

# Angle Recoding 2. Wu

## 3. MVR

# 20180824 Thr

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## ② MVR (Modified Vector Rotational)

two modifications

① **repetition** of elementary angles

each micro-rotation of elementary angle  
can be performed repeatedly

- more possible combinations
- smaller  $\xi_m$

② **confinement** of total micro-rotation number

confine the iteration number

in the micro-rotation phase  
to  $R_m$  ( $R_m \ll W$ )

The role of  $R_m$  is quite similar  
to the **number of non-zero digit**  
 $N_D$  in CSD recoding scheme

the angle quantization error

$$\xi_{m, \text{MVR}} \triangleq \theta - \sum_{i=0}^{R_m-1} \alpha(i) a(s(i))$$

the rotational sequence

$$s(i) \in \{0, 1, \dots, W-1\}$$

the micro-rotation angle  
in the  $i$ -th iteration

the directional sequence

$$\alpha(i) \in \{-1, 0, +1\}$$

the direction of the  $i$ -th  
micro-rotation of  $a(s(i))$

$$\alpha(i) a(s(i)) = \tilde{\theta}(j)$$

the rotational sequence

$$s(i) \in \{0, 1, \dots, W-1\} \quad [0, 3, 6, 7]$$

the directional sequence

$$\alpha(i) \in \{-1, 0, +1\} \quad [1, -1, -1, 1]$$

$$\text{atan}(2^0) - \text{atan}(2^{-3}) - \text{atan}(2^{-6}) + \text{atan}(2^{-7})$$

$$\alpha(i) \alpha(s(i)) = \tilde{\theta}(j)$$

MVR-CORDIC Algorithm with $R_n = 4$	Greedy Algorithm	3	$\bar{\alpha} = [1 \ -1 \ -1 \ -1]$ $\bar{s} = [0 \ 3 \ 6 \ 7]$	$5.2891 \cdot 10^{-4}$
	Semi-greedy Algorithm ( $D = 2$ )	4	$\bar{\alpha} = [1 \ -1 \ -1 \ 1]$ $\bar{s} = [0 \ 3 \ 5 \ 7]$	$5.2033 \cdot 10^{-4}$
	TBS Algorithm	5	$\bar{\alpha} = [1 \ 1 \ -1 \ -1]$ $\bar{s} = [1 \ 2 \ 4 \ 7]$	$2.5911 \cdot 10^{-4}$

```
>> sum(atan(2.^(-s)) .* a)
```

```
ans = 0.65323
```

```
>> pi
```

```
ans = 3.1416
```

```
>> 13*pi/32
```

```
ans = 1.2763
```

```
>> a
```

```
a =
```

```
1 -1 -1 1
```

```
>> a = [1, -1, -1, -1]
```

```
a =
```

```
1 -1 -1 -1
```

```
>> sum(atan(2.^(-s)) .* a)
```

```
ans = 0.63761
```

# AQ & MVR CORDIC

$$\xi_{m, MVR} \triangleq \theta - \left[ \sum_{j=0}^{R_m-1} \alpha(j) a(s(j)) \right]$$

the rotational sequence  $s(j)$

$$j = 0, 1, 2, \dots, R_m-1$$



$$s(j) \in \{0, 1, \dots, W-1\} \quad \text{rotational sequence}$$

determines the micro-rotation angle  $a(s(j))$   
in the  $j$ -th iteration

the directional sequence  $\alpha(j)$

$$\alpha(j) \in \{-1, 0, +1\}$$

controls the direction of the  $j$ -th  
micro-rotation of  $a(s(j))$

$$\alpha(j) a(s(j)) = \tilde{\theta}(j)$$

$i = 0, 1, 2, 3, \dots, W-1$	
$s(j) = 0, 1, 2, 3, \dots, W-1$	rotational sequence
$\alpha(j) = -1, 0, 0, +1, \dots, -1$	directional sequence
$j = 0, -, -, 1, \dots, R_m-1$	effective iteration number
$R_m \ll W$	

sub-angle  $(\alpha(j) a(s(j))) \sim \tilde{\theta}(j)$

$$\xi_{M,AR} = \theta - \left[ \sum_{j=0}^{N'-1} \tan^{-1}(\alpha(j) \cdot 2^{-s(j)}) \right]$$
$$= \theta - \left[ \sum_{j=0}^{N'-1} \tilde{\theta}(j) \right], \quad \tilde{\theta}(j) = \tan^{-1}(\alpha(j) \cdot 2^{-s(j)})$$

$$N' \triangleq \sum_{j=0}^{N-1} |\mu(j)| \quad \text{the effective iteration number}$$

EAS formed by MVR-CORDIC  
is the same as AR  
also performs AQ

the EAS consists of all possible values of  $\tilde{\theta}(j)$

the EAS  $S_1$  in AR

$$S_1 = \{ \tan^{-1}(\alpha^* \cdot 2^{-s^*}) : \alpha^* \in \{-1, 0, +1\}, s^* \in \{0, 1, \dots, N-1\} \}$$

The major difference

1) the total number of sub-angles  $N_A$

the total iteration number

in the micro-rotation phase

is kept fixed to a pre-defined value of  $R_m$

$$N_A = R_m$$

2) the sub-angle  $\theta_i$  corresponds to  $\alpha^{(j)} a(s^{(j)})$

$$\theta_j = \alpha^{(j)} a(s^{(j)}) = \tilde{\theta}_j$$



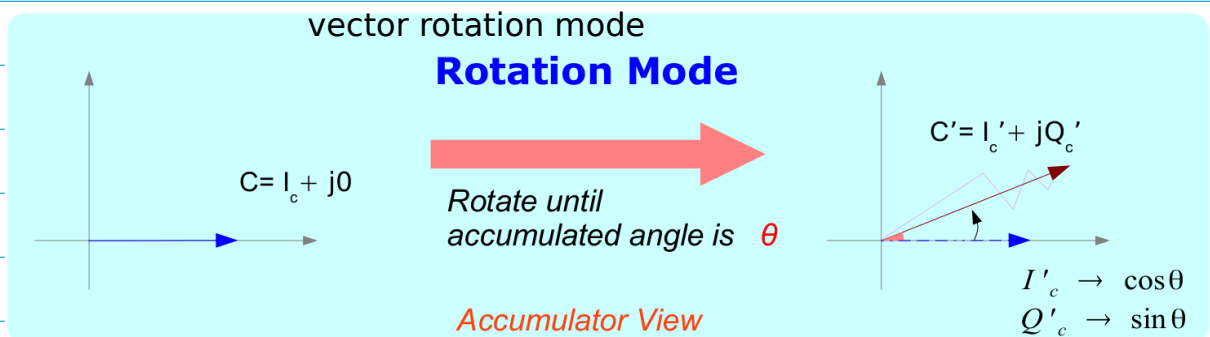
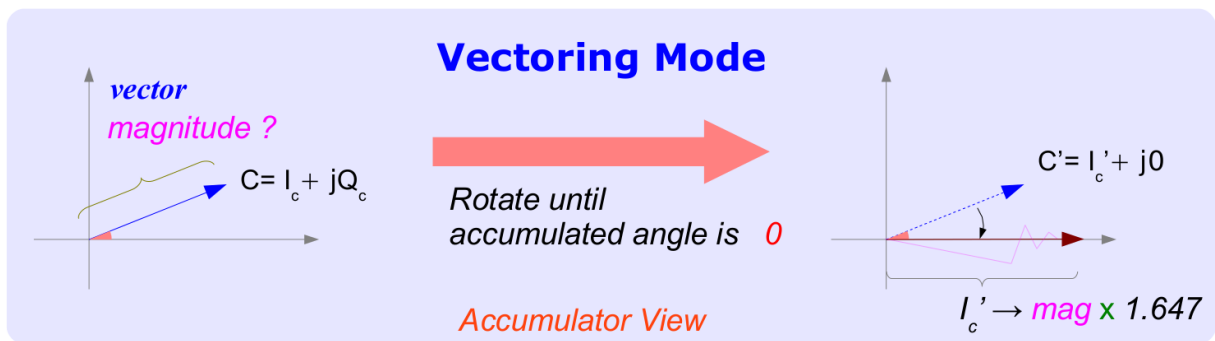
# MVR (Modified Vector Rotation)

1) Repeat of Elementary Angles  $\theta_i, \theta_i$

2) fixed total micro-rotation Number  $R_m$

\* Vector Rotation Mode

\* and the rotation angles are known in advance



# Modified Vector Rotational MVR CORDIC

- reduce the iteration number
- maintaining the SQNR performance
- modifying the basic microrotation procedure

## Three Searching Algorithm

- ① the selective prerotation
- ② the selective scaling
- ③ iteration-tradeoff scheme

# Optimization Problem

EAS point of view

Given  $\theta$ , find the combination of  $R_m$  elementary angles from EAS  $S_i$ , such that the angle quantization error  $|\xi_{m, \text{MUR}}|$  is minimized.

Semi-greedy algorithm

trade offs between computational complexities  
and performance

Key issue in the MVR-CORDIC  
is to find the best sequences of  
 $s(i)$  and  $\alpha(i)$  to minimize  $|\xi_m|$   
subject to the constraint that  
the total iteration number is confined to  $R_m$

- 1) Greedy Algorithm
- 2) Exhaustive Algorithm
- 3) Semigreedy Algorithm

# 1) Greedy Algorithm

try to approach the target rotation angle,  $\theta$ , step by step  
in each step, decisions are made on  $\alpha(i)$  and  $s(i)$   
by choosing the best combination of  $\alpha(i)$  and  $s(i)$   
so as to minimize  $|\xi_m|$

$\alpha(i)$  and  $s(i)$  are determined such that  
the error function  $J(i) = |\theta(i) - \alpha(i) a(s(i))|$  is minimized

$\theta(i)$  : the residue angle in the  $i$ -th step

$$\theta(i) = \theta - \sum_{m=0}^{i-1} \alpha(m) a(s(m))$$

the searching is terminated

if no further improvements can be found

$$J(i) \geq J(i-1)$$

$\alpha(R_m-1)$  and  $s(R_m-1)$  are determined

at the end of the searching

the greedy algorithm terminates

only when the residue angle error

cannot be further reduced.

Initialization:

given  $\theta$ ,  $w$ ,  $R_m$

$$\theta^{(i)} = \theta - \sum_{n=0}^{i-1} \alpha^{(n)} a(s^{(n)})$$

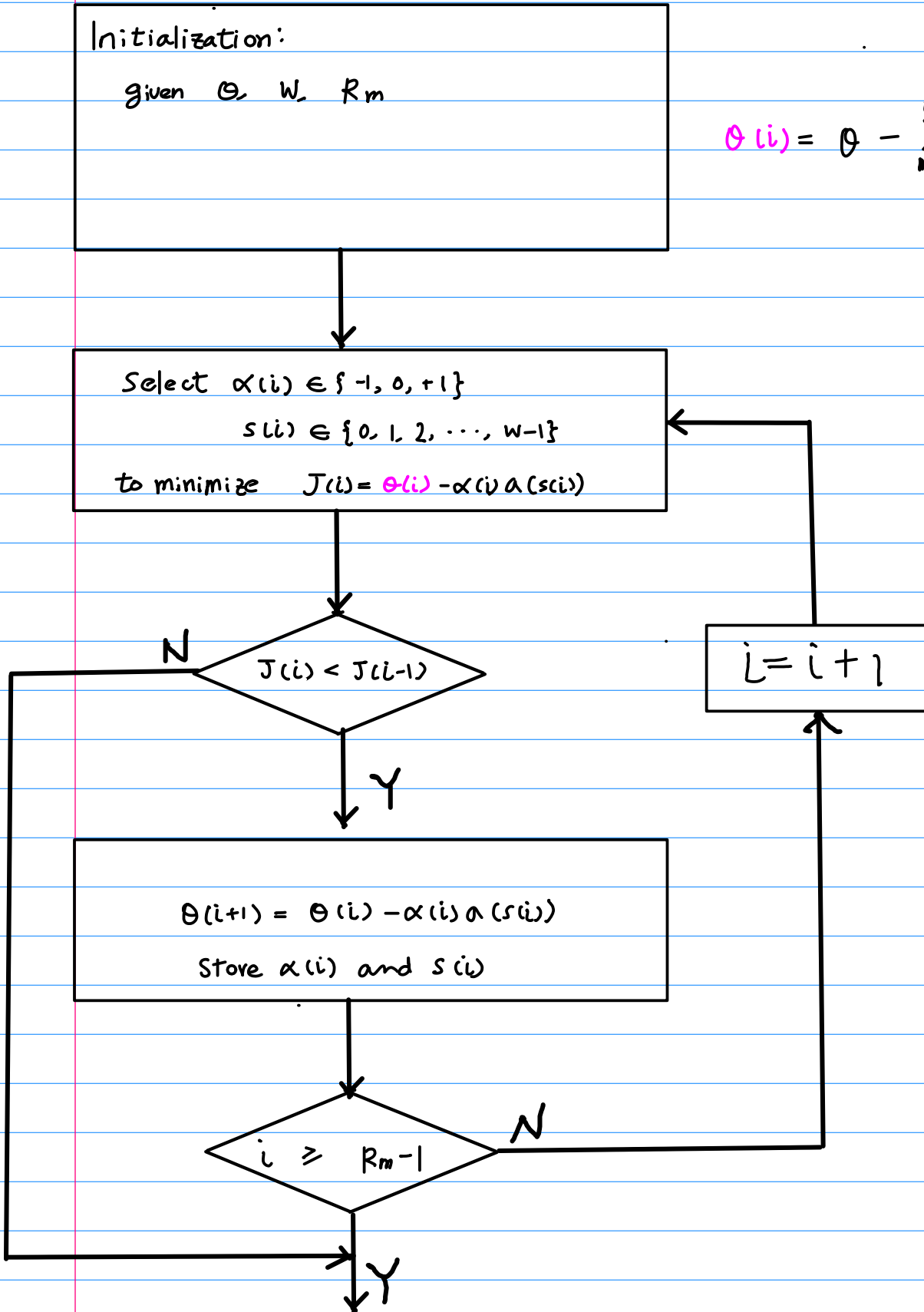
Select  $\alpha^{(i)} \in \{-1, 0, +1\}$   
 $s^{(i)} \in \{0, 1, 2, \dots, w-1\}$   
to minimize  $J^{(i)} = \theta^{(i)} - \alpha^{(i)} a(s^{(i)})$

N  
Y  
 $J^{(i)} < J^{(i-1)}$

$\theta^{(i+1)} = \theta^{(i)} - \alpha^{(i)} a(s^{(i)})$   
Store  $\alpha^{(i)}$  and  $s^{(i)}$

N  
Y  
 $i \geq R_m - 1$

$i = i + 1$



## 2) Exhaustive Algorithm

search for the entire solution space

all possible combinations of

$$\sum_{i=0}^{R_m-1} \alpha(i) a(s(i))$$

in a single step

decisions for  $\alpha(i)$  and  $s(i)$ ,  $0 \leq i \leq R_m-1$

by minimizing the error function

$$J = \left| 0 - \sum_{i=0}^{R_m-1} \alpha(i) a(s(i)) \right|$$

global optimal solution

Initialization:

given  $\Theta, W, R_m$

let  $\theta(0) = \Theta,$

$i = 0$

$J(-1) = \infty$

Select  $\alpha(i) \in \{-1, 0, +1\}$

$s(i) \in \{0, 1, 2, \dots, W-1\}$

for  $0 \leq i \leq R_m - 1$

to minimize  $J(i) = \theta - \sum_{l=0}^{R_m-1} \alpha(l) a(s(i))$

$$(3 \cdot W) \cdot (3 \cdot W) \dots (3 \cdot W) \\ = 3^{R_m} \cdot W^{R_m}$$

Store  $\alpha(i)$  and  $s(i)$

for  $0 \leq i \leq R_m - 1$





in the  $i$ -th block

decision of  $\alpha(k)$  and  $s(k)$  for  $iD \leq k \leq (i+1)D-1$

$$\text{minimizes } J = \left| \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k)) \right|$$

$$\text{where } \theta(i) = \theta - \sum_{m=0}^{i-1} \left[ \sum_{k=mD}^{(m+1)D-1} \alpha(k) a(s(k)) \right]$$

the residue angle in the  $i$ -th step

$$s = \left\lceil \frac{R_m}{D} \right\rceil$$

$$\theta(i) = \theta - \left[ \sum_{k=0D}^{1D-1} \alpha(k) a(s(k)) + \sum_{k=1D}^{2D-1} \alpha(k) a(s(k)) + \dots + \sum_{k=(i-1)D}^{iD-1} \alpha(k) a(s(k)) \right]$$

Initialization:

given  $\theta, W, R_m$

let  $\theta(0) = \theta,$

$i = 0$

$J(-1) = \infty$

Select  $\alpha(k) \in \{-1, 0, +1\}$

$s(k) \in \{0, 1, 2, \dots, W-1\}$

for  $iD \leq k \leq (i+1)D - 1$

to minimize  $J(i) = \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k))$

N  
 $J(i) < J(i-1)$

$\theta(i+1) = \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k))$

Store  $\alpha(k), s(k)$

$i \geq \lceil \frac{R_m}{D} \rceil - 1$   
N

Y

$i = i + 1$

