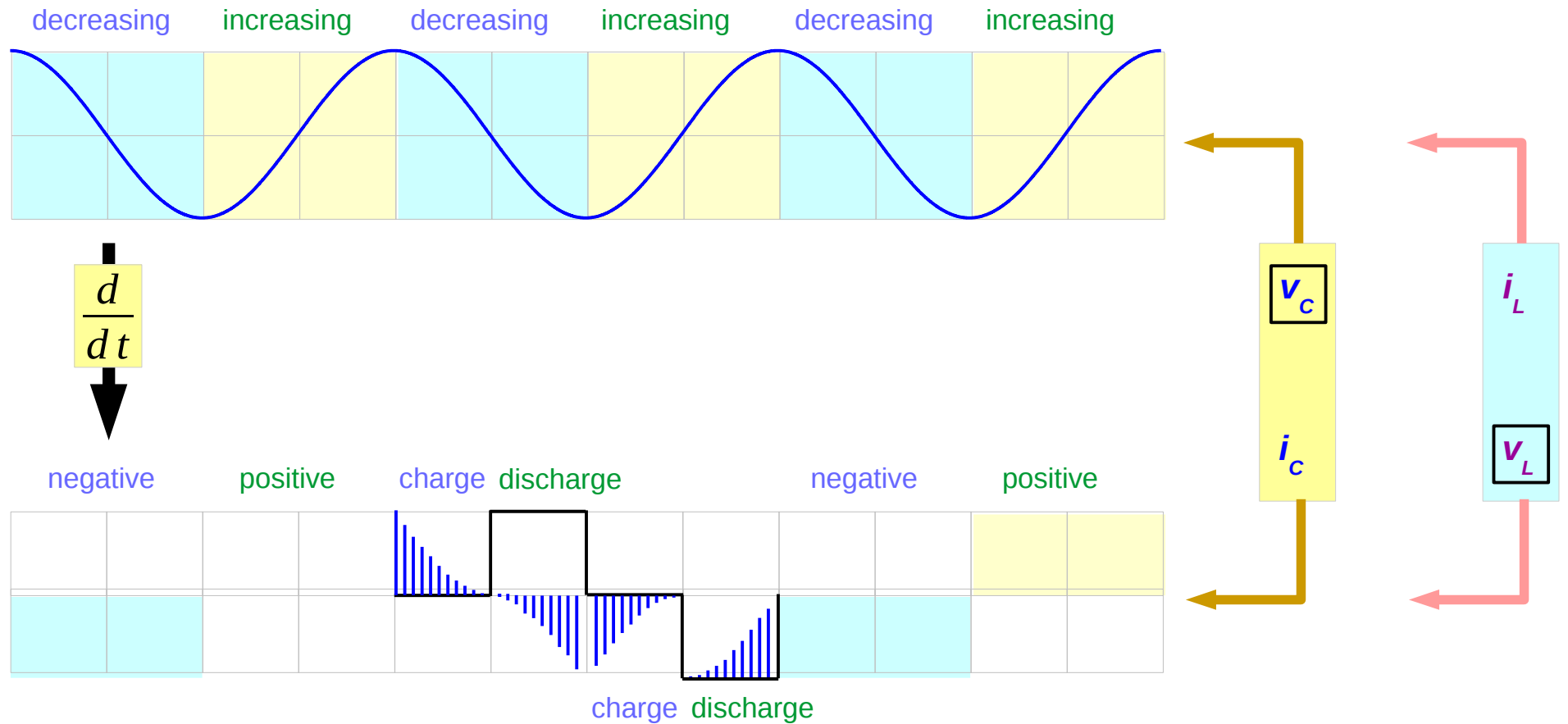
discharge



Evercharging signal pairs

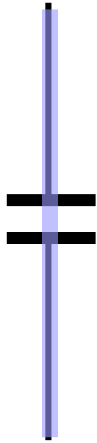


Evercharging signal pairs



I leads V by 90°

*Initial
charge*

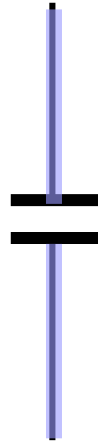


SHORT

V = 0

I : peak

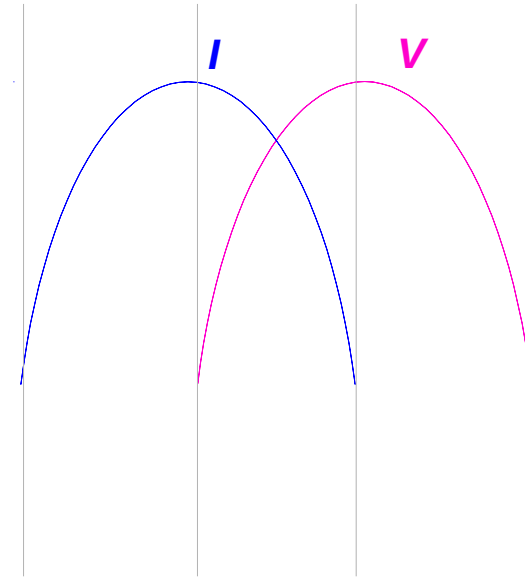
*Full
charge*



OPEN

I = 0

V : peak



References

[1] <http://en.wikipedia.org/>

[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003