

Relation (H.1)

20170509

Copyright (c) 2017 Young W. Lim.

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

Based on "Discrete Mathematics and Its Applications"
by Rosen

$$R_1 = \{(a,b) \mid a \leq b\},$$

$$R_2 = \{(a,b) \mid a > b\},$$

$$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\},$$

$$R_4 = \{(a,b) \mid a = b\},$$

$$R_5 = \{(a,b) \mid a = b + 1\},$$

$$R_6 = \{(a,b) \mid a + b \leq 3\}.$$

R_1	0	1	2	3
0	\leq	\leq	\leq	\leq
1		\leq	\leq	\leq
2			\leq	\leq
3				\leq

R_2	0	1	2	3
0				
1	$>$			
2	$>$	$>$		
3	$>$	$>$	$>$	

R_3	0	1	2	3
0	\pm			
1		\pm		
2			\pm	
3				\pm

R_4	0	1	2	3
0	$=$			
1		$=$		
2			$=$	
3				$=$

R_5	0	1	2	3
0				
1	$-$			
2		$-$		
3			$-$	

R_6	0	1	2	3
0	0	1	2	3
1	1	2	3	4
2	2	3	4	5
3	3	4	5	6

Reflexive
 Symmetric
 Anti-Symmetric
 Transitive

$A \times A$

$$A = \{0, 1, 2, 3\}$$

$$\text{Cartesian Product} = \{(0, 0), (0, 1), (0, 2), (0, 3), \\ (1, 0), (1, 1), (1, 2), (1, 3), \\ (2, 0), (2, 1), (2, 2), (2, 3), \\ (3, 0), (3, 1), (3, 2), (3, 3)\}$$

$$R_1 = \{(a, b) \mid a \leq b\}$$

$$= \{(0, 0), (0, 1), (0, 2), (0, 3), \\ (1, 1), (1, 2), (1, 3), \\ (2, 2), (2, 3), \\ (3, 3)\}$$

$$R_2 = \{(a, b) \mid a > b\}$$

 $= \{$

$$(1, 0), \\ (2, 0), (2, 1), \\ (3, 0), (3, 1), (3, 2)\}$$

$$R_3 = \{(a, b) \mid a = b \text{ or } a = -b\}$$

$$= \{(0, 0),$$

$$(1, 1),$$

$$(2, 2),$$

$$(3, 3)\}$$

$$= \{(0, 0),$$

$$(1, 1),$$

$$(2, 2),$$

$$(3, 3)\}$$

$$R_4 = \{ (a, b) \mid a = b \}$$

$$= \{ (0, 0),$$

$$(1, 1),$$

$$(2, 2),$$

$$(3, 3) \}$$

$$R_5 = \{ (a, b) \mid a = b + 1 \}$$

$$= \{$$

$$(1, 0),$$

$$(2, 1),$$

$$(3, 2)$$

$$\}$$

$$R_6 = \{ (a, b) \mid a + b \leq 3 \}$$

$$= \{ (0, 0), (0, 1), (0, 2), (0, 3),$$

$$(1, 0), (1, 1), (1, 2),$$

$$(2, 0), (2, 1),$$

$$(3, 0)$$

$$\}$$

Reflexive Relations

$$R_1 = \{(a,b) \mid a \leq b\},$$

$$R_2 = \{(a,b) \mid a > b\},$$

$$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\},$$

$$R_4 = \{(a,b) \mid a = b\},$$

$$R_5 = \{(a,b) \mid a = b + 1\},$$

$$R_6 = \{(a,b) \mid a + b \leq 3\}.$$

Reflexive

	0	1	2	3
0	<=	<=	<=	<=
1		<=	<=	<=
2			<=	<=
3				<=

	0	1	2	3
0				
1	>			
2	>	>		
3	>	>	>	

Reflexive

	0	1	2	3
0	≠			
1		≠		
2			≠	
3				≠

Reflexive

	0	1	2	3
0	≠			
-1		≠		
-2			≠	
-3				≠

Reflexive

	0	1	2	3
0	=			
1		=		
2			=	
3				=

	0	1	2	3
0				
1	+1			
2		+1		
3			+1	

	0	1	2	3
0	0	1	2	3
1	1	2	3	4
2	2	3	4	5
3	3	4	5	6

Symmetric Relations

$$R_1 = \{(a,b) \mid a \leq b\},$$

$$R_2 = \{(a,b) \mid a > b\},$$

$$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\},$$

$$R_4 = \{(a,b) \mid a = b\},$$

$$R_5 = \{(a,b) \mid a = b + 1\},$$

$$R_6 = \{(a,b) \mid a + b \leq 3\}.$$

Symmetric

	0	1	2	3
0	≤	≤	≤	≤
1		≤	≤	≤
2			≤	≤
3				≤

	0	1	2	3
0				
1		>		
2		>	>	
3		>	>	>

	0	1	2	3
0	±			
1		±		
2			±	
3				±

Symmetric

$$R_4$$

	0	1	2	3
0	=			
1		=		
2			=	
3				=

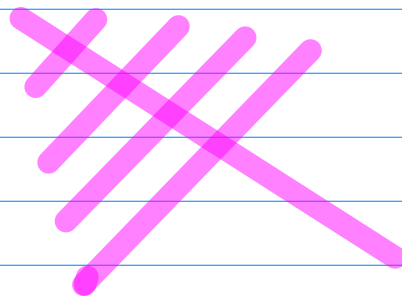
	0	1	2	3
0				
1	+1			
2		+1		
3			+1	

Symmetric

	0	1	2	3
0	0	1	2	3
1	1	2	3	4
2	2	3	4	5
3	3	4	5	6

Symmetric

	0	1	2	3
0	±			
-1		±		
-2			±	
-3				±



$$R_6$$

	0	1	2	3
0	(0,0)	(0,1)	(0,2)	
1	(1,0)	(1,1)		
2	(2,0)			
3				

- (0,0) (1,0) (2,0)
- (0,1) (1,1)
- (0,2)

$$R_1 = \left\{ \begin{array}{cccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ (0,0) & (0,1) & (0,2) & (1,0) & (1,1) & (2,0) \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ (0,0) & (1,0) & (2,0) & (0,1) & (1,1) & (0,2) \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ R_b & R_b & R_b & R_b & R_b & R_b \end{array} \right\}$$

Definition: R is symmetric iff $(b,a) \in R$ whenever $(a,b) \in R$ for all $a,b \in A$. Written symbolically, R is symmetric if and only if

$$\forall x \forall y [(x,y) \in R \rightarrow (y,x) \in R]$$

$$R_4 = \{ (a, b) \mid a = b \}$$

$$R_4 = \{ (0,0), (1,1), (2,2), (3,3) \}$$

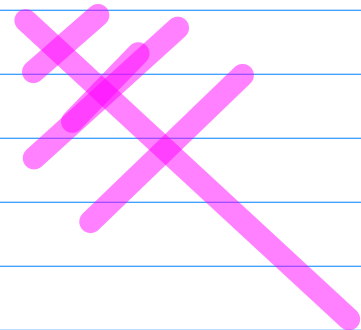
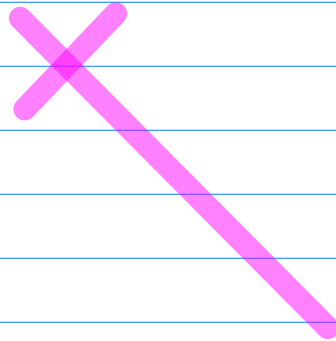
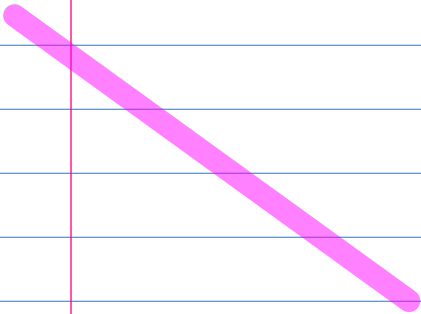


$(0,0)$ $(1,1)$ $(2,2)$ $(3,3)$



R_4 R_4 R_0 R_4

Symmetri: G



$$R_1 = \{(a,b) \mid a \leq b\},$$

$$R_2 = \{(a,b) \mid a > b\},$$

$$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\},$$

$$R_4 = \{(a,b) \mid a = b\},$$

$$R_5 = \{(a,b) \mid a = b + 1\},$$

$$R_6 = \{(a,b) \mid a + b \leq 3\}.$$

AntiSymmetric

	0	1	2	3
0	\leq	\leq	\leq	\leq
1	\leq	\leq	\leq	\leq
2	\leq	\leq	\leq	\leq
3	\leq	\leq	\leq	\leq

AntiSymmetric

	0	1	2	3
0	$>$	$>$	$>$	$>$
1	$>$	$>$	$>$	$>$
2	$>$	$>$	$>$	$>$
3	$>$	$>$	$>$	$>$

AntiSymmetric

	0	1	2	3
0	\neq			
1	\neq	\neq		
2	\neq	\neq	\neq	
3	\neq	\neq	\neq	\neq

AntiSymmetric

	0	1	2	3
0	$=$			
1	$=$	$=$		
2	$=$	$=$	$=$	
3	$=$	$=$	$=$	$=$

AntiSymmetric

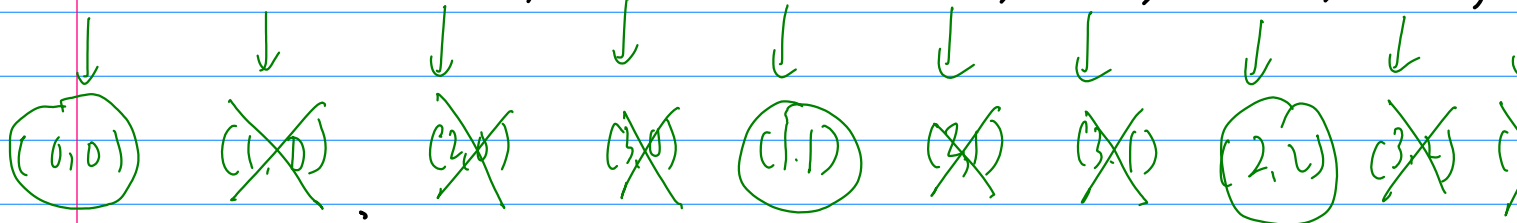
	0	1	2	3
0	$+1$			
1	$+1$	$+1$		
2	$+1$	$+1$	$+1$	
3	$+1$	$+1$	$+1$	$+1$

	0	1	2	3
0	0	1	2	3
1	1	2	3	4
2	2	3	4	5
3	3	4	5	6

	0	1	2	3
0	\neq			
-1	\neq	\neq		
-2	\neq	\neq	\neq	
-3	\neq	\neq	\neq	\neq

$$R_1 = \{ (a,b) \mid a \leq b \}$$

$$= \{ (0,0), (0,1), (0,2), (0,3), (1,1), (1,2), (1,3), (2,2), (2,3), (3,3) \}$$



Definition: A relation R on a set A such that for all $a, b \in A$ if $(a,b) \in R$ and $(b,a) \in R$, then $a = b$ is called *antisymmetric*.

Written symbolically, R is antisymmetric if and only if

$$\forall x \forall y [(x,y) \in R \wedge (y,x) \in R \rightarrow x = y]$$

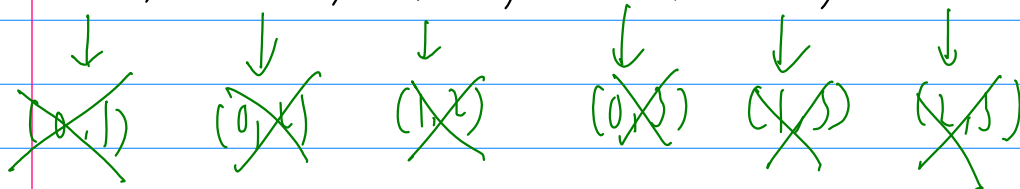
$p \rightarrow q$

T	T
T	F
F	T
F	F

T
F
T
T

$$R_2 = \{ (a,b) \mid a > b \}$$

$$= \{ (1,0), (2,0), (2,1), (3,0), (3,1), (3,2) \}$$



$$R_1 = \{(a,b) \mid a \leq b\},$$

$$R_2 = \{(a,b) \mid a > b\},$$

$$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\},$$

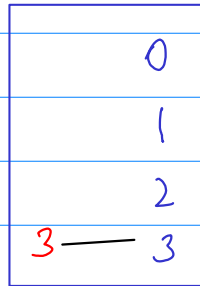
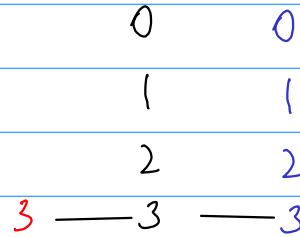
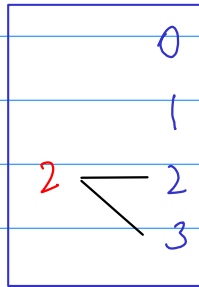
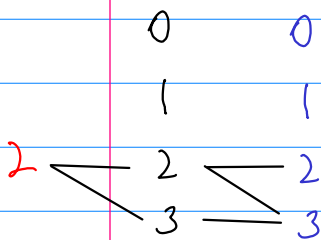
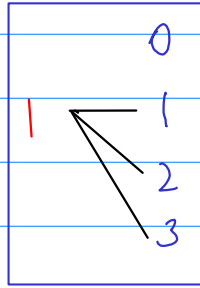
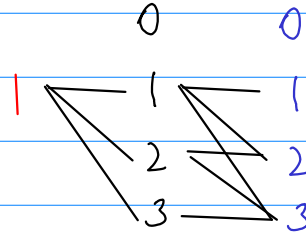
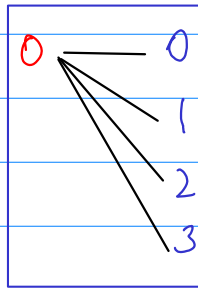
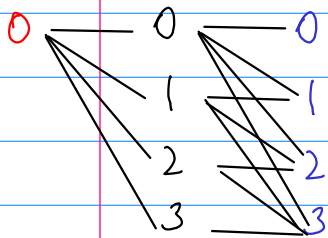
$$R_4 = \{(a,b) \mid a = b\},$$

$$R_5 = \{(a,b) \mid a = b + 1\},$$

$$R_6 = \{(a,b) \mid a + b \leq 3\}.$$

21

	0	1	2	3
0	≤	≤	≤	≤
1		≤	≤	≤
2			≤	≤
3				≤



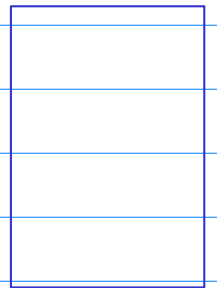
	0	1	2	3
0				
1	>			
2	>	>		
3	>	>	>	

0

0 0
1 1
2 2
3 3

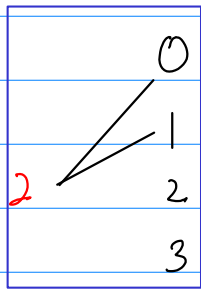


0 0
1 1
2 2
3 3



2

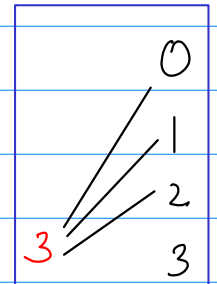
0 0
1 1
2 2
3 3



3

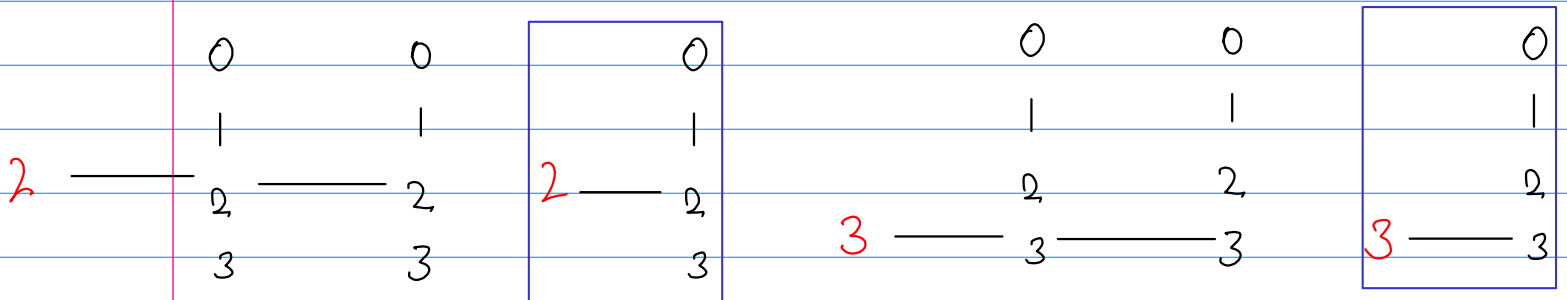
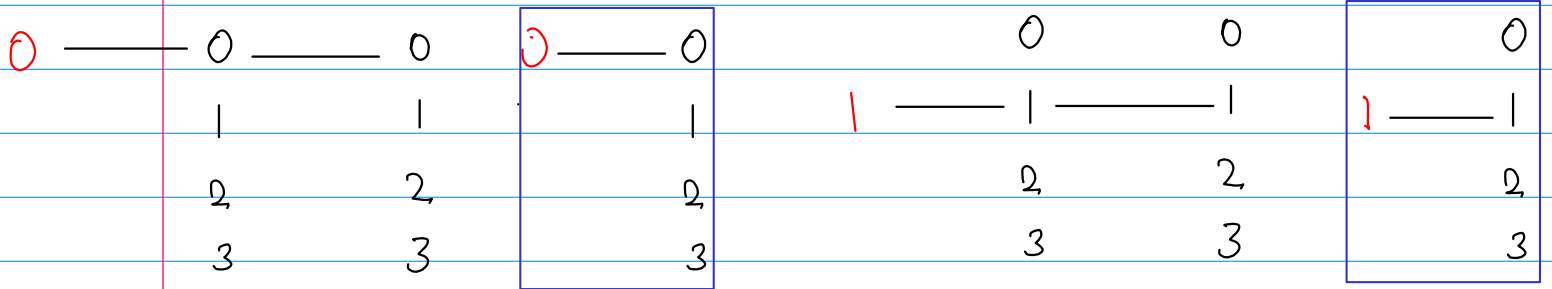
0 0
1 1
2 2
3 3

3

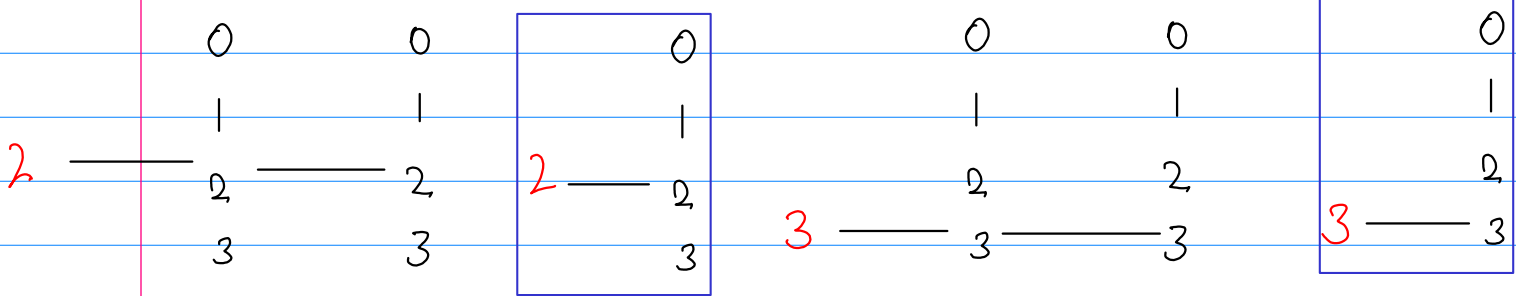
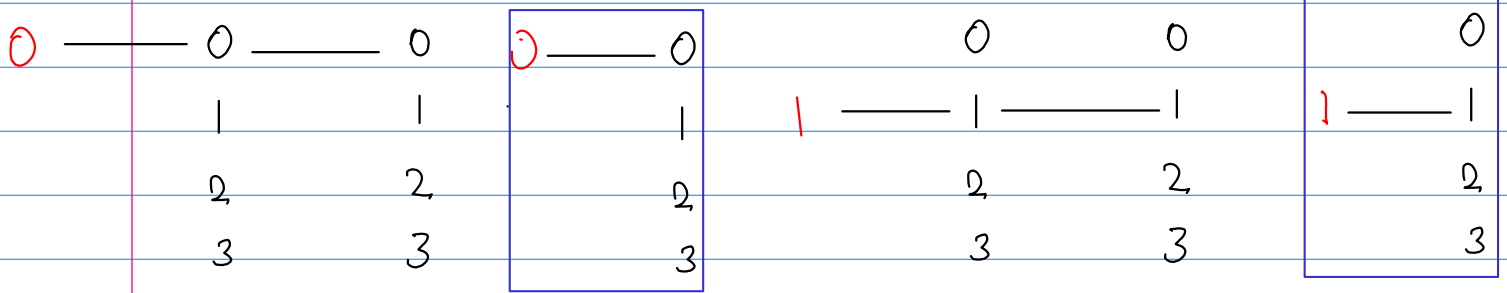


	0	1	2	3
0	±			
1		±		
2			±	
3				±

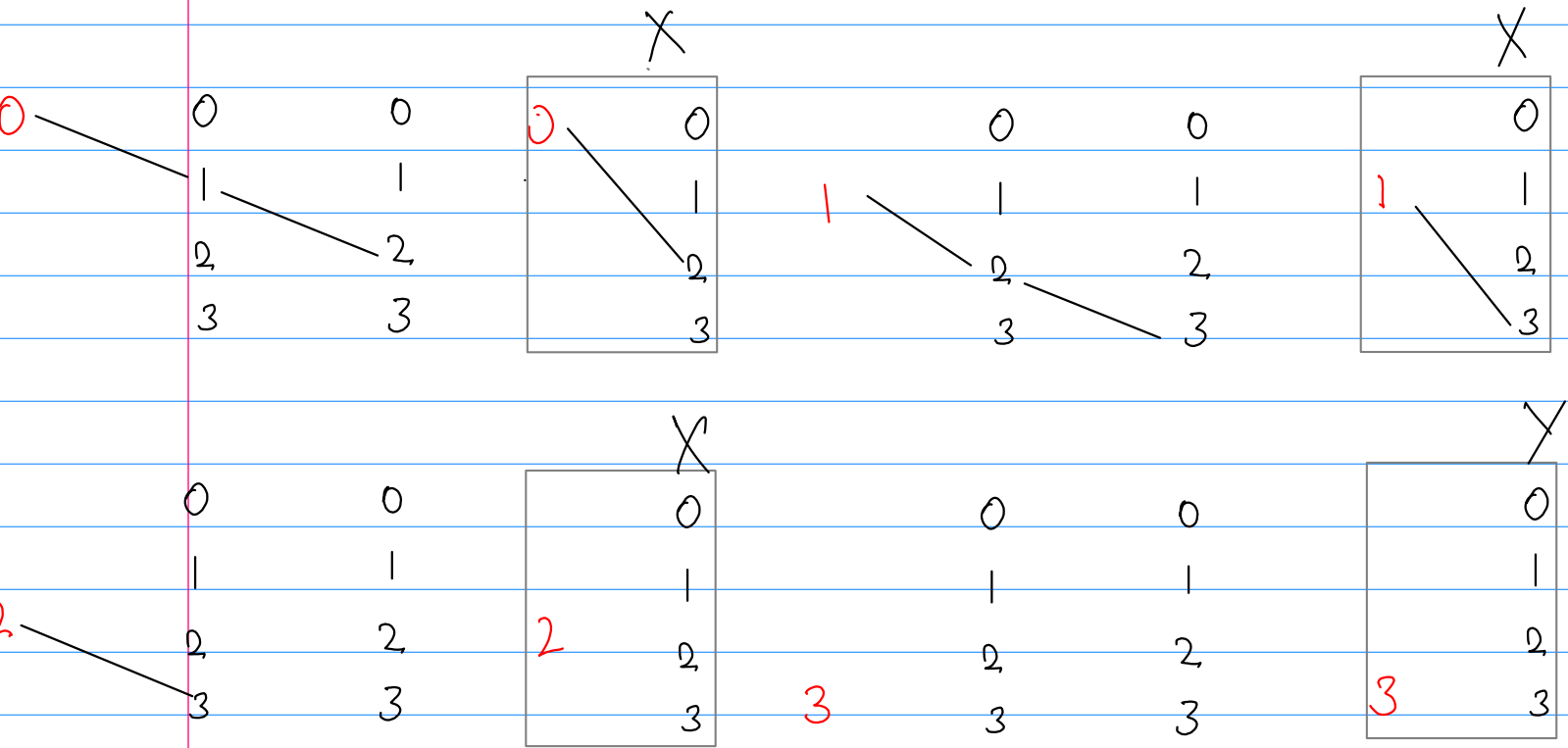
	0	1	2	3
0	±			
-1		±		
-2			±	
-3				±



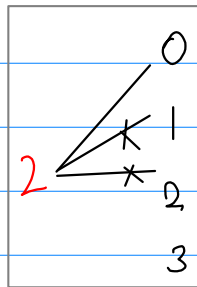
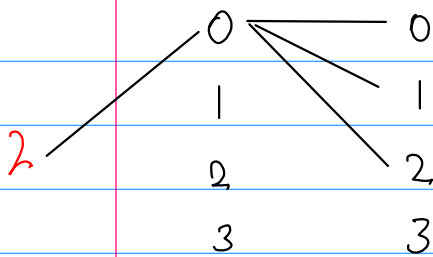
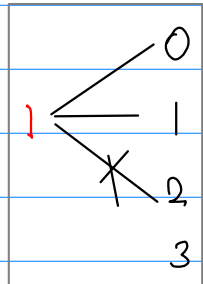
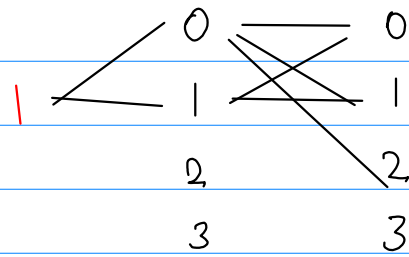
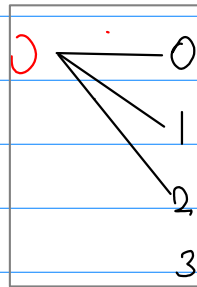
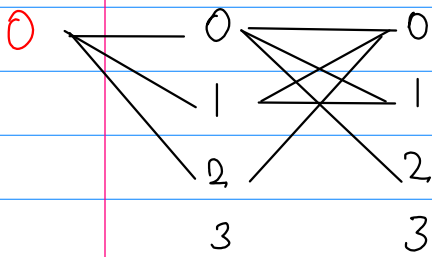
	0	1	2	3
0	1			
1		=		
2			=	
3				=



	0	1	2	3
0		+1		
1			+1	
2				+1
3				



	0	1	2	3
0	0	1	2	3
1	1	2	3	4
2	2	3	4	5
3	3	4	5	6



3

