

Monad Overview (3B)

Based on

[Haskell in 5 steps](https://wiki.haskell.org/Haskell_in_5_steps)

https://wiki.haskell.org/Haskell_in_5_steps

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Based on

[Haskell in 5 steps](https://wiki.haskell.org/Haskell_in_5_steps)

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Monadic Effect

```
class Monad m where
  return :: a -> m a
  (>>=) :: m a -> (a -> m b) -> m b
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/IO

<https://stackoverflow.com/questions/2488646/why-are-side-effects-modeled-as-monads-in-haskell>

<https://stackoverflow.com/questions/7840126/why-monads-how-does-it-resolve-side-effects>

<https://stackoverflow.com/questions/2488646/why-are-side-effects-modeled-as-monads-in-haskell>

<https://www.cs.hmc.edu/~adavidso/monads.pdf>

Monadic Operations

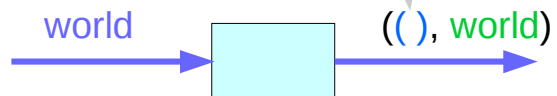
Monadic operations tend to have types which look like

`val-in-type-1 -> ... -> val-in-type-n -> effect-monad val-out-type`

`put :: s -> (State s) ()`



`putStr :: String -> IO ()`



returning a function as a value
executable function
executing an action (**effect-monad**)
produce a result **val-out-type**

<https://stackoverflow.com/questions/16892570/what-is-in-haskell-exactly>

Monadic Operations

`val-in-type-1 -> ... -> val-in-type-n` -> `effect-monad` `val-out-type`

where the **return type** is a type application:
a type with a parameter type

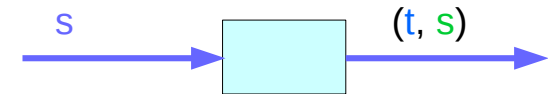
effect-monad

an executable function
giving information about which **effects** are possible

val-out-type

the argument of the executable function
the type of the **result** produced by the function
(the result of executing the function)

returning a function as a value



```
put :: s -> (State s) ()
```

```
putStr :: String -> IO ()
```

<https://stackoverflow.com/questions/16892570/what-is-in-haskell-exactly>

Monadic Operations – put, putStrLn

```
put :: s -> State s ()
```

```
put :: s -> (State s) ()
```

one value input type **s**
the effect-monad **State s**
the value output type **()**

the operation is used *only for its effect*;
the *value delivered* is *uninteresting*

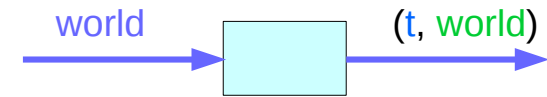
```
putStrLn :: String -> IO ()
```

delivers a string to stdout but does not return anything exciting.

<https://stackoverflow.com/questions/16892570/what-is-in-haskell-exactly>

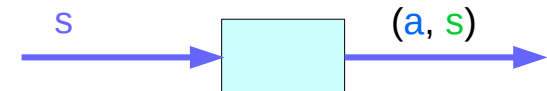
IO t and State s a types

```
type IO t = World -> (t, World)      type synonym
```



```
newtype State s a = State { runState :: s -> (a, s) }
```

s : the type of the state,
 a : the type of the produced result
 $s \rightarrow (a, s)$: function type



Monad Definition

```
class Monad m where
  return :: a -> m a
  (>=) :: m a -> (a -> m b) -> m b
  (>>) :: m a -> m b -> m b
  fail :: String -> m a
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads

Maybe Monad Instance

```
instance Monad Maybe where
  return x = Just x
  Nothing >=> f = Nothing
  Just x >=> f = f x
  fail _ = Nothing
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads

IO Monad Instance

```
instance Monad IO where
  m >> k  = m >>= \_ -> k
  return  = returnIO
  (>>=)   = bindIO
  fail s  = failIO s

returnIO :: a -> IO a
returnIO x = IO $ \ s -> (# s, x #)

bindIO :: IO a -> (a -> IO b) -> IO b
bindIO (IO m) k
  = IO $ \ s -> case m s of (# new_s, a #)
    -> unIO (k a) new_s
```

<https://stackoverflow.com/questions/9244538/what-are-the-definitions-for-and-return-for-the-io-monad>

State Monad Instance

```
instance Monad (State s) where

return :: a -> State s a
return x = state (\ s -> (x, s) )

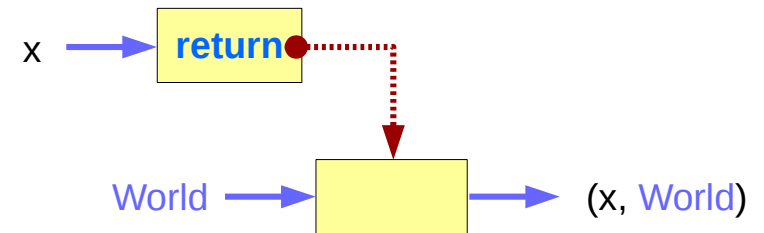
(>>=) :: State s a -> (a -> State s b) -> State s b
p >>= k = q where
  p' = runState p      -- p' :: s -> (a, s)
  k' = runState . k    -- k' :: a -> s -> (b, s)
  q' s0 = (y, s2) where -- q' :: s -> (b, s)
    (x, s1) = p' s0    -- (x, s1) :: (a, s)
    (y, s2) = k' x s1 -- (y, s2) :: (b, s)
  q = State q'
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

Monad IO - return

The **return** function takes x
and gives back a function
that takes a **World**
and returns x along with the new, **updated World (=World)**
formed by not modifying the **World** it was given

return x world = (x, world)



<https://www.cs.hmc.edu/~adavidso/monads.pdf>

Monad IO - >>=

the expression (**ioX >>= f**) has

type `World -> (t, World)`

a function **ioX** that takes **w0** of the type `World`,
which is used to extract **x** from its **IO** monad.

x gets passed to **f**, resulting in another **IO** monad,
which again is a function that takes **w1** of the type `World`
and returns a **y** and a new, updated `World`.

the implementation of bind

<https://www.cs.hmc.edu/~adavidso/monads.pdf>

IO Monad >>= Implementation

We give IO the World

$w0 :: \text{World}$

we got back the World

$w1 :: \text{World}$

from getting x out of its monad,

$x :: t$

and the thing IO gives back to us is

the y with

$y :: t$

a final version of the World

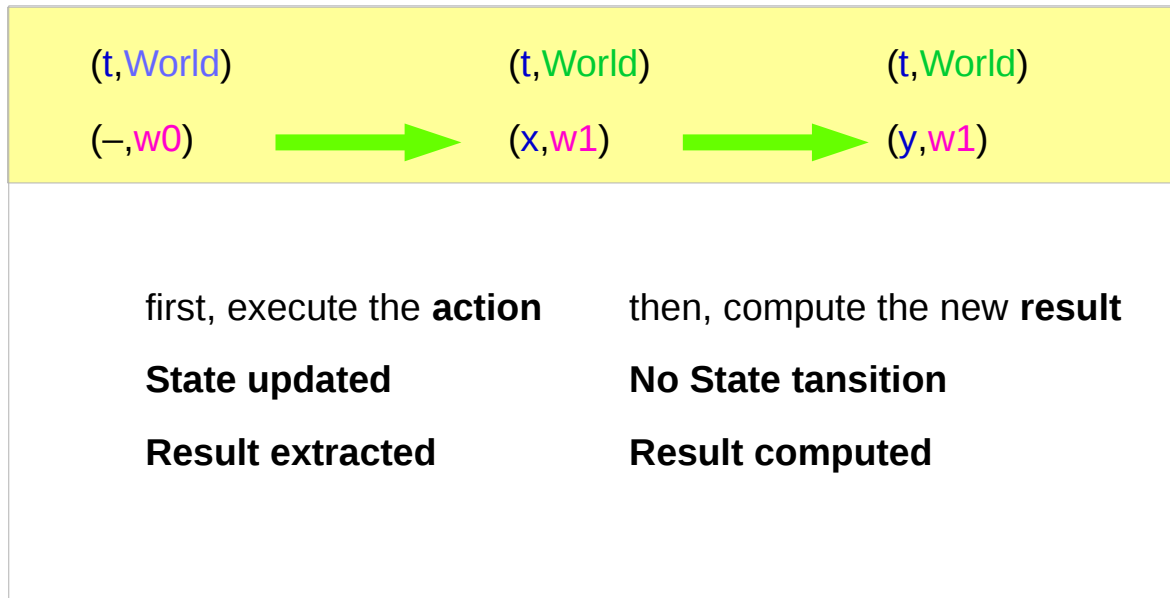
$w1 :: \text{World}$

.

the implementation of bind

<https://www.cs.hmc.edu/~adavidso/monads.pdf>

IO Monad >>= Implementation



the implementation of bind

<https://www.cs.hmc.edu/~adavidso/monads.pdf>

IO Monad Implementation

```
instance Monad IO where
```

```
return x w0 = (x, w0)
```

```
(ioX >>= f) w0 =
```

```
  let (x, w1) = ioX w0
```

```
  in f x w1      -- has type (t, World)
```

```
type IO t = World -> (t, World)
```

type synonym

<https://www.cs.hmc.edu/~adavidso/monads.pdf>

IO Monad Implementation

```
instance Monad IO where
```

```
  return x w0 = (x, w0)
```

```
(ioX >>= f) w0 =
```

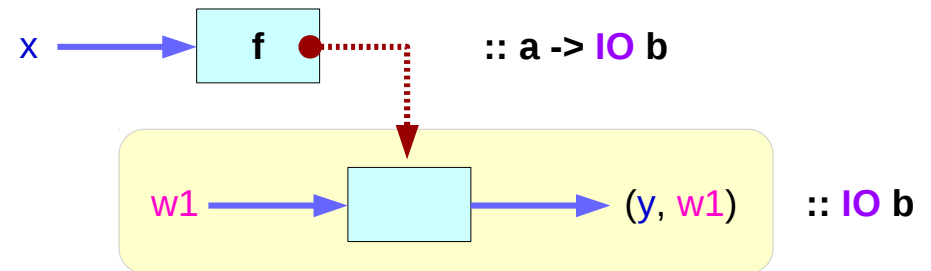
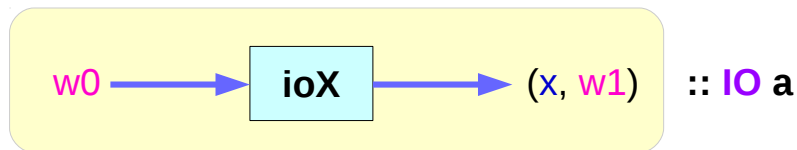
```
  let (x, w1) = ioX w0
```

```
  in f x w1      -- has type (t, World)
```

```
ioX >>= f :: IO a -> (a -> IO b) -> IO b
```

```
type IO t = World -> (t, World)
```

type synonym



<https://www.cs.hmc.edu/~adavidso/monads.pdf>

IO Monad Implementation

$\text{ioX} \gg= f \ :: \ \text{IO } a \rightarrow (a \rightarrow \text{IO } b) \rightarrow \text{IO } b$

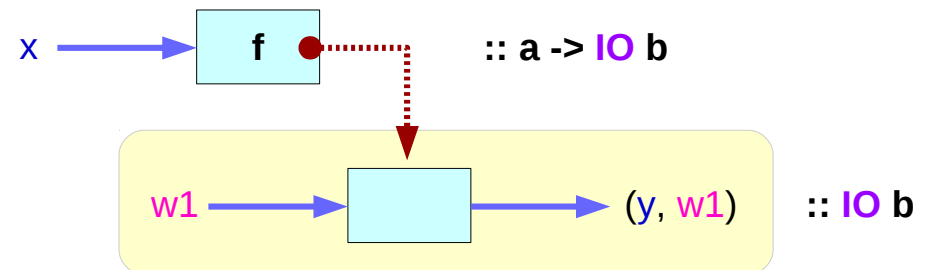
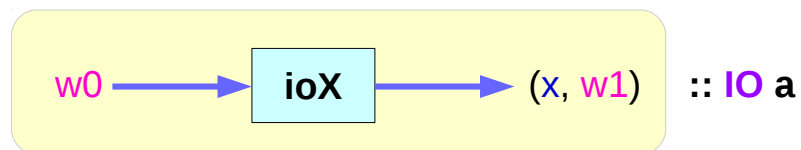
$\text{ioX} \ :: \ \text{IO } a \quad w0 \ :: \ \text{World} \quad x \ :: \ a$

$f \ :: \ a \rightarrow \text{IO } b \quad w1 \ :: \ \text{World}$

$\text{ioX } w0 \ :: \ \text{IO } a \ \text{World} \quad \longrightarrow \quad (x, w1)$

$f \ x \ :: \ \text{IO } b$

$f \ x \ w1 \ :: \ \text{IO } b \ \text{World} \quad \longrightarrow \quad (y, w1)$



<https://www.cs.hmc.edu/~adavidso/monads.pdf>

IO Monad Implementation

$\text{ioX} \gg= f \ :: \ \text{IO } a \rightarrow (a \rightarrow \text{IO } b) \rightarrow \text{IO } b$

$\text{ioX} \ :: \ \text{IO } a$

$f \ :: \ a \rightarrow \text{IO } b$

$w_0 \ :: \ \text{World}$

$x \ :: \ a$

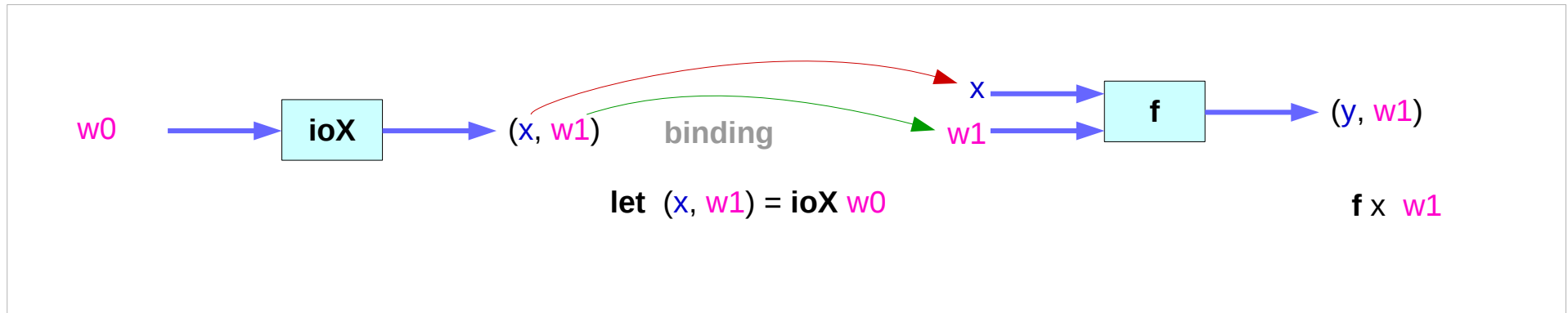
$w_1 \ :: \ \text{World}$

internal
variables

$\text{ioX } w_0 \ :: \ \text{IO } a \ \text{World} \quad \longrightarrow \quad (x, w_1)$

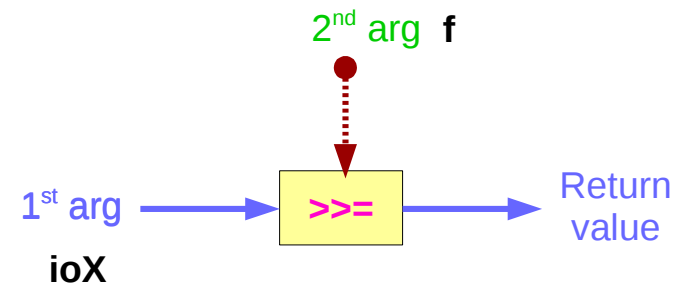
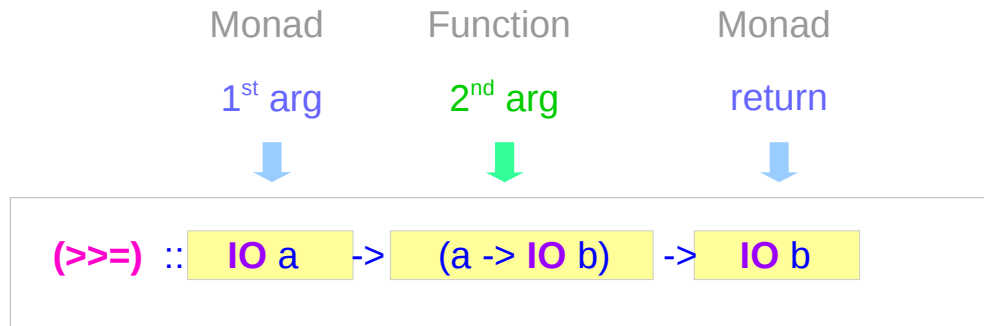
$f \ x \ :: \ a \rightarrow a \rightarrow \text{IO } b$

$f \ x \ w_1 \ :: \ \text{IO } b \ \text{World} \quad \longrightarrow \quad (y, w_1)$



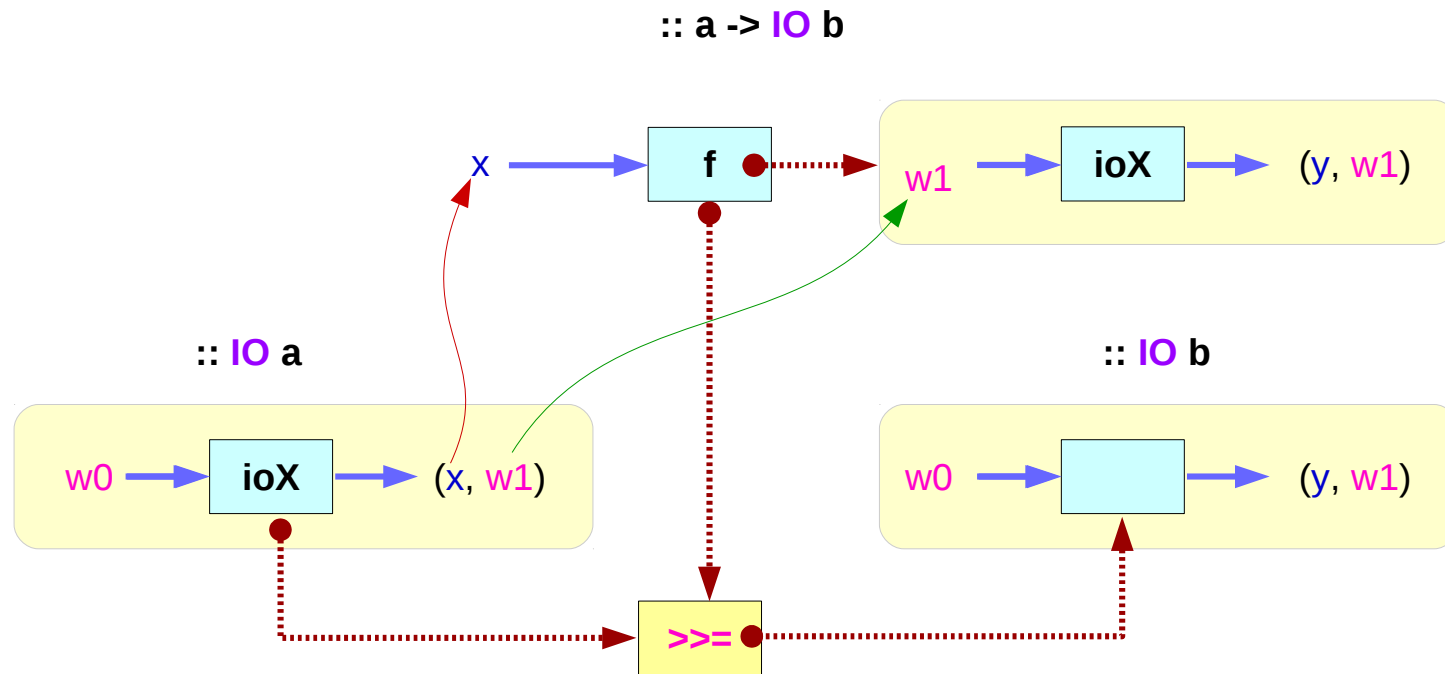
<https://www.cs.hmc.edu/~adavidso/monads.pdf>

IO Monad



```
(ioX >>= f) w0 =  
  let (x, w1) = ioX w0  
  in f x w1      -- has type (t, World)
```

IO Monad



Monad IO and Monad ST

instance Monad IO where

```
return x world = (x, world)
```

```
(ioX >>= f) w0 =
```

```
  let (x, w1) = ioX w0
```

```
  in  f x w1           -- has type (t, World)
```

instance Monad ST where

```
-- return :: a -> ST a
```

```
return x = \s -> (x,s)
```

```
-- (>>=) :: ST a -> (a -> ST b) -> ST b
```

```
st >>= f = \s -> let (x,s') = st s
                  in f x s'
```

```
type IO t = World -> (t, World)
```

type synonym

<https://www.cs.hmc.edu/~adavidso/monads.pdf>

State Transformers ST

instance **Monad ST** where

```
-- return :: a -> ST a
```

```
return x = \s -> (x,s)
```

```
-- (>>=) :: ST a -> (a -> ST b) -> ST b
```

```
st >>= f = \s -> let (x,s') = st s in f x s'
```

>>= provides a means of sequencing **state transformers**:

st >>= f applies the **state transformer st** to an initial state **s**,

then applies the function **f** to the resulting value **x**

to give a second **state transformer (f x)**,

which is then applied to the modified state **s'** to give the final result:

```
st >>= f = \s -> f x s'
```

```
where (x,s') = st s
```

```
st >>= f = \s -> (y,s')
```

```
where (x,s') = st s
```

```
(y,s') = f x s'
```

```
(x,s') = st s
```

```
f x s'
```

<https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html>

References

- [1] <ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf>
- [2] <https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf>