Laurent Series and z-Transform Examples case 5.A

20171230

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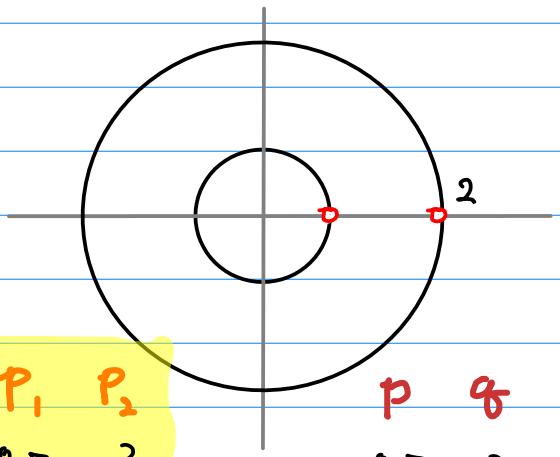
relative values

$p_1 < p_2$

fixed values

$p = 0.5$ $q = 2$

$f(z)$



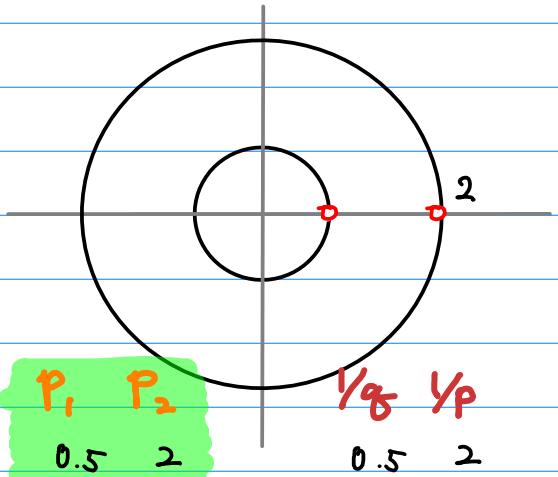
p_1 p_2

0.5 2

p q

0.5 2

$X(z)$



p_1 p_2

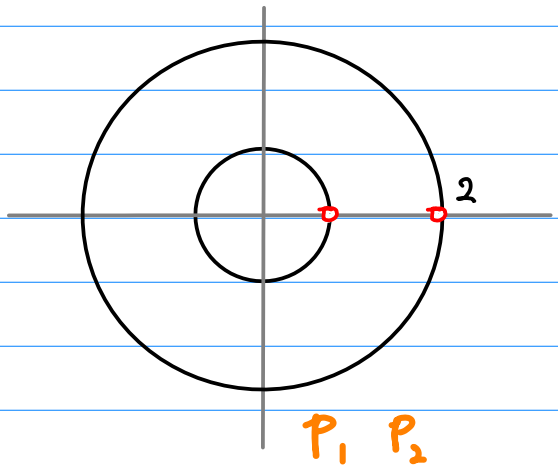
0.5 2

$1/q$ $1/p$

0.5 2

$f(z)$ $\quad p_1 < p_2$

$p_1 \leqslant p_2 \leqslant 1$ $\quad 2^{n-1}$		$1 \leqslant p_1 \leqslant p_2$ $\quad \left(\frac{1}{2}\right)^{n+1}$		
$\left(\frac{1}{p_2}\right)^{n-1} - \left(\frac{1}{p_1}\right)^{n-1}$	$(n > 0)$	$\left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1}$	$(n \geqslant 0)$	$\lvert z \rvert < p_1$
0	$(n \leqslant 0)$	0	$(n < 0)$	
0	$(n > 0)$	0	$(n \geqslant 0)$	$\lvert z \rvert > p_2$
$\left(\frac{1}{p_1}\right)^{n-1} - \left(\frac{1}{p_2}\right)^{n-1}$	$(n \leqslant 0)$	$\left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1}$	$(n < 0)$	
$\left(\frac{1}{p_2}\right)^{n-1}$	$(n > 0)$	$\left(\frac{1}{p_2}\right)^{n+1}$	$(n \geqslant 0)$	$p_1 < \lvert z \rvert < p_2$
$\left(\frac{1}{p_1}\right)^{n-1}$	$(n \leqslant 0)$	$\left(\frac{1}{p_1}\right)^{n+1}$	$(n < 0)$	

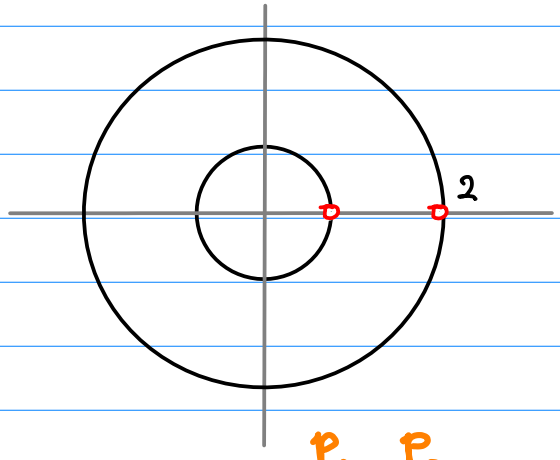


$p_1 \quad p_2$

$0.5 \quad 2$

$\frac{1}{p_1}$	$\frac{1}{p_2}$
2	0.5

2^{n-1}



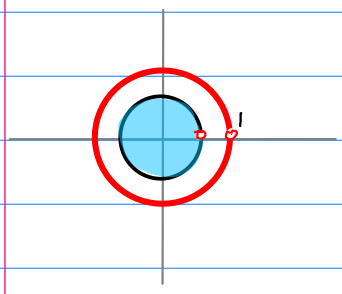
$p_1 \quad p_2$

$1 \quad 2$

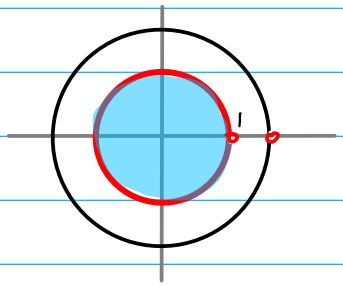
$\frac{1}{p_1}$	$\frac{1}{p_2}$
2	0.5

$\left(\frac{1}{2}\right)^{n+1}$

$p_1 < p_2 < 1$ $\quad 2^{n-1}$ $\quad 1 < p_1 < p_2$ $\quad \left(\frac{1}{2}\right)^{n+1}$

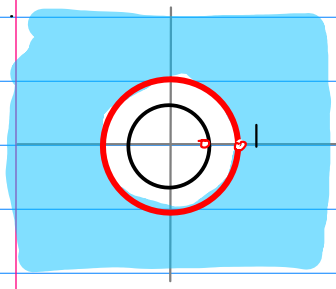


$n > 0$

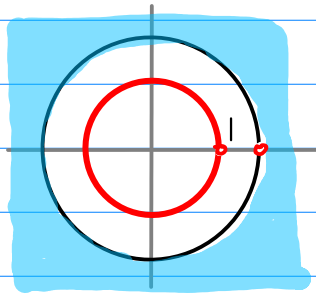


$n \geq 0$

$|z| < p_1$

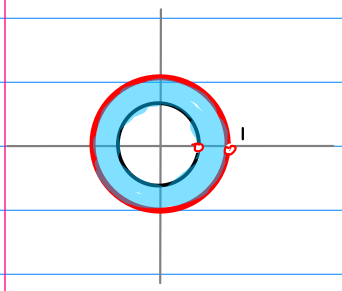


$n \leq 0$



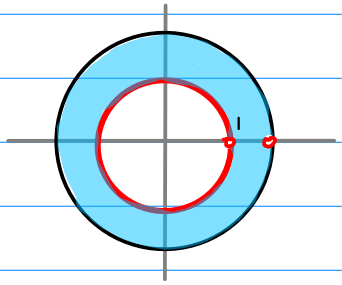
$n < 0$

$|z| > p_2$



$n > 0$

$n \leq 0$



$n \geq 0$

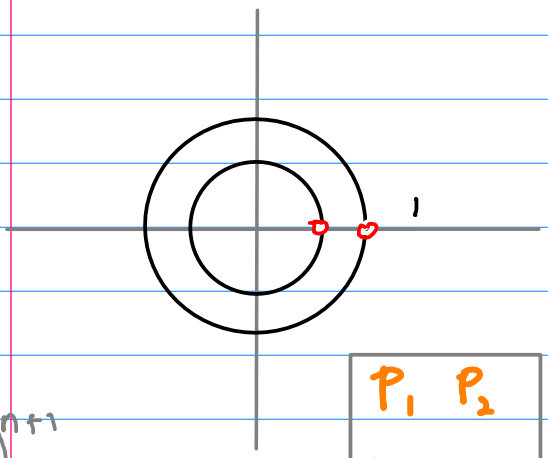
$n < 0$

$p_1 < |z| < p_2$

Unit Circle $\in$ ROC $\rightarrow$ $\leq, \geq$

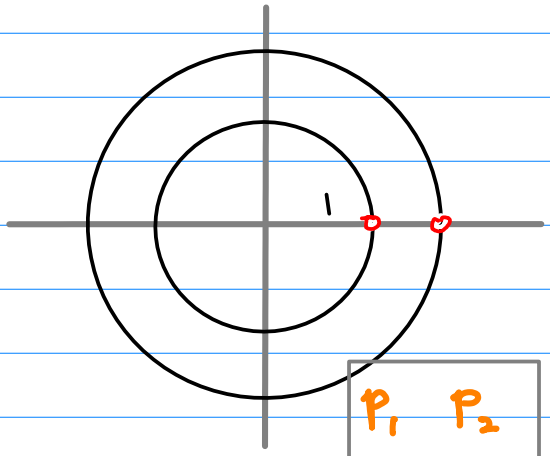
$X(z)$ $\quad p_1 < p_2$

$p_1 < p_2 < 1$ $\left(\frac{1}{2}\right)^{n+1}$		$1 < p_1 < p_2$ 2^{n-1}		
0 $(p_1)^{n+1} - (p_2)^{n+1}$	$(n \geq 0)$ $(n < 0)$	0 $(p_1)^{n-1} - (p_2)^{n-1}$	$(n > 0)$ $(n \leq 0)$	$\lvert z \rvert < p_1$
$(p_2)^{n+1} - (p_1)^{n+1}$ 0	$(n \geq 0)$ $(n < 0)$	$(p_2)^{n-1} - (p_1)^{n-1}$ 0	$(n > 0)$ $(n \leq 0)$	$\lvert z \rvert > p_2$
$(p_2)^{n+1}$ $(p_1)^{n+1}$	$(n \geq 0)$ $(n < 0)$	$(p_2)^{n-1}$ $(p_1)^{n-1}$	$(n > 0)$ $(n \leq 0)$	$p_1 < \lvert z \rvert < p_2$



$\left(\frac{1}{2}\right)^{n+1}$

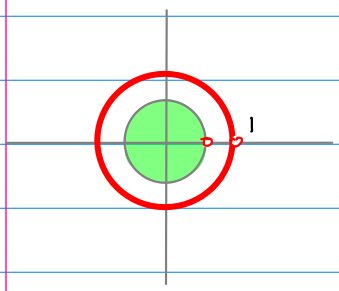
p_1	p_2
0.5	1.0
$\frac{1}{p_1}$	$\frac{1}{p_2}$
2	1



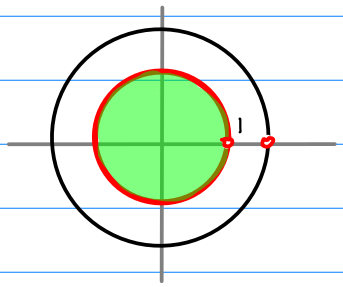
2^{n-1}

p_1	p_2
1	2
$\frac{1}{p_1}$	$\frac{1}{p_2}$
1	0.5

$p_1 \leqslant p_2 \leqslant 1$ $\quad 2^{n-1}$ $\quad 1 \leqslant p_1 \leqslant p_2$ $\quad \left(\frac{1}{2}\right)^{n+1}$

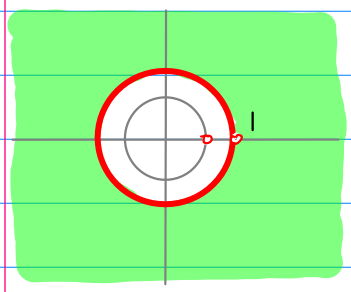


$n < 0$

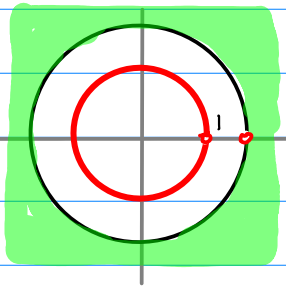


$n \leqslant 0$

$|z| < p_1$

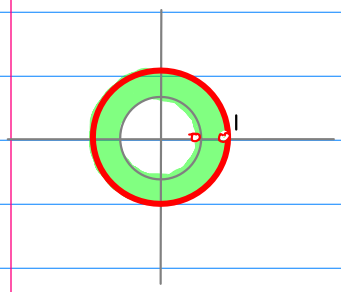


$n \geqslant 0$

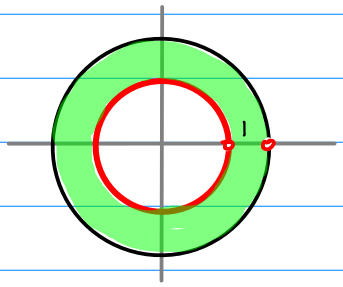


$n > 0$

$|z| > p_2$



$n \geqslant 0$
$n < 0$



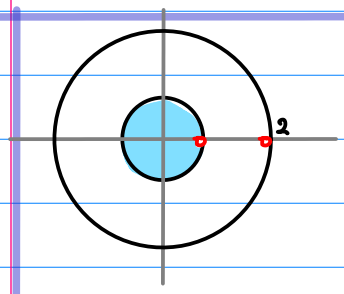
$n > 0$
$n \leqslant 0$

$p_1 < |z| < p_2$

Unit Circle $\in$ ROC $\rightarrow$ $\leqslant, \geqslant$

(A-1) $f(z)$, a_n

Ⅰ



$\left(\frac{1}{2}\right)^{n+1} - 1 \quad (n \geq 0)$

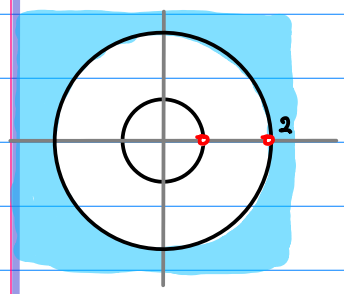
$0 \quad (n < 0)$

$p_1 = 1$, $p_2 = 2$

$\left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} \quad (n \geq 0)$

$0 \quad (n < 0)$

Ⅱ



$0 \quad (n \geq 0)$

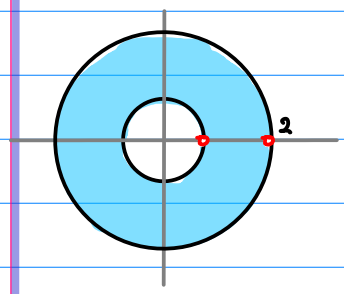
$1 - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$

$p_1 = 1$, $p_2 = 2$

$0 \quad (n \geq 0)$

$\left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} \quad (n < 0)$

Ⅲ



$\left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$

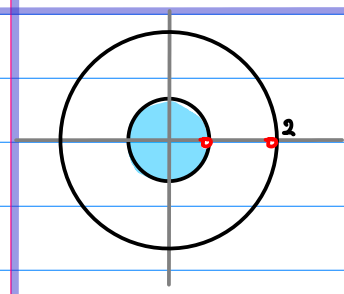
$1 \quad (n < 0)$

$p_1 = 1$, $p_2 = 2$

$\left(\frac{1}{p_2}\right)^{n+1} \quad (n \geq 0)$

$\left(\frac{1}{p_1}\right)^{n+1} \quad (n < 0)$

Ⅰ



$1 - 2^{n-1} \quad (n > 0)$

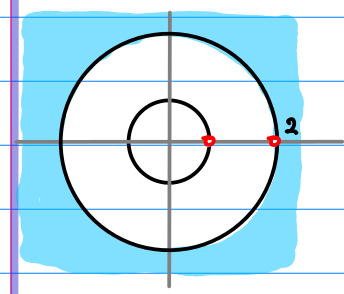
$0 \quad (n \leq 0)$

$p_1 = 0.5$, $p_2 = 1$

$\left(\frac{1}{p_2}\right)^{n-1} - \left(\frac{1}{p_1}\right)^{n-1} \quad (n > 0)$

$0 \quad (n \leq 0)$

Ⅱ



$0 \quad (n > 0)$

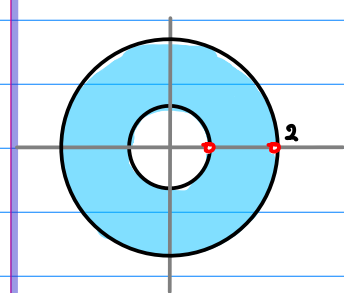
$2^{n-1} - 1 \quad (n \leq 0)$

$p_1 = 0.5$, $p_2 = 1$

$0 \quad (n > 0)$

$\left(\frac{1}{p_1}\right)^{n-1} - \left(\frac{1}{p_2}\right)^{n-1} \quad (n \leq 0)$

Ⅲ



$1 \quad (n > 0)$

$2^{n-1} \quad (n \leq 0)$

$p_1 = 0.5$, $p_2 = 1$

$\left(\frac{1}{p_2}\right)^{n-1} \quad (n > 0)$

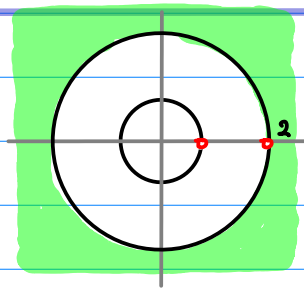
$\left(\frac{1}{p_1}\right)^{n-1} \quad (n \leq 0)$

(B-1) $f(z) \overset{Z^{-1}}{\longleftrightarrow} X(z)$ $\quad a_n = x_n$

(I)

$p_1 = 0.5$, $p_2 = 1$

$$\begin{matrix} (p_1)^{n+1} - (p_2)^{n+1} \\ 0 \end{matrix}$$

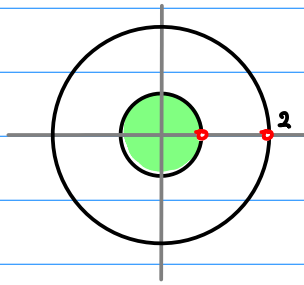


$$\begin{matrix} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{matrix}$$

(II)

$p_1 = 0.5$, $p_2 = 1$

$$\begin{matrix} 0 \\ (p_2)^{n+1} - (p_1)^{n+1} \end{matrix}$$

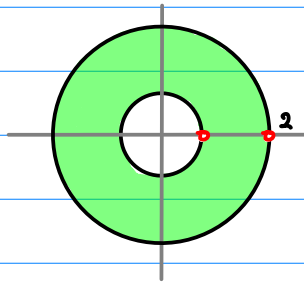


$$\begin{matrix} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{matrix}$$

(III)

$p_1 = 0.5$, $p_2 = 1$

$$\begin{matrix} (p_1)^{n+1} \\ (p_2)^{n+1} \end{matrix}$$

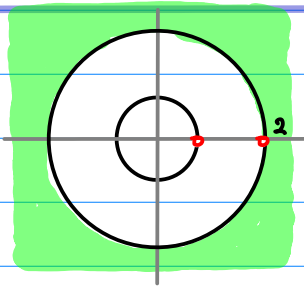


$$\begin{matrix} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{matrix}$$

(I)

$p_1 = 1$, $p_2 = 2$

$$\begin{matrix} (p_1)^{n-1} - (p_2)^{n-1} \\ 0 \end{matrix}$$

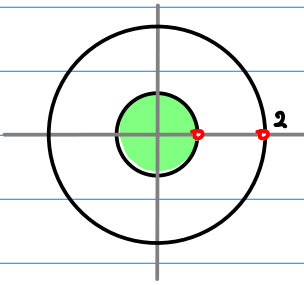


$$\begin{matrix} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{matrix}$$

(II)

$p_1 = 1$, $p_2 = 2$

$$\begin{matrix} 0 \\ (p_2)^{n-1} - (p_1)^{n+1} \end{matrix}$$

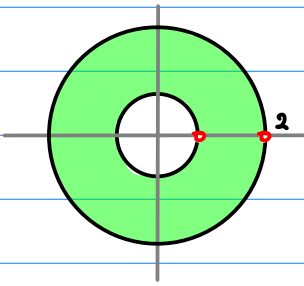


$$\begin{matrix} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{matrix}$$

(III)

$p_1 = 1$, $p_2 = 2$

$$\begin{matrix} (p_1)^{n-1} \\ (p_2)^{n-1} \end{matrix}$$



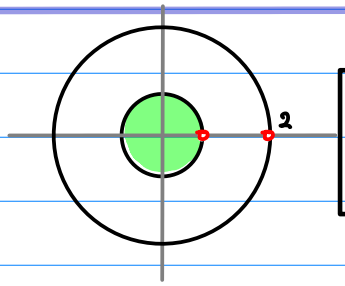
$$\begin{matrix} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{matrix}$$

(C-1) $f(z) = X(z)$ $\quad a_n \overset{-n}{\Longleftrightarrow} x_n$

(I)

$p_1 = 1$, $p_2 = 2$

$$\begin{matrix} 0 \\ (p_2)^{n-1} - (p_1)^{n-1} \end{matrix}$$

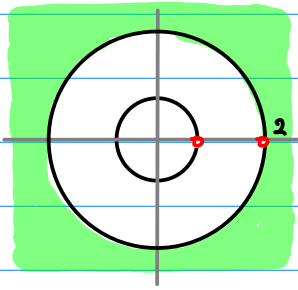


$$\begin{matrix} 0 & (n>0) \\ 2^{n-1} - 1 & (n \leq 0) \end{matrix}$$

(II)

$p_1 = 1$, $p_2 = 2$

$$\begin{matrix} (p_1)^{n-1} - (p_2)^{n-1} \\ 0 \end{matrix}$$

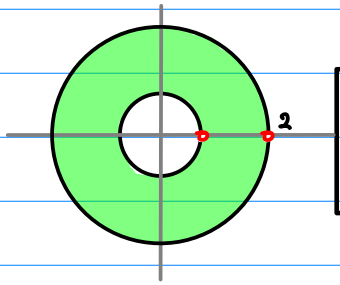


$$\begin{matrix} 1 - 2^{n-1} & (n>0) \\ 0 & (n \leq 0) \end{matrix}$$

(III)

$p_1 = 1$, $p_2 = 2$

$$\begin{matrix} (p_1)^{n-1} \\ (p_2)^{n-1} \end{matrix}$$

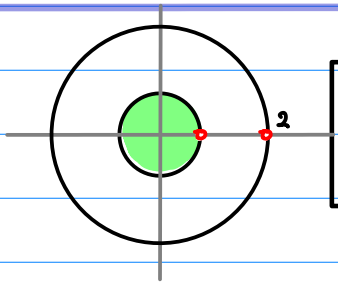


$$\begin{matrix} 1 & (n>0) \\ 2^{n-1} & (n \leq 0) \end{matrix}$$

(I)

$p_1 = 0.5$, $p_2 = 1$

$$\begin{matrix} 0 \\ (p_2)^{n+1} - (p_1)^{n+1} \end{matrix}$$

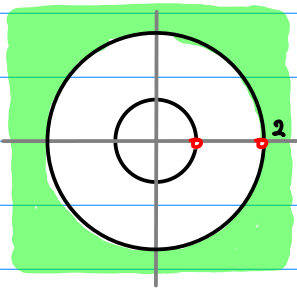


$$\begin{matrix} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n<0) \end{matrix}$$

(II)

$p_1 = 0.5$, $p_2 = 1$

$$\begin{matrix} (p_1)^{n+1} - (p_2)^{n+1} \\ 0 \end{matrix}$$

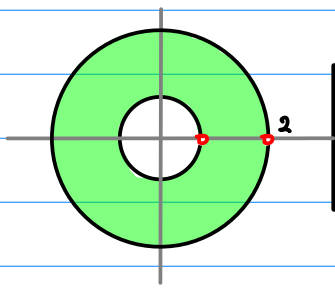


$$\begin{matrix} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n<0) \end{matrix}$$

(III)

$p_1 = 0.5$, $p_2 = 1$

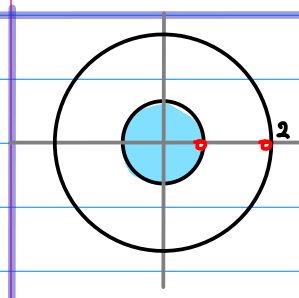
$$\begin{matrix} (p_1)^{n+1} \\ (p_2)^{n+1} \end{matrix}$$



$$\begin{matrix} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n<0) \end{matrix}$$

(A-2) $f(z) \overset{Z^{-1}}{\longleftrightarrow} X(z)$ $\qquad a_n = x_n$

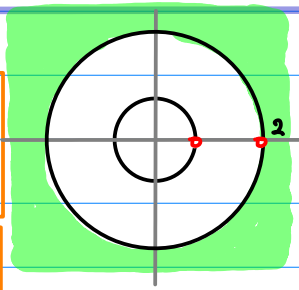
I



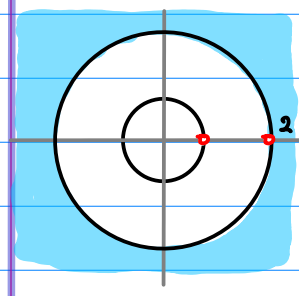
$\left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1}$ $(n \geq 0)$ $(n \geq 0)$ $\left(p_1\right)^{n+1} - \left(p_2\right)^{n+1}$

0 $(n < 0)$ $(n < 0)$ 0

$p_1 = 1$, $p_2 = 2$ $\qquad$ $p_1 = 0.5$, $p_2 = 1$



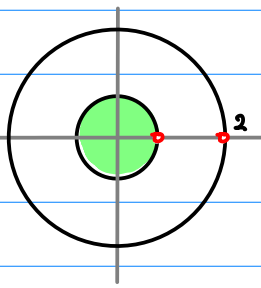
II



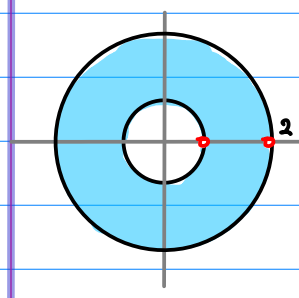
0 $(n \geq 0)$ $(n \geq 0)$ 0

$\left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1}$ $(n < 0)$ $(n < 0)$ $\left(p_2\right)^{n+1} - \left(p_1\right)^{n+1}$

$p_1 = 1$, $p_2 = 2$ $\qquad$ $p_1 = 0.5$, $p_2 = 1$



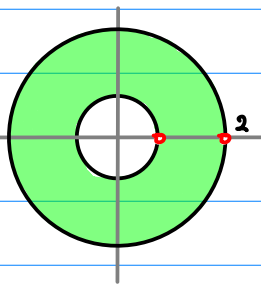
III



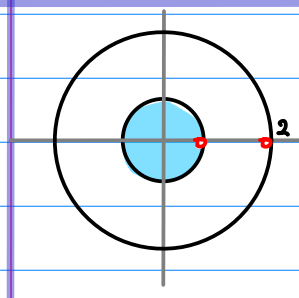
$\left(\frac{1}{p_2}\right)^{n+1}$ $(n \geq 0)$ $(n \geq 0)$ $\left(p_1\right)^{n+1}$

$\left(\frac{1}{p_1}\right)^{n+1}$ $(n < 0)$ $(n < 0)$ $\left(p_2\right)^{n+1}$

$p_1 = 1$, $p_2 = 2$ $\qquad$ $p_1 = 0.5$, $p_2 = 1$



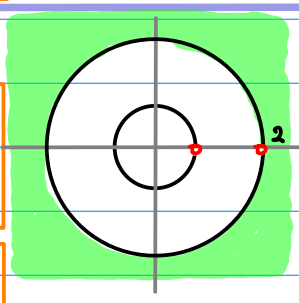
I



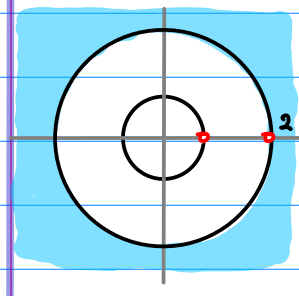
$\left(\frac{1}{p_2}\right)^{n-1} - \left(\frac{1}{p_1}\right)^{n-1}$ $(n > 0)$ $(n > 0)$ $\left(p_1\right)^{n-1} - \left(p_2\right)^{n-1}$

0 $(n \leq 0)$ $(n \leq 0)$ 0

$p_1 = 0.5$, $p_2 = 1$ $\qquad$ $p_1 = 1$, $p_2 = 2$



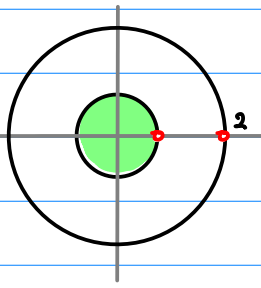
II



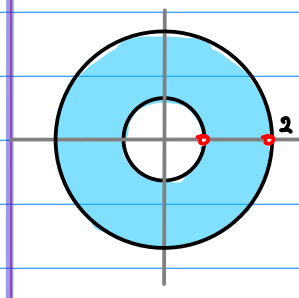
0 $(n > 0)$ $(n > 0)$ 0

$\left(\frac{1}{p_1}\right)^{n-1} - \left(\frac{1}{p_2}\right)^{n-1}$ $(n \leq 0)$ $(n \leq 0)$ $\left(p_2\right)^{n-1} - \left(p_1\right)^{n-1}$

$p_1 = 0.5$, $p_2 = 1$ $\qquad$ $p_1 = 1$, $p_2 = 2$



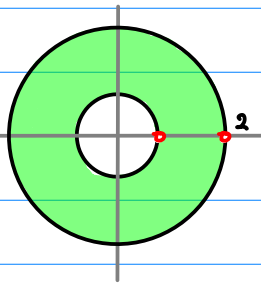
III



$\left(\frac{1}{p_2}\right)^{n-1}$ $(n > 0)$ $(n > 0)$ $\left(p_1\right)^{n-1}$

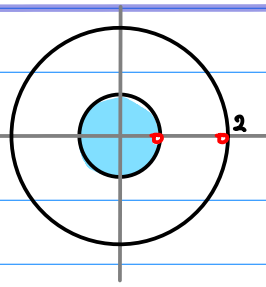
$\left(\frac{1}{p_1}\right)^{n-1}$ $(n \leq 0)$ $(n \leq 0)$ $\left(p_2\right)^{n-1}$

$p_1 = 0.5$, $p_2 = 1$ $\qquad$ $p_1 = 1$, $p_2 = 2$



(B-2) $f(z) = X(z)$ $a_n \overset{-n}{\Longleftrightarrow} x_n$

I



$\left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1}$ $(n \geq 0)$ $(n > 0)$

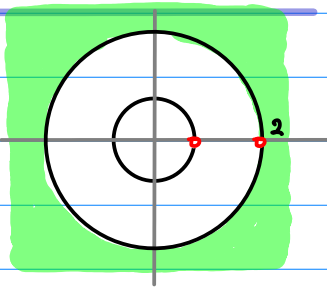
0 $(n < 0)$ $(n \leq 0)$

$p_1 = 1$
$p_2 = 2$

0

$\left(p_2\right)^{n-1} - \left(p_1\right)^{n-1}$

$p_1 = 1$
$p_2 = 2$



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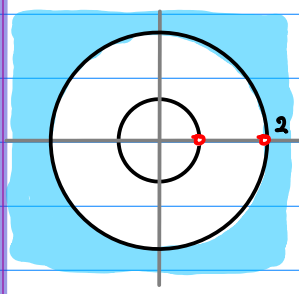


0 $(n \geq 0)$ $(n > 0)$

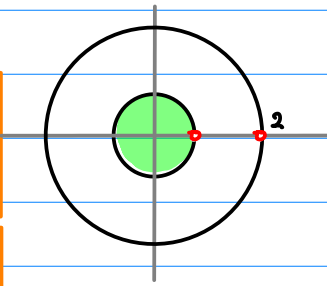
$\left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1}$ $(n < 0)$ $(n \leq 0)$

$p_1 = 1$
$p_2 = 2$

$\left(p_1\right)^{n-1} - \left(p_2\right)^{n-1}$

0

$p_1 = 1$
$p_2 = 2$



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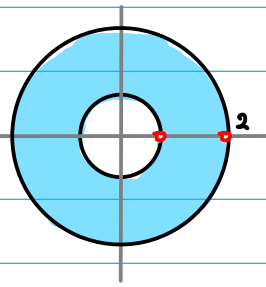


$\left(\frac{1}{p_2}\right)^{n+1}$ $(n \geq 0)$ $(n > 0)$

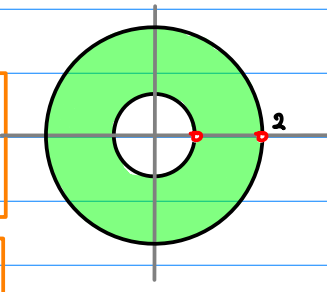
$\left(\frac{1}{p_1}\right)^{n+1}$ $(n < 0)$ $(n \leq 0)$

$p_1 = 1$
$p_2 = 2$

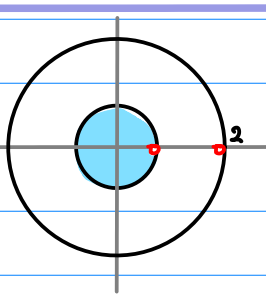
$\left(p_1\right)^{n-1}$

$\left(p_2\right)^{n-1}$

$p_1 = 1$
$p_2 = 2$



I



$\left(\frac{1}{p_2}\right)^{n-1} - \left(\frac{1}{p_1}\right)^{n-1}$ $(n > 0)$ $(n \geq 0)$

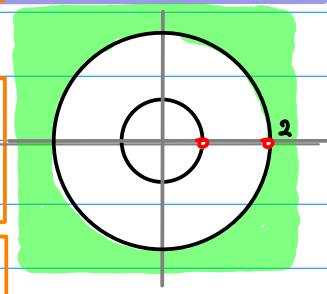
0 $(n \leq 0)$ $(n < 0)$

$p_1 = 0.5$
$p_2 = 1$

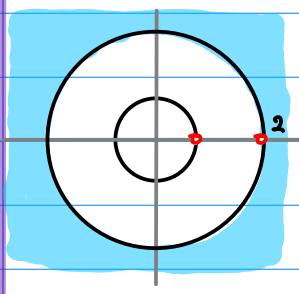
0

$\left(p_2\right)^{n+1} - \left(p_1\right)^{n+1}$

$p_1 = 0.5$
$p_2 = 1$



II



0 $(n > 0)$ $(n \geq 0)$

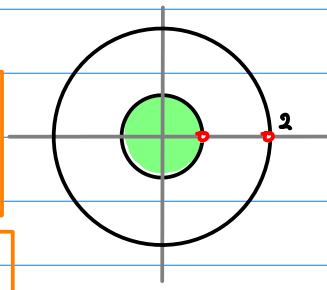
$\left(\frac{1}{p_1}\right)^{n-1} - \left(\frac{1}{p_2}\right)^{n-1}$ $(n \leq 0)$ $(n < 0)$

$p_1 = 0.5$
$p_2 = 1$

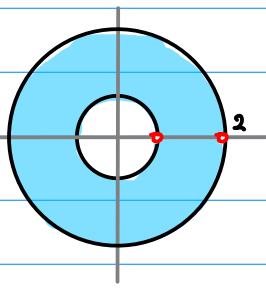
$\left(p_1\right)^{n+1} - \left(p_2\right)^{n+1}$

0

$p_1 = 0.5$
$p_2 = 1$



III



$\left(\frac{1}{p_2}\right)^{n-1}$ $(n > 0)$ $(n \geq 0)$

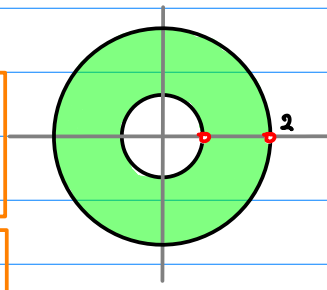
$\left(\frac{1}{p_1}\right)^{n-1}$ $(n \leq 0)$ $(n < 0)$

$p_1 = 0.5$
$p_2 = 1$

$\left(p_1\right)^{n+1}$

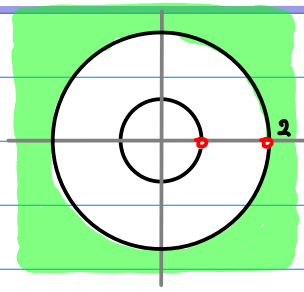
$\left(p_2\right)^{n+1}$

$p_1 = 0.5$
$p_2 = 1$

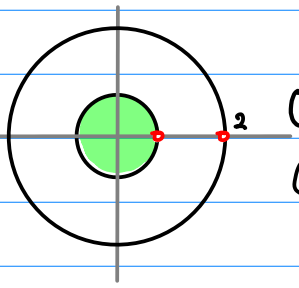


(C-2) $a_n = x_n$ $\quad a_n \overset{-n}{\longleftrightarrow} x_n$

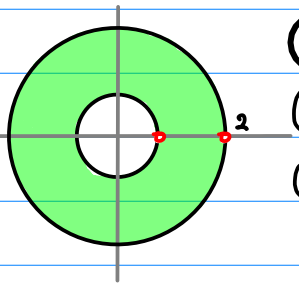
① $(n \geq 0)$ $(p_1)^{n+1} - (p_2)^{n+1}$ $(n < 0)$ 0 — $p_1 = 0.5$, $p_2 = 1$



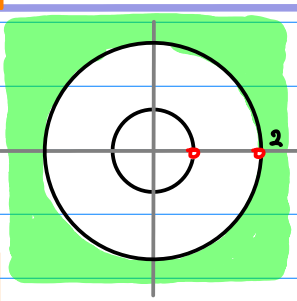
② $(n \geq 0)$ 0 $(n < 0)$ $(p_2)^{n+1} - (p_1)^{n+1}$ — $p_1 = 0.5$, $p_2 = 1$



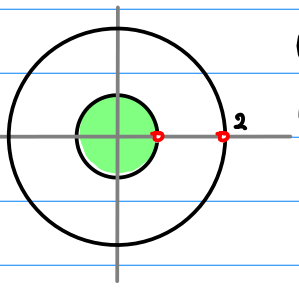
③ $(n \geq 0)$ $(p_1)^{n+1}$ $(n < 0)$ $(p_2)^{n+1}$ — $p_1 = 0.5$, $p_2 = 1$



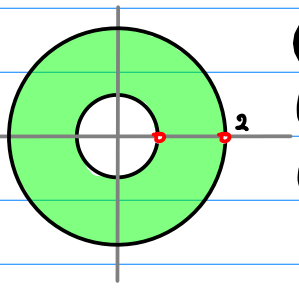
④ $(n > 0)$ $(p_1)^{n-1} - (p_2)^{n-1}$ $(n \leq 0)$ 0 — $p_1 = 1$, $p_2 = 2$



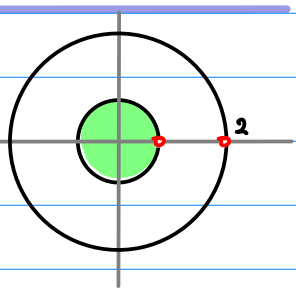
⑤ $(n > 0)$ 0 $(n \leq 0)$ $(p_2)^{n-1} - (p_1)^{n-1}$ — $p_1 = 1$, $p_2 = 2$



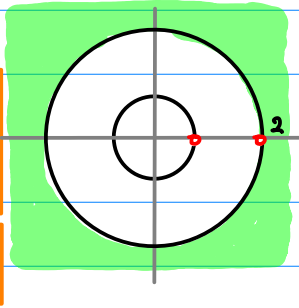
⑥ $(n > 0)$ $(p_1)^{n-1}$ $(n \leq 0)$ $(p_2)^{n-1}$ — $p_1 = 1$, $p_2 = 2$



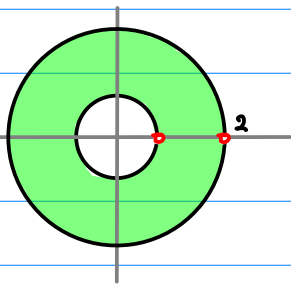
⑦ $(n > 0)$ 0 $(n \leq 0)$ $(p_2)^{n-1} - (p_1)^{n-1}$ — $p_1 = 1$, $p_2 = 2$



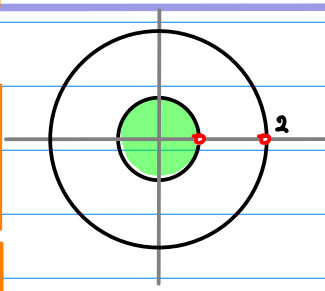
⑧ $(n > 0)$ $(p_1)^{n-1} - (p_2)^{n-1}$ $(n \leq 0)$ 0 — $p_1 = 1$, $p_2 = 2$



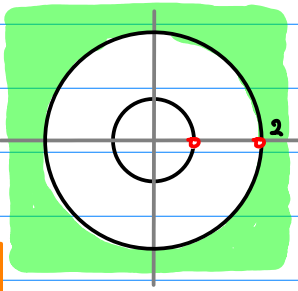
⑨ $(n > 0)$ $(p_1)^{n-1}$ $(n \leq 0)$ $(p_2)^{n-1}$ — $p_1 = 1$, $p_2 = 2$



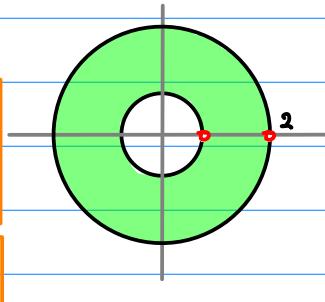
⑩ $(n \geq 0)$ 0 $(n < 0)$ $(p_2)^{n+1} - (p_1)^{n+1}$ — $p_1 = 0.5$, $p_2 = 1$



⑪ $(n \geq 0)$ $(p_1)^{n+1} - (p_2)^{n+1}$ $(n < 0)$ 0 — $p_1 = 0.5$, $p_2 = 1$



⑫ $(n \geq 0)$ $(p_1)^{n+1}$ $(n < 0)$ $(p_2)^{n+1}$ — $p_1 = 0.5$, $p_2 = 1$



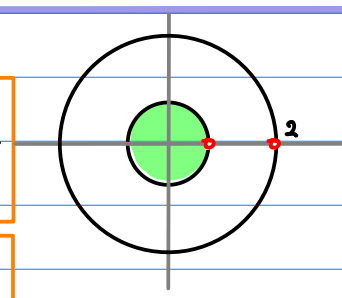
(C-3) $a_n = x_n$ $\qquad$ $a_n \overset{-n}{\longleftrightarrow} x_n$

2

$(n \geq 0)$ 0

$(n < 0)$ $(p_2)^{n+1} - (p_1)^{n+1}$

$p_1 = 0.5$
$p_2 = 1$

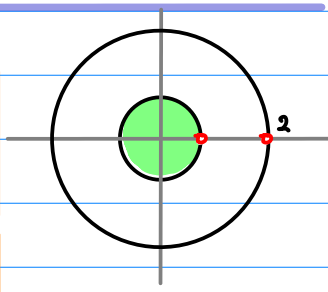


7

$(n > 0)$ 0

$n \leq 0)$ $(p_2)^{n-1} - (p_1)^{n-1}$

$p_1 = 1$
$p_2 = 2$

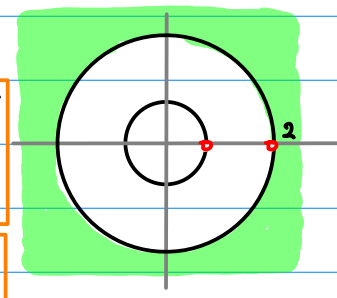


1

$(n \geq 0)$ $(p_1)^{n+1} - (p_2)^{n+1}$

$(n < 0)$ 0

$p_1 = 0.5$
$p_2 = 1$

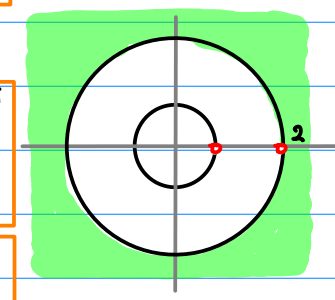


8

$(n > 0)$ $(p_1)^{n-1} - (p_2)^{n-1}$

$(n \leq 0)$ 0

$p_1 = 1$
$p_2 = 2$

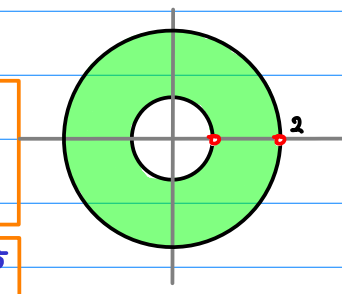


3

$(n \geq 0)$ $(p_1)^{n+1}$

$(n < 0)$ $(p_2)^{n+1}$

$p_1 = 0.5$
$p_2 = 1$

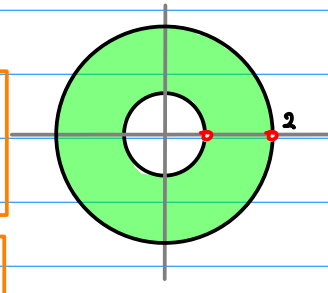


9

$(n > 0)$ $(p_1)^{n-1}$

$(n \leq 0)$ $(p_2)^{n-1}$

$p_1 = 1$
$p_2 = 2$

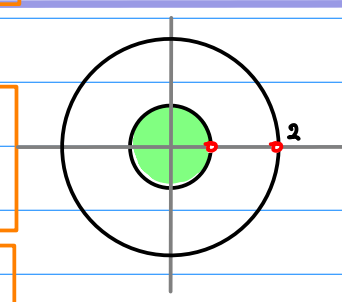


5

$(n > 0)$ 0

$(n \leq 0)$ $(p_2)^{n-1} - (p_1)^{n-1}$

$p_1 = 1$
$p_2 = 2$

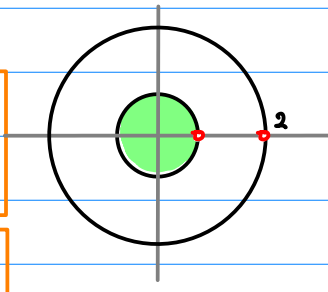


10

$(n \geq 0)$ 0

$(n < 0)$ $(p_2)^{n+1} - (p_1)^{n+1}$

$p_1 = 0.5$
$p_2 = 1$

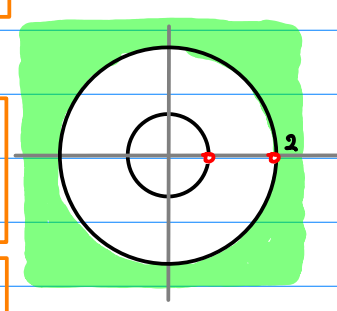


4

$(n > 0)$ $(p_1)^{n-1} - (p_2)^{n-1}$

$(n \leq 0)$ 0

$p_1 = 1$
$p_2 = 2$

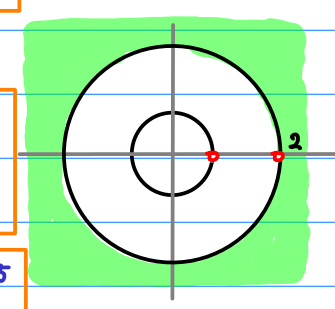


11

$(n \geq 0)$ $(p_1)^{n+1} - (p_2)^{n+1}$

$(n < 0)$ 0

$p_1 = 0.5$
$p_2 = 1$

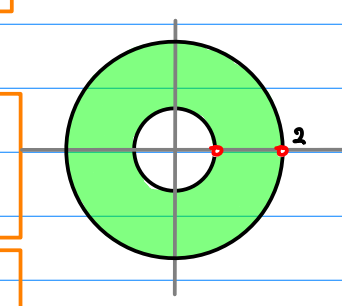


6

$(n > 0)$ $(p_1)^{n-1}$

$(n \leq 0)$ $(p_2)^{n-1}$

$p_1 = 1$
$p_2 = 2$

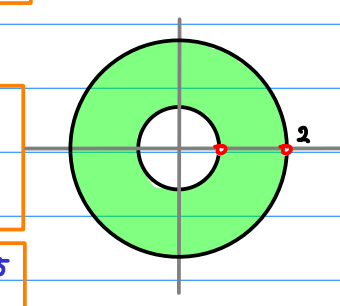


12

$(n \geq 0)$ $(p_1)^{n+1}$

$(n < 0)$ $(p_2)^{n+1}$

$p_1 = 0.5$
$p_2 = 1$



(C-4)

$n \geq 0$
$n < 0$

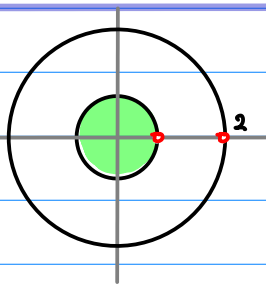
$n > 0$
$n \leq 0$

②

$(n \geq 0)$ 0

$(n < 0)$ $(p_2)^{n+1} - (p_1)^{n+1}$

$p_1 = 0.5$, $p_2 = 1$

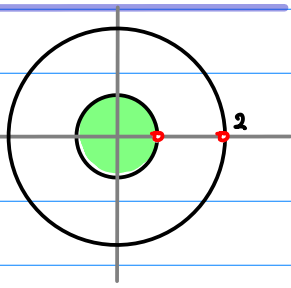


⑦

$(n > 0)$ 0

$(n \leq 0)$ $(p_2)^{n-1} - (p_1)^{n-1}$

$p_1 = 1$, $p_2 = 2$

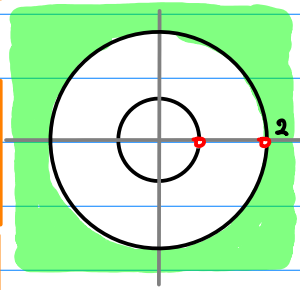


①

$(n \geq 0)$ $(p_1)^{n+1} - (p_2)^{n+1}$

$(n < 0)$ 0

$p_1 = 0.5$, $p_2 = 1$

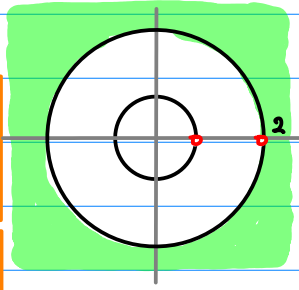


⑧

$(n > 0)$ $(p_1)^{n-1} - (p_2)^{n-1}$

$(n \leq 0)$ 0

$p_1 = 1$, $p_2 = 2$

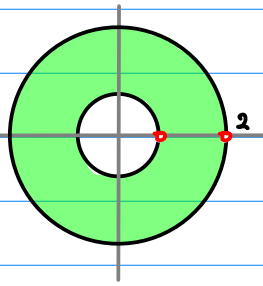


③

$(n \geq 0)$ $(p_1)^{n+1}$

$(n < 0)$ $(p_2)^{n+1}$

$p_1 = 0.5$, $p_p = 1$

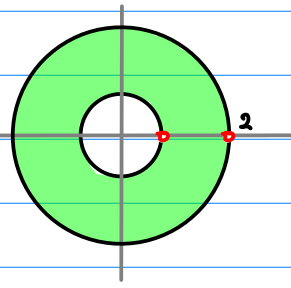


⑨

$(n > 0)$ $(p_1)^{n-1}$

$(n \leq 0)$ $(p_2)^{n-1}$

$p_1 = 1$, $p_{2a} = 2$

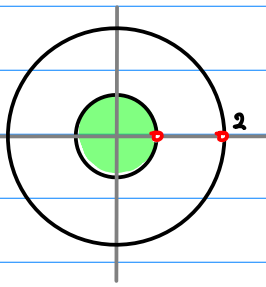


⑩

$(n \geq 0)$ 0

$(n < 0)$ $(p_2)^{n+1} - (p_1)^{n+1}$

$p_1 = 0.5$, $p_2 = 1$

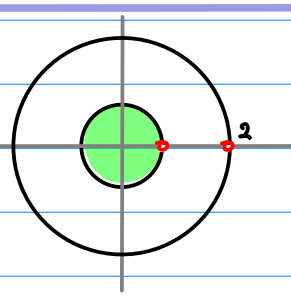


⑤

$n > 0)$ 0

$(n \leq 0)$ $(p_2)^{n-1} - (p_1)^{n-1}$

$p_1 = 1$, $p_2 = 2$

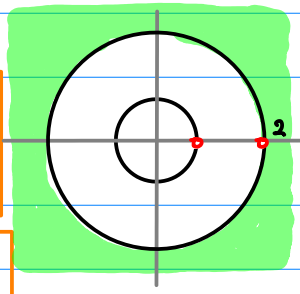


⑪

$(n \geq 0)$ $(p_1)^{n+1} - (p_2)^{n+1}$

$(n < 0)$ 0

$p_1 = 0.5$, $p_2 = 1$

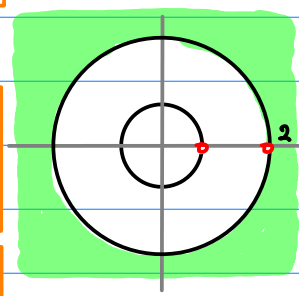


④

$(n > 0)$ $(p_1)^{n-1} - (p_2)^{n-1}$

$(n \leq 0)$ 0

$p_1 = 1$, $p_2 = 2$

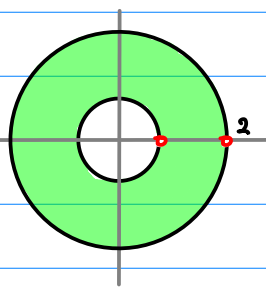


⑫

$(n \geq 0)$ $(p_1)^{n+1}$

$(n < 0)$ $(p_2)^{n+1}$

$p_1 = 0.5$, $p_2 = 1$

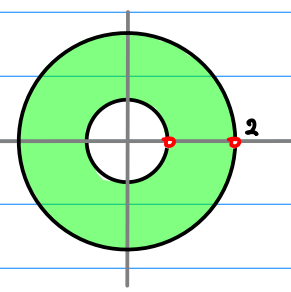


⑥

$(n > 0)$ $(p_1)^{n-1}$

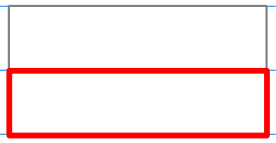
$(n \leq 0)$ $(p_2)^{n-1}$

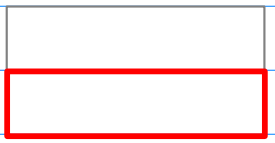
$p_1 = 1$, $p_2 = 2$

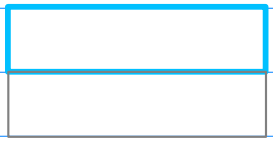


$n \geq 0$	$n > 0$
$n < 0$	$n \leq 0$
0	0
$(p_2)^{n+1} - (p_1)^{n+1}$	$(p_2)^{n-1} - (p_1)^{n-1}$
$(p_1)^{n+1} - (p_2)^{n+1}$	$(p_1)^{n-1} - (p_2)^{n-1}$
0	0
$(p_1)^{n+1}$	$(p_1)^{n-1}$
$(p_2)^{n+1}$	$(p_2)^{n-1}$
$p_1 = 0.5 \quad p = 1$	$p_1 = 1 \quad p_2 = 2$
$(\ )^{n+1}$	$(\ )^{n-1}$

$n \geq 0$	$n > 0$
$n < 0$	$n \leq 0$
$p_1 = 0.5$, $p = 1$	$p_1 = 1$, $p_2 = 2$
$(p_1)^{n+1}$, $(p_2)^{n+1}$	$(p_1)^{n-1}$, $(p_2)^{n-1}$

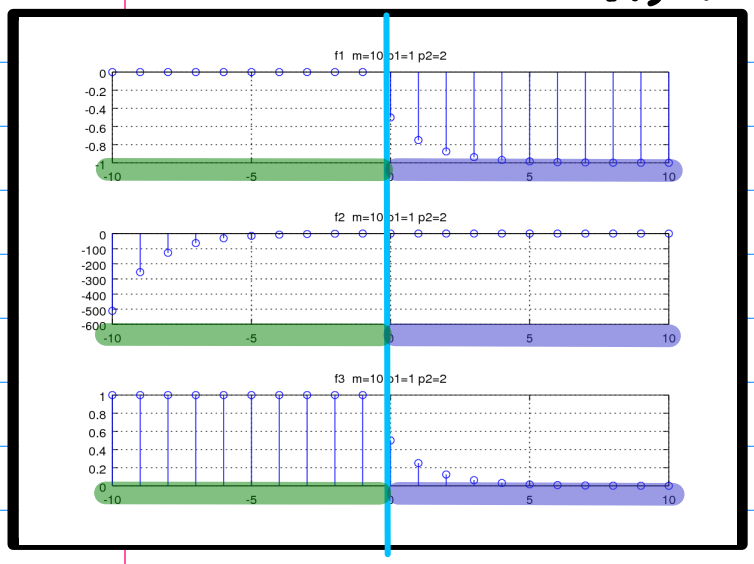






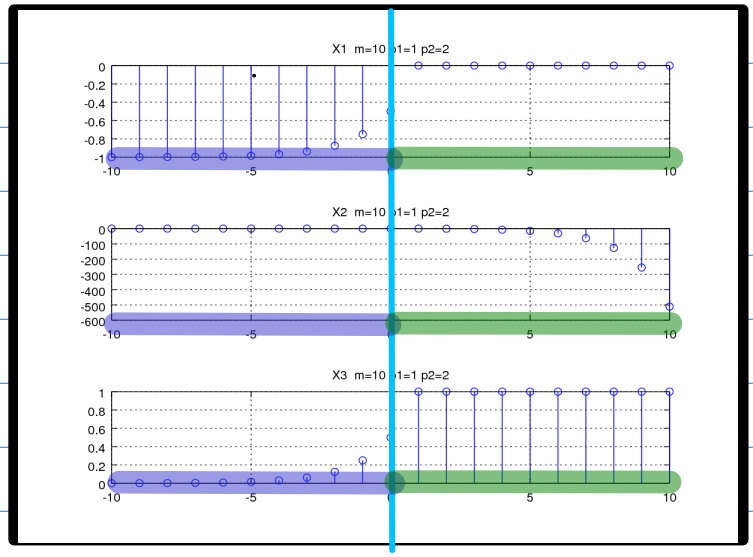
f(z)

(1, 2)

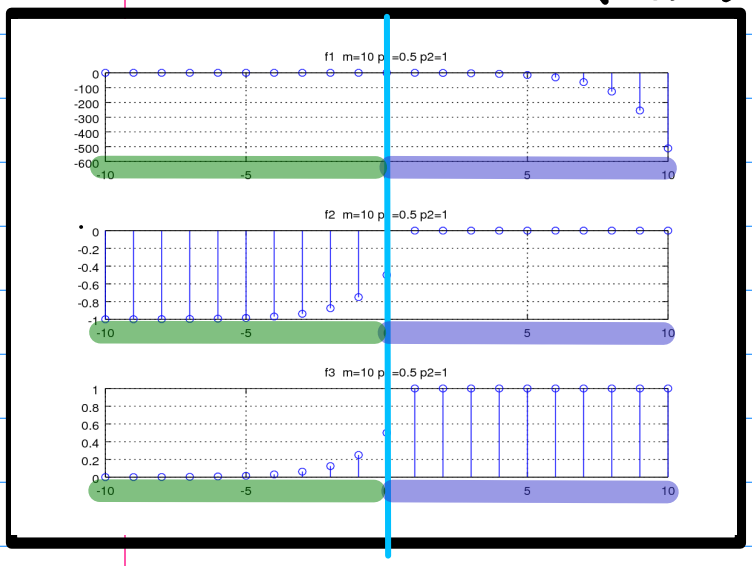


X(z)

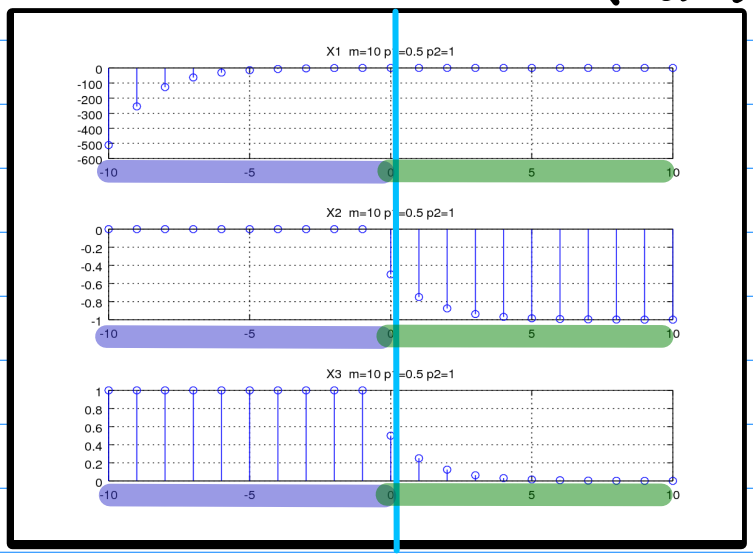
(1, 2)



(0.5, 1)



(0.5, 1)



$\left(\frac{1}{2}\right)^{n+1} - 1 \quad (n \geq 0)$

$0 \quad (n < 0)$

$0 \quad (n \geq 0)$

$1 - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$

$\left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$

$1 \quad (n < 0)$

$0 \quad (n > 0)$

$2^{n-1} - 1 \quad (n \leq 0)$

$1 - 2^{n-1} \quad (n > 0)$

$0 \quad (n \leq 0)$

$1 \quad (n > 0)$

$2^{n-1} \quad (n \leq 0)$

$0 \quad \left(\frac{1}{2}\right)^{n+1} - 1$

$1 - \left(\frac{1}{2}\right)^{n+1} \quad 0$

$1 \quad \left(\frac{1}{2}\right)^{n+1}$

$2^{n-1} - 1 \quad 0$

$0 \quad 1 - 2^{n-1}$

$2^{n-1} \quad 1$

$1 - 2^{n-1} \quad (n > 0)$

$0 \quad (n \leq 0)$

$0 \quad (n \geq 0)$

$1 - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$

$0 \quad (n > 0)$

$2^{n-1} - 1 \quad (n \leq 0)$

$\left(\frac{1}{2}\right)^{n+1} - 1 \quad (n \geq 0)$

$0 \quad (n < 0)$

$1 \quad (n > 0)$

$2^{n-1} \quad (n \leq 0)$

$\left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$

$1 \quad (n < 0)$

$0 \quad 1 - 2^{n-1}$

$2^{n-1} - 1 \quad 0$

$2^{n-1} \quad 1$

$1 - \left(\frac{1}{2}\right)^{n+1} \quad 0$

$0 \quad \left(\frac{1}{2}\right)^{n+1} - 1$

$1 \quad \left(\frac{1}{2}\right)^{n+1}$

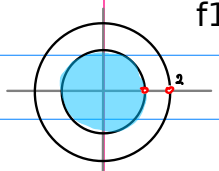
plotseq1(m, p1, p2) : Laurent

[-m, +m]

```
% Laurent Series and sequences
function plotseq1(m=1, p1=2, p2=2.1)


if (p1 >=1  && p2 >= 1 && p1 < p2)
  t1n = -m: -1;
  t1p =  0: m;
  t1 = [t1n, t1p];
  f1 = [zeros(1,m), ((1/p2).^(t1p+1) - (1/p1).^(t1p+1))];
  f2 = [((1/p1).^(t1n+1) -(1/p2).^(t1n+1)), zeros(1,m+1)];
  f3 = [(1/p1).^(t1n+1), (1/p2).^(t1p+1)];
endif

if (p1 <= 1 && p2 <= 1 && p1 < p2)
  t1n = -m: 0;
  t1p = +1: m;
  t1 = [t1n, t1p];
  f1 = [zeros(1,m+1), ((1/p2).^(t1p-1) - (1/p1).^(t1p-1))];
  f2 = [((1/p1).^(t1n-1) -(1/p2).^(t1n-1)), zeros(1,m)];
  f3 = [(1/p1).^(t1n-1), (1/p2).^(t1p-1)];
endif

subplot(3, 1, 1);
stem(t1, f1);
grid on
%axis([0, m])
title(sprintf("f1  m=%d p1=%g p2=%g", m, p1, p2))

subplot(3, 1, 2);
stem(t1, f2);
grid on
%axis([0, m])
title(sprintf("f2  m=%d p1=%g p2=%g", m, p1, p2))

subplot(3, 1, 3);
stem(t1, f3);
grid on
%axis([0, m])
title(sprintf("f3  m=%d p1=%g p2=%g", m, p1, p2))

endfunction
```

f(z) Laurent

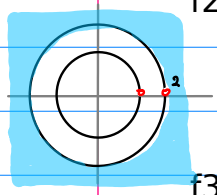
```
if (p1 >=1  && p2 >= 1 && p1 < p2)
  t1n = -m: -1;
  t1p =  0: m;
  t1 = [t1n, t1p];

  f1 = [zeros(1,m), ((1/p2).^(t1p+1) - (1/p1).^(t1p+1))];
```



$$\begin{cases} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} \\ 0 \end{cases}$$

$$\begin{matrix} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} & (n \geq 0) \\ 0 & (n < 0) \end{matrix}$$

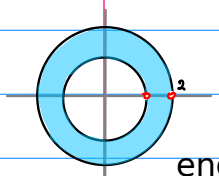
```
  f2 = [((1/p1).^(t1n+1) -(1/p2).^(t1n+1)), zeros(1,m+1)];
```



$$\begin{cases} 0 \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} \end{cases}$$

$$\begin{matrix} 0 & (n \geq 0) \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} & (n < 0) \end{matrix}$$

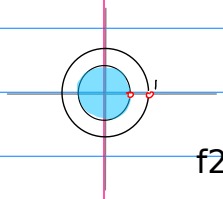
```
  f3 = [(1/p1).^(t1n+1), (1/p2).^(t1p+1)];
```



$$\begin{cases} \left(\frac{1}{p_2}\right)^{n+1} \\ \left(\frac{1}{p_1}\right)^{n+1} \end{cases}$$

$$\begin{matrix} \left(\frac{1}{p_2}\right)^{n+1} & (n \geq 0) \\ \left(\frac{1}{p_1}\right)^{n+1} & (n < 0) \end{matrix}$$

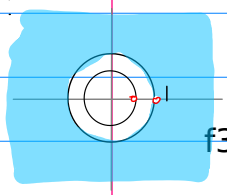
```
endif

if (p1 <= 1 && p2 <= 1 && p1 < p2)
  t1n = -m: 0;
  t1p = +1: m;
  t1 = [t1n, t1p];

  f1 = [zeros(1,m+1), ((1/p2).^(t1p-1) - (1/p1).^(t1p-1))];
```



$$\begin{cases} \left(\frac{1}{p_2}\right)^{n-1} - \left(\frac{1}{p_1}\right)^{n-1} \\ 0 \end{cases}$$

$$\begin{matrix} \left(\frac{1}{p_2}\right)^{n-1} - \left(\frac{1}{p_1}\right)^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{matrix}$$

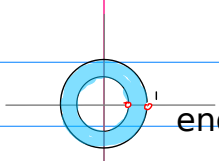
```
  f2 = [((1/p1).^(t1n-1) -(1/p2).^(t1n-1)), zeros(1,m)];
```



$$\begin{cases} 0 \\ \left(\frac{1}{p_1}\right)^{n-1} - \left(\frac{1}{p_2}\right)^{n-1} \end{cases}$$

$$\begin{matrix} 0 & (n > 0) \\ \left(\frac{1}{p_1}\right)^{n-1} - \left(\frac{1}{p_2}\right)^{n-1} & (n \leq 0) \end{matrix}$$

```
  f3 = [(1/p1).^(t1n-1), (1/p2).^(t1p-1)];
```



$$\begin{cases} \left(\frac{1}{p_2}\right)^{n-1} \\ \left(\frac{1}{p_1}\right)^{n-1} \end{cases}$$

$$\begin{matrix} \left(\frac{1}{p_2}\right)^{n-1} & (n > 0) \\ \left(\frac{1}{p_1}\right)^{n-1} & (n \leq 0) \end{matrix}$$

```
endif
```

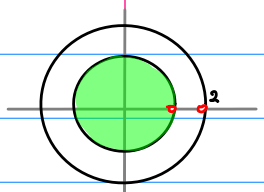
plotseq2(m, p1, p2) : Z-trans

[-m, +m]

```
% z-Transform and sequences
function plotseq2(m=1, p1=2, p2=2.1)

cla;

if (p1 <= 1 && p2 <= 1 && p1 < p2)
  t1n = -m: -1;
  t1p =  0: m;
  t1 = [t1n, t1p];
  X1 = [((p2).^(t1n+1) -(p1).^(t1n+1)), zeros(1,m+1)];
  X2 = [zeros(1,m), ((p1).^(t1p+1) - (p2).^(t1p+1))];
  X3 = [(p2).^(t1n+1), (p1).^(t1p+1)];
endif

if (p1 >= 1 && p2 >= 1 && p1 < p2)
  t1n = -m: 0;
  t1p = +1: m;
  t1 = [t1n, t1p];
  X1 = [((p2).^(t1n-1) -(p1).^(t1n-1)), zeros(1,m)];
  X2 = [zeros(1,m+1), ((p1).^(t1p-1) - (p2).^(t1p-1))];
  X3 = [(p2).^(t1n-1), (p1).^(t1p-1)];
endif


subplot(3, 1, 1);
stem(t1, X1);
grid on
%axis([0, m])
title(sprintf("X1  m=%d p1=%g p2=%g", m, p1, p2))

subplot(3, 1, 2);
stem(t1, X2);
grid on
%axis([0, m])
title(sprintf("X2  m=%d p1=%g p2=%g", m, p1, p2))

subplot(3, 1, 3);
stem(t1, X3);
grid on
%axis([0, m])
title(sprintf("X3  m=%d p1=%g p2=%g", m, p1, p2))

endfunction
```

X(z) z-Trans

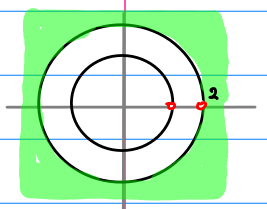
```
if (p1 <= 1 && p2 <= 1 && p1 < p2)
  t1n = -m: -1;
  t1p =  0: m;
  t1 = [t1n, t1p];

  X1 = [zeros(1,m), ((p1).^(t1p+1) - (p2).^(t1p+1))];
```



$$\begin{cases} 0 \\ (p_2)^{n+1} - (p_1)^{n+1} \end{cases}$$

$$\begin{cases} (n>0) & 0 \\ (n\leq 0) & (p_2)^{n-1} - (p_1)^{n-1} \end{cases}$$

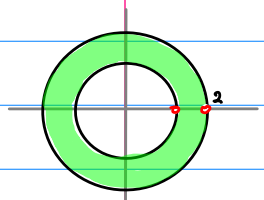
```
  X2 = [((p2).^(t1n+1) -(p1).^(t1n+1)), zeros(1,m+1)];
```



$$\begin{cases} (p_1)^{n+1} - (p_2)^{n+1} \\ 0 \end{cases}$$

$$\begin{cases} (n>0) & (p_1)^{n-1} - (p_2)^{n-1} \\ (n\leq 0) & 0 \end{cases}$$

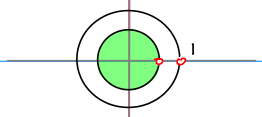
```
  X3 = [(p2).^(t1n+1), (p1).^(t1p+1)];
```



$$\begin{cases} (p_1)^{n+1} \\ (p_2)^{n+1} \end{cases}$$

$$\begin{cases} (n>0) & (p_1)^{n-1} \\ (n\leq 0) & (p_2)^{n-1} \end{cases}$$

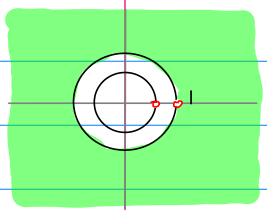
```
endif

if (p1 >= 1 && p2 >= 1 && p1 < p2)
  t1n = -m: 0;
  t1p = +1: m;
  t1 = [t1n, t1p];

  X1 = [((p2).^(t1n-1) -(p1).^(t1n-1)), zeros(1,m)];
```



$$\begin{cases} 0 \\ (p_2)^{n-1} - (p_1)^{n-1} \end{cases}$$

$$\begin{cases} (n\geq 0) & 0 \\ (n<0) & (p_2)^{n+1} - (p_1)^{n+1} \end{cases}$$

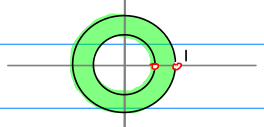
```
  X2 = [zeros(1,m+1), ((p1).^(t1p-1) - (p2).^(t1p-1))];
```



$$\begin{cases} (p_1)^{n-1} - (p_2)^{n-1} \\ 0 \end{cases}$$

$$\begin{cases} (n\geq 0) & (p_1)^{n+1} - (p_2)^{n+1} \\ (n<0) & 0 \end{cases}$$

```
  X3 = [(p2).^(t1n-1), (p1).^(t1p-1)];
```

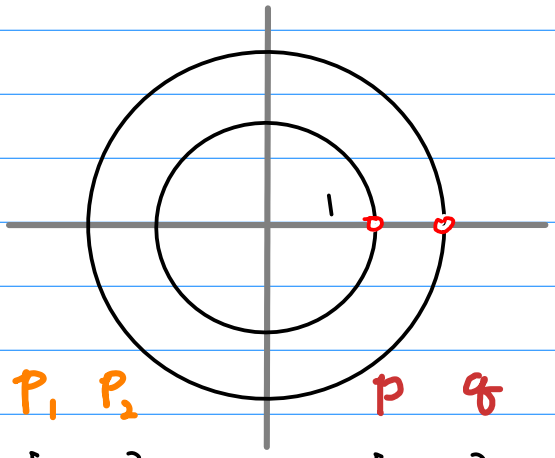


$$\begin{cases} (p_1)^{n-1} \\ (p_2)^{n-1} \end{cases}$$

$$\begin{cases} (n\geq 0) & (p_1)^{n+1} \\ (n<0) & (p_2)^{n+1} \end{cases}$$

```
endif
```

$f(z)$ $X(z)$



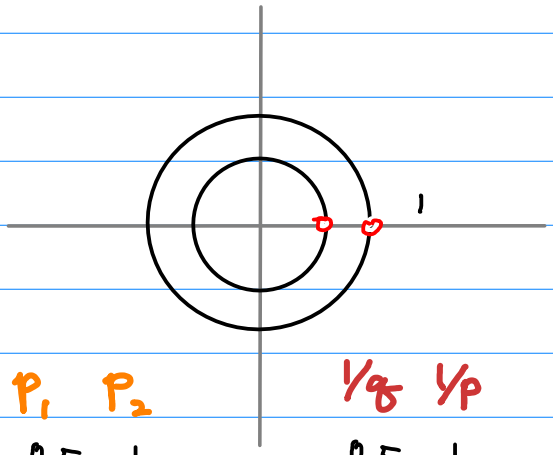
p_1	p_2
1	2

$\frac{1}{p_1}$	$\frac{1}{p_2}$
1	0.5

$\left(\frac{1}{2}\right)^{n+1}$

p	q
1	2

$\frac{1}{p}$	$\frac{1}{q}$
1	0.5



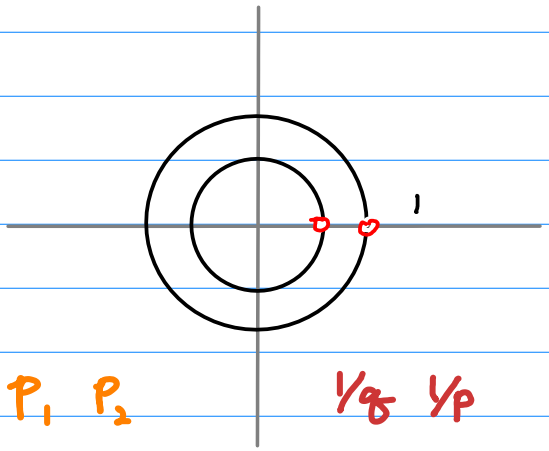
p_1	p_2
0.5	1

p_1	p_2
0.5	1

$1/q$	$1/p$
0.5	1

$\frac{1}{q}$	$\frac{1}{p}$
0.5	1

$\left(\frac{1}{2}\right)^{n+1}$



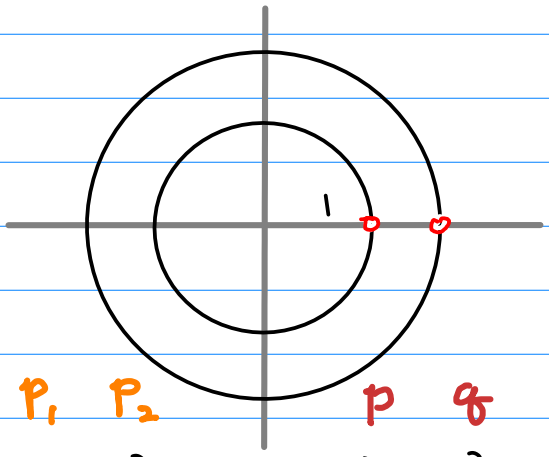
p_1	p_2
0.5	1

$\frac{1}{p_1}$	$\frac{1}{p_2}$
2	1

2^{n-1}

$1/q$	$1/p$
0.5	1

q	p
2	1



p_1	p_2
1	2

p_1	p_2
1	2

p	q
1	2

p	q
1	2

2^{n-1}

relative values

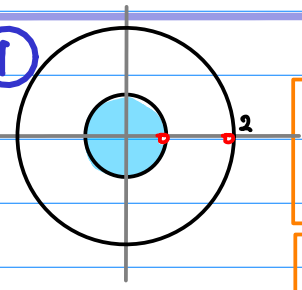
$p_1 < p_2$

fixed values

$p = 0.5$ $q = 2$

I

1



$\left(\frac{1}{q}\right)^{n+1} - \left(\frac{1}{p}\right)^{n+1}$ $(n \geq 0)$ $(n > 0)$

0 $(n < 0)$ $(n \leq 0)$

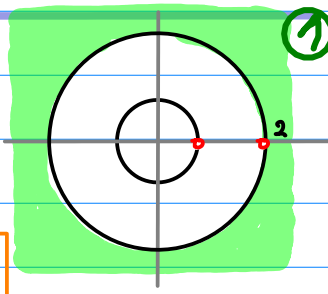
$p_1 = 1 = p$
$p_2 = 2 = q$

0

$\left(\frac{1}{q}\right)^{n+1} - \left(\frac{1}{p}\right)^{n+1}$

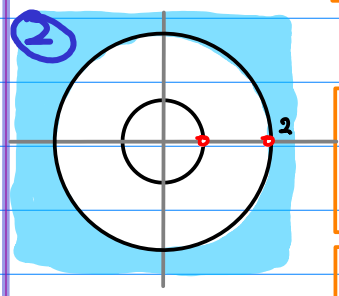
$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

7



II

2



0 $(n \geq 0)$ $(n > 0)$

$\left(\frac{1}{p}\right)^{n+1} - \left(\frac{1}{q}\right)^{n+1}$ $(n < 0)$ $(n \leq 0)$

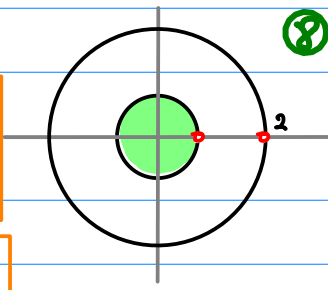
$p_1 = 1 = p$
$p_2 = 2 = q$

$\left(\frac{1}{p}\right)^{n+1} - \left(\frac{1}{q}\right)^{n+1}$

0

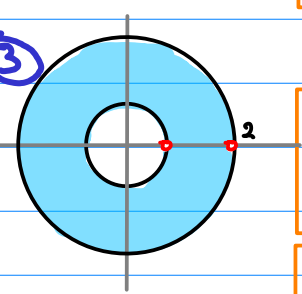
$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

8



III

3



$\left(\frac{1}{q}\right)^{n+1}$ $(n \geq 0)$ $(n > 0)$

$\left(\frac{1}{p}\right)^{n+1}$ $(n < 0)$ $(n \leq 0)$

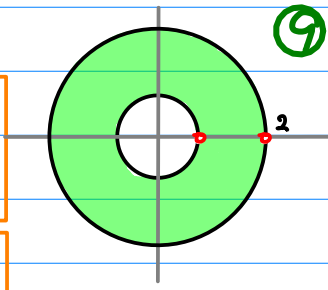
$p_1 = 1 = p$
$p_2 = 2 = q$

$\left(\frac{1}{q}\right)^{n+1}$

$\left(\frac{1}{p}\right)^{n+1}$

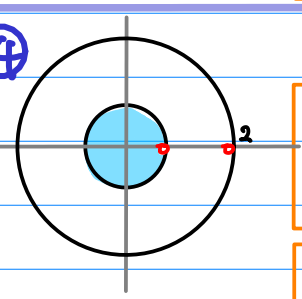
$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

9



I

4



$(p)^{n-1} - (q)^{n-1}$ $(n > 0)$ $(n \geq 0)$

0 $(n \leq 0)$ $(n < 0)$

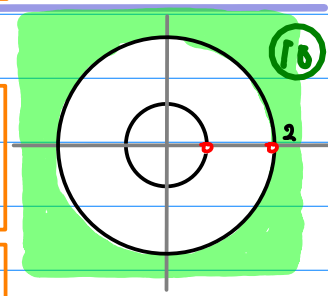
$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

0

$(p)^{n-1} - (q)^{n-1}$

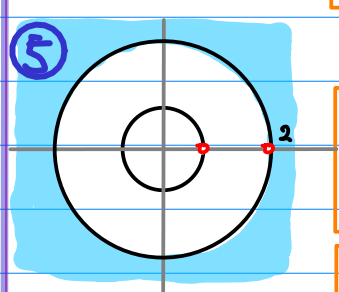
$p_1 = 1 = p$
$p_2 = 2 = q$

10



II

5



0 $(n > 0)$ $(n \geq 0)$

$(q)^{n-1} - (p)^{n-1}$ $(n \leq 0)$ $(n < 0)$

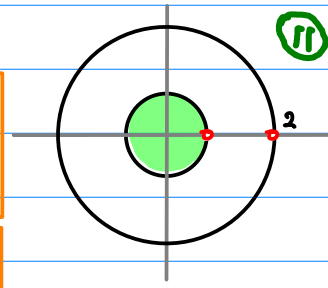
$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

$(q)^{n-1} - (p)^{n-1}$

0

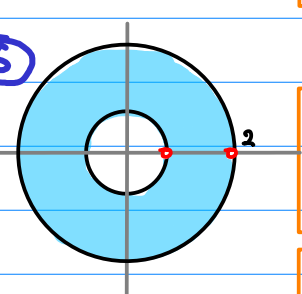
$p_1 = 1 = p$
$p_2 = 2 = q$

11



III

6



$(p)^{n-1}$ $(n > 0)$ $(n \geq 0)$

$(q)^{n-1}$ $(n \leq 0)$ $(n < 0)$

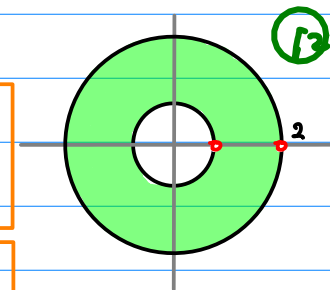
$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

$(p)^{n-1}$

$(q)^{n-1}$

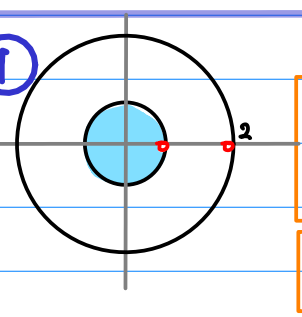
$p_1 = 1 = p$
$p_2 = 2 = q$

13



I

1



$\left(\frac{1}{q}\right)^{n+1} - \left(\frac{1}{p}\right)^{n+1}$ $(n \geq 0)$ $(n > 0)$

0 $(n < 0)$ $(n \leq 0)$

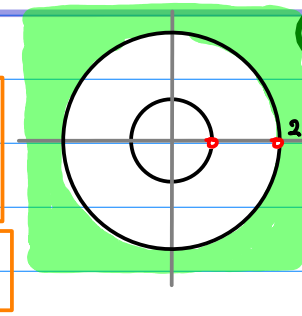
$p_1 = 1 = p$
$p_2 = 2 = q$

0

$\left(\frac{1}{q}\right)^{n+1} - \left(\frac{1}{p}\right)^{n+1}$

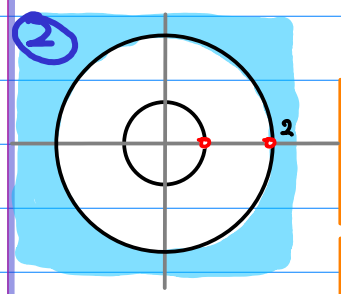
$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

7



II

2



0 $(n \geq 0)$ $(n > 0)$

$\left(\frac{1}{p}\right)^{n+1} - \left(\frac{1}{q}\right)^{n+1}$ $(n < 0)$ $(n \leq 0)$

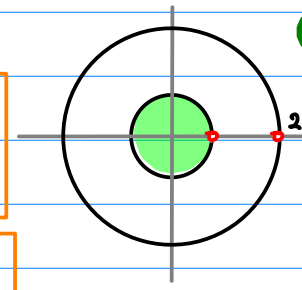
$p_1 = 1 = p$
$p_2 = 2 = q$

$\left(\frac{1}{p}\right)^{n+1} - \left(\frac{1}{q}\right)^{n+1}$

0

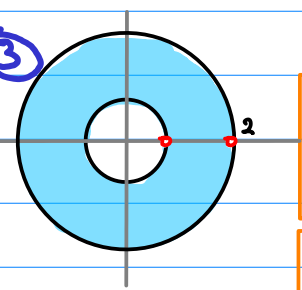
$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

8



III

3



$\left(\frac{1}{q}\right)^{n+1}$ $(n \geq 0)$ $(n > 0)$

$\left(\frac{1}{p}\right)^{n+1}$ $(n < 0)$ $(n \leq 0)$

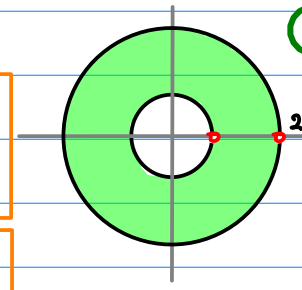
$p_1 = 1 = p$
$p_2 = 2 = q$

$\left(\frac{1}{q}\right)^{n+1}$

$\left(\frac{1}{p}\right)^{n+1}$

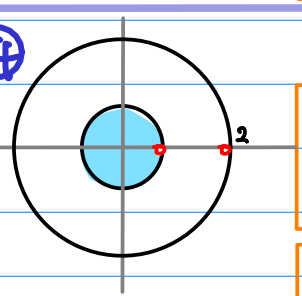
$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

9



I

4



$(p)^{n-1} - (q)^{n-1}$ $(n > 0)$ $(n \geq 0)$

0 $(n \leq 0)$ $(n < 0)$

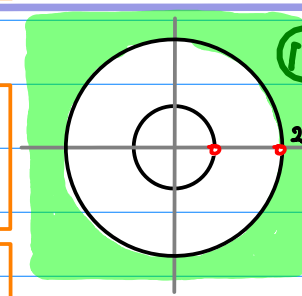
$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

0

$(p)^{n-1} - (q)^{n-1}$

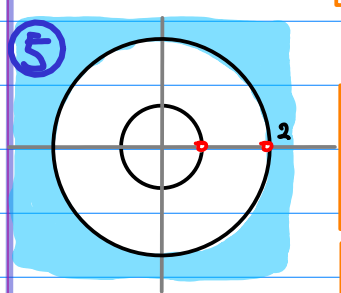
$p_1 = 1 = p$
$p_2 = 2 = q$

10



II

5



0 $(n > 0)$ $(n \geq 0)$

$(q)^{n-1} - (p)^{n-1}$ $(n \leq 0)$ $(n < 0)$

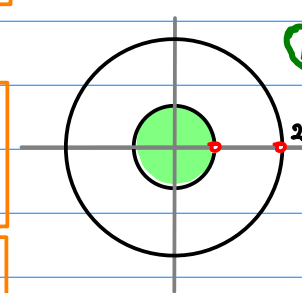
$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

$(q)^{n-1} - (p)^{n-1}$

0

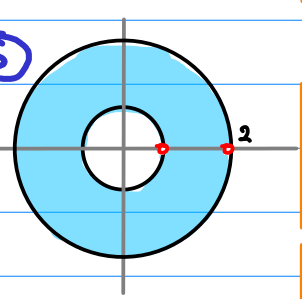
$p_1 = 1 = p$
$p_2 = 2 = q$

11



III

6



$(p)^{n-1}$ $(n > 0)$ $(n \geq 0)$

$(q)^{n-1}$ $(n \leq 0)$ $(n < 0)$

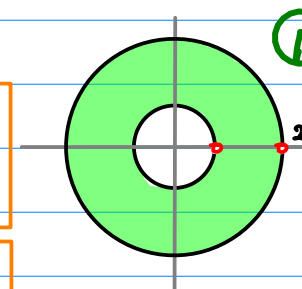
$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

$(p)^{n-1}$

$(q)^{n-1}$

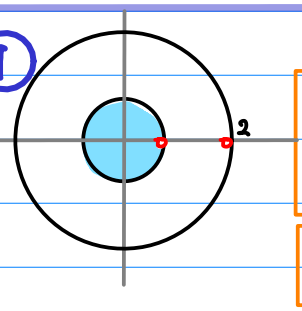
$p_1 = 1 = p$
$p_2 = 2 = q$

12



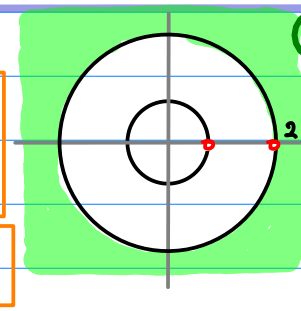
$$f(z) \xleftrightarrow{z^{-1}} X(z) \qquad a_n = x_n$$

I ①

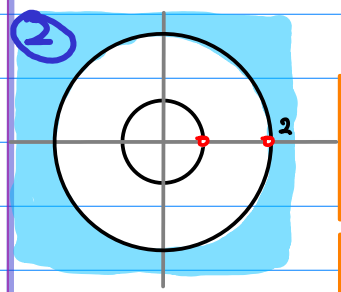


$\left(\frac{1}{q}\right)^{n+1} - \left(\frac{1}{p}\right)^{n+1}$	$(n \geq 0)$ $(n > 0)$	0
0	$(n < 0)$ $(n \leq 0)$	$\left(\frac{1}{q}\right)^{n+1} - \left(\frac{1}{p}\right)^{n+1}$
$p_1 = 1 = p$, $p_2 = 2 = q$		$p_1 = 0.5 = 1/q$, $p_2 = 1 = 1/p$

⑦

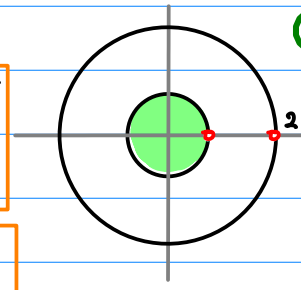


II ②

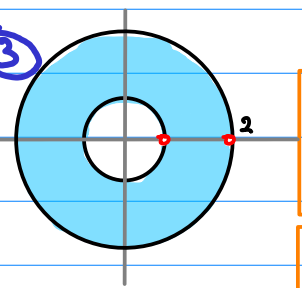


0	$(n \geq 0)$ $(n > 0)$	$\left(\frac{1}{p}\right)^{n+1} - \left(\frac{1}{q}\right)^{n+1}$
$\left(\frac{1}{p}\right)^{n+1} - \left(\frac{1}{q}\right)^{n+1}$	$(n < 0)$ $(n \leq 0)$	0
$p_1 = 1 = p$, $p_2 = 2 = q$		$p_1 = 0.5 = 1/q$, $p_2 = 1 = 1/p$

⑧

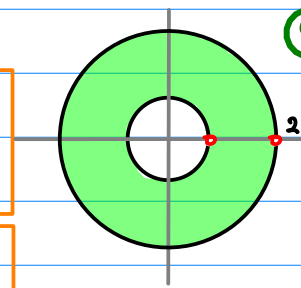


III ③

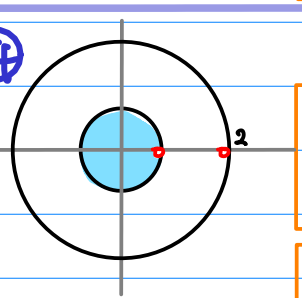


$\left(\frac{1}{q}\right)^{n+1}$	$(n \geq 0)$ $(n > 0)$	$\left(\frac{1}{q}\right)^{n+1}$
$\left(\frac{1}{p}\right)^{n+1}$	$(n < 0)$ $(n \leq 0)$	$\left(\frac{1}{p}\right)^{n+1}$
$p_1 = 1 = p$, $p_2 = 2 = q$		$p_1 = 0.5 = 1/q$, $p_2 = 1 = 1/p$

⑨

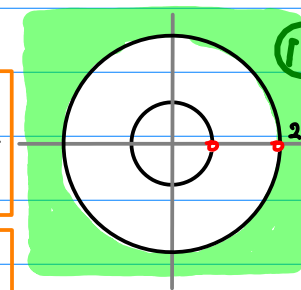


I ④

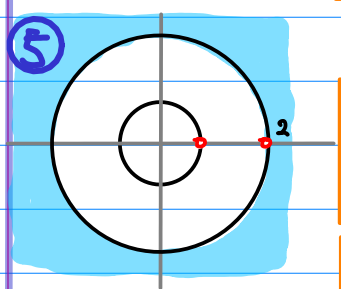


$(p)^{n-1} - (q)^{n-1}$	$(n > 0)$ $(n \geq 0)$	0
0	$(n \leq 0)$ $(n < 0)$	$(p)^{n-1} - (q)^{n-1}$
$p_1 = 0.5 = 1/q$, $p_2 = 1 = 1/p$		$p_1 = 1 = p$, $p_2 = 2 = q$

⑩

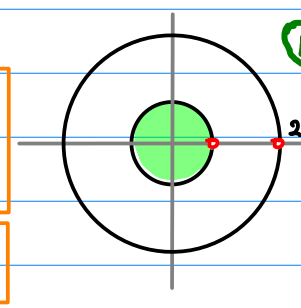


II ⑤

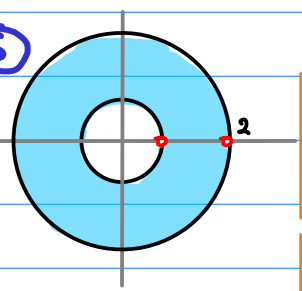


0	$(n > 0)$ $(n \geq 0)$	$(q)^{n-1} - (p)^{n-1}$
$(q)^{n-1} - (p)^{n-1}$	$(n \leq 0)$ $(n < 0)$	0
$p_1 = 0.5 = 1/q$, $p_2 = 1 = 1/p$		$p_1 = 1 = p$, $p_2 = 2 = q$

⑪

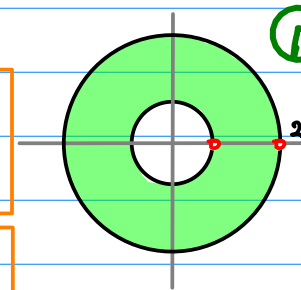


III ⑥



$(p)^{n-1}$	$(n > 0)$ $(n \geq 0)$	$(p)^{n-1}$
$(q)^{n-1}$	$(n \leq 0)$ $(n < 0)$	$(q)^{n-1}$
$p_1 = 0.5 = 1/q$, $p_2 = 1 = 1/p$		$p_1 = 1 = p$, $p_2 = 2 = q$

⑫



$n \geq 0$

$n < 0$

$n > 0$

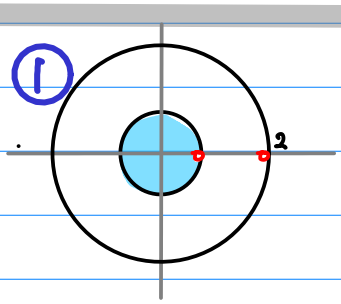
$n \leq 0$

1

$\left(\frac{1}{q}\right)^{n+1} - \left(\frac{1}{p}\right)^{n+1}$

0

1, 2

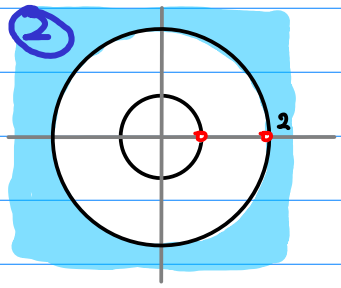


2

0

$\left(\frac{1}{p}\right)^{n+1} - \left(\frac{1}{q}\right)^{n+1}$

1, 2

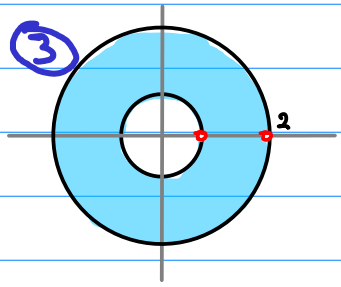


3

$\left(\frac{1}{q}\right)^{n+1}$

$\left(\frac{1}{p}\right)^{n+1}$

1, 2

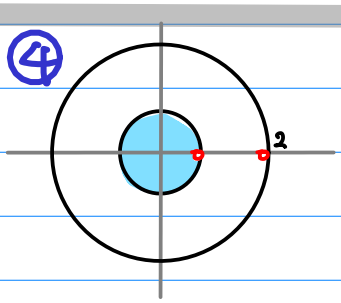


4

$(p)^{n-1} - (q)^{n-1}$

0

0.5, 1.0

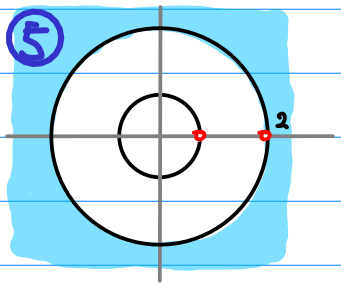


5

0

$(q)^{n-1} - (p)^{n-1}$

0.5, 1.0

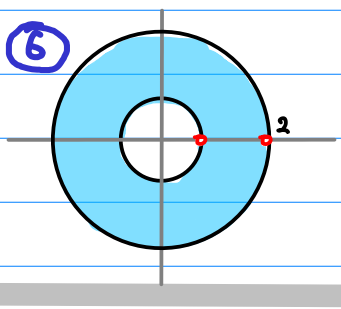


6

$(p)^{n-1}$

$(q)^{n-1}$

0.5, 1.0

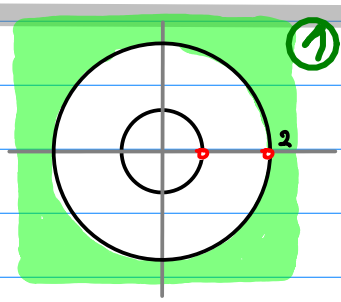


7

0

$\left(\frac{1}{q}\right)^{n+1} - \left(\frac{1}{p}\right)^{n+1}$

0.5, 1.0

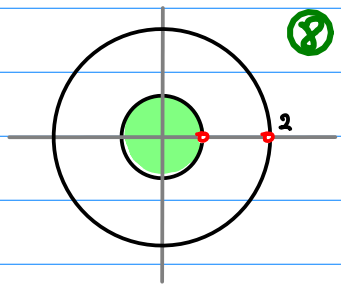


8

$\left(\frac{1}{p}\right)^{n+1} - \left(\frac{1}{q}\right)^{n+1}$

0

0.5, 1.0

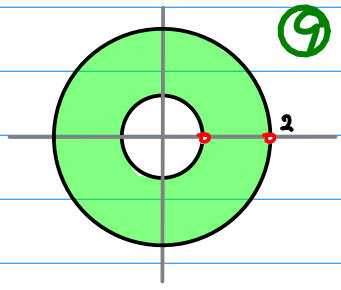


9

$\left(\frac{1}{p}\right)^{n+1}$

$\left(\frac{1}{q}\right)^{n+1}$

0.5, 1.0

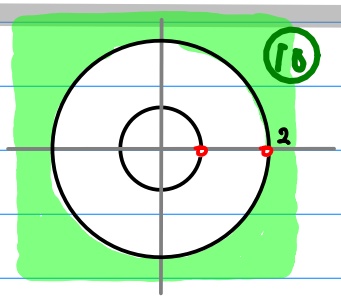


10

0

$(p)^{n-1} - (q)^{n-1}$

1, 2

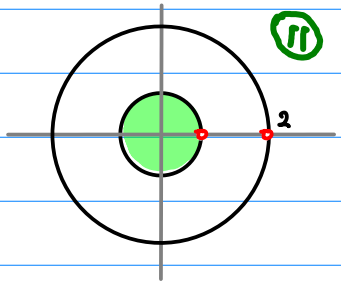


11

$(q)^{n-1} - (p)^{n-1}$

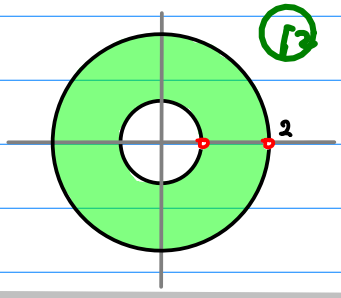
0

1, 2



12

$(q)^{n-1}$

$(p)^{n-1}$

1, 2