Applications of Arrays (1A)

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- Viewing an **array** as a **pointer**

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`int a[4] ;` an array **a**

generalization

`int (*a)` view **a** as a pointer

virtual pointer
- no real memory location
- constraints :
  value(&**a**) = value(**a**)

- Viewing a **pointer** as an **array**

`int (*a) ;` a pointer **a**

a specific instance

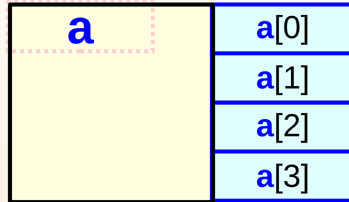
`int a[N]` view **a** as an array

N is not fixed

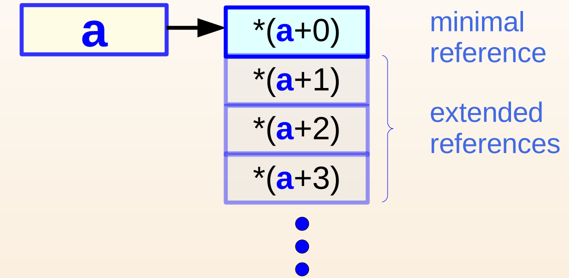
sizeof(**a**) is
not the size of the array
but of a pointer variable

Array a and pointer a

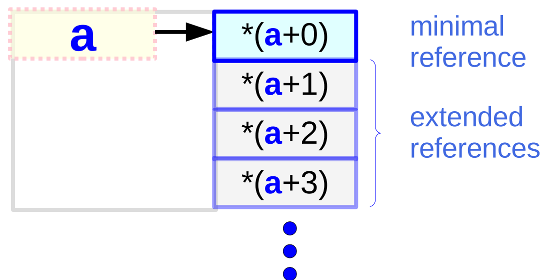
int a[4] ; an array a



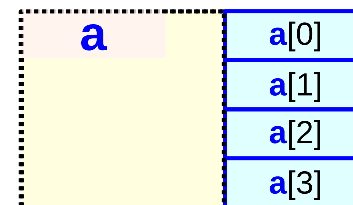
int (*a) ; a pointer a



int (*a) a as a pointer



int a[N] a as an array



Array **a** and pointer **a**

int **a**[4] ;   an array **a**

- type(**a**)   = int [4]
- sizeof(**a**)   = an array size (16 bytes)
- value(&**a**)   = value(**a**)
- fixed number of elements

int (****a**)   **a** as a pointer

a is not a real pointer

- sizeof(**a**)   = an array size
- value(&**a**)   = value(**a**)

int (***a**) ;   a pointer **a**

- type(**a**)   = int (*)
- sizeof(**a**)   = a pointer size (4 bytes)
- value(&**a**)   ≠ value(**a**)
- variable number of elements

int **a**[N]   **a** as an array

a is not a real array

- sizeof(**a**)   ≠ an array size
= a pointer size
- value(&**a**)   ≠ value(**a**)
= assigned address

Relationship between array and array pointer types

`int b[4][2] ;` declare a **2-d** array b

generalization

`int (*b) [2]` b as a **1-d** array pointer

`int (*b)[2] ;` declare a **1-d** array pointer b

a specific instance

`int b[N][2]` b as a **2-d** array

`int a[4] ;` declare a **1-d** array a

generalization

`int (*a)` a as a **0-d** array pointer

`int (*a) ;` declare a **0-d** array pointer a

a specific instance

`int a[N]` a as a **1-d** array

Array **b** and array pointer **b**

int **b**[4][2] ;   **2-d** array **b**

- type(**b**)   = int [4]
- sizeof(**b**)   = an array size (32 bytes)
- value(&**b**)   = value(**b**)
- fixed number of elements

int (*) [2]   **b** as a **1-d** array pointer

b is not a real pointer

- sizeof(**b**)   = an array size
- value(&**b**)   = value(**b**)

int (***b**) [2] ;   **1-d** array pointer **b**

- type(**b**)   = int (*)
- sizeof(**b**)   = a pointer size (4 bytes)
- value(&**b**)   ≠ value(**b**)
- variable number of elements

int [N][2]   **b** as a **2-d** array

b is not a real array

- sizeof(**b**)   ≠ an array size
  = a pointer size
- value(&**b**)   ≠ value(**b**)
  = assigned address

Array b and array pointer b

int b[4][2] ; 2-d array b

b	b[0]	b[0][0]
		b[0][1]
	b[1]	b[1][0]
		b[1][1]
	b[2]	b[2][0]
		b[2][1]
	b[3]	b[3][0]
		b[3][1]

int (*) [2] b as a 1-d array pointer

b →

*(b+0)	(*(b+0))[0]
	(*(b+0))[1]
*(b+1)	(*(b+1))[0]
	(*(b+1))[1]
*(b+2)	(*(b+2))[0]
	(*(b+2))[1]
*(b+3)	(*(b+3))[0]
	(*(b+3))[1]

•••

minimal reference

extended references

virtual pointer
- no real memory location
- constraints :
  &b = b

int (*b) [2] ; 1-d array pointer b

b →

*(b+0)	(*(b+0))[0]
	(*(b+0))[1]
*(b+1)	(*(b+1))[0]
	(*(b+1))[1]
*(b+2)	(*(b+2))[0]
	(*(b+2))[1]
*(b+3)	(*(b+3))[0]
	(*(b+3))[1]

•••

minimal reference

extended references

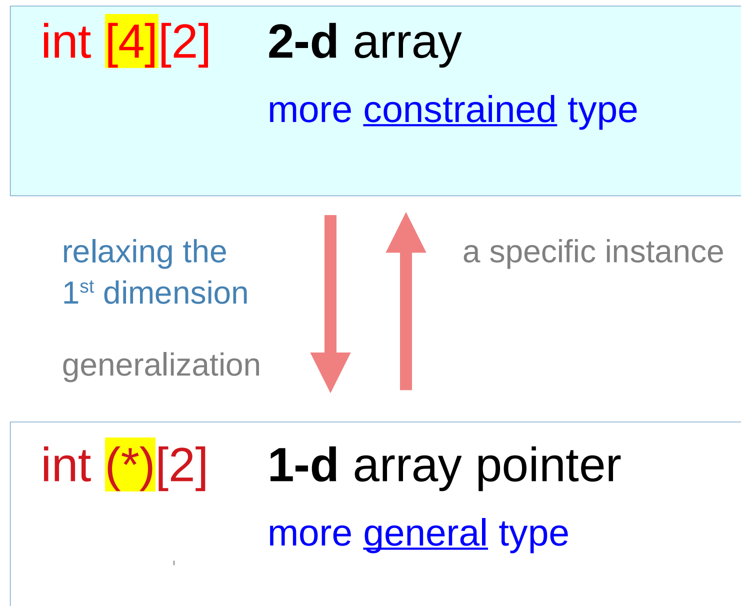
int [N][2] b as a 2-d array

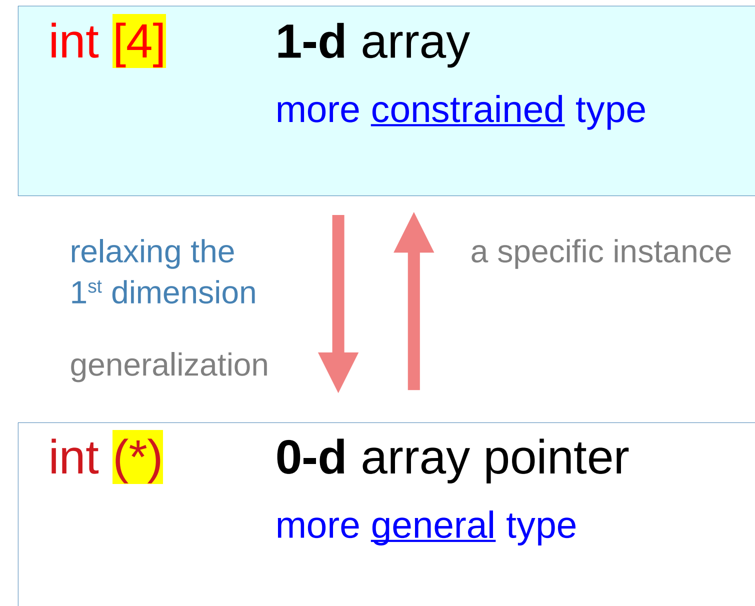
b	b[0]	b[0][0]
		b[0][1]
	b[1]	b[1][0]
		b[1][1]
	b[2]	b[2][0]
		b[2][1]
	b[3]	b[3][0]
		b[3][1]

N is not fixed to 4

sizeof(b) is not the size of the array but the size of a pointer variable

Dual type - relaxing the 1st dimension of an array

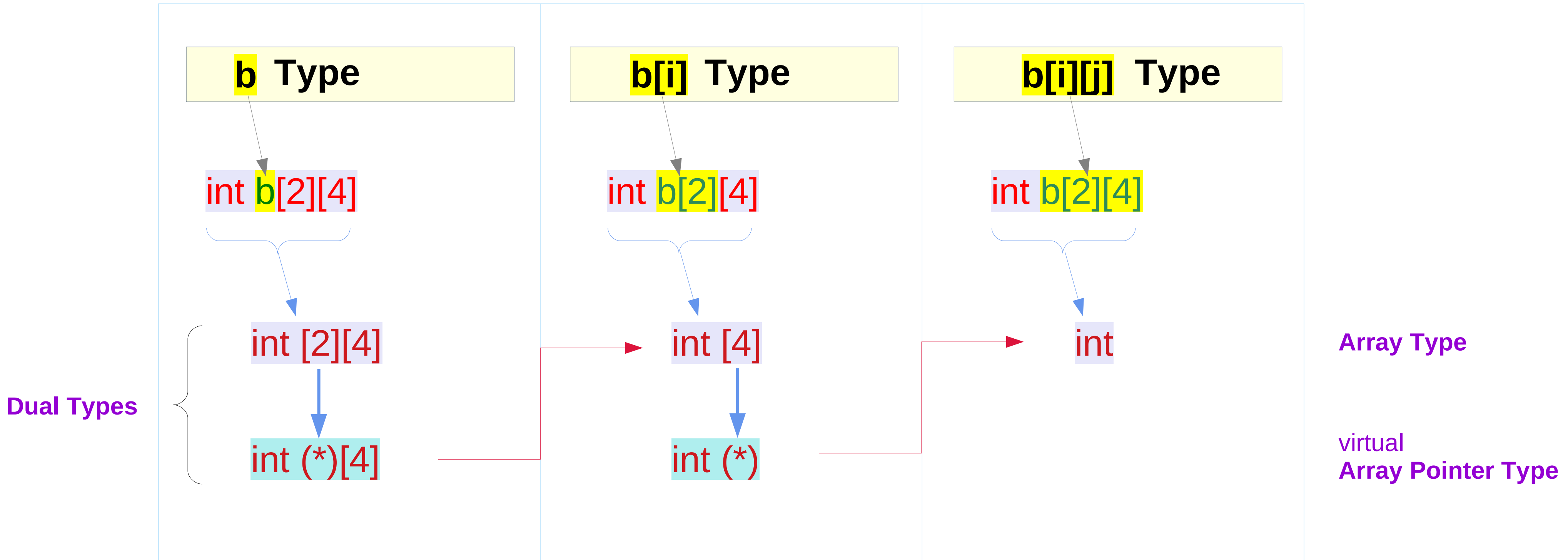




Subarray types in a **2-d** array

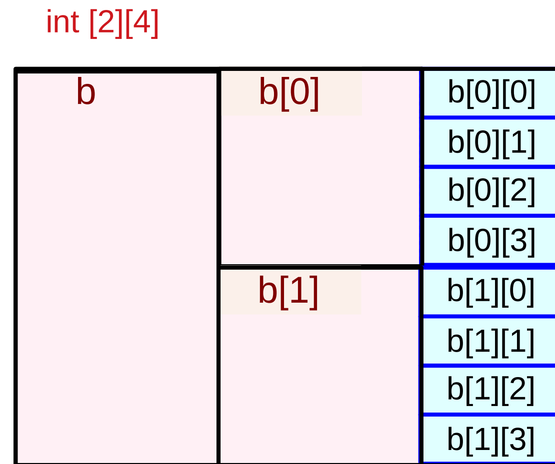
```
int b[2][4];
```

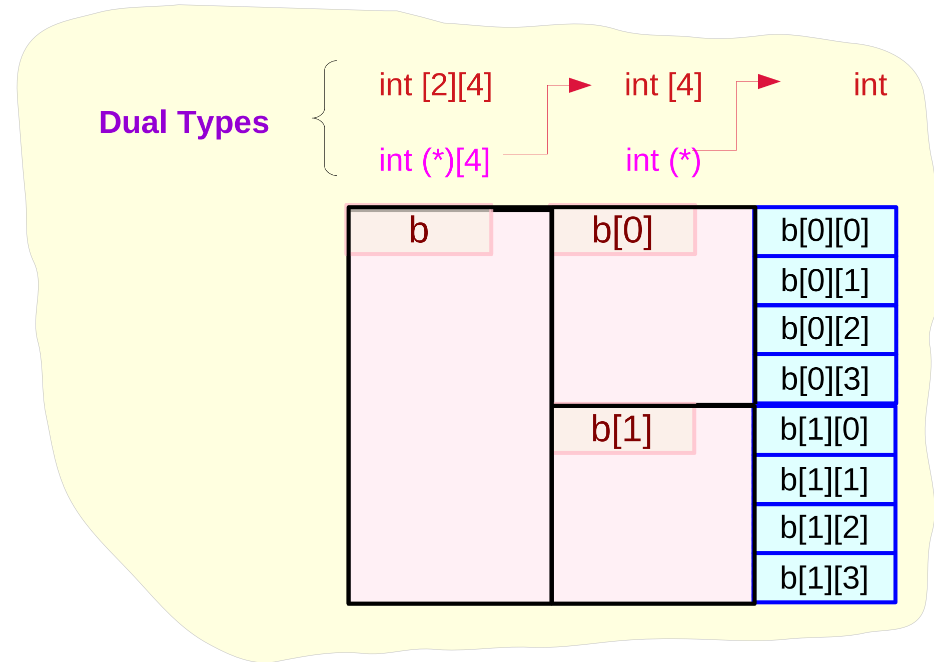
2-d array b

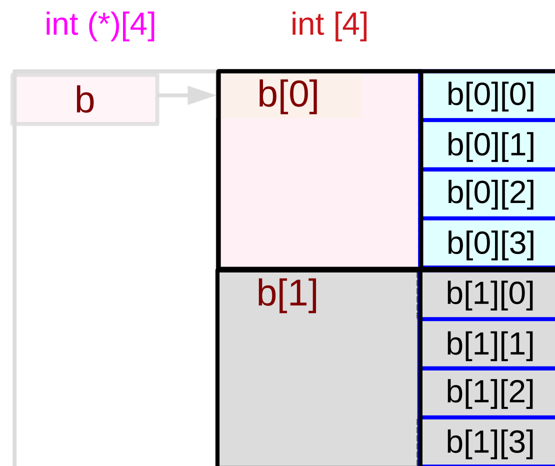


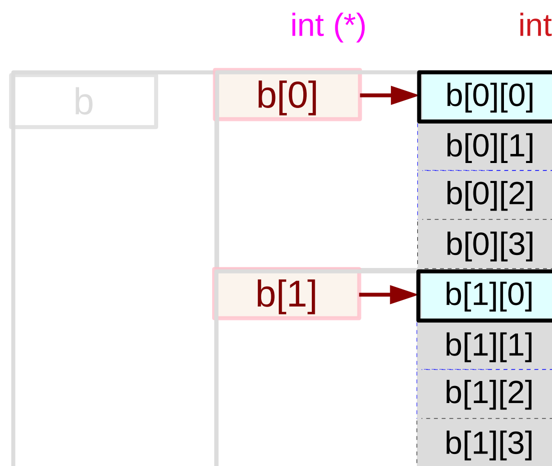
Dual types in a **2-d** array

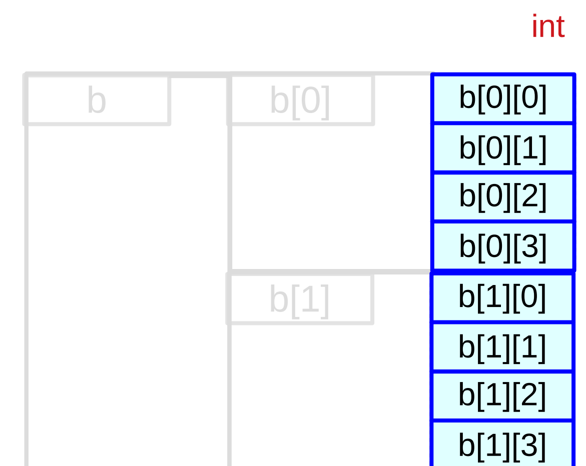
int b[2][4];   **2-d** array b











Subarray type examples

```
int a[4];
```

			relaxed type	virtual
a	int [4]	**1-d** array type	int (*)	**0-d** array pointer type
a[i]	int	**0-d** array type		

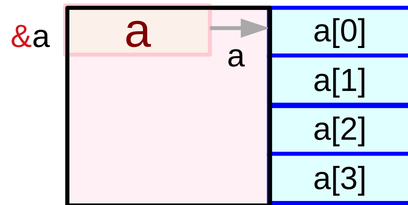
```
int b[2][4];
```

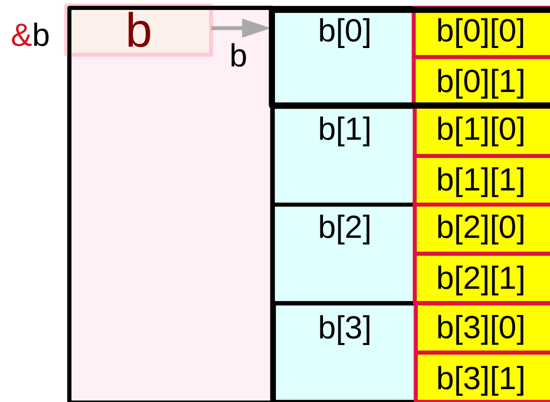
			relaxed type	virtual
b	int [2][4]	**2-d** array type	int (*)[4]	**1-d** array pointer type
b[i]	int [4]	**1-d** array type	int (*)	**0-d** array pointer type
b[i][j]	int	**0-d** array type		

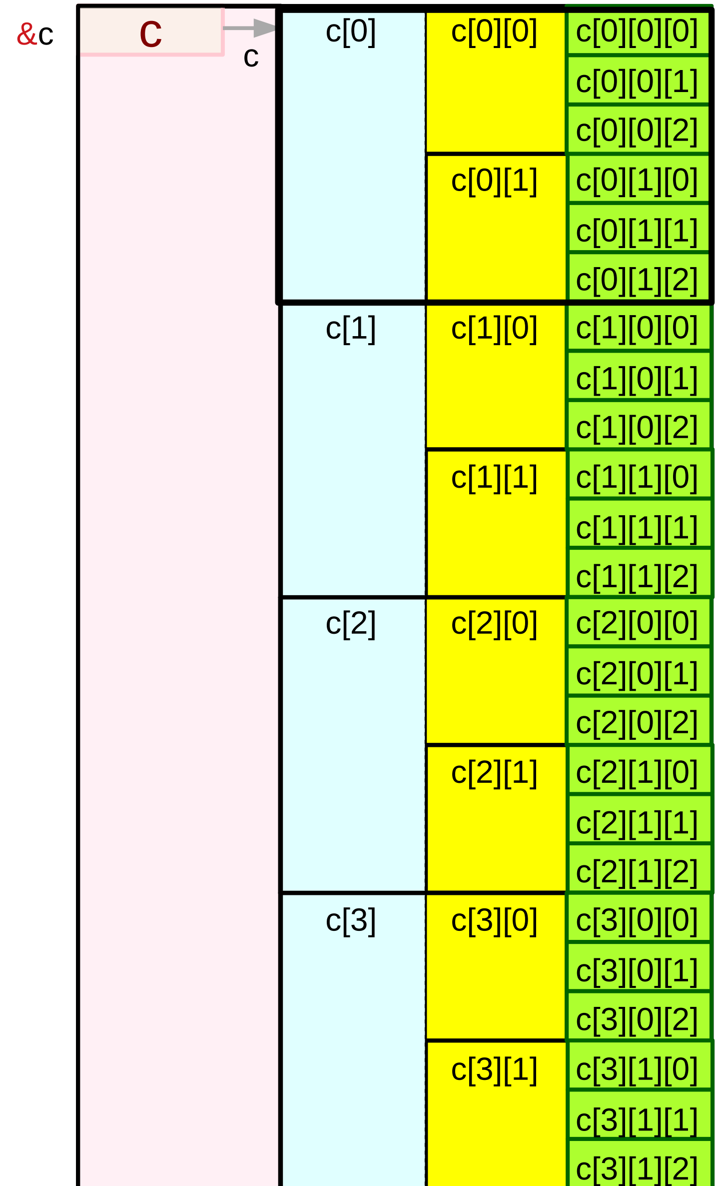
```
int c[4][2][3];
```

			relaxed type	virtual
c	int [4][2][3]	**3-d** array type	int (*)[2][3]	**2-d** array pointer type
c[i]	int [4][2]	**2-d** array type	int (*)[2]	**1-d** array pointer type
c[i][j]	int [4]	**1-d** array type	int (*)	**0-d** array pointer type
c[i][j][k]	int	**0-d** array type		

Types of **a**, **b**, **c** arrays





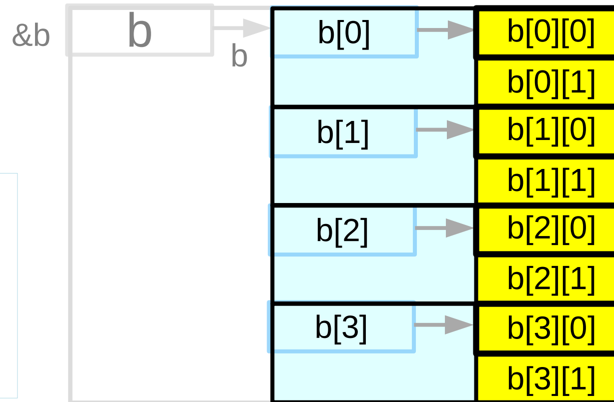



```
int a[4];
int b[2][4];
int c[4][2][3];
```

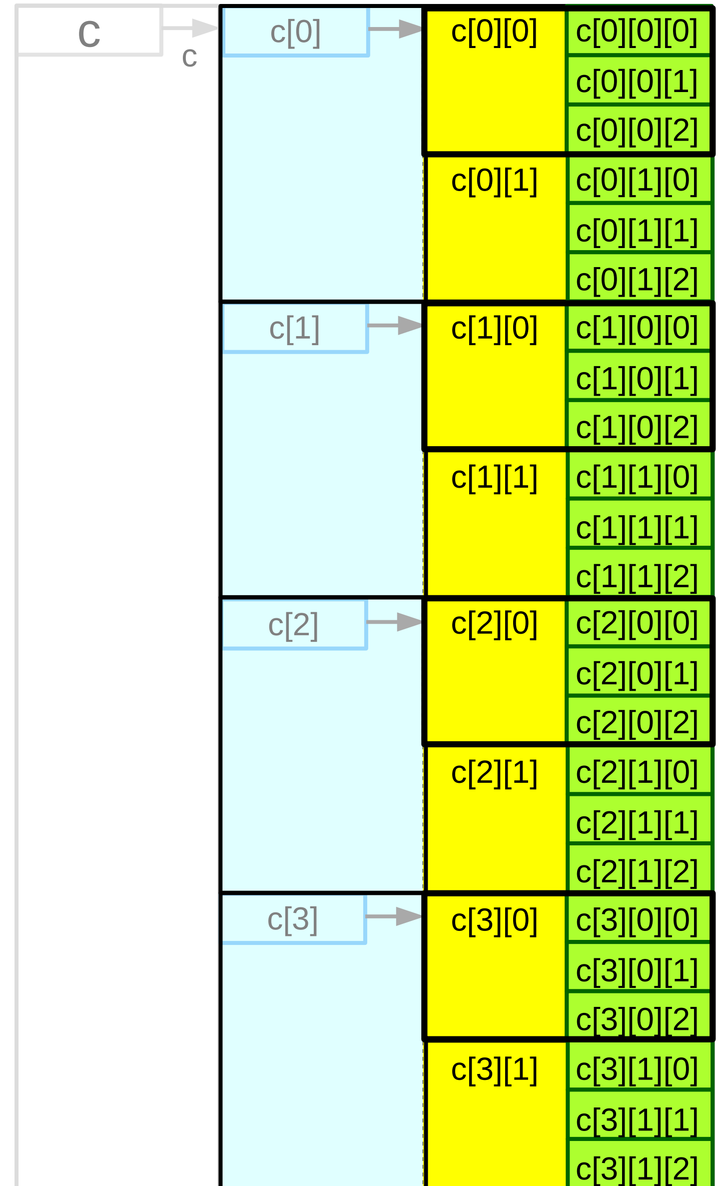
dual types

int [4] int (*)	**1-d** array **a** **0-d** array pointer **a** (virtual)	**a[i]** ***(a+i)**
int [4][2]; int (*)[2];	**2-d** array **b** **1-d** array pointer **b** (virtual)	**b[i]** ***(b+i)**
int [4][2][3]; int (*)[2][3];	**3-d** array **c** **2-d** array pointer **c** (virtual)	**c[i]** ***(c+i)**

Types of **b[i]**, **c[i]** subarrays



int a[4];
int b[2][4];
int c[4][2][3];



dual types

int [2] int (*)	**1-d** array **b[i]** **0-d** array pointer **b[i]** (virtual)	**b[i][j]** ***(b[i]+j)**
int [2][3]; int (*)[3];	**2-d** array **c[i]** **1-d** array pointer **c[i]** (virtual)	**c[i][j]** ***(c[i]+j)**

Types of **c[i][j]** subarrays

```
int a[4];
int b[2][4];
int c[4][2][3];
```

dual types

int [3]	**1-d** array **c[i][j]**	**c[i][j][k]**
int (*)	**0-d** array pointer **c[i][j]** (virtual)	***(c[i][j]+k)**

Types of a **4-d** array and its subarrays

int **d**[4][2][3][4];

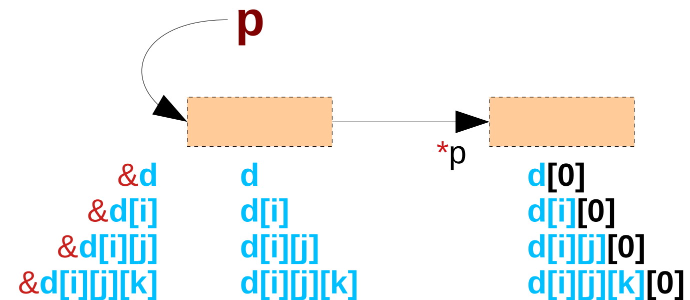
types

d	consider d[4][2][3][4]	int [4][2][3][4]	⇨	**4-d** array
	relax the 1st dimension	int (*)[2][3][4]	⇨	**3-d** array pointer (virtual)
d[i]	consider d[i][2][3][4]	int [2][3][4]	⇨	**3-d** array
	relax the 1st dimension	int (*)[3][4]	⇨	**2-d** array pointer (virtual)
d[i][j]	consider d[i][j][3][4]	int [3][4]	⇨	**2-d** array
	relax the 1st dimension	int (*)[4]	⇨	**1-d** array pointer (virtual)
d[i][j][k]	consider d[i][j][k][4]	int [4]	⇨	**1-d** array
	relax the 1st dimension	int (*)	⇨	**0-d** array pointer (virtual)

i,j,k are specific index values i=[0..3], j=[0..1], k=[0..2]

Initializing ***n*-d** array pointers with ***n*-d** subarrays

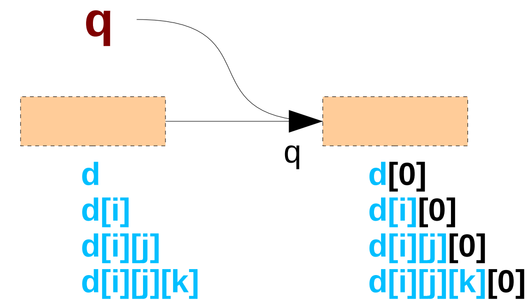
int d[4][2][3][4];



d	**4-d** array	d[4][2][3][4]	p = &d	abstract data
p	**4-d** array pointer	(*p)[4][2][3][4]	int (*p)[4][2][3][4] = &d;	
			(*p)[i][j][k][l] ≡ d[i][j][k][l]	
d[i]	**3-d** array	d[i][2][3][4]	p = &d[i]	abstract data
p	**3-d** array pointer	(*p)[2][3][4]	int (*p)[3][4] = &d[i];	
			(*p)[j][k][l] ≡ d[i][j][k][l] given i	
d[i][j]	**2-d** array	d[i][j][3][4]	p = &d[i][j]	abstract data
p	**2-d** array pointer	(*p)[3][4]	int (*p)[4] = &d[i][j];	
			(*p)[k][l] ≡ d[i][j][k][l] given i, j	
d[i][j][k]	**1-d** array	d[i][j][k][4]	p = &d[i][j][k]	abstract data
p	**1-d** array pointer	(*p)[4]	int (*p) = &d[i][j][k];	
			(*p)[l] ≡ d[i][j][k][l] given i, j, k	

Initializing (*n*-1)-d array pointers with *n*-d subarrays

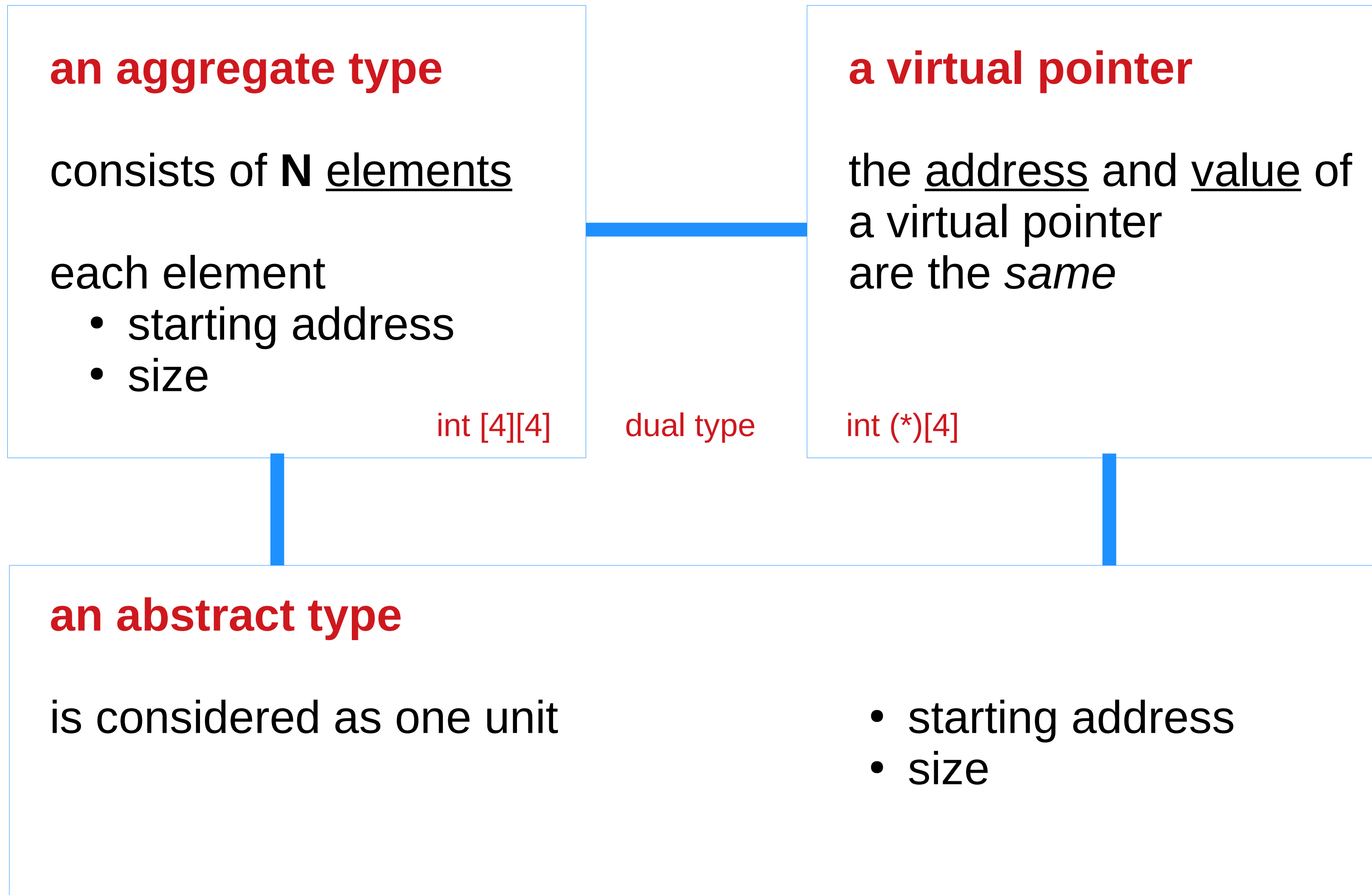
int d[4][2][3][4];



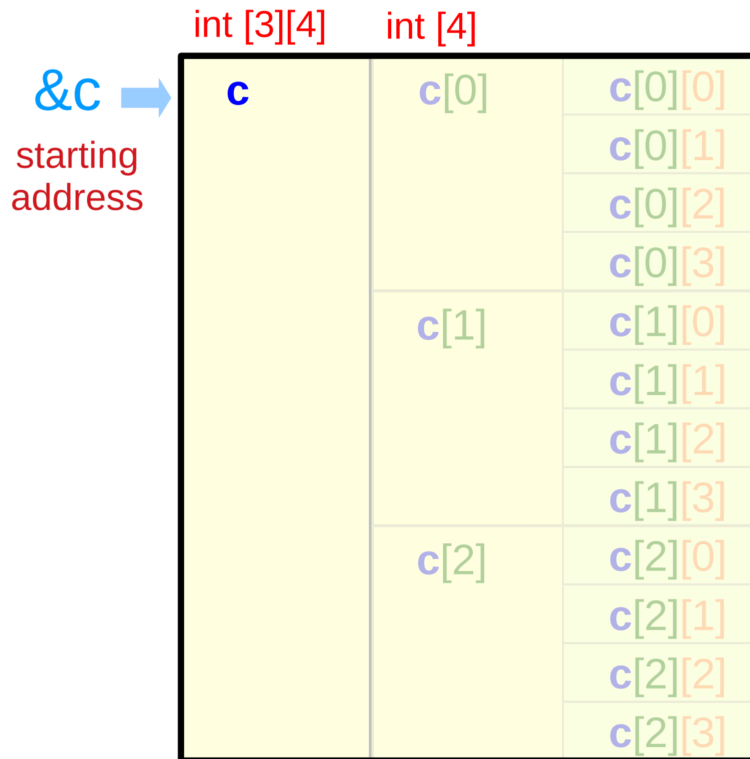
d	**4-d** array	d[4][2][3][4]	q = d	virtual pointer
q	**3-d** array pointer	(*q)[2][3][4]	int (*q)[2][3][4] = d;	
			q[i][j][k][l] ≡ d[i][j][k][l]	
d[i]	**3-d** array	d[i][2][3][4]	q = d[i]	virtual pointer
q	**2-d** array pointer	(*q)[3][4]	int (*q)[3][4] = d[i];	
			q[j][k][l] ≡ d[i][j][k][l]	given i
d[i][j]	**2-d** array	d[i][j][3][4]	q = d[i][j]	virtual pointer
q	**1-d** array pointer	(*q)[4]	int (*q)[4] = d[i][j];	
			q[k][l] ≡ d[i][j][k][l]	given i, j
d[i][j][k]	**1-d** array	d[i][j][k][4]	q = d[i][j][k]	virtual pointer
q	**0-d** array pointer	(*q)	int (*q) = d[i][j][k];	
			q[l] ≡ d[i][j][k][l]	given i, j, k

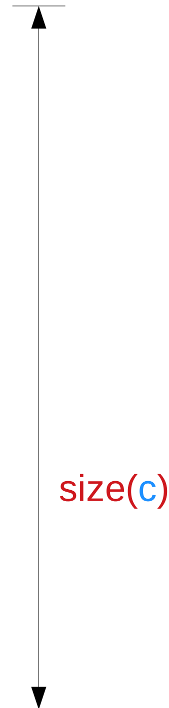
Aggregate Data Types
Abstract Data Types
Virtual Array Pointers

Aggregate data type



Abstract data **c**

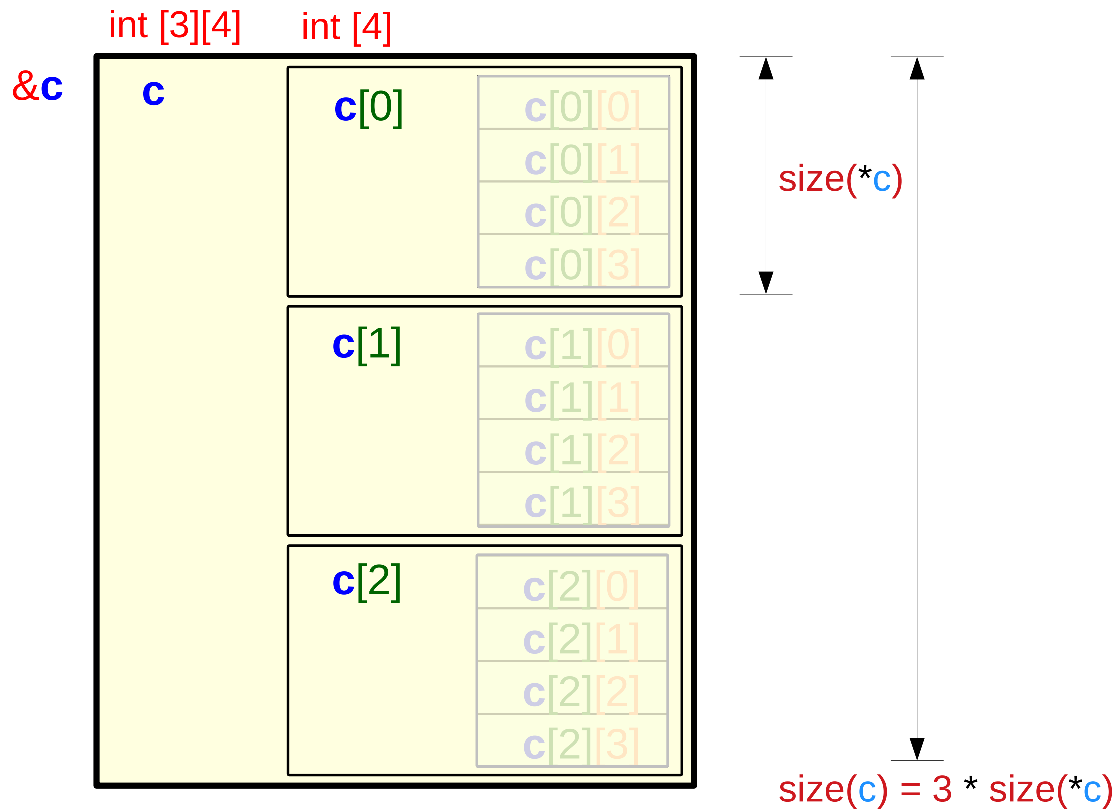




size(c)

an abstract data	c
- start address	&c
- size	sizeof(c)

Aggregate data c

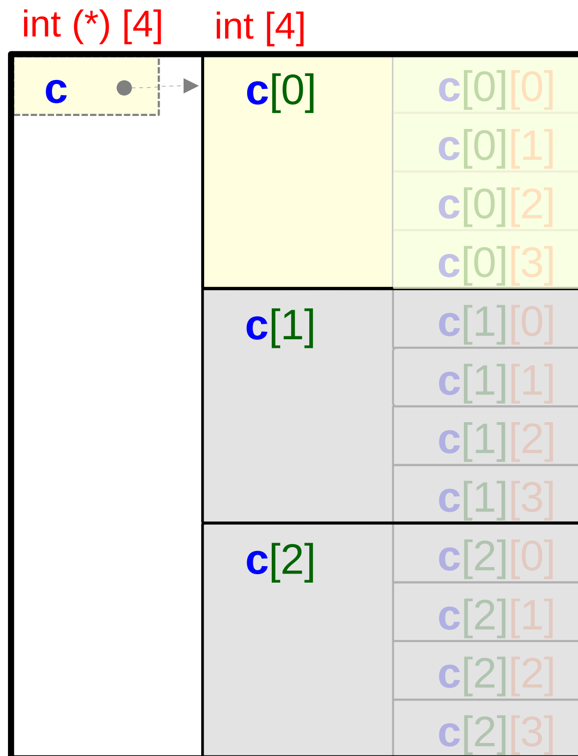


an aggregate type data c

- 1st element c[0]
- 2nd element c[1]
- 3rd element c[2]

Virtual pointer c

&c = c = &c[0]



size(*c)

a virtual pointer c
- pointer address &c
- pointer value c = &c[0]

with the constraint
c = &c

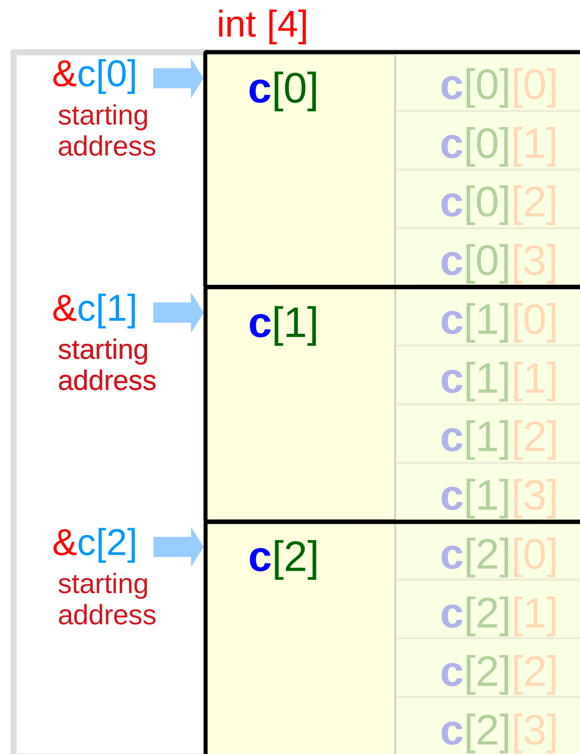
an abstract data c[0] = *c
- start address &c[0] = c
- size sizeof(c[0])

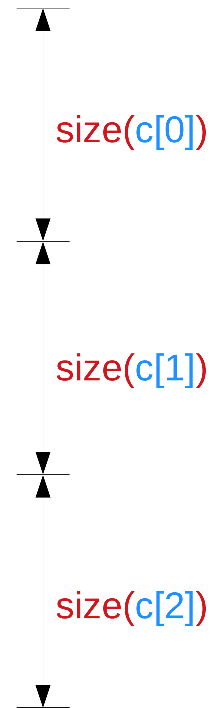
virtual pointer c points
to abstract data c[0]

virtual pointers

- no physical memory locations are allocated
- address and data have the same value

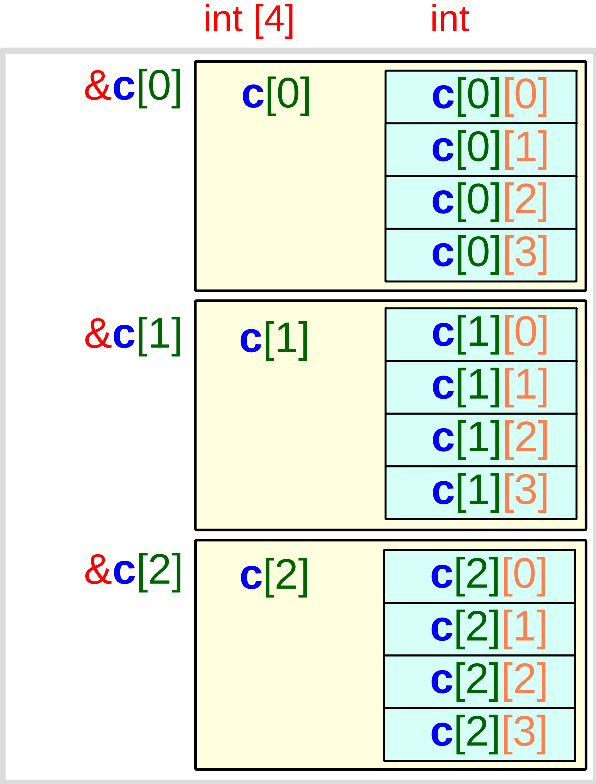
Abstract data **c[i]**

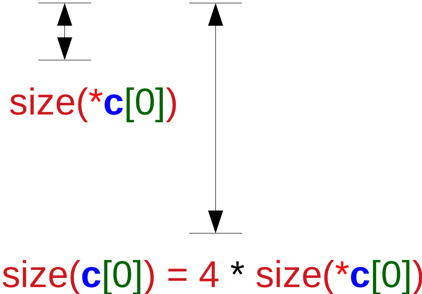




- an abstract data c[0]
  - start address &c[0]
  - size sizeof(c[0])
- an abstract data c[1]
  - start address &c[1]
  - size sizeof(c[1])
- an abstract data c[2]
  - start address &c[2]
  - size sizeof(c[2])

Aggregate data c[i]

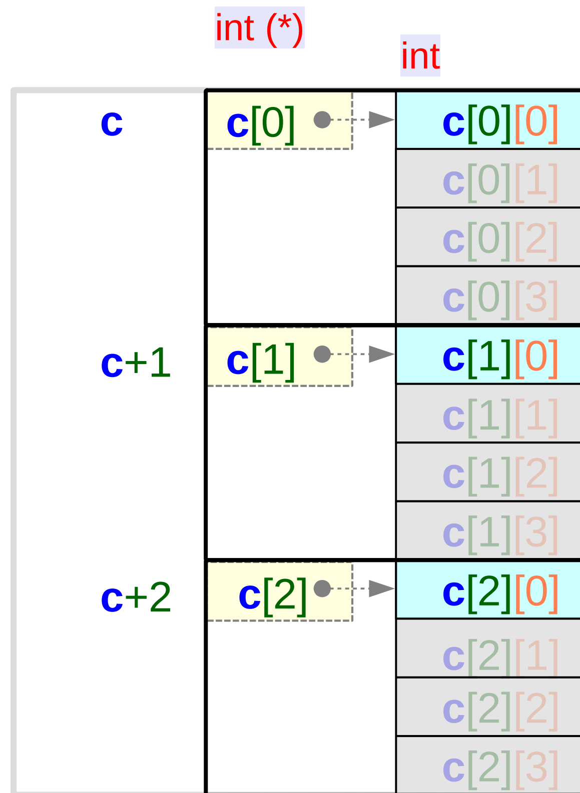




an aggregate type data c[i]

- 1st element c[i][0]
- 2nd element c[i][1]
- 3rd element c[i][2]
- 4th element c[i][3]

Virtual pointer **c[i]**



a virtual pointer c[i]
- pointer address &c[i]
- pointer value c+i = &c[i]

with the constraint
c[i] = &c[i]

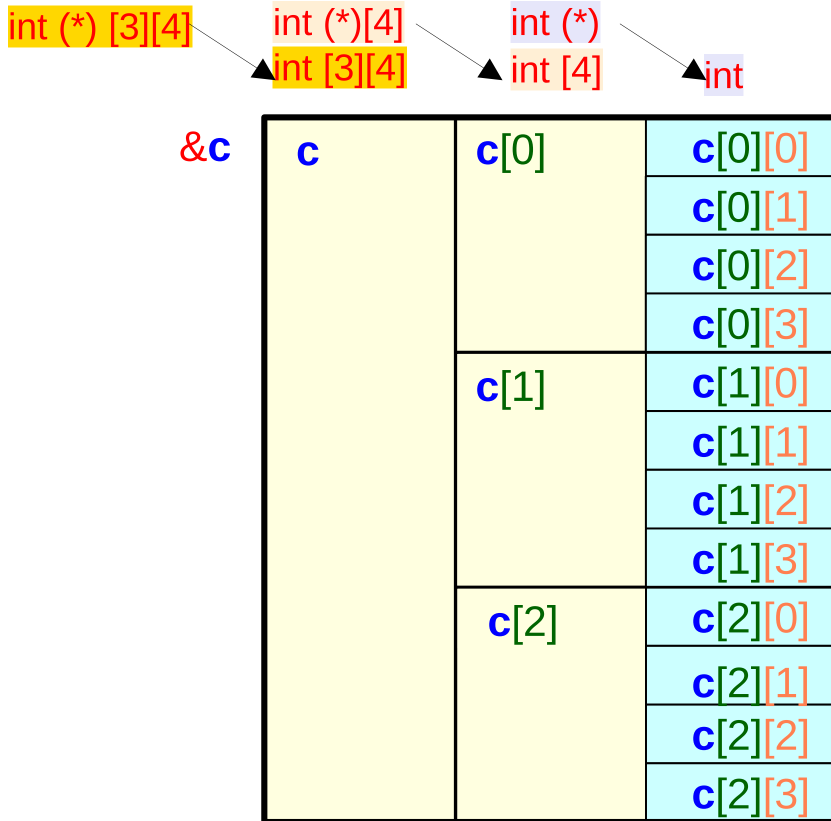
an primitive data c[i][0] = *c[i]
- start address &c[i][0] = c[i]
- size sizeof(c[i][0])

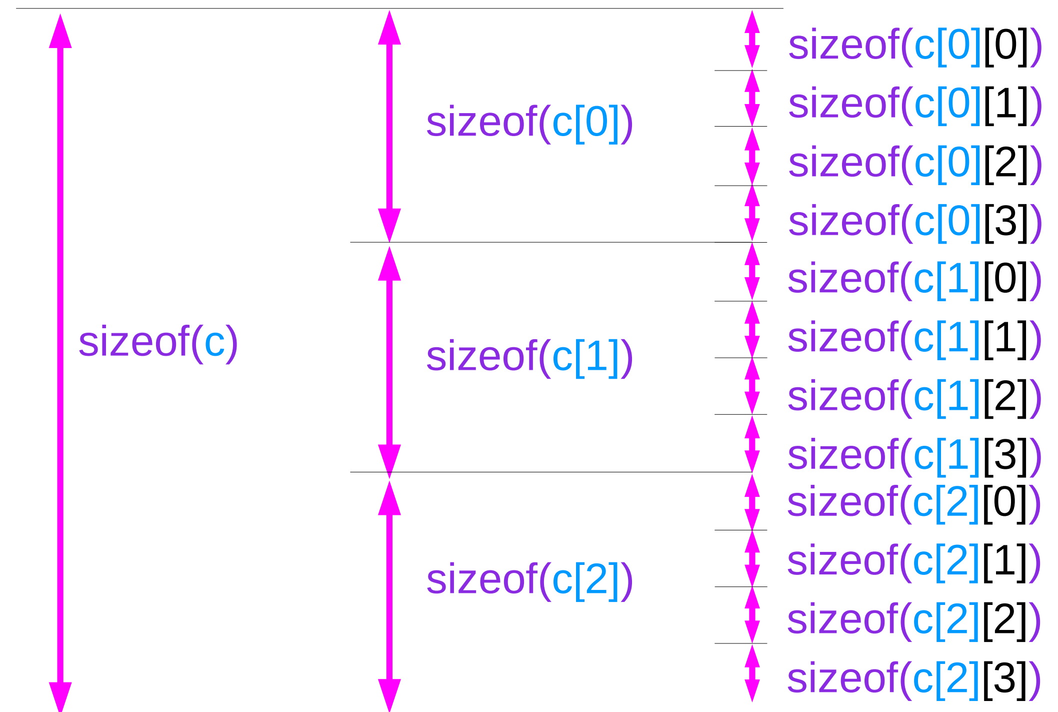
virtual pointer **c**[i]
points to primitive data **c**[i][0]

virtual pointers
- no physical memory locations are allocated
- address and data have the same value

A **2-d** array and its **1-d** sub-arrays – a size view

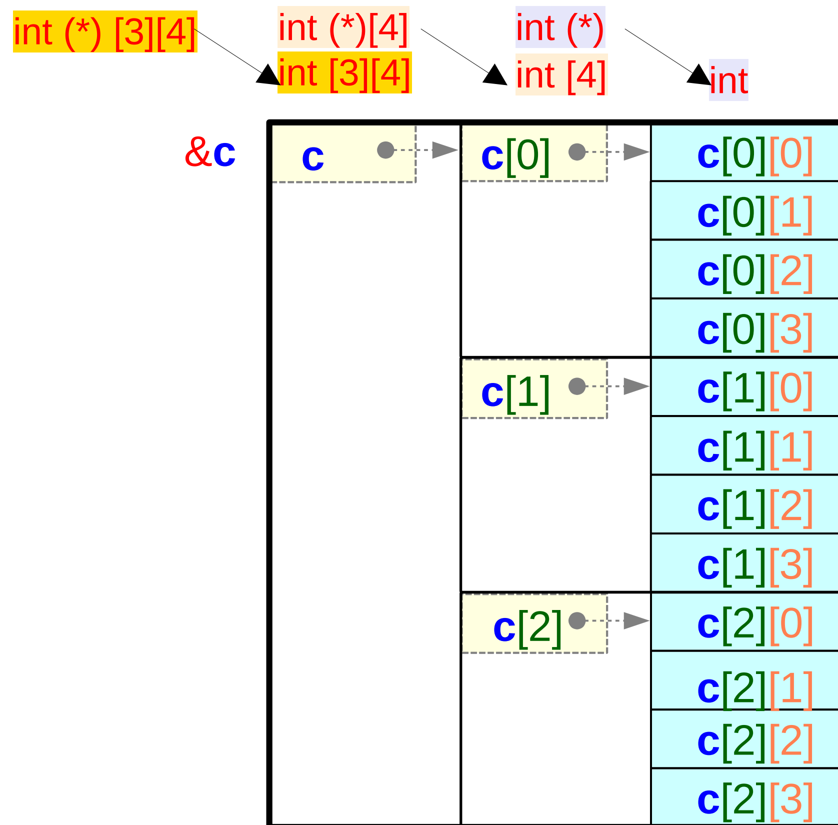
int c[3][4];





A **2-d** array and its **1-d** sub-arrays – a virtual pointer view

int c[3][4];



value(c) = value(c[0]) = value(&c[0][0])
value(&c) = value(&c[0]) = value(&c[0][0])

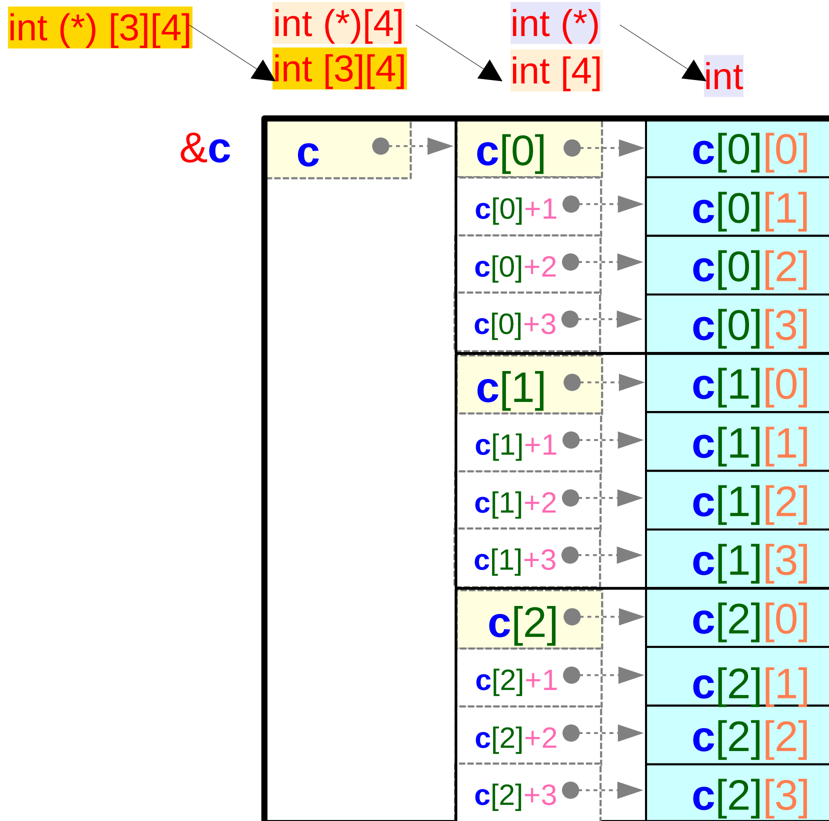
value(c[1]) = value(&c[1][0])
value(&c[1]) = value(&c[1][0])

value(c[2]) = value(&c[2][0])
value(&c[2]) = value(&c[2][0])

address(c) = address(c[0]) = address(c[0][0])
address(c[1]) = address(c[1][0])
address(c[2]) = address(c[2][0])

A **2-d** array and its **1-d** sub-arrays – size relation

int c[3][4];



sizeof(c) = sizeof(c[0]) * 3 … *leading element*
sizeof(c+1) = pointer size (4/8 bytes)
sizeof(c+2) = pointer size (4/8 bytes)

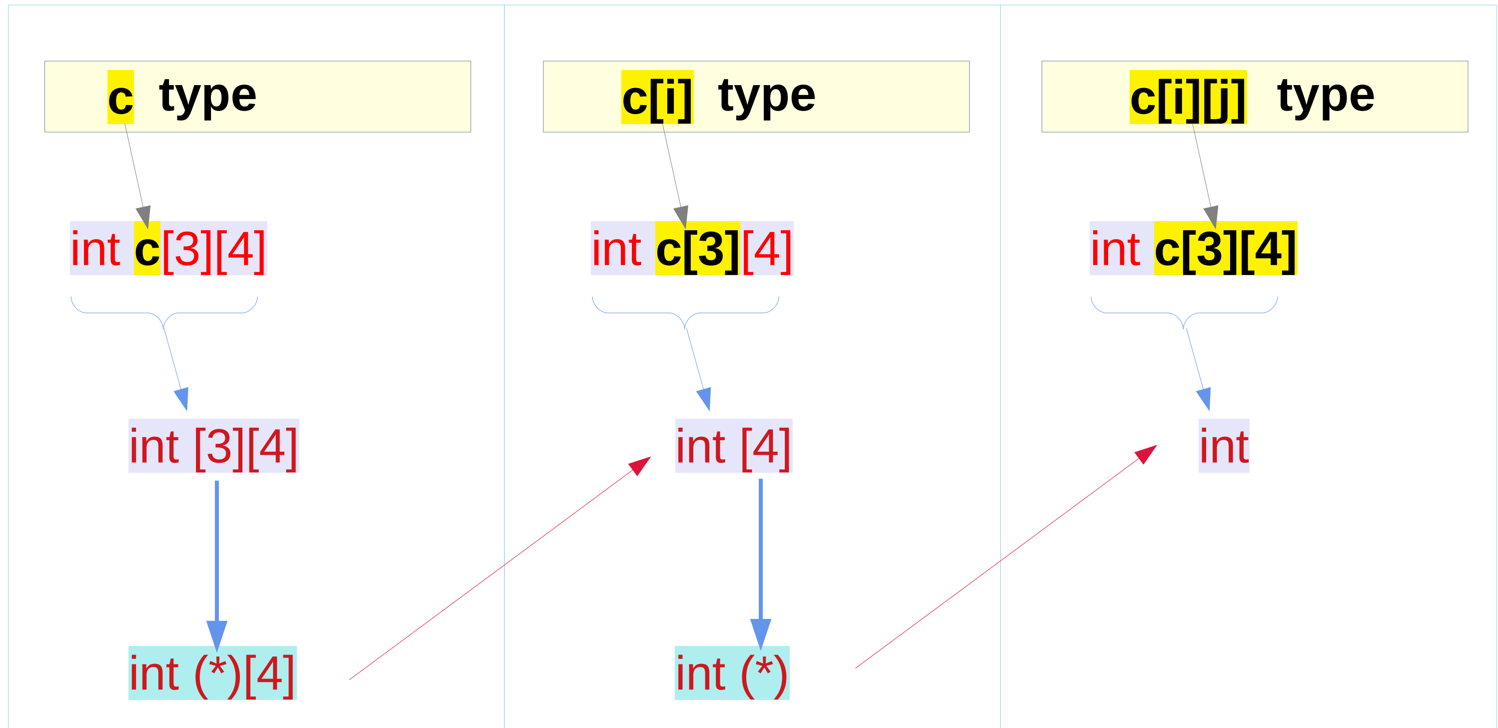
sizeof(c[0]) = sizeof(c[0][0]) * 4 … *leading element*
sizeof(c[0]+1) = pointer size (4/8 bytes)
sizeof(c[0]+2) = pointer size (4/8 bytes)
sizeof(c[0]+3) = pointer size (4/8 bytes)

sizeof(c[1]) = sizeof(c[1][0]) * 4 … *leading element*
sizeof(c[1]+1) = pointer size (4/8 bytes)
sizeof(c[1]+2) = pointer size (4/8 bytes)
sizeof(c[1]+3) = pointer size (4/8 bytes)

sizeof(c[2]) = sizeof(c[2][0]) * 4 … *leading element*
sizeof(c[2]+1) = pointer size (4/8 bytes)
sizeof(c[2]+2) = pointer size (4/8 bytes)
sizeof(c[2]+3) = pointer size (4/8 bytes)

Sub-array types in a **2-d** array

int c[3][4]; **2-d** array c



Array Type

relaxing the
1st dimension

Array Pointer Type

Dual Types

- **Identifying nested arrays**

  in a 2-d array declaration

Nested arrays in a 2-d array declaration

int c[3] [4] ;

int c[3] [4] ;

c : a 3 element array
c[i] : each element

int c[3] [4] ;

c[i]'s type 1 : int [4]
an array of 4 integers

relaxed dimension

int c[3] [4] ;

c[i]'s type 2: int (*)
a pointer to an integer

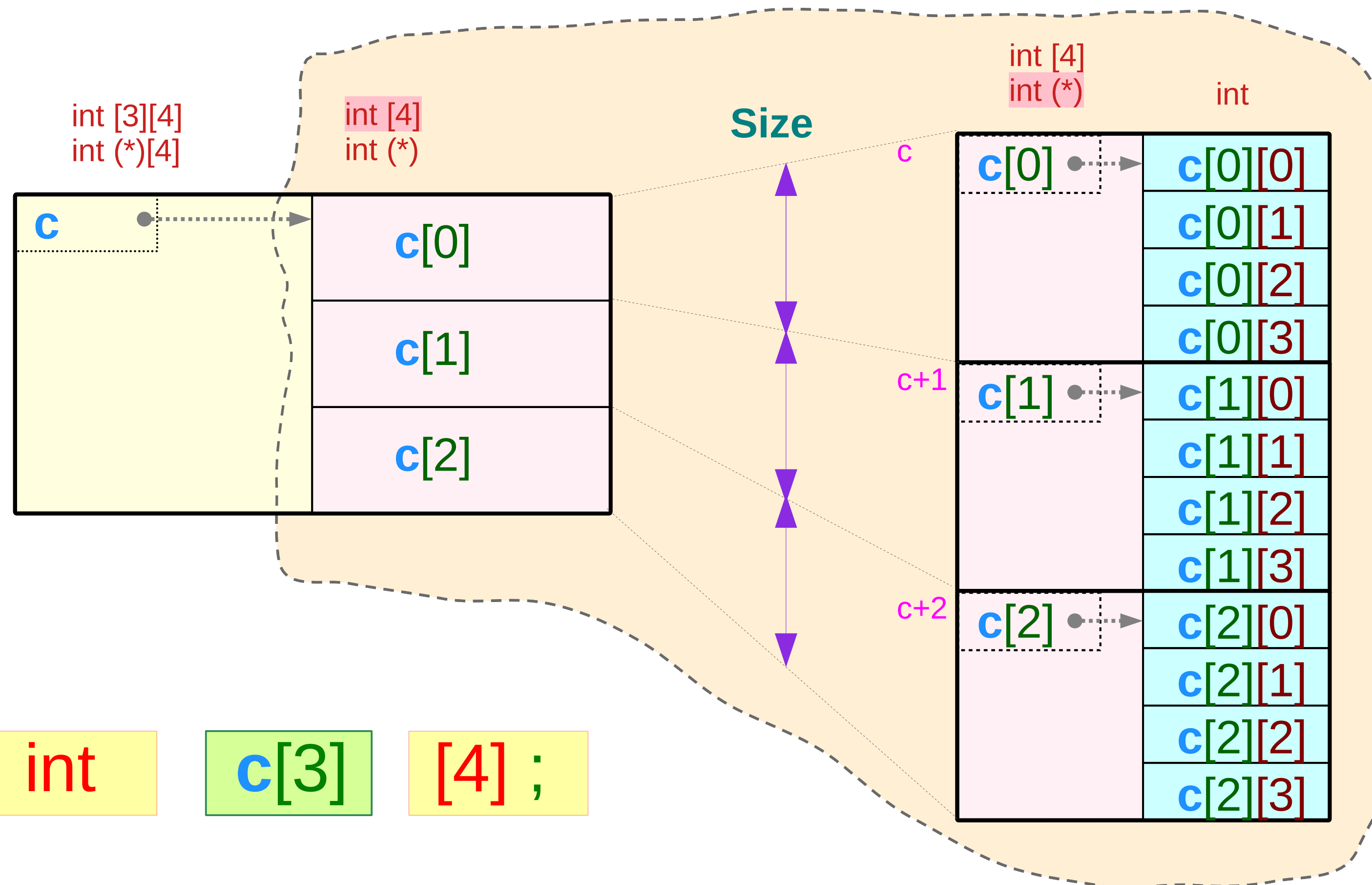
Nested arrays

c[3]

c : a 3 element array
c[i] : each element

int [4] ;

c[i]'s type 1 : int [4]
c[i]'s type 2 : int (*)



Address

&c[0][0] ➡ c[0] ➡ c

&c[1][0] ➡ c[1]

&c[2][0] ➡ c[2]

c : 3-element array

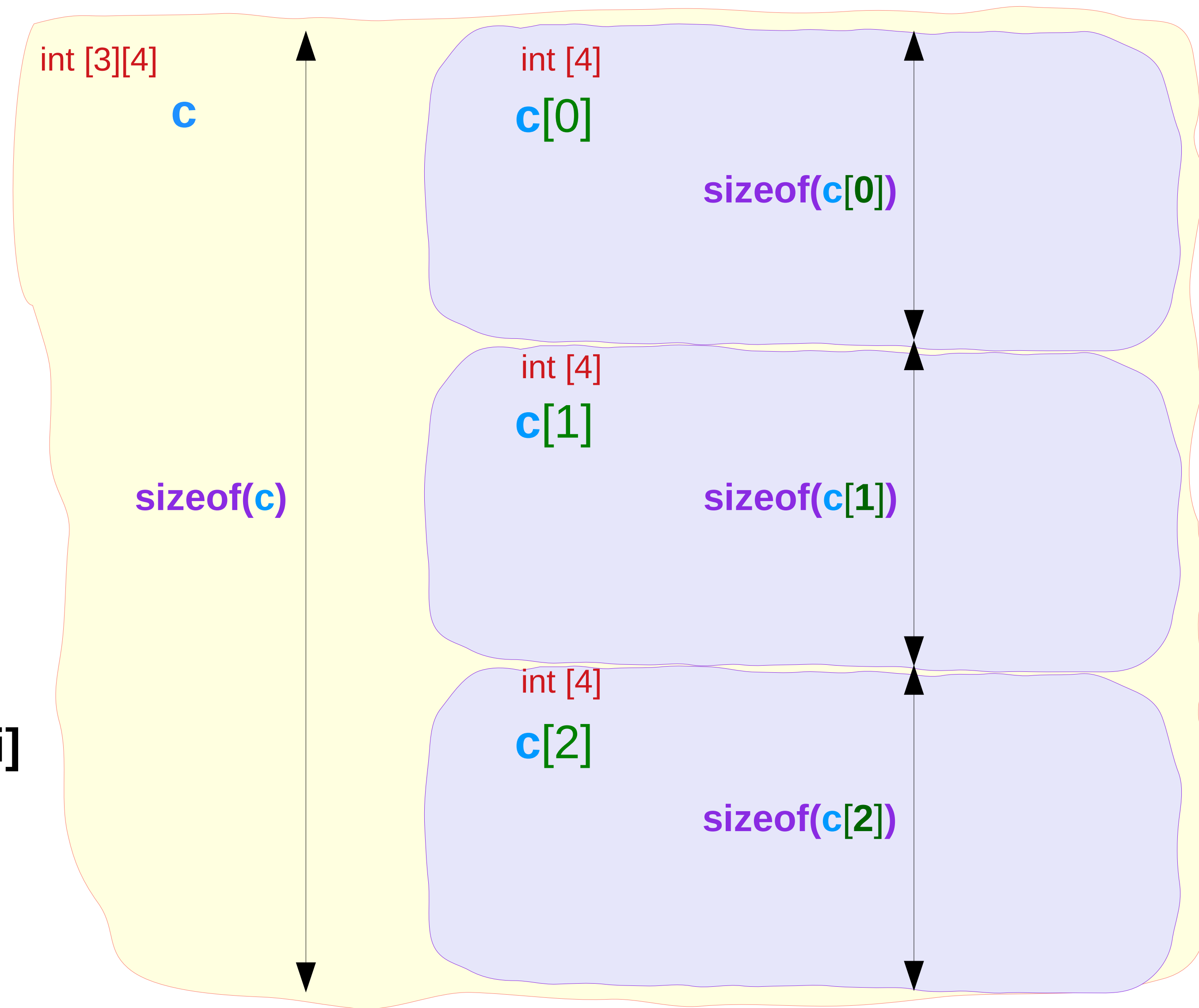
c	**2-d** array	int [3][4]
c[i]	**1-d** array	int [4]

```
int c [3] [4] ;
```

3-element array c

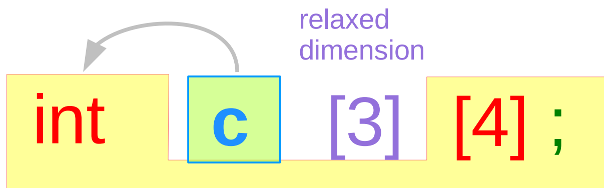
abstract data element **c[i]**

each element **c[i]** has the
1-d array type int [4]



c : pointer to a 4-element array

c	**1-d** array pointer	int (*)[4]
c[i]	**1-d** array	int [4]



pointer c

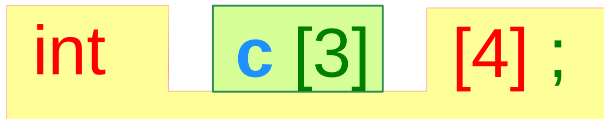
abstract data element **c[0]**

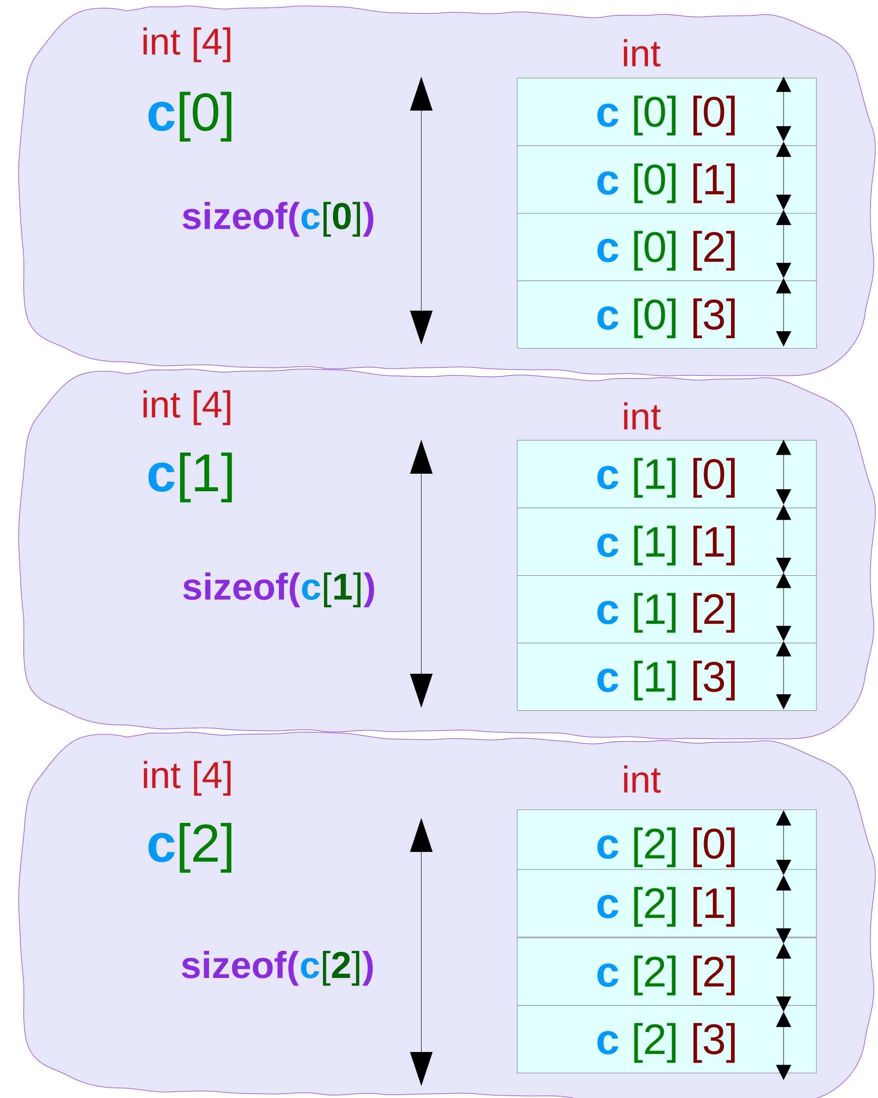
each element **c[i]** has the
1-d array type int [4]

int (*) [4]
c
c+0
int [4]
c[0]
sizeof(c[0])
c+1
int [4]
c[1]
sizeof(c[1])
c+2
int [4]
c[2]
sizeof(c[2])

c[i] : 4-element array

c[i]	**1-d** array	int [4]
c[i][j]	**0-d** array	int





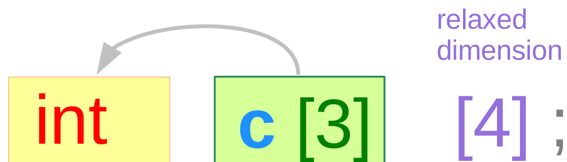
4-element array c[i]

primitive data element **c[i][j]**

each element **c[i][j]** has
the primitive type int

c[i] : pointer to a primitive data

c[i]	**0-d** array pointer	int (*)
c[i][j]	**0-d** array	int



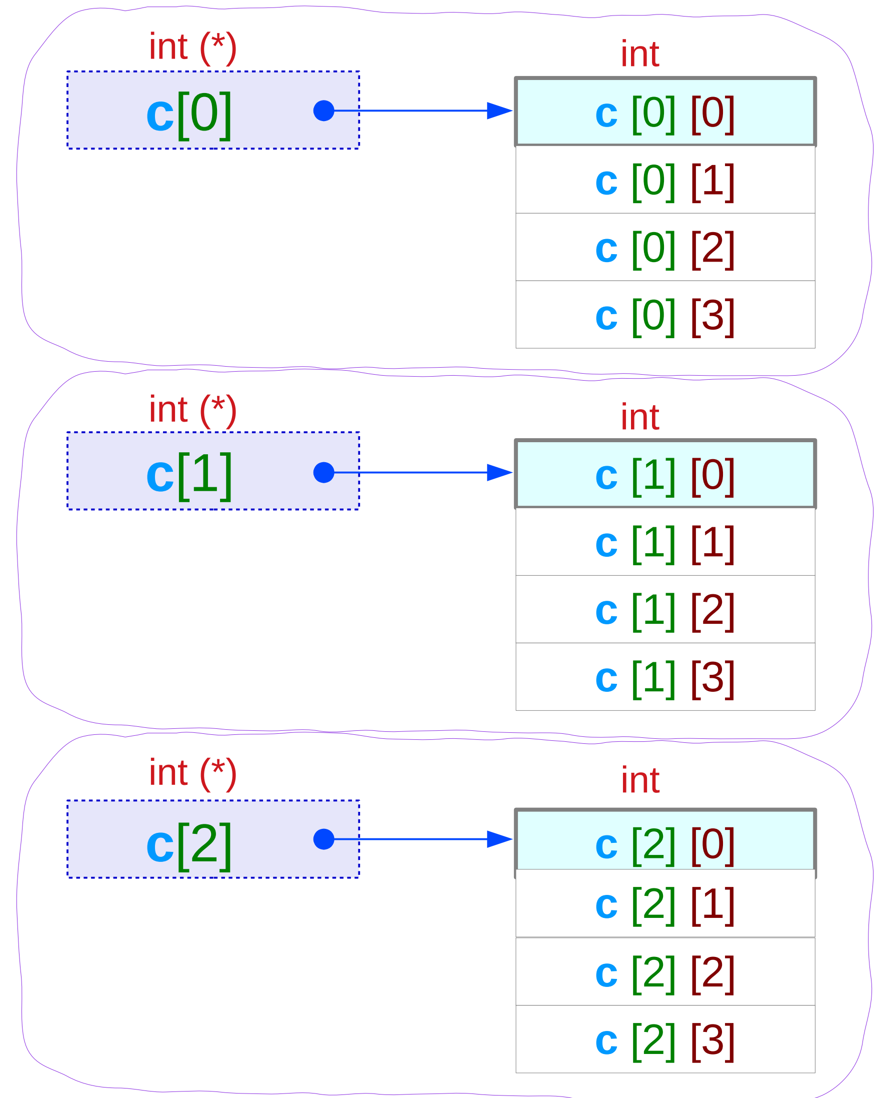
relaxed
dimension

[4] ;

pointer c[i]

primitive data element **c[i][0]**

each element **c[i][j]** has
the primitive type int



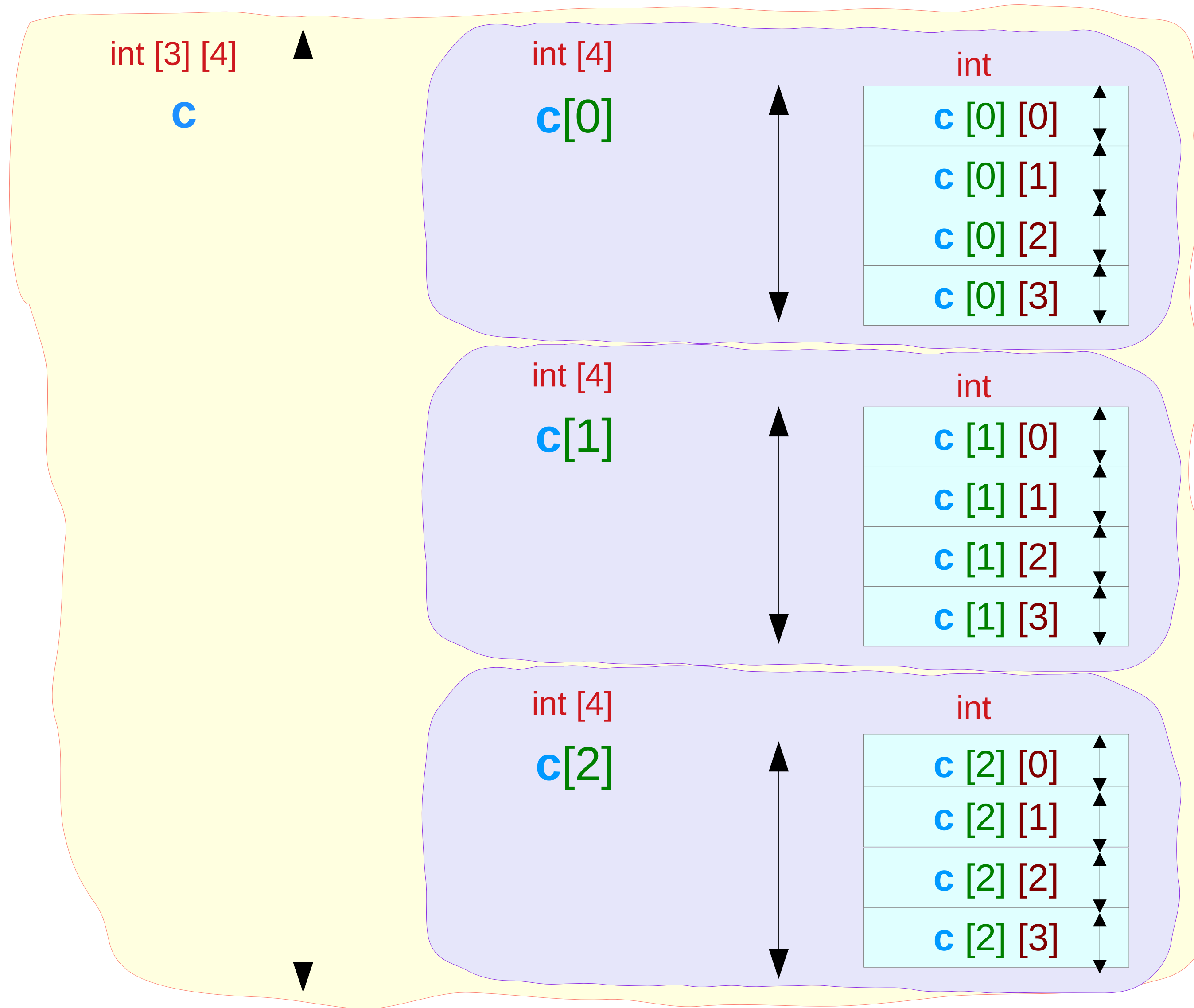
Recursive data view

c	**2-d** array	int [3][4]
c	**1-d** array pointer	int (*)[4]
c[i]	**1-d** array	int [4]
c[i]	**0-d** array pointer	int (*)
c[i][j]	**0-d** array	int

int c[3] [4] ;

3-element array c

4-element array c[i]



Pointer view

c	**2-d** array	int [3][4]
c	**1-d** array pointer	int (*)[4]
c[i]	**1-d** array	int [4]
c[i]	**0-d** array pointer	int (*)
c[i][j]	**0-d** array	int

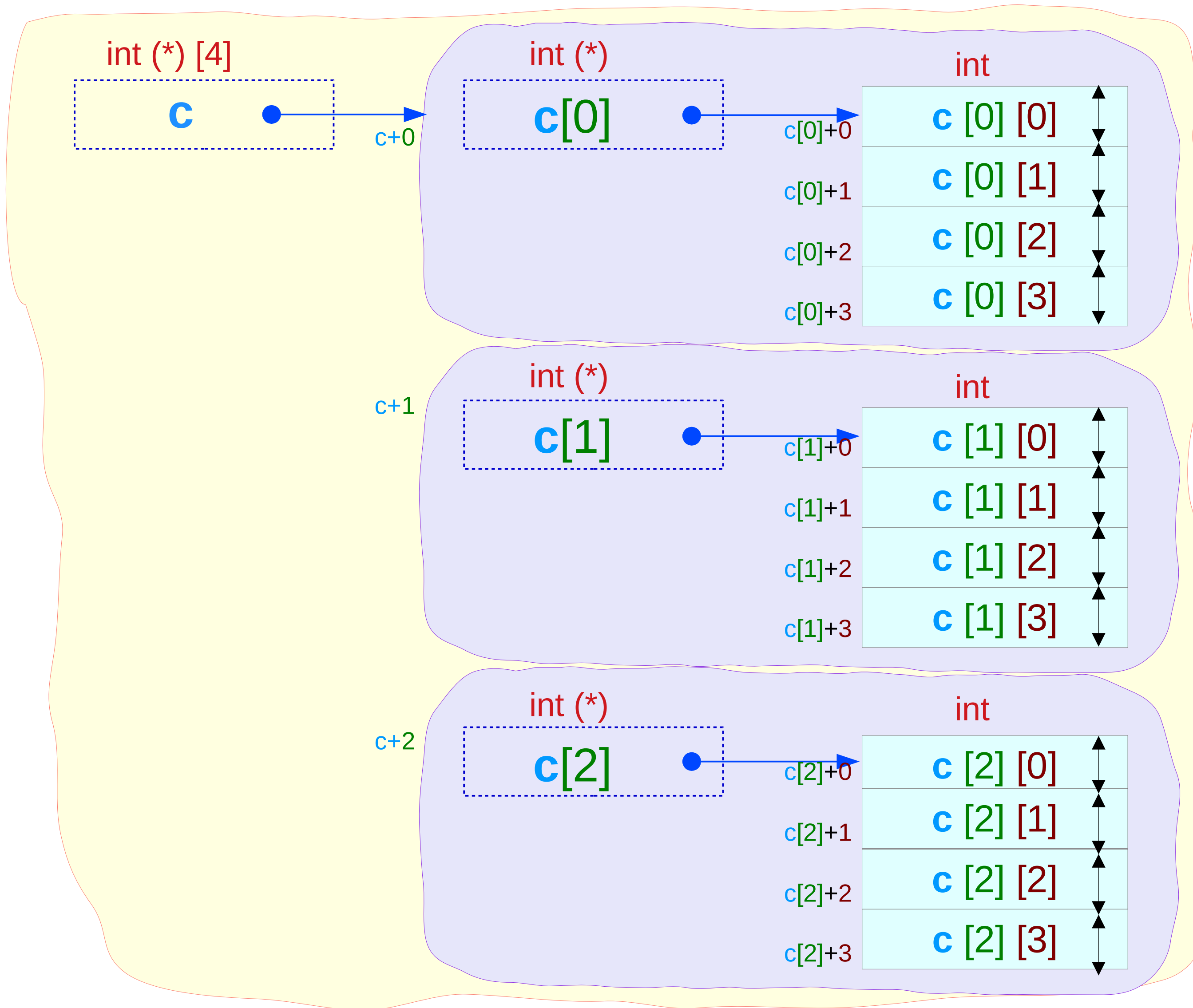
int c[3] [4] ;

v(c) = v(c[0]) = v(&c[0][0])

v(c[1]) = v(&c[1][0])

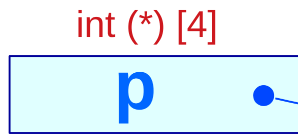
v(c[2]) = v(&c[2][0])

v ≡ value



1-d array pointer

int (*p) [4] ;



int c[3] [4] ;

int (*) [4]
c
p = c;

c+0
c+1
c+2

int (*)
c[0]
c[1]
c[2]

int

c[0]+0
c[0]+1
c[0]+2
c[0]+3
c[1]+0
c[1]+1
c[1]+2
c[1]+3
c[2]+0
c[2]+1
c[2]+2
c[2]+3

c [0] [0]
c [0] [1]
c [0] [2]
c [0] [3]
c [1] [0]
c [1] [1]
c [1] [2]
c [1] [3]
c [2] [0]
c [2] [1]
c [2] [2]
c [2] [3]

v(c) = v(c[0]) = v(&c[0][0])

v(c[1]) = v(&c[1][0])

v(c[2]) = v(&c[2][0])

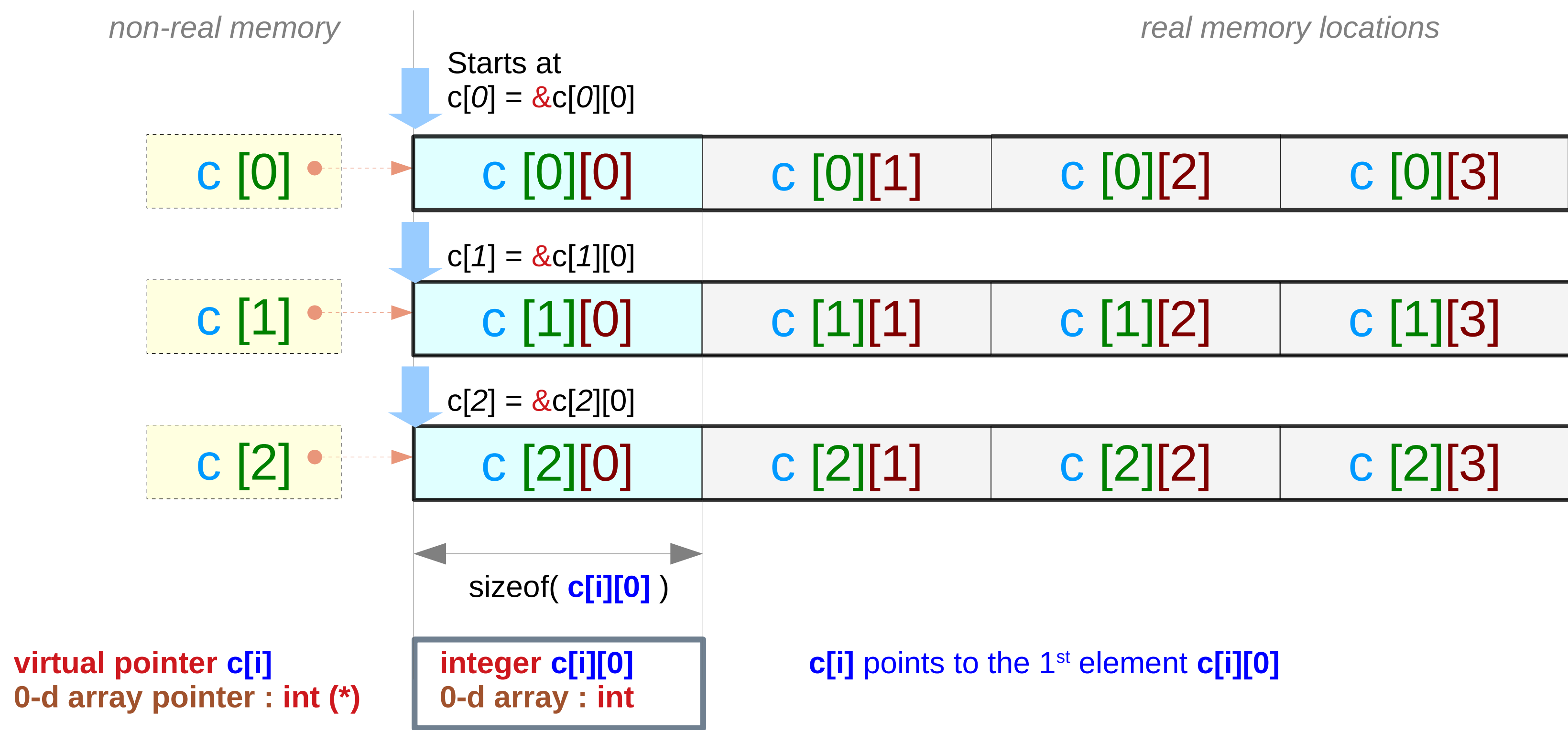
v ≡ value

Pointer **c[i]** and integer **c[i][0]**

```
int c[3] [4];
```

non-real pointer **c[i]** : value(**c[i]**)= &**c[i][0]**

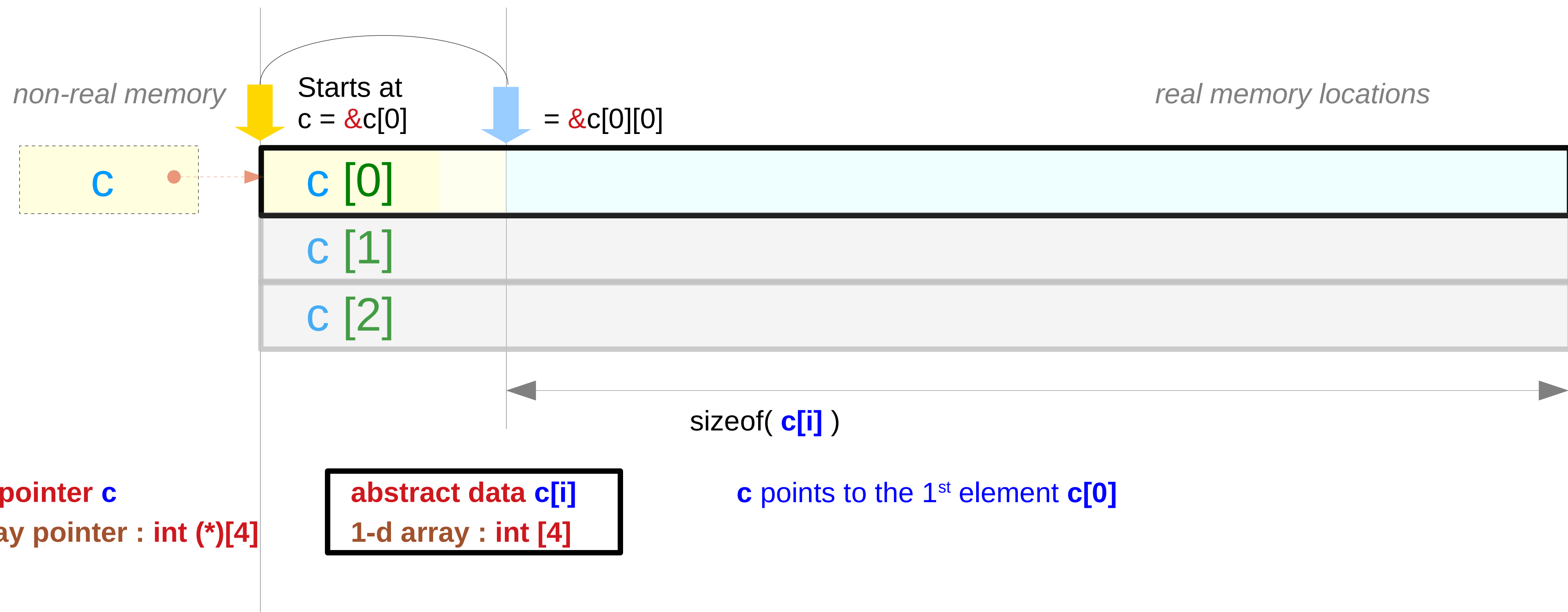
0-d array pointer



Pointer **c** and abstract data **c[i]**

```
int c [3] [4];
```

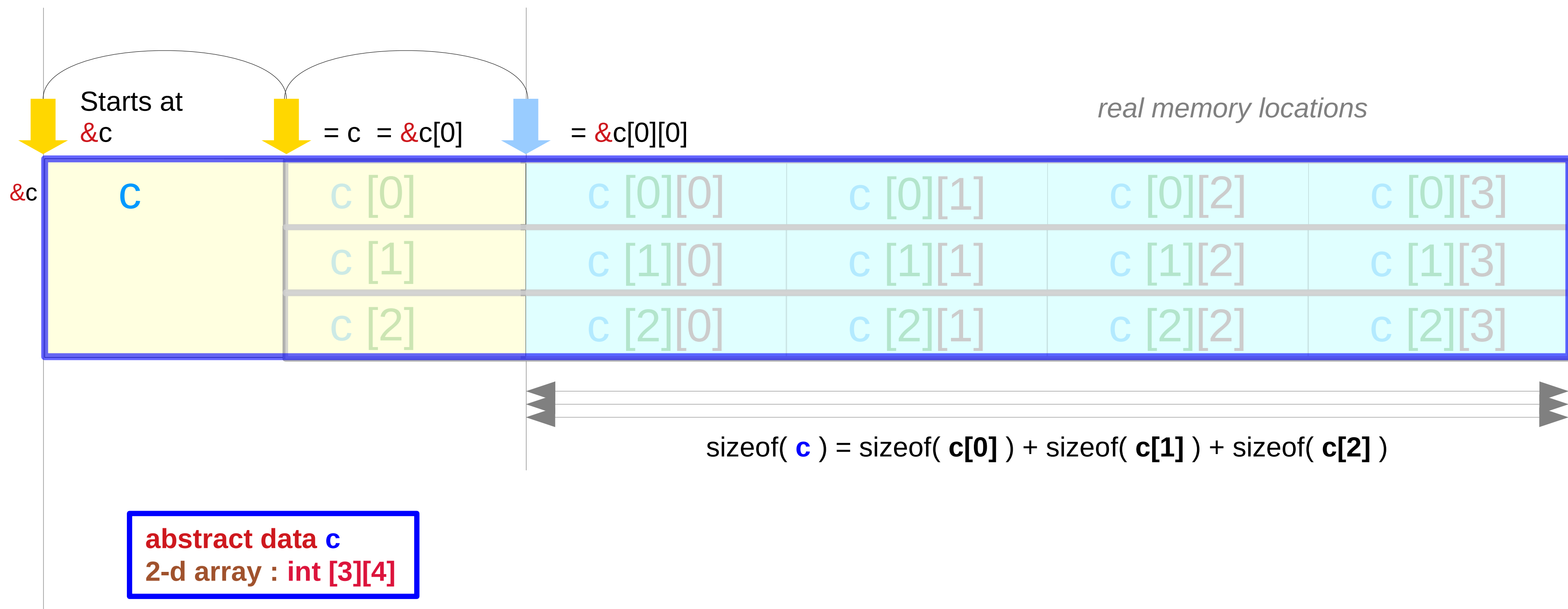
non-real pointer **c** : value(**c**) = &**c[0]** = &**c[0][0]** — **1-d** array pointer
abstract data **c[i]** : sizeof(**c[i]**) = 4 * sizeof(**int**) — **1-d** array



Abstract data **c**

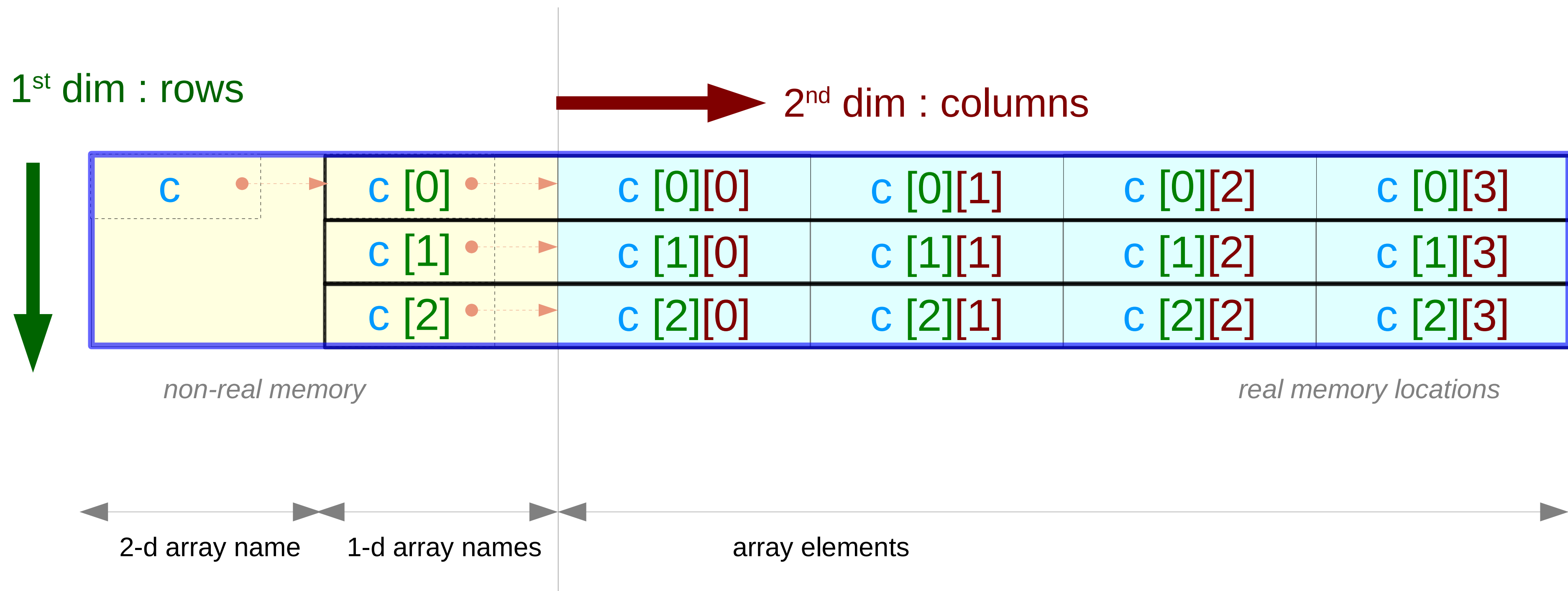
```
int c [3] [4];
```

abstract data **c:** sizeof(**c**) = 3 * sizeof(**c[i]**) **2-d** array



Rows and columns of a **2-d** array **c**

```
int c[3] [4];
```



Determining types of sub-arrays

from the declaration of an array

Types of array names

int a [4] ;

a is the name of the 1-d array int [4]

sizeof(a) = 4 * 4

[3] is declared;
[0], [1], [2] are used

int c [3] [4] ;

c[i] is the name of the 1-d subarray int [4]

sizeof(c[i]) = 4 * 4

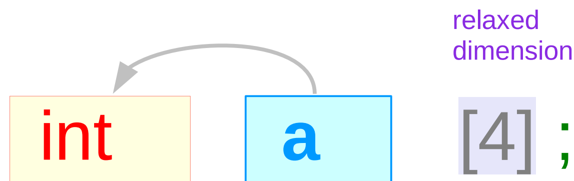
int c [3] [4]

c is the name of the 2-d array int [3][4]

sizeof(c) = 3 * 4 * 4

Values of array names

relaxed dimension



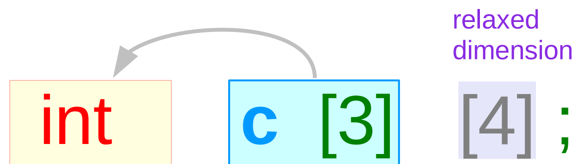
[4] ;

the value of **a** is the starting address of an array with 4 elements of **int** type — **int (*)**

a: pointer to the first element — **a = &a[0]**

[3] is declared;
[0], [1], [2] are used

relaxed dimension

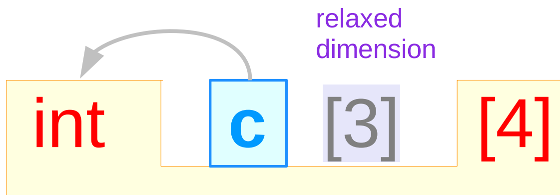


[4] ;

each value of **c[i]** is the starting address of an array with 4 elements of **int** type — **int (*)**

c[i]: pointer to the first element — **c[i] = &c[i][0]**

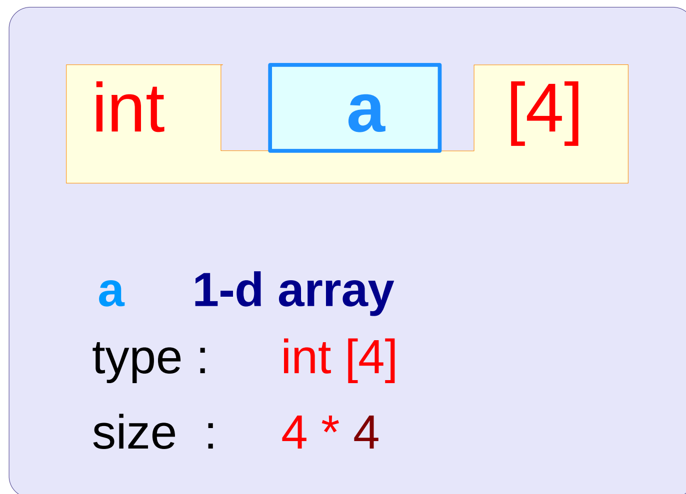
relaxed dimension

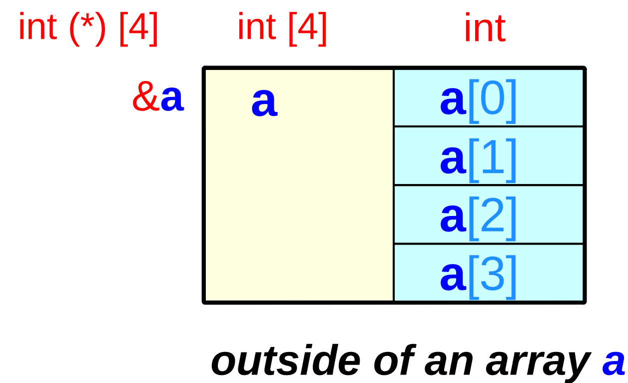


the value of **c** is the starting address of an array with 3 elements of **int [4]** type — **int (*) [4]**

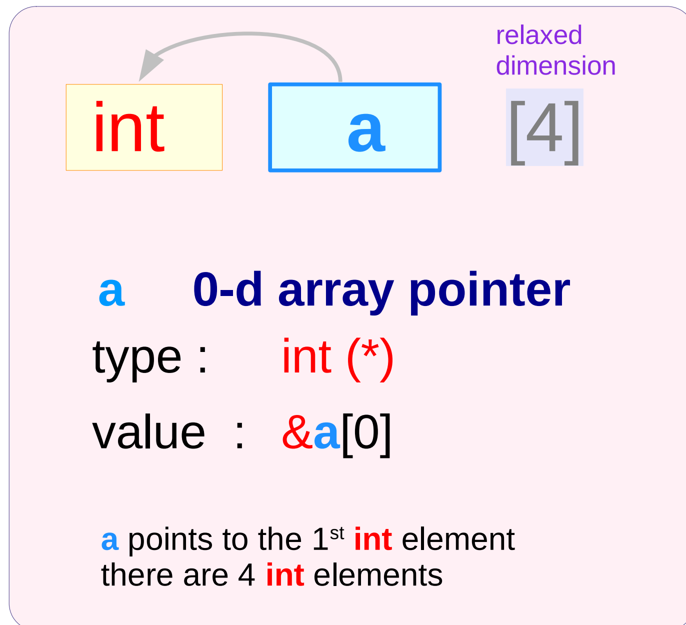
c: pointer to the first element — **c = &c[0]**

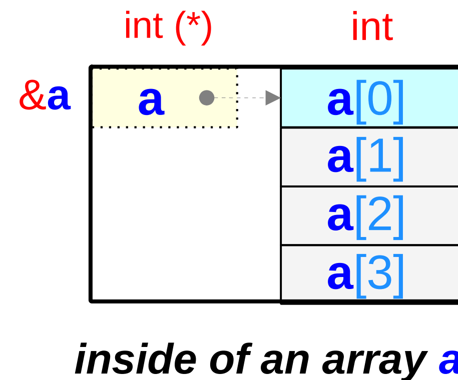
Array and pointer types in a **1-d** array





outside of an array a

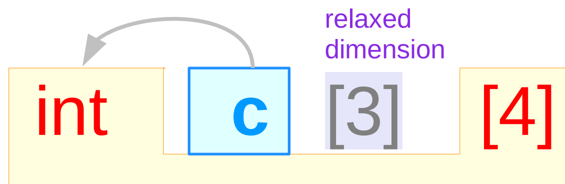




inside of an array a

2-d array type

&c



c 1-d array pointer

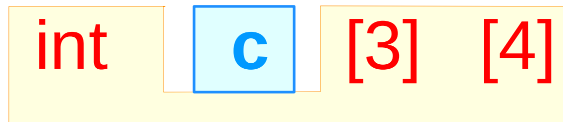
type : int (*) [4]

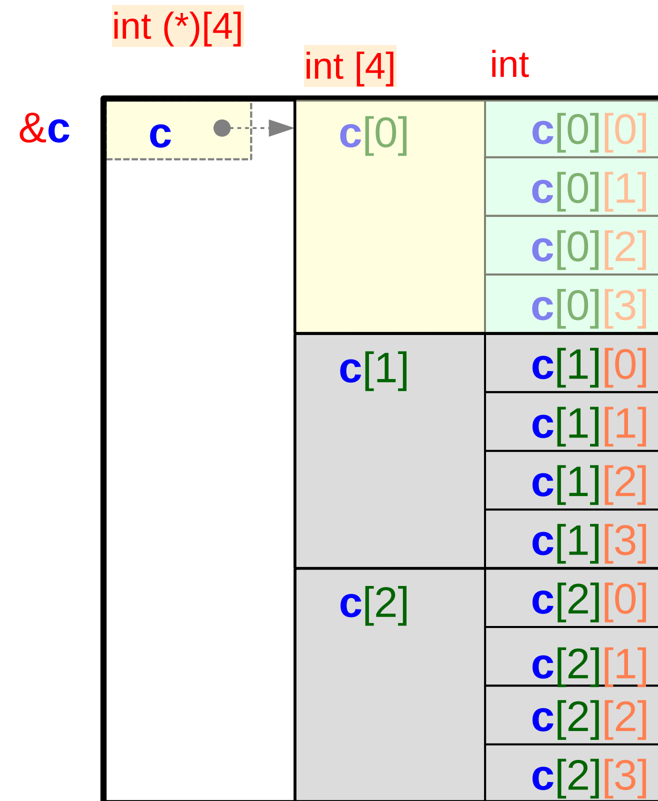
value : c = &c[0][0]

int (*) [3][4]	int [3][4]	int [4]	int
&c	c	c[0]	c[0][0]
			c[0][1]
			c[0][2]
			c[0][3]
		c[1]	c[1][0]
			c[1][1]
			c[1][2]
			c[1][3]
		c[2]	c[2][0]
			c[2][1]
			c[2][2]
			c[2][3]

outside of an array c
(*c as an abstract data*)

1-d array pointer type





c 2-d array

type : int [3][4]

size : 3 * 4 * 4

relaxed dimension

int c [3] [4]

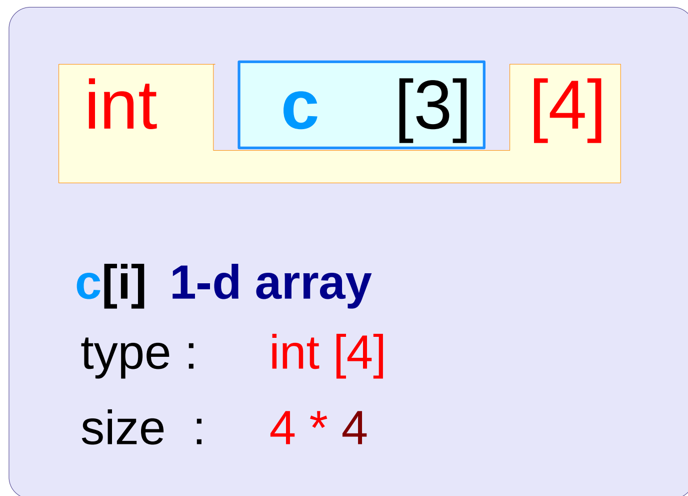
c 1-d array pointer

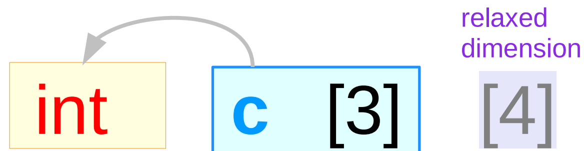
type : int (*) [4]

value : c = &c[0][0]

c points to the 1st **int [4]** element
There are 3 **int [4]** elements

1-d array type

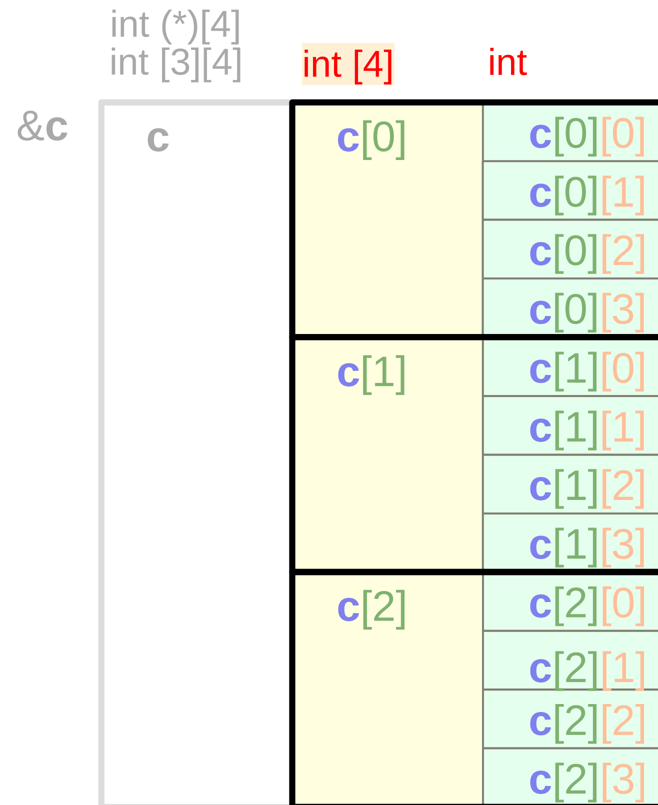




c[i] 0-d array pointer

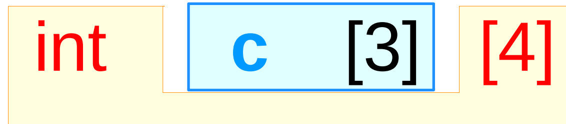
type : int (*)

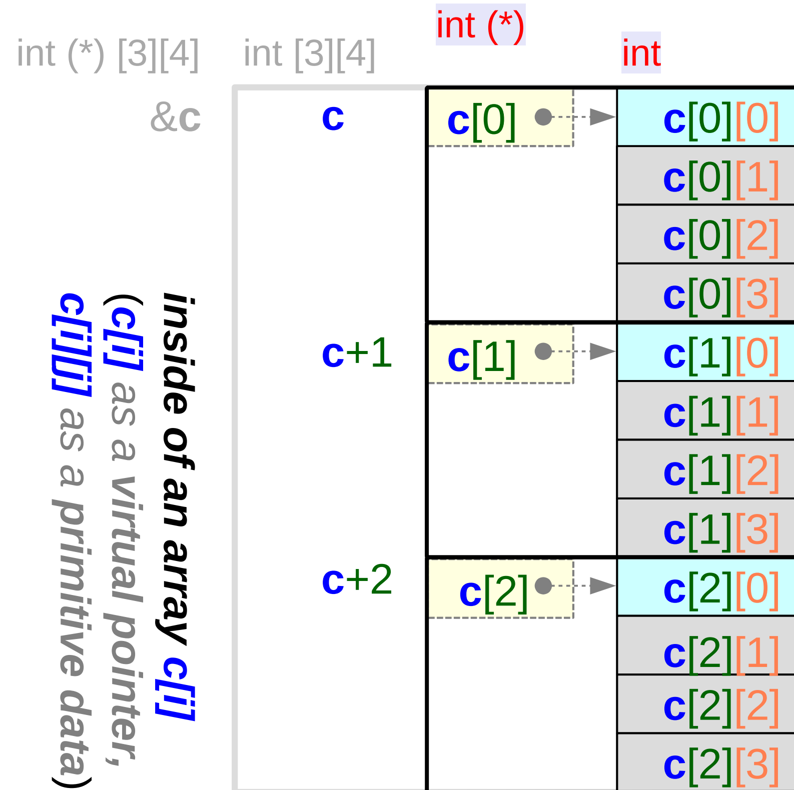
value : c[*i*] = &c[*i*][0]



c[i] points to the 1st **int** element
There are 4 **int** elements

0-d array pointer type





c[i] 1-d array

type : int [4]

size : 4 * 4

int c [3] [4]

relaxed dimension

c[i] 0-d array pointer

type : int (*)

value : c[*i*] = &c[*i*][0]

c[i] points to the 1st **int** element
There are 4 **int** elements

Types in a **2-d** array

int c [3] [4]

c **2-d array**

type : int [3][4]

size : 3 * 4 * 4

relaxing the 1st dimension

int c [3] [4]

relaxed dimension

c **1-d array pointer** (virtual)

type : int (*) [4]

value : &c[0][0]

int c [3] [4]

c[i] **1-d array**

type : int [4]

size : 4 * 4

relaxing the 1st dimension

int c [3] [4]

relaxed dimension

c[i] **0-d array pointer** (virtual)

type : int (*)

value : &c[*i*][0]

The name of a 2-d array

```
int         a [4];

int     c [4] [4];
```

1. the name of the nested array (recursive definition)

2. a double pointer

3. a pointer to an array

2-d array c and 1-d array q

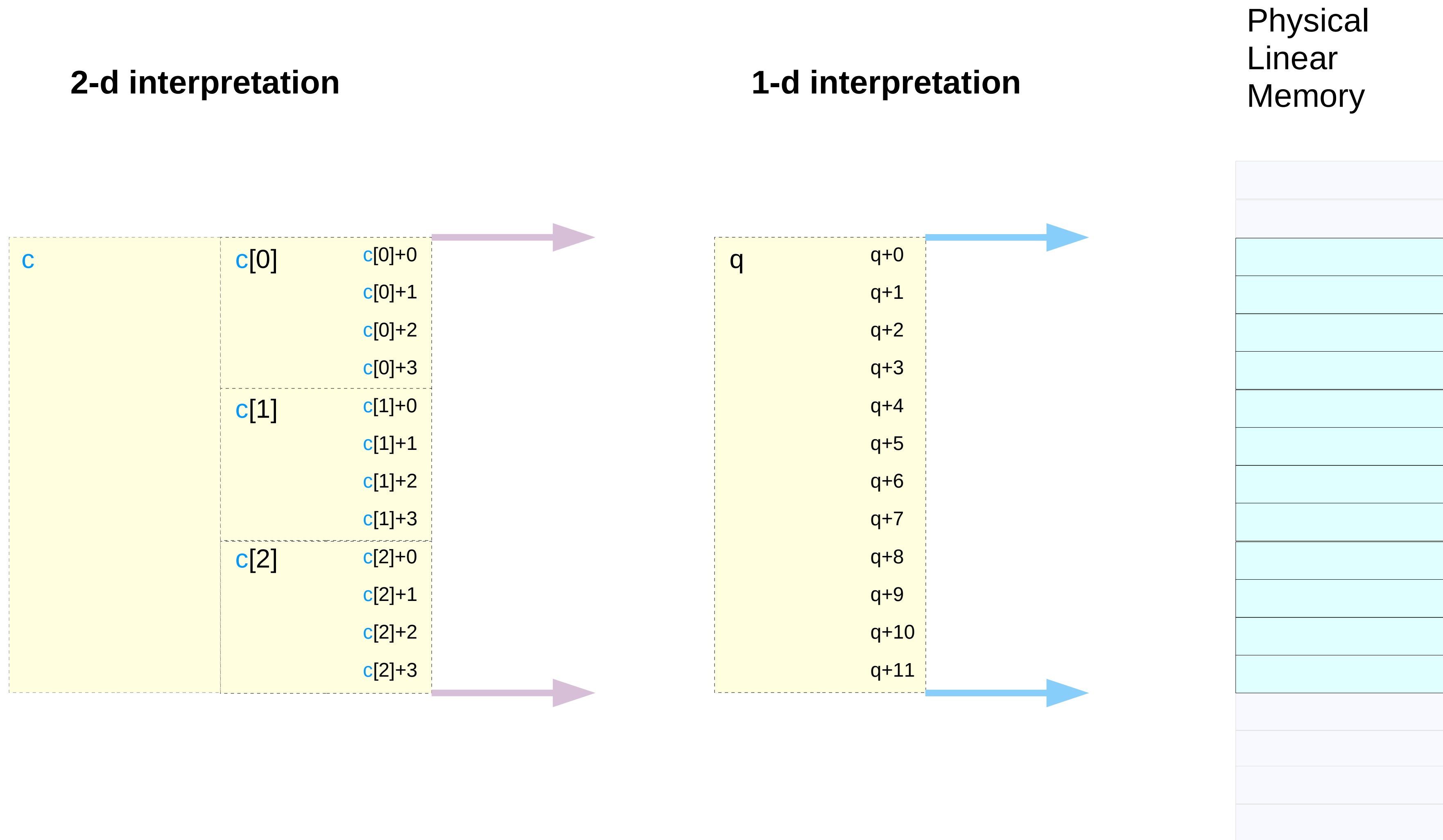
```
int c [3] [4];
```

```
int q [3*4];
```

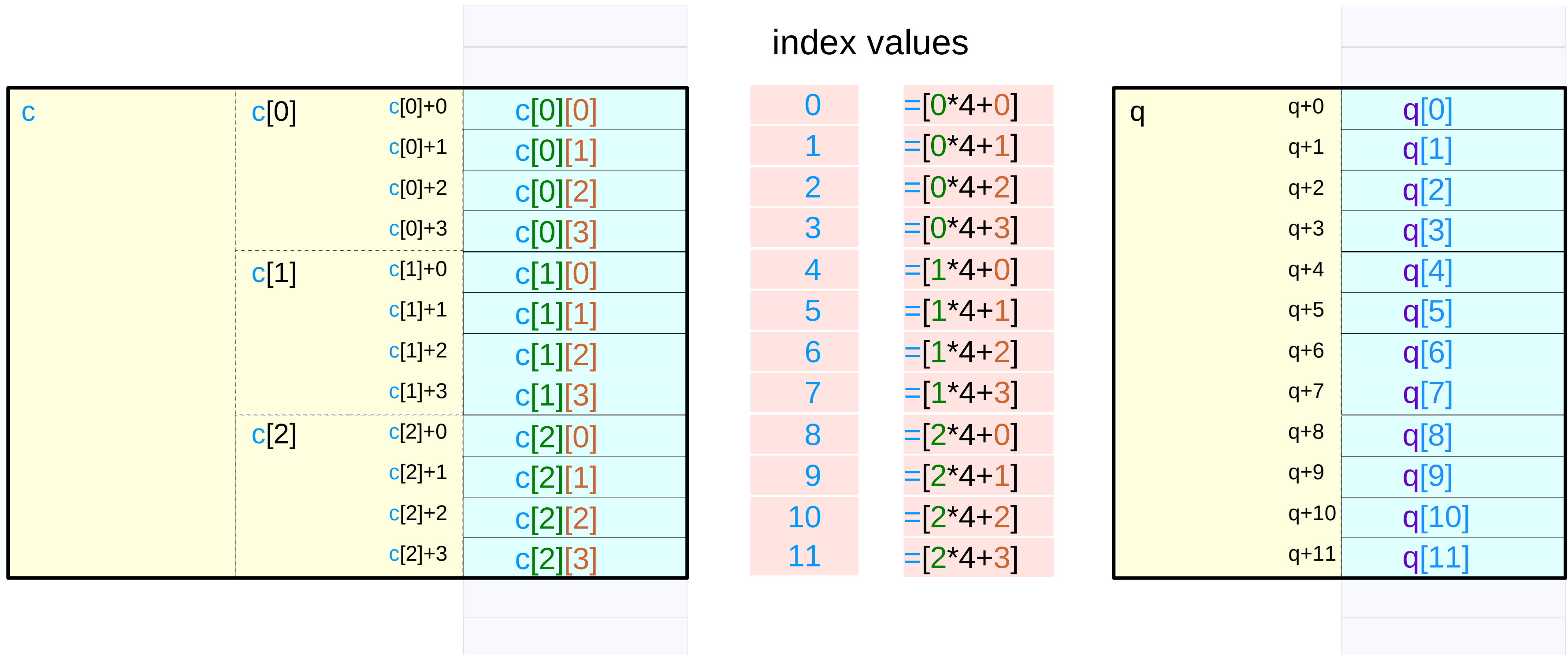
c	c[0]	c[0]+0	c[0][0]
		c[0]+1	c[0][1]
		c[0]+2	c[0][2]
		c[0]+3	c[0][3]
	c[1]	c[1]+0	c[1][0]
		c[1]+1	c[1][1]
		c[1]+2	c[1][2]
		c[1]+3	c[1][3]
	c[2]	c[2]+0	c[2][0]
		c[2]+1	c[2][1]
		c[2]+2	c[2][2]
		c[2]+3	c[2][3]

q	q+0	q[0*4+0]
	q+1	q[0*4+1]
	q+2	q[0*4+2]
	q+3	q[0*4+3]
	q+4	q[1*4+0]
	q+5	q[1*4+1]
	q+6	q[1*4+2]
	q+7	q[1*4+3]
	q+8	q[2*4+0]
	q+9	q[2*4+1]
	q+10	q[2*4+2]
	q+11	q[2*4+3]

2-d and 1-d interpretations of linear memories

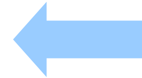


A **2-d** array stored as a **1-d** array (row major order)



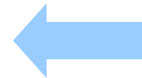
2-d array access via a single pointer

```
int     *p = c[0] ;
```



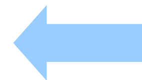
```
int c [3][4];
```

```
p[ i*4 + j ]
```



```
c[ i ][ j ]
```

```
*(p+ i*4 + j)
```



```
*(*(c+i)+ j)
```

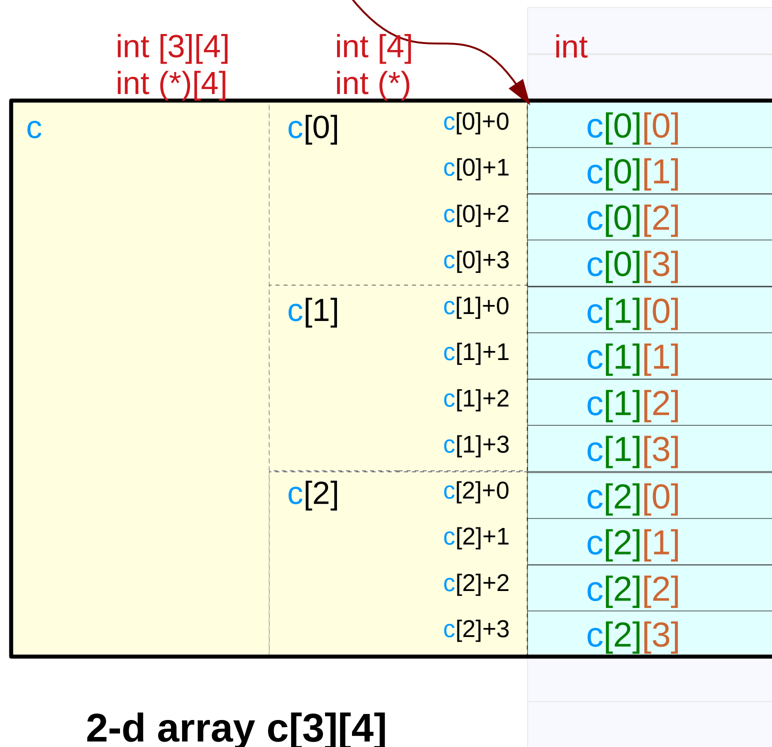
```
*(p +k)        i = k / 4;
               j = k % 4;
```

View a **2-d** array as a **1-d** array

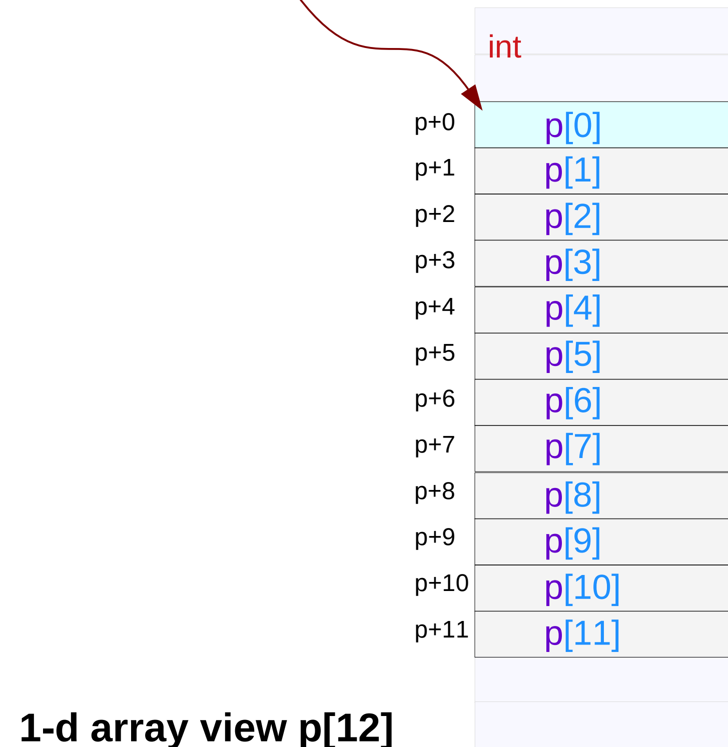
```
int c [3][4];
```

```
int *p = c[0];
```

c, c[0], &c[0][0]



2-d array c[3][4]



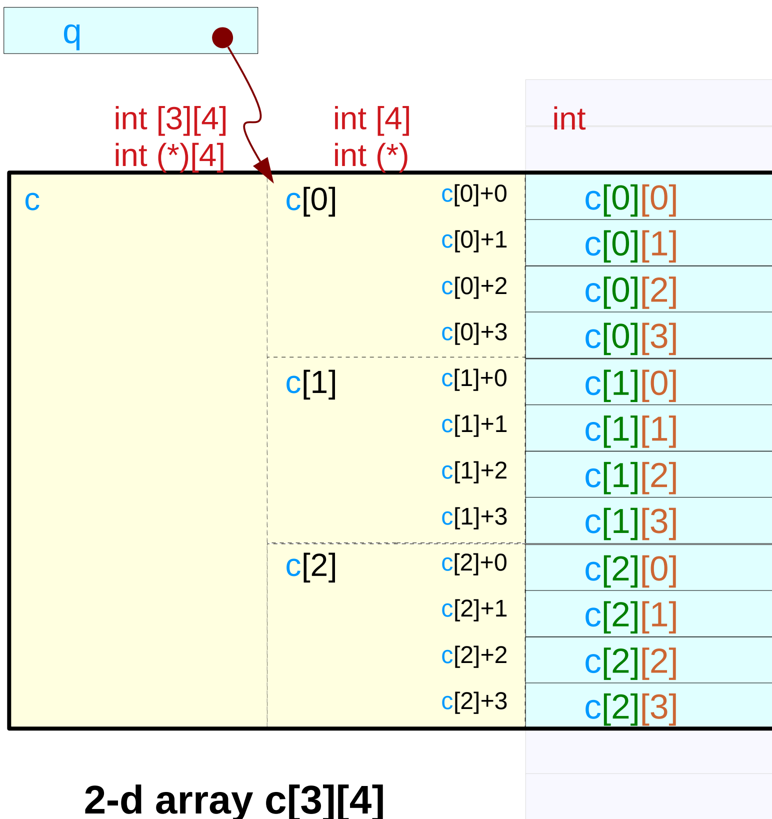
1-d array view p[12]

View a **2-d** array as another **2-d** array

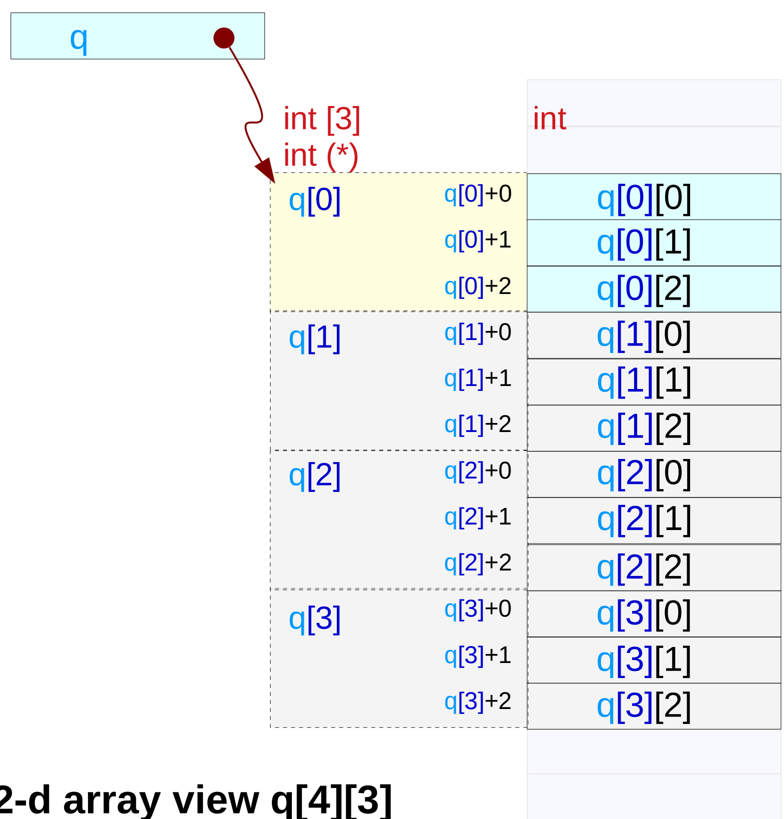
int c [3][4];

int (*q) [3] = (int (*) [3]) c;

c, c[0], &c[0][0]



2-d array c[3][4]



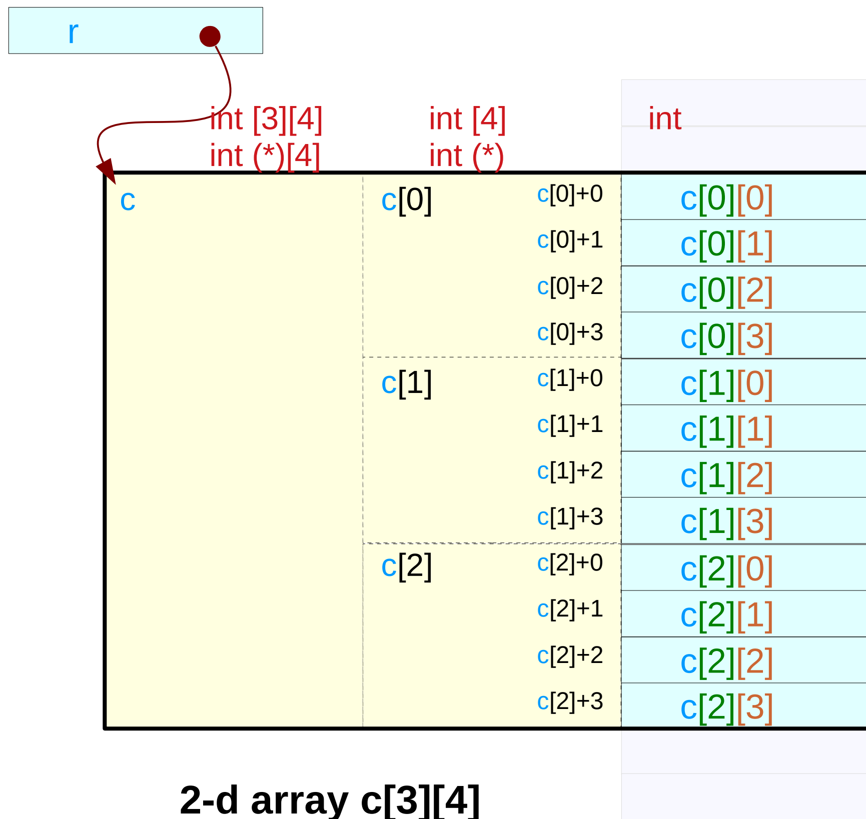
2-d array view q[4][3]

A **2-d** array stored as a **1-d** array (row major order)

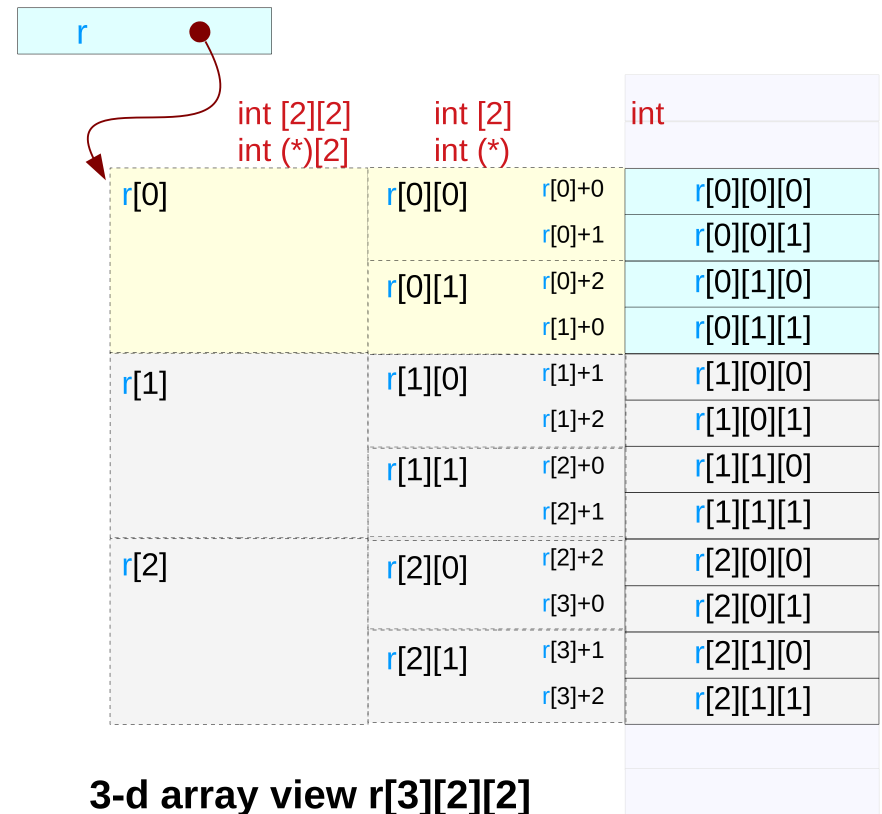
int c [3] [4];

int (*r) [2][2] = (int (*) [2][2]) c;

c, c[0], &c[0][0]



2-d array c[3][4]



3-d array view r[3][2][2]

2-d array access via pointers

```
int c [3][4];
```

```
int    *p = c[0] ;
```

1. recursive pointers

```
c [ i ][ j ]
```

```
(*(c+i))[ j ]
```

→ `int (*p)[4];`

```
*(c[ i ]+ j)
```

```
*(*(c+i)+ j)
```

→ `int **q;`

2. linear array pointers

```
p[ i*4 + j ]
```

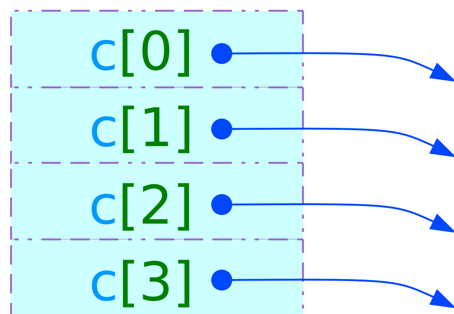
```
*(p+ i*4 + j )
```

Static Allocation of a 2-d Array

int A [3][4];

A in %eax,
i in %edx,
j in %ecx

```
sall   $2, %ecx                ;; j * 4
leal   (%edx, %edx, 2), %edx   ;; i * 3
leal   (%ecx, %edx, 4), %edx   ;; j * 4 + i * 12
movl   (%eax, %edx), %eax      ;; read M[ XA+4(3i +j) ]
```

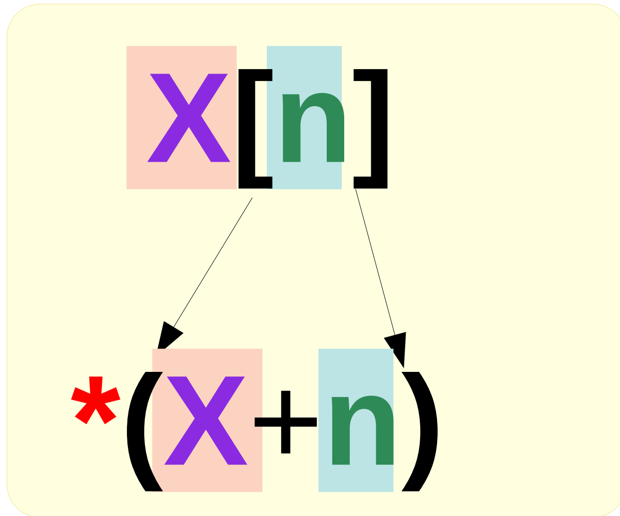


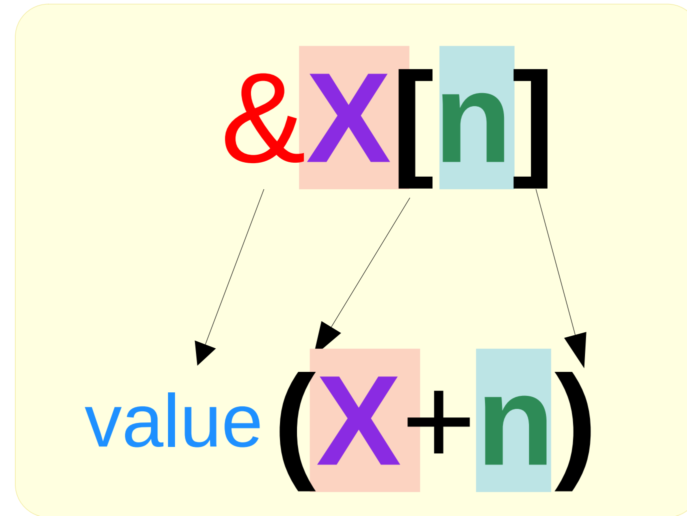
The pointer array :
not allocated
in the memory

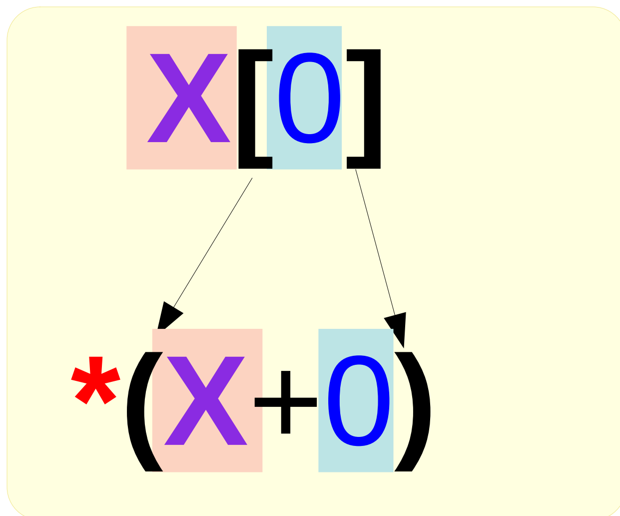
c[0]+0	*(c [0]+0)
c[0]+1	*(c [0]+1)
c[0]+2	*(c [0]+2)
c[0]+3	*(c [0]+3)
c[1]+0	*(c [1]+0)
c[1]+1	*(c [1]+1)
c[1]+2	*(c [1]+2)
c[1]+3	*(c [1]+3)
c[2]+0	*(c [2]+0)
c[2]+1	*(c [2]+1)
c[2]+2	*(c [2]+2)
c[2]+3	*(c [2]+3)

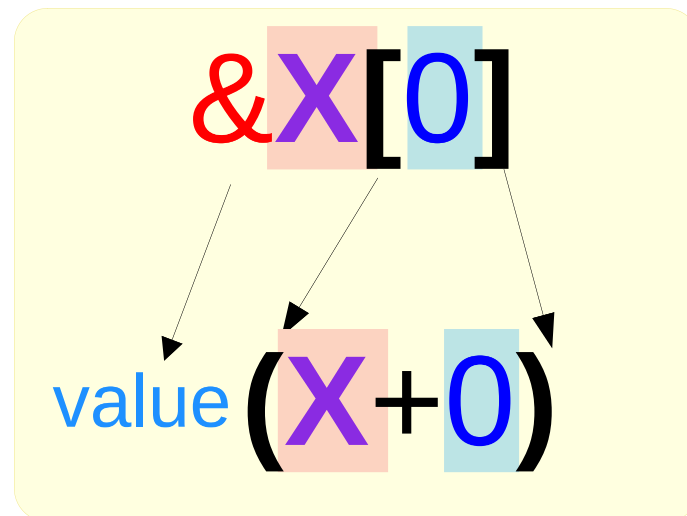
Pointers, arrays, and operator precedence

Equivalences between *, &, and [ ] operators









Operator Precedence of * and [ ]

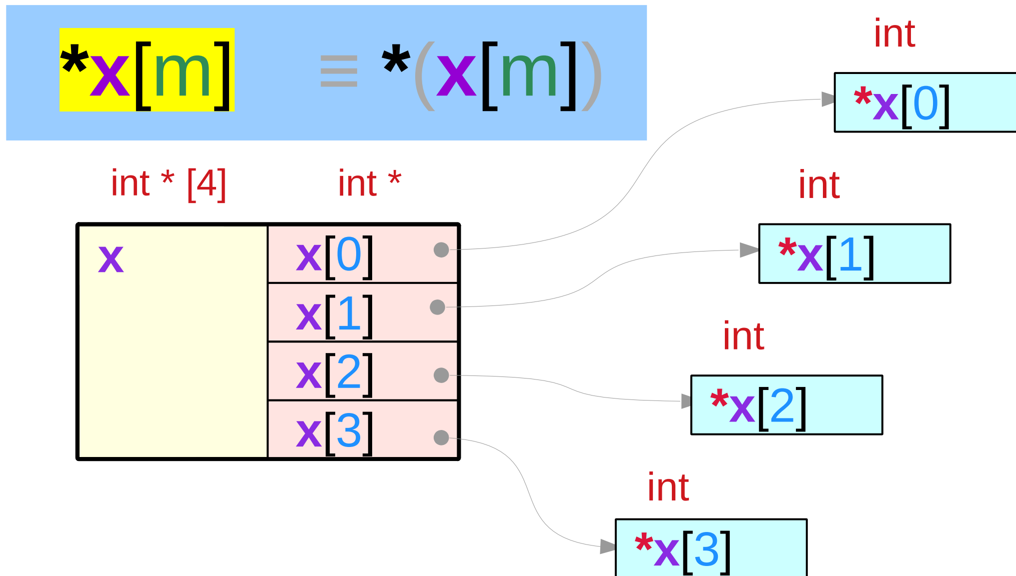
*x[m]	≡	*(x[m])	[ ] has a higher priority than *
x[m][n]	≡	(x[m])[n]	[ ] has left-to-right associativity
**x	≡	*(*x)	* has right-to-left associativity

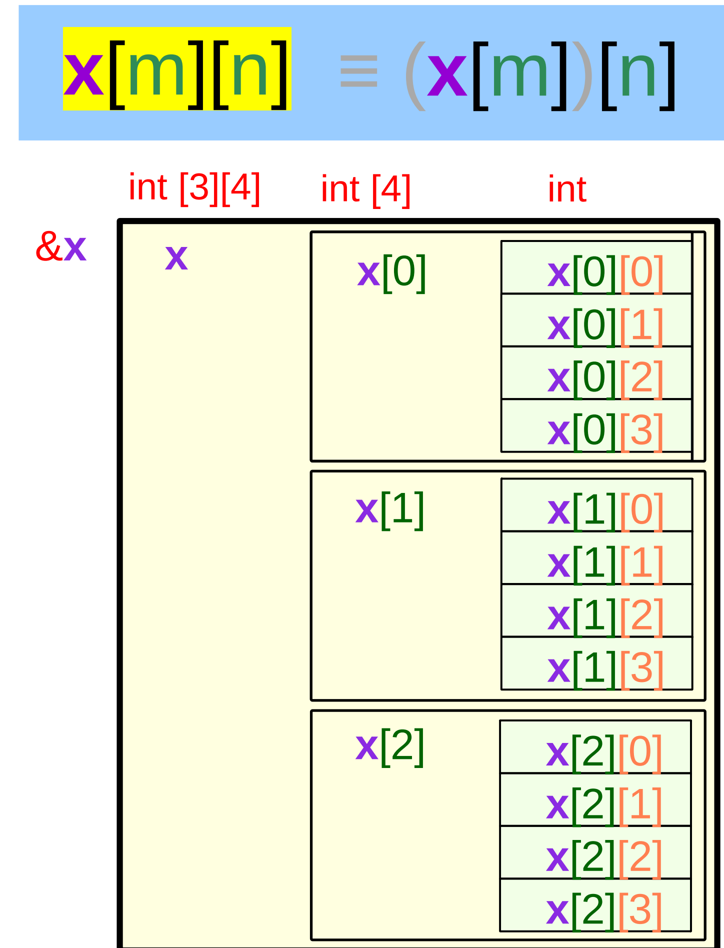
(*x)[m][n] ⟺ ((*x)[m])[n]

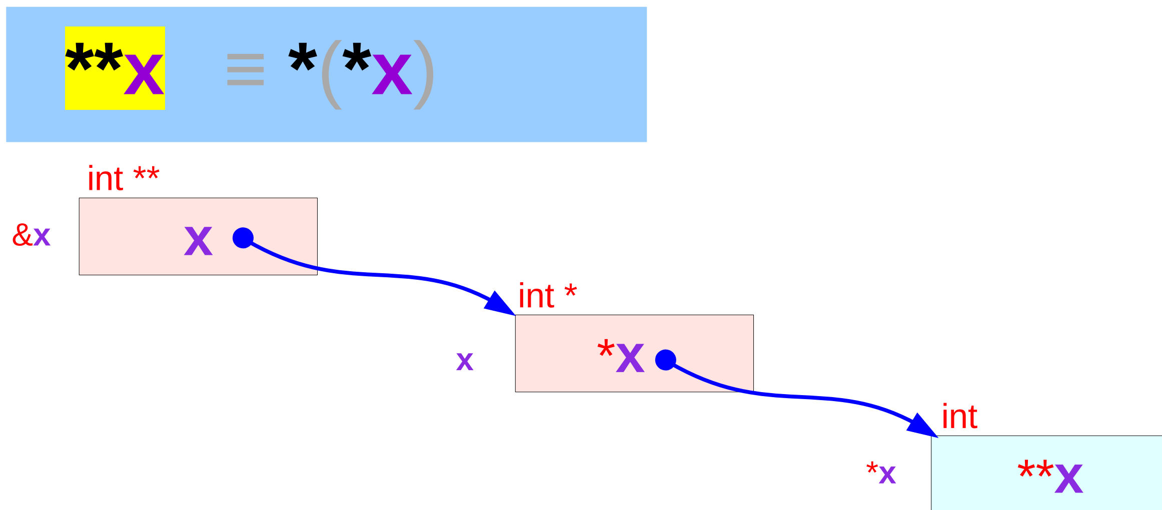
red parentheses ( ) must not be removed
gray parentheses ( ) can be removed

(*x[m])[n] ⟺ (*(x[m]))[n]

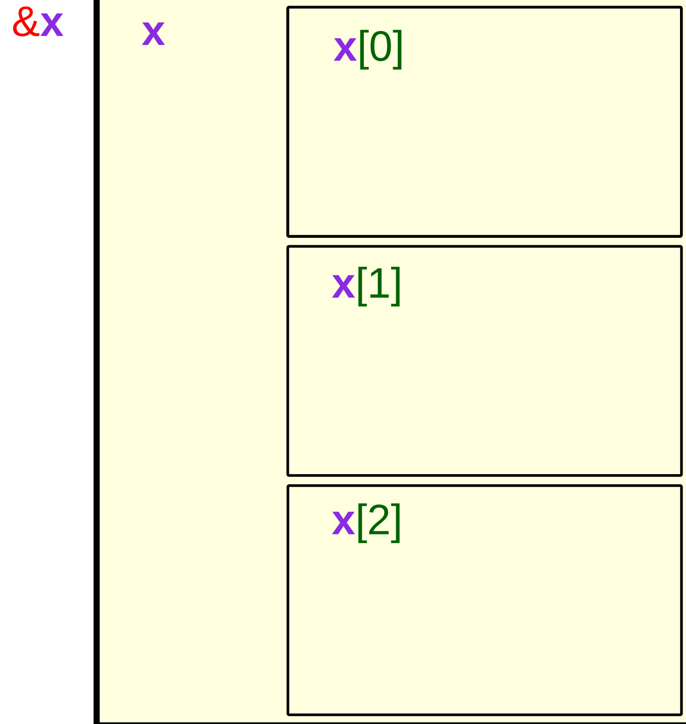
Operator Precedence of * and [ ]

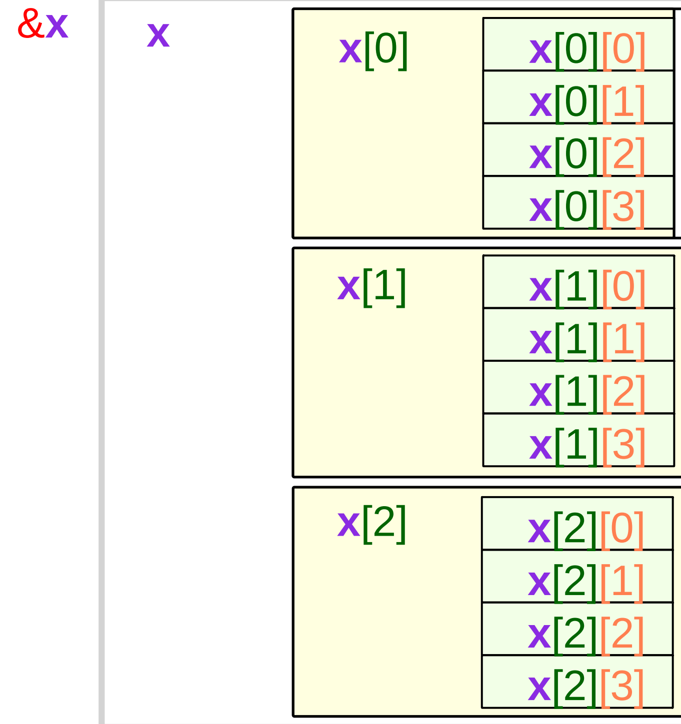




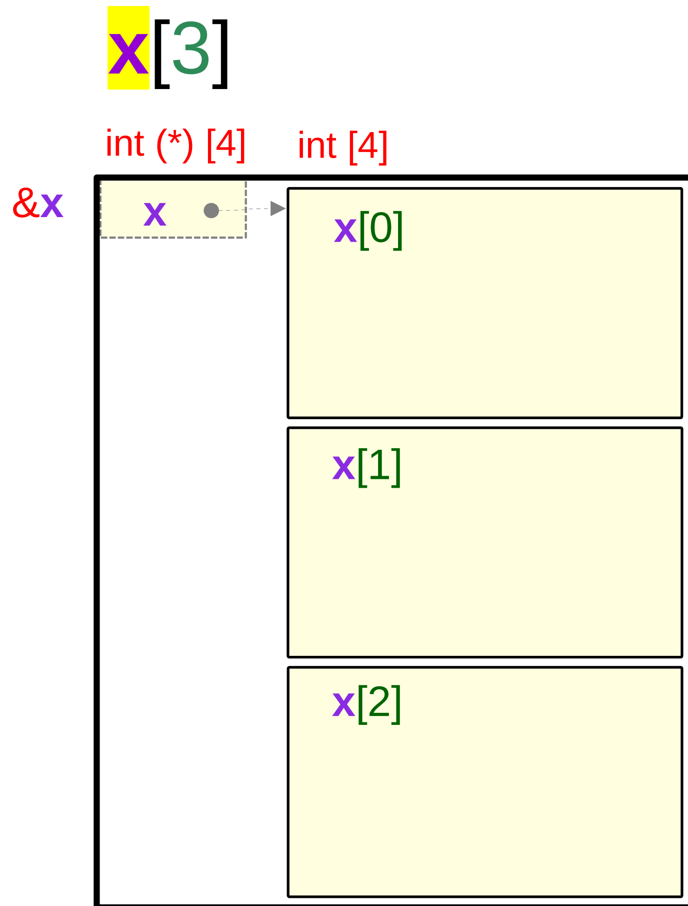


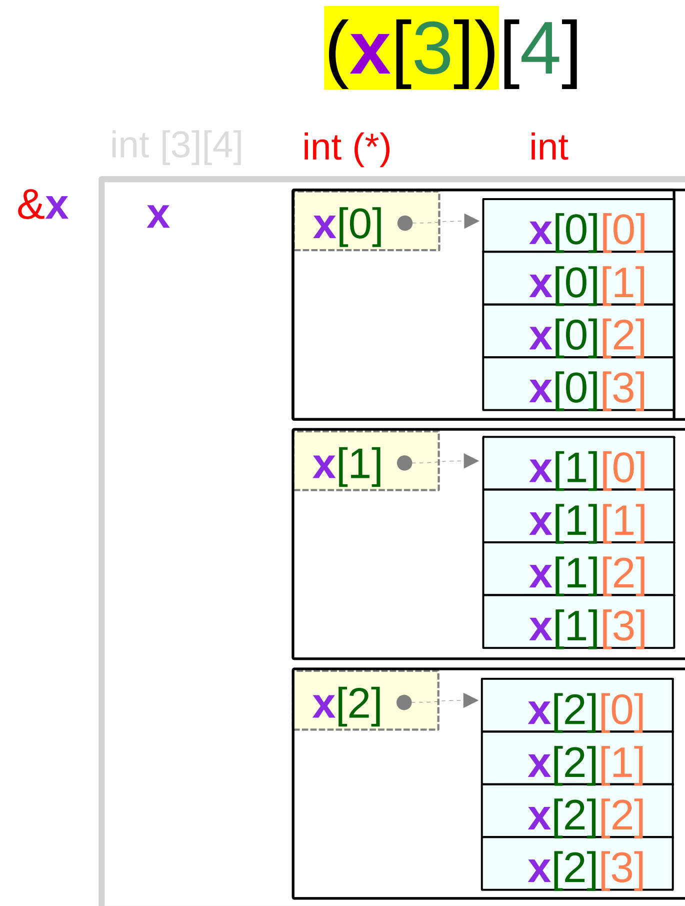
Abstract Data x and x[i]





Virtual Pointers **x** and **x[i]**

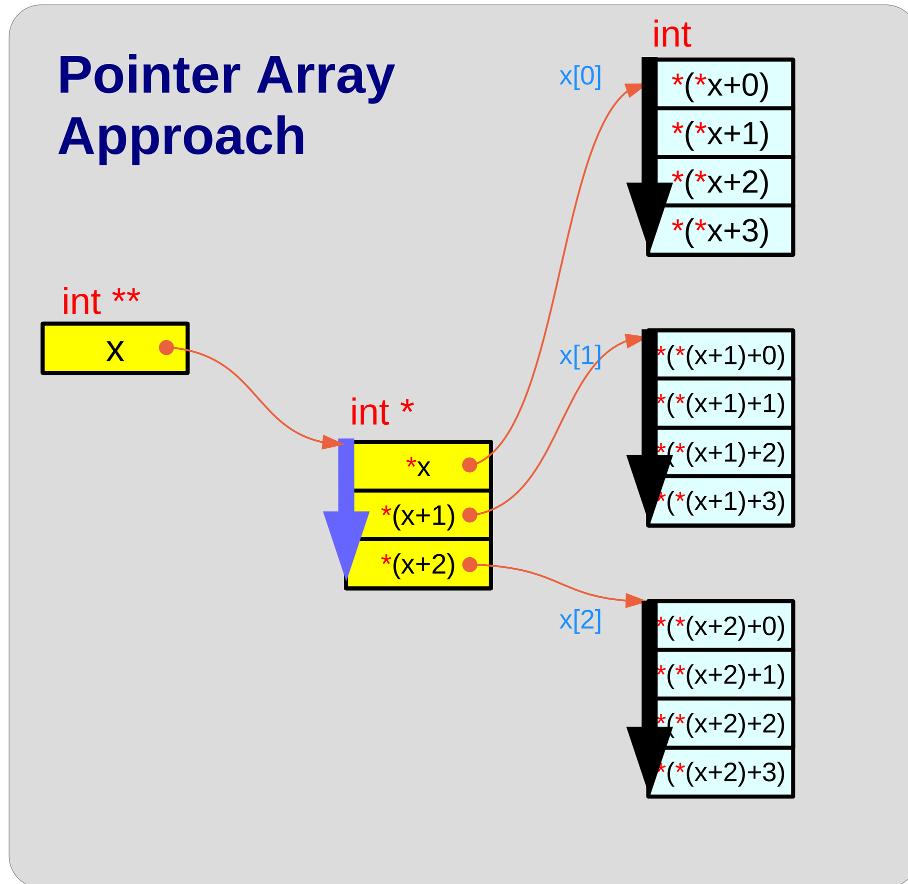


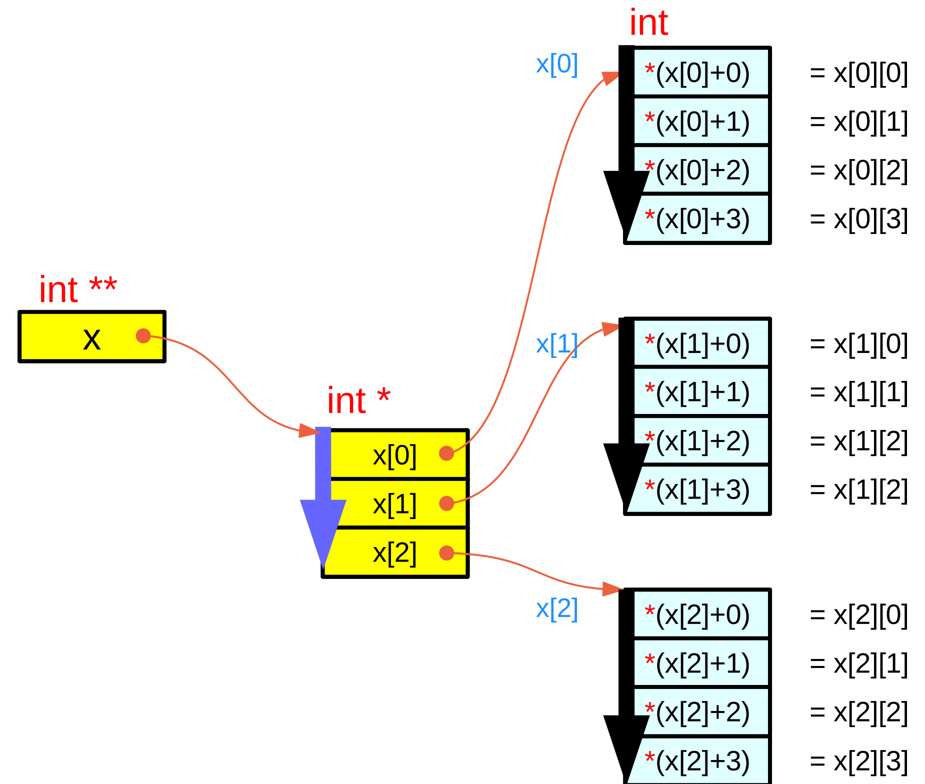


array name **x** ………...virtual pointer
array element **x**[**i**] …... abstract data

array name **x**[**i**] ……… virtual pointer
array element **x**[**i**][**j**] …. primitive data

* into [ ] notations – Pointer Array Approach



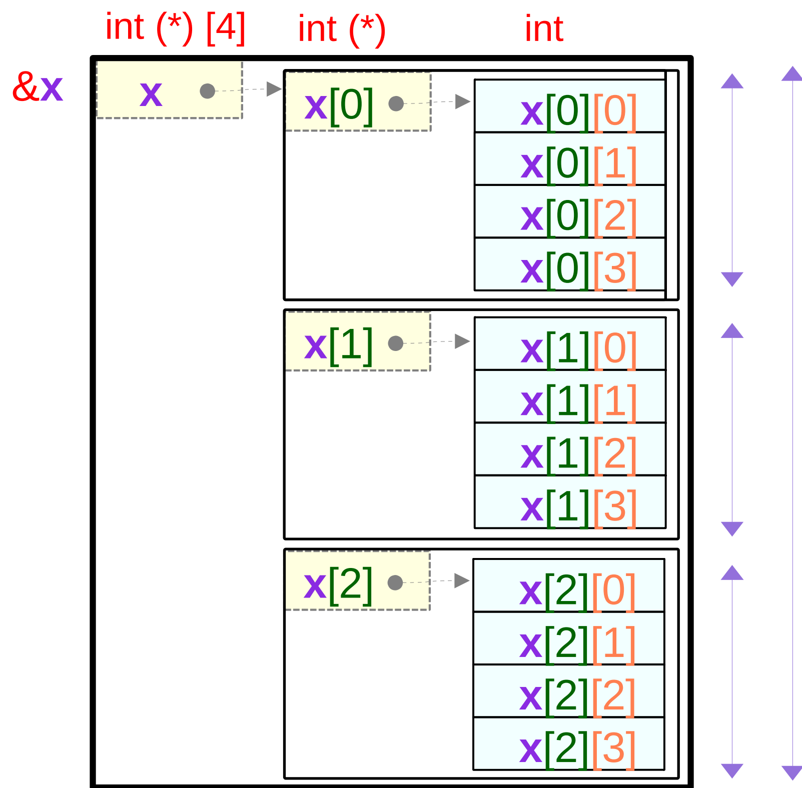


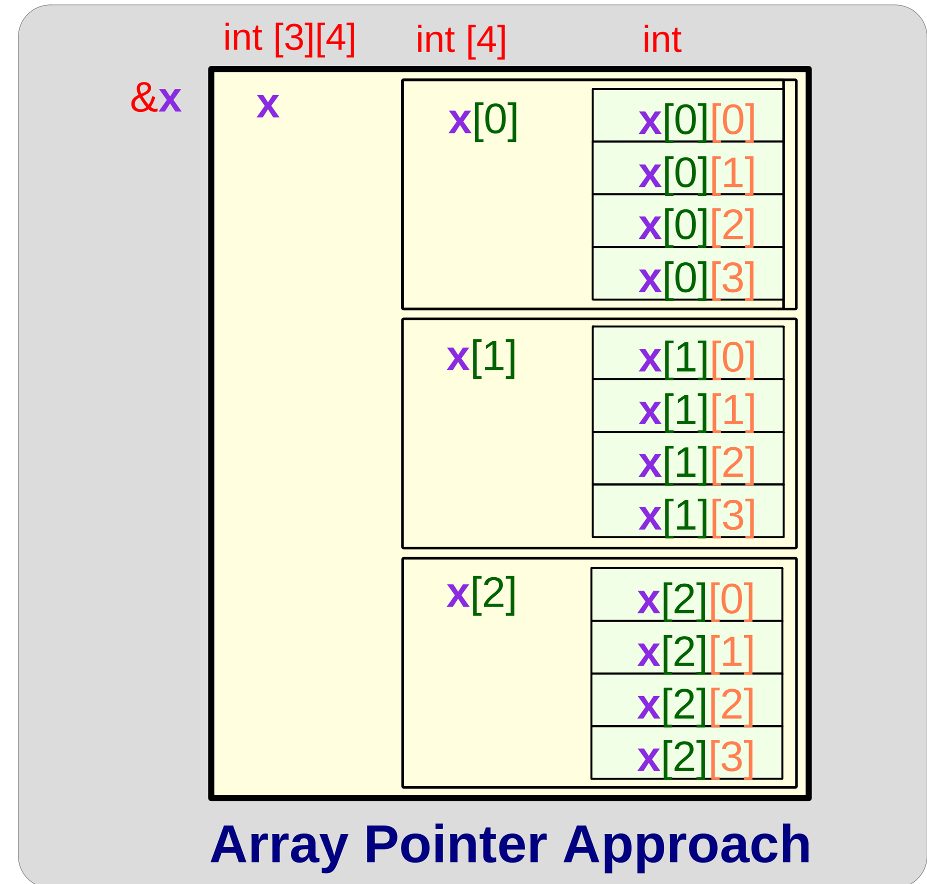
C expression *(*(x+i)+j)

x[i][j]

Math expression $*(*(x+i)_{1\cdot 4}+j)_{1\cdot 4}$

* and [ ] notations – Array Pointer Approach



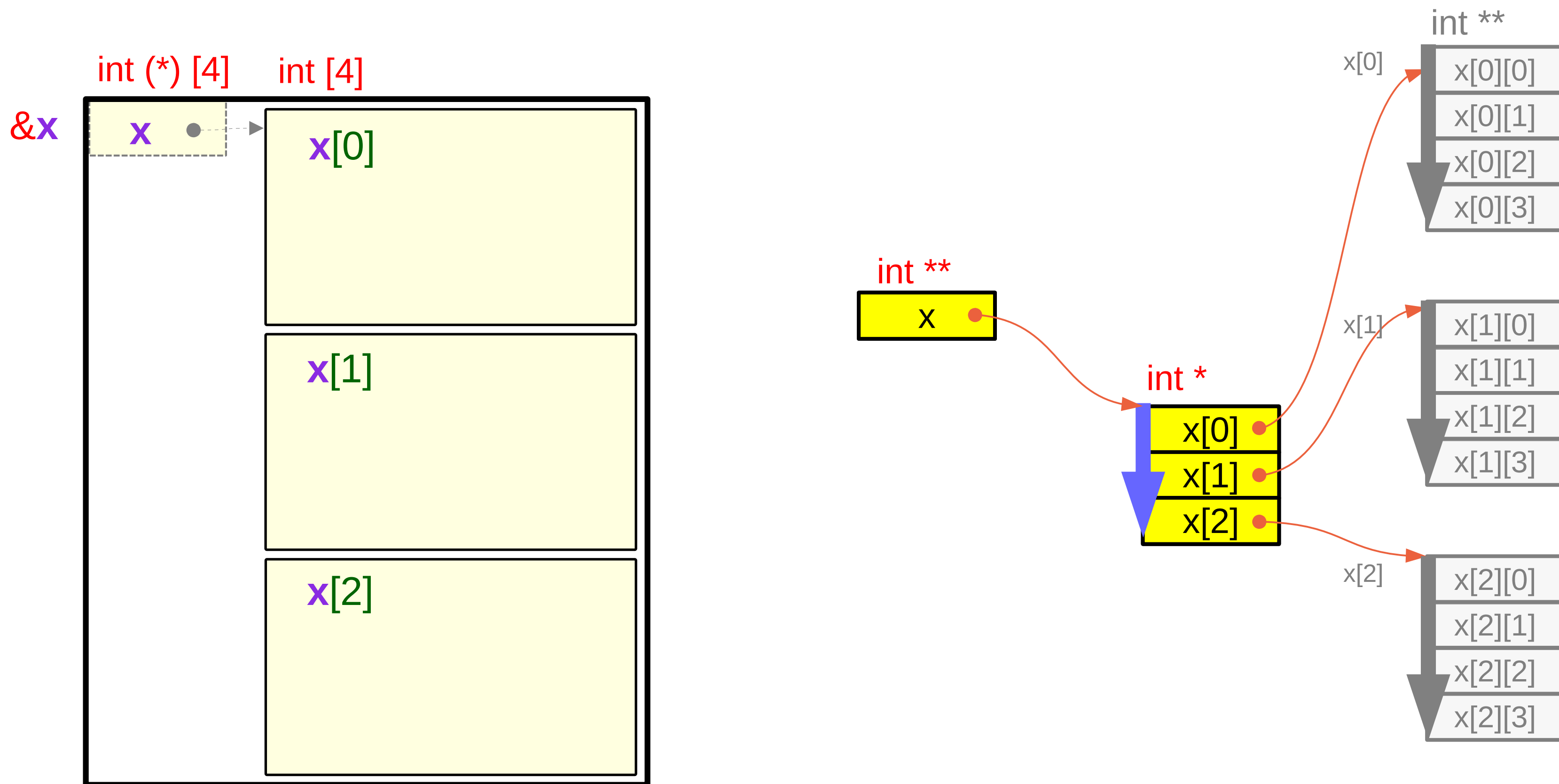


C expression *(*(x+i)+j)

x[i][j]

Math expression $*(*(x+i)_{4\cdot 4}+j)_{1\cdot 4}$

Virtual pointers vs. real pointers (1)



&x = x   sizeof(x) = 3 * sizeof(*x)

x[i] = *(x + i)$_{4\cdot 4}$

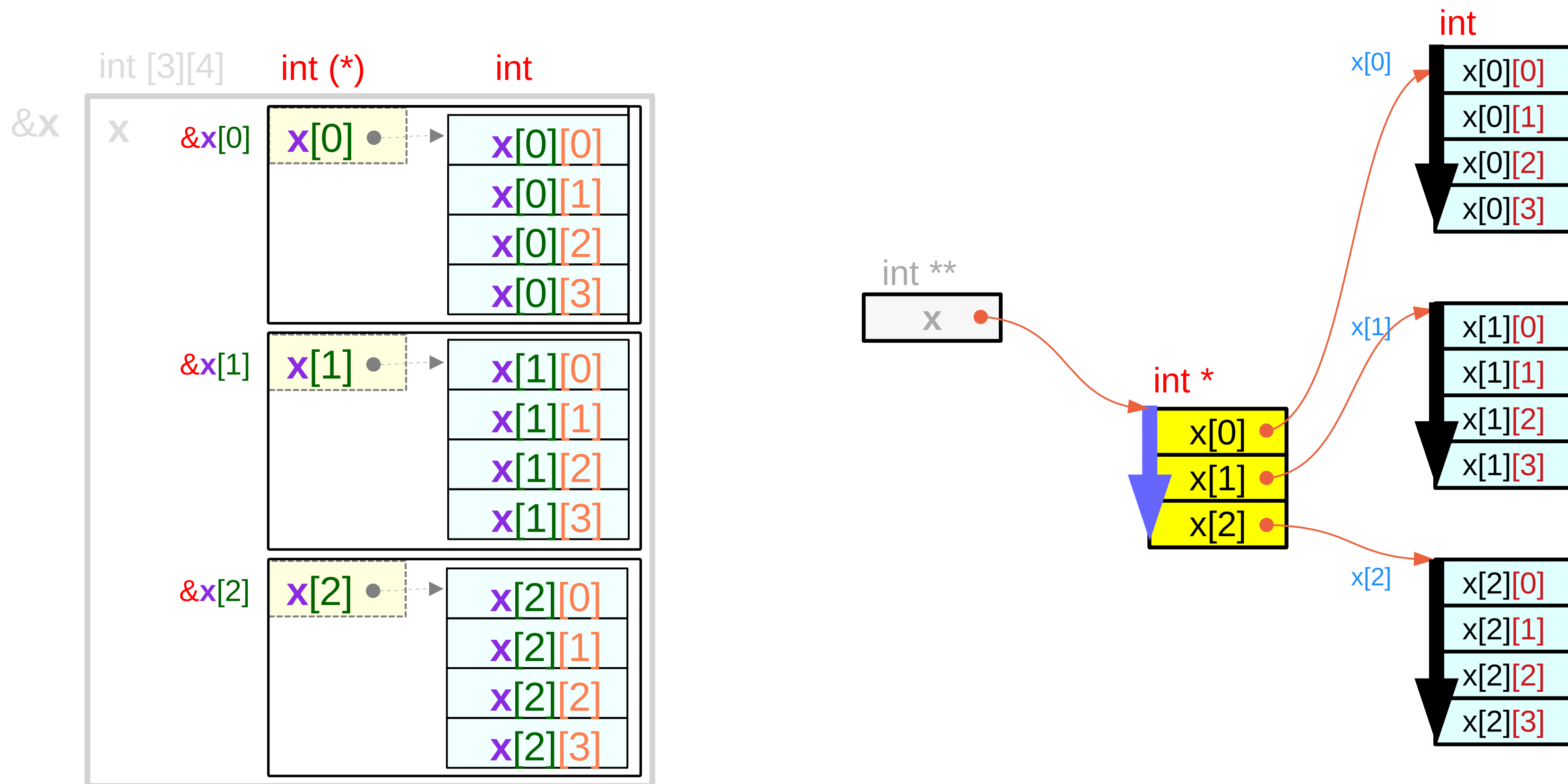
value(x+i) = value(x) + i * sizeof(*x)
= value(x) + i * 4 *4

&x ≠ x   sizeof(x) = sizeof(*x) = 4

x[i] = *(x + i)$_{1\cdot 4}$

value(x+i) = value(x) + i * sizeof(*x)
= value(x) + i * 4

Virtual pointers vs. real pointers (2)



&x[i] = x[i]   sizeof(x[i]) = 4 * sizeof(*x[i])

x[i] = *(x + i)$_{1\cdot 4}$

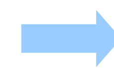
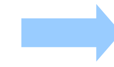
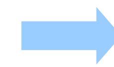
value(x[i]+j) = value(x[i]) + j * sizeof(*x[i])
= value(x[i]) + j * 4

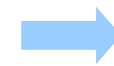
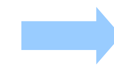
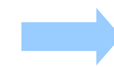
&x[i] = x[i]   sizeof(x[i]) = sizeof(*x[i]) = 4

x[i][j] = *(x[i] + j)$_{1\cdot 4}$

value(x[i]+j) = value(x[i]) + j * sizeof(*x[i])
= value(x[i]) + j * 4

Left-to-right and right-to-left associative operators

p[i]	≡	p[i]		*(p+i)
p[i][j]	≡	(p[i])[j]		*(*(p+i)+j)
p[i][j][k]	≡	((p[i])[j])[k]		*(*(*(p+i)+j)+k)

*p	≡	*(p)		p[0]
**p	≡	*(*(p))		(p[0])[0]
***p	≡	*(*(*(p)))		((p[0])[0])[0]

Relaxing the outermost dimension

p[i]	≡	*(p+i)
p[i][j]	≡	*(p[i]+j)
p[i][j][k]	≡	*(p[i][j]+k)

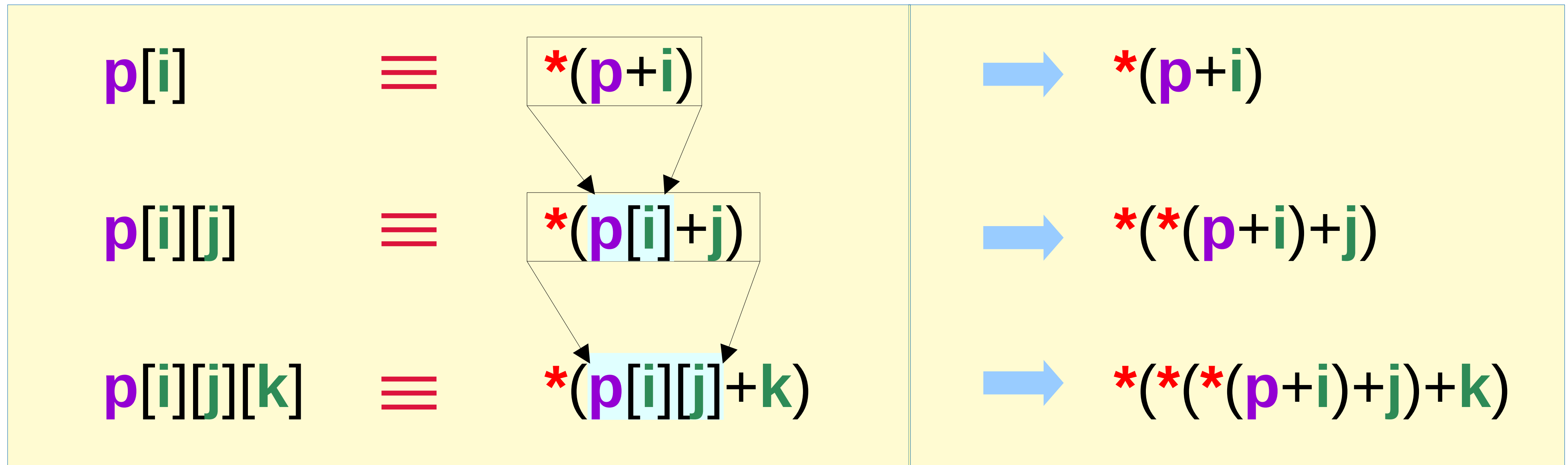
&p[i]	≡	value(p+i)
&p[i][j]	≡	value(p[i]+j)
&p[i][j][k]	≡	value(p[i][j]+k)

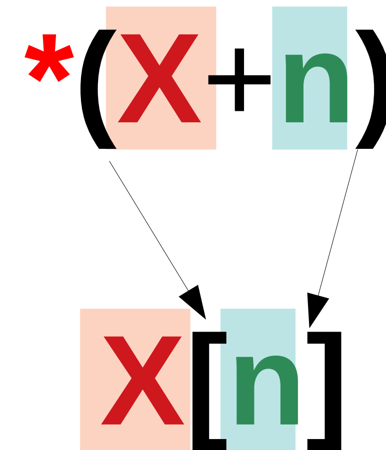
p[0]	≡	*p
p[i][0]	≡	*p[i]
p[i][j][0]	≡	*p[i][j]

&p[0]	≡	value(p)
&p[i][0]	≡	value(p[i])
&p[i][j][0]	≡	value(p[i][j])

valid for proper **i**, **j**, **k** values

Relaxing all the dimensions





valid for proper **i**, **j**, **k** values

Equivalences on relaxing all the dimensions

p[i]	≡	*(p+i)
p[i][j]	≡	*(*(p+i)+j)
p[i][j][k]	≡	*(*(*(p+i)+j)+k)

&p[i]	≡	value(p+i)
&p[i][j]	≡	value(*(p+i)+j)
&p[i][j][k]	≡	value(*(*(p+i)+j)+k)

*value(X) = *X

p[0]	≡	*p
p[0][0]	≡	**p
p[0][0][0]	≡	***p

&p[0]	≡	value(p)
&p[0][0]	≡	value(*p)
&p[0][0][0]	≡	value(**p)

valid for proper **i**, **j**, **k** values

Address Calculation (1) Array Pointer Approach

int c [2][3][4] ;

c[i]	≡ *(c + i)
c[i][j]	≡ *(c[i] + j)
c[i][j][k]	≡ *(c[i][j] + k)

&c[i]	≡ value(c + i)
&c[i][j]	≡ value(c[i] + j)
&c[i][j][k]	≡ value(c[i][j] + k)

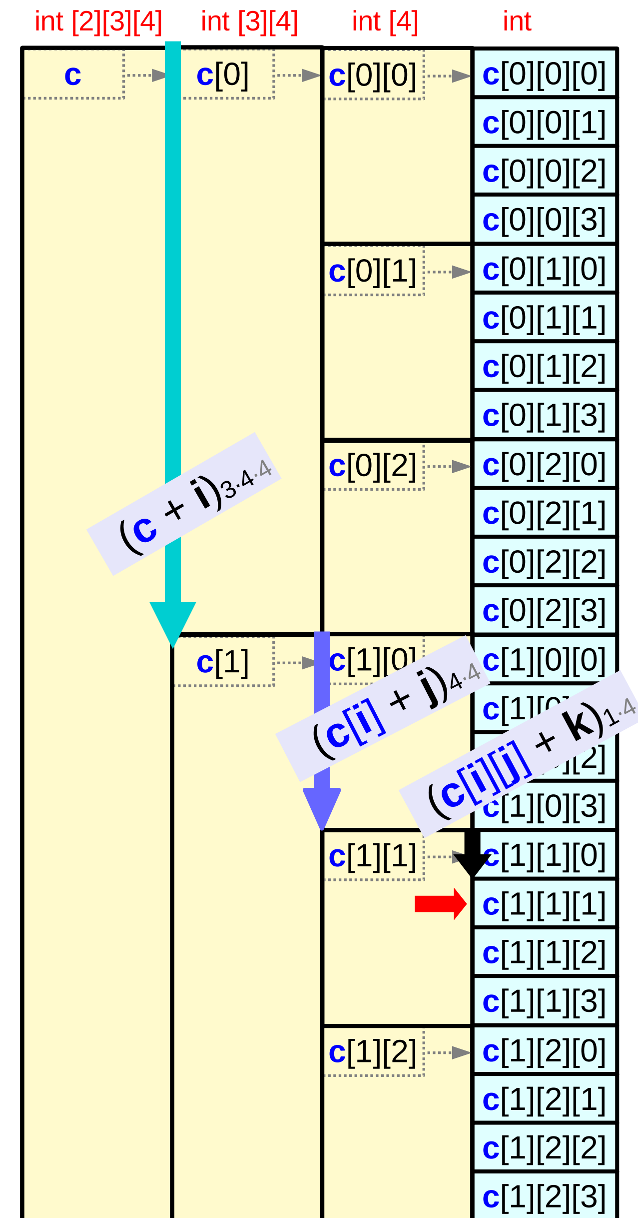
address replication

value(c[i][j][k]) ≠ value(&c[i][j][k]) ← primitive data & address

value(c[i][j])	= value(&c[i][j])	= value(&c[i][j][0])
value(c[i])	= value(&c[i])	= value(&c[i][0][0])
value(c)	= value(&c)	= value(&c[0][0][0])

skip i elements of c
skip j elements of c[i]
skip k elements of c[i][j]

skip i*3*4 primitive elements of c
skip j*4 primitive elements of c
skip k primitive elements of c



Address Calculation (2) Pointer Array Approach

```
int ** c [2] ;
int *  b [2*3] ;
int    a [2*3*4] ;
```

b[j] ≡ (a+j*4)

*(b[j]+k) = *(a+j*4+k);

b[j][k] ≡ a[j*4+k]

c[i] ≡ (b+i*3)

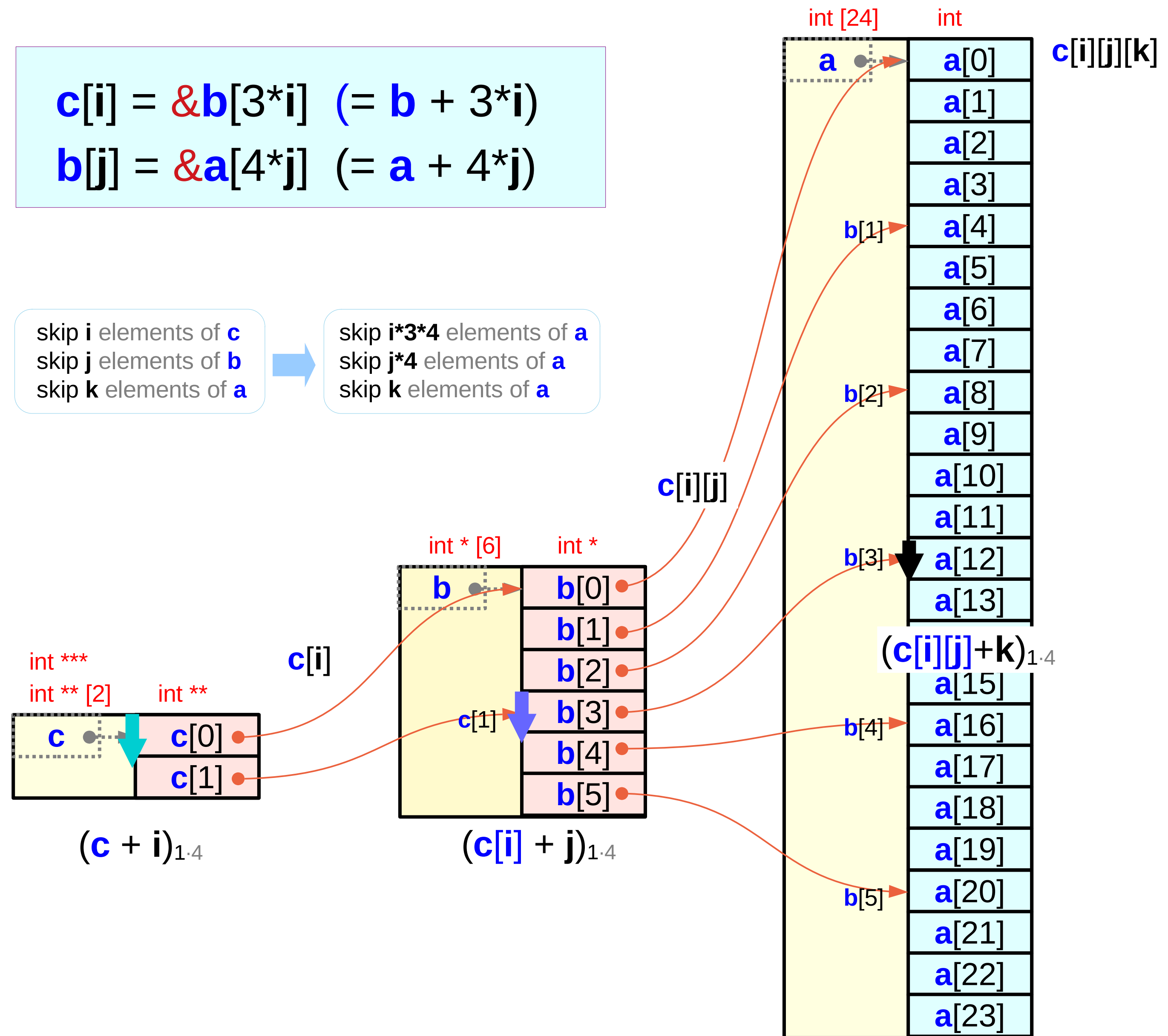
*(c[i]+j) = *(b+i*3+j);

c[i][j] ≡ b[i*3+j]

c[i][j] ≡ (a+(i*3+j)*4)

*(c[i][j]+k) = *(a+(i*3+j)*4+k);

c[i][j][j] ≡ a[(i*3+j)*4+k]

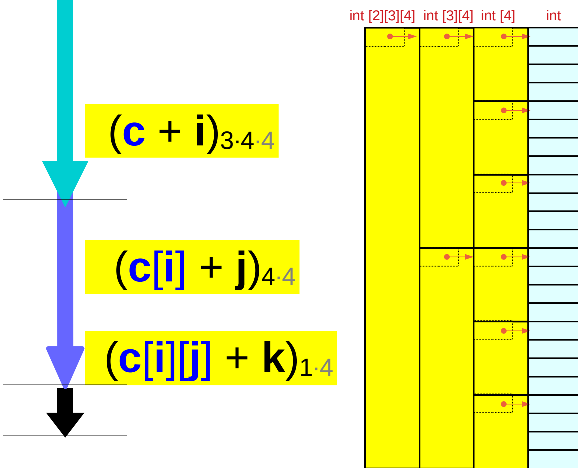


Address Calculation (3)

value(**c** + **i**) = value(**c**) + **i** * 3 * 4 * 4
value(**c[i]** + **j**) = value(**c[i]**) + **j** * 4 * 4
value(**c[i][j]** + **k**) = value(**c[i][j]**) + **k** * 4

value(**c** + **i**) = value(**c**) + **i** * sizeof(*c)
value(**c[i]** + **j**) = value(**c[i]**) + **j** * sizeof(*c[i])
value(**c[i][j]** + **k**) = value(**c[i][j]**) + **k** * sizeof(*c[i][j])

Array Pointer Approach



$(c + i)_{3 \cdot 4 \cdot 4}$

$(c[i] + j)_{4 \cdot 4}$

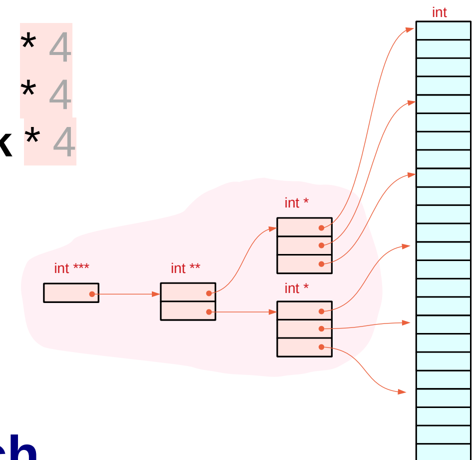
$(c[i][j] + k)_{1 \cdot 4}$

c[i] ≡ *(**c** + **i**)
c[i][j] ≡ *(**c[i]** + **j**)
c[i][j][k] ≡ *(**c[i][j]** + **k**)

&**c[i]** ≡ value(**c** + **i**)
&**c[i][j]** ≡ value(**c[i]** + **j**)
&**c[i][j][k]** ≡ value(**c[i][j]** + **k**)

value(**c** + **i**) = value(**c**) + **i** * 4
value(**c[i]** + **j**) = value(**c[i]**) + **j** * 4
value(**c[i][j]** + **k**) = value(**c[i][j]**) + **k** * 4

$(c + i)_{1 \cdot 4}$ $(c[i] + j)_{1 \cdot 4}$ $(c[i][j] + k)_{1 \cdot 4}$



Pointer Array Approach

Subscript [ ] and dereference * notations (1a)

p[i] ≡ *(p+i)

from **p**, skip
i·M·N integers

&p[i] = value((**p** + **i**)$_{M \cdot N \cdot 4}$)
= value(**p** + **i** * M·N·4)

p[i][j] ≡ *(*(p+i)+j)

from **p[i]**, skip
j·N integers

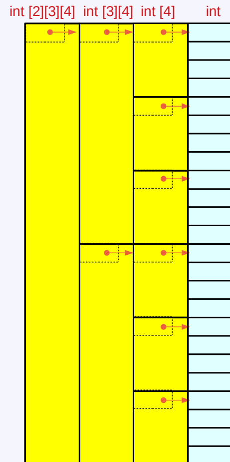
&p[i][j] = value((**p[i]** + **j**)$_{N \cdot 4}$)
= value(**p[i]** + **j** * N·4)

p[i][j][k] ≡ *(*(*(p+i)+j)+k)

from **p[i][j]**, skip
k integers

&p[i][j][k] = value((**p[i][j]**+**k**)$_{1 \cdot 4}$)
= value(**p[i][j]** + **k** * 1·4)

int p[L][M][N]



Array Pointer Approach

address replications

value(**p[i]**) = &**p[i]** = value(**p** + **i**)

value(**p[i][j]**) = &**p[i][j]** = value(**p[i]** + **j**)

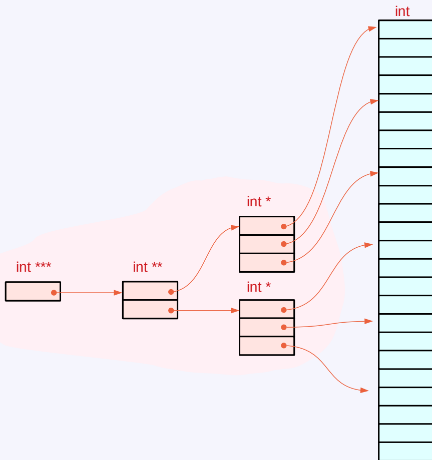
value(**p[i][j][k]**) ≠ &**p[i][j][k]** = value(**p[i][j]** + **k**)

&**p[i][j][k]** = value(**p** + **i** *M·N·4 + **j** *N·4 + **k** * 4)

p[i][j][k] = *value(**p** + **i** *M·N·4 + **j** *N·4 + **k** * 4)

Subscript [ ] and dereference * notations (1b)

p[i]	≡	*(p+i)	skip **i** pointers from **p**	&**p**[**i**] = value(**p** + **i**)$_{1\cdot4}$ = value(**p** + **i** *4)
p[i][j]	≡	*(*(p+i)+j)	skip **j** pointers from **p**[**i**]	&**p**[**i**][**j**] = value(**p**[**i**] + **j**)$_{1\cdot4}$ = value(**p**[**i**] + **j** *4)
p[i][j][k]	≡	*(*(*(p+i)+j)+k)	skip **k** integers from **p**[**i**][**j**]	&**p**[**i**][**j**][**k**] = value(**p**[**i**][**j**] + **k**)$_{1\cdot4}$ = value(**p**[**i**][**j**] + **k** *4)



int ** p [L] ;
int * q [L·M] ;
int r [L·M·N] ;

Pointer Array Approach

address dereferences

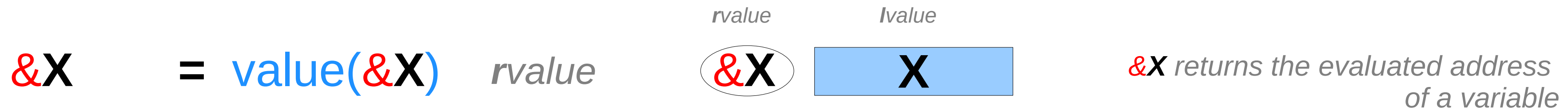
value(**p**[**i**]) = *(&**p**[**i**]) = *value(**p** + **i**)

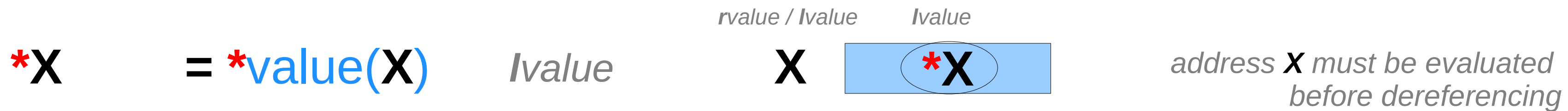
value(**p**[**i**][**j**]) = *(&**p**[**i**][**j**]) = *value(**p**[**i**] + **j**)

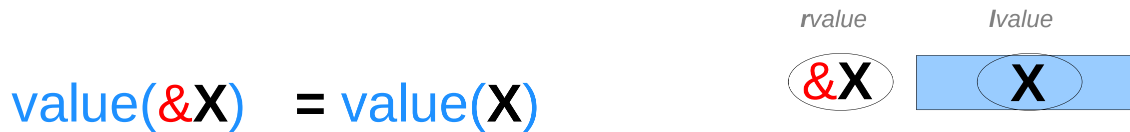
value(**p**[**i**][**j**][**k**]) = *(&**p**[**i**][**j**][**k**]) = *value(**p**[**i**][**j**] + **k**)

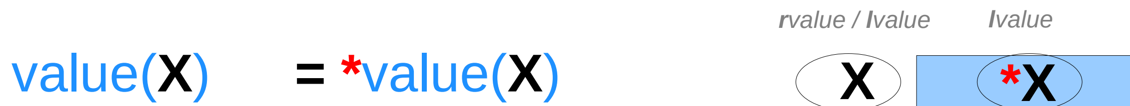
&**p**[**i**][**j**][**k**] = value(*value(*value(**p**+**i***4)+**j***4)+**k***4)

Subscript [ ] and dereference * notations (1a)





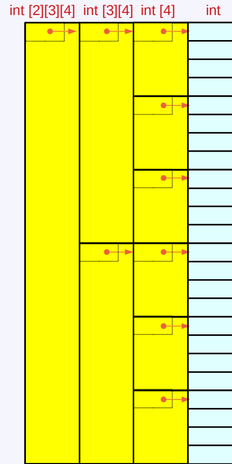




Subscript [ ] and dereference * notations (1a)

int p[L][M][N]

Array Pointer Approach



address replications

value(p[i]) = &p[i] = value(p + i)

value(p[i][j]) = &p[i][j] = value(p[i] + j)

value(p[i][j][k]) ≠ &p[i][j][k] = value(p[i][j] + k)

&p[i][j][k] = value(p + i *M·N·4 + j *N·4 + k * 4)

p[i][j][k] = *value(p + i *M·N·4 + j *N·4 + k * 4)

abstract data	int [3][4]	value(p[i])	= &p[i]	= value(p + i)	int (*)[3][4]	virtual pointer
abstract data	int [4]	value(p[i][j])	= &p[i][j]	= value(p[i] + j)	int (*)[4]	virtual pointer
primitive data	int	value(p[i][j][k])	≠ &p[i][j][k]	= value(p[i][j] + k)	int (*)	virtual pointer

p[i][j][k] = *value(*value(*value(p+i)+j)+k) → *address replications*

= value(value(value(p + i)$_{3·4·4}$) + j)$_{4·4}$ + k)$_{4}$

= value(p + i *3·4·4 + j *4·4 + k *4)

Subscript [ ] and dereference * notations (2)

p[i]	≡	*(p+i)
p[i][j]	≡	*(*(p+i)+j)
p[i][j][k]	≡	*(*(*(p+i)+j)+k)

C Expressions

&p[i]	≡	value(p+i)
&p[i][j]	≡	value(*(p+i)+j)
&p[i][j][k]	≡	value(*(*(p+i)+j)+k)

C Expressions

int p [L][M][N] ;

value(&X) = value(X) (address replication)

p[i]	→	$*(p+i)_{M \cdot N \cdot 4}$
p[i][j]	→	$*(*(p+i)_{M \cdot N \cdot 4}+j)_{N \cdot 4}$
p[i][j][k]	→	$*(*(*(p+i)_{M \cdot N \cdot 4}+j)_{N \cdot 4}+k)_{1 \cdot 4}$

Math Expressions

&p[i]	→	$value(p+i)_{M \cdot N \cdot 4}$
&p[i][j]	→	$value((p+i)_{M \cdot N \cdot 4}+j)_{N \cdot 4}$
&p[i][j][k]	→	$value(((p+i)_{M \cdot N \cdot 4}+j)_{N \cdot 4}+k)_{1 \cdot 4}$

Math Expressions

int ** p[L], * q[L·M], r[L·M·N] ;

*value(X) = *X

p[i]	→	$*(p+i)_{1 \cdot 4}$
p[i][j]	→	$*(*(p+i)_{1 \cdot 4}+j)_{1 \cdot 4}$
p[i][j][k]	→	$*(*(*(p+i)_{1 \cdot 4}+j)_{1 \cdot 4}+k)_{1 \cdot 4}$

Math Expressions

&p[i]	→	$value(p+i)_{1 \cdot 4}$
&p[i][j]	→	$value(*(p+i)_{1 \cdot 4}+j)_{1 \cdot 4}$
&p[i][j][k]	→	$value(*(*(p+i)_{1 \cdot 4}+j)_{1 \cdot 4}+k)_{1 \cdot 4}$

Math Expressions

Subscript [ ] and dereference * notations (3)

int p [L][M][N] ;

value(&X) = value(X) (address replication)

&p[i] = value((p + i)$_{M·N·4}$) = value(p + i * M·N·4)

&p[i][j] = value((p[i] + j)$_{N·4}$) = value(p[i] + j * N·4)

&p[i][j][k] = value((p[i][j]+k)$_{1·4}$) = value(p[i][j] + k * 1·4)

= value(p + i * M·N·4 + j * N·4 + k * 4)

&p[i]	→	value(p+i)$_{M·N·4}$
&p[i][j]	→	value((p+i)$_{M·N·4}$+j)$_{N·4}$
&p[i][j][k]	→	value(((p+i)$_{M·N·4}$+j)$_{N·4}$+k)$_{1·4}$

Math Expressions

int ** p[L], * q[L·M], r[L·M·N] ;

*value(X) = *X

&p[i] = value(p + I)$_{1·4}$ = value(p + i * 1·4)

&p[i][j] = value(p[i] + j)$_{1·4}$ = value(p[i] + j * 1·4)

&p[i][j][k] = value(p[i][j] + k)$_{1·4}$ = value(p[i][j] + k * 1·4)

= value(*value(*value(p +i*4) +j*4) +k*4)

&p[i]	→	value(p+i)$_{1·4}$
&p[i][j]	→	value(*(p+i)$_{1·4}$+j)$_{1·4}$
&p[i][j][k]	→	value(*(*(p+i)$_{1·4}$+j)$_{1·4}$+k)$_{1·4}$

Math Expressions

Operator Precedence

Precedence	Operator	Description	Associativity
1	++ -- () [] . -> (type){list}	Suffix/postfix increment and decrement Function call Array subscripting Structure and union member access member access through pointer Compound literal(C99)	Left-to-right (((x[m])[n])[p])
2	++ -- + - ! ~ (type) * & sizeof _Alignof	Prefix increment and decrement Unary plus and minus Logical NOT and bitwise NOT Type cast Indirection (dereference) Address-of Size-of Alignment requirement(C11)	Right-to-left *(*(*(*x)))

https://en.cppreference.com/w/c/language/operator_precedence

Limitations

No index Range Checking

Array Size must be a constant expression

- Variable Array Size

Arrays cannot be Copied or Compared

Aggregate Initialization and Global Arrays

Precedence Rule

Index Type Must be Integral

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