Angle Recoding 2. Wu
4. EEAS

20180903 Mon

③ Extended EAS-based CORDIC

$$S_2 = \{ \tan^{-1}(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*}) : \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\},\ s_0^*, s_1^* \in \{0, 1, \cdots, W-1\} \}$$

$$\theta_i = \tan^{-1}(\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)})$$

the angle quantization error

$$\boxed{|\xi_{m,EAAS}| \triangleq \left| \theta - \sum_{j=0}^{R_m-1} \tan^{-1}(\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)}) \right|}$$

Generalized EEAS Scheme

$$S_d = \left\{ \tan^{-1}\left(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*} + \cdots + \alpha_{d-1}^* \cdot 2^{-s_{d-1}^*}\right) : \begin{array}{l} \alpha_0^*, \alpha_1^*, \cdots, \alpha_{d-1}^* \in \{-1, 0, +1\}, \\ s_0^*, s_1^*, \cdots, s_{d-1}^* \in \{0, 1, \cdots, W-1\} \end{array} \right\}$$

Extended EAS (EEAS) - Wu

more flexible way of decomposing the rotation angle

better
the number of iterations
the error performance

$$S_{EAS} = \{ (\sigma \cdot \tan^{-1}(2^{-r})) : \sigma \in \{+1, 0, -1\}, \ r \in \{1, 2, \cdots, n-1\} \}$$

$$S_{EEAS} = \{ (\sigma_1 \cdot \tan^{-1}(2^{-r_1}) + \sigma_2 \cdot \tan^{-1}(2^{-r_2})) : \sigma_1, \sigma_2 \in \{+1, 0, -1\}, \ r_1, r_2 \in \{1, 2, \cdots, n-1\} \}$$

The pse do-rotation
for i-th micro rotations

$$x_{i+1} = x_i - [\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)}]\, y_i$$
$$y_{i+1} = y_i + [\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)}]\, x_i$$

The pseudo-rotated vector $[x_{R_m}, y_{R_m}]$
after R_m (the required number of micro-rotations)

needs to be scaled by a factor $K = \prod K_i$

$$K_i = \left[1 + \left(\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)} \right)^2 \right]^{-\frac{1}{2}}$$

$$\tilde{x}_{i+1} = \tilde{x}_i - [k_1(i) \cdot 2^{-s_1(i)} + k_2(i) \cdot 2^{-s_2(i)}]\, \tilde{y}_i$$
$$\tilde{y}_{i+1} = \tilde{y}_i + [k_1(i) \cdot 2^{-s_1(i)} + k_2(i) \cdot 2^{-s_2(i)}]\, \tilde{x}_i$$

$$\tilde{x}_0 = x_{R_m} \qquad k_1, k_2 \in \{-1, 0, 1\}$$
$$\tilde{y}_0 = y_{R_m} \qquad s_1, s_2 \in \{1, 2, \cdots, n-1\}$$

[21] C.-S. Wu, A.-Y. Wu, and C.-H. Lin, "A high-performance/low-latency vector rotational CORDIC architecture based on extended elementary angle set and trellis-based searching schemes," *IEEE Trans. Circuits Syst. II: Anal. Digital Signal Process.*, vol. 50, no. 9, pp. 589–601, Sep. 2003.

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A Unified View for Vector Rotational CORDIC Algorithms and Architectures Based on Angle Quantization Approach

An-Yeu Wu and Cheng-Shing Wu

AR : to approximate θ
with the combination
of selected angle elements
from a pre-defined **EAS**
(Elementary Angle Set)

EAS : all possible values of $\theta(j)$

EAS $S_1 = \{ \tan^{-1}(\alpha^* \cdot 2^{-s^*}) : \alpha^* \in \{-1, 0, +1\},$
$s^* \in \{0, 1, \cdots, N-1\} \}$

EAS S_1 consists of $\tan^{-1}$(Single signed power of two)
$\tan^{-1}$(Single SPT)
$\tan^{-1}(\alpha^* \cdot 2^{-s^*})$

SPT-based digital filter design

to increase the coefficient resolution

→ imploy more SPT terms to represent filter coefficients

[12] H. Samueli, "An improved search algorithm for the design of multiplierless FIR filters with power-of-two coefficients," *IEEE Trans. Circuits Syst.*, vol. 36, pp. 1044–1047, July 1989.

[13] Y. C. Lim, R. Yang, D. Li, and J. Song, "Signed power-of-two term allocation scheme for the design of digital filters," *IEEE Trans. Circuits Syst. II*, vol. 46, pp. 577–584, May 1999.

EAS $\hat{S}_1$ consists of $\tan^{-1}$(Single signed power of two)

$\tan^{-1}$(Single SPT)

$\tan^{-1}(\alpha^* \cdot 2^{-s^*})$

EAS $\hat{S}_2$ consists of $\tan^{-1}$(two signed power of two)

$\tan^{-1}$(two SPT)

$\tan^{-1}(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*})$

Two Signed-Power-of-Two terms

$$S_2 = \left\{ \tan^{-1}\left(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*}\right) : \begin{array}{l} \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\} \\ s_0^*, s_1^* \in \{0, 1, \cdots, w-1\} \end{array} \right\}$$

$$\{\alpha_0^*, \alpha_1^*\} = \{-1, 0, +1\} \times \{-1, 0, +1\}$$

$$= \left\{ \begin{array}{lll} (-1, -1), & (-1, 0), & (-1, +1), \\ (0, -1), & (0, 0), & (0, +1), \\ (+1, -1), & (+1, 0), & (+1, +1) \end{array} \right\}$$

$$\{2^{-s_0^*}, 2^{-s_1^*}\} = \{2^0, 2^{-1}, 2^{-2}\} \times \{2^0, 2^{-1}, 2^{-2}\}$$

$$= \left\{ \begin{array}{lll} (2^0, 2^0), & (2^0, 2^{-1}), & (2^0, 2^{-2}), \\ (2^{-1}, 2^0), & (2^{-1}, 2^{-1}), & (2^{-1}, 2^{-2}), \\ (2^{-2}, 2^0), & (2^{-2}, 2^{-1}), & (2^{-2}, 2^{-2}), \end{array} \right\}$$

(-1, -1)	−	−	2^0	2^0
(-1, 0)	−	0	2^0	2^{-1}
(-1, +1)	−	+	2^0	2^{-2}
(0, -1)	0	−	2^{-1}	2^0
(0, 0)	0	0	2^{-1}	2^{-1}
(0, +1)	0	+	2^{-1}	2^{-2}
(+1, -1)	+	−	2^{-2}	2^0
(+1, 0)	+	0	2^{-2}	2^{-1}
(+1, +1)	+	+	2^{-2}	2^{-2}

(-1, -1)	−	−	2^0	2^0
(-1, 0)	−	0	2^0	2^{-1}
(-1, +1)	−	+	2^0	2^{-2}
(0, -1)	0	−	2^{-1}	2^0
(0, 0)	0	0	2^{-1}	2^{-1}
(0, +1)	0	+	2^{-1}	2^{-2}
(+1, -1)	+	−	2^{-2}	2^0
(+1, 0)	+	0	2^{-2}	2^{-1}
(+1, +1)	+	+	2^{-2}	2^{-2}

2^{0} -2^{0}

(-1, -1)	-2^{0} -2^{0}	-2^{0} -2^{-1}	-2^{0} -2^{-2}
(-1, 0)	-2^{0} $0\,2^{0}$	-2^{0} $0\,2^{-1}$	-2^{0} $0\,2^{-2}$
(-1, +1)	-2^{0} $+2^{0}$	-2^{0} $+2^{-1}$	-2^{0} $+2^{-2}$
(0, -1)	$0\,2^{0}$ -2^{0}	$0\,2^{0}$ -2^{-1}	$0\,2^{0}$ -2^{-2}
(0, 0)	$0\,2^{0}$ $0\,2^{0}$	$0\,2^{0}$ $0\,2^{-1}$	$0\,2^{0}$ $0\,2^{-2}$
(0, +1)	$0\,2^{0}$ $+2^{0}$	$0\,2^{0}$ $+2^{-1}$	$0\,2^{0}$ $+2^{-2}$
(+1, -1)	$+2^{0}$ -2^{0}	$+2^{0}$ -2^{-1}	$+2^{0}$ -2^{-2}
(+1, 0)	$+2^{0}$ $0\,2^{0}$	$+2^{0}$ $0\,2^{-1}$	$+2^{0}$ $0\,2^{-2}$
(+1, +1)	$+2^{0}$ $+2^{0}$	$+2^{0}$ $+2^{-1}$	$+2^{0}$ $+2^{-2}$

(-1, -1)	-2^{-1} -2^{0}	-2^{-1} -2^{-1}	-2^{-1} -2^{-2}
(-1, 0)	-2^{-1} $0\,2^{0}$	-2^{-1} $0\,2^{-1}$	-2^{-1} $0\,2^{-2}$
(-1, +1)	-2^{-1} $+2^{0}$	-2^{-1} $+2^{-1}$	-2^{-1} $+2^{-2}$
(0, -1)	$0\,2^{-1}$ -2^{0}	$0\,2^{-1}$ -2^{-1}	$0\,2^{-1}$ -2^{-2}
(0, 0)	$0\,2^{-1}$ $0\,2^{0}$	$0\,2^{-1}$ $0\,2^{-1}$	$0\,2^{-1}$ $0\,2^{-2}$
(0, +1)	$0\,2^{-1}$ $+2^{0}$	$0\,2^{-1}$ $+2^{-1}$	$0\,2^{-1}$ $+2^{-2}$
(+1, -1)	$+2^{-1}$ -2^{0}	$+2^{-1}$ -2^{-1}	$+2^{-1}$ -2^{-2}
(+1, 0)	$+2^{-1}$ $0\,2^{0}$	$+2^{-1}$ $0\,2^{-1}$	$+2^{-1}$ $0\,2^{-2}$
(+1, +1)	$+2^{-1}$ $+2^{0}$	$+2^{-1}$ $+2^{-1}$	$+2^{-1}$ $+2^{-2}$

(-1, -1)	-2^{-2} -2^{0}	-2^{-2} -2^{-1}	-2^{-2} -2^{-2}
(-1, 0)	-2^{-2} $0\,2^{0}$	-2^{-2} $0\,2^{-1}$	-2^{-2} $0\,2^{-2}$
(-1, +1)	-2^{-2} $+2^{0}$	-2^{-2} $+2^{-1}$	-2^{-2} $+2^{-2}$
(0, -1)	$0\,2^{-2}$ -2^{0}	$0\,2^{-2}$ -2^{-1}	$0\,2^{-2}$ -2^{-2}
(0, 0)	$0\,2^{-2}$ $0\,2^{0}$	$0\,2^{-2}$ $0\,2^{-1}$	$0\,2^{-2}$ $0\,2^{-2}$
(0, +1)	$0\,2^{-2}$ $+2^{0}$	$0\,2^{-2}$ $+2^{-1}$	$0\,2^{-2}$ $+2^{-2}$
(+1, -1)	$+2^{-2}$ -2^{0}	$+2^{-2}$ -2^{-1}	$+2^{-2}$ -2^{-2}
(+1, 0)	$+2^{-2}$ $0\,2^{0}$	$+2^{-2}$ $0\,2^{-1}$	$+2^{-2}$ $0\,2^{-2}$
(+1, +1)	$+2^{-2}$ $+2^{0}$	$+2^{-2}$ $+2^{-1}$	$+2^{-2}$ $+2^{-2}$

(-1, -1)	-2^0 -2^0	-2^0 -2^{-1}	-2^0 -2^{-2}
(-1, 0)	-2^0	-2^0	$0 2^{-2}$
(-1, +1)		-2^0 $+2^{-1}$	-2^0 $+2^{-2}$
(0, -1)	-2^0	-2^{-1}	-2^{-2}
(0, 0)			
(0, +1)	$+2^0$	$+2^{-1}$	$+2^{-2}$
(+1, -1)		$+2^0$ -2^{-1}	$+2^0$ -2^{-2}
(+1, 0)	$+2^0$	$+2^0$	$+2^0$
(+1, +1)	$+2^0$ $+2^0$	$+2^0$ $+2^{-1}$	$+2^0$ $+2^{-2}$

(-1, -1)	-2^0 -2^{-1}	-2^0	-2^{-1} -2^{-2}
(-1, 0)	-2^{-1}	-2^{-1}	-2^{-1}
(-1, +1)	-2^0 $+2^{-1}$		-2^{-1} $+2^{-2}$
(0, -1)	-2^0	-2^{-1}	-2^{-2}
(0, 0)			
(0, +1)	$+2^0$	$+2^{-1}$	$+2^{-2}$
(+1, -1)	$+2^0$ -2^{-1}		$+2^{-1}$ -2^{-2}
(+1, 0)	$+2^{-1}$	$+2^{-1}$	$+2^{-1}$
(+1, +1)	$+2^0$ $+2^{-1}$	$+20$	$+2^{-1}$ $+2^{-2}$

(-1, -1)	-2^0 -2^{-2}	-2^{-1} -2^{-2}	-2^{-1}
(-1, 0)	-2^{-2}	-2^{-2}	-2^{-2}
(-1, +1)	-2^0 $+2^{-2}$	-2^{-1} $+2^{-2}$	
(0, -1)	-2^0	-2^{-1}	-2^{-2}
(0, 0)			
(0, +1)	$+2^0$	$+2^{-1}$	$+2^{-2}$
(+1, -1)	$+2^0$ -2^{-2}	$+2^{-1}$ -2^{-2}	
(+1, 0)	$+2^{-2}$	$+2^{-2}$	$+2^{-2}$
(+1, +1)	$+2^0$ $+2^{-2}$	$+2^{-1}$ $+2^{-2}$	$+2^{-1}$

(-1, -1)	$-2^0 \quad -2^0$	$-2^0 \quad -2^{-1}$	$-2^0 \quad -2^{-2}$
(-1, 0)	-2^0	-2^0	-2^0
(-1, +1)		$\boxed{-2^0 \quad +2^{-1}}$	$\boxed{-2^0 \quad +2^{-2}}$
(0, -1)	-2^0	-2^{-1}	-2^{-2}
(0, 0)			
(0, +1)	$+2^0$	$+2^{-1}$	$+2^{-2}$
(+1, -1)		$\boxed{+2^0 \quad -2^{-1}}$	$\boxed{+2^0 \quad -2^{-2}}$
(+1, 0)	$+2^0$	$+2^0$	$+2^0$
(+1, +1)	$+2^0 \quad +2^0$	$+2^0 \quad +2^{-1}$	$+2^0 \quad +2^{-2}$

(-1, -1)	$-2^0 \quad -2^{-1}$	-2^0	$-2^{-1} \quad -2^{-2}$
(-1, 0)	-2^{-1}	-2^{-1}	-2^{-1}
(-1, +1)	$-2^0 \quad +2^{-1}$		$\boxed{-2^{-1} \quad +2^{-2}}$
(0, -1)	-2^0	-2^{-1}	-2^{-2}
(0, 0)			
(0, +1)	$+2^0$	$+2^{-1}$	$+2^{-2}$
(+1, -1)	$+2^0 \quad -2^{-1}$		$\boxed{+2^{-1} \quad -2^{-2}}$
(+1, 0)	$+2^{-1}$	$+2^{-1}$	$+2^{-1}$
(+1, +1)	$+2^0 \quad +2^{-1}$	$+2^0$	$+2^{-1} \quad +2^{-2}$

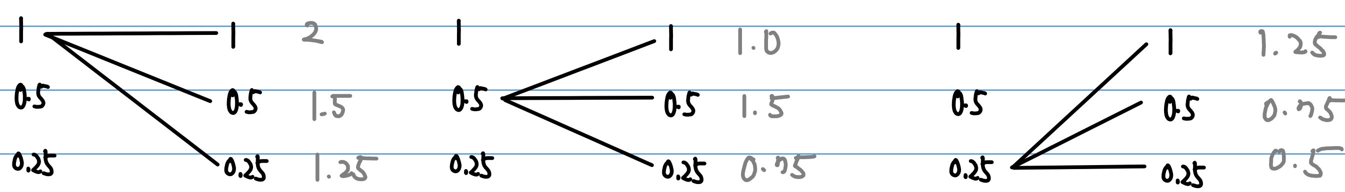
(-1, -1)	$-2^0 \quad -2^{-2}$	$-2^{-1} \quad -2^{-2}$	-2^{-1}
(-1, 0)	-2^{-2}	-2^{-2}	-2^{-2}
(-1, +1)	$\boxed{-2^0 \quad +2^{-2}}$	$-2^{-1} \quad +2^{-2}$	
(0, -1)	-2^0	-2^{-1}	-2^{-2}
(0, 0)			
(0, +1)	$+2^0$	$+2^{-1}$	$+2^{-2}$
(+1, -1)	$\boxed{+2^0 \quad -2^{-2}}$	$+2^{-1} \quad -2^{-2}$	
(+1, 0)	$+2^{-2}$	$+2^{-2}$	$+2^{-2}$
(+1, +1)	$+2^0 \quad +2^{-2}$	$+2^{-1} \quad +2^{-2}$	$+2^{-1}$

S_1

1	$1 = 2^{-0}$	$\tan^{-1}(2^{-0})$
0.5	$\frac{1}{2} = 2^{-1}$	$\tan^{-1}(2^{-1})$
0.25	$\frac{1}{4} = 2^{-2}$	$\tan^{-1}(2^{-2})$

S_2

2	$1+1 = 2^{-0} + 2^{-0}$	$\pm\tan^{-1}(2^{-0} + 2^{-0})$
1.5	$1+\frac{1}{2} = 2^{-0} + 2^{-1}$	$\pm\tan^{-1}(2^{-0} + 2^{-1})$
1.25	$1+\frac{1}{4} = 2^{-0} + 2^{-2}$	$\pm\tan^{-1}(2^{-0} + 2^{-2})$
1.0	$1 = 2^{-0}$	$\pm\tan^{-1}(2^{-0})$
0.75	$\frac{1}{2}+\frac{1}{4} = 2^{-1} + 2^{-2}$	$\pm\tan^{-1}(2^{-1} + 2^{-2})$
0.5	$\frac{1}{2} = 2^{-1}$	$\pm\tan^{-1}(2^{-1})$
0.25	$\frac{1}{4} = 2^{-2}$	$\pm\tan^{-1}(2^{-2})$



$2^{-0}, \; 2^{-1}, \; 2^{-2}$ $\qquad \{0, 1, 2\} = \{0, 1, W-1\}$

$W=3$

$s_0^*, s_1^* \in \{0, 1, 2\}$

$2^{s_0^*}, 2^{s_1^*} \in \{2^{-0}, 2^{-1}, 2^{-2}\}$

as the word size w increases,
the size of the set S_2 increases exponentially

$$\theta_i = \tan^{-1}\left(\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)}\right)$$

R_m: the number of the subangle N_A

$$S_2 = \{ \theta_i \mid i = 0, 1, \cdots R_m \}$$

$$S_2 = \{ \tan^{-1}\left(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*}\right) : \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\}, \; s_0^*, s_1^* \in \{0, 1, \cdots, w-1\} \}$$

the optimization problem of the EEAS-based CORDIC algorithm

given θ and R_m

find $\alpha_0(j)$, $\alpha_1(j)$, $s_0(j)$, and $s_1(j)$
the combination of elementary angles
from EEAS $\hat{S}_2$

minimize the angle quantization error

$$|\xi_{m,\,EEAS}| \triangleq \theta - \sum_{j=0}^{R_m-1} \tan^{-1}\left(\alpha_0(j)\, 2^{-s_0(j)} + \alpha_1(j)\, 2^{-s_1(j)}\right)$$

EAS S_1

elementary angle

```
r(1) = atan(-2^{-0})
r(2) = atan(-2^{-1})
r(3) = atan(-2^{-2})
r(4) = atan(         0 )
r(5) = atan( 2^{-2})
r(6) = atan( 2^{-1})
r(7) = atan( 2^{-0})
```

EEAS S_2

```
r(1)   = atan(-2^{-0} - 2^{-0})
r(2)   = atan(-2^{-0} - 2^{-1})
r(3)   = atan(-2^{-0} - 2^{-2})
r(4)   = atan(-2^{-0}          )
r(5)   = atan(-2^{-1} - 2^{-2})
r(6)   = atan(-2^{-1}          )
r(7)   = atan(-2^{-2}          )
r(8)   = atan(        0        )
r(9)   = atan( 2^{-2}          )
r(10) = atan( 2^{-1}          )
r(11) = atan( 2^{-1}+ 2^{-2})
r(12) = atan( 2^{-0}          )
r(13) = atan( 2^{-0}+ 2^{-2})
r(15) = atan( 2^{-0}+ 2^{-1})
r(16) = atan( 2^{-0}+ 2^{-0})
```

given $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix}$

$$\begin{bmatrix} x(j+1) \\ y(j+1) \end{bmatrix} = \begin{bmatrix} 1 & \alpha_0(j)\,2^{-S_0(j)} + \alpha_1(j)\,2^{-S_1(j)} \\ \alpha_0(j)\,2^{-S_0(j)} + \alpha_1(j)\,2^{-S_1(j)} & 1 \end{bmatrix} \begin{bmatrix} x(j) \\ y(j) \end{bmatrix}$$

$$\begin{bmatrix} x_f \\ y_f \end{bmatrix} = P \begin{bmatrix} x(R_m) \\ y(R_m) \end{bmatrix} = \frac{1}{\prod_{j=0}^{R_m-1} \sqrt{1+[\alpha_0(j)\cdot 2^{-S_0(j)} + \alpha_1(j)\cdot 2^{-S_1(j)}]^2}} \begin{bmatrix} x(R_m) \\ y(R_m) \end{bmatrix}$$

micro rotation procedure

the scaling operation

4 additions

increased hardware

reduced iteration steps

W = 16

elementary angle

r(1) = atan(2^{-1})
r(2) = atan(2^{-2})
r(3) = atan(2^{-3})
r(4) = atan(2^{-4})
r(5) = atan(2^{-5})
r(6) = atan(2^{-6})
r(7) = atan(2^{-7})
r(8) = atan(2^{-8})
r(9) = atan(2^{-9})
r(10) = atan(2^{-10})
r(11) = atan(2^{-11})
r(12) = atan(2^{-12})
r(13) = atan(2^{-13})
r(14) = atan(2^{-14})
r(15) = atan(2^{-15})
r(16) = atan(0)
r(17) = atan(2^{-15})
r(18) = atan(2^{-14})
r(19) = atan(2^{-13})
r(20) = atan(2^{-12})
r(21) = atan(2^{-11})
r(22) = atan(2^{-10})
r(23) = atan(2^{-9})
r(24) = atan(2^{-8})
r(26) = atan(2^{-7})
r(27) = atan(2^{-6})
r(28) = atan(2^{-4})
r(29) = atan(2^{-4})
r(30) = atan(2^{-3})
r(31) = atan(2^{-2})
r(32) = atan(2^{-1})