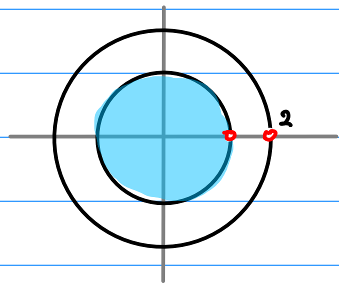
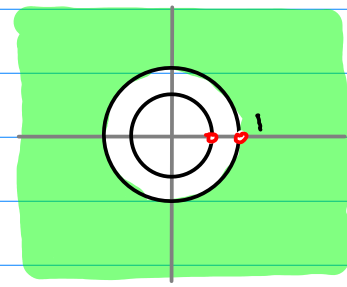
Laurent Series and z-Transform Examples case 0.A

20171208

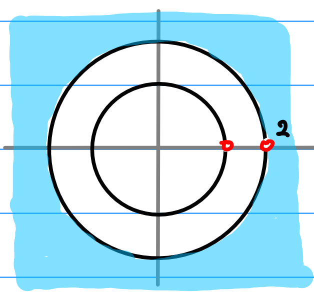
1. A

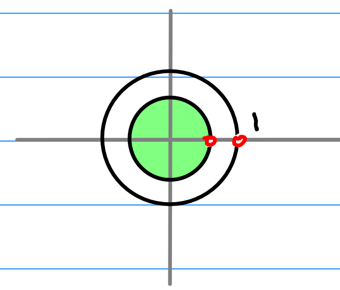
$$f(z) = \frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} X(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$



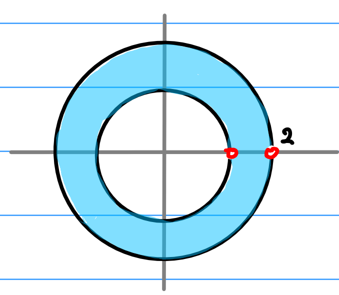


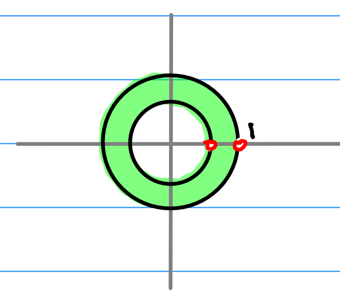
$$\sum_{n=0}^{\infty}\left[\left(\tfrac{1}{2}\right)^{n+1}-1\right]z^{n} \xrightarrow{z^{-1}} \sum_{n=0}^{\infty}\left[\left(\tfrac{1}{2}\right)^{n+1}-1\right]z^{-n}$$





$$\sum_{n=-1}^{-\infty}\left[1-\left(\tfrac{1}{2}\right)^{n+1}\right]z^{n} \xrightarrow{z^{-1}} \sum_{n=-1}^{-\infty}\left[1-\left(\tfrac{1}{2}\right)^{n+1}\right]z^{-n}$$

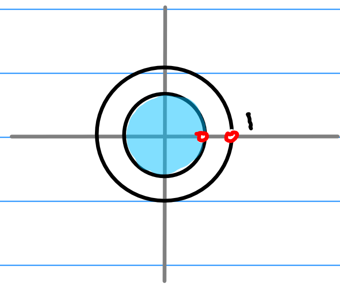


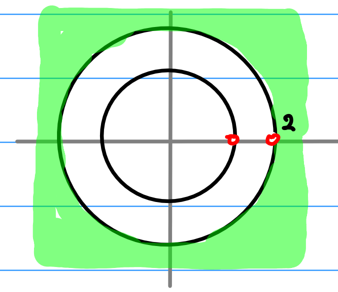


$$\sum_{n=-1}^{-\infty} 1\cdot z^{n} + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1} z^{n} \xrightarrow{z^{-1}} \sum_{n=-1}^{-\infty} 1\cdot z^{-n} + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1} z^{-n}$$

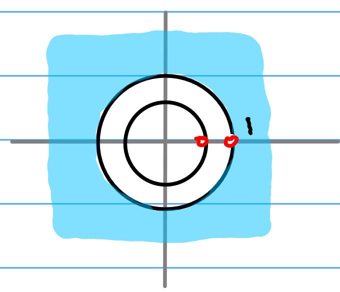
2.A

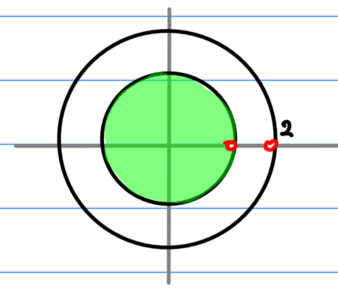
$$f(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} X(z) = \frac{-1}{(z-1)(z-2)}$$



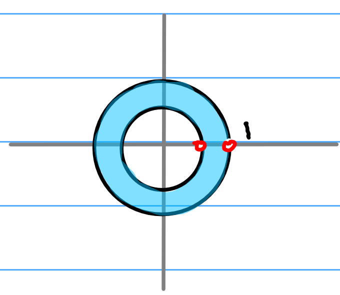


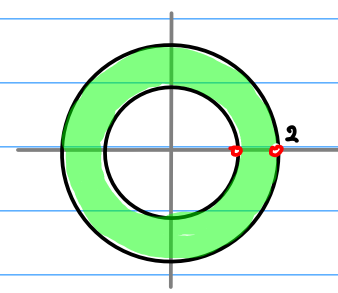
$$\sum_{n=1}^{\infty} \left[1 - 2^{n-1}\right] z^{n} \xrightarrow{z^{-1}} \sum_{n=1}^{\infty} \left[1 - 2^{n-1}\right] z^{-n}$$





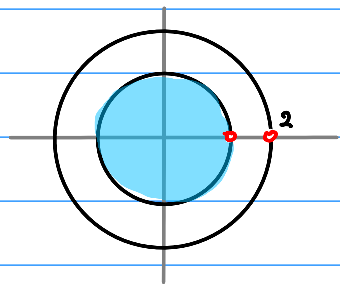
$$\sum_{n=0}^{-\infty} \left[2^{n-1} - 1\right] z^{n} \xrightarrow{z^{-1}} \sum_{n=0}^{-\infty} \left[2^{n-1} - 1\right] z^{-n}$$

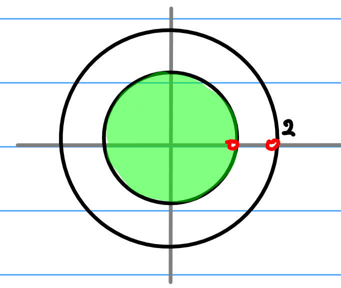




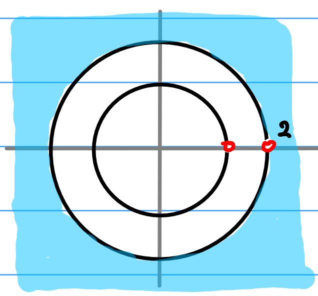
$$\sum_{n=1}^{\infty} 1 \cdot z^{n} + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{n} \xrightarrow{z^{-1}} \sum_{n=1}^{\infty} 1 \cdot z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n}$$

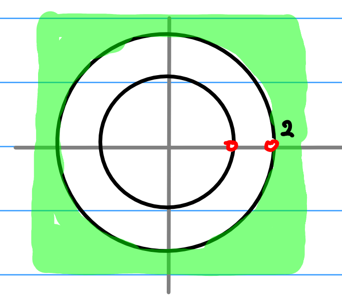
3 . A $f(z) = \frac{-1}{(z-1)(z-2)}$ $=\!=$ $X(z) = \frac{-1}{(z-1)(z-2)}$



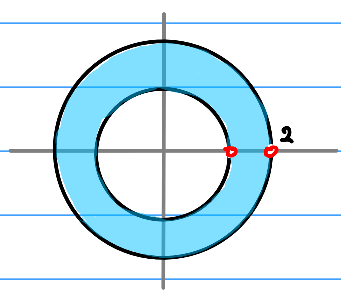


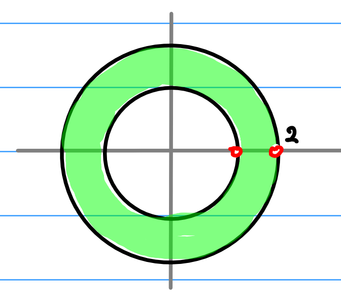
$$\sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1 \right] z^{n} \xrightarrow{-n} \sum_{n=0}^{-\infty} \left[2^{n-1} - 1 \right] z^{-n}$$





$$\sum_{n=-1}^{-\infty} \left[1 - \left(\tfrac{1}{2}\right)^{n+1} \right] z^{n} \xrightarrow{-n} \sum_{n=1}^{\infty} \left[1 - 2^{n-1} \right] z^{-n}$$

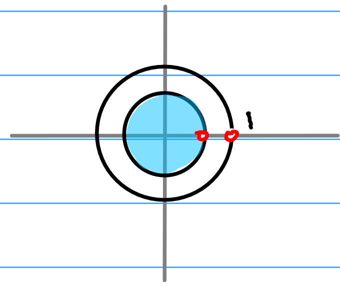


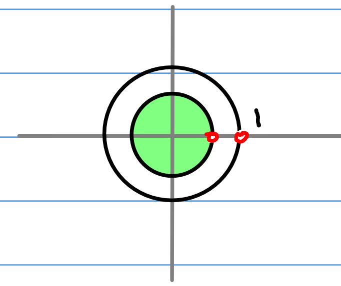


$$\sum_{n=-1}^{-\infty} z^{n} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^{n} \xrightarrow{-n} + \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

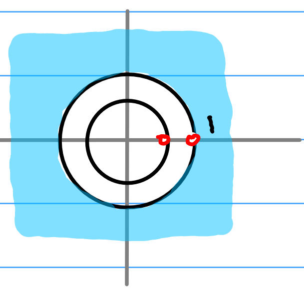
4.A

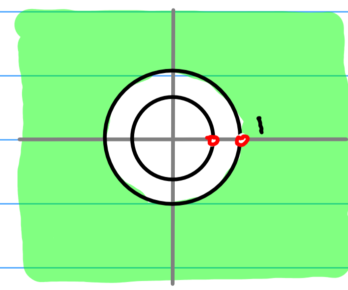
$$f(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)} = X(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$



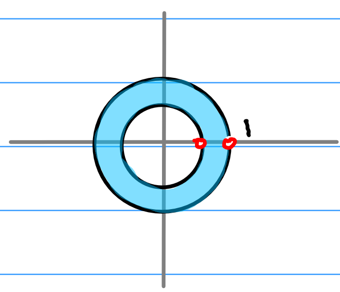


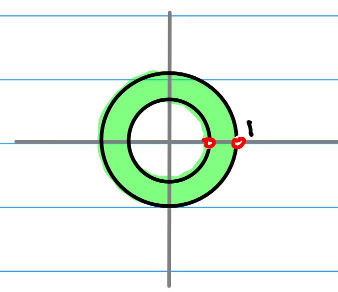
$$\sum_{n=1}^{\infty}\left[1-2^{n-1}\right]z^{n} \xrightarrow{-n} \sum_{n=-1}^{\infty}\left[1-\left(\tfrac{1}{2}\right)^{n+1}\right]z^{-n}$$





$$\sum_{n=-1}^{-\infty}\left[2^{n-1}-1\right]z^{n} \xrightarrow{-n} \sum_{n=1}^{\infty}\left[\left(\tfrac{1}{2}\right)^{n+1}-1\right]z^{-n}$$





$$+\sum_{n=1}^{\infty}z^{n} + \sum_{n=0}^{-\infty}2^{n-1}z^{n} \xrightarrow{-n} +\sum_{n=-1}^{-\infty}z^{-n} + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}z^{-n}$$

1. A $\quad f(z) = \dfrac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} X(z) = \dfrac{-0.5\, z^2}{(z-1)(z-0.5)}$

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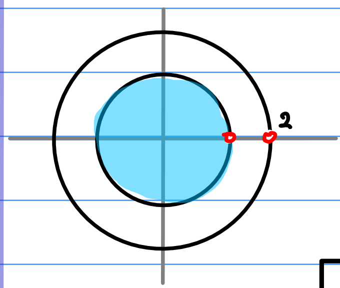

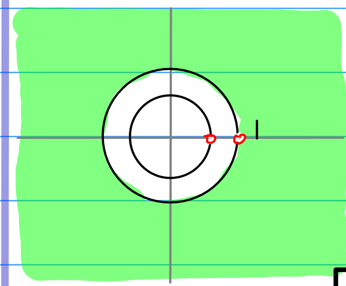

$$a_n = \boxed{\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}} \quad = \quad x_n = \boxed{\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}}$$

$$f(z) = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^n \qquad X(z) = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^{-n}$$

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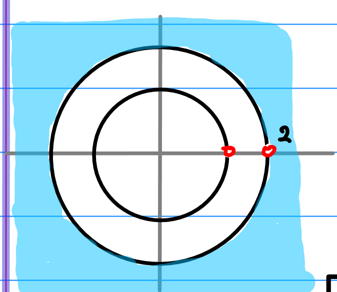

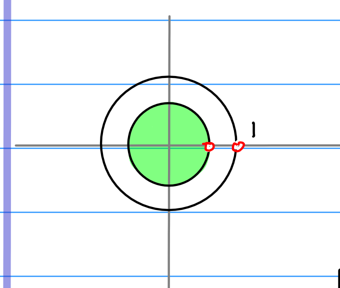

$$a_n = \boxed{\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}} \quad = \quad x_n = \boxed{\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}}$$

$$f(z) = \sum_{n=-1}^{-\infty} \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) z^n \qquad X(z) = \sum_{n=-1}^{-\infty} \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) z^{-n}$$

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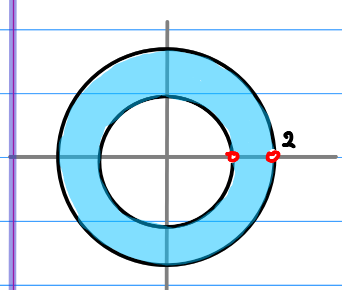

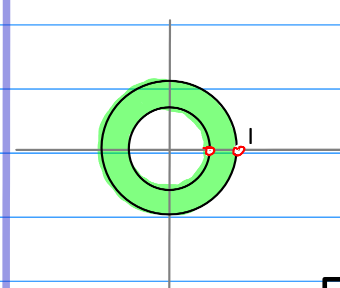

$$a_n = \boxed{\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}} \quad = \quad x_n = \boxed{\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}}$$

$$f(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n \qquad X(z) = \sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n}$$

2.A

$$f(z) = \frac{-0.5\, z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} X(z) = \frac{-1}{(z-1)(z-2)}$$

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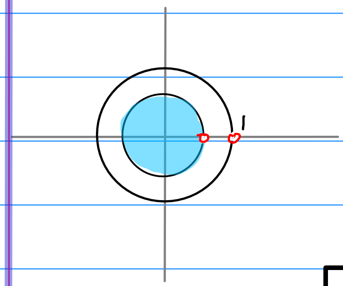

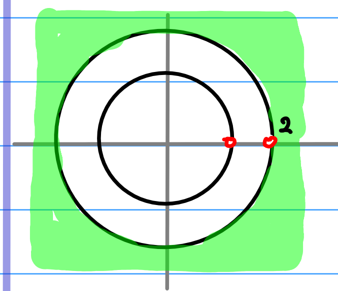

$$a_n = \boxed{\begin{matrix} [1-2^{n-1}] & (n>0) \\ 0 & (n\le 0) \end{matrix}} \quad = \quad x_n = \boxed{\begin{matrix} [1-2^{n-1}] & (n>0) \\ 0 & (n\le 0) \end{matrix}}$$

$$f(z) = \sum_{n=1}^{\infty} [1-2^{n-1}]\, z^n \qquad X(z) = \sum_{n=1}^{\infty} [1-2^{n-1}]\, z^{-n}$$

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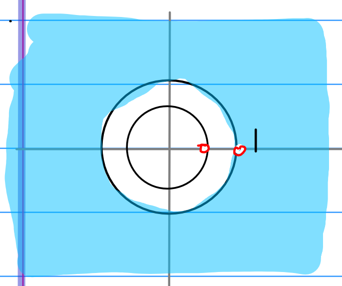

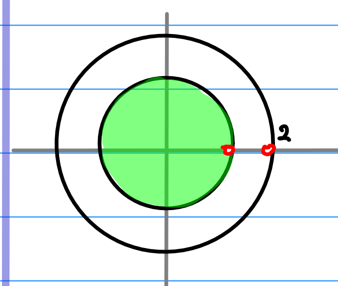

$$a_n = \boxed{\begin{matrix} 0 & (n>0) \\ [2^{n-1}-1] & (n\le 0) \end{matrix}} \quad = \quad x_n = \boxed{\begin{matrix} 0 & (n>0) \\ [2^{n-1}-1] & (n\le 0) \end{matrix}}$$

$$f(z) = \sum_{n=0}^{-\infty} [2^{n-1}-1]\, z^n \qquad X(z) = \sum_{n=0}^{-\infty} [2^{n-1}-1]\, z^{-n}$$

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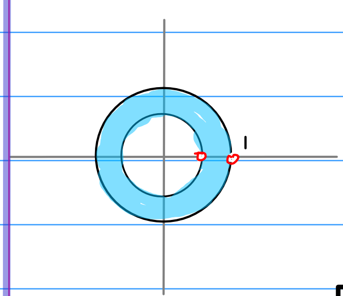

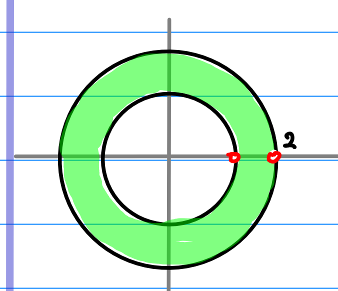

$$a_n = \boxed{\begin{matrix} 1 & (n>0) \\ 2^{n-1} & (n\le 0) \end{matrix}} \quad = \quad x_n = \boxed{\begin{matrix} 1 & (n>0) \\ 2^{n-1} & (n\le 0) \end{matrix}}$$

$$f(z) = \sum_{n=1}^{\infty} 1\cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^n \qquad X(z) = \sum_{n=1}^{\infty} 1\cdot z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^{-n}$$

3 . A $f(z) = \frac{-1}{(z-1)(z-2)}$ $=$ $X(z) = \frac{-1}{(z-1)(z-2)}$

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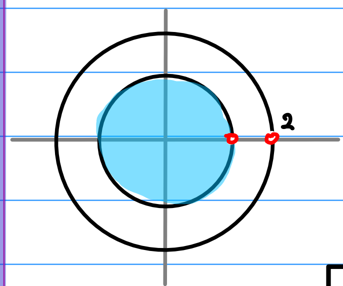

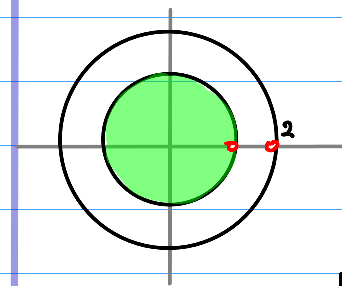

$a_n = \boxed{\begin{array}{ll} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{array}}$ $\xrightarrow{-n}$ $x_n = \boxed{\begin{array}{ll} 0 & (n > 0) \\ [2^{n-1} - 1] & (n \leq 0) \end{array}}$

$f(z) = \sum_{n=0}^{\infty} [(\frac{1}{2})^{n+1} - 1] z^n$

$X(z) = \sum_{n=0}^{-\infty} [2^{n-1} - 1] z^{-n}$

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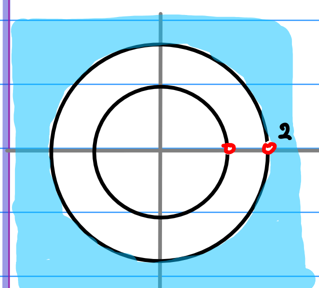

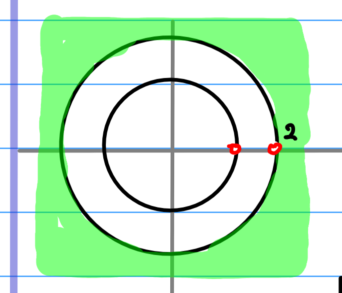

$a_n = \boxed{\begin{array}{ll} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{array}}$ $\xrightarrow{-n}$ $x_n = \boxed{\begin{array}{ll} [1 - 2^{n-1}] & (n > 0) \\ 0 & (n \leq 0) \end{array}}$

$f(z) = \sum_{n=-1}^{-\infty} [1 - (\frac{1}{2})^{n+1}] z^n$

$X(z) = \sum_{n=1}^{\infty} [1 - 2^{n-1}] z^{-n}$

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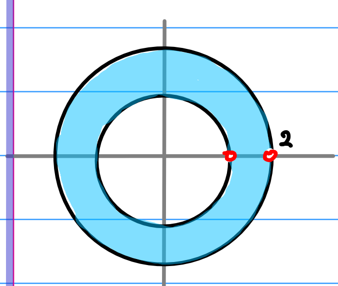

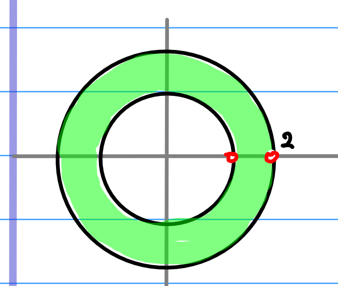

$a_n = \boxed{\begin{array}{ll} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{array}}$ $\xrightarrow{-n}$ $x_n = \boxed{\begin{array}{ll} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{array}}$

$-n$

$f(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n$

$X(z) = + \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$

4.A $\quad f(z)=\dfrac{-0.5z^2}{(z-1)(z-0.5)} \quad = \quad X(z)=\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

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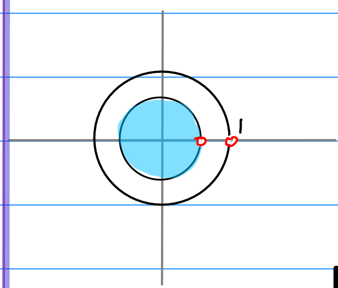

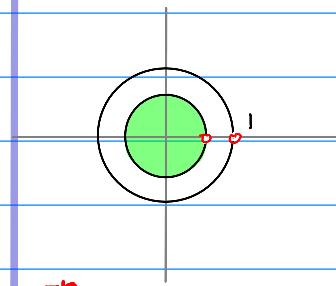

$$a_n=\boxed{\begin{matrix}[1-2^{n-1}] & (n>0)\\ 0 & (n\le 0)\end{matrix}} \xrightarrow{-n} x_n=\boxed{\begin{matrix}0 & (n\ge 0)\\ 1-(\frac{1}{2})^{n+1} & (n<0)\end{matrix}}$$

$$f(z)=\sum_{n=1}^{\infty}[1-2^{n-1}]z^n \qquad X(z)=\sum_{n=-1}^{\infty}[1-(\tfrac{1}{2})^{n+1}]z^{-n}$$

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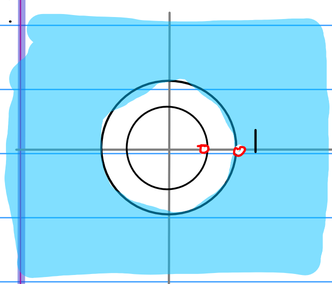

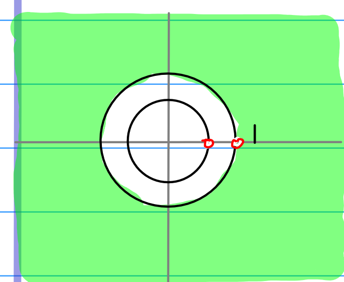

$$a_n=\boxed{\begin{matrix}0 & (n>0)\\ [2^{n-1}-1] & (n\le 0)\end{matrix}} \xrightarrow{-n} x_n=\boxed{\begin{matrix}(\frac{1}{2})^{n+1}-1 & (n\ge 0)\\ 0 & (n<0)\end{matrix}}$$

$$f(z)=\sum_{n=-1}^{-\infty}[2^{n-1}-1]z^n \qquad X(z)=\sum_{n=1}^{\infty}[(\tfrac{1}{2})^{n+1}-1]z^{-n}$$

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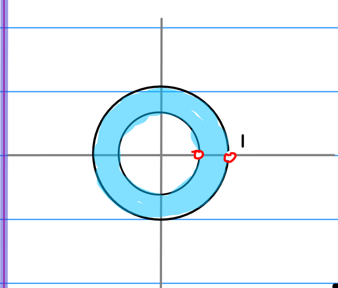
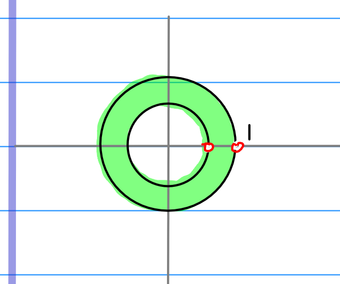
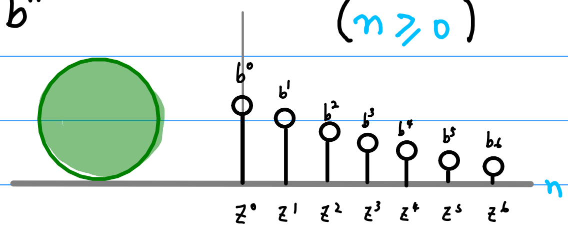




$$a_n=\boxed{\begin{matrix}1 & (n>0)\\ 2^{n-1} & (n\le 0)\end{matrix}} \xrightarrow{-n} x_n=\boxed{\begin{matrix}(\frac{1}{2})^{n+1} & (n\ge 0)\\ 1 & (n<0)\end{matrix}}$$

$$f(z)=+\sum_{n=1}^{\infty}z^n+\sum_{n=0}^{-\infty}2^{n-1}z^n \qquad X(z)=+\sum_{n=-1}^{-\infty}z^{-n}+\sum_{n=0}^{\infty}(\tfrac{1}{2})^{n+1}z^{-n}$$

b^n $(n \geqslant 0)$



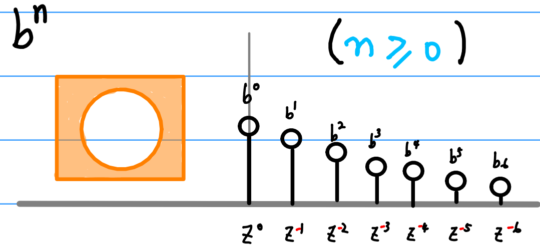
$$X(z^{-1}) = \frac{z^{-1}}{z^{-1} - 0.5} \qquad |z| < 2$$

$$f(z) = \frac{2}{2-z} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^n$$

$$a_n = \left(\frac{1}{2}\right)^n$$

$$= p^{-n} \qquad \boxed{p = 2}$$

b^n $(n \geqslant 0)$



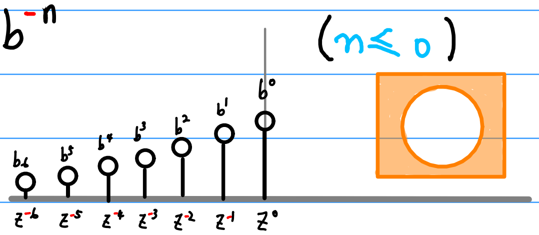
$$|z| > \frac{1}{2}$$

$$X(z) = \frac{z}{z - 0.5} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$$

$$x_n = \left(\frac{1}{2}\right)^n$$

$$= p^n \qquad \boxed{p = \frac{1}{2}}$$

b^{-n} $(n \leqslant 0)$



$$X(z^{-1}) = \frac{2}{2 - z^{-1}} \qquad |z| > \frac{1}{2}$$

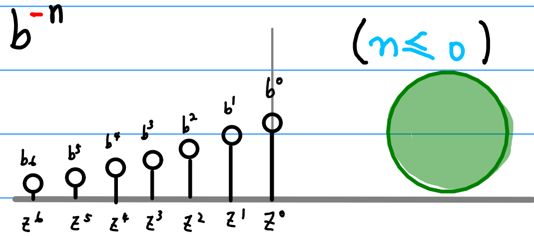
$$f(z) = \frac{z}{z - 0.5} = \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{-n} z^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$$

$$a_n = \left(\frac{1}{2}\right)^{-n}$$

$$= p^{-n} \qquad \boxed{p = \frac{1}{2}}$$

b^{-n} $(n \leqslant 0)$



$$|z| < 2$$

$$X(z) = \frac{2}{2 - z} = \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{-n} z^{-n}$$

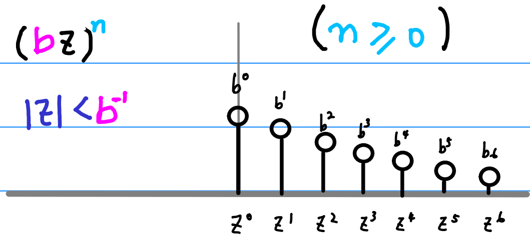
$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^n$$

$$x_n = \left(\frac{1}{2}\right)^{-n}$$

$$= p^n \qquad \boxed{p = 2}$$

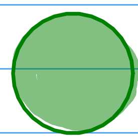
$(bz)^n$ $\quad (n \geqslant 0)$

$|z| < b^{-1}$



$$f(z) = \frac{1}{1-bz} = \frac{b^{-1}}{b^{-1}-z}$$

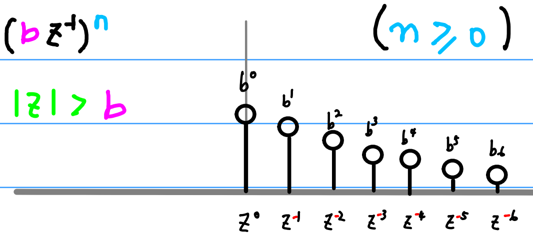
$$a_n = b^n$$
$$= p^{-n}$$



$$\boxed{p = b^{-1}}$$

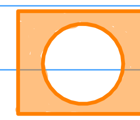
$(bz^{-1})^n$ $\quad (n \geqslant 0)$

$|z| > b$



$$X(z) = \frac{1}{1-b/z} = \frac{z}{z-b}$$

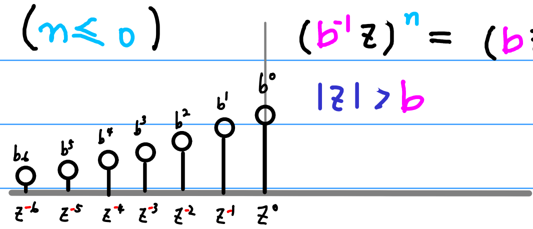
$$x_n = b^n$$
$$= p^n$$



$$\boxed{p = b}$$

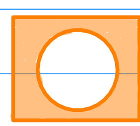
$(n \leqslant 0)$ $\quad (b^{-1}z)^n = (bz^{-1})^{-n}$

$|z| > b$



$$f(z) = \frac{1}{1-(bz^{-1})} = \frac{z}{z-b}$$

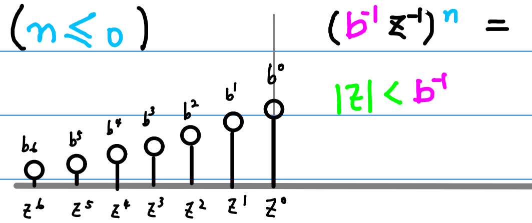
$$a_n = b^{-n}$$
$$= p^{-n}$$



$$\boxed{p = b}$$

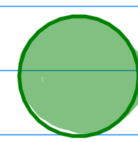
$(n \leqslant 0)$ $\quad (b^{-1}z^{-1})^n = (bz)^{-n}$

$|z| < b^{-1}$



$$X(z) = \frac{1}{1-(bz)} = \frac{b^{-1}}{b^{-1}-z}$$

$$x_n = b^{-n}$$
$$= p^n$$

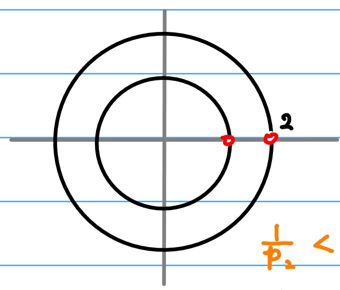


$$\boxed{p = b^{-1}}$$

$$f(z) \xrightarrow{z^{-1}} X(z)$$

1.A

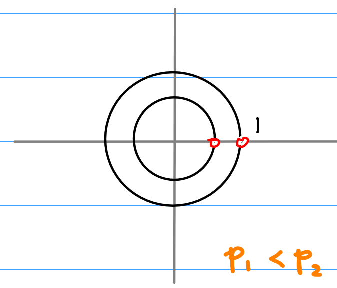
$$\frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$



$\frac{1}{p_2} < \frac{1}{p_1}$

$\frac{1}{2} \quad \frac{1}{1}$

$\boxed{\begin{array}{l} p_1 = 1 \\ p_2 = 2 \end{array}}$



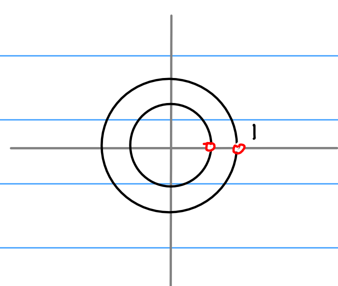
$p_1 < p_2$

$0.5 \quad 1.0$

$\boxed{\begin{array}{l} p_1 = 0.5 \\ p_2 = 1 \end{array}}$

2.A

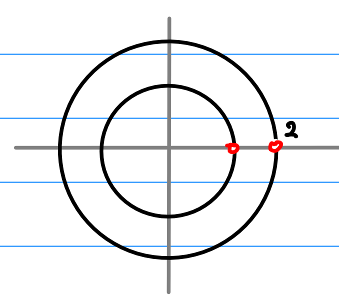
$$\frac{-0.5\,z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} \frac{-1}{(z-1)(z-2)}$$



$\frac{1}{p_2} < \frac{1}{p_1}$

$\frac{1}{1} \quad \frac{1}{1/2}$

$\boxed{\begin{array}{l} p_1 = 0.5 \\ p_2 = 1 \end{array}}$



$p_1 < p_2$

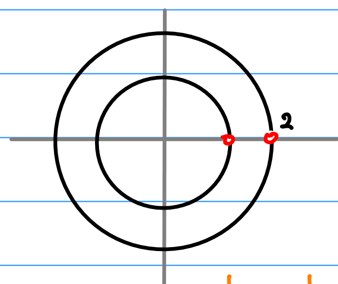
$1 \quad 2$

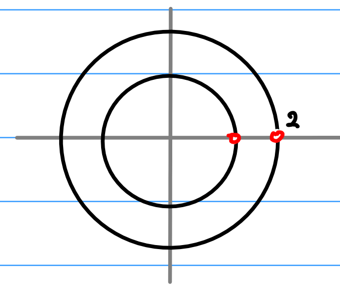
$\boxed{\begin{array}{l} p_1 = 1 \\ p_2 = 2 \end{array}}$

$$f(z) = X(z)$$

3.A

$$\frac{-1}{(z-1)(z-2)} = \frac{-1}{(z-1)(z-2)}$$





$\frac{1}{p_2} < \frac{1}{p_1}$ $\quad \boxed{p_1 = 1,\ p_2 = 2}$

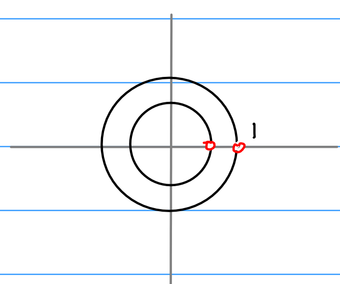
$\frac{1}{2} \quad \frac{1}{1}$

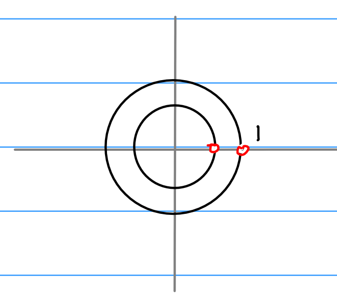
$p_1 < p_2$ $\quad \boxed{p_1 = 1,\ p_2 = 2}$

$1 \quad 2$

4.A

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = \frac{-0.5z^2}{(z-1)(z-0.5)}$$





$\frac{1}{p_2} < \frac{1}{p_1}$ $\quad \boxed{p_1 = 0.5,\ p_2 = 1}$

$\frac{1}{1} \quad \frac{1}{1/2}$

$p_1 < p_2$ $\quad \boxed{p_1 = 0.5,\ p_2 = 1}$

$1 \quad 2$

$$f(z) \overset{Z^{-1}}{\longleftrightarrow} X(z) \qquad a_n = x_n$$

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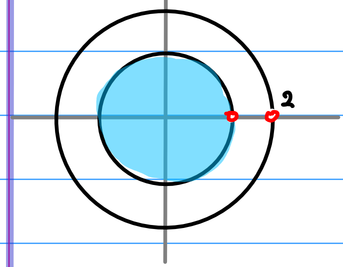
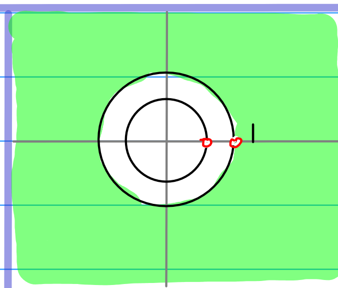


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$



$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

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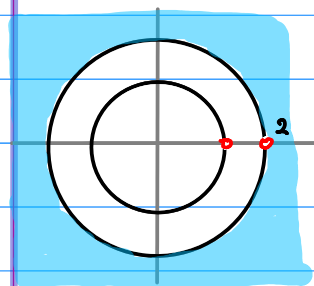
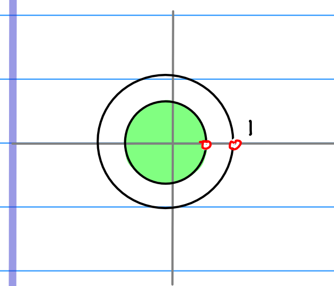


$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$



$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

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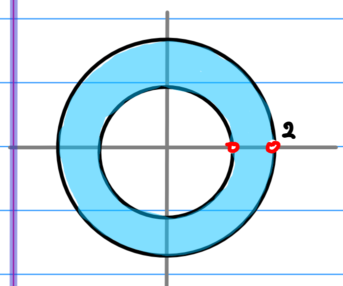
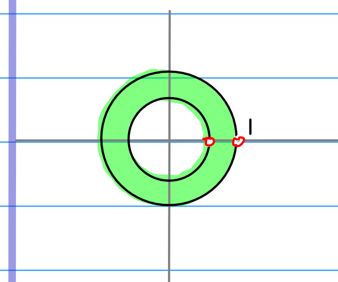


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$



$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

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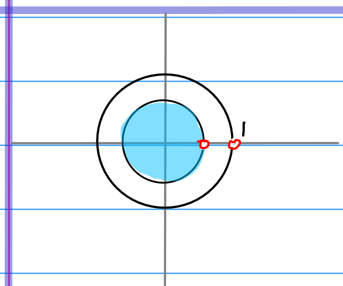
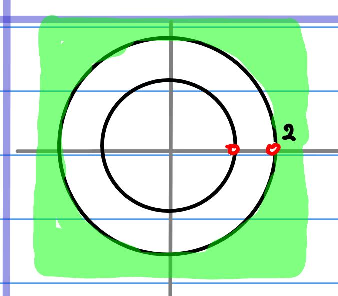
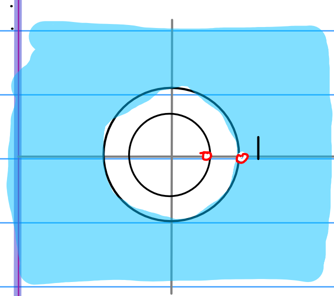


$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

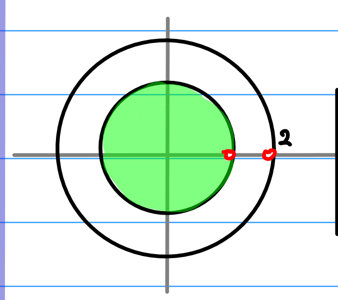


$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

Ⅱ



$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$



$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

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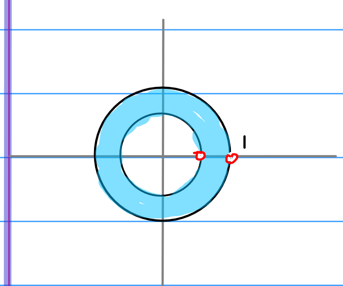
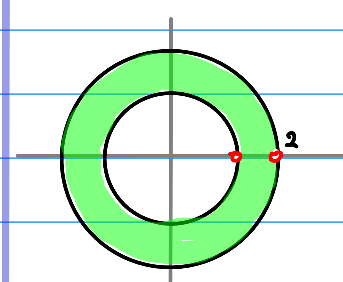


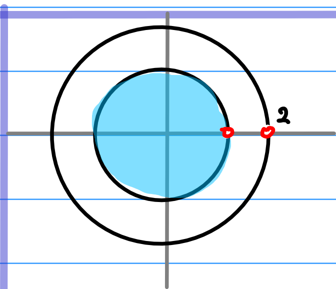
$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$



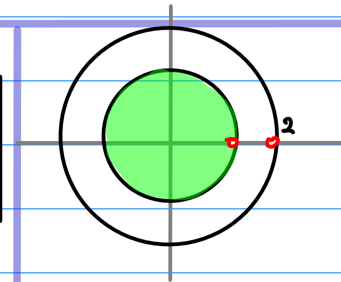
$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$f(z) = X(z) \qquad a_n \xleftrightarrow{-n} x_n$$

Ⅰ



$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$



$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

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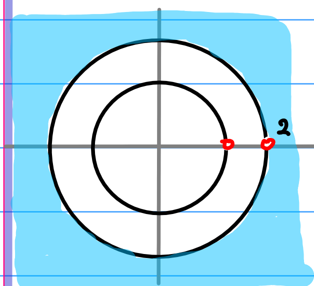
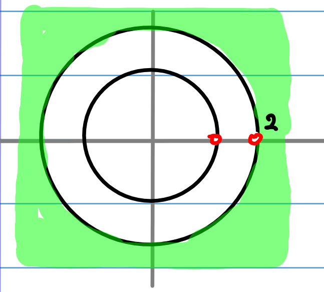


$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$



$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

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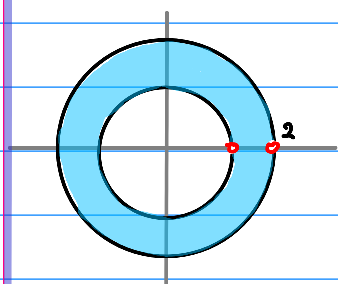
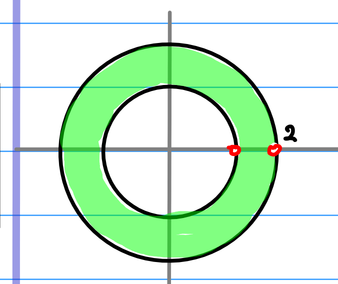
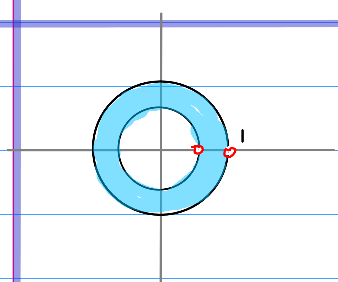


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

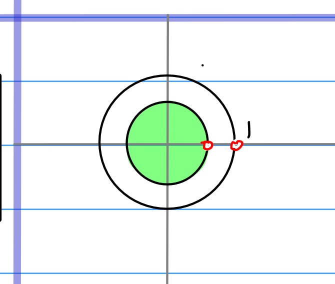


$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

Ⅰ



$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$



$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

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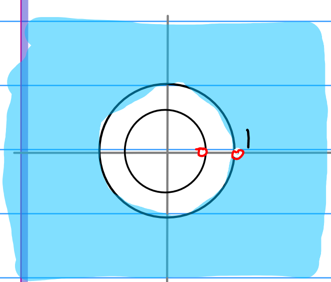
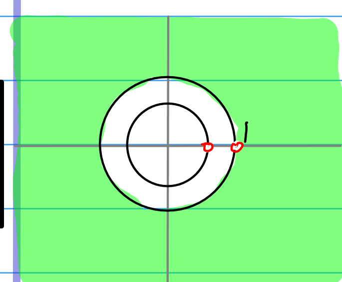


$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$



$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

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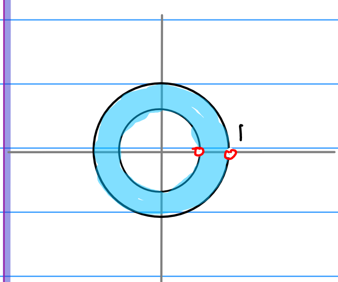
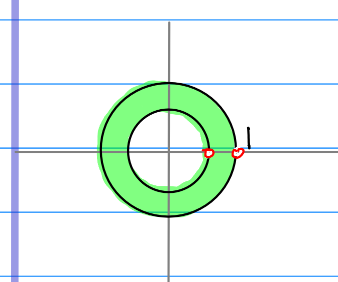


$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$



$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) \xleftrightarrow{Z^{-1}} X(z) \qquad a_n = x_n$$

Ⅰ

$P_1 = 1$, $P_2 = 2$

$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$P_1 = 0.5$, $P_2 = 1$

$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$*-1$ $\qquad$ $*-1$

Ⅱ

$P_1 = 1$, $P_2 = 2$

$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

$P_1 = 0.5$, $P_2 = 1$

$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

Ⅲ

$P_1 = 1$, $P_2 = 2$

$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$P_1 = 0.5$, $P_2 = 1$

$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

Ⅰ

$P_1 = 0.5$, $P_2 = 1$

$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$P_1 = 1$, $P_2 = 2$

$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$*-1$ $\qquad$ $*-1$

Ⅱ

$P_1 = 0.5$, $P_2 = 1$

$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

$P_1 = 1$, $P_2 = 2$

$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

Ⅲ

$P_1 = 0.5$, $P_2 = 1$

$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$P_1 = 1$, $P_2 = 2$

$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$f(z) = X(z)$ $\qquad$ $a_n \xleftrightarrow{-n} x_n$

		a_n	x_n	
I	$p_1 = 1$, $p_2 = 2$	$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$	$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$	$p_1 = 1$, $p_2 = 2$
II	$p_1 = 1$, $p_2 = 2$	$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$	$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$	$p_1 = 1$, $p_2 = 2$
III	$p_1 = 1$, $p_2 = 2$	$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$	$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$	$p_1 = 1$, $p_2 = 2$

$* -1$ $\qquad$ $* -1$

		a_n	x_n	
I	$p_1 = 0.5$, $p_2 = 1$	$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$	$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$	$p_1 = 0.5$, $p_2 = 1$
II	$p_1 = 0.5$, $p_2 = 1$	$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$	$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$	$p_1 = 0.5$, $p_2 = 1$
III	$p_1 = 0.5$, $p_2 = 1$	$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$	$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$	$p_1 = 0.5$, $p_2 = 1$

$* -1$ $\qquad$ $* -1$

$$\boxed{\begin{array}{ll} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{array}} \longrightarrow \boxed{\begin{array}{ll} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{array}}$$

n	-3	-2	-1	0	1	2	3	4
$\left(\frac{1}{2}\right)^{n+1}$	$\left(\frac{1}{2}\right)^{-3+1}$	$\left(\frac{1}{2}\right)^{-2+1}$	$\left(\frac{1}{2}\right)^{-1+1}$	$\left(\frac{1}{2}\right)^{0+1}$	$\left(\frac{1}{2}\right)^{1+1}$	$\left(\frac{1}{2}\right)^{2+1}$	$\left(\frac{1}{2}\right)^{3+1}$	$\left(\frac{1}{2}\right)^{4+1}$
	$\left(\frac{1}{2}\right)^{-2}$	$\left(\frac{1}{2}\right)^{-1}$	$\left(\frac{1}{2}\right)^{0}$	$\left(\frac{1}{2}\right)^{1}$	$\left(\frac{1}{2}\right)^{2}$	$\left(\frac{1}{2}\right)^{3}$	$\left(\frac{1}{2}\right)^{4}$	$\left(\frac{1}{2}\right)^{5}$
2^{-n-1}	2^{2}	2^{1}	2^{0}	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
$-n$	$+3$	$+2$	$+1$	0	-1	-2	-3	-4
2^{-n-1}	2^{3-1}	2^{2-1}	2^{1-1}	2^{0-1}	2^{-1-1}	2^{-2-1}	2^{-3-1}	

n'	m	-2	-1	0	$+1$	$+2$	$+3$
$2^{n'-1}$	2^{m-1}	2^{-2-1}	2^{-1-1}	2^{0-1}	2^{1-1}	2^{2-1}	2^{3-1}

n	-3	-2	-1	0	1	2	3
2^n	2^{-3}	2^{-2}	2^{-1}	2^0	2^1	2^2	2^3
$-n$	3	2	1	0	-1	-2	-3
2^{-n}	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}

rearrange

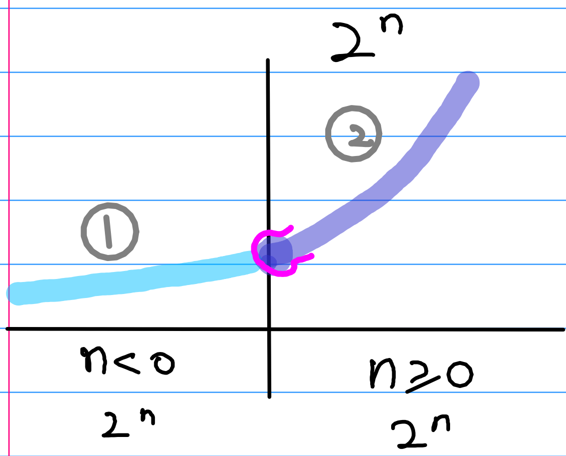
n' m	$-n$	-3	-2	-1	0	1	2	3
$2^{n'}$ 2^m	2^{-n}	2^{-3}	2^{-2}	2^{-1}	2^0	2^1	2^2	2^3

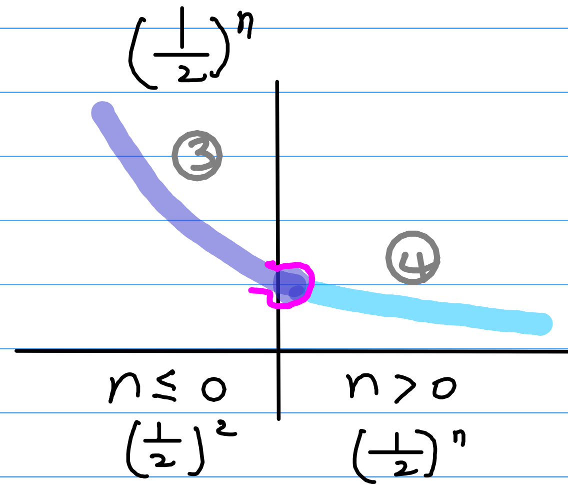
$$\begin{array}{ccc} & \vdots & \\ 2 & \longrightarrow & 2^2 \\ 1 & \longrightarrow & 2^1 \\ 0 & \longrightarrow & 2^0 \\ -1 & \longrightarrow & 2^{-1} \\ -2 & \longrightarrow & 2^{-2} \\ & \vdots & \end{array} \qquad \begin{array}{ccc} & \vdots & \\ -2 & \longrightarrow & 2^{-2} \\ -1 & \longrightarrow & 2^{-1} \\ 0 & \longrightarrow & 2^0 \\ 1 & \longrightarrow & 2^1 \\ 2 & \longrightarrow & 2^2 \\ & \vdots & \end{array}$$

the same function

n	-3	-2	-1	0	1	2	3
2^n	2^{-3}	2^{-2}	2^{-1}	2^0	2^1	2^2	2^3
n	-3	-2	-1	0	1	2	3
2^{-n}	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}

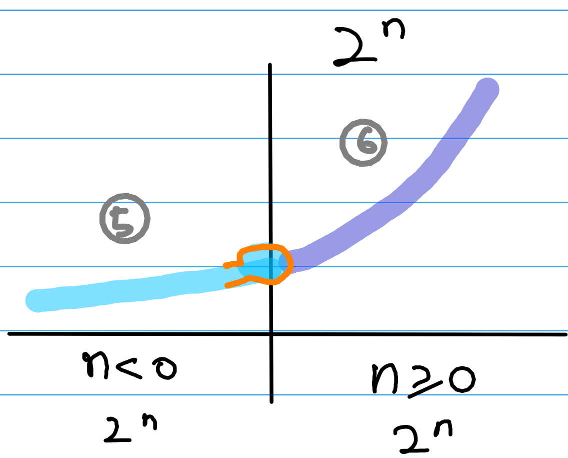
different functions

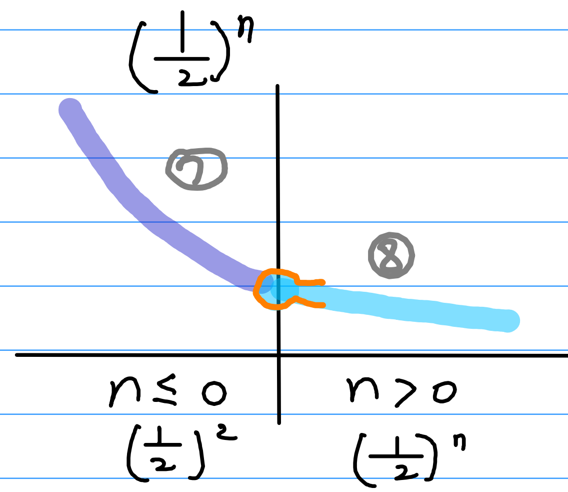




① 2^n $(n<0)$ ③ $(\frac{1}{2})^n$ $(n\leq 0)$

② 2^n $(n\geq 0)$ ④ $(\frac{1}{2})^n$ $(n>0)$





⑤ 2^n $(n\leq 0)$ ⑦ $(\frac{1}{2})^n$ $(n<0)$

⑥ 2^n $(n>0)$ ⑧ $(\frac{1}{2})^n$ $(n\geq 0)$

$$2^{n-1} \xleftrightarrow{-n} 2^{-n-1} = \left(\frac{1}{2}\right)^{n+1}$$

$$2^{n-1} \quad (n < 0) \qquad -n \qquad \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

$$2^{n-1} \quad (n \geq 0) \qquad -n \qquad \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

$$2^{n-1} \quad (n \leq 0) \qquad -n \qquad \left(\frac{1}{2}\right)^{n+1} \quad (n > 0)$$

$$2^{n-1} \quad (n > 0) \qquad -n \qquad \left(\frac{1}{2}\right)^{n+1} \quad (n \leq 0)$$