# Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
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# Outline

1 Statistical Indepence

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# Statistical independence

## for 2 random variables $X$ and $Y$

### Definition

two events $A$ and B are statistically independent iff

$$P(A \cap B) = P(A)P(B)$$

Let $A = \{X \leq x\}$and $B = \{Y \leq y\}$
the two random variables $X$ and Y are statistically independent iff

$$P\{X \leq x, Y \leq y\} = P\{X \leq x\}P\{Y \leq y\}$$

$$F_{X,Y}(x, y) = F_X(x)F_Y(y)$$

$$f_{X,Y}(x, y) = f_X(x)f_Y(y)$$

# Conditional distribution function
for 2 random variables $X$ and $Y$

## Definition

the conditional distribution function of random variables $X$ and $Y$

$$F_X(x|B) = F_X(x|Y \le y) = \frac{P\{X \le x, Y \le y\}}{P\{Y \le y\}} = \frac{F_{X,Y}(x,y)}{F_Y(y)}$$

if $X$ and $Y$ are statisitcally independent

$$F_X(x|B) = F_X(x|Y \le y) = F_X(x)$$
$$F_Y(y|A) = F_Y(y|X \le x) = F_Y(y)$$
$$f_X(x|B) = f_X(x|Y \le y) = f_X(x)$$
$$f_Y(y|A) = f_Y(y|X \le x) = f_Y(y)$$

# Statistical independence
## of $N$ random variables $X_1, X_2, ..., X_N$ and $Y$

### Definition

events $A_i = \{X_i \le x_i\} \qquad i = 1, 2, ..., N$
where $x_i$ are real numbers
the random variables $X_1, X_2, ..., X_N$
are said to be statistically independent iff

$$P(A_1 \cap A_2 \cap ... \cap A_N) = P(A_1)P(A_2)...P(A_N)$$

# Statistical independence

## of $N$ random variables $X_1, X_2, ..., X_N$ and $Y$

### Definition

It can be shown that
if $X_1, X_2, ..., X_N$ are statistically independent
then any set that consists of $X_i$'s is independent of any other sets

for example, consider 4 random variables $X_1, X_2, X_3, X_4$
$X_4$ is statistically independent of $X_1 + X_2 + X_3$
$X_3$ is statistically independent of $X_1 + X_2$