

DFT Octave Codes (0B)

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Based on

M.J. Roberts, Fundamentals of Signals and Systems

S.K. Mitra, Digital Signal Processing : a computer-based approach 2nd ed

S.D. Stearns, Digital Signal Processing with Examples in MATLAB

B.D Storey, Computing Fourier Series and Power Spectrum with MATLAB

B Ninness, Spectral Analysis using the FFT

U of Rhode Island, ELE 436, FFT Tutorial

fft(x)

fft (x)

- Compute the discrete Fourier transform of **x** using a Fast Fourier Transform (FFT) algorithm.
- The FFT is calculated along the first non-singleton dimension of the array.
- if **x** is a matrix, fft (**x**) computes the FFT for each column of **x**.

fft(x, n)

fft (x, n)

- If called with two arguments, **n** is expected to be an integer specifying the number of elements of **x** to use, or an empty matrix to specify that its value should be ignored.
- If **n** is larger than the dimension (the number of data) along which the FFT is calculated, then **x** is resized and padded with zeros.
- If **n** is smaller than the dimension (the number of data) along which the FFT is calculated, then **x** is truncated.

fft(x, n, dim)

fft (x, n, dim)

- If called with three arguments, dim is an integer specifying the dimension of the matrix along which the FFT is performed

A Cosine Waveform

```
n= [0:29];  
x= cos(2*pi*(n/10));
```

$$nT_s = n \cdot \frac{1}{10}$$

```
x= cos((2/10)*pi*n);
```

$$nT_s = n \cdot 1$$

$$\omega_0 nT_s = 2\pi f_0 nT_s = \frac{2\pi}{T_0} nT_s = 2\pi n \frac{T_s}{T_0}$$

$$\omega_0 t = 2\pi f t$$

$$\omega_0 nT_s = 2\pi f_0 nT_s = 2\pi \cdot 1 \cdot n \cdot \frac{1}{10}$$

$$\omega_0 nT_s = 2\pi f_0 nT_s = 2\pi \cdot \frac{1}{10} \cdot n \cdot 1$$

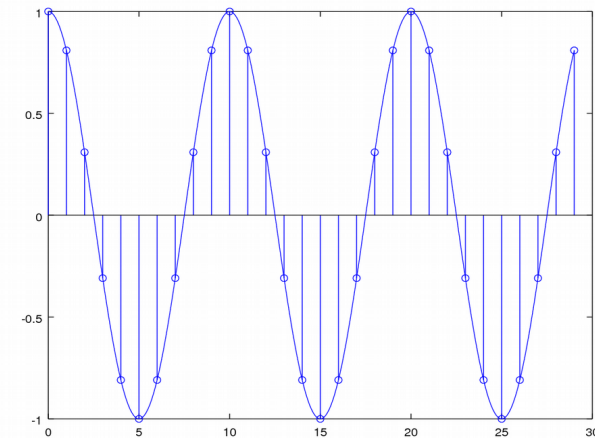
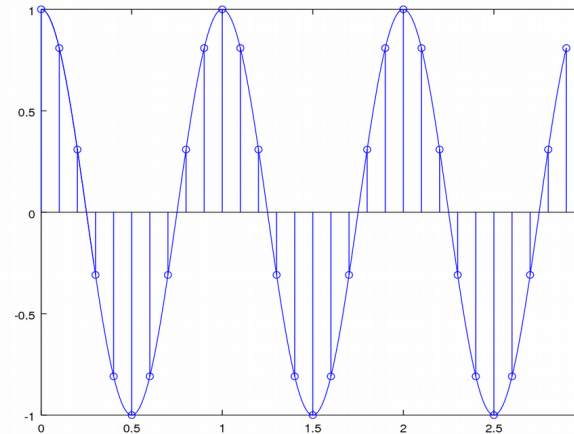
$$f_0 = 1 \quad T_0 = 1 \quad T_s = 0.1$$

$$f_0 = 0.1 \quad T_0 = 10 \quad T_s = 1$$

Many waveforms share the same sampled data

x

```
1.00000  
0.80902  
0.30902  
-0.30902  
-0.80902  
-1.00000  
-0.80902  
-0.30902  
0.30902  
0.80902  
1.00000  
0.80902  
0.30902  
-0.30902  
-0.80902  
-1.00000  
-0.80902  
-0.30902  
0.30902  
0.80902  
1.00000  
0.80902  
0.30902  
-0.30902  
-0.80902  
-1.00000  
-0.80902  
-0.30902  
0.30902  
0.80902
```



U of Rhode Island, ELE 436, FFT Tutorial

Cosine Wave 1

```
n = [0:29];  
x = cos(2*pi*n/10);
```

```
t = [0:29]/10;  
y = cos(2*pi*t);  
stem(t, y)  
hold on  
t2 = [0:290]/100;  
y2 = cos(2*pi*t2);  
plot(t2, y2)
```

```
t = [0:29];  
y = cos(0.2*pi*t);  
stem(t, y)  
hold on  
t2 = [0:290]/10;  
y2 = cos(0.2*pi*t2);  
plot(t2, y2)
```

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \quad f_0 = 1 \quad T_0 = 1 \quad T_s = 0.1$$

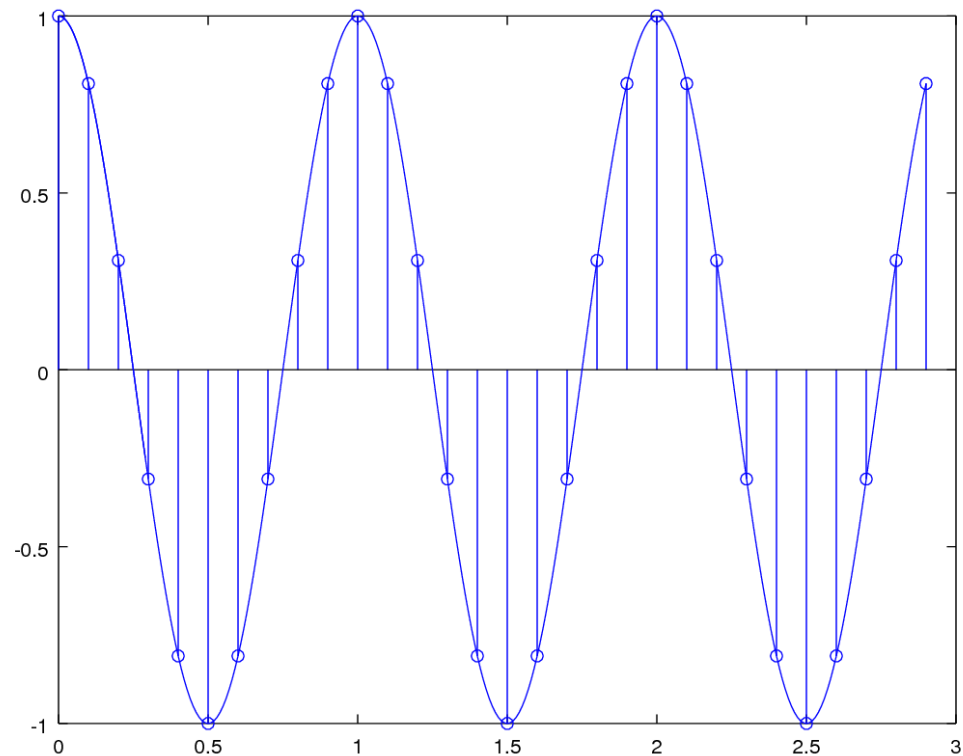
$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \quad f_0 = 0.1 \quad T_0 = 10 \quad T_s = 1$$

Cosine Wave 1

```
n = [0:29];  
x = cos(2*pi*n/10);
```

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \quad f_0 = 1 \quad T_0 = 1 \quad T_s = 0.1$$

```
t = [0:29]/10;  
y = cos(2*pi*t);  
stem(t, y)  
hold on  
t2 = [0:290]/100;  
y2 = cos(2*pi*t2);  
plot(t2, y2)
```



U of Rhode Island, ELE 436, FFT Tutorial

Cosine Wave 2

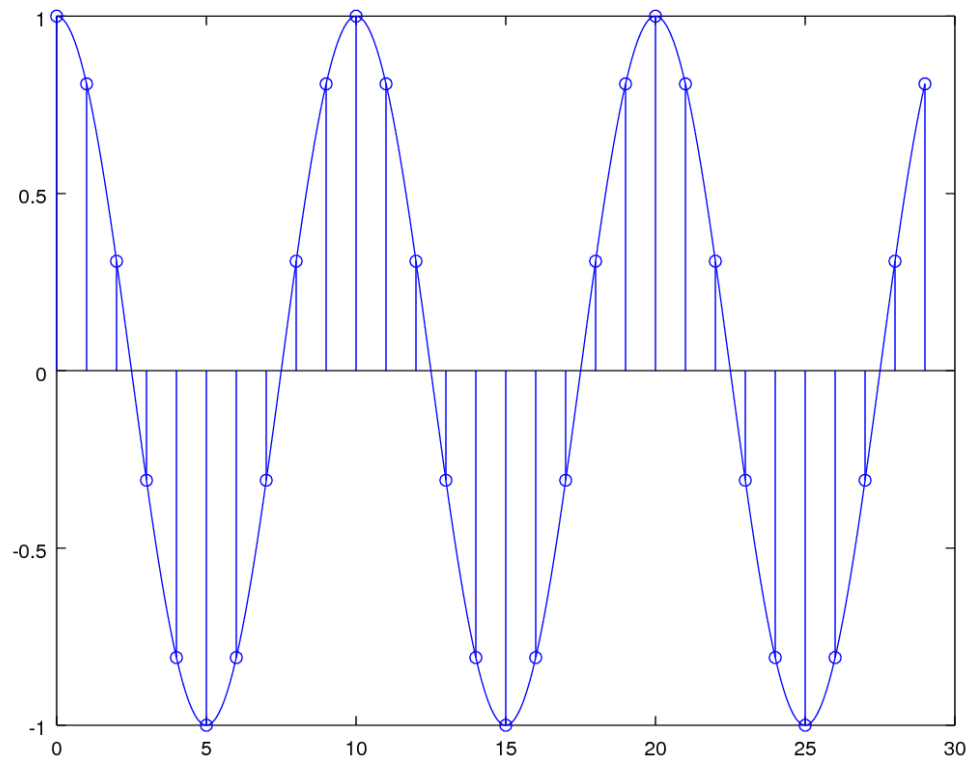
```
n = [0:29];  
x = cos(2*pi*n/10);
```

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \quad f_0 = 0.1 \quad T_0 = 10 \quad T_s = 1$$

```
t = [0:29];  
y = cos(0.2*pi*t);  
stem(t, y)  
hold on  
t2 = [0:290]/10;  
y2 = cos(0.2*pi*t2);  
plot(t2, y2)
```

$$\omega_0 n T_s = 2\pi f_0 n T_s = 2\pi \cdot \frac{1}{10} \cdot n \cdot 1$$

$$f_0 = 0.1 \quad T_0 = 10 \quad T_s = 1$$



U of Rhode Island, ELE 436, FFT Tutorial

Sampled Sinusoids

$$g[n] = A e^{\beta n}$$

$$g[n] = A z^n \quad z = e^{\beta}$$

$$g[n] = A \cos(2\pi n/N_0 + \theta)$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$g[n] = A \cos(\Omega_0 n + \theta)$$

$$1/N_0$$

$$= F_0$$

$$= \Omega_0/2\pi$$

$$2\pi/N_0$$

$$= 2\pi F_0$$

$$= \Omega_0$$

M.J. Roberts, Fundamentals of Signals and Systems

Sampling Period and Frequency

$$g(t) = A \cos(2\pi f_0 t + \theta)$$

$$g[n] = g(nT_s)$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$g(nT_s) = A \cos(2\pi f_0 T_s n + \theta)$$

$$F_0 = f_0 T_s = f_0 / f_s$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$T_s = \frac{1}{f_s}$$

sampling period

$$\frac{1}{T_s} = f_s$$

sampling frequency
sampling rate

M.J. Roberts, Fundamentals of Signals and Systems

Periodic Condition of a Sampled Signal

$$2\pi F_0 n = 2\pi m$$

$$F_0 n = m \quad \text{Integers } n, m$$

$$F_0 = \frac{m}{n}$$

$$F_0 = \frac{m}{n} = \frac{f_0}{f_s}$$

Fundamental Frequency
Sampling Frequency

Rational Number $F_0 = \frac{m}{n}$

$$g(nT_s) = A \cos(2\pi f_0 T_s n + \theta)$$

$$F_0 = f_0 T_s = f_0 / f_s$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

Periodic Condition Examples

$$g(t) = A \cos(2\pi f_0 t + \theta)$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$g[t] = 4 \cos\left(\frac{72\pi t}{19}\right) = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot t\right)$$

$$g[n] = 4 \cos\left(\frac{72\pi n}{19}\right) = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot n\right) \quad T_s = 1$$

$$g(t) = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot (t + T_0)\right)$$

$$T_0 = \frac{19}{36} \quad \text{Fundamental period of } g(t)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot (n + N_0)\right)$$

$$N_0 = 19 \quad \text{Fundamental period of } g[n]$$

$$N_0 \neq \frac{1}{F_0} \quad \frac{N_0}{q} = \frac{1}{F_0} \quad \frac{q}{N_0} = F_0$$

M.J. Roberts, Fundamentals of Signals and Systems

Periodic Condition Examples

$$g[n] = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot (n + N_0)\right)$$

$$\frac{36}{19} \cdot (n + N_0)$$

integer

$$\frac{1}{19} \cdot N_0 = k$$

integer

$$N_0$$

integer

$N_0 = 19$ Fundamental period of $g[n]$

$$g(t) = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot (t + T_0)\right)$$

$$\frac{36}{19} \cdot (t + T_0)$$

integer

$$\frac{36}{19} \cdot T_0 = k$$

integer

$$T_0$$

~~integer~~

$T_0 = \frac{19}{36}$ Fundamental period of $g(t)$

M.J. Roberts, Fundamentals of Signals and Systems

Periodic Condition Examples

$$g[n] = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot n\right)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{1}{19} \cdot n\right)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{2}{19} \cdot n\right)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{3}{19} \cdot n\right)$$

```
clf
n = [0:36]; t = [0:3600]/100;
y1 = 4*cos(2*pi*(1/19)*n);
y2 = 4*cos(2*pi*(2/19)*n);
y3 = 4*cos(2*pi*(3/19)*n);
y4 = 4*cos(2*pi*(36/19)*n);
yt1 = 4*cos(2*pi*(1/19)*t);
yt2 = 4*cos(2*pi*(2/19)*t);
yt3 = 4*cos(2*pi*(3/19)*t);
yt4 = 4*cos(2*pi*(36/19)*t);
```

```
subplot(4,1,1);
stem(n, y1); hold on;
plot(t, yt1);
subplot(4,1,2);
stem(n, y2); hold on;
plot(t, yt2);
subplot(4,1,3);
stem(n, y3); hold on;
plot(t, yt3);
subplot(4,1,4);
stem(n, y4); hold on;
plot(t, yt4);
```

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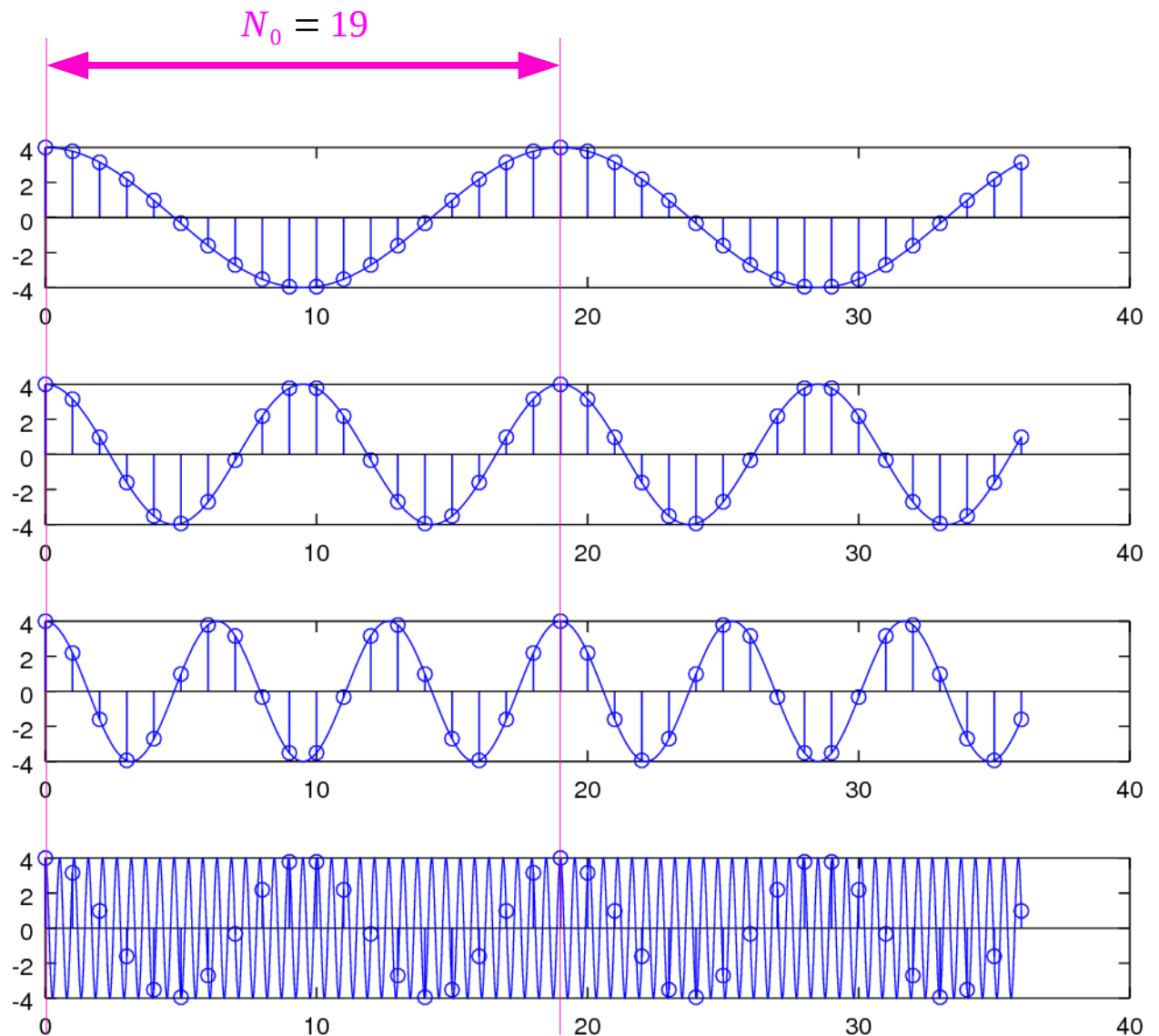
Periodic Condition Examples

$$g[n] = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot n\right)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{1}{19} \cdot n\right)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{2}{19} \cdot n\right)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{3}{19} \cdot n\right)$$



Periodic Condition Examples

$$g(t) = A \cos(2\pi f_0 t + \theta)$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$g[n] = 4 \cos\left(\frac{72\pi n}{19}\right)$$

$$g_1(t) = 4 \cos\left(\frac{72\pi t}{19}\right)$$

$$t = nT_{s1} = n \cdot 1$$

$$g_2(t) = 4 \cos\left(\frac{1}{2} \frac{72\pi 2t}{19}\right)$$

$$2t = nT_{s2} = n \cdot 2$$

$$g_3(t) = 4 \cos\left(\frac{1}{3} \frac{72\pi 3t}{19}\right)$$

$$3t = nT_{3s} = n \cdot 3$$

Periodic Condition Examples

$$g(t) = A \cos(2\pi f_0 t + \theta)$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$\begin{aligned} g[n] &= 4 \cos\left(\frac{72\pi n}{19}\right) \\ &= 4 \cos\left(2\pi \left(\frac{36}{19}\right) n\right) \\ &= 4 \cos\left(2\pi \left(\frac{36}{19} \cdot (n + N_0)\right)\right) \\ &\quad \text{smallest } N_0 = 19 \end{aligned}$$

$$2\pi F_0 n = 2\pi m$$

$$\frac{36}{19} n = m$$

smallest $n = 19$

$$\frac{36}{19} = \frac{m}{n}$$

$$\frac{36}{19} = \frac{m}{n} = \frac{f_0}{f_s}$$

$$F_0 = \frac{q}{N_0}$$

$$1/N_0$$

$$= F_0$$

$$= \Omega_0/2\pi$$

M.J. Roberts, Fundamentals of Signals and Systems

FFT of a cosine (N=64, 128, 256)

```
n= [0:29];  
x= cos(2*pi*n/10);
```

```
N1= 64;  
N2= 128;  
N3= 256;
```

```
X1= abs(fft(x,N1));  
X2= abs(fft(x,N2));  
X3= abs(fft(x,N3));
```

```
F1= [0: N1-1]/N1;  
F2= [0: N2-1]/N2;  
F3= [0: N3-1]/N3;
```

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \quad f_0 = 0.1 \quad T_0 = 10$$

N = 30



zero padding

N1 = 64



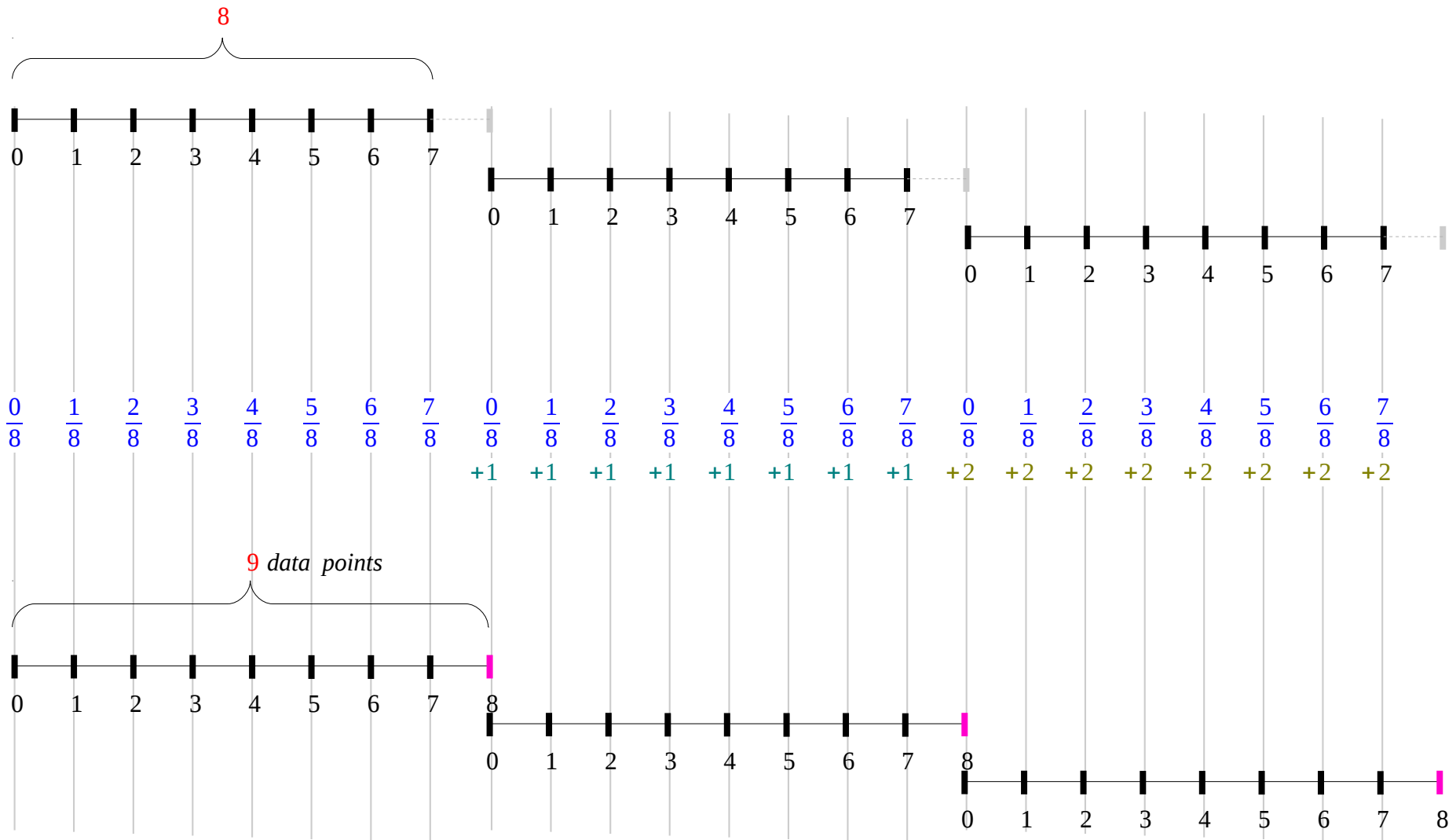
N2 = 128



N3 = 256

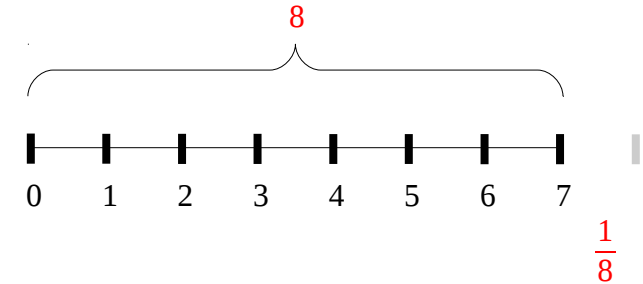


Linearly Spaced Elements (1)



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Linearly Spaced Elements (2)



```
F1= [0: (N1-1)]/N1;
```

```
F2= [0: (N2-1)]/N2;
```

```
F3= [0: (N3-1)]/N3;
```

```
F1= 0 : 1/N1 : (N1-1)/N1;
```

```
F2= 0 : 1/N2 : (N2-1)/N2;
```

```
F3= 0 : 1/N3 : (N3-1)/N3;
```

```
F1= linspace(0, (N1-1)/N1, N1);
```

```
F2= linspace(0, (N2-1)/N2, N2);
```

```
F3= linspace(0, (N3-1)/N3, N3);
```

```
F1= [0: 7]/8;
```

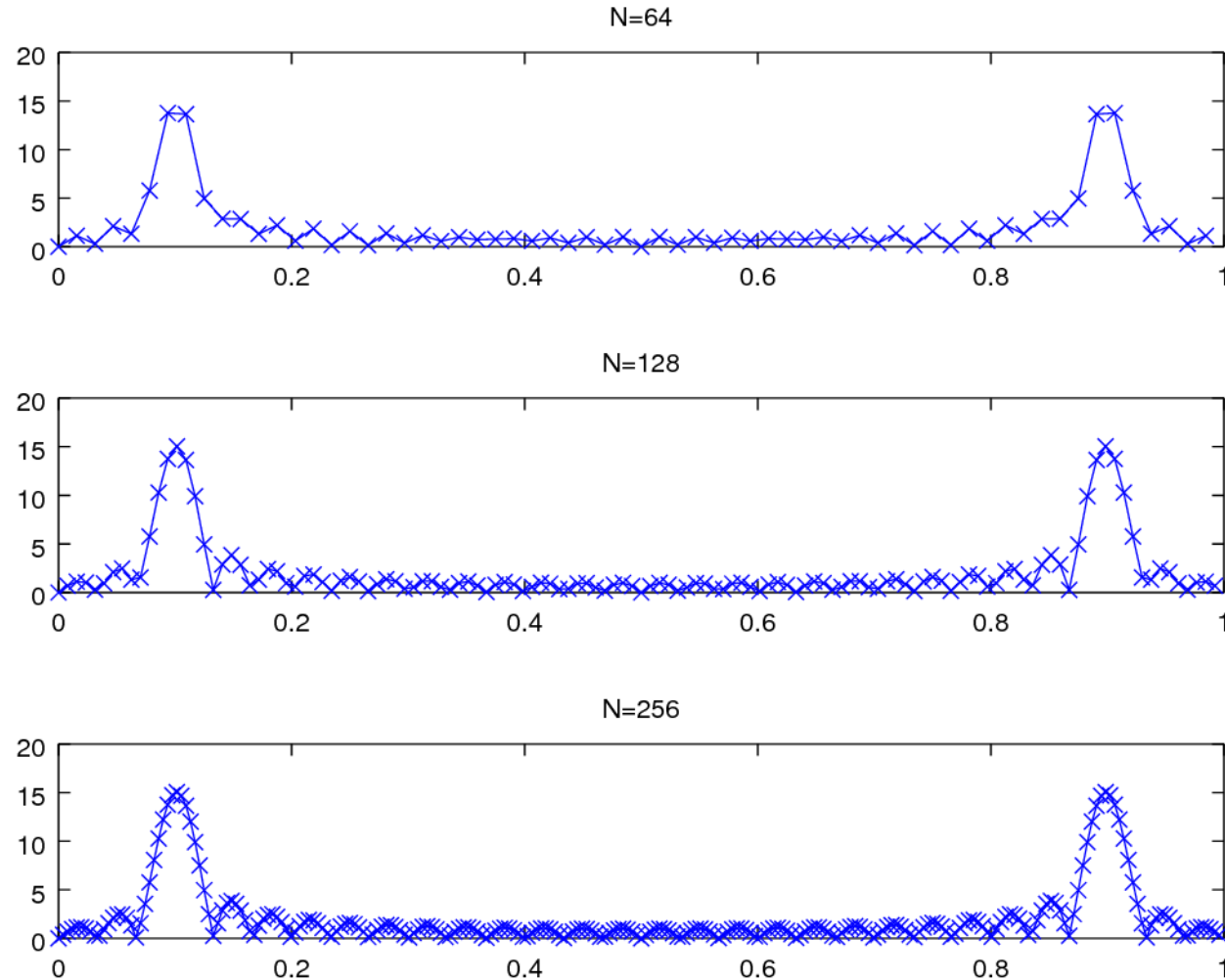
```
F1= 0 : 1/8 : 7/8;
```

```
F1= linspace(0, 7/8, 8);
```

FFT of a cosine (N=64, 128, 256) - plot

```
subplot(3,1,1);  
plot(F1, X1, '-x'), title('N=64'), axis([0 1 0 20]);  
subplot(3,1,2);  
plot(F2, X2, '-x'), title('N=128'), axis([0 1 0 20]);  
subplot(3,1,3);  
plot(F3, X3, '-x'), title('N=256'), axis([0 1 0 20]);
```


FFT of a cosine (N=64, 128, 256)- results



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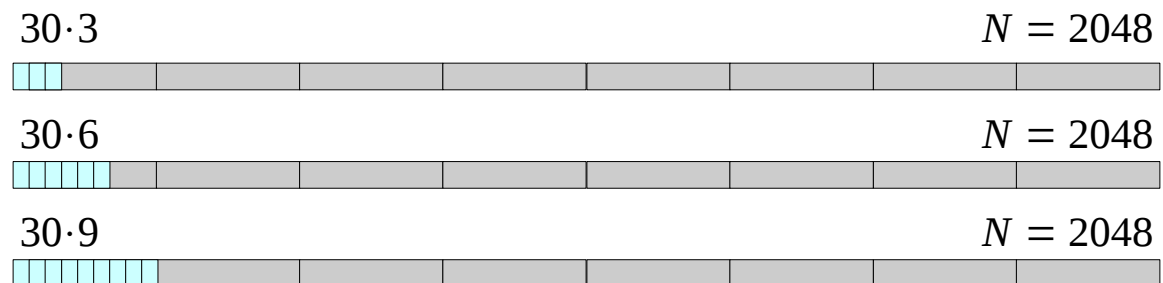
FFT of a cosine (3, 6, 9 periods)

```
n = [0:29];  
x1 = cos(2*pi*n/10);    % 3 periods  
x2 = [x1 x1];          % 6 periods  
x3 = [x1 x1 x1];       % 9 periods
```

```
N = 2048;
```

```
X1 = abs(fft(x1,N));  
X2 = abs(fft(x2,N));  
X3 = abs(fft(x3,N));
```

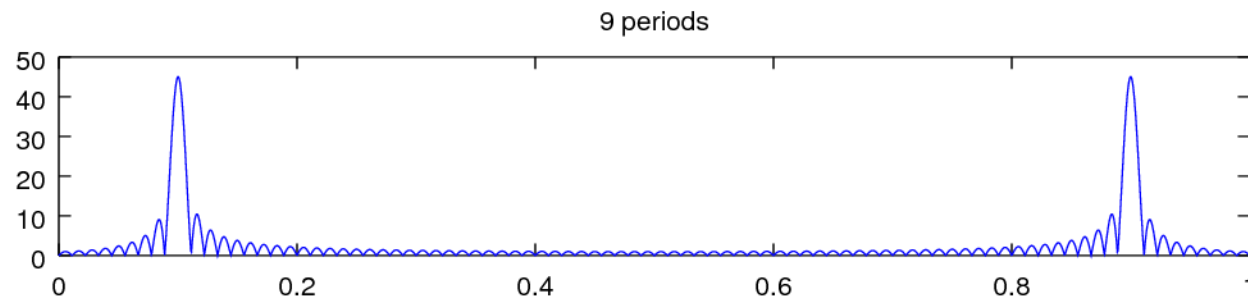
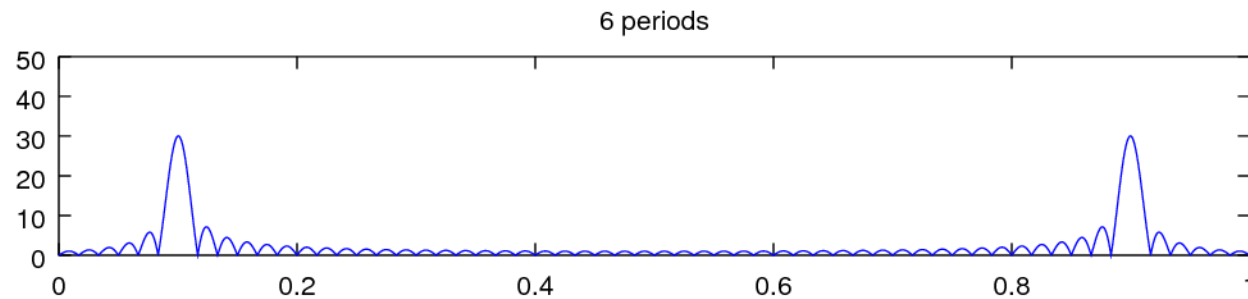
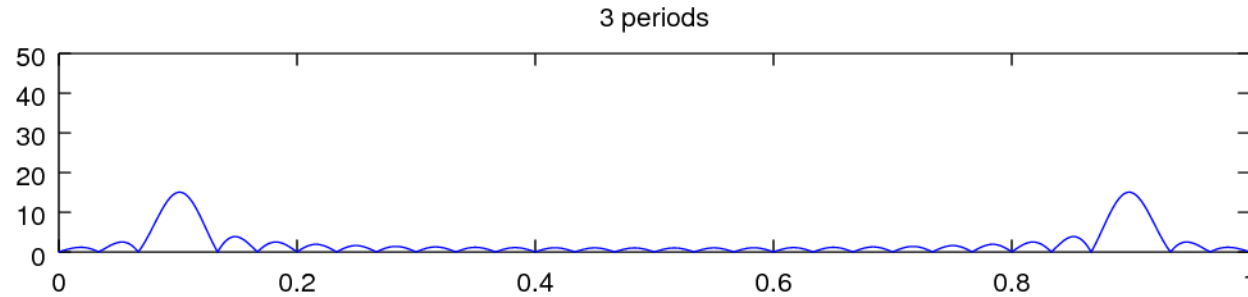
```
F = [0:N-1]/N;
```



FFT of a cosine (3, 6, 9 periods) – plot

```
subplot(3,1,1);  
plot(F, X1), title('3 periods'), axis([0 1 0 50]);  
subplot(3,1,2);  
plot(F, X2), title('6 periods'), axis([0 1 0 50]);  
subplot(3,1,3);  
plot(F, X3), title('9 periods'), axis([0 1 0 50]);
```

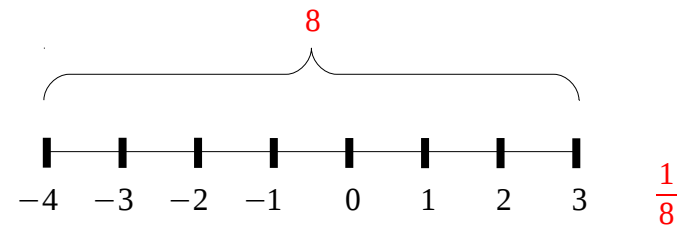
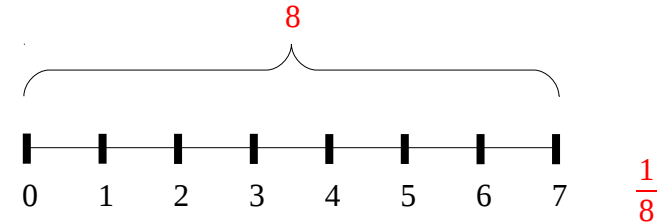
FFT of a cosine (3, 6, 9 periods) – results



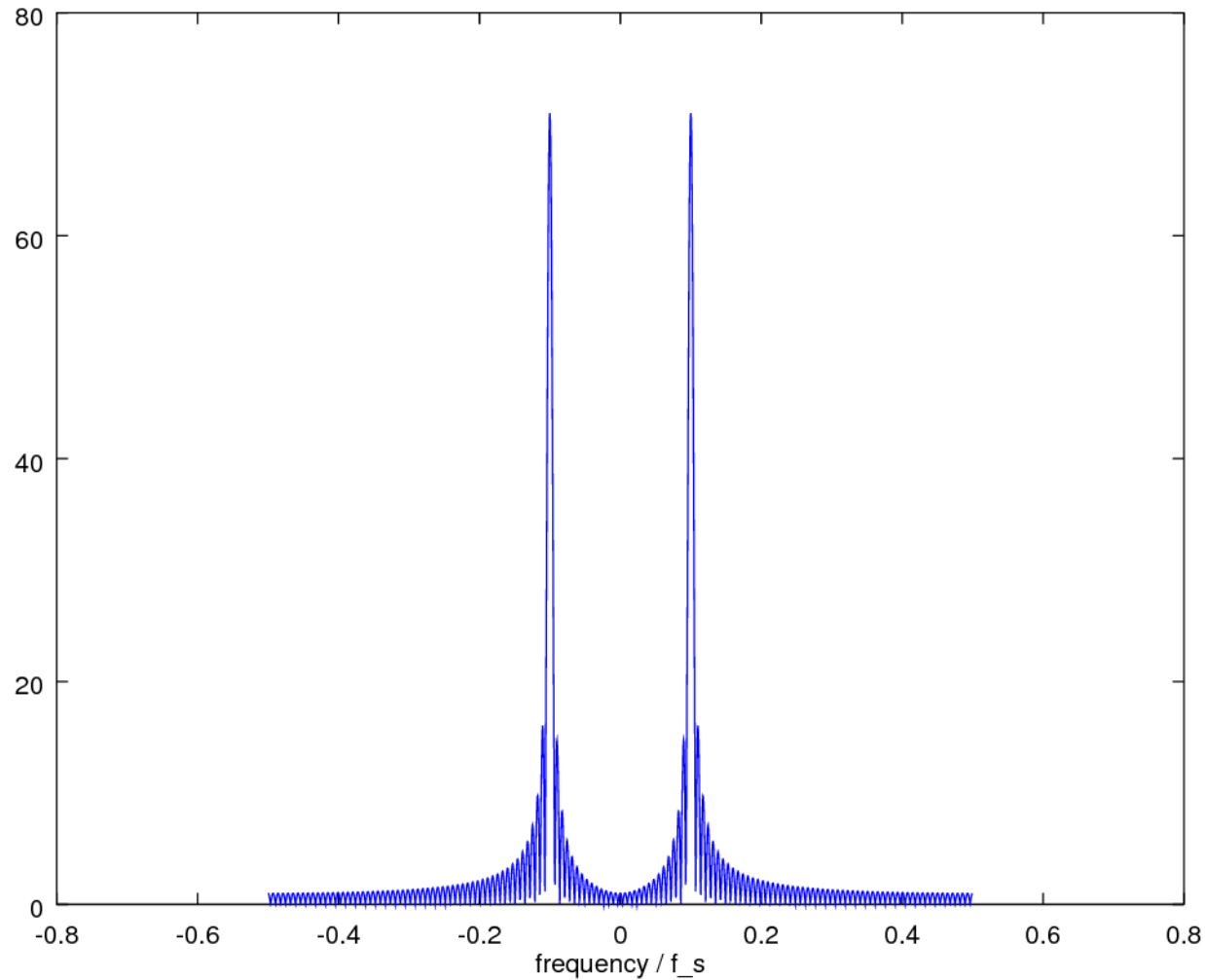
U of Rhode Island, ELE 436, FFT Tutorial

FFT Spectrum Analysis

```
n = [0: 140];  
x1= cos(2*pi*n/10);  
  
N= 2048;  
  
X = abs(fft(x1,N));  
X = fftshift(X);  
  
F = [-N/2:N/2-1]/N;  
  
plot(F, X);  
xlabel('frequency / f_s');
```



FFT Spectrum Analysis

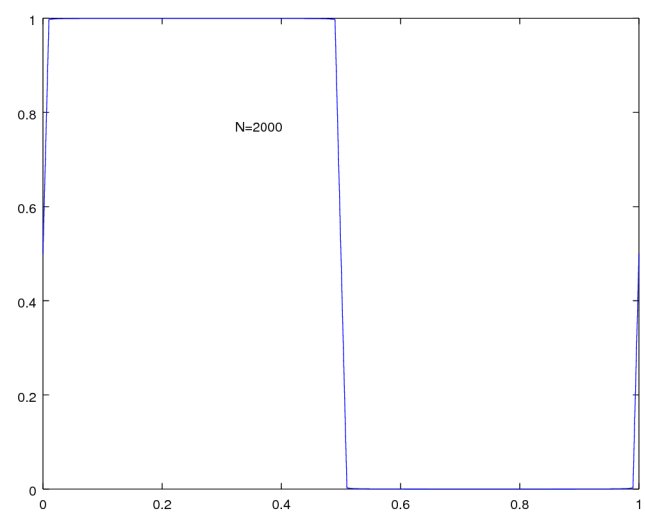
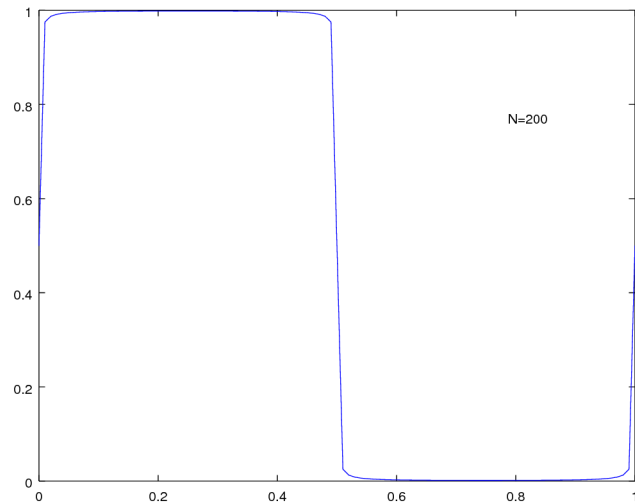
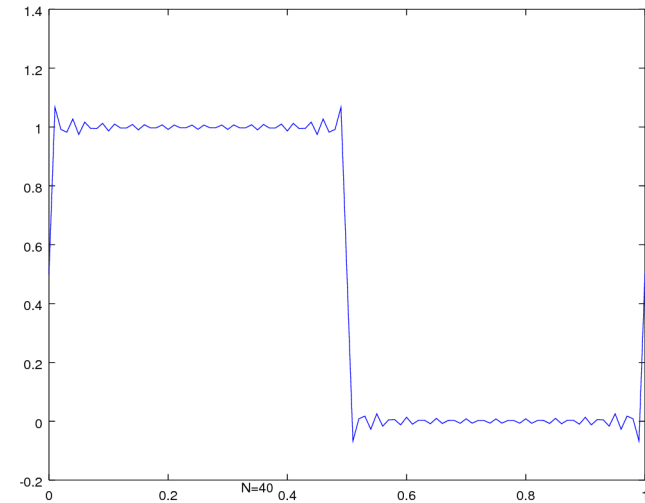
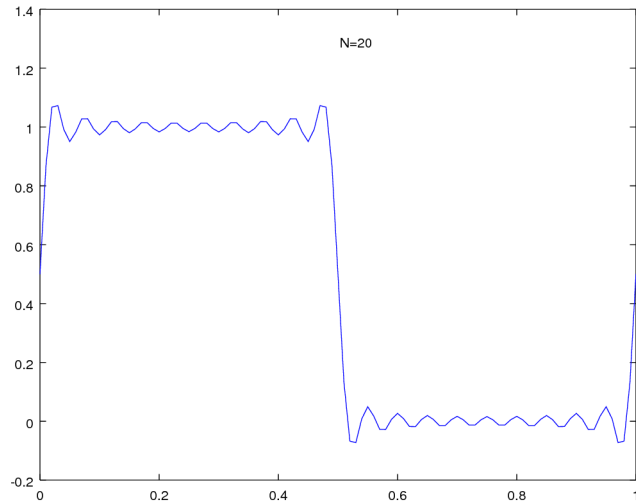


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Normalized ω_s and ω_0

```
N = 200;  
x = [0:100]/100;  
f = ones(1,101)*1/2;  
for i = 1:2:N  
    a = 2/pi/i;  
    f = f + a*sin(2*pi*i*x);  
end  
  
plot(x, f);
```

Normalized ω_s and ω_0



Normalized ω_s and ω_0

```
N = 8;  
t = [0:N-1]'/N;  
f = sin(2*pi*t);  
p = abs(fft(f))/(N/2);  
p = p(1:N/2).^2
```

Power Spectrum

```
N = 10000;  
T = 3.4;  
t = [0:N-1]/N;  
t = t*T;  
  
f = sin(2*pi*10*t);  
  
p = abs(fft(f))/(N/2);  
  
q = p(1:N/2).^2;  
freq = [0:N/2-1]/T;  
  
semilogy(freq,q);  
axis([0 20 0 1]);
```

Normalized ω_s and ω_0

```
Fs = 44100;  
y = wavrecord(5*Fs, Fs);  
wavplay(y, Fs);
```

DTFT Computation Example

```
k = input('the number of frequency points =');
num = input('the numerator coefficients =');
den = input('the denominator coefficients =');

w = 0 : pi/k : pi;
h = freqz(num, den, w);

% plot(w/pi, real(h));
% plot(w/pi, imag(h));
% plot(w/pi, abs(h));
% plot(w/pi, angle(h));
```

Mitra, Digital Signal Processing 2nd ed

DTFT Computation Example - plot real & imag

```
subplot(2,2,1)
plot(w/pi, real(h)); grid
title('real part');
xlabel('normalized angular frequency');
ylabel('Amplitude');
```

```
subplot(2,2,2)
plot(w/pi, imag(h)); grid
title('imaginary part');
xlabel('normalized angular frequency');
ylabel('Amplitude');
```

Mitra, Digital Signal Processing 2nd ed

DTFT Computation Example - plot mag & phase

```
subplot(2,2,3)
plot(w/pi, abs(h)); grid
title('magnitude spectrum');
xlabel('normalized angular frequency');
ylabel('Magnitude');
```

```
subplot(2,2,4)
plot(w/pi, angle(h)); grid
title('phase spectrum');
xlabel('normalized angular frequency');
ylabel('phase, radians');
```

DTFT Computation Example - input data

k = 256

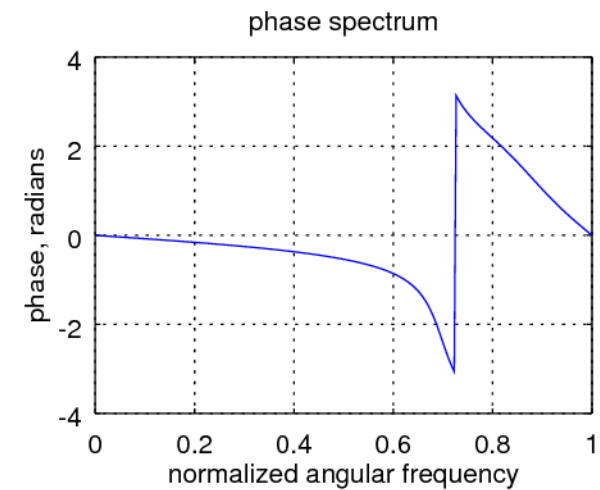
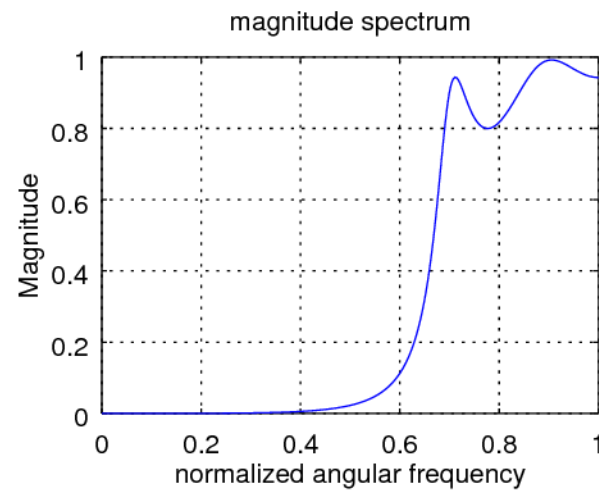
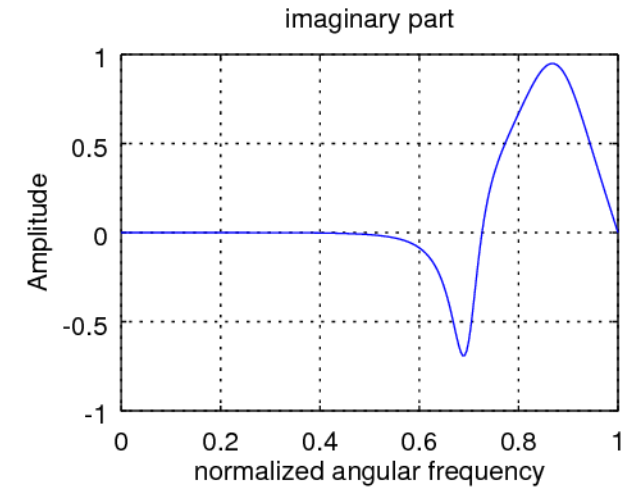
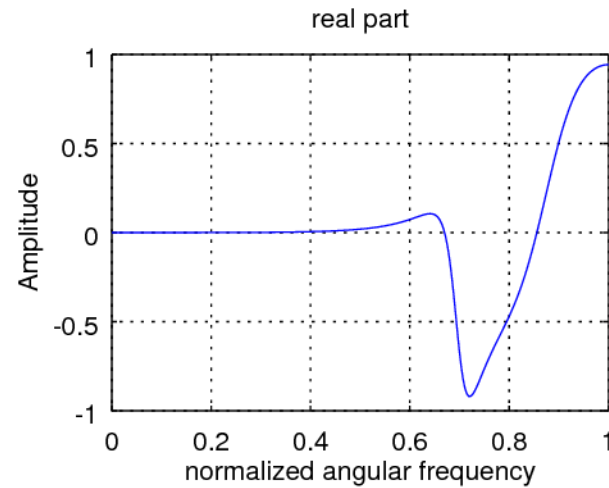
Num = [0.008 -0.033 0.05 -0.033 0.008]

Den = [1 2.37 2.7 1.6 0.41]

$$H(e^{-j\omega}) = \frac{0.008 - 0.033e^{-j\omega} + 0.05e^{-j2\omega} - 0.033e^{-j3\omega} + 0.008e^{-j4\omega}}{1 + 2.37e^{-j\omega} + 2.7e^{-j2\omega} + 1.6e^{-j3\omega} + 0.41e^{-j4\omega}}$$

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DTFT Computation Example - resulting plots



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Rect FFT

```
N = input('the length of the sequence = ');
M = input('the length of the DFT = ');

u = [ones(1,N)];
U = fft(u, M);

% t = 0:1:N-1;
% k = 0:1:M-1;
```

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Rect FFT - plot

```
t = 0:1:N-1;
stem(t, u);
title('Original time domain sequence');
xlabel('Time index n'); ylabel('Amplitude');
Pause

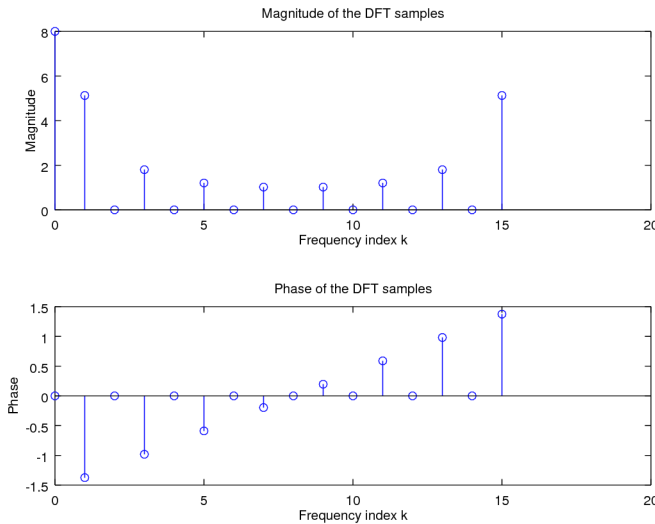
subplot(2,1,1)
k = 0:1:M-1;
stem(k, abs(U));
title('Magnitude of the DFT samples');
xlabel('Frequency index k'); ylabel('Magnitude');

subplot(2,1,2)
k = 0:1:M-1;
stem(k, angle(U));
title('Phase of the DFT samples');
xlabel('Frequency index k'); ylabel('Phase');
```

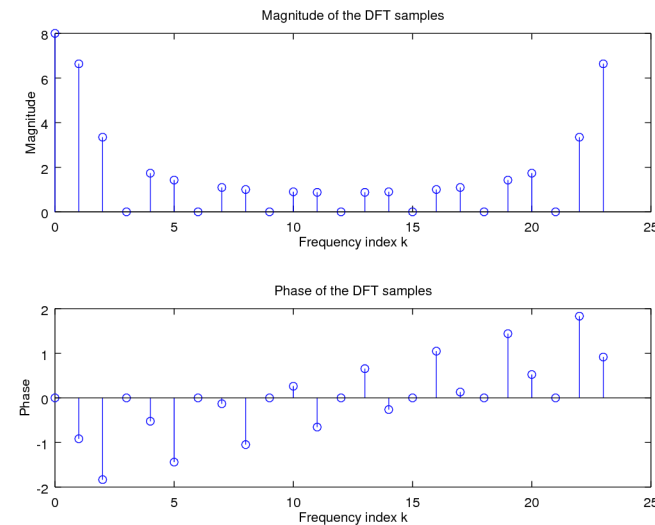
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Rect FFT - resulting plots

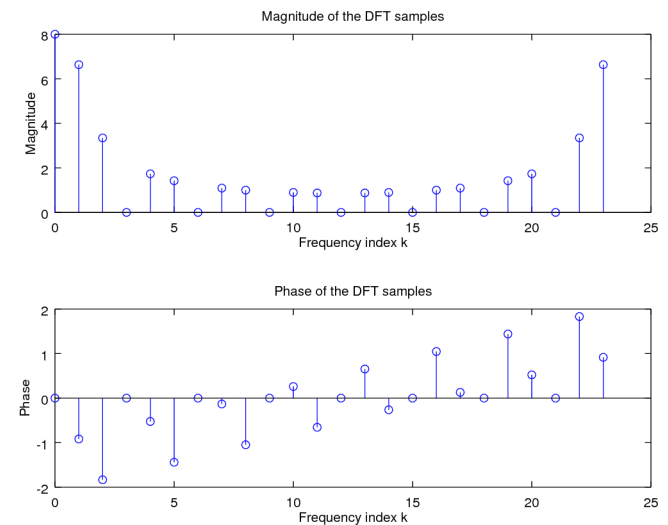
$N=8$
 $M=16$



$N=8$
 $M=24$



$N=8$
 $M=32$



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Ramp IDFT

```
clear
clf

K= input('the length of the DFT = ');
N= input('the length of the IDFT = ');

k= 1:K;
U= (k-1)/K;

u= ifft(U, N);
```

Ramp IDFT – plot

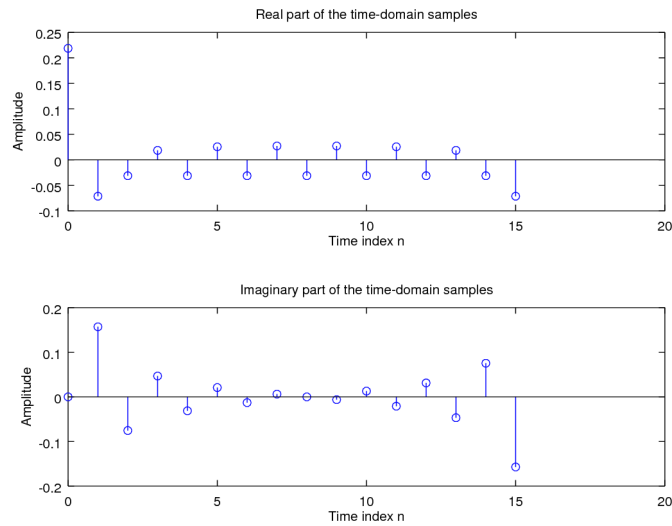
```
stem(k-1, U);
xlabel('Frequency index k');
ylabel('Amplitude');
title('Original DFT samples');
pause

subplot(2,1,1);
n= 0:1:N-1;
stem(n, real(u));
title('Real part of the time-domain samples');
xlabel('Time index n');
ylabel('Amplitude');

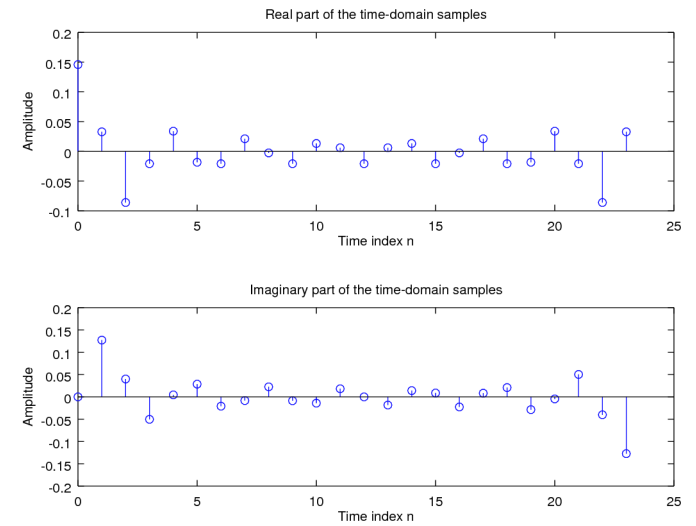
subplot(2,1,2);
n= 0:1:N-1;
stem(n, imag(u));
title('Imaginary part of the time-domain samples');
xlabel('Time index n');
ylabel('Amplitude');
```

Ramp IDFT – resulting plots

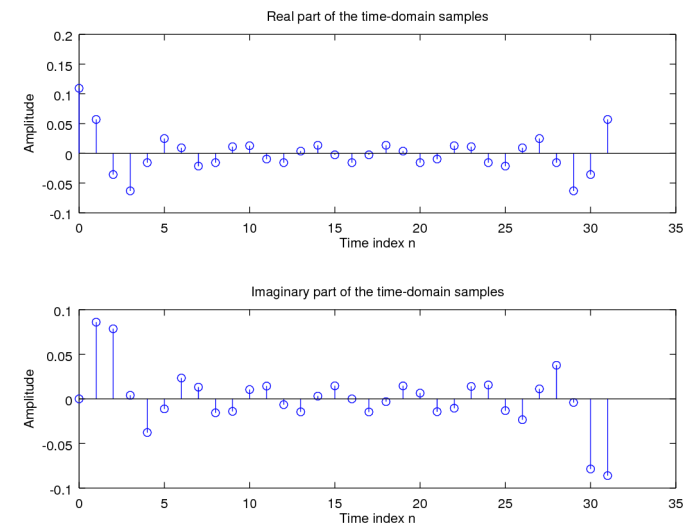
K=8
N=16



K=8
N=24



K=8
N=24



Numerical Computation of DTFT

```
clear
clf

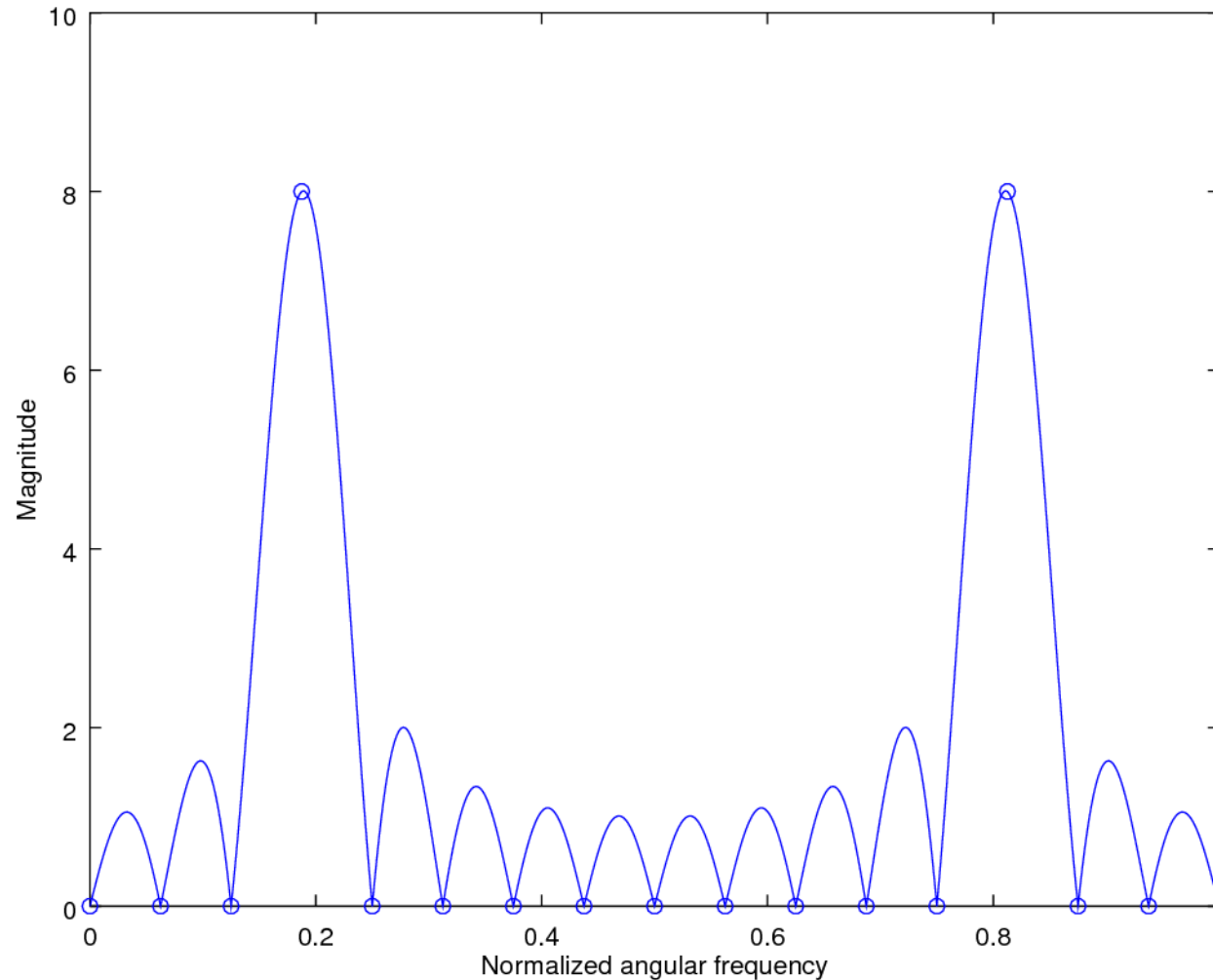
k = 0:15;
x = cos(2*pi*k*3/16);

X = fft(x);
XE = fft(x, 512);

L = 0:511;
plot(L/512, abs(XE));
hold

plot(k/16, abs(X), 'o');
xlabel('Normalized angular frequency');
ylabel('Magnitude');
```

Numerical Computation of DTFT – results



Correlation

Correlation

```
n = 0:127;
```

```
x = [ones(1,25), -ones(1,25), zeros(1,78)];
```

```
y = [0:24, 25:-1:1, zeros(1,78)]/25;
```

```
yy = [y, y];
```

```
for m=0:127
```

```
    phi(m+1) = (2/128)*x*yy(m+1:m+128);
```

```
end
```

DFT

```
N = 50;
```

```
n = [0:N-1];
```

```
m = [0:N-1];
```

```
x = [zeros(1,28), ones(1,12), zeros(1, N-40)];
```

```
X = x*exp(-j*2*pi*m'*n/N);
```

```
flops(0);
```

```
fft(x,N);
```

```
f(N)=flops;
```

```
n = 0:99;  
x = sin(2*pi*(n-50.5)/5)./(n-50.5);  
X = fftshift(fft(x));  
amplitude = abs(X);  
phase = unwrap(angle(X));
```

```
N1 = 32; N2 = 128;  
x = [ones(1,N1/2), -ones(1,N1/2)];  
X = fft(x, N1);  
Y = fft(x, N2);
```

```
X = fft(x);  
Y = [X(1:(N+1)/2), zeros(1,K*N), X((N+1)/2+1:N)];  
Y = (K+1)*ifft(Y);
```

Numerical Computation of DTFT

A

Numerical Computation of DTFT

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References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] M.J. Roberts, Fundamentals of Signals and Systems
- [4] S.J. Orfanidis, Introduction to Signal Processing
- [5] K. Shin, et al., Fundamentals of Signal Processing for Sound and Vibration Engineerings

- [6] A “graphical interpretation” of the DFT and FFT, by Steve Mann