

# Multiple Random Variables

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Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles, Jr. and B. Shi

# Outline

## 1 Statistical Independence

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# Statistical independence

for 2 random variables  $X$  and  $Y$

## Definition

two events  $A$  and  $B$  are statistically independent iff

$$P(A \cap B) = P(A)P(B)$$

Let  $A = \{X \leq x\}$  and  $B = \{Y \leq y\}$

the two random variables  $X$  and  $Y$  are statistically independent iff

$$P\{X \leq x, Y \leq y\} = P\{X \leq x\}P\{Y \leq y\}$$

$$F_{X,Y}(x,y) = F_X(x)F_Y(y)$$

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# Conditional distribution function

for 2 random variables  $X$  and  $Y$

## Definition

the conditional distribution function of random variables  $X$  and  $Y$

$$F_X(x|B) = F_X(x|Y \leq y) = \frac{P\{X \leq x, Y \leq y\}}{P\{Y \leq y\}} = \frac{F_{X,Y}(x,y)}{F_Y(y)}$$

if  $X$  and  $Y$  are statistically independent

$$F_X(x|B) = F_X(x|Y \leq y) = F_X(x)$$

$$F_Y(y|A) = F_Y(y|X \leq x) = F_Y(y)$$

$$f_X(x|B) = f_X(x|Y \leq y) = f_X(x)$$

$$f_Y(y|A) = f_Y(y|X \leq x) = f_Y(y)$$

# Statistical independence

of  $N$  random variables  $X_1, X_2, \dots, X_N$  and  $Y$

## Definition

events  $A_i = \{X_i \leq x_i\}$        $i = 1, 2, \dots, N$

where  $x_i$  are real numbers

the random variables  $X_1, X_2, \dots, X_N$

are said to be statistically independent iff

$$P(A_1 \cap A_2 \cap \dots \cap A_N) = P(A_1)P(A_2)\dots P(A_N)$$

# Statistical independence

of  $N$  random variables  $X_1, X_2, \dots, X_N$  and  $Y$

## Definition

It can be shown that

if  $X_1, X_2, \dots, X_N$  are statistically independent

then **any set** that consists of  $X_i$ 's is independent of **any other sets**

for example, consider 4 random variables  $X_1, X_2, X_3, X_4$

$X_4$  is statistically independent of  $X_1 + X_2 + X_3$

$X_3$  is statistically independent of  $X_1 + X_2$





