Propositional Logic – Semantics (3A)

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Based on

Contemporary Artificial Intelligence,
R.E. Neapolitan & X. Jiang

Logic and Its Applications,
Burkey & Foxley

Semantics

Gives meaning to the propositions

Consists of rules for assigning either the value **T** or **F** to every proposition

The truth value of a proposition

If a proposition has truth value **T**, we say it is true
Otherwise, we say it is false

Semantic Rules

1. the logical value **True** ← the value T always
   the logical value **False** ← the value F always

2. Every **atomic proposition** ← a value T or F

   The set of all these assignments constitues a **model** or **possible world**

   All possible worlds (assignments) are **permissable**

3. The truth values of arbitrary propositions connected with **connectives**
   are given by the connective **truth tables**.

4. The truth value for **compound propositions** are determined recursively
   using the truth tables according to the following rules

   (a) the grouping () has highest precedence
   (b) the precedence order : ¬, ∧, ∨, → , ↔
   (c) binary connectives : from left to right

Connective Truth Tables

from en.wikipedia.org

P	Q	$P \wedge Q$	$P \vee Q$	$P \underline{\vee} Q$	$P \underline{\wedge} Q$	$P \Rightarrow Q$	$P \Leftarrow Q$	$P \Leftrightarrow Q$
T	T	T	T	F	T	T	T	T
T	F	F	T	T	F	F	T	F
F	T	F	T	T	F	T	F	F
F	F	F	F	F	T	T	T	T

T = true, F = false

$\wedge$ = AND (logical conjunction)

$\vee$ = OR (logical disjunction)

$\underline{\vee}$ = XOR (exclusive or)

$\underline{\wedge}$ = XNOR (exclusive nor)

$\rightarrow$ = conditional "if-then"

$\leftarrow$ = conditional "(then)-if"

$\Longleftrightarrow$ biconditional or "if-and-only-if" is logically equivalent to $\underline{\wedge}$: XNOR (exclusive nor).

Semantic Rule Purposes

The semantics for propositional logic

- assign truth values to all propositions

- could use different truth tables from the conventional ones
- but it must provide a way to reflect the real world
  - ➔ to allow reasoning

- The purpose is to make statements about the real world
- and to reason with these statements
- the semantics must reflect the way humans reason
  with the statements in the world

- some difficulty with $A \rightarrow B$

What a proposition denote

A: It is raining.

B: Professor N is 5 feet tall.

Suppose that

It is **not** raining (A is false)

Professor N is **6 feet** actually (B is false)

A→ B is true according to the truth table

Proposition A denotes it is raining currently.

Proposition A does not denote it is raining an hour ago, tomorrow, a year ago…

Proposition B denotes Professor N is 6 feet

Proposition B does not denote Professor N is 7 feet, 5 feet, …

A → B : if it is not raining currently (could rain yesterday),

Then professor

What A → B denotes

A: It is raining. (false)

B: Professor N is 5 feet tall. (false)

A → B : does not mean that

If it rains some day,

Then Professor N will be 5 feet.

Only concerns

The proposition that there is rain currently (false)

Suppose A → B is true

After finding A is true, then B must be true

After finding A is false, then we do not know whether B is true or false

Semantics of Implication Rule

Suppose A → B is true
If A is true then B must be true

Suppose A → B is false
If A is true then B must be false

Suppose A → B is true
If A is false then we do not know whether B is true or false

A	B	A→B
T	T	T
T	F	F
F	T	T
F	F	T

Semantics of Implication Rule

Suppose A → B is true
If B is true then A must be true

B implies A
This is not what we want

A	B	A→B
T	T	T
T	F	F
F	T	F
F	F	F

Semantics of Implication Rule

Suppose $A \rightarrow B$ is true
If A is false then B must be false

False A implies false B
This is not what we want

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	F
F	F	T

References

[1] en.wikipedia.org
[2] en.wiktionary.org
[3] U. Endriss, “Lecture Notes : Introduction to Prolog Programming”
[4] http://www.learnprolognow.org/ Learn Prolog Now!
[5] http://www.csupomona.edu/~jrfisher/www/prolog_tutorial
[6] www.cse.unsw.edu.au/~billw/cs9414/notes/prolog/intro.html
[7] www.cse.unsw.edu.au/~billw/dictionaries/prolog/negation.html
[8] http://ilppp.cs.lth.se/, P. Nugues,` An Intro to Lang Processing with Perl and Prolog