Relation (H.1)

20170509

Based on "Discrete Mathematics and Its Applications" by Rosen

$R_1 = \{(a,b) \mid a \leq b\}$, $R_4 = \{(a,b) \mid a = b\}$,

$R_2 = \{(a,b) \mid a > b\}$, $R_5 = \{(a,b) \mid a = b + 1\}$,

$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\}$, $R_6 = \{(a,b) \mid a + b \leq 3\}$.

R1	0	1	2	3
0	≤	≤	≤	≤
1		≤	≤	≤
2			≤	≤
3				≤

R2	0	1	2	3
0				
1	>			
2	>	>		
3	>	>	>	

R3	0	1	2	3
0	±			
1		±		
2			±	
3				±

R4	0	1	2	3
0	=			
1		=		
2			=	
3				=

R5	0	1	2	3
0				
1	-1			
2		-1		
3			-1	

R6	0	1	2	3
0	0	1	2	3
1	1	2	3	4
2	2	3	4	5
3	3	4	5	6

Reflexive
Symmetric
Anti-Symmetric
Transitive

$A \times A \qquad A = \{0, 1, 2, 3\}$

Catesian Product $= \{(0,0), (0,1), (0,2), (0,3),$
$(1,0), (1,1), (1,2), (1,3),$
$(2,0), (2,1), (2,2), (2,3),$
$(3,0), (3,1), (3,2), (3,3)\}$

$R_1 = \{(a,b) \mid a \leq b\}$

$= \{(0,0), (0,1), (0,2), (0,3),$
$(1,1), (1,2), (1,3),$
$(2,2), (2,3),$
$(3,3)\}$

$R_2 = \{(a,b) \mid a > b\}$
$= \{$
$(1,0),$
$(2,0), (2,1),$
$(3,0), (3,1), (3,2) \}$

$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\}$
$= \{(0,0),$
$(1,1),$
$(2,2),$
$(3,3)\}$

$= \{(0,0),$
$(-1,1),$
$(-2,2),$
$(-3,3)\}$

$$R_4 = \{ (a,b) \mid a=b \}$$
$$= \{ (0,0), (1,1), (2,2), (3,3) \}$$

$$R_5 = \{ (a,b) \mid a = b+1 \}$$
$$= \{ (1,0), (2,1), (3,2) \}$$

$$R_6 = \{ (a,b) \mid a+b \leq 3 \}$$
$$= \{ (0,0), (0,1), (0,2), (0,3), (1,0), (1,1), (1,2), (2,0), (2,1), (3,0) \}$$

Reflexive Relations

$R_1 = \{(a,b) \mid a \leq b\}$, $R_4 = \{(a,b) \mid a = b\}$,

$R_2 = \{(a,b) \mid a > b\}$, $R_5 = \{(a,b) \mid a = b + 1\}$,

$R_3 = \{(a,b) \mid a = b$ or $a = -b\}$, $R_6 = \{(a,b) \mid a + b \leq 3\}$.

Reflexive

	0	1	2	3
0	≤	≤	≤	≤
1		≤	≤	≤
2			≤	≤
3				≤

	0	1	2	3
0				
1	>			
2	>	>		
3	>	>	>	

Reflexive

	0	1	2	3
0	1			
1		1		
2			1	
3				1

Reflexive

	0	1	2	3
0	1			
-1		1		
-2			1	
-3				1

Reflexive

	0	1	2	3
0	=			
1		=		
2			=	
3				=

	0	1	2	3
0				
1	+1			
2		+1		
3			+1	

	0	1	2	3
0	0	1	2	3
1	1	2	3	4
2	2	3	4	5
3	3	4	5	6

Symmetric Relations

$R_1 = \{(a,b) \mid a \le b\}$, $R_4 = \{(a,b) \mid a = b\}$,
$R_2 = \{(a,b) \mid a > b\}$, $R_5 = \{(a,b) \mid a = b + 1\}$,
$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\}$, $R_6 = \{(a,b) \mid a + b \le 3\}$.

	0	1	2	3
0	≤	≤	≤	≤
1		≤	≤	≤
2			≤	≤
3				≤

	0	1	2	3
0				
1	>			
2	>	>		
3	>	>	>	

Symmetric

	0	1	2	3
0	±			
1		±		
2			±	
3				±

Symmetric

R_4	0	1	2	3
0	=			
1		=		
2			=	
3				=

	0	1	2	3
0				
1	+1			
2		+1		
3			+1	

Symmetric

	0	1	2	3
0	0	1	2	3
1	1	2	3	4
2	2	3	4	5
3	3	4	5	6

Symmetric

	0	1	2	3
0	±			
- 1		±		
- 2			±	
- 3				±

R_6	0	1	2	3
0	(0,0)	(0,1)	(0,2)	
1	(1,0)	(1,1)		
2	(2,0)			
3				

(0,0) (1,0) (2,0)
(0,1) (1,1)
(0,2)

$$
\begin{array}{ccccccc}
R1 = \{ & (0,0) & (0,1) & (0,2) & (1,0) & (1,1) & (2,0) \; \} \\
 & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
 & (0,0) & (1,0) & (2,0) & (0,1) & (1,1) & (0,2) \\
 & \Cap & \Cap & \Cap & \Cap & \Cap & \Cap \\
 & R1 & R1 & R1 & R1 & R1 & R1
\end{array}
$$

Definition: R is symmetric iff $(b,a) \in R$ whenever $(a,b) \in R$ for all $a,b \in A$. Written symbolically, R is symmetric if and only if

$$\forall x \forall y \, [(x,y) \in R \longrightarrow (y,x) \in R]$$

$R_4 = \{(a, b) \mid a = b\}$

$R_4 = \{(0,0), (1,1), (2,2), (3,3)\}$

$(0,0)$ $(1,1)$ $(2,2)$ $(3,3)$

R_4 R_4 R_4 R_4

Symmetric

$R_1 = \{(a,b) \mid a \le b\}$, $R_4 = \{(a,b) \mid a = b\}$,

$R_2 = \{(a,b) \mid a > b\}$, $R_5 = \{(a,b) \mid a = b + 1\}$,

$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\}$, $R_6 = \{(a,b) \mid a + b \le 3\}$.

Anti Symmetric

	0	1	2	3
0	≤	≤	≤	≤
1	0	≤	≤	≤
2	0		≤	≤
3				≤

Anti Symmetric

	0	1	2	3
0				
1	>			
2	>	>		
3	>	>	>	

Anti Symmetric

	0	1	2	3
0	±			
1		±		
2			±	
3				±

Anti Symmetric

	0	1	2	3
0	=			
1		=		
2			=	
3				=

Anti Symmetric

	0	1	2	3
0		+1		
1			+1	
2				+1
3				

	0	1	2	3
0	0	1	2	3
1	1	2	3	4
2	2	3	4	5
3	3	4	5	6

	0	1	2	3
0	±			
−1		±		
−2			±	
−3				±

$R1 = \{(a,b) \mid a \leq b\}$

$= \{(0,0), (0,1), (0,2), (0,3), (1,1), (1,2), (1,3), (2,2), (2,3), (3$

$(0,0)$ $\cancel{(1,0)}$ $\cancel{(2,0)}$ $\cancel{(3,0)}$ $(1,1)$ $\cancel{(2,1)}$ $\cancel{(3,1)}$ $(2,2)$ $\cancel{(3,2)}$ $($

Definition: A relation R on a set A such that for all $a,b \in A$ if $(a,b) \in R$ and $(b,a) \in R$, then $a = b$ is called *antisymmetric*.
Written symbolically, R is antisymmetric if and only if
$\forall x \forall y\, [(x,y) \in R \wedge (y,x) \in R \longrightarrow x = y]$

p	$\to$	q
T	T	T
T	F	F
F	T	T
F	T	F

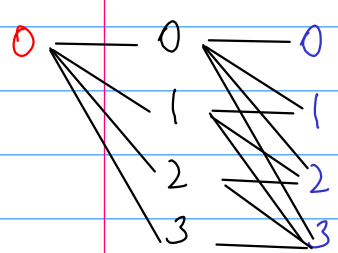
$R_2 = \{(a,b) \mid a > b\}$

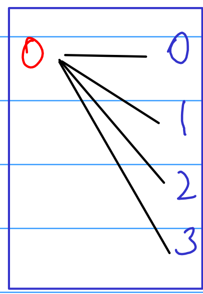
$= \{(1,0), (2,0), (2,1), (3,0), (3,1), (3,2)\}$

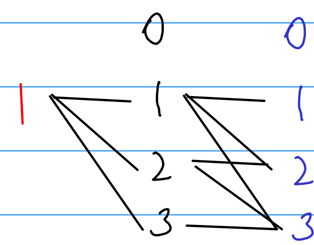
$\cancel{(0,1)}$ $\cancel{(0,2)}$ $\cancel{(1,2)}$ $\cancel{(0,3)}$ $\cancel{(1,3)}$ $\cancel{(2,3)}$

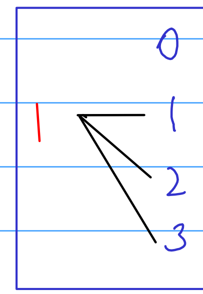
$R_1 = \{(a,b) \mid a \le b\}$, $R_4 = \{(a,b) \mid a = b\}$,
$R_2 = \{(a,b) \mid a > b\}$, $R_5 = \{(a,b) \mid a = b + 1\}$,
$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\}$, $R_6 = \{(a,b) \mid a + b \le 3\}$.

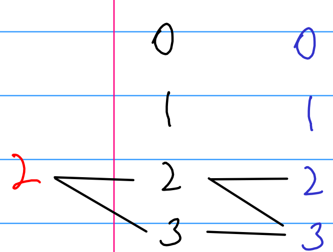
R1	0	1	2	3
0	$\le$	$\le$	$\le$	$\le$
1		$\le$	$\le$	$\le$
2			$\le$	$\le$
3				$\le$

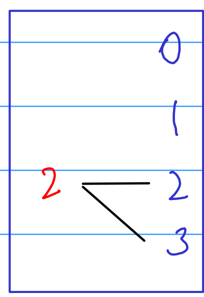


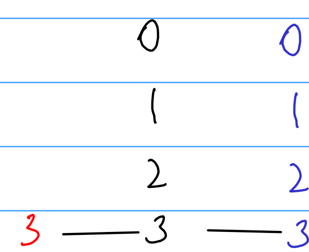


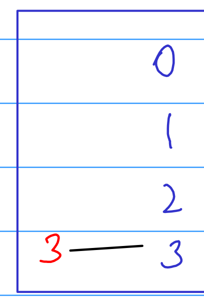




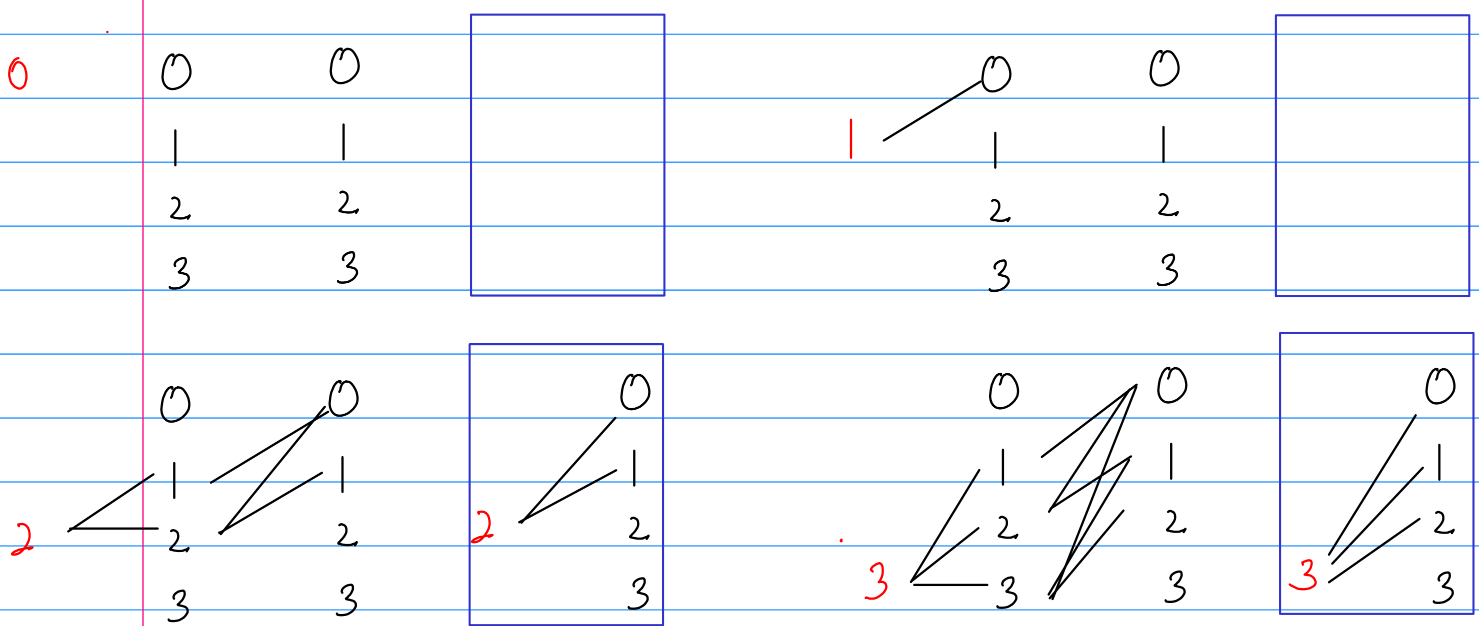






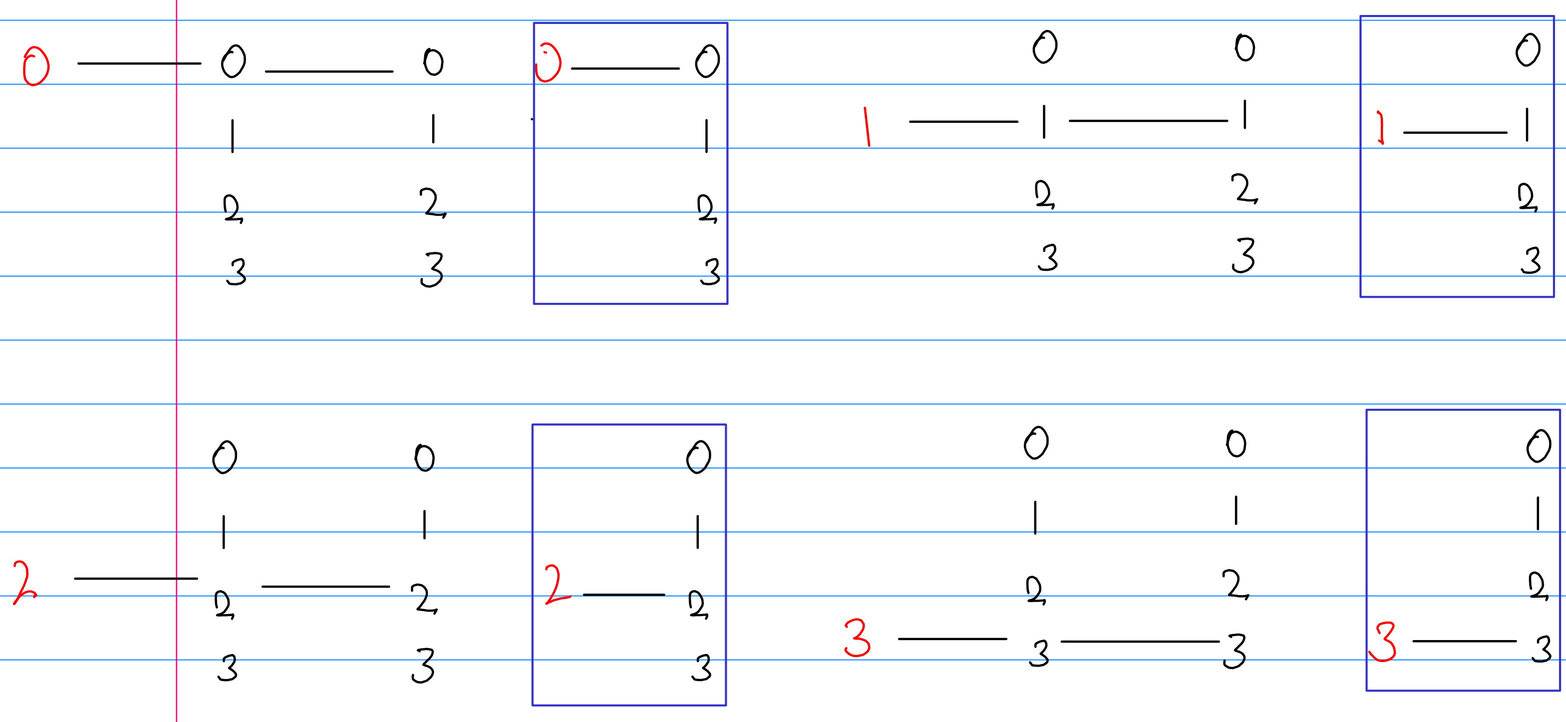


	0	1	2	3
0				
1	>			
2	>	>		
3	>	>	>	

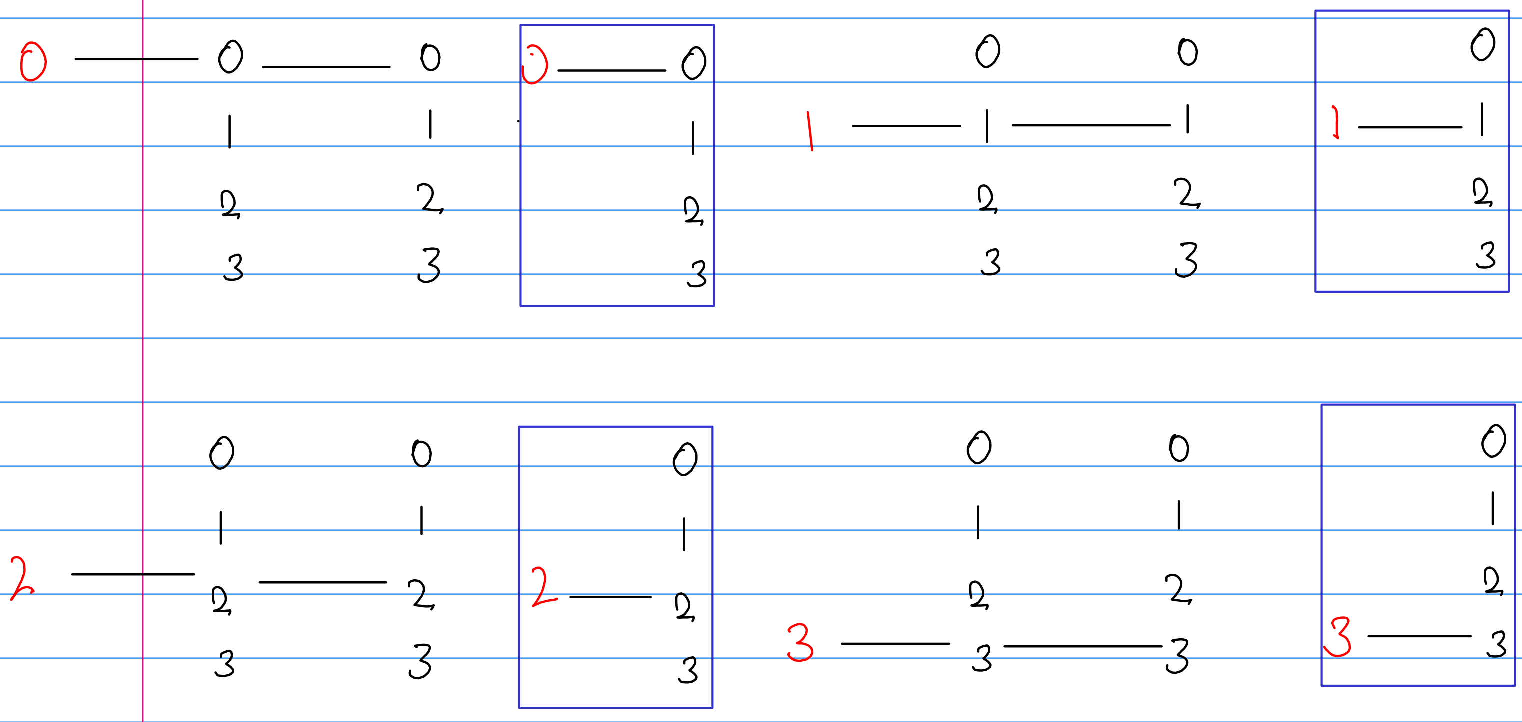


	0	1	2	3
0	±			
1		±		
2			±	
3				±

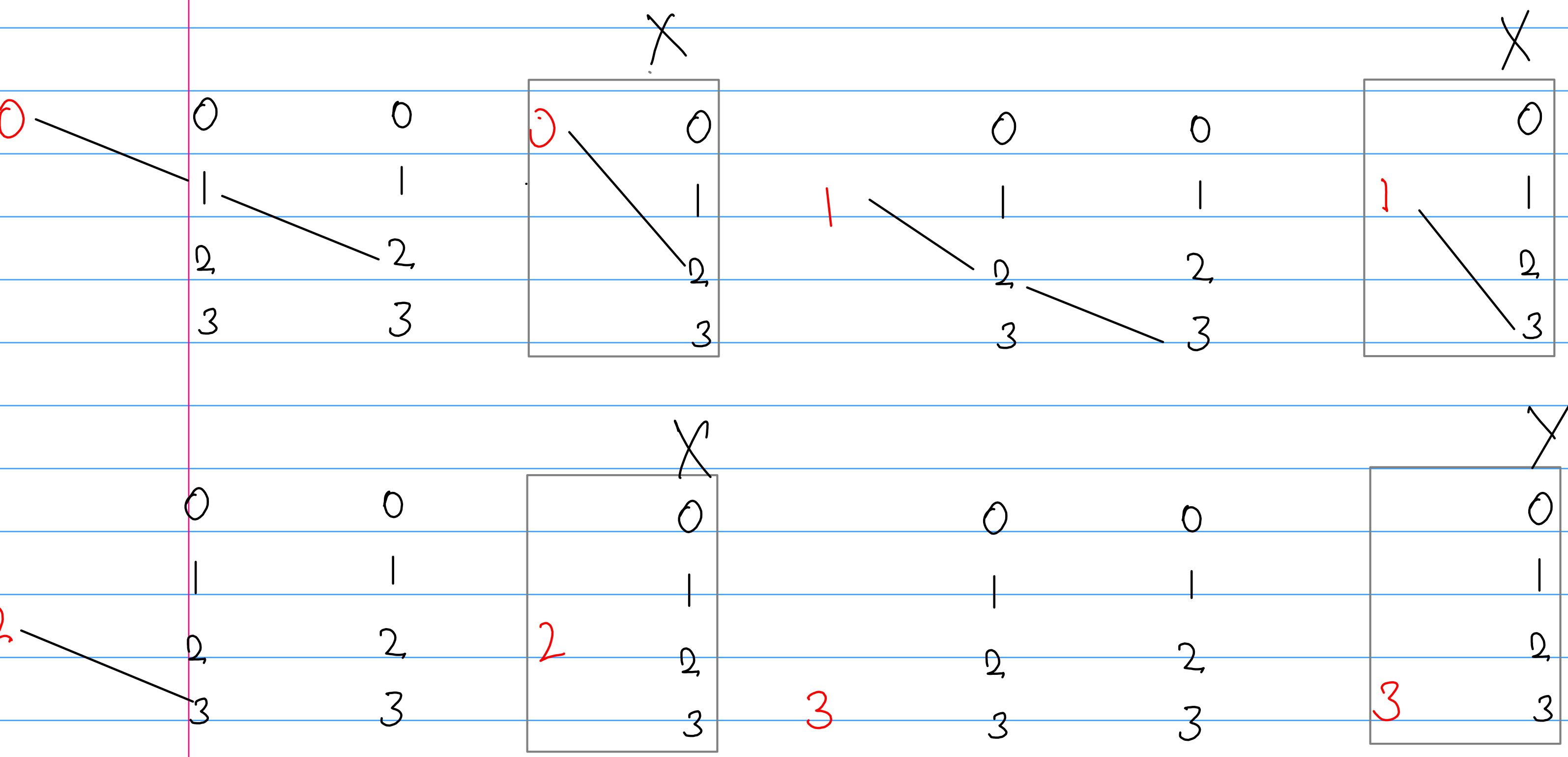
	0	1	2	3
0	±			
- 1		±		
- 2			±	
- 3				±



	0	1	2	3
0	=			
1		=		
2			=	
3				=



	0	1	2	3
0		+1		
1			+1	
2				+1
3				



	0	1	2	3
0	0	1	2	3
1	1	2	3	4
2	2	3	4	5
3	3	4	5	6