# First Order ODEs (H.1)

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1차 미분 방정식 ch 2.

- ⓐ Separable eq. 변수분리 2.2
- ⓑ Linear eq 선형방정식 2.3
- ⓒ Exact eq 완전방정식 2.4

↑

1st order differential eq

Solution : $y(x)$ : $y$는 $x$의 함수

$y'$ $\cancel{y''}$ $\cancel{y'''}$ 1차 미분 방정식

$y$, $x$

ODE Ordinary Differential Equation 상미분 방정식

PDE Partial Differential Equation 편미분 방정식

$\dfrac{df}{dx}$ $\quad$ $f(x)$ ..... 2차원 그림

$\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}$ $\quad$ $f(x, y)$ ... 3차원 그림

partial derivative.

$$\frac{dy}{dx} = 6y^2x = (6x)(y^2)$$

$$y' = \frac{3x^2+4x-4}{2y-4} = (3x^2+4x-4)\left(\frac{1}{2y-4}\right)$$

$$y' = \frac{xy^3}{\sqrt{1+x^2}} = \left(\frac{x}{\sqrt{1+x^2}}\right)(y^3)$$

$$y' = e^{-y}(2x-4) = (2x-4)(e^{-y})$$

$$\frac{dr}{d\theta} = \frac{r^2}{\theta}$$ $r(\theta)$ 함수를 $\theta$에 대하여 미분

$$\frac{d}{d\theta} r(\theta) = \frac{r^2(\theta)}{\theta} \Rightarrow r(\theta)?$$

$$\frac{dr}{d\theta} = \frac{r^2}{\theta} = \left(\frac{1}{\theta}\right)(r^2)$$

$y(t)$?

$$\frac{dy}{dt} = e^{y-t}\sec(y)(1+t^2) = \left(e^{-t}(1+t^2)\right)\left(e^{y}\sec(y)\right)$$

1차 미분 방정식 $\underbrace{y'}_{}$ = $\underbrace{g(x, y)}_{}$   $y(x) = ?$

x에 대해서 y를 미분   x, y의 수식

$$y' = g_1(x) \times g_2(y)$$

$\underline{y'}$ (y 식) = (x 식)

(y 식) $\boxed{\dfrac{dy}{dx}\, dx}$ = (x 식) $dx$

접선의 기울기

(y 식) $dy$ = (x 식) $dx$

$\int$ (y 식) $dy$ = $\int$ (x 식) $dx$

1차 미분 방정식 $\underbrace{y'}_{x에 대해서\ y를\ 미분} = \underbrace{g(x, y)}_{x,\ y의\ 수식}$ $y(x)$?

Linear Eq

$$\underset{\substack{x의 수식 \\ or\ 상수}}{a_1(x)} \boxed{y'} + \underset{\substack{x의 수식 \\ or\ 상수}}{a_2(x)} \boxed{y} = \underset{x의 수식}{g(x)}$$

$$1 \cdot \boxed{y'} + \underset{\substack{x의 수식 \\ or\ 상수}}{p(x)} \boxed{y} = \underset{x의 수식}{g(x)}$$

1. $\boxed{y'} + p(x)\boxed{y} = g(x)$

$$\frac{dv}{dt} = 9.8 - 0.196v \qquad v(t) = ?$$

$$1\ \frac{dv}{dt} + 0.196v = 9.8$$

$$\cos(x)\boxed{y'} + \sin(x)\boxed{y} = 2\cos^3(x)\sin(x) - 1$$

$$\boxed{y'} + \frac{\sin(x)}{\cos(x)}\boxed{y} = 2\cos^2(x)\sin(x) - \frac{1}{\cos(x)}$$

$$t\boxed{y'} + 2\boxed{y} = t^2 - t + 1$$

$$\boxed{y'} + \frac{2}{t}\boxed{y} = t - 1 + \frac{1}{t}$$

$$t\boxed{y'} - 2\boxed{y} = t^5\sin(2t) - t^3 + 4t^4$$

$$1\ \boxed{y'} - \frac{2}{t}\boxed{y} = t^4\sin(2t) - t^2 + 4t^3$$

$$2\boxed{y'} - \boxed{y} = 4\sin(3t) \qquad y(t)\ ?$$

$$1\ \boxed{y'} - \frac{1}{2}\boxed{y} = 2\sin(3t)$$

homogeneous eq

$y' + p(x)\, y = 0$ sol $y_h$

$\Leftrightarrow \quad y_h' + p(x)\, y_h = 0$

non-homogeneous eq

$y' + p(x)\, y = Q(x)$ sol $y_p$

$\Leftrightarrow \quad y_p' + p(x)\, y_p = Q(x)$

$y_h' + p(x)\, y_h = 0$

$y_p' + p(x)\, y_p = Q(x)$

$(y_p + y_h)' + p(x)(y_p + y_h) = Q(x)$

$\Leftrightarrow \quad y' + p(x)\, y = Q(x)$ sol $(y_p + y_h)$

homogeneous eq

$$y' + p(x)\, y = 0 \quad \text{sol} \quad y_h = c \cdot y_1$$

$$= c \cdot e^{-\int p(x)\,dx}$$

non-homogeneous eq

$$y' + p(x)\, y = Q(x) \quad \text{sol} \quad y_p = u \cdot y_1$$

$$= u(x)\, e^{-\int p(x)\,dx}$$

$$\boxed{1\cdot y' + \boxed{P(x)}\, y = Q(x)}$$

$$y_h = c \cdot e^{-\int \boxed{P(x)}\,dx}$$

$$y_p = u(x)\, e^{-\int \boxed{P(x)}\,dx}$$

$$\boxed{e^{+\int \boxed{P(x)}\,dx}}\ \boxed{\left(1\cdot y' + \boxed{P(x)}\, y\right) = \left(Q(x)\right)}\ \boxed{e^{+\int \boxed{P(x)}\,dx}}$$

$$\left(\boxed{e^{+\int \boxed{P(x)}\,dx}\cdot}\ \boxed{y_p}\right)' = Q(x)\, e^{+\int \boxed{P(x)}\,dx}$$

$$e^{+\int \boxed{P(x)}\,dx}\cdot y_p = \int Q(x)\, e^{+\int \boxed{P(x)}\,dx}\,dx$$

$$y_p = e^{-\int \boxed{P(x)}\,dx}\cdot \int Q(x)\, e^{+\int \boxed{P(x)}\,dx}\,dx$$

$$y = y_h + y_p$$

$$y = c \cdot e^{-\int \boxed{P(x)}\,dx} + e^{-\int \boxed{P(x)}\,dx}\cdot \int Q(x)\, e^{+\int \boxed{P(x)}\,dx}\,dx$$

Calculus 1

Review : Exponential

Review : Logarithmic

Derivatives : Trig Derivatives

Differential Equation

First Order : Linear Equation

First Order Differential Equations (1P.pdf) .....

Linear Equation (2A.pdf)

cf) Partial Derivatives (9.4 Zill & Wright)

# Exact Equation

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$\boxed{y(x) = ?}$ ... 1 variable

$f(x,y)$ ... 2 variables

$\boxed{\text{Exact}} \rightarrow f(x,y)$ exists

total differential

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

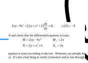
$$= M(x,y)\,dx + N(x,y)\,dy = 0$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

$$\frac{\partial}{\partial y}(M) = \frac{\partial}{\partial x}(N)$$

$f(x.y)$ $\qquad df = 0 \qquad \boxed{f(x.y) = c}$

$\downarrow$

 $y(x) \leftarrow$ Curve $\leftarrow$ Surface $\cap$ 평면

$$(2xy - 9x^2) + (2y + x^2 + 1)\frac{dy}{dx} = 0$$

$$(2xy - 9x^2)\,dx + (2y + x^2 + 1)\,dy = 0$$

$$\boxed{M}\,dx + \boxed{N}\,dy = 0$$

$$\boxed{\frac{\partial f}{\partial x}}\,dx + \boxed{\frac{\partial f}{\partial y}}\,dy = 0$$

Exact?

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$2x = 2x \qquad \therefore \text{ exact}$$

$$f(x,y) = \int \frac{\partial f}{\partial x}\,dx + c$$

$y$

$$= \int \frac{\partial f}{\partial x}\,dx + g(y)$$

$$= \int (2xy - 9x^2)\,dx + g(y)$$

$$f(x,y) = x^2y - 3x^3 + g(y) \quad ?$$

$$\frac{\partial f}{\partial y} = \cancel{x^2} + g'(y) = (2y + \cancel{x^2} + 1)$$

$$g'(y) = 2y + 1 \qquad g'(y) = y^2 + y$$

$$f(x,y) = x^2y - 3x^3 + y^2 + y$$

$$df = 0$$

$$f(x,y) = x^2y - 3x^3 + y^2 + y = C$$

$$\underline{y(x)}$$

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$$\boxed{y(x) = ?}$$ ... 1 variable

$f(x,y)$ ... 2 variables

$$\underset{\frac{\partial f}{\partial x}}{\underset{\parallel}{M(x,y)}}\,dx + \underset{\frac{\partial f}{\partial y}}{\underset{\parallel}{N(x,y)}}\,dy = 0$$

① Verify "exactness"

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

② $\underset{\frac{\partial f}{\partial x}}{\underset{\parallel}{M(x,y)}}$

$$f(x,y) = \int M(x,y)\,dx + g(y)$$

③ $\underset{\frac{\partial f}{\partial y}}{\underset{\parallel}{N(x,y)}}$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[\int M(x,y)\,dx\right] + g'(y)$$

$$= N(x,y)$$

④ $f(x,y) = c$

f의 surface $\cap$ 평면 $\Rightarrow$ $y(x)$