Characteristics of Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

Expected Value

two random variables

Definition

the expected value of $g(x, y)$ is given by

$$\overline{g} = E[g(X, Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x, y) f_{X,Y}(x, y) dxdy$$

where$g(x, y)$is some function of two random variables X and Y

Expected Value
N random variables

Definition

for N random variables $X_1, X_2, ..., X_N$,
the expected value of $g(X_1, X_2, ..., X_N)$ is given by

$$\overline{g} = E[g(X_1, X_2, ..., X_N)]$$

$$= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(x_1, ..., x_N) f_{x_1, ..., x_N}(x_1, ..., x_N) dx_1 \cdots dx_N$$

where $g(X_1, X_2, ..., X_N)$ is some function of
N random variables $X_1, X_2, ..., X_N$

Expected Value

N random variables to a single random variable

If $g(X_1, X_2, ..., X_N) = g(X_1)$, then

$$\overline{g} = E[g(X_1, X_2, ..., X_N)]$$

$$= \int_{-\infty}^{\infty} g(x_1) f_{x_1}(x_1) dx_1 = E[g(X_1)]$$

$$\overline{g} = E[g(X_1, X_2, ..., X_N)] = E[g(X_1)]$$

Joint Moments about the Origin
2 random variables

Definition

joint moment about the origin $m_{\{nk\}}$is defined by

$$m_{\{nk\}} = E[X^n Y^k] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^n y^k f_{X,Y}(x,y)dxdy$$

the second moment $m_{\{11\}} = E[XY]$ is called the correlation $R_{\{XY\}}$ of X and Y

$$R_{\{XY\}} = m_{\{11\}} = E[X^1 Y^1] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^1 y^1 f_{X,Y}(x,y)dxdy$$

if the correlation $R_{\{XY\}}$ can be written as $R_{\{XY\}} = E[X]E[Y]$, then X and Y are uncorrelated
statistical independence of X and Y is sufficient to guarantee they are uncorrelated