Applications of Arrays (1A)

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- Viewing an **array** as a **pointer**

- Viewing a **pointer** as an **array**

- Viewing an **array** as a **pointer**

int **a**[4] ; an array **a**

generalization

int (*__a__) view **a** as a pointer

virtual pointer
- no real memory location
- constraints :
  value(&**a**) = value(**a**)

- Viewing a **pointer** as an **array**

int (*__a__) ; a pointer **a**

a specific instance

int **a**[N] view **a** as an array

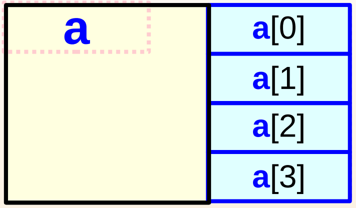
N is not fixed

sizeof(**a**) is
not the size of the array
but of a pointer variable

Array **a** and pointer **a**

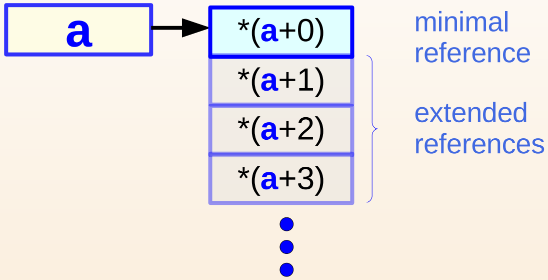
int **a**[4] ;

an array **a**



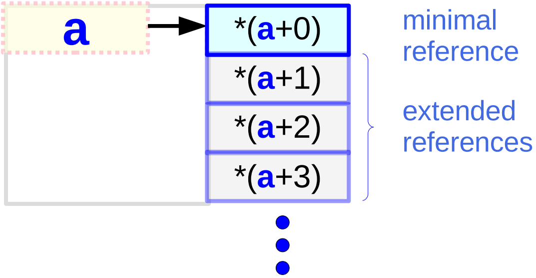
int (***a**) ;

a pointer **a**



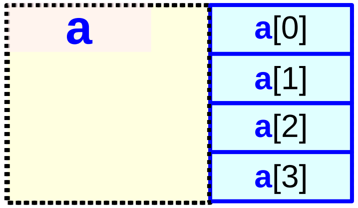
int (***a**)

a as a pointer



int **a**[N]

a as an array



Array **a** and pointer **a**

`int a[4] ;` an array **a**

- type(**a**) = int [4]
- sizeof(**a**) = an array size (16 bytes)
- value(&**a**) = value(**a**)
- fixed number of elements

`int (*a)` **a** as a pointer

a is not a real pointer

- sizeof(**a**) = an array size
- value(&**a**) = value(**a**)

`int (*a) ;` a pointer **a**

- type(**a**) = int (*)
- sizeof(**a**) = a pointer size (4 bytes)
- value(&**a**) ≠ value(**a**)
- variable number of elements

`int a[N]` **a** as an array

a is not a real array

- sizeof(**a**) ≠ an array size
  = a pointer size
- value(&**a**) ≠ value(**a**)
  = assigned address

Relationship between array and array pointer types

int **b**[4][2] ; declare a **2-d** array b

generalization

int (***b**) [2] **b** as a **1-d** array pointer

int **a**[4] ; declare a **1-d** array **a**

generalization

int (***a**) **a** as a **0-d** array pointer

int (***b**)[2] ; declare a **1-d** array pointer **b**

a specific instance

int **b**[N][2] **b** as a **2-d** array

int (***a**) ; declare a **0-d** array pointer **a**

a specific instance

int **a**[N] **a** as a **1-d** array

Array **b** and array pointer **b**

`int b[4][2] ;` **2-d** array **b**

- type(**b**) = int [4]
- sizeof(**b**) = an <u>array</u> size (32 bytes)
- value(&**b**) = value(**b**)
- <u>fixed</u> number of elements

`int (*) [2]` **b** as a **1-d** array pointer

b is <u>not</u> a real <u>pointer</u>

- sizeof(**b**) = an array size
- value(&**b**) = value(**b**)

`int (*b) [2] ;` **1-d** array pointer **b**

- type(**b**) = int (*)
- sizeof(**b**) = a <u>pointer</u> size (4 bytes)
- value(&**b**) ≠ value(**b**)
- <u>variable</u> number of elements

`int [N][2]` **b** as a **2-d** array

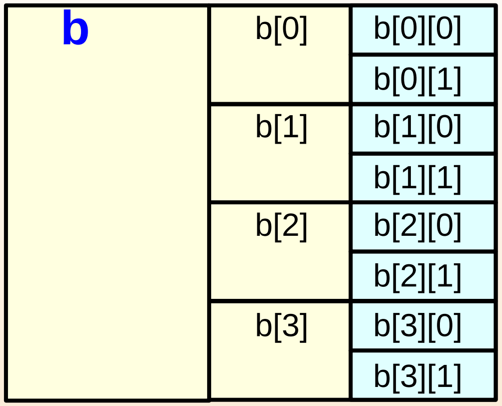
b is <u>not</u> a real <u>array</u>

- sizeof(**b**) ≠ an array size
  = a pointer size
- value(&**b**) ≠ value(**b**)
  = assigned address

Array **b** and array pointer **b**

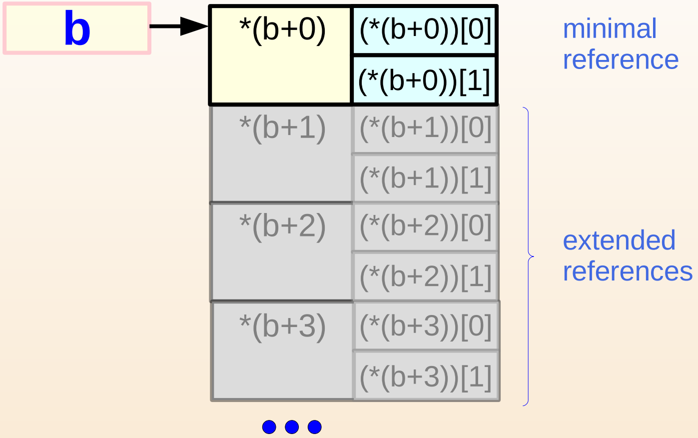
int **b**[4][2] ;

2-d array **b**



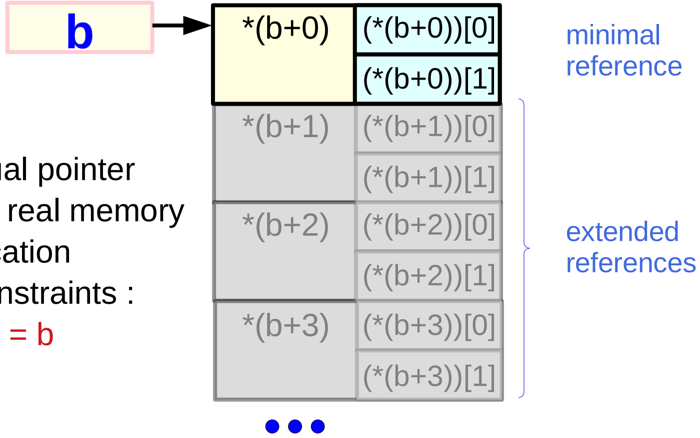
int (***b**) [2] ;

1-d array pointer **b**



int (*) [2]

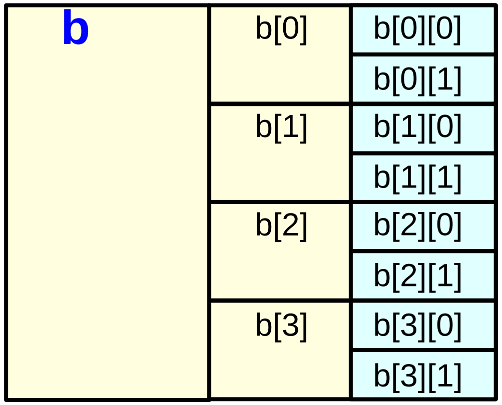
b as a **1-d** array pointer



virtual pointer
- no real memory
  location
- constraints :
  &b = b

int [N][2]

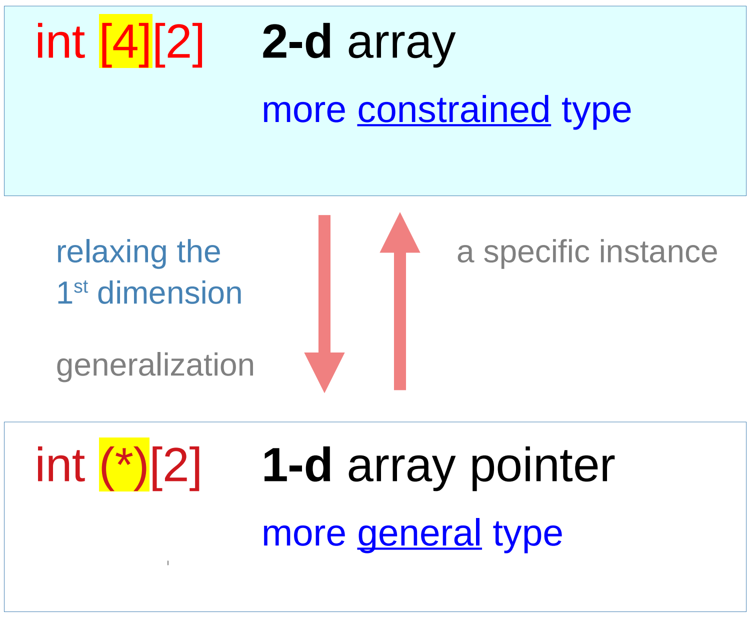
b as a **2-d** array



N is not fixed
to 4

sizeof(b) is not
the size of the array
but the size of a
pointer variable

Dual type - relaxing the 1st dimension of an array



int [4][2] **2-d** array

more constrained type

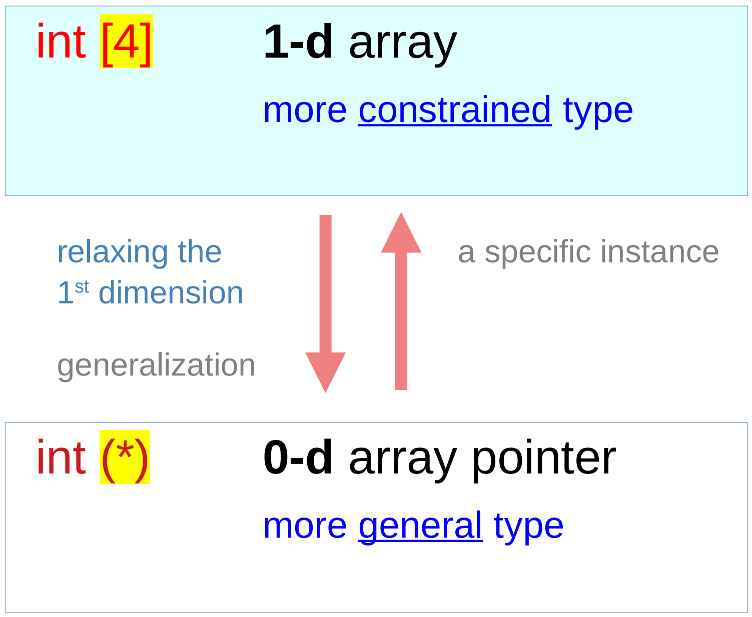
relaxing the 1st dimension

generalization

a specific instance

int (*)[2] **1-d** array pointer

more general type



int [4] **1-d** array

more constrained type

relaxing the 1st dimension

generalization

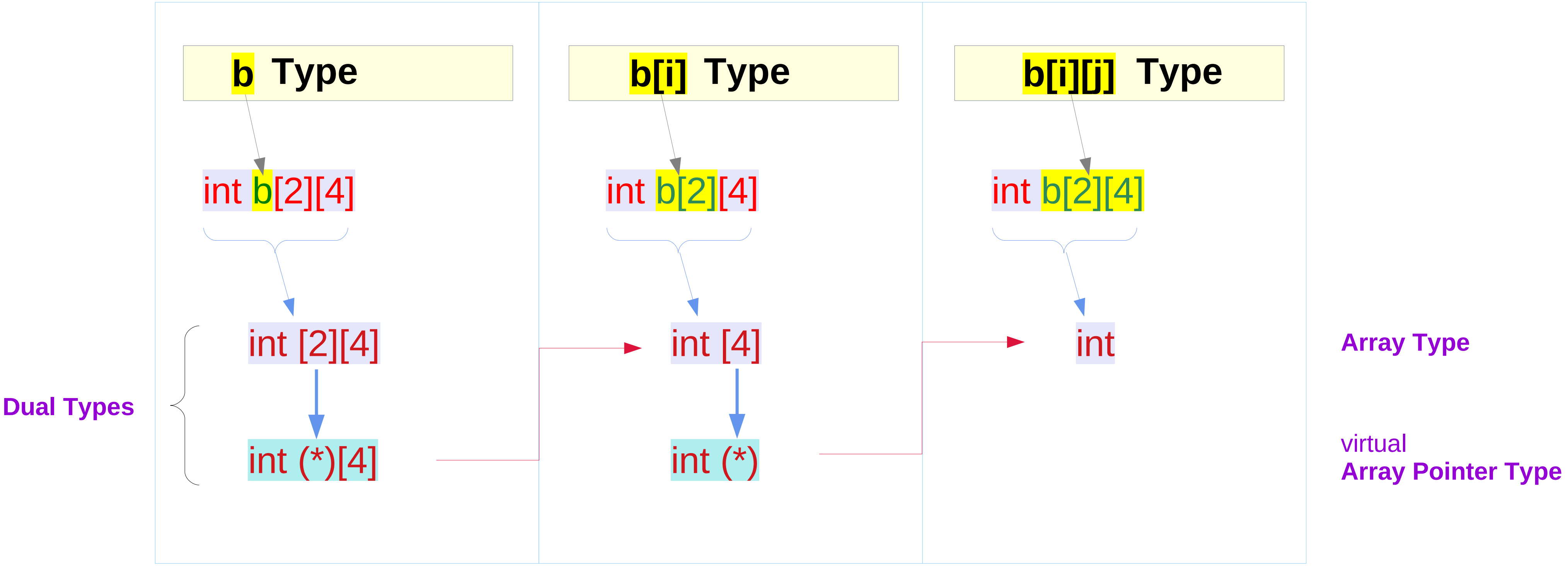
a specific instance

int (*) **0-d** array pointer

more general type

Subarray types in a **2-d** array

`int b[2][4];` **2-d** array b



b Type

int b[2][4]

int [2][4]

int (*)[4]

b[i] Type

int b[2][4]

int [4]

int (*)

b[i][j] Type

int b[2][4]

int

Dual Types

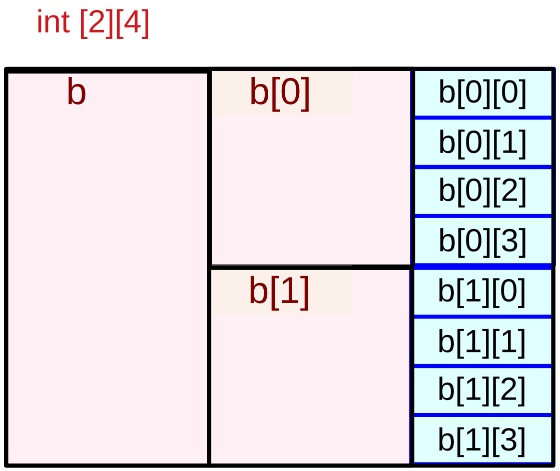
Array Type

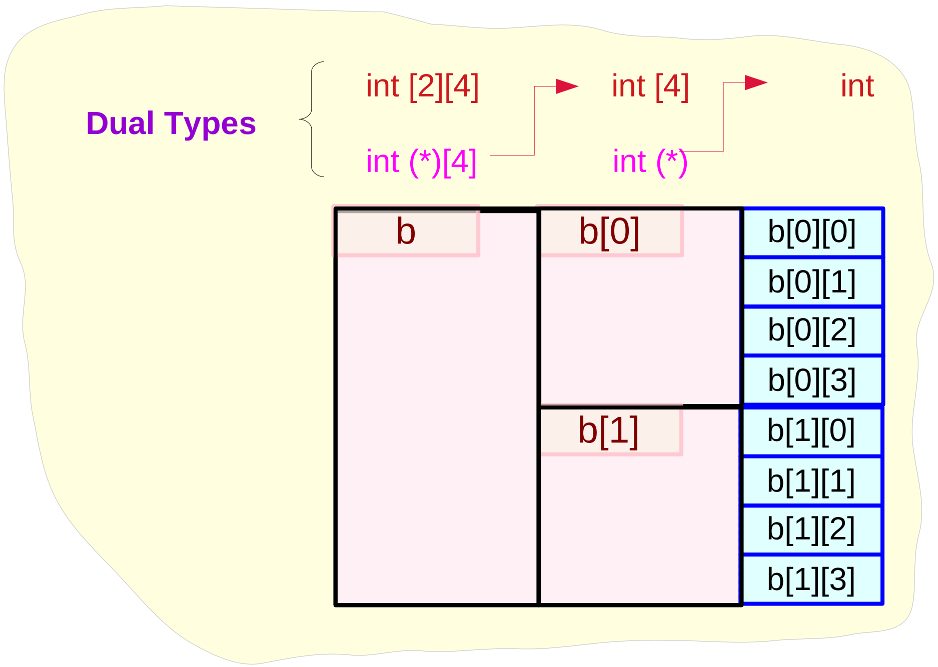
virtual
Array Pointer Type

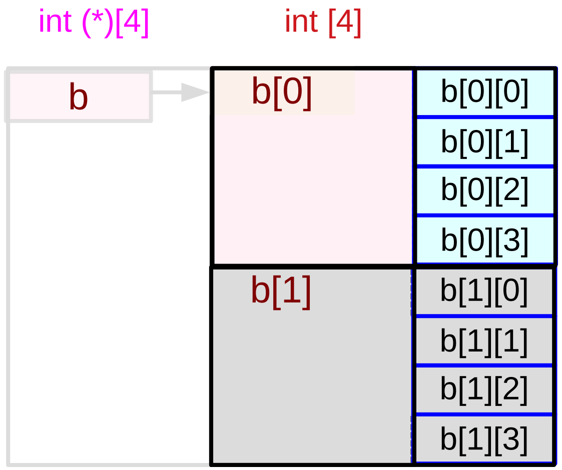
Dual types in a **2-d** array

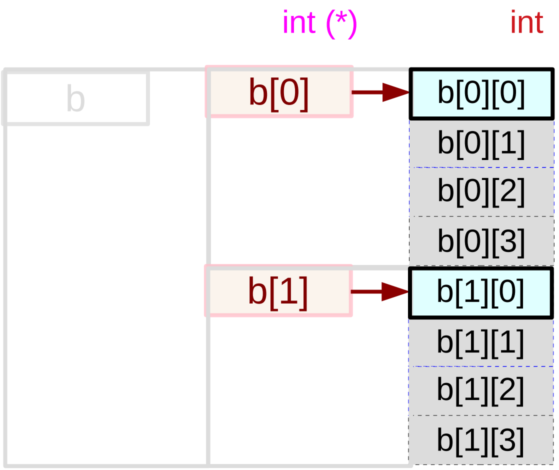
int b[2][4];

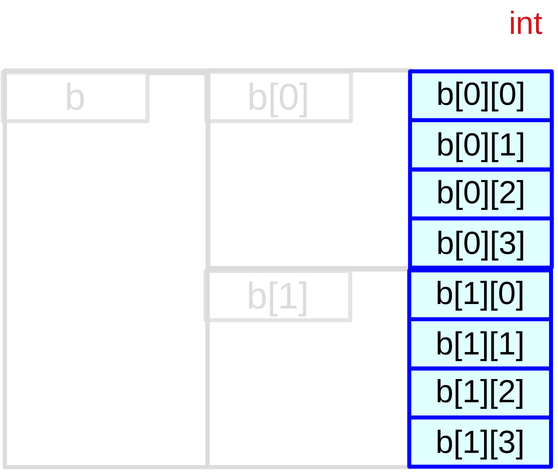
2-d array b











Subarray type examples

`int a[4];`

			relaxed type	virtual
a	int [4]	**1-d** array type	int (*)	**0-d** array pointer type
a[i]	int	**0-d** array type		

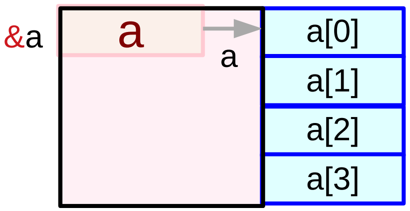
`int b[2][4];`

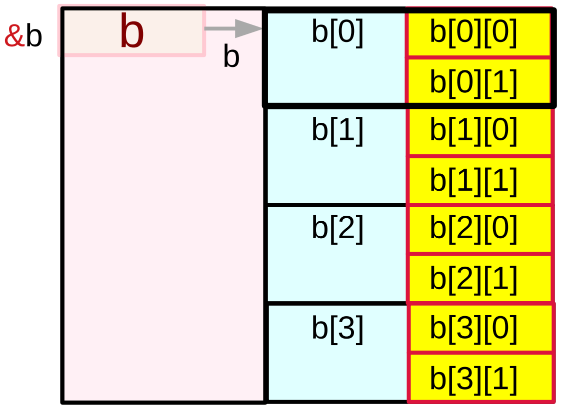
			relaxed type	virtual
b	int [2][4]	**2-d** array type	int (*)[4]	**1-d** array pointer type
b[i]	int [4]	**1-d** array type	int (*)	**0-d** array pointer type
b[i][j]	int	**0-d** array type		

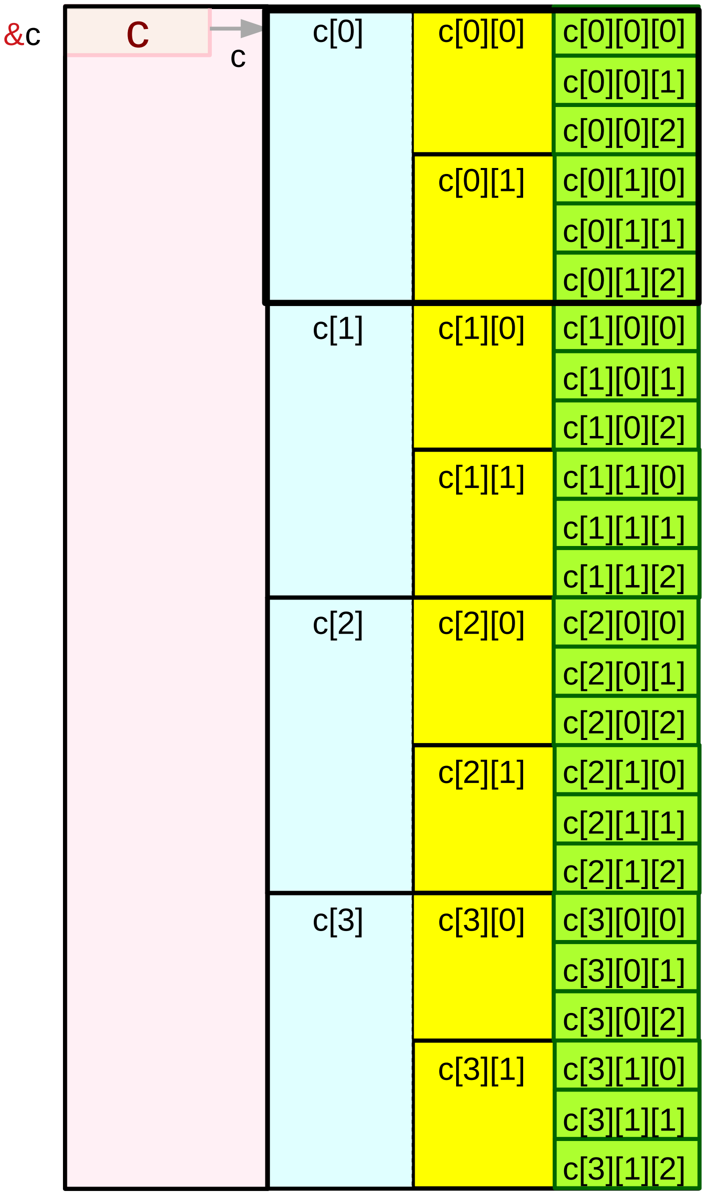
`int c[4][2][3];`

			relaxed type	virtual
c	int [4][2][3]	**3-d** array type	int (*)[2][3]	**2-d** array pointer type
c[i]	int [4][2]	**2-d** array type	int (*)[2]	**1-d** array pointer type
c[i][j]	int [4]	**1-d** array type	int (*)	**0-d** array pointer type
c[i][j][k]	int	**0-d** array type		

Types of **a**, **b**, **c** arrays





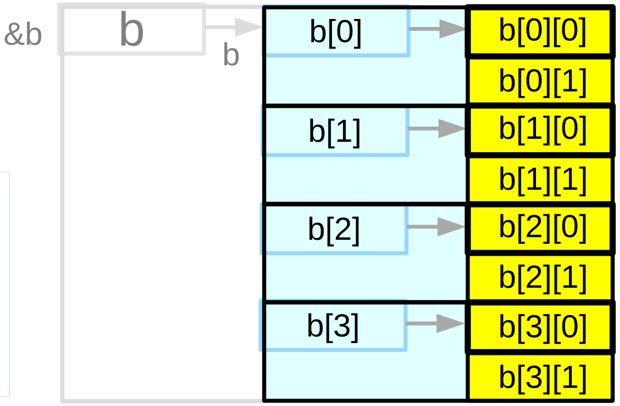


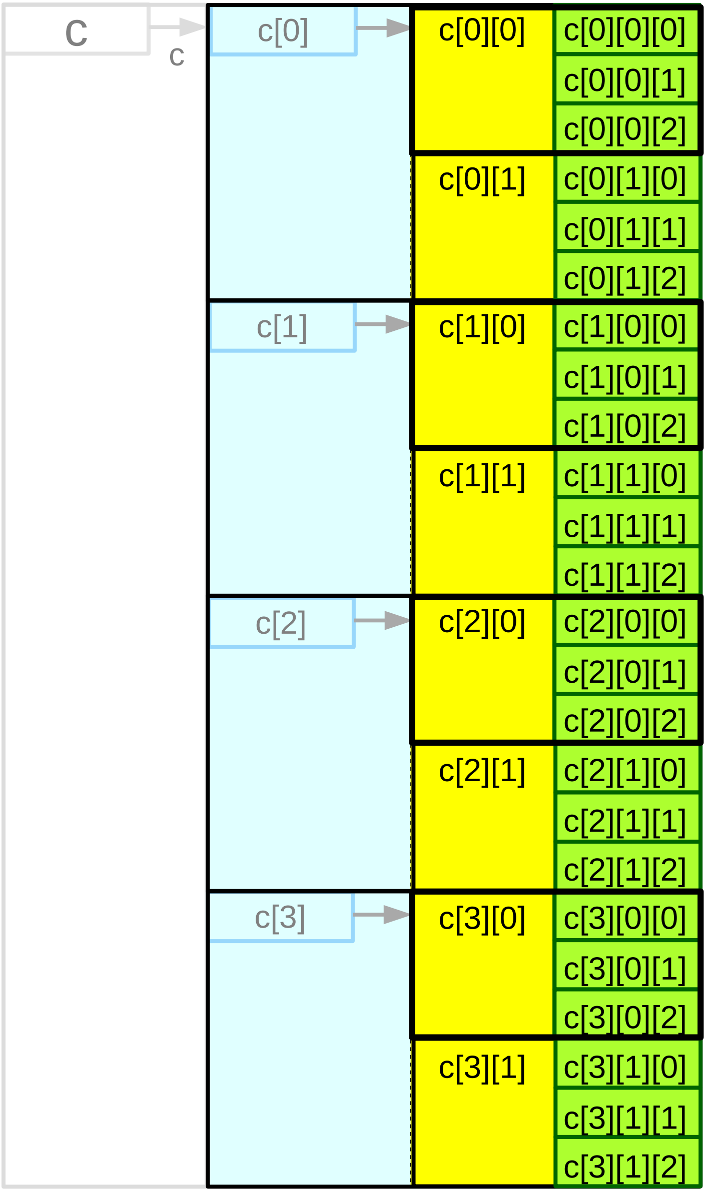
```
int a[4];
int b[2][4];
int c[4][2][3];
```

dual types

int [4]	**1-d** array **a**	**a[i]**
int (*)	**0-d** array pointer **a** (virtual)	***(a+i)**
int [4][2];	**2-d** array **b**	**b[i]**
int (*)[2];	**1-d** array pointer **b** (virtual)	***(b+i)**
int [4][2][3];	**3-d** array **c**	**c[i]**
int (*)[2][3];	**2-d** array pointer **c** (virtual)	***(c+i)**

Types of **b[i]**, **c[i]** subarrays





```
int a[4];
int b[2][4];
int c[4][2][3];
```

dual types

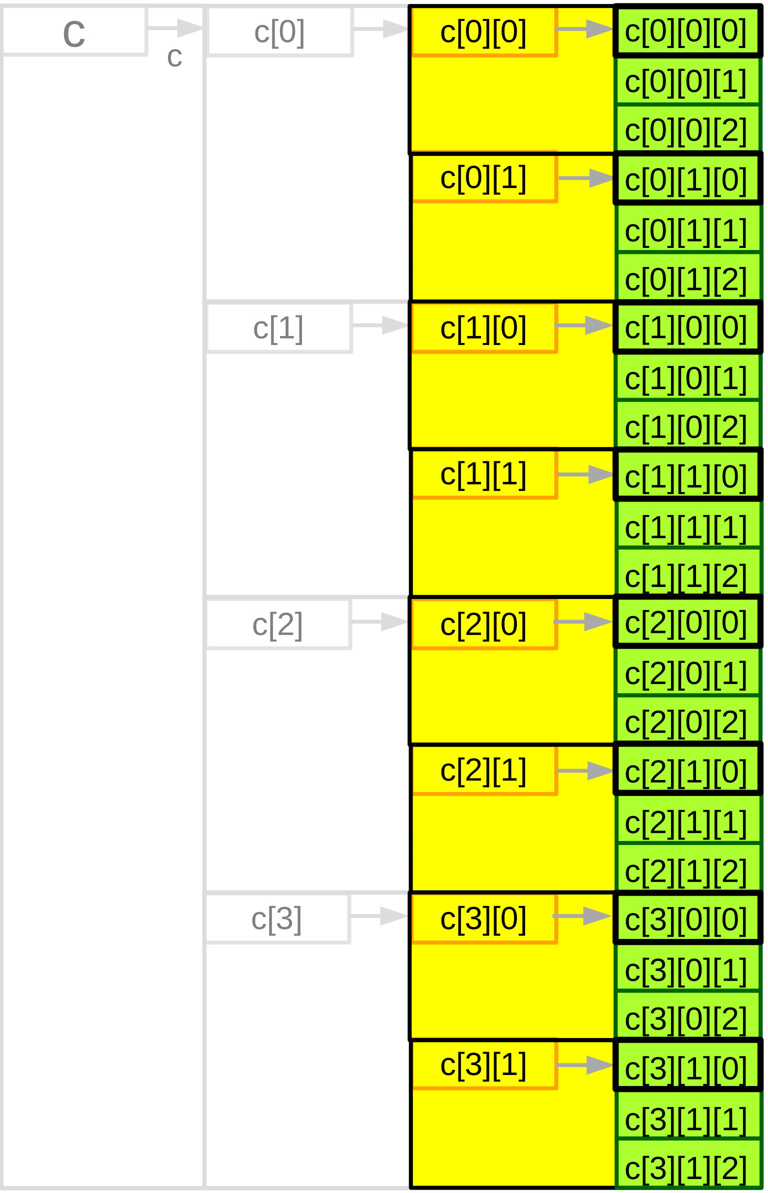
int [2]	**1-d** array **b[i]**	**b[i][j]**
int (*)	**0-d** array pointer **b[i]** (virtual)	***(b[i]+j)**
int [2][3];	**2-d** array **c[i]**	**c[i][j]**
int (*)[3];	**1-d** array pointer **c[i]** (virtual)	***(c[i]+j)**

Types of **c[i][j]** subarrays

```
int a[4];
int b[2][4];
int c[4][2][3];
```

dual types

int [3]	**1-d** array **c[i][j]**	**c[i][j][k]**
int (*)	**0-d** array pointer **c[i][j]** (virtual)	***(c[i][j]+k)**



Types of a **4-d** array and its subarrays

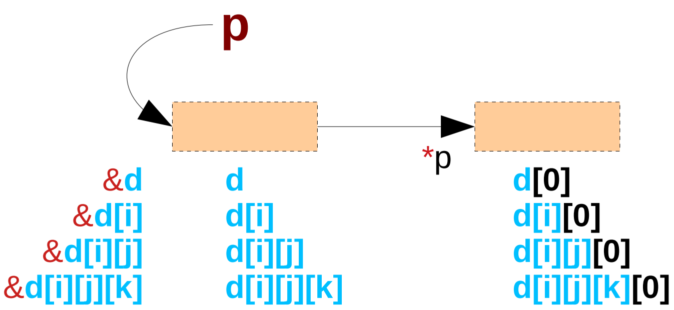
int **d**[4][2][3][4];

			types		
d	consider d[4][2][3][4]	→	int [4][2][3][4]	→	**4-d** array
	relax the 1st dimension	→	int (*)[2][3][4]	→	**3-d** array pointer (virtual)
d[i]	consider d[i][2][3][4]	→	int [2][3][4]	→	**3-d** array
	relax the 1st dimension	→	int (*)[3][4]	→	**2-d** array pointer (virtual)
d[i][j]	consider d[i][j][3][4]	→	int [3][4]	→	**2-d** array
	relax the 1st dimension	→	int (*)[4]	→	**1-d** array pointer (virtual)
d[i][j][k]	consider d[i][j][k][4]	→	int [4]	→	**1-d** array
	relax the 1st dimension	→	int (*)	→	**0-d** array pointer (virtual)

i,j,k are specific index values i=[0..3], j=[0..1], k=[0..2]

Initializing ***n*-d** array pointers with ***n*-d** subarrays

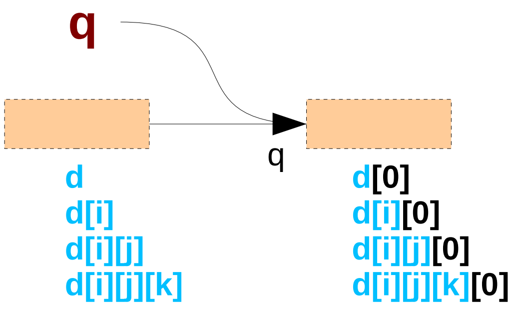
int d[4][2][3][4];



d	**4-d** array	d[4][2][3][4]	p = &d	abstract data
p	**4-d** array pointer	(*p)[4][2][3][4]	int (*p)[4][2][3][4] = &d;	
			(*p)[i][j][k][l] ≡ d[i][j][k][l]	
d[i]	**3-d** array	d[i][2][3][4]	p = &d[i]	abstract data
p	**3-d** array pointer	(*p)[2][3][4]	int (*p)[3][4] = &d[i];	
			(*p)[j][k][l] ≡ d[i][j][k][l] given i	
d[i][j]	**2-d** array	d[i][j][3][4]	p = &d[i][j]	abstract data
p	**2-d** array pointer	(*p)[3][4]	int (*p)[4] = &d[i][j];	
			(*p)[k][l] ≡ d[i][j][k][l] given i, j	
d[i][j][k]	**1-d** array	d[i][j][k][4]	p = &d[i][j][k]	abstract data
p	**1-d** array pointer	(*p)[4]	int (*p) = &d[i][j][k];	
			(*p)[l] ≡ d[i][j][k][l] given i, j, k	

Initializing **(*n*-1)-d** array pointers with ***n*-d** subarrays

int **d**[4][2][3][4];



q

q

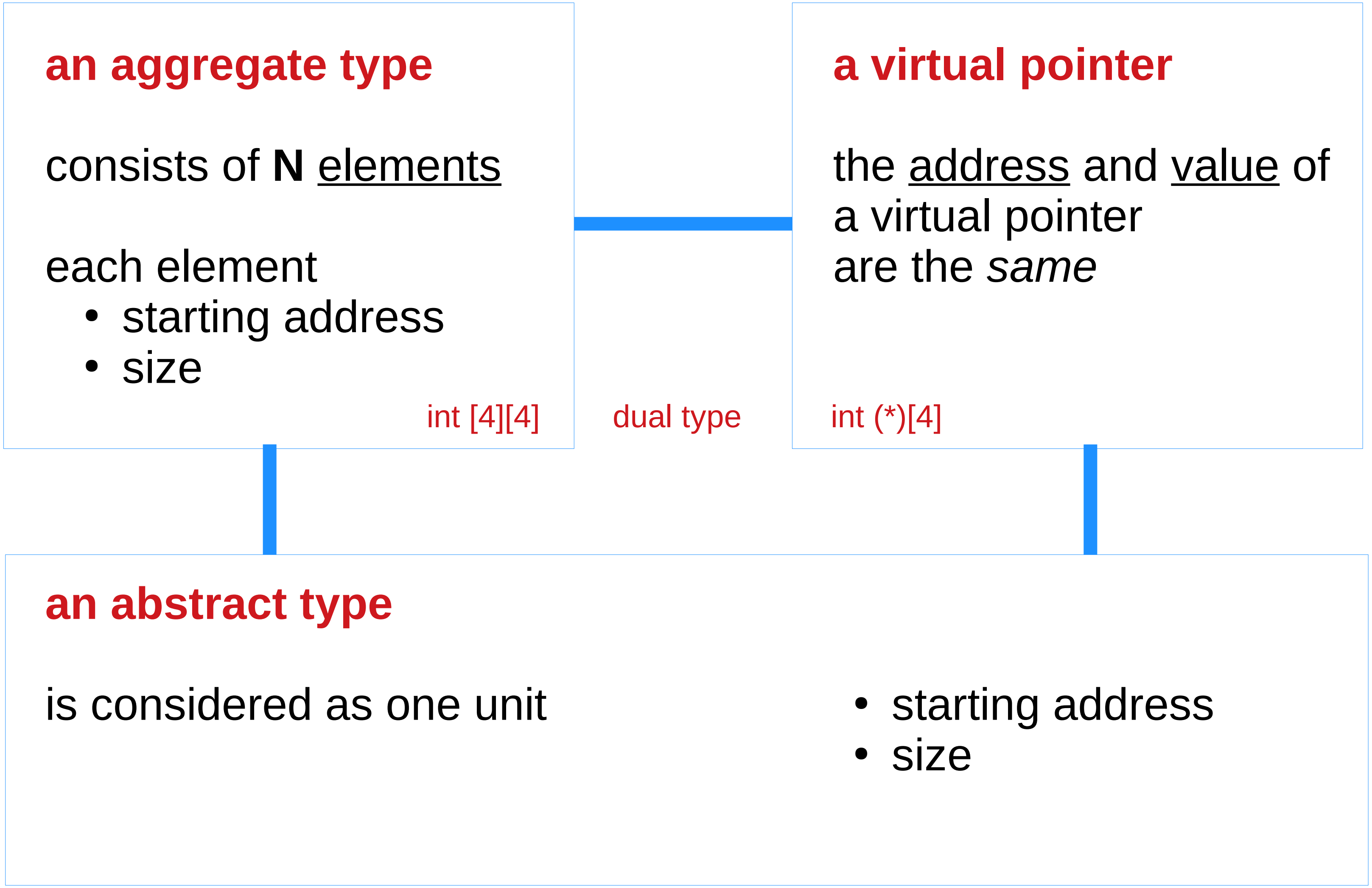
d
d[i]
d[i][j]
d[i][j][k]

d[0]
d[i][0]
d[i][j][0]
d[i][j][k][0]

d	**4-d** array	d[4][2][3][4]	**q** = **d**	virtual pointer
q	**3-d** array pointer	(*q)[2][3][4]	int (*q)[2][3][4] = **d**;	
			q[i][j][k][l] ≡ **d**[i][j][k][l]	
d[i]	**3-d** array	d[i][2][3][4]	**q** = **d[i]**	virtual pointer
q	**2-d** array pointer	(*q)[3][4]	int (*q)[3][4] = **d[i]**;	
			q[j][k][l] ≡ **d[i]**[j][k][l]	given i
d[i][j]	**2-d** array	d[i][j][3][4]	**q** = **d[i][j]**	virtual pointer
q	**1-d** array pointer	(*q)[4]	int (*q)[4] = **d[i][j]**;	
			q[k][l] ≡ **d[i][j]**[k][l]	given i, j
d[i][j][k]	**1-d** array	d[i][j][k][4]	**q** = **d[i][j][k]**	virtual pointer
q	**0-d** array pointer	(*q)	int (*q) = **d[i][j][k]**;	
			q[l] ≡ **d[i][j][k]**[l]	given i, j, k

Aggregate Data Types
Abstract Data Types
Virtual Array Pointers

Aggregate data type



an aggregate type

consists of **N** elements

each element
- starting address
- size

int [4][4]

dual type

a virtual pointer

the address and value of
a virtual pointer
are the *same*

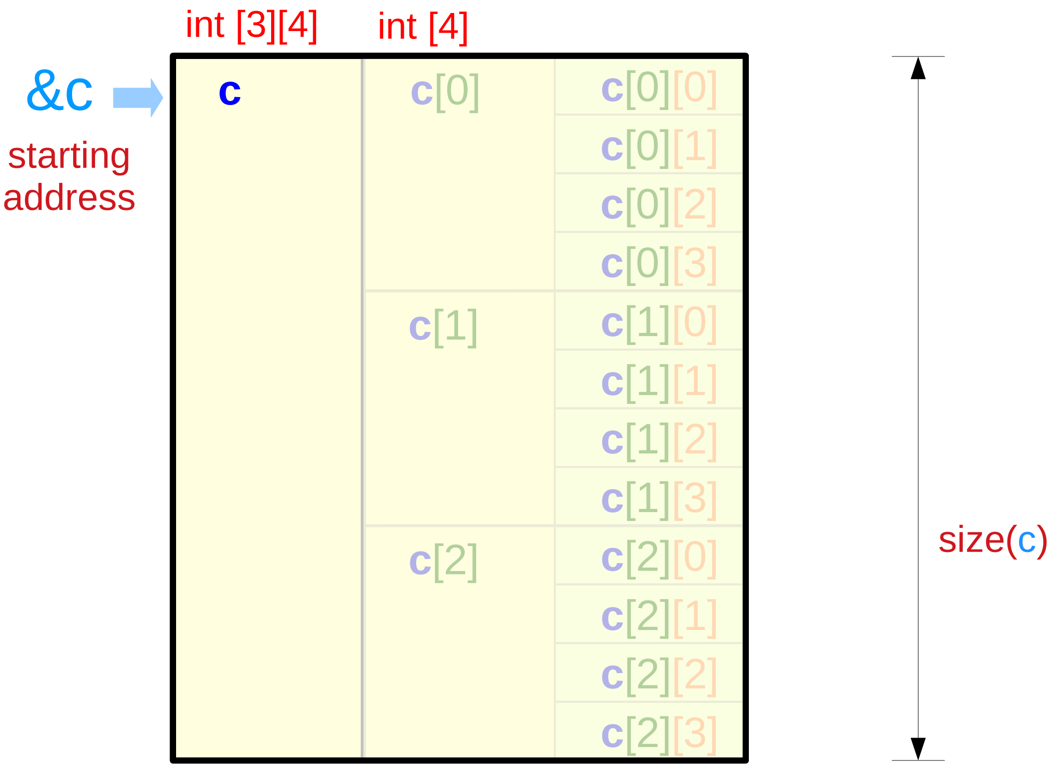
int (*)[4]

an abstract type

is considered as one unit

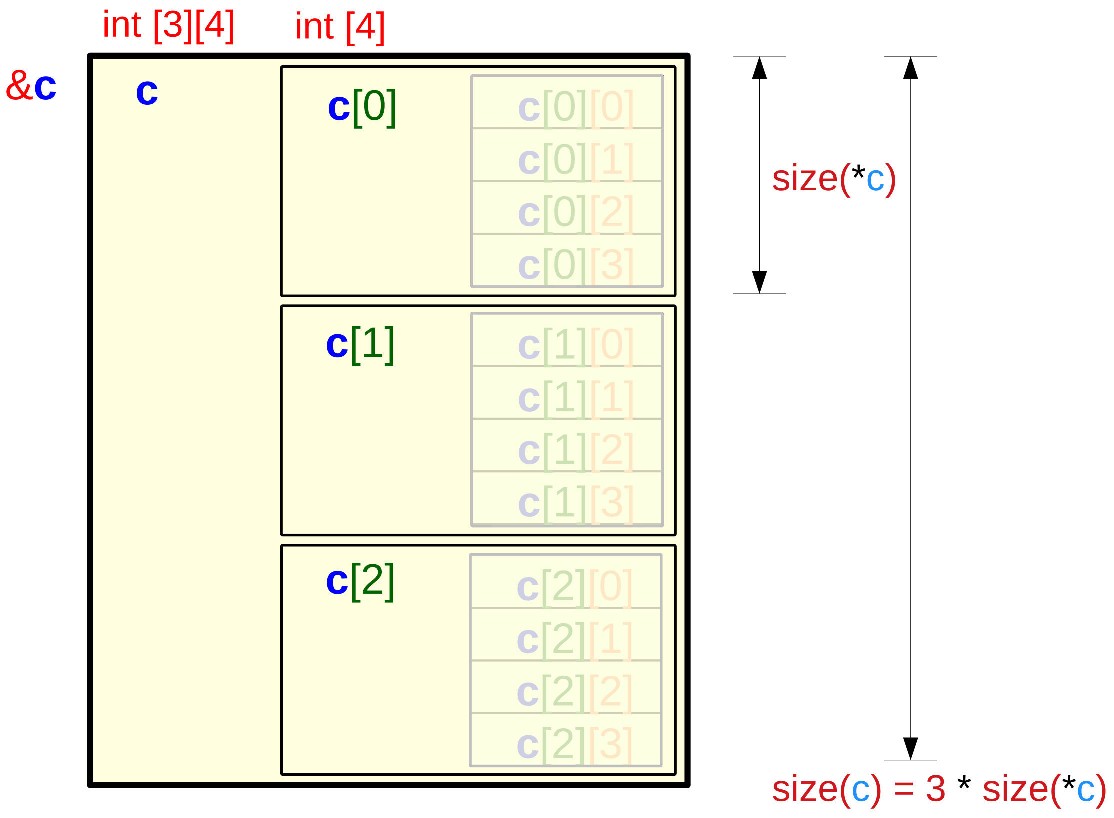
- starting address
- size

Abstract data **c**



an abstract data	c
- start address	&c
- size	sizeof(c)

Aggregate data **c**

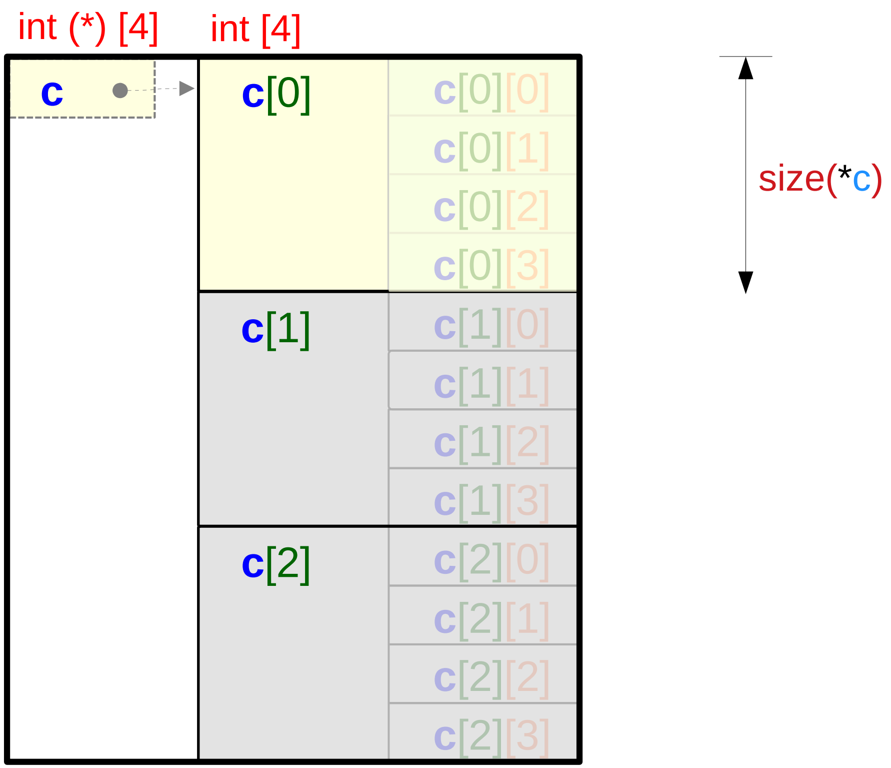


an aggregate type data **c**

- 1st element **c**[0]
- 2nd element **c**[1]
- 3rd element **c**[2]

Virtual pointer **c**

&c = c = &c[0]



a virtual pointer **c**
- pointer address &**c**
- pointer value **c** = &**c**[0]

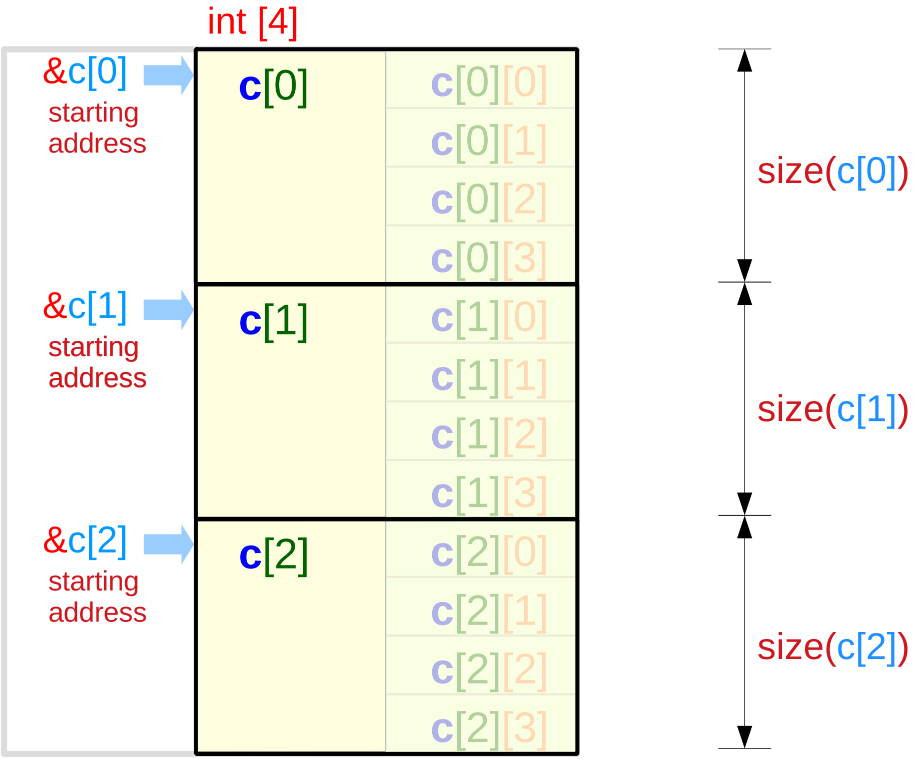
with the constraint
c = &**c**

an abstract data **c**[0] = ***c**
- start address &**c**[0] = **c**
- size sizeof(**c**[0])

virtual pointer **c** points
to abstract data **c**[0]

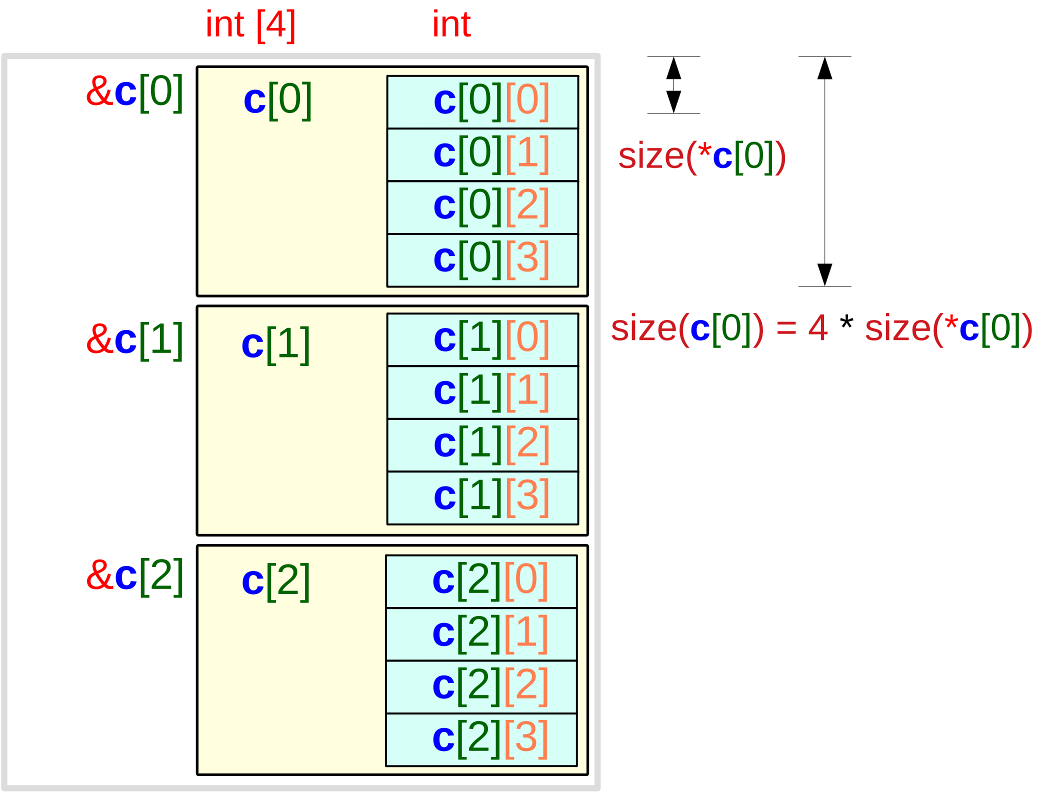
virtual pointers
- no physical memory locations are allocated
- address and data have the same value

Abstract data **c[i]**



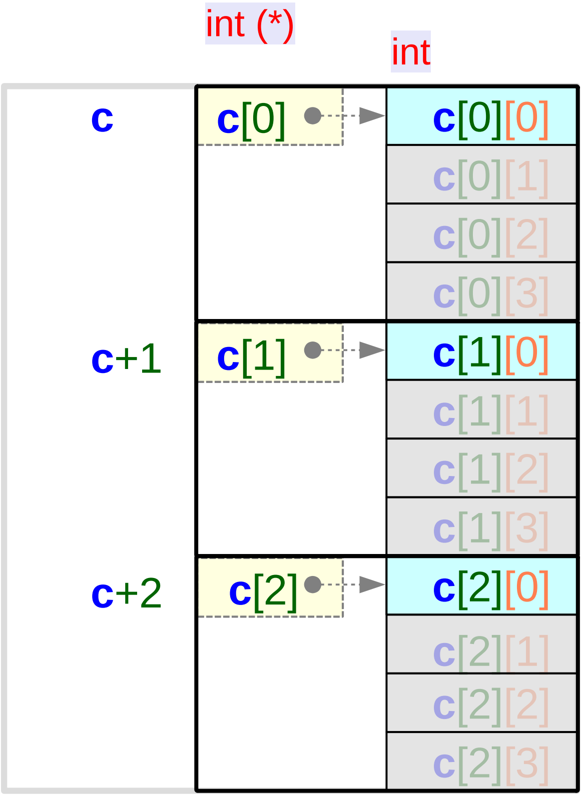
- an abstract data	c[0]
- start address	&c[0]
- size	sizeof(c[0])
- an abstract data	c[1]
- start address	&c[1]
- size	sizeof(c[1])
- an abstract data	c[2]
- start address	&c[2]
- size	sizeof(c[2])

Aggregate data **c[i]**



an aggregate type data	**c**[i]
- 1st element	**c**[i][0]
- 2nd element	**c**[i][1]
- 3rd element	**c**[i][2]
- 4th element	**c**[i][3]

Virtual pointer c[i]



a virtual pointer c[i]
- pointer address &c[i]
- pointer value c+i = &c[i]

with the constraint
c[i] = &c[i]

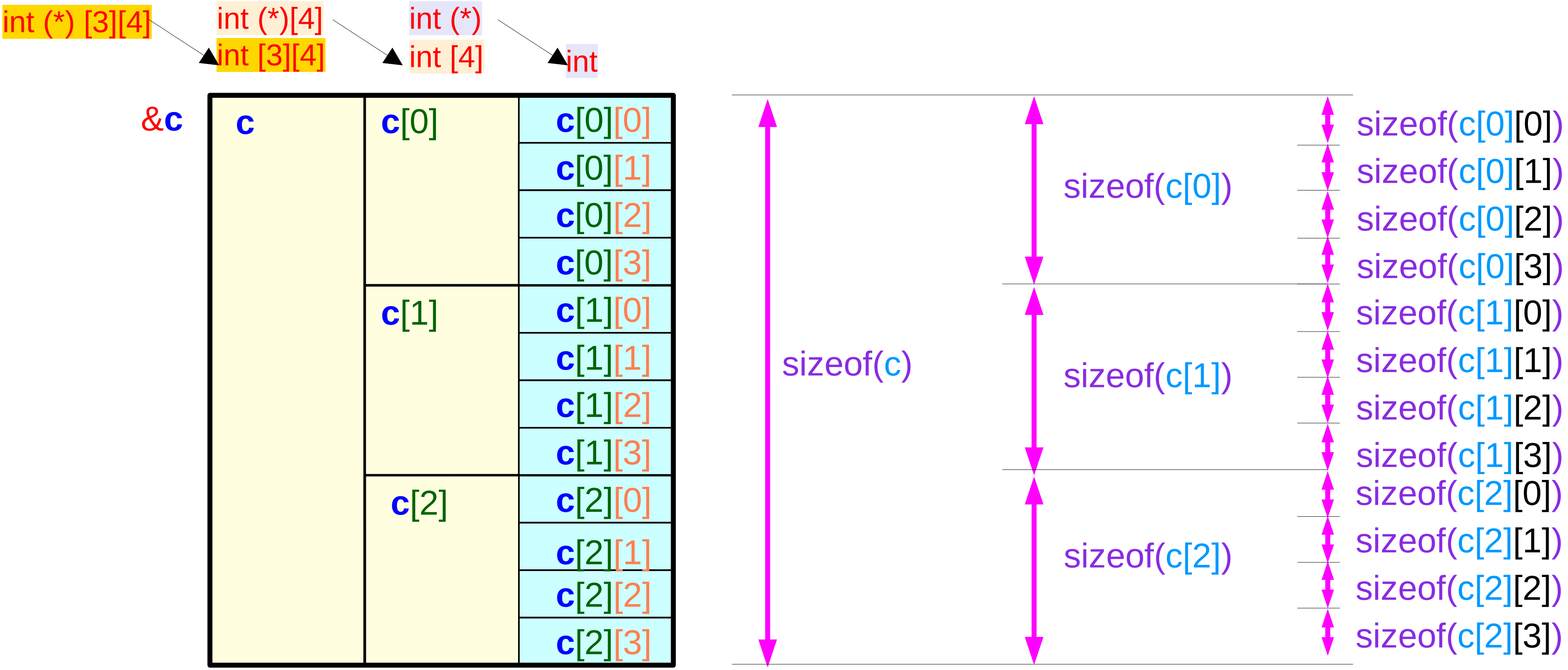
an primitive data c[i][0] = *c[i]
- start address &c[i][0] = c[i]
- size sizeof(c[i][0])

virtual pointer c[i]
points to primitive data c[i][0]

virtual pointers
- no physical memory locations are allocated
- address and data have the same value

A **2-d** array and its **1-d** sub-arrays – a size view

`int c[3][4];`



int (*) [3][4]

int (*)[4]

int [3][4]

int (*)

int [4]

int

&c

c

c[0]

c[1]

c[2]

c[0][0]
c[0][1]
c[0][2]
c[0][3]
c[1][0]
c[1][1]
c[1][2]
c[1][3]
c[2][0]
c[2][1]
c[2][2]
c[2][3]

sizeof(c)

sizeof(c[0])

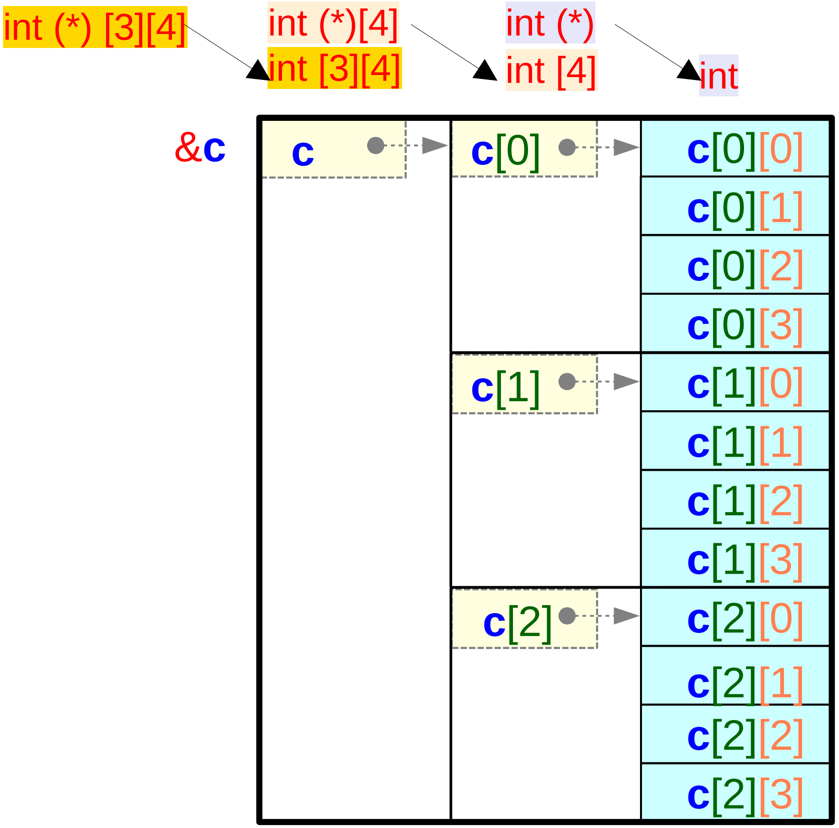
sizeof(c[1])

sizeof(c[2])

sizeof(c[0][0])
sizeof(c[0][1])
sizeof(c[0][2])
sizeof(c[0][3])
sizeof(c[1][0])
sizeof(c[1][1])
sizeof(c[1][2])
sizeof(c[1][3])
sizeof(c[2][0])
sizeof(c[2][1])
sizeof(c[2][2])
sizeof(c[2][3])

A **2-d** array and its **1-d** sub-arrays – a virtual pointer view

`int c[3][4];`



value(c) = value(c[0]) = value(&c[0][0])
value(&c) = value(&c[0]) = value(&c[0][0])

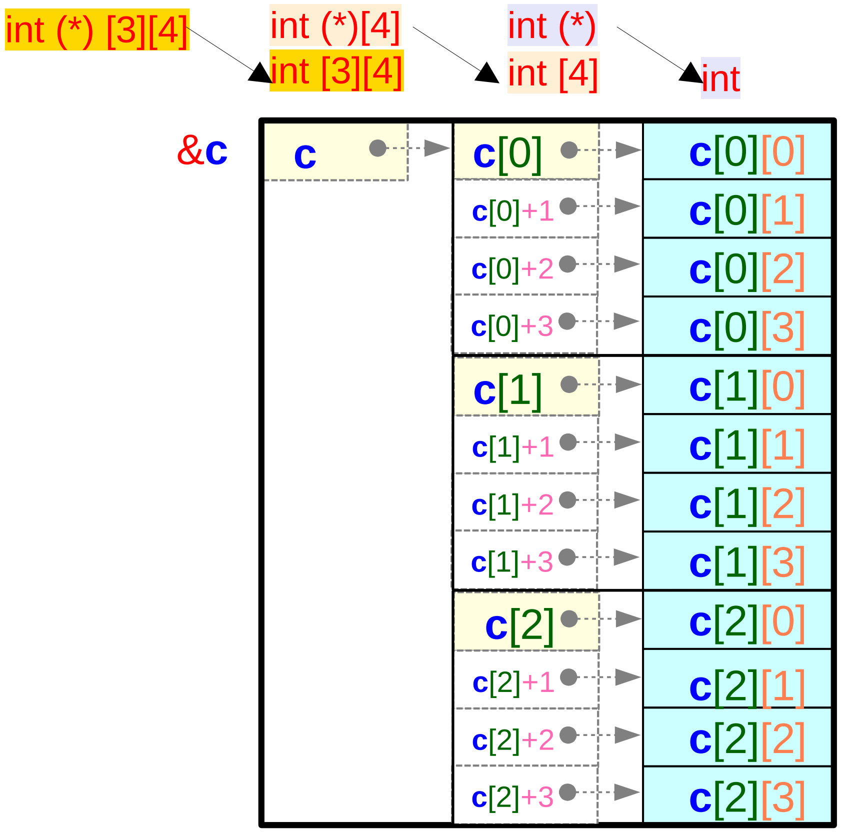
value(c[1]) = value(&c[1][0])
value(&c[1]) = value(&c[1][0])

value(c[2]) = value(&c[2][0])
value(&c[2]) = value(&c[2][0])

address(c) = address(c[0]) = address(c[0][0])
address(c[1]) = address(c[1][0])
address(c[2]) = address(c[2][0])

A **2-d** array and its **1-d** sub-arrays – size relation

int c[3][4];



sizeof(c) = sizeof(c[0]) * 3 … *leading element*
sizeof(c+1) = pointer size (4/8 bytes)
sizeof(c+2) = pointer size (4/8 bytes)

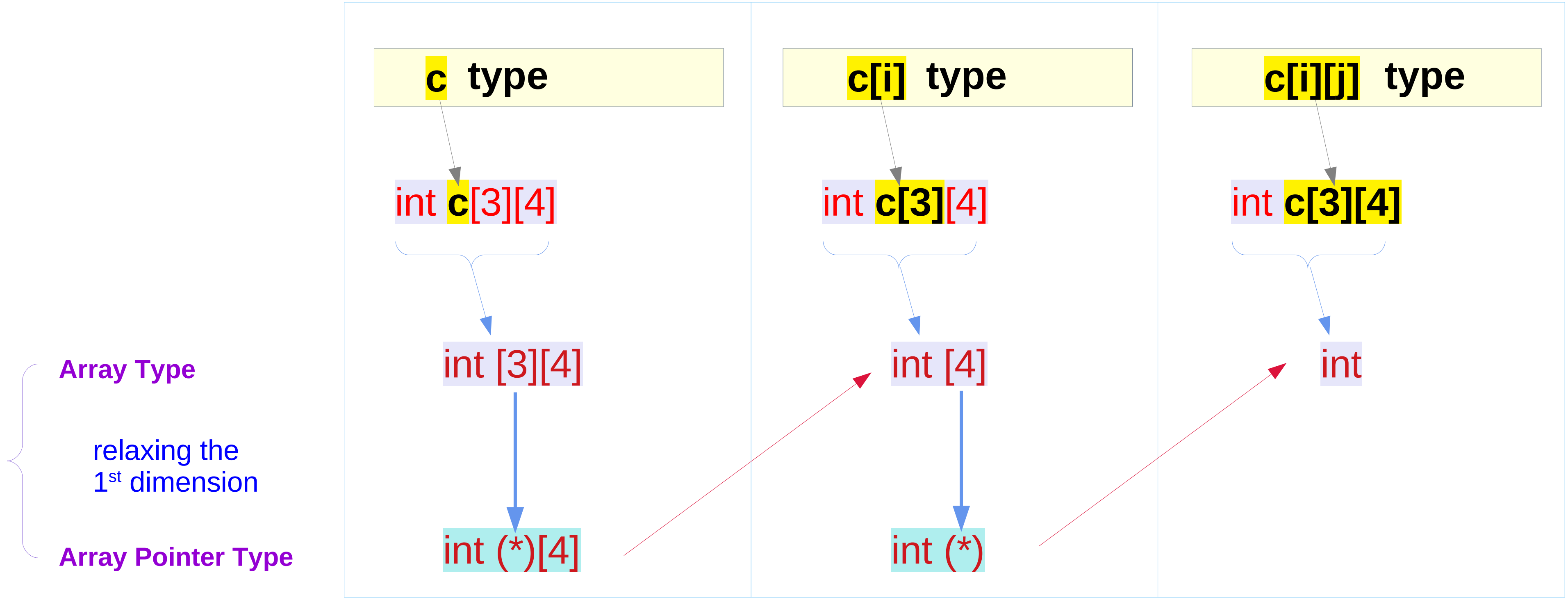
sizeof(c[0]) = sizeof(c[0][0]) * 4 … *leading element*
sizeof(c[0]+1) = pointer size (4/8 bytes)
sizeof(c[0]+2) = pointer size (4/8 bytes)
sizeof(c[0]+3) = pointer size (4/8 bytes)

sizeof(c[1]) = sizeof(c[1][0]) * 4 … *leading element*
sizeof(c[1]+1) = pointer size (4/8 bytes)
sizeof(c[1]+2) = pointer size (4/8 bytes)
sizeof(c[1]+3) = pointer size (4/8 bytes)

sizeof(c[2]) = sizeof(c[2][0]) * 4 … *leading element*
sizeof(c[2]+1) = pointer size (4/8 bytes)
sizeof(c[2]+2) = pointer size (4/8 bytes)
sizeof(c[2]+3) = pointer size (4/8 bytes)

Sub-array types in a **2-d** array

`int c[3][4];` **2-d** array c



c type

int **c**[3][4]

int [3][4]

int (*)[4]

c[i] type

int **c[3]**[4]

int [4]

int (*)

c[i][j] type

int **c[3][4]**

int

Array Type

relaxing the
1st dimension

Array Pointer Type

Dual Types

- **Identifying nested arrays**

  in a 2-d array declaration

Nested arrays in a 2-d array declaration

`int c[3] [4] ;`

`int c[3] [4] ;`

c : a 3 element array

c[i] : each element

`int c[3] [4] ;`

c[i]'s type 1 : int [4]

an array of 4 integers

relaxed dimension

`int c[3] [4] ;`

c[i]'s type 2: int (*)

a pointer to an integer

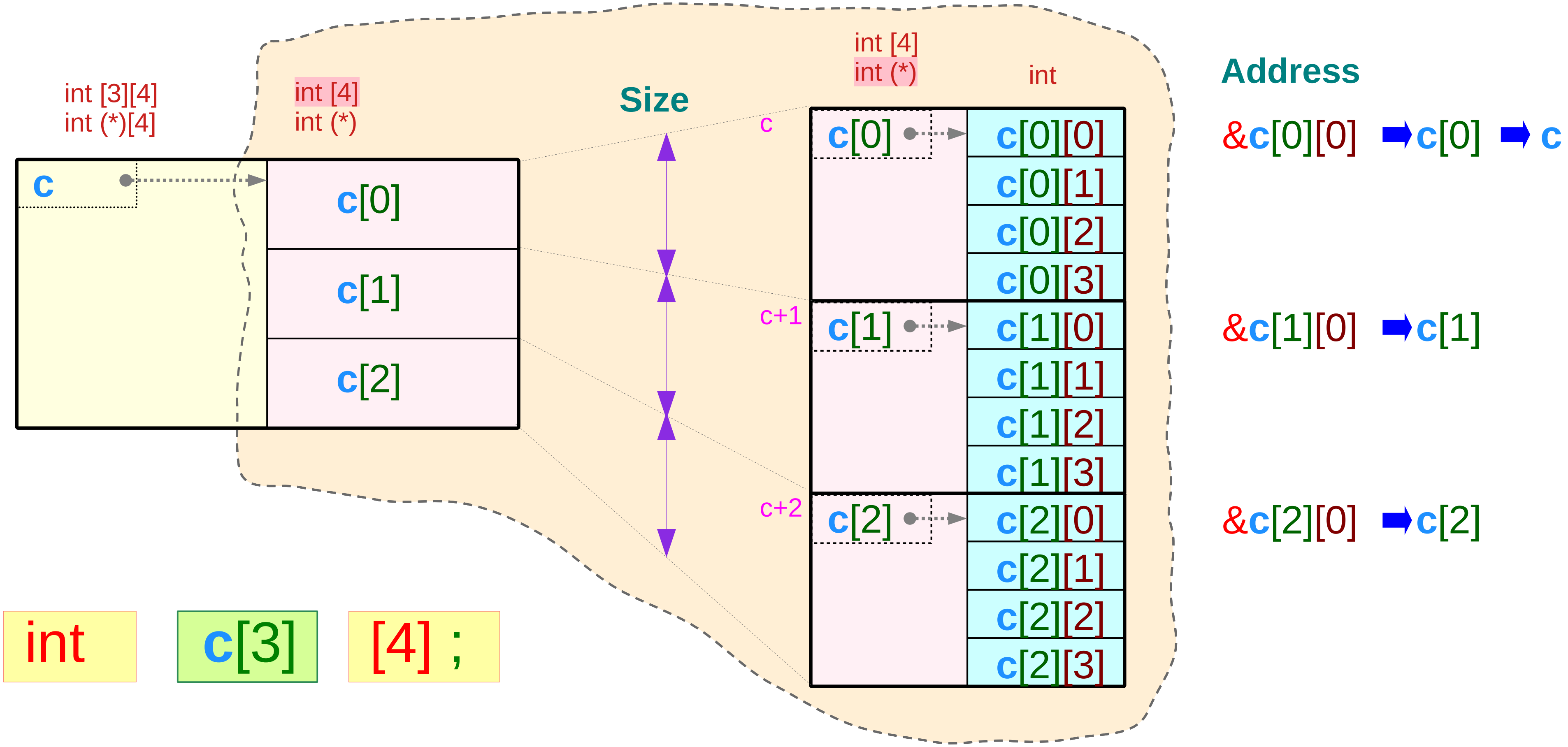
Nested arrays

c[3]

c : a 3 element array
c[i] : each element

int [4] ;

c[i]'s type 1 : int [4]
c[i]'s type 2 : int (*)



int c[3] [4] ;

c : 3-element array

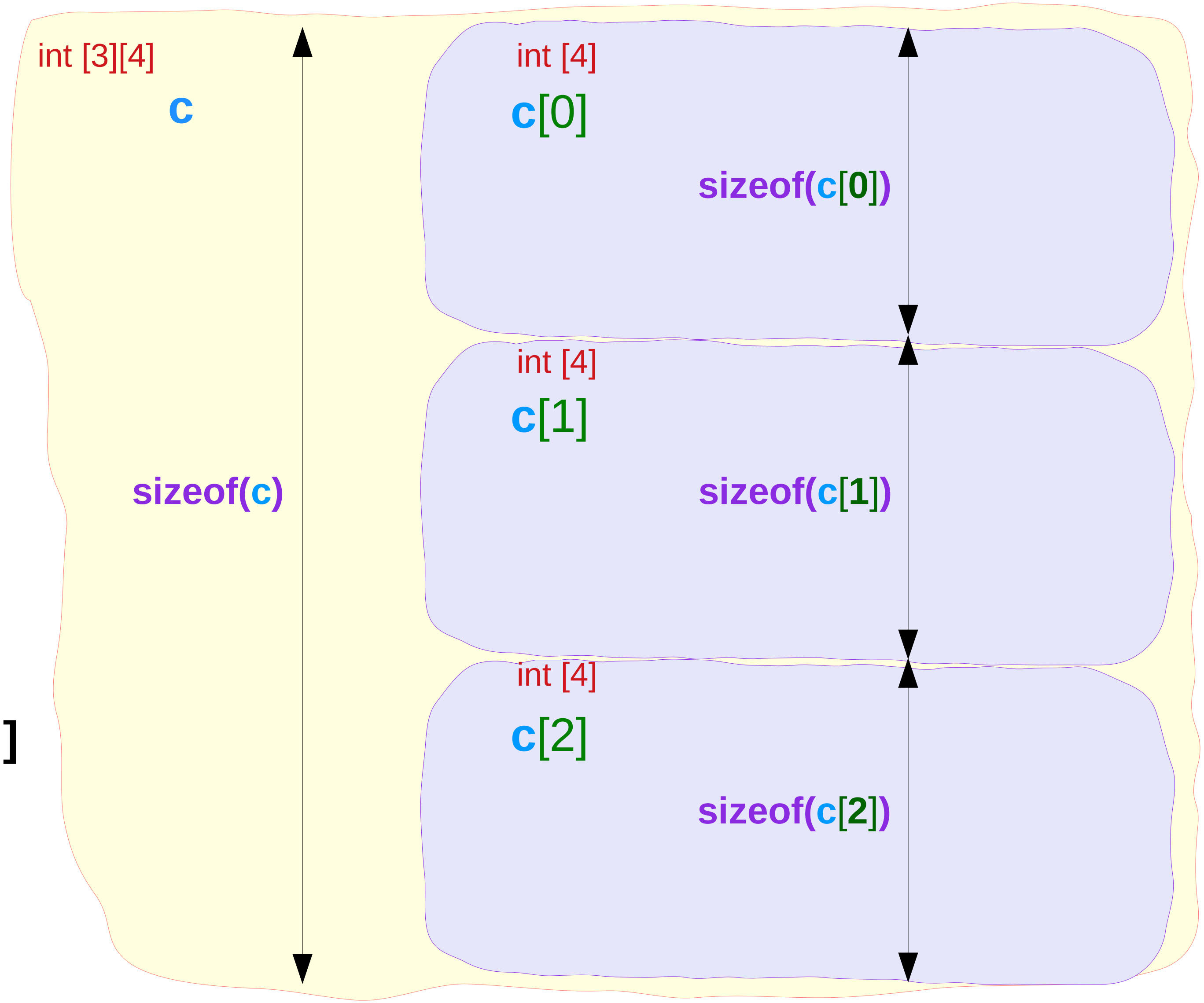
c	**2-d** array	int [3][4]
c[i]	**1-d** array	int [4]

`int c [3] [4] ;`

3-element array c

abstract data element **c[i]**

each element **c[i]** has the
1-d array type int [4]



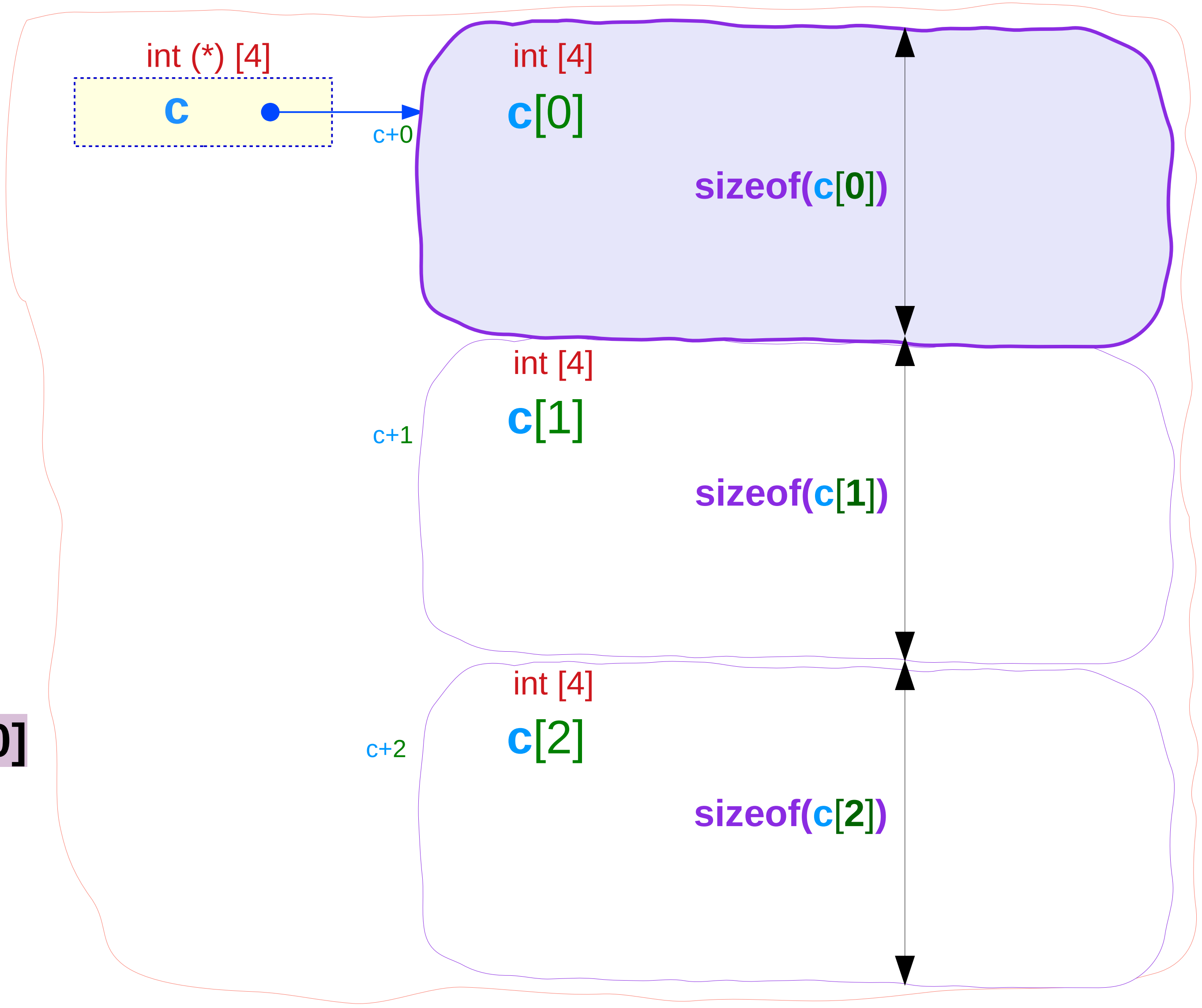
int [3][4]
c
sizeof(c)

int [4]
c[0]
sizeof(c[0])

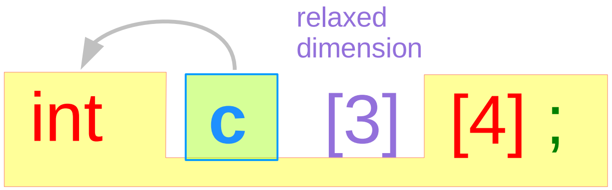
int [4]
c[1]
sizeof(c[1])

int [4]
c[2]
sizeof(c[2])

c : pointer to a 4-element array



c	**1-d** array pointer	int (*)[4]
c[i]	**1-d** array	int [4]



pointer c

abstract data element **c[0]**

each element **c[i]** has the
1-d array type int [4]

c[i] : 4-element array

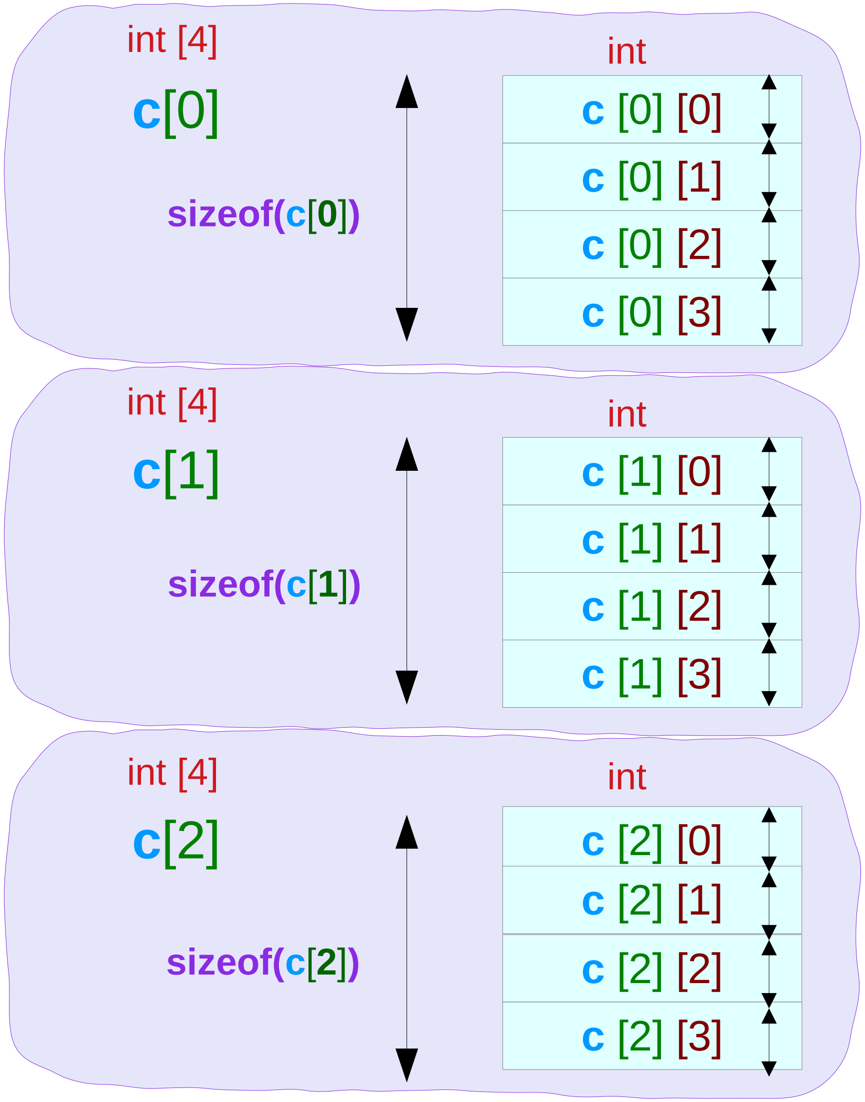
c[i]	**1-d** array	int [4]
c[i][j]	**0-d** array	int

int c [3] [4] ;

4-element array c[i]

primitive data element **c[i][j]**

each element **c[i][j]** has
the primitive type int



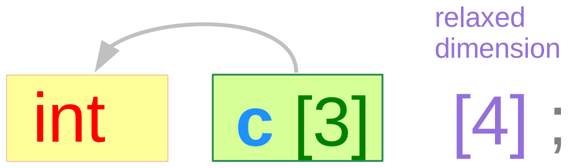
int [4]
c[0]
sizeof(c[0])
int
c [0] [0]
c [0] [1]
c [0] [2]
c [0] [3]

int [4]
c[1]
sizeof(c[1])
int
c [1] [0]
c [1] [1]
c [1] [2]
c [1] [3]

int [4]
c[2]
sizeof(c[2])
int
c [2] [0]
c [2] [1]
c [2] [2]
c [2] [3]

c[i] : pointer to a primitive data

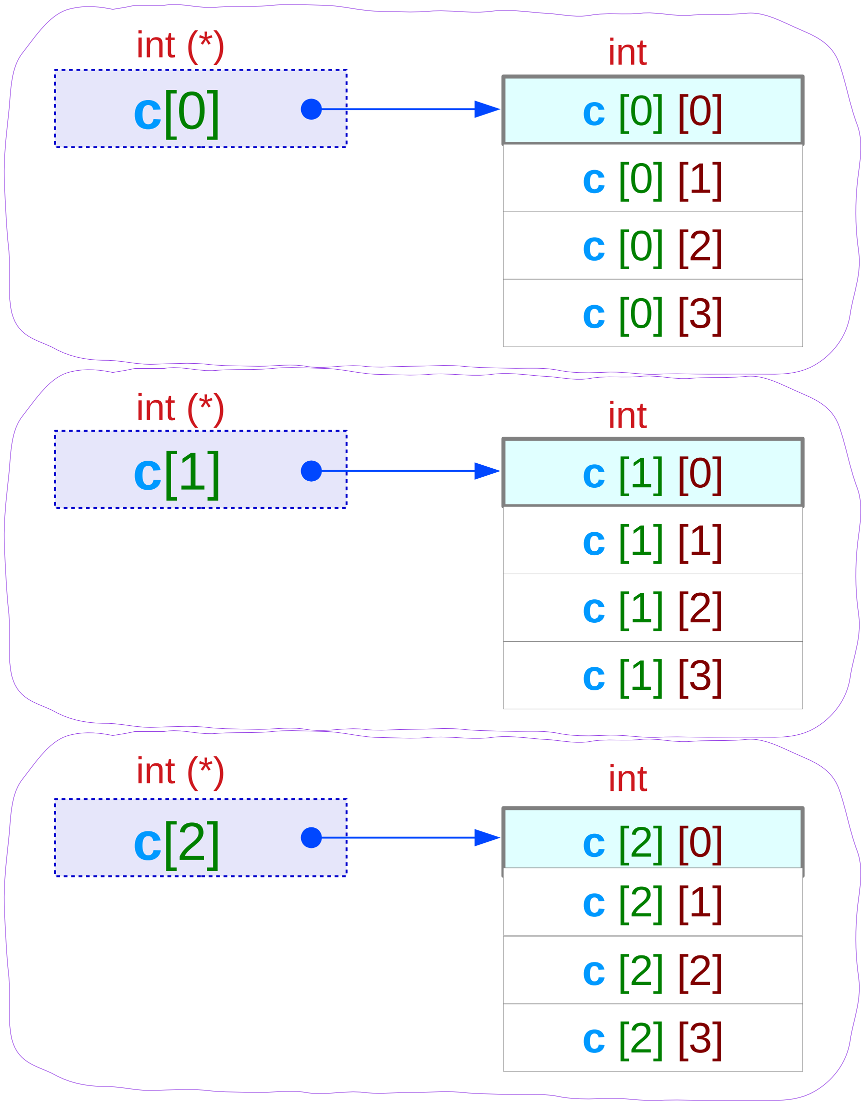
c[i]	**0-d** array pointer	int (*)
c[i][j]	**0-d** array	int



pointer c[i]

primitive data element **c[i][0]**

each element **c[i][j]** has
the primitive type int



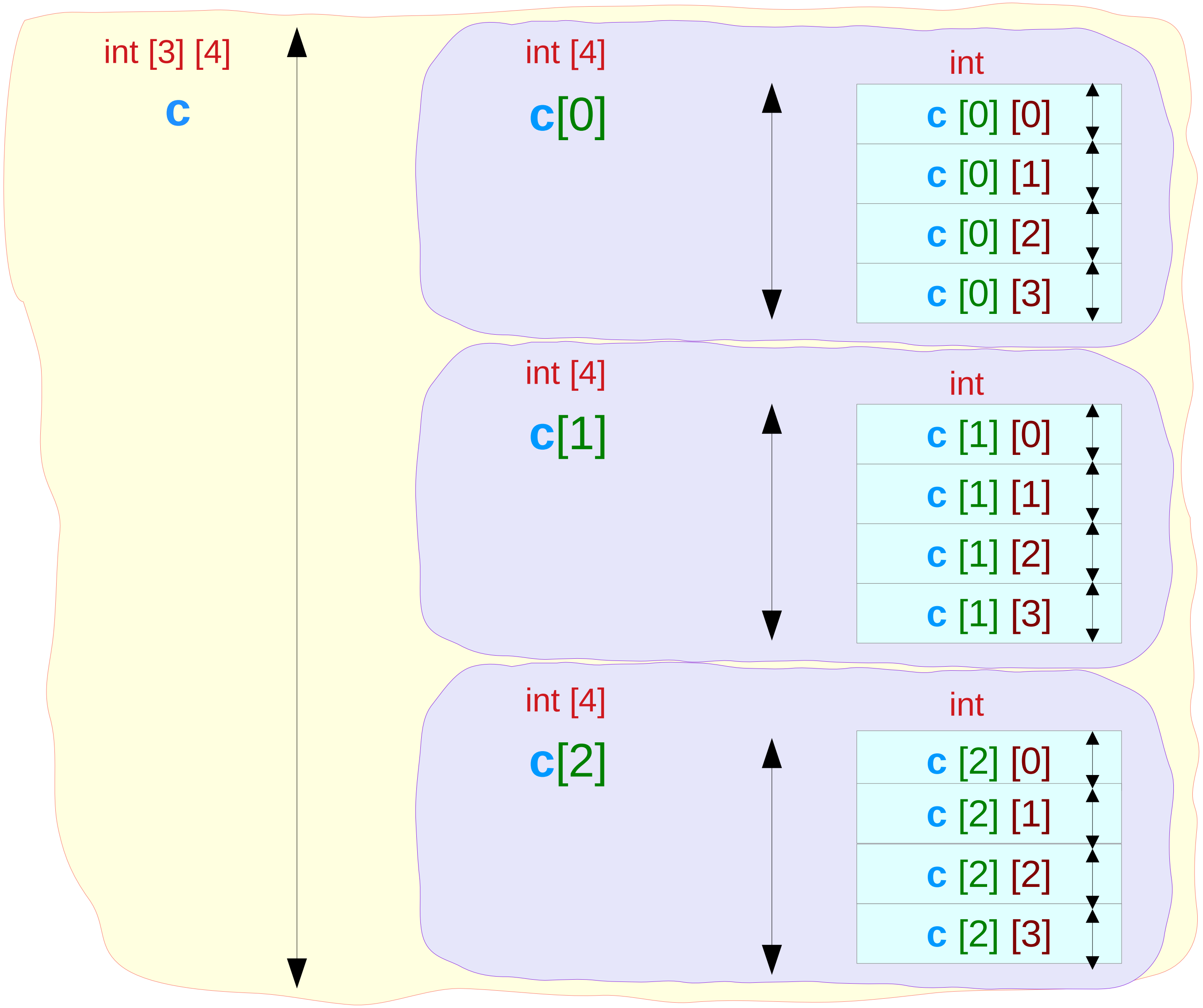
Recursive data view

c	**2-d** array	int [3][4]
c	**1-d** array pointer	int (*)[4]
c[i]	**1-d** array	int [4]
c[i]	**0-d** array pointer	int (*)
c[i][j]	**0-d** array	int

int c[3] [4] ;

3-element array c

4-element array c[i]



Pointer view

c	**2-d** array	int [3][4]
c	**1-d** array pointer	int (*)[4]
c[i]	**1-d** array	int [4]
c[i]	**0-d** array pointer	int (*)
c[i][j]	**0-d** array	int

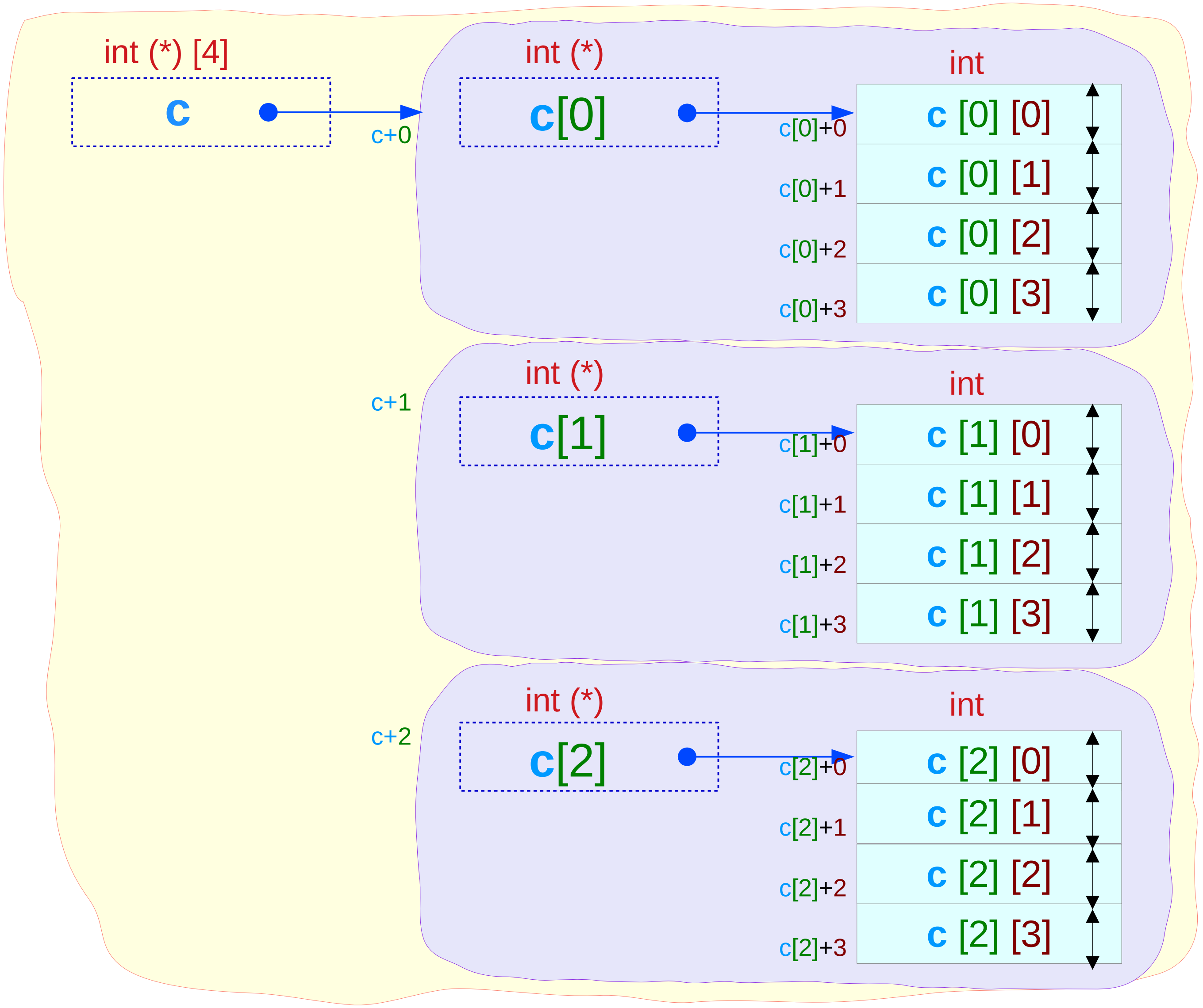
int c[3] [4] ;

v(**c**) = v(**c**[0]) = v(&**c**[0][0])

v(**c**[1]) = v(&**c**[1][0])

v(**c**[2]) = v(&**c**[2][0])

v ≡ value



1-d array pointer

int (*p) [4] ;

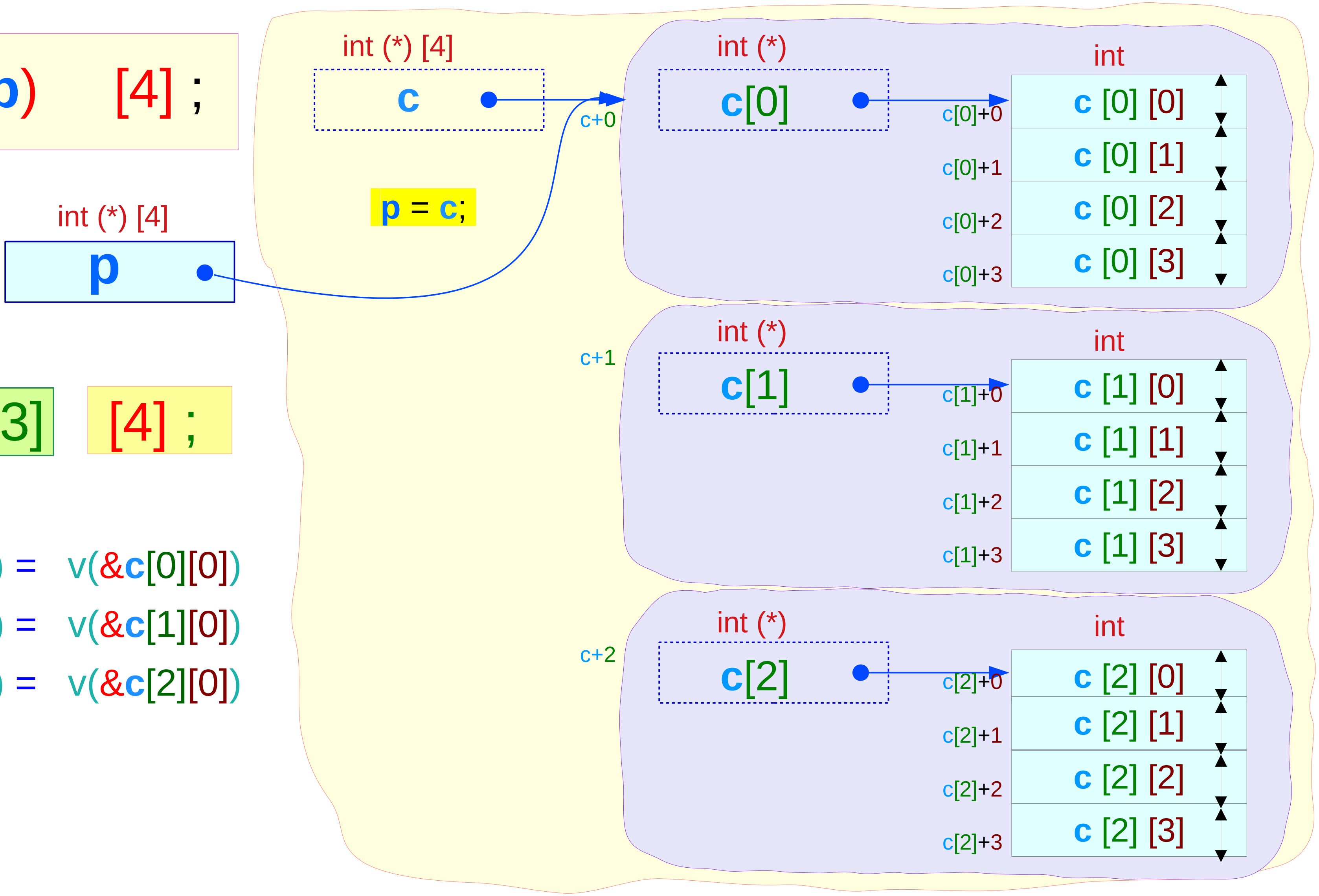
int c[3] [4] ;

v(c) = v(c[0]) = v(&c[0][0])

v(c[1]) = v(&c[1][0])

v(c[2]) = v(&c[2][0])

v ≡ value

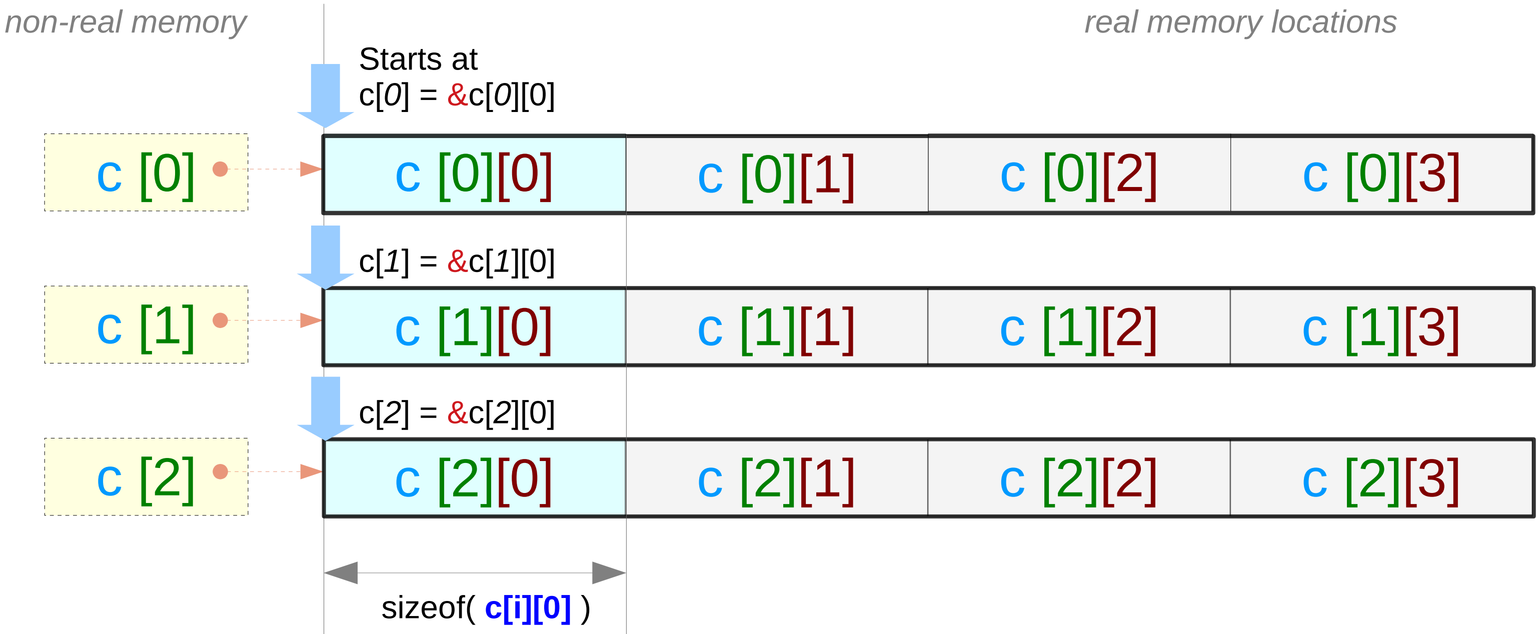


Pointer **c[i]** and integer **c[i][0]**

int **c[3]** [4];

non-real pointer **c[i]** : value(**c[i]**)= &**c[i][0]**

0-d array pointer



virtual pointer c[i]
0-d array pointer : int (*)

integer c[i][0]
0-d array : int

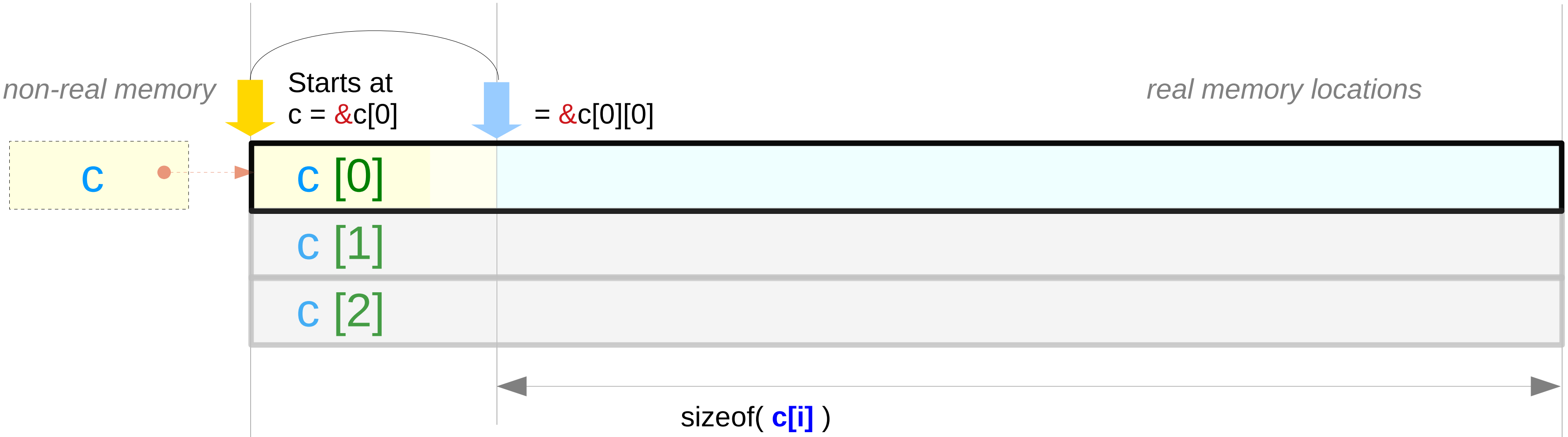
c[i] points to the 1st element **c[i][0]**

Pointer **c** and abstract data **c[i]**

```
int c [3] [4];
```

non-real pointer **c** : value(**c**) = &**c[0]** = &**c[0][0]** — **1-d** array pointer

abstract data **c[i]** : sizeof(**c[i]**) = 4 * sizeof(**int**) — **1-d** array



virtual pointer c

1-d array pointer : int (*)[4]

abstract data c[i]

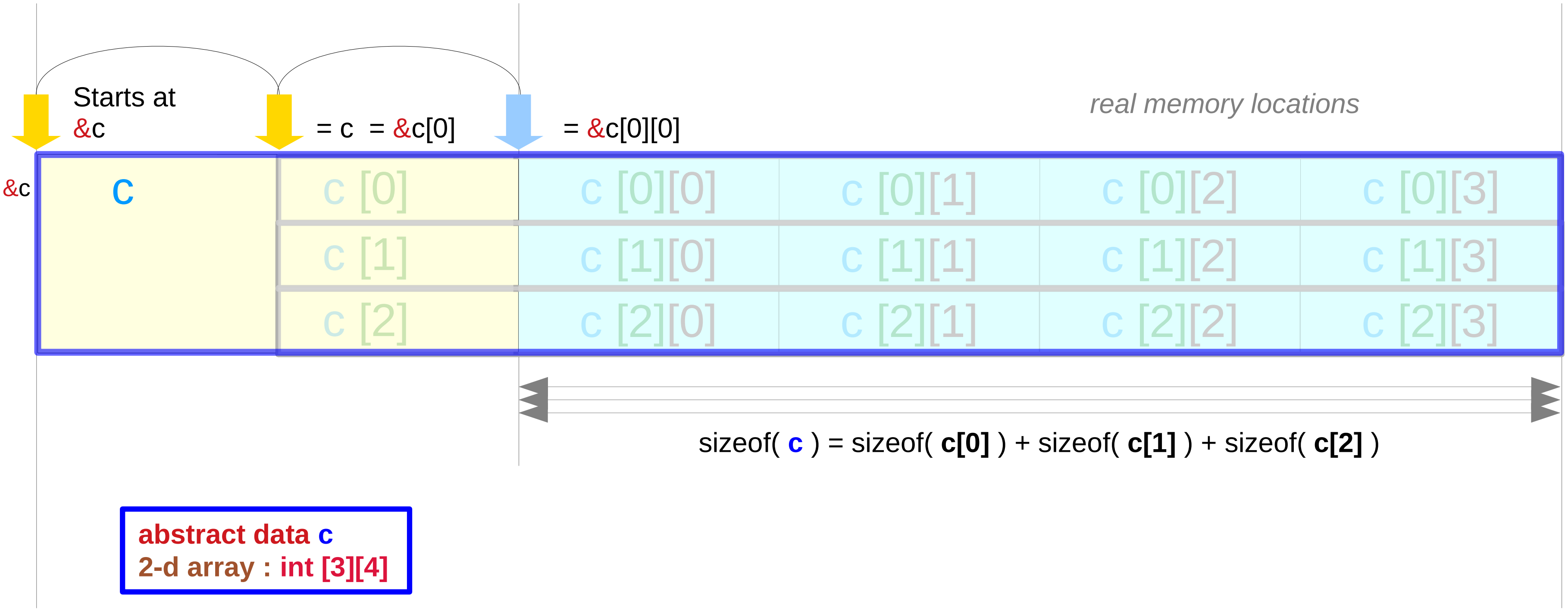
1-d array : int [4]

c points to the 1st element **c[0]**

Abstract data **c**

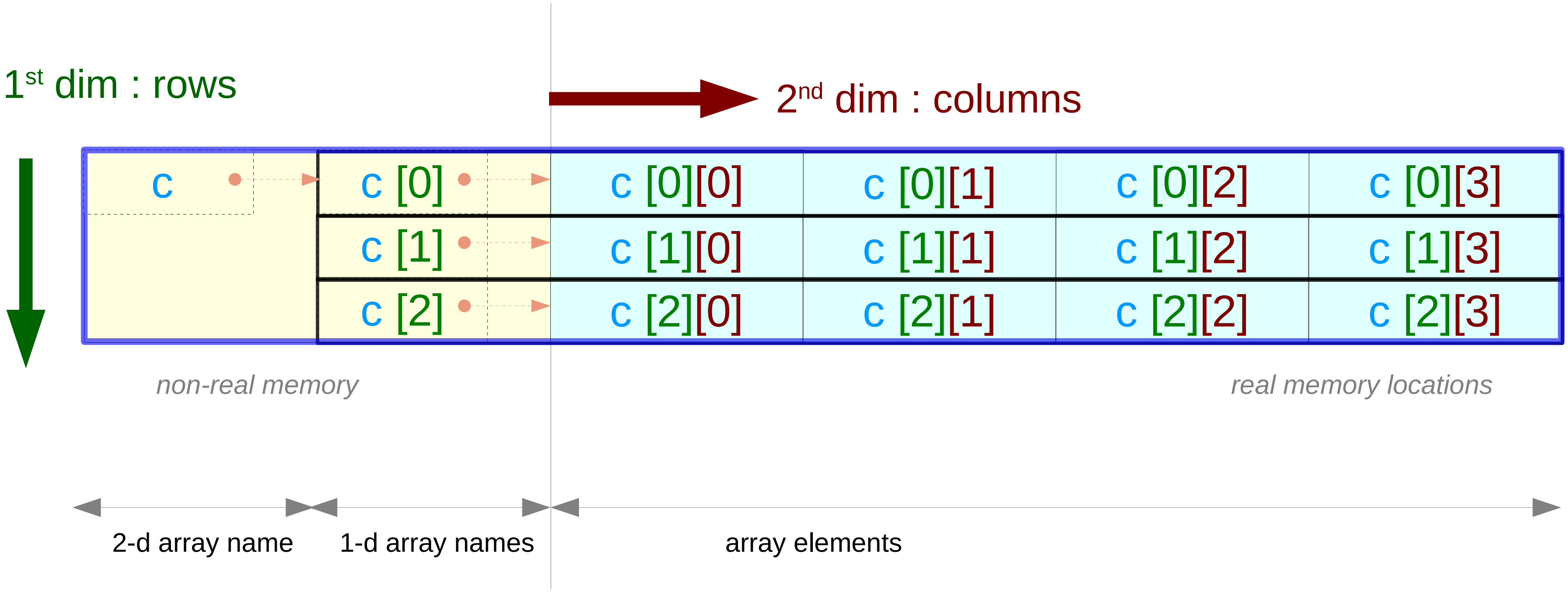
```
int c [3] [4];
```

abstract data **c:** sizeof(**c**) = 3 * sizeof(**c[i]**) **2-d** array



Rows and columns of a **2-d** array **c**

```
int c[3] [4];
```



1st dim : rows

2nd dim : columns

c	c [0]	c [0][0]	c [0][1]	c [0][2]	c [0][3]
	c [1]	c [1][0]	c [1][1]	c [1][2]	c [1][3]
	c [2]	c [2][0]	c [2][1]	c [2][2]	c [2][3]

non-real memory

real memory locations

2-d array name

1-d array names

array elements

The name of a 2-d array

```
int         a [4];

int     c [4] [4];
```

1. the name of the nested array (recursive definition)

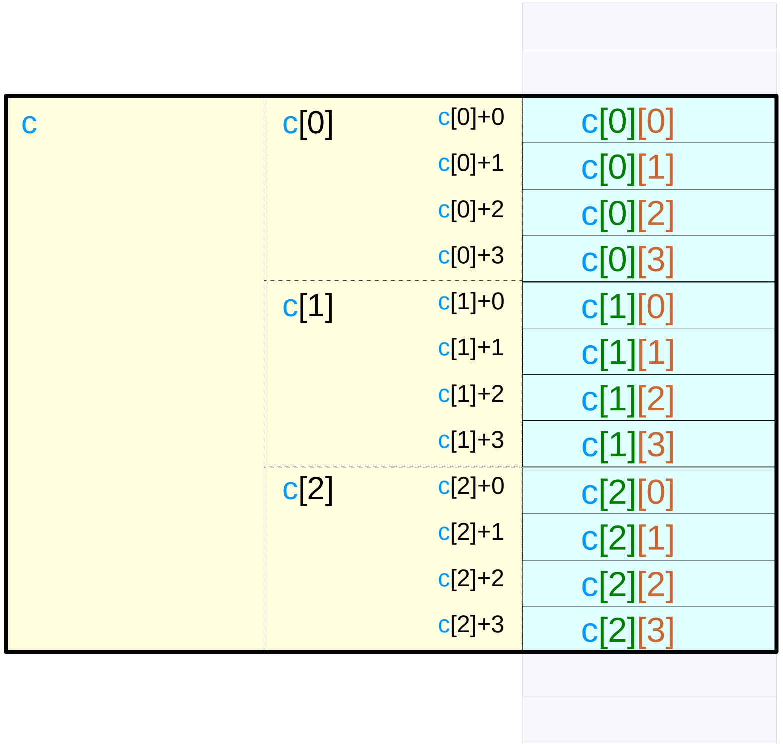
2. a double pointer

3. a pointer to an array

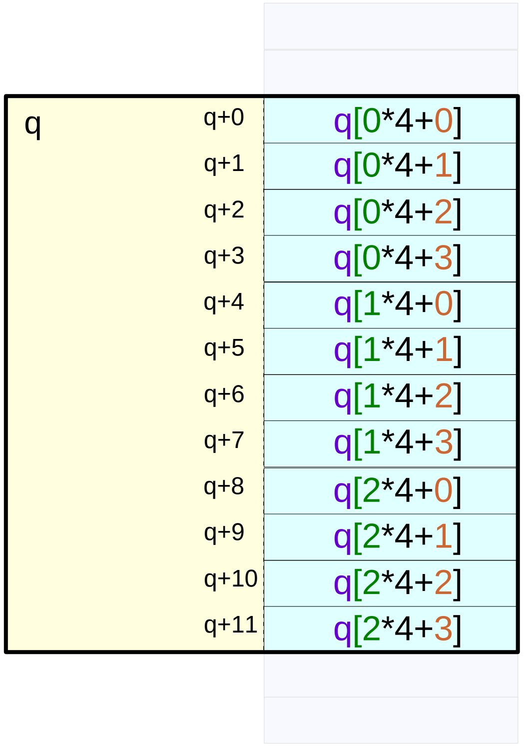
2-d array **c** and **1-d** array **q**

```
int c [3] [4];
```

```
int q [3*4];
```

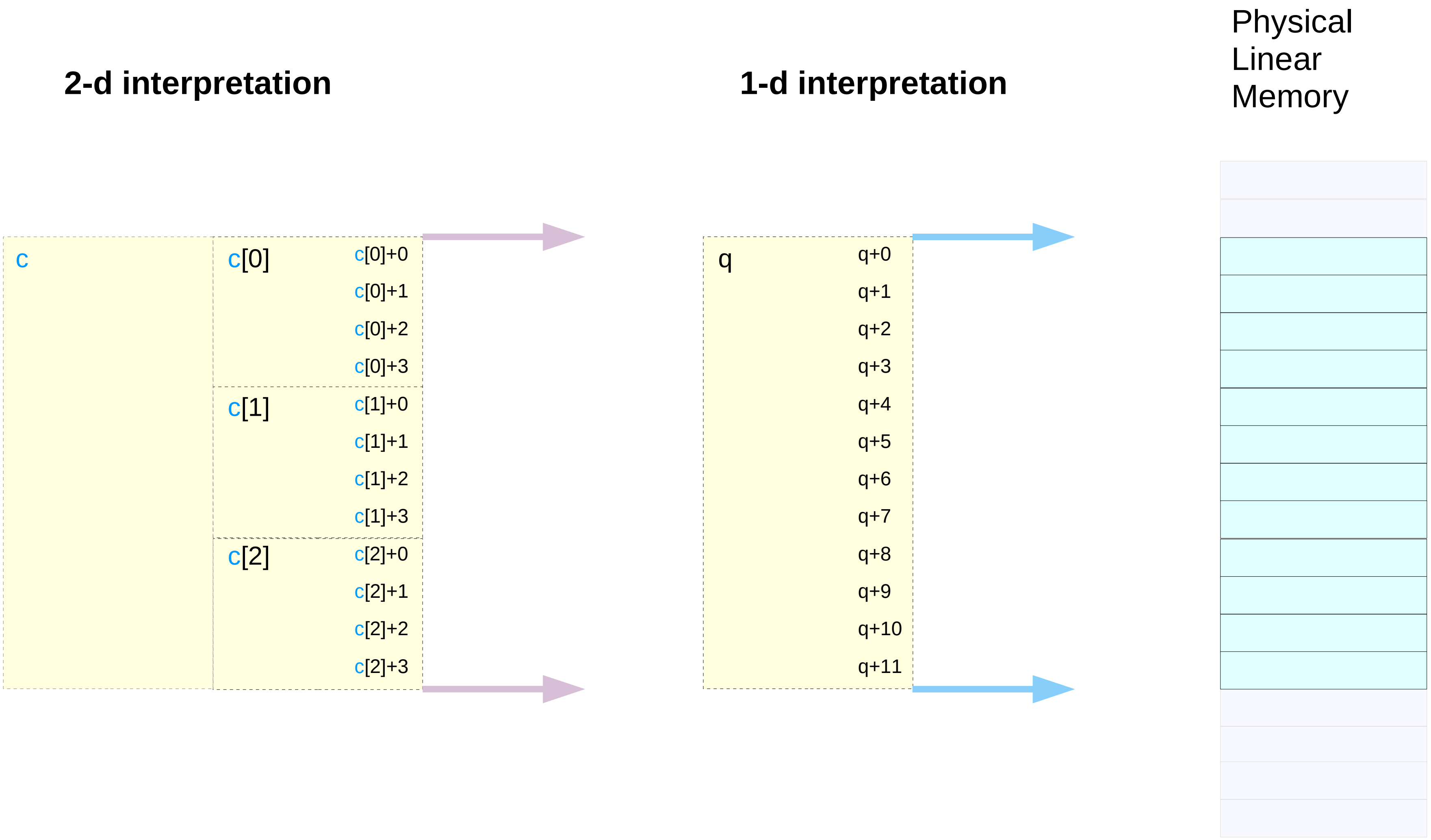


c	c[0]	c[0]+0	c[0][0]
		c[0]+1	c[0][1]
		c[0]+2	c[0][2]
		c[0]+3	c[0][3]
	c[1]	c[1]+0	c[1][0]
		c[1]+1	c[1][1]
		c[1]+2	c[1][2]
		c[1]+3	c[1][3]
	c[2]	c[2]+0	c[2][0]
		c[2]+1	c[2][1]
		c[2]+2	c[2][2]
		c[2]+3	c[2][3]



q	q+0	q[0*4+0]
	q+1	q[0*4+1]
	q+2	q[0*4+2]
	q+3	q[0*4+3]
	q+4	q[1*4+0]
	q+5	q[1*4+1]
	q+6	q[1*4+2]
	q+7	q[1*4+3]
	q+8	q[2*4+0]
	q+9	q[2*4+1]
	q+10	q[2*4+2]
	q+11	q[2*4+3]

2-d and **1-d** interpretations of linear memories



2-d interpretation

1-d interpretation

Physical
Linear
Memory

c

c[0]
c[0]+0
c[0]+1
c[0]+2
c[0]+3

c[1]
c[1]+0
c[1]+1
c[1]+2
c[1]+3

c[2]
c[2]+0
c[2]+1
c[2]+2
c[2]+3

q
q+0
q+1
q+2
q+3
q+4
q+5
q+6
q+7
q+8
q+9
q+10
q+11

A **2-d** array stored as a **1-d** array (row major order)

int c [3] [4];

c[i][j]

[i*4+j]

[k]

c	c[0]	c[0]+0	c[0][0]
		c[0]+1	c[0][1]
		c[0]+2	c[0][2]
		c[0]+3	c[0][3]
	c[1]	c[1]+0	c[1][0]
		c[1]+1	c[1][1]
		c[1]+2	c[1][2]
		c[1]+3	c[1][3]
	c[2]	c[2]+0	c[2][0]
		c[2]+1	c[2][1]
		c[2]+2	c[2][2]
		c[2]+3	c[2][3]

index values

0	=[0*4+0]
1	=[0*4+1]
2	=[0*4+2]
3	=[0*4+3]
4	=[1*4+0]
5	=[1*4+1]
6	=[1*4+2]
7	=[1*4+3]
8	=[2*4+0]
9	=[2*4+1]
10	=[2*4+2]
11	=[2*4+3]

q	q+0	q[0]
	q+1	q[1]
	q+2	q[2]
	q+3	q[3]
	q+4	q[4]
	q+5	q[5]
	q+6	q[6]
	q+7	q[7]
	q+8	q[8]
	q+9	q[9]
	q+10	q[10]
	q+11	q[11]

2-d array access via a single pointer

```
int      *p = c[0] ;
```

```
int c [3][4];
```

```
p[ i*4 + j ]
```

```
c[ i ][ j ]
```

```
*(p+ i*4 + j)
```

```
*(*(c+i)+ j)
```

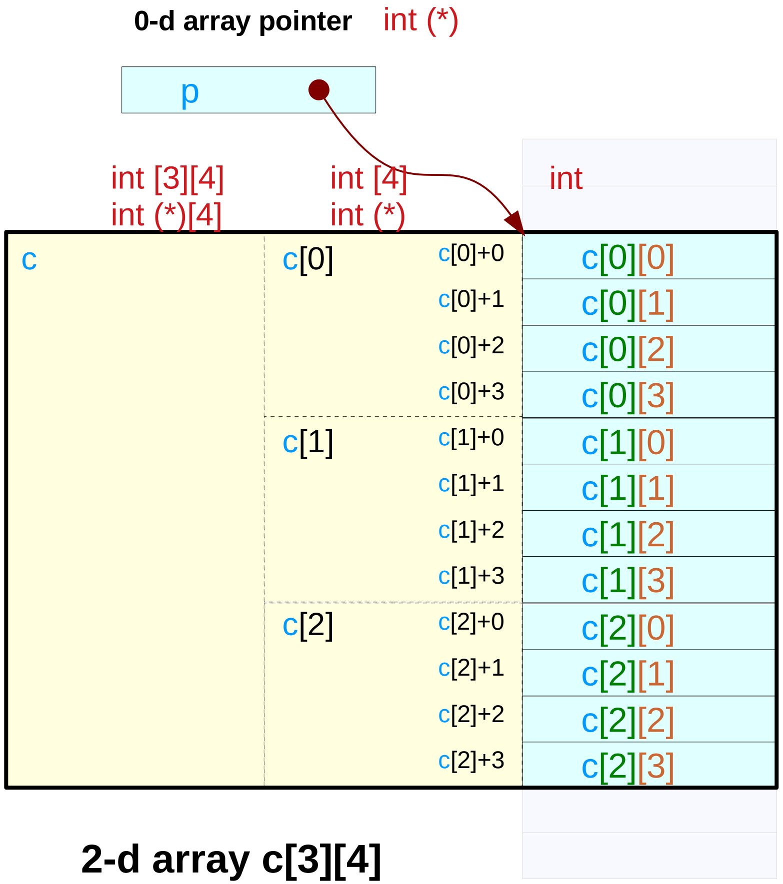
```
*(p +k)        i = k / 4;
               j = k % 4;
```

View a **2-d** array as a **1-d** array

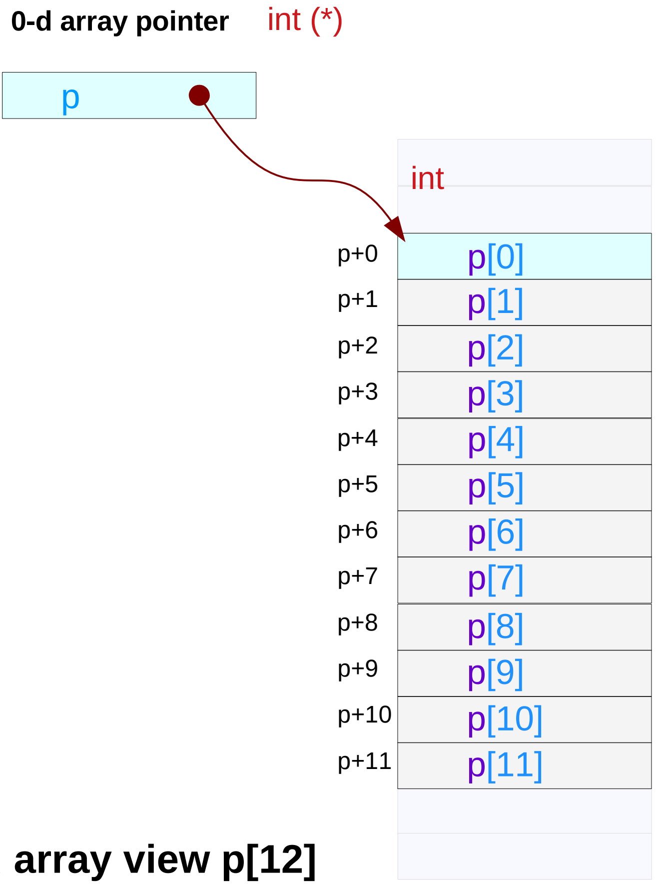
`int c [3][4];`

`int *p = c[0];`

c, c[0],
&c[0][0]



2-d array c[3][4]



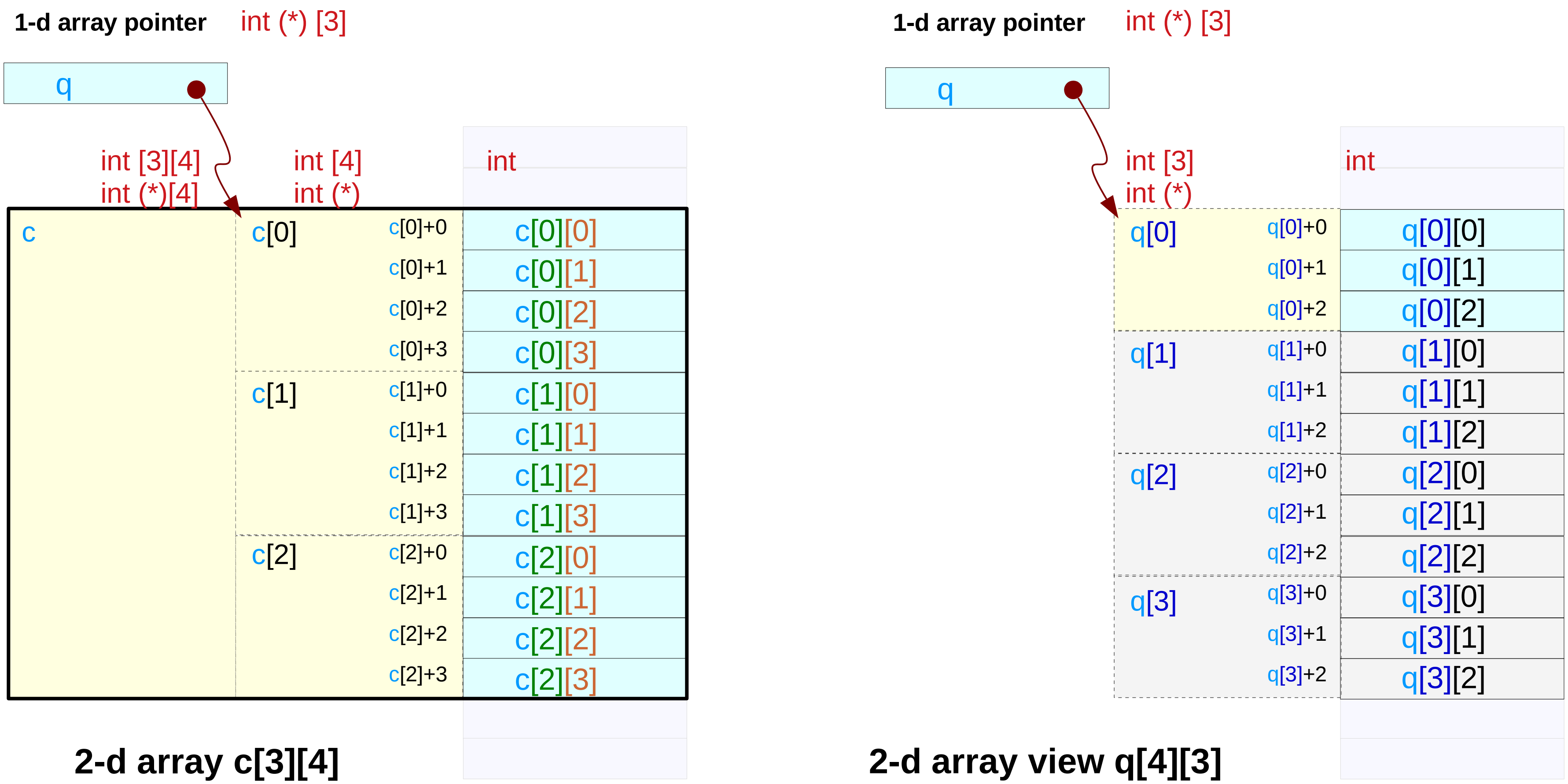
1-d array view p[12]

View a **2-d** array as another **2-d** array

`int c [3][4];`

`int (*q) [3] = (int (*) [3]) c;`

c, c[0], &c[0][0]



2-d array c[3][4]

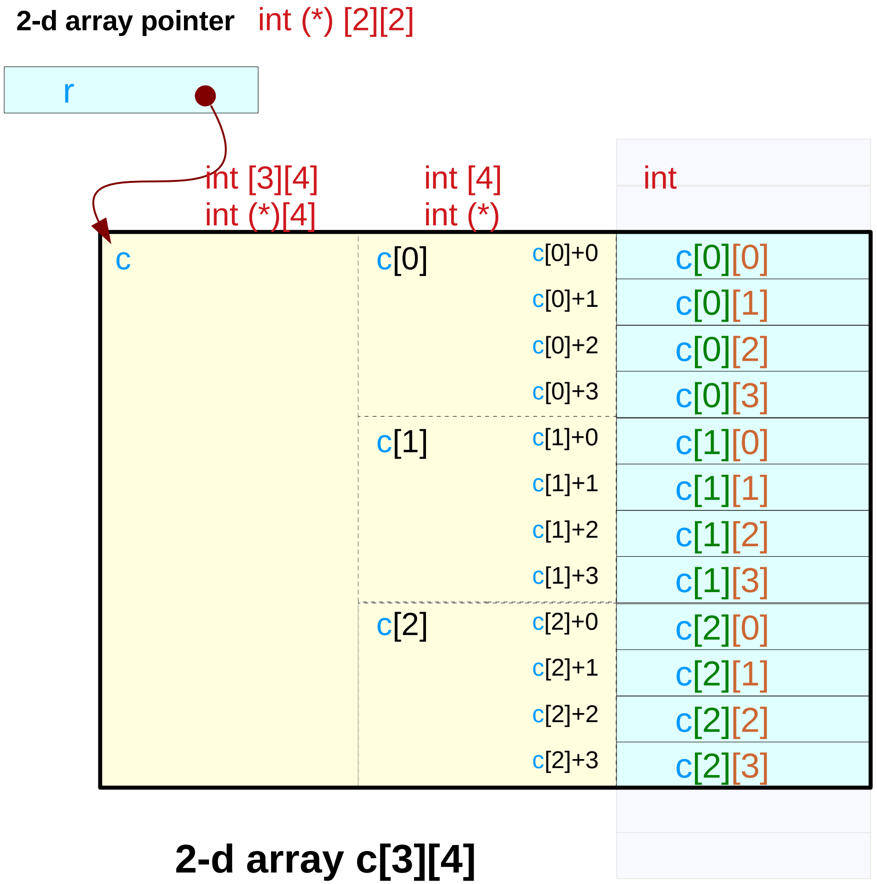
2-d array view q[4][3]

A **2-d** array stored as a **1-d** array (row major order)

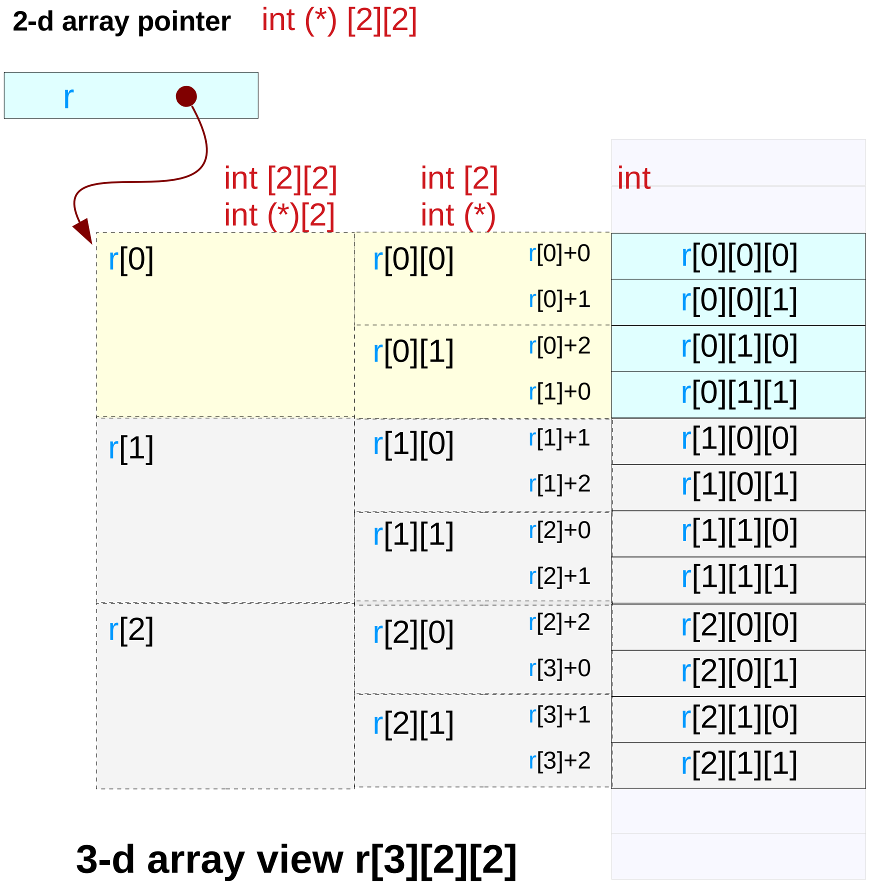
`int c [3] [4];`

`int (*r) [2][2] = (int (*) [2][2]) c;`

c, c[0],
&c[0][0]



2-d array c[3][4]



3-d array view r[3][2][2]

2-d array access via pointers

```
int c [3][4];
```

```
int      *p = c[0] ;
```

1. recursive pointers

```
c [ i ][ j ]
```

```
(*(c+i))[ j ]
```

→ `int (*p)[4];`

```
*(c[ i ]+ j)
```

```
*(*(c+i)+ j)
```

→ `int **q;`

2. linear array pointers

```
p[ i*4 + j ]
```

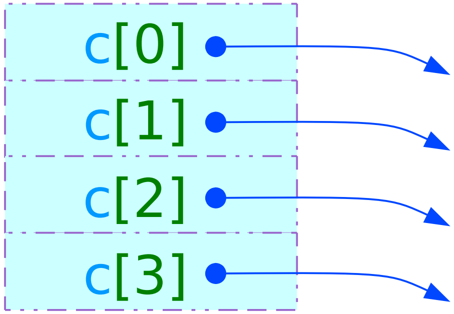
```
*(p+ i*4 + j )
```

Static Allocation of a 2-d Array

int A [3][4];

A in %eax,
i in %edx,
j in %ecx

```
sall    $2, %ecx                     ;; j * 4
leal    (%edx, %edx, 2), %edx        ;; i * 3
leal    (%ecx, %edx, 4), %edx        ;; j * 4 + i * 12
movl    (%eax, %edx), %eax           ;; read M[ XA+4(3i +j) ]
```



The pointer array :
not allocated
in the memory

c[0]+0	*(c [0]+0)
c[0]+1	*(c [0]+1)
c[0]+2	*(c [0]+2)
c[0]+3	*(c [0]+3)
c[1]+0	*(c [1]+0)
c[1]+1	*(c [1]+1)
c[1]+2	*(c [1]+2)
c[1]+3	*(c [1]+3)
c[2]+0	*(c [2]+0)
c[2]+1	*(c [2]+1)
c[2]+2	*(c [2]+2)
c[2]+3	*(c [2]+3)

Pointers, arrays, and operator precedence

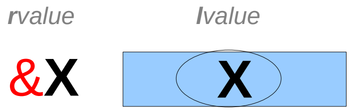
Address-of & and dereference * operators

Address-of operation

&X = value(&X) *rvalue*

C Expressions

Mixed Expressions



&X evaluates the address value of a variable X

& is a mathematical operator (the inverse operator of *)

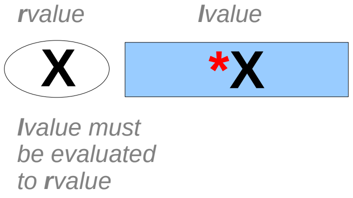
value(&X) = value(value(&X)) = value(&X) = &X

Dereference operation

*X = *value(X) *lvalue*

C Expressions

Mixed Expressions



X must be evaluated to an address before de-referencing

Equivalences in address replications

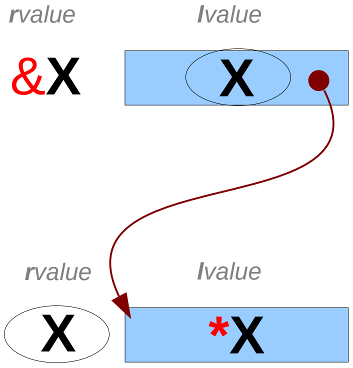
Eqivalences in Address Replications

*a pointer variable **X***

value(&**X**) ≡ value(**X**)

*at the pointed address **X***

value(**X**) ≡ *value(**X**)



lvalue must be evaluated to rvalue

*&**X** and **X** have different types but have the same value*

***X** and ***X** have different types but have the same value*

value(&**X**) ≡ value(**X**) ≡ *value(**X**)

Equivalences in array notations

X[n]

*(X+n)

&X[n]

value (X+n)

C operator &
≠ inverse of *

math operator &
= inverse of *

value(&**X**[**n**])
= value(&*(**X**+**n**))
= value(**X**+**n**)

X[0]

*(X+0)

&X[0]

value (X+0)

C operator &
≠ inverse of *

math operator &
= inverse of *

value(&**X**[0])
= value(&*(**X**+0))
= value(**X**+0)

Pointer Arithmetic

• increment / decrement	**++X, --X, X++, X--**
• addition of an integer	**X + i**
• subtraction of an integer	**X – i**
• subtracting two pointers of the same type	**X – Y**
• comparison of pointers	**==, !=, >, >=, <, <=**
• adding two pointers are not allowed	~~**X + Y**~~

pointer variables: **X**, **Y**

integer compatible variables : **i**

(**int**, **short**, **char**, ...)

Pointer Addition / Subtraction

pointer variables: **X**, **Y**

primitive variables : **A, B**

value(**X**) is used to avoid confusion between pointer additions and arithmetic additions

X + A

X – A

the variable **A** must have integer compatible types, otherwise error

X + Y error!

X – Y o.k.

value(**A**) = **A** primitive variable **A**

value(**X**) ≠ **X** pointer variable **X**

value(**X + A**) = value(**X**) $+_a$ **A** * sizeof(***X**)

value(**X – A**) = value(**X**) $-_a$ **A** * sizeof(***X**)

value(**X + Y**) error!

value(**X – Y**) = value(**X**) $-_a$ value(**Y**)

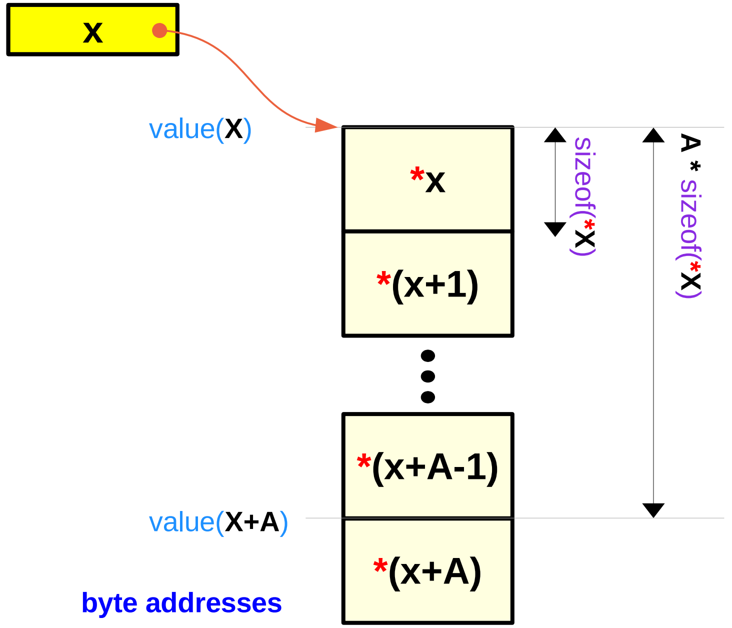
$+_a$ is used to denote arithmetic additions

$-_a$ is used to denote arithmetic subtractions

Pointer Addition / Subtraction

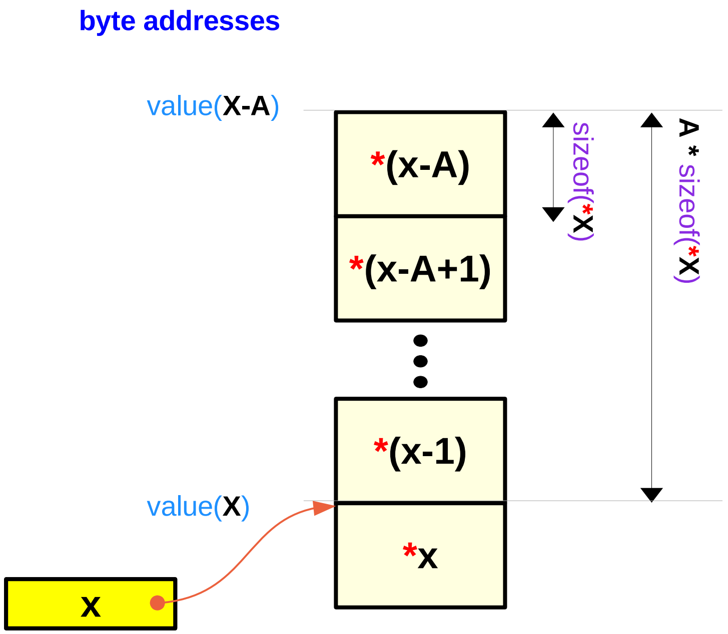
X + A : pointer addition

$$\text{value}(X + A) = \text{value}(X) +_a A*\text{sizeof}(*X)$$



X – A : pointer subtraction

$$\text{value}(X - A) = \text{value}(X) -_a A*\text{sizeof}(*X)$$



Pointer variables vs. primitive variables

pointer variables: **X**	value(**X**) ≠ **X**
integer compatible variables : **A**	value(**A**) = **A**

X	C expression	+ → pointer additions − → pointer subtractions	value(**X** + **A**) = value(**X**) $+_a$ **A***sizeof(***X**) value(**X** – **A**) = value(**X**) $-_a$ **A***sizeof(***X**)
A	C expression	+ → pointer additions − → pointer subtractions *for a pointer type operand*	value(**X** + **A**) = value(**X**) $+_a$ **A***sizeof(***X**) value(**X** – **A**) = value(**X**) $-_a$ **A***sizeof(***X**)
		+ → arithmetic additions $+_a$ − → arithmetic subtractions $-_a$ *for a non pointer type operand*	
value(**X**)	Math expression	+ → arithmetic additions $+_a$ − → arithmetic subtractions $-_a$	
value(**A**)	Math expression	+ → arithmetic additions $+_a$ − → arithmetic subtractions $-_a$	

Recursive application of value()

$\text{value}(\text{value}(\mathbf{A})) = \text{value}(\mathbf{A}) = \mathbf{A}$

$\text{value}(\text{value}(\mathbf{X})) = \text{value}(\mathbf{X}) \neq \mathbf{X}$

$\text{value}(\text{value}(\mathbf{X} + \mathbf{i}))$

$= \text{value}(\text{value}(\mathbf{X}) +_a \mathbf{i} * \text{sizeof}(*\mathbf{X}))$

$= \text{value}(\text{value}(\mathbf{X})) +_a \text{value}(\mathbf{i} * \text{sizeof}(*\mathbf{X}))$

$= \text{value}(\mathbf{X}) +_a \mathbf{i} * \text{sizeof}(*\mathbf{X})$

$= \text{value}(\mathbf{X}) + \mathbf{i} * \text{sizeof}(*\mathbf{X})$ in math expression

Operator Precedence of * and [ ]

*x[m] ≡ *(x[m])

[ ] has a higher priority than *

x[m][n] ≡ (x[m])[n]

[ ] has left-to-right associativity

**x ≡ *(*x)

* has right-to-left associativity

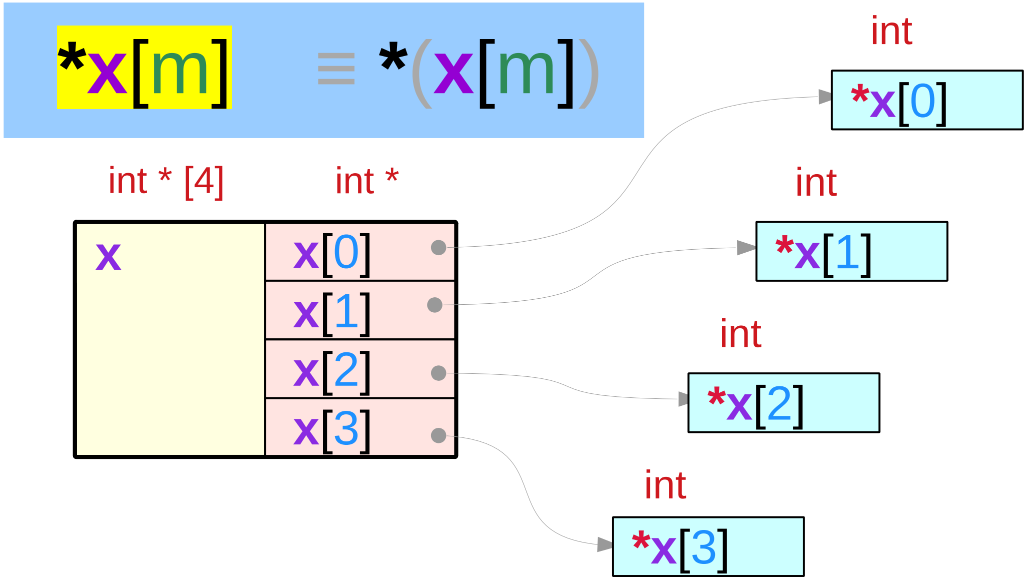
(*x)[m][n] ⟷ ((*x)[m])[n]

(*x[m])[n] ⟷ (*(x[m]))[n]

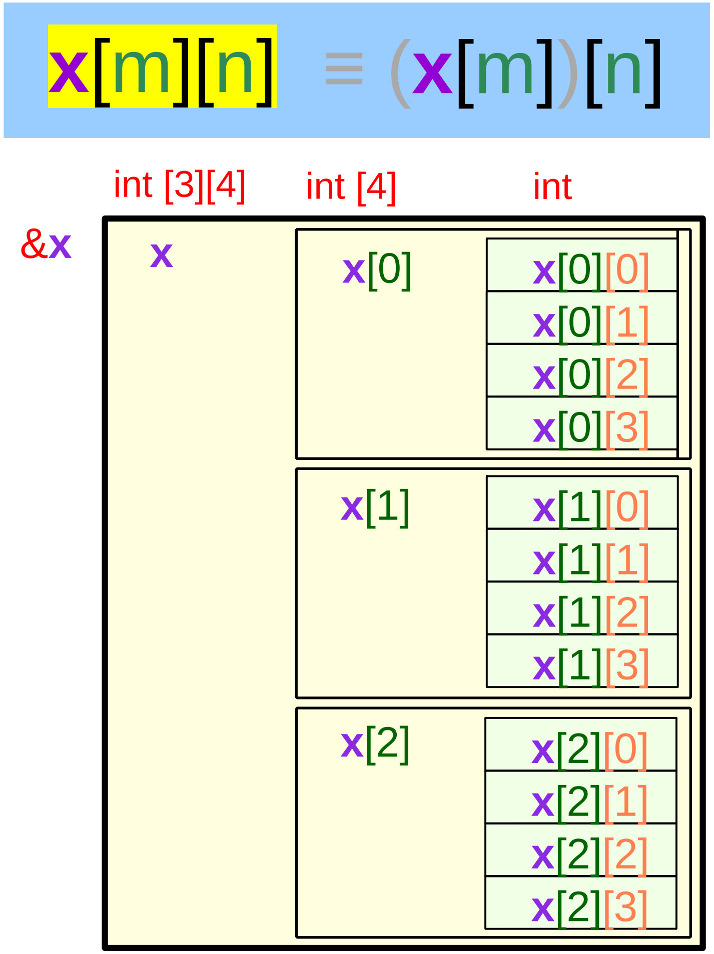
red parentheses ( ) must <u>not</u> be removed

gray parentheses ( ) can be removed

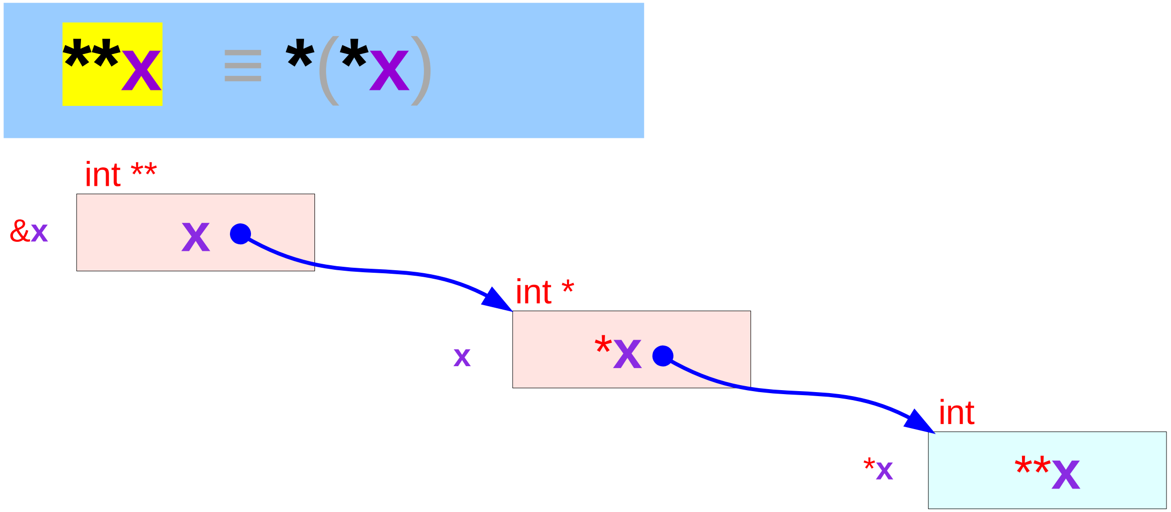
Operator Precedence of * and [ ]



*x[m] ≡ *(x[m])



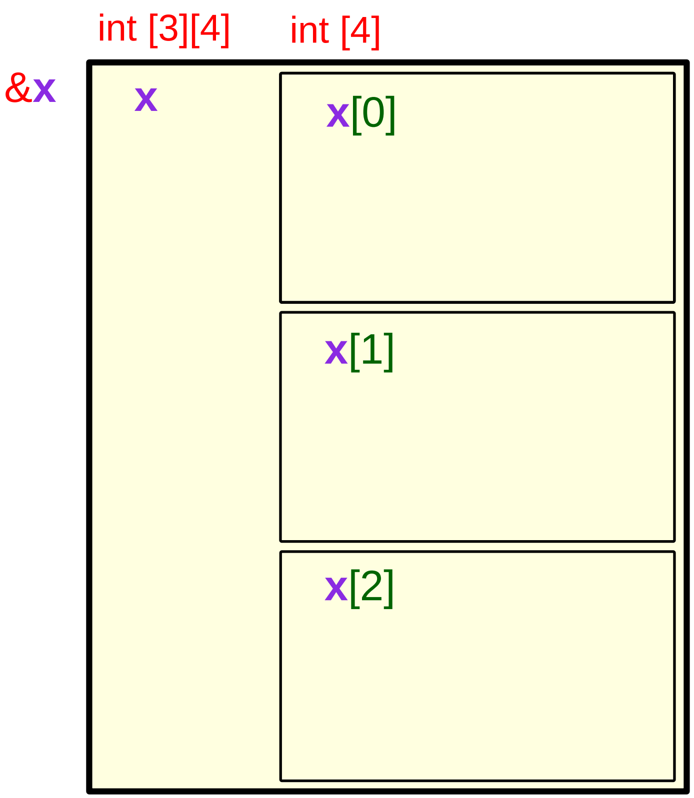
x[m][n] ≡ (x[m])[n]



**x ≡ *(*x)

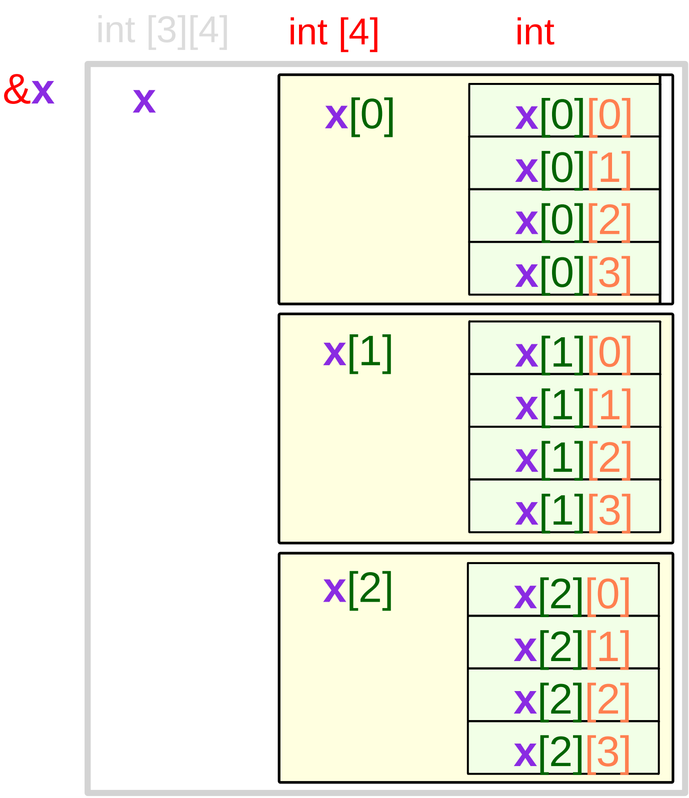
Abstract Data **x** and **x[i]**

x[3] x has 3 elements


int [3][4] int [4]
&x
x
x[0]
x[1]
x[2]

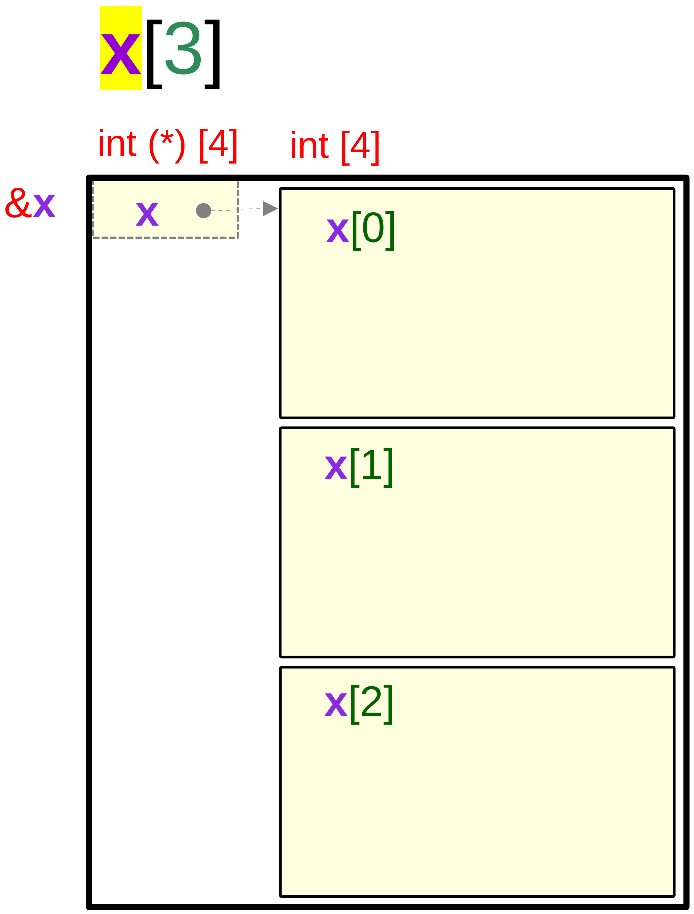
array element **x[i]**

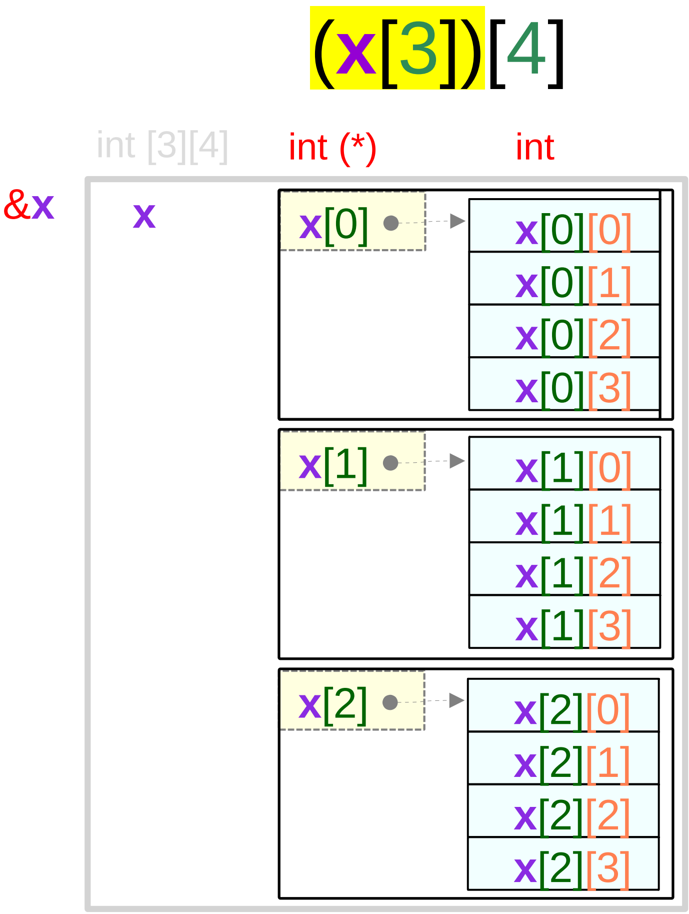
(**x**[3])[4] each **x[i]** has 4 elements


int [3][4] int [4] int
&x
x
x[0] x[0][0] x[0][1] x[0][2] x[0][3]
x[1] x[1][0] x[1][1] x[1][2] x[1][3]
x[2] x[2][0] x[2][1] x[2][2] x[2][3]

array name **x[i][j]**

Virtual Pointers **x** and **x[i]**

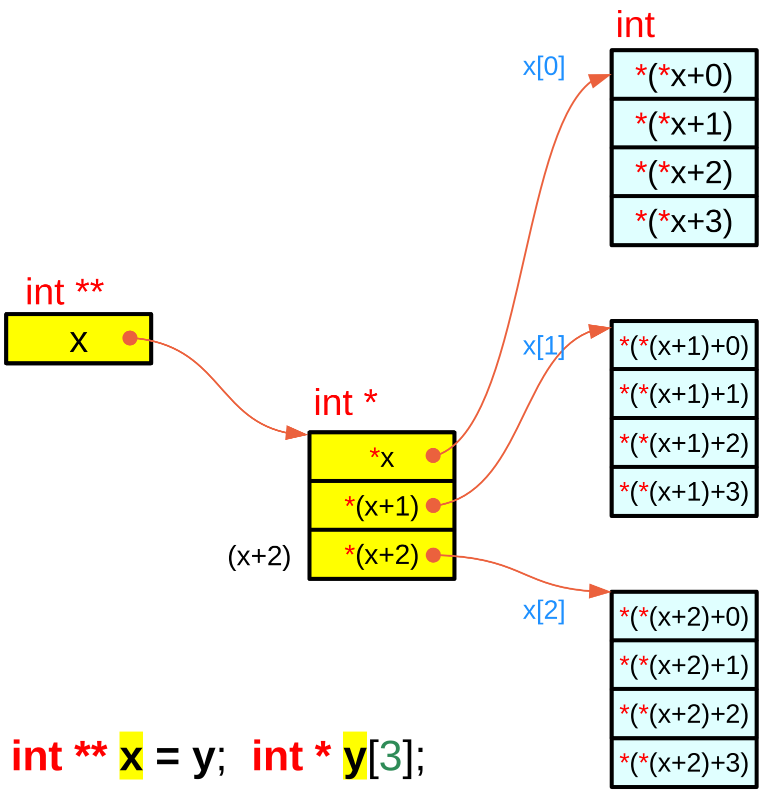


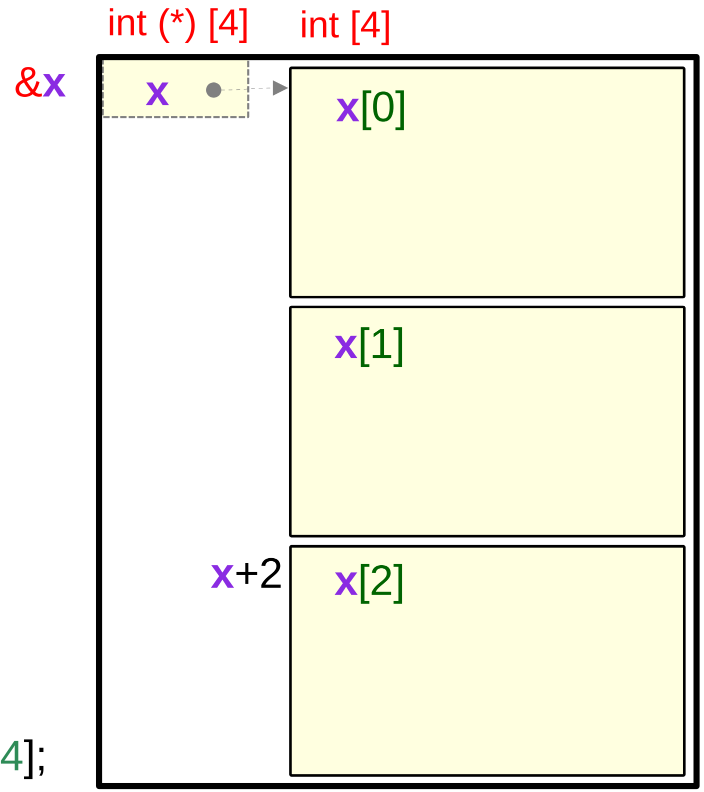


array name **x** ………...virtual pointer
array element **x**[**i**] …... abstract data

array name **x**[**i**] ……… virtual pointer
array element **x**[**i**][**j**] …. primitive data

Virtual Pointers **x** and **x[i]**





int ** x = y; **int * y**[3];

int x[3][4];

Pointer Array Approach

value(**x** + **i**)

= value(**x**) $+_a$ **i** * sizeof(***x**)

Array Pointer Approach

value(**x** + **i**)

= value(**x**) $+_a$ **i** * sizeof(**x**[0])

Base and offset in byte addresses

pointer variable X

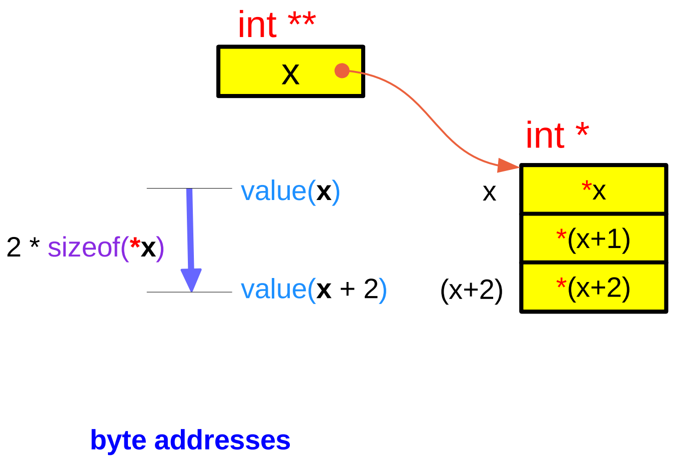
value(**x** + **i**) pointer addition +

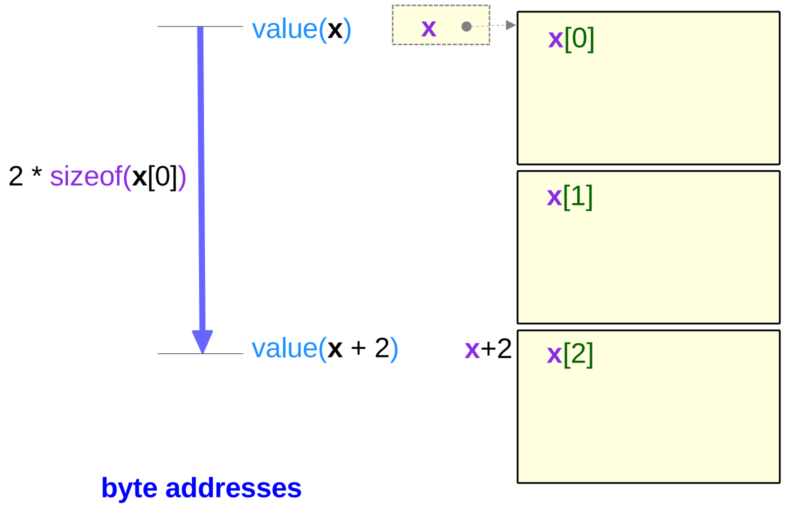
= value(**x**) $+_a$ **i** * sizeof(***x**) arithmetic addition $+_a$

array variable X

value(**x** + **i**) pointer addition +

= value(**x**) $+_a$ **i** * sizeof(**x**[0]) arithmetic addition $+_a$





int ** x = y; int * y[3];

int x[3][4];

Base and offset in byte address (1)

pointer variable **x**

$\text{value}(x + i)$ pointer addition +

$= \text{value}(x) \; +_a \; i * \text{sizeof}(*x)$ arithmetic addition $+_a$

$= \text{value}(x) \; + \; i * \text{sizeof}(*x)$ in math expression

value(**x**) may be involved
in only arithmetic additions / subtractions,
therefore the arithmetic addition $+_a$
can be replaced with the ordinary +

$= \text{value}(x) \; +_a \; i * sz,$ $sz = \text{sizeof}(*x)$

$= \text{value}(x + i)_{sz}$

`int ** x = y;  int * y[3];`

array variable **x**

$\text{value}(\&x[i])$ pointer addition +

$= \text{value}(x) \; +_a \; i * \text{sizeof}(x[0])$ arithmetic addition $+_a$

$= \text{value}(x) \; + \; i * \text{sizeof}(x[0])$ in math expression

$= \text{value}(x) \; +_a \; i * sz,$ $sz = \text{sizeof}(x[0])$

$= \text{value}(x + i)_{sz}$

`int x[3][4];`

Base and offset in byte address (2)

pointer variable **X**

value(*(x + i) + j)

value(value(x)) = value(x)

= value(*(x + i)) $\boxed{+_a}$ j * sizeof(**x)

= *value(x + i) $\boxed{+_a}$ j * sizeof(**x)

= *(value(x) $\boxed{+_a}$ i * sizeof(*x)) $\boxed{+_a}$ j * sizeof(**x)

= *(value(x) + i * sizeof(*x)) + j * sizeof(**x)

in math expressions

= *(value(x) + i * s1) + j * s2

= $(*(x + i)_{s1} + j)_{s2}$

s1 = sizeof(*x), s2 = sizeof(**x)

int ** x = y; int * y[3];

array variable **X**

value(x[i] + j)

address replications

= value(x[i]) $\boxed{+_a}$ j *sizeof(x[i][j])

= *(value(x) $\boxed{+_a}$ i *sizeof(x[0])) $\boxed{+_a}$ j*sizeof(x[0][0])

= (value(x) $\boxed{+_a}$ i *sizeof(x[0])) $\boxed{+_a}$ j *sizeof(x[0][0])

= value(x) + i * sizeof(x[0]) + j * sizeof(x[0][0])

in math expressions

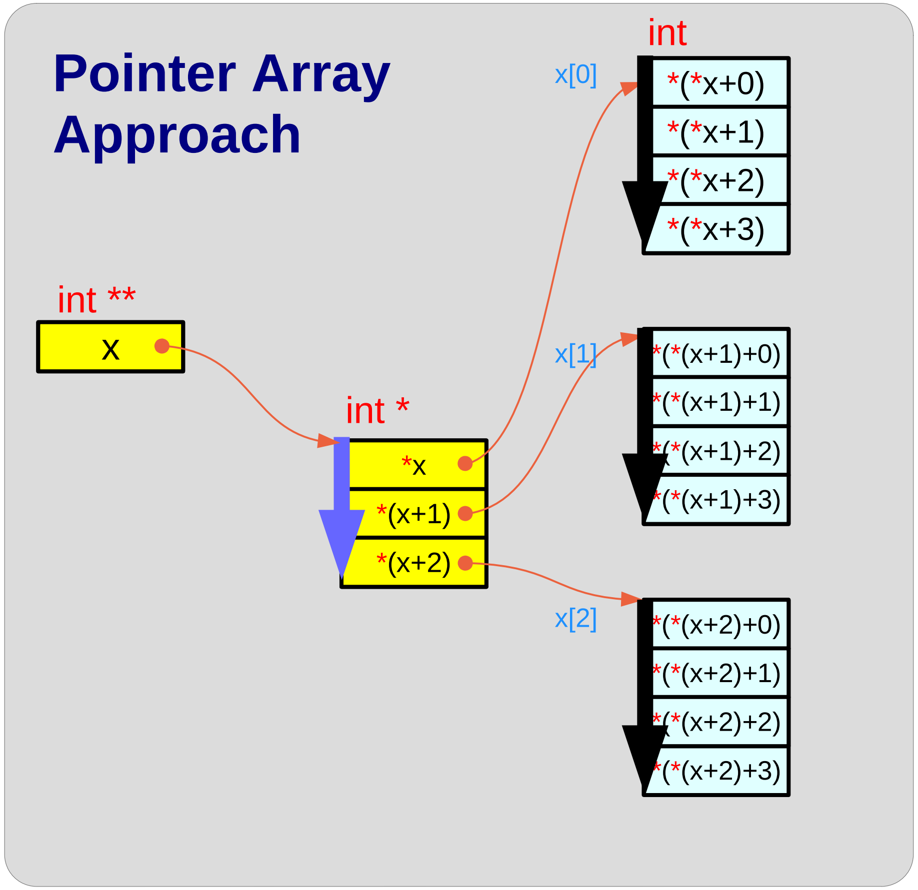
= value(x) + i * s1 + j * s2

= $(*(x + i)_{s1} + j)_{s2}$

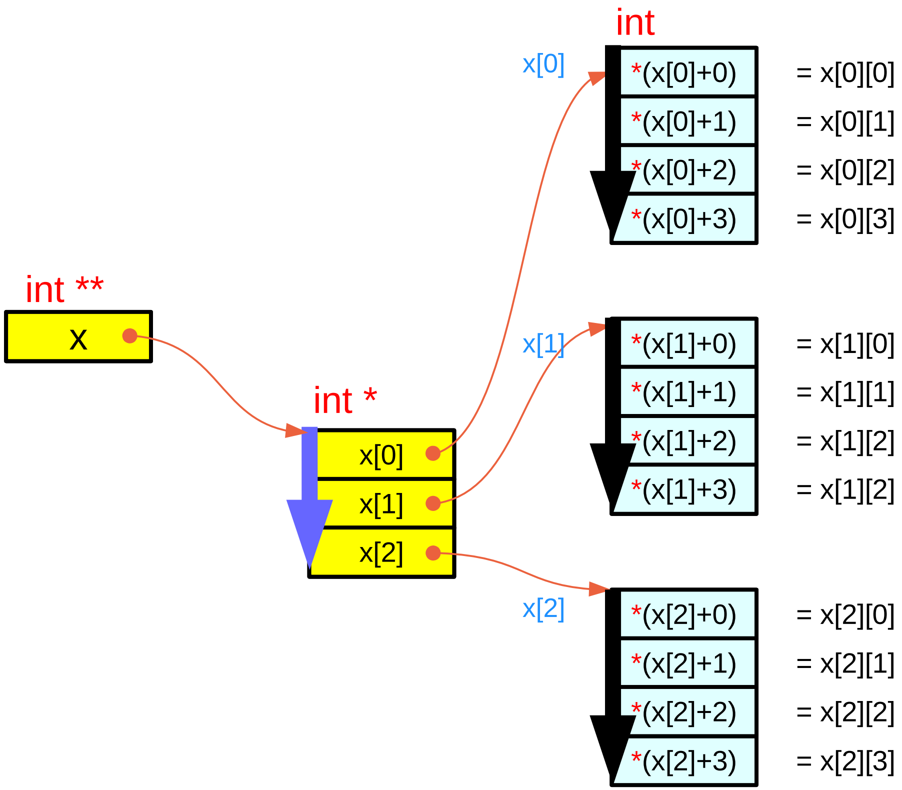
s1 = sizeof(x[0]), s2 = sizeof(x[0][0])

int x[3][4];

* into [ ] notations – Pointer Array Approach



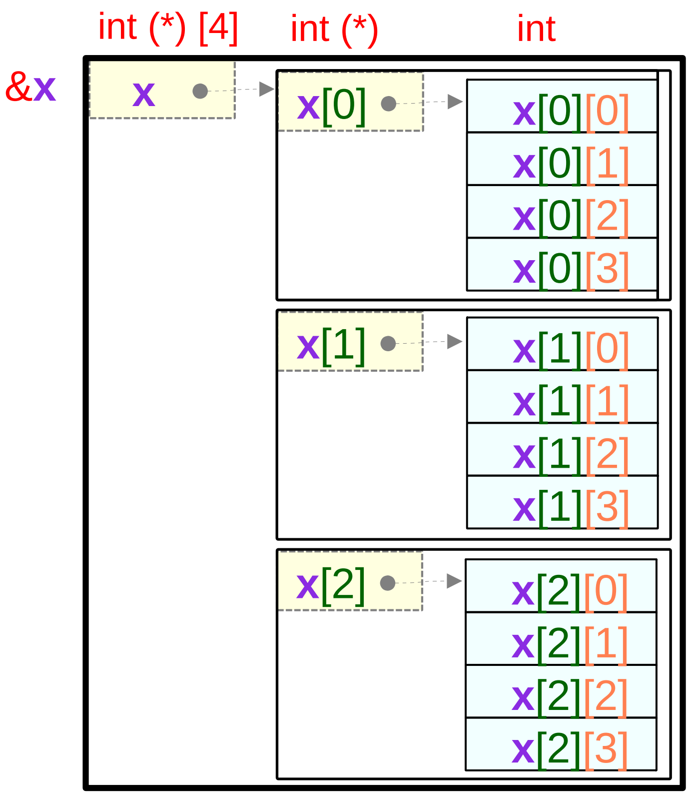
Pointer Array Approach

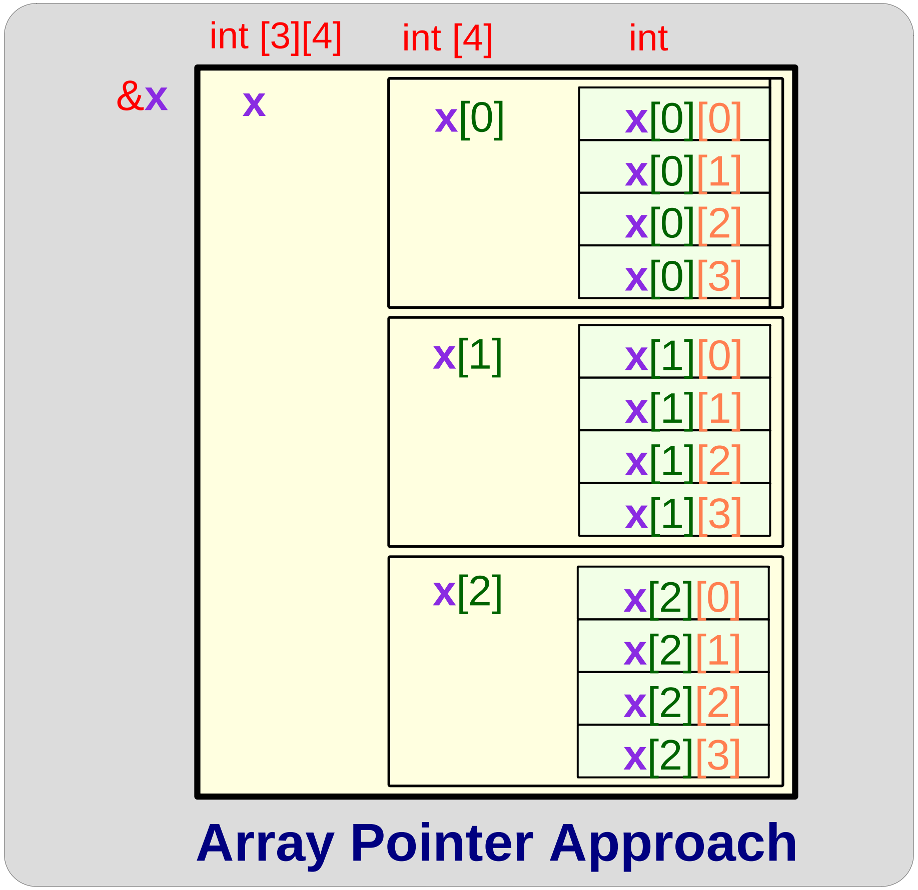


C expression $*(*(x+i)+j) \longrightarrow x[i][j]$

Math expression $*(*(x+i)_{1\cdot 4}+j)_{1\cdot 4}$

* and [ ] notations – Array Pointer Approach



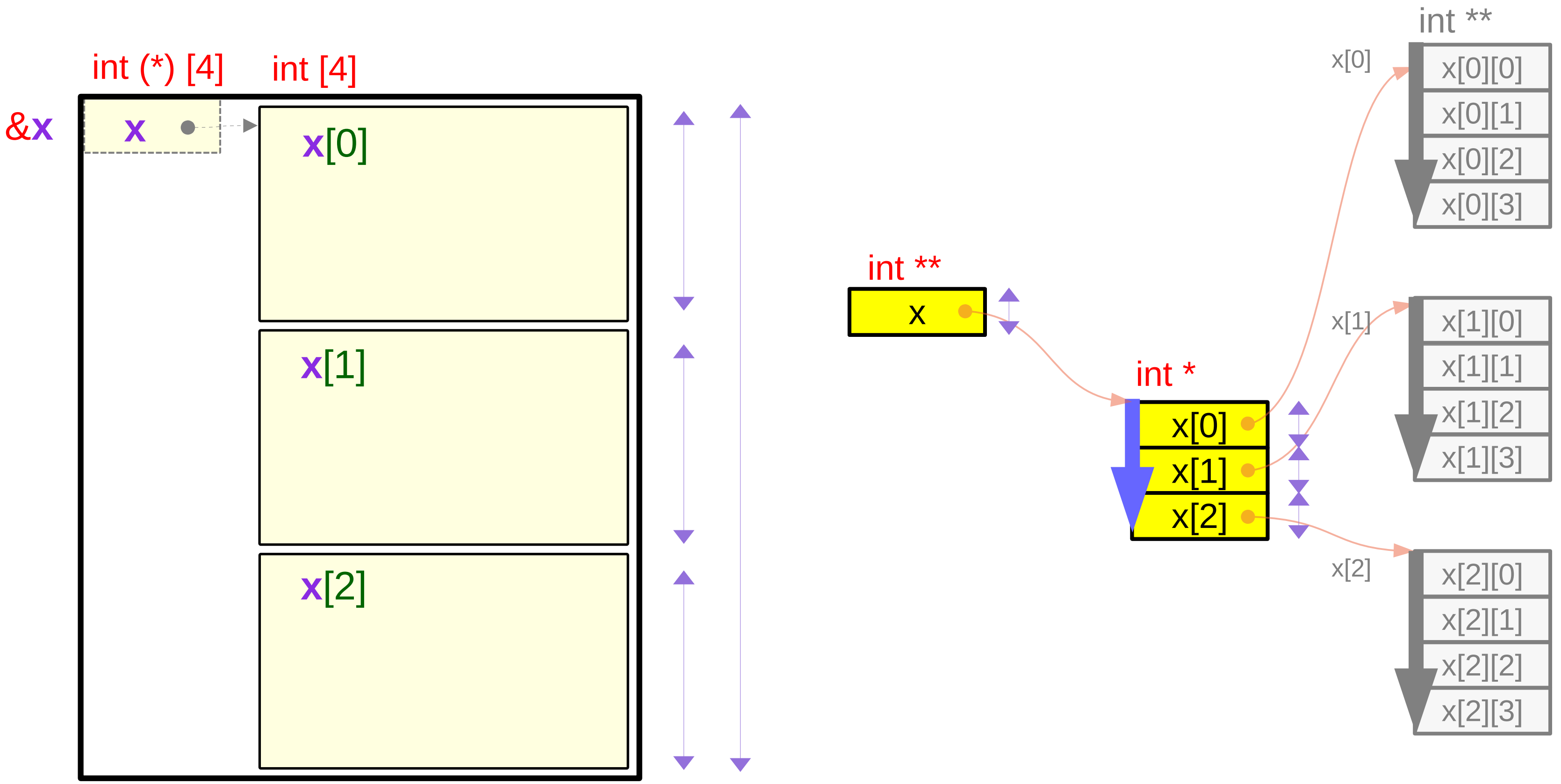


Array Pointer Approach

C expression $*(*(x+i)+j)$ ← $x[i][j]$

Math expression $*(*(x+i)_{4\cdot 4}+j)_{1\cdot 4}$

Virtual pointers vs. real pointers (1)



value(**&x**) = value(**x**) address replications

sizeof(**x**) = 3 * sizeof(***x**) abstract data size

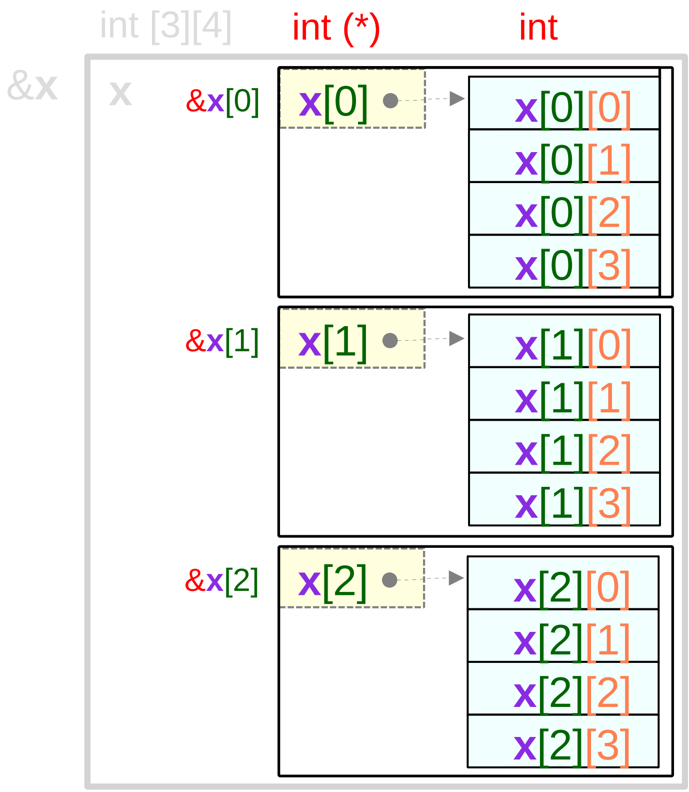
value(**x+i**) = value(**x**) + **i** * sizeof(***x**)
= value(**x**) + **i** * 4 *4 pointer arithmetic

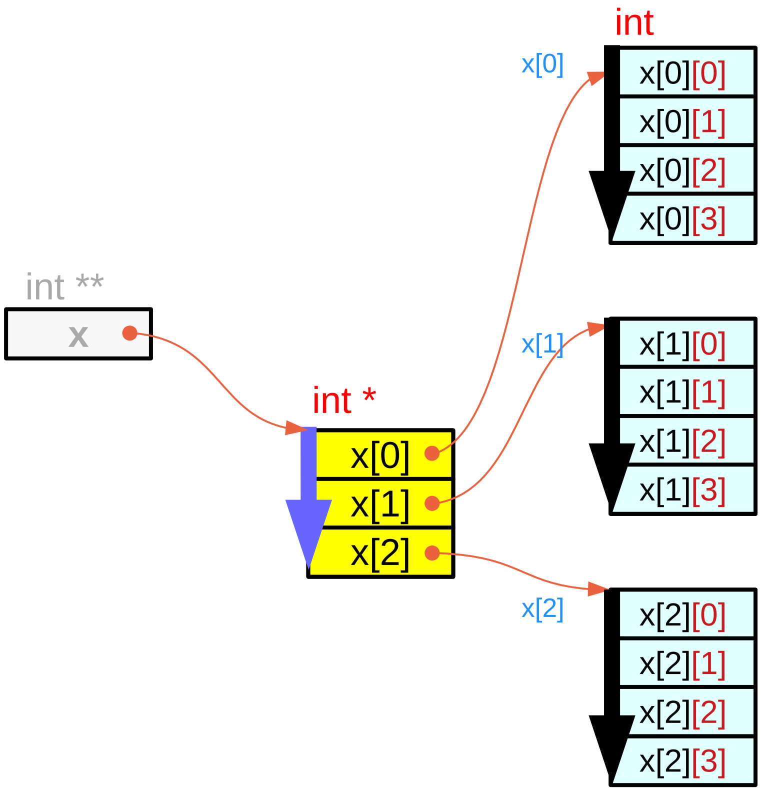
value(**&x**) ≠ value(**x**) physical memory

sizeof(**x**) = sizeof(***x**) = 4 pointer size

value(**x+i**) = value(**x**) + **i** * sizeof(***x**)
= value(**x**) + **i** * 4 pointer arithmetic

Virtual pointers vs. real pointers (2)





value(&x[i]) = value(x[i]) address replications

sizeof(x[i]) = 4 * sizeof(*x[i]) abstract data size

value(x[i]+j) = value(x[i]) + j * sizeof(*x[i])
= value(x[i]) + j * 4 pointer arithmetic

value(&x[i]) ≠ value(x[i]) physical memory

sizeof(x[i]) = sizeof(*x[i]) = 4 pointer size

value(x[i]+j) = value(x[i]) + j * sizeof(*x[i])
= value(x[i]) + j * 4 pointer arithmetic

Relaxing the outermost dimension

p[i] ≡ *(p+i)
p[i][j] ≡ *(p[i]+j)
p[i][j][k] ≡ *(p[i][j]+k)

&p[i] ≡ value(p+i)
&p[i][j] ≡ value(p[i]+j)
&p[i][j][k] ≡ value(p[i][j]+k)

*value(**X**) = ***X**

p[0] ≡ *p
p[i][0] ≡ *p[i]
p[i][j][0] ≡ *p[i][j]

&p[0] ≡ value(p)
&p[i][0] ≡ value(p[i])
&p[i][j][0] ≡ value(p[i][j])

valid for proper **i**, **j**, **k** values

Relaxing all the dimensions

p[i]	≡	*(p+i)	→	*(p+i)
p[i][j]	≡	*(p[i]+j)	→	*(*(p+i)+j)
p[i][j][k]	≡	*(p[i][j]+k)	→	*(*(*(p+i)+j)+k)

*(X+n) ≡ X[n]

*(X+n)

X[n]

valid for proper **i**, **j**, **k** values

Left-to-right and right-to-left associative operators

p[i] ≡ p[i] ⇒ *(p+i)

p[i][j] ≡ (p[i])[j] ⇒ *(*(p+i)+j)

p[i][j][k] ≡ ((p[i])[j])[k] ⇒ *(*(*(p+i)+j)+k)

*p ≡ *(p) ⇒ p[0]

**p ≡ *(*(p)) ⇒ (p[0])[0]

***p ≡ *(*(*(p))) ⇒ ((p[0])[0])[0]

Address Calculation (1) Array Pointer Approach

```
int   c [2][3][4] ;
```

c[i] ≡ *(c + i)
c[i][j] ≡ *(c[i] + j)
c[i][j][k] ≡ *(c[i][j] + k)

&c[i] ≡ value(c + i)
&c[i][j] ≡ value(c[i] + j)
&c[i][j][k] ≡ value(c[i][j] + k)

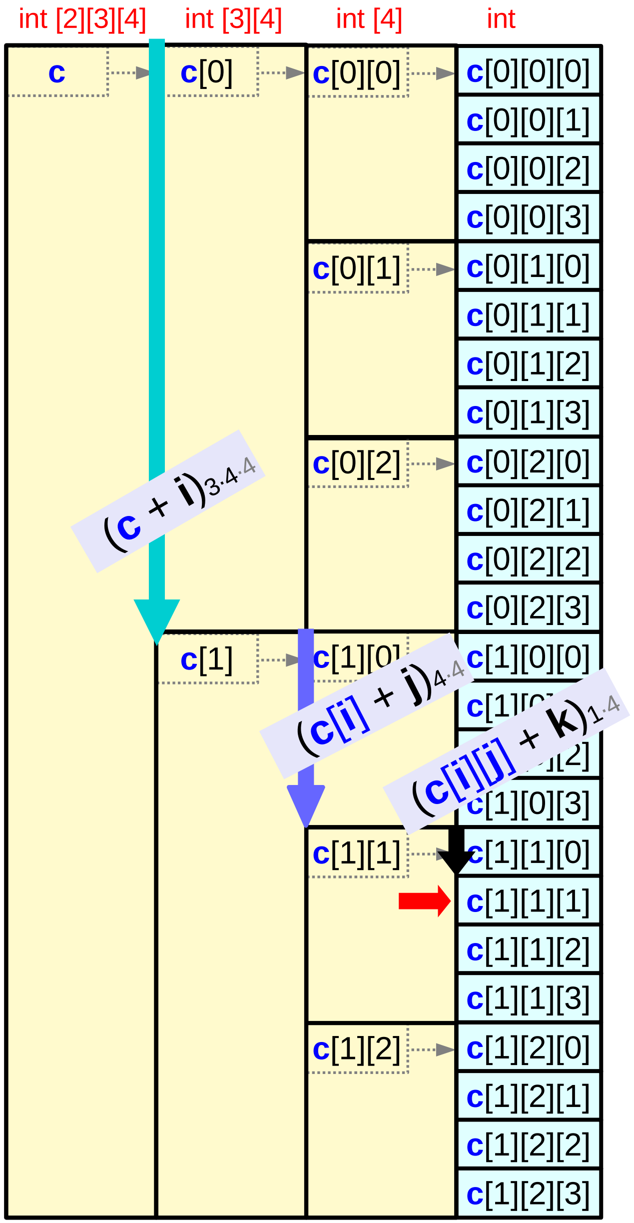
address replication

value(c[i][j][k]) ≠ value(&c[i][j][k]) ← primitive data & address

value(c[i][j]) = value(&c[i][j]) = value(&c[i][j][0])
value(c[i]) = value(&c[i]) = value(&c[i][0][0])
value(c) = value(&c) = value(&c[0][0][0])

skip **i** elements of **c**
skip **j** elements of **c[i]**
skip **k** elements of **c[i][j]**

skip **i*3*4** primitive elements of **c**
skip **j*4** primitive elements of **c**
skip **k** primitive elements of **c**



Address Calculation (2) Pointer Array Approach

```
int ** c [2] ;
int *  b [2*3] ;
int    a [2*3*4] ;
```

c[i] = &b[3*i] (= b + 3*i)

b[j] = &a[4*j] (= a + 4*j)

b[j] ≡ (a+j*4)

*(b[j]+k) = *(a+j*4+k);

b[j][k] ≡ a[j*4+k]

c[i] ≡ (b+i*3)

*(c[i]+j) = *(b+i*3+j);

c[i][j] ≡ b[i*3+j]

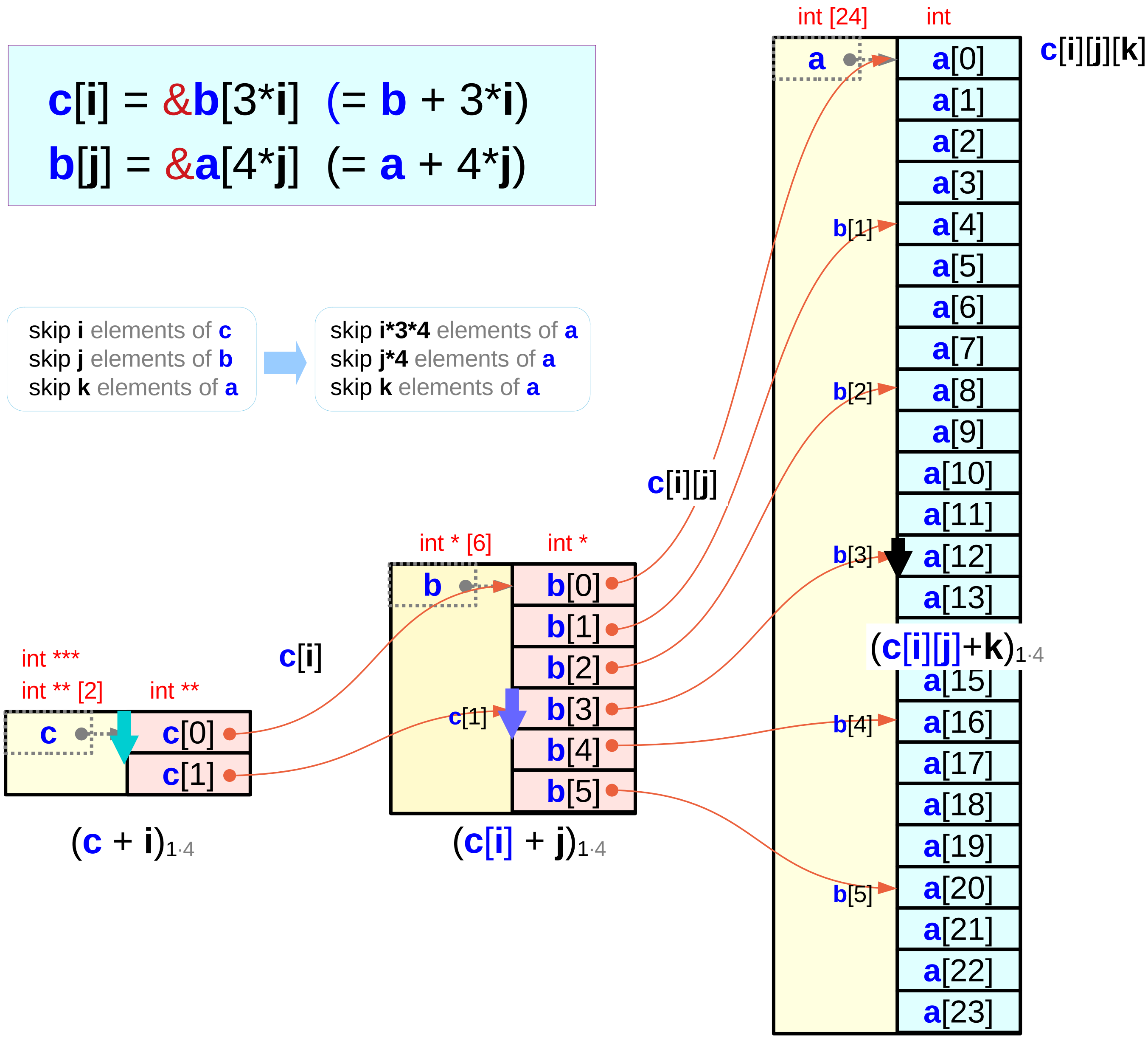
c[i][j] ≡ (a+(i*3+j)*4)

*(c[i][j]+k) = *(a+(i*3+j)*4+k);

c[i][j][j] ≡ a[(i*3+j)*4+k]

skip **i** elements of **c**
skip **j** elements of **b**
skip **k** elements of **a**

skip **i*3*4** elements of **a**
skip **j*4** elements of **a**
skip **k** elements of **a**



Address Calculation (3)

$\mathrm{value}(c + i) = \mathrm{value}(c) + i * 3 * 4 * 4$

$\mathrm{value}(c[i] + j) = \mathrm{value}(c[i]) + j * 4 * 4$

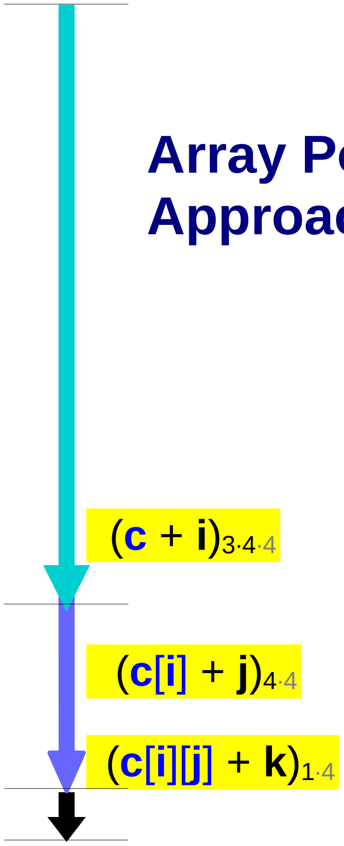
$\mathrm{value}(c[i][j] + k) = \mathrm{value}(c[i][j]) + k * 4$

$\mathrm{value}(c + i) = \mathrm{value}(c) + i * \mathrm{sizeof}(*c)$

$\mathrm{value}(c[i] + j) = \mathrm{value}(c[i]) + j * \mathrm{sizeof}(*c[i])$

$\mathrm{value}(c[i][j] + k) = \mathrm{value}(c[i][j]) + k * \mathrm{sizeof}(*c[i][j])$

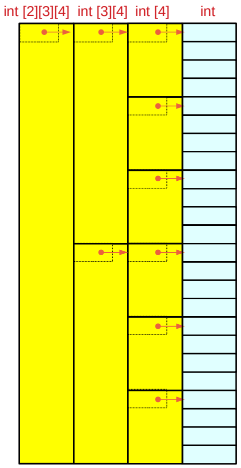
Array Pointer Approach



$(c + i)_{3 \cdot 4 \cdot 4}$

$(c[i] + j)_{4 \cdot 4}$

$(c[i][j] + k)_{1 \cdot 4}$



$c[i] \equiv *(c + i)$

$c[i][j] \equiv *(c[i] + j)$

$c[i][j][k] \equiv *(c[i][j] + k)$

$\&c[i] \equiv \mathrm{value}(c + i)$

$\&c[i][j] \equiv \mathrm{value}(c[i] + j)$

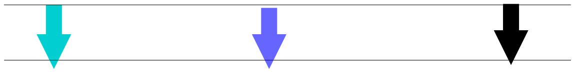
$\&c[i][j][k] \equiv \mathrm{value}(c[i][j] + k)$

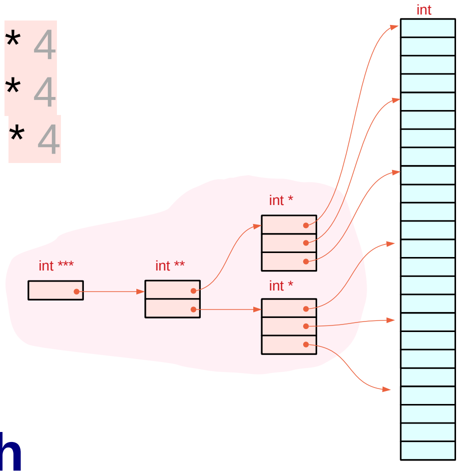
$\mathrm{value}(c + i) = \mathrm{value}(c) + i * 4$

$\mathrm{value}(c[i] + j) = \mathrm{value}(c[i]) + j * 4$

$\mathrm{value}(c[i][j] + k) = \mathrm{value}(c[i][j]) + k * 4$

$(c + i)_{1 \cdot 4}$ $(c[i] + j)_{1 \cdot 4}$ $(c[i][j] + k)_{1 \cdot 4}$





Pointer Array Approach

Subscript [ ] and dereference * notations (1a)

p[i] ≡ *(p+i)

from **p**, skip
i·M·N integers

&**p**[**i**] = value(**p** + **i**)$_{M\cdot N\cdot 4}$
= value(**p**) + **i** * M·N·4

p[i][j] ≡ *(*(p+i)+j)

from **p**[**i**], skip
j·N integers

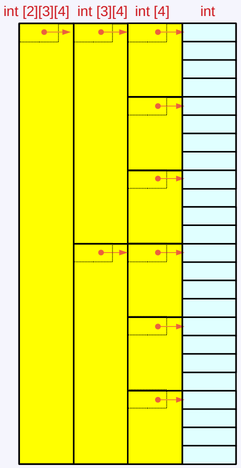
&**p**[**i**][**j**] = value(**p**[**i**] + **j**)$_{N\cdot 4}$
= value(**p**[**i**]) + **j** * N·4

p[i][j][k] ≡ *(*(*(p+i)+j)+k)

from **p**[**i**][**j**], skip
k integers

&**p**[**i**][**j**][**k**] = value(**p**[**i**][**j**]+**k**)$_{1\cdot 4}$
= value(**p**[**i**][**j**]) + **k** * 1·4

int p[L][M][N]



Array Pointer Approach

address replications

value(**p**[**i**]) = &**p**[**i**] = value(**p** + **i**)

value(**p**[**i**][**j**]) = &**p**[**i**][**j**] = value(**p**[**i**] + **j**)

value(**p**[**i**][**j**][**k**]) ≠ &**p**[**i**][**j**][**k**] = value(**p**[**i**][**j**] + **k**)

&**p**[**i**][**j**][**k**] = value(**p**) + **i** *M·N·4 + **j** *N·4 + **k** * 4

p[**i**][**j**][**k**] = *(value(**p**) + **i** *M·N·4 + **j** *N·4 + **k** * 4)

Subscript [ ] and dereference * notations (1b)

p[i] ≡ *(p+i)

p[i][j] ≡ *(*(p+i)+j)

p[i][j][k] ≡ *(*(*(p+i)+j)+k)

skip **i** pointers
from **p**

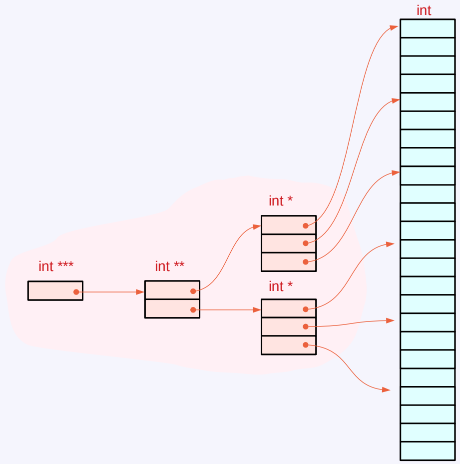
&**p**[**i**] = value(**p** + **i**)$_{1\cdot4}$
= value(**p**) + **i** *4

skip **j** pointers
from **p[i]**

&**p**[**i**][**j**] = value(**p[i]** + **j**)$_{1\cdot4}$
= value(**p[i]**) + **j** *4

skip **k** integers
from **p[i][j]**

&**p**[**i**][**j**][**k**] = value(**p[i][j]** + **k**)$_{1\cdot4}$
= value(**p[i][j]**) + **k** *4



int ** p [L] ;
int * q [L·M] ;
int r [L·M·N] ;

Pointer Array Approach

address dereferences

value(**p[i]**) = *(&**p[i]**) = *value(**p** + **i**)

value(**p[i][j]**) = *(&**p[i][j]**) = *value(**p[i]** + **j**)

value(**p[i][j][k]**) = *(&**p[i][j][k]**) = *value(**p[i][j]** + **k**)

p[i][j][k] =*value(*value(*(value(**p**) +**i***4) +**j***4) +**k***4)

&**p[i][j][k]** = value(*value(*(value(**p**) +**i***4) +**j***4) +**k***4)

Subscript [ ] and dereference * notations (1a)

$$p[i][j][k] = *\mathrm{value}(*\mathrm{value}(*\mathrm{value}(p + i) + j) + k) \xrightarrow[\text{address replications}]{} = \mathrm{value}(\mathrm{value}(\mathrm{value}(p + i)_{3\cdot4\cdot4} + j)_{4\cdot4} + k)_{4}$$

$$= \mathrm{value}(\mathrm{value}((\mathrm{value}(p) + i *3\cdot4\cdot4) + j)_{4\cdot4} + k)_{4}$$

$$= \mathrm{value}((\mathrm{value}(p) + i *3\cdot4\cdot4 + j *4\cdot4) + k)_{4}$$

$$= (\mathrm{value}(p) + i *3\cdot4\cdot4 + j *4\cdot4 + k *4)$$

$$p[i][j][k] = *\mathrm{value}(*\mathrm{value}(*\mathrm{value}(p + i) + j) + k) \xrightarrow{} = *\mathrm{value}(*\mathrm{value}(*\mathrm{value}(p + i)_{4} + j)_{4} + k)_{4}$$

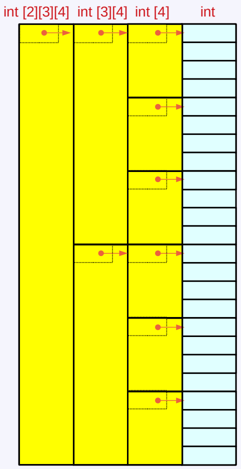
$$= *\mathrm{value}(*\mathrm{value}(*(\mathrm{value}(p) + i *4) + j)_{4} + k)_{4}$$

$$= *\mathrm{value}(*\mathrm{value}(*(\mathrm{value}(p) + i *4) + j*4) + k)_{4}$$

$$= *\mathrm{value}(*\mathrm{value}(*(\mathrm{value}(p) + i *4) + j*4) + k*4)$$

Subscript [ ] and dereference * notations (1a)

int p[L][M][N]



Array Pointer Approach

address dereferences

value(p[i]) = *(&p[i]) = *value(p + i)

value(p[i][j]) = *(&p[i][j]) = *value(p[i] + j)

value(p[i][j][k]) = *(&p[i][j][k]) = *value(p[i][j] + k)

&p[i][j][k] = value(*value(*(value(p) +i*4) +j*4) +k*4)

p[i][j][k] =*value(*value(*(value(p) +i*4) +j*4) +k*4)

abstract data	int [3][4]	value(p[i])	= &p[i]	= value(p + i)	int (*)[3][4]	virtual pointer
abstract data	int [4]	value(p[i][j])	= &p[i][j]	= value(p[i] + j)	int (*)[4]	virtual pointer
primitive data	int	value(p[i][j][k])	≠ &p[i][j][k]	= value(p[i][j] + k)	int (*)	virtual pointer

p[i][j][k] = *value(*value(*value(p+i)+j)+k)

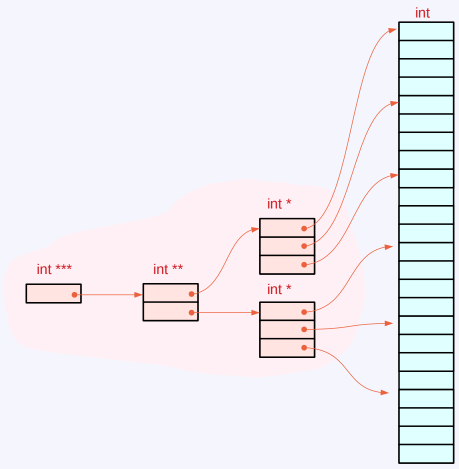
address replications

$= \text{value}(\text{value}(\text{value}(p + i)_{3\cdot 4\cdot 4}) + j)_{4\cdot 4} + k)_4$

$= \text{value}(p + i *3\cdot 4\cdot 4 + j *4\cdot 4 + k *4)$

Subscript [ ] and dereference * notations (1a)

int p[L][M][N]



Pointer Array Approach

address replications

value(p[i]) = &p[i] = value(p + i)

value(p[i][j]) = &p[i][j] = value(p[i] + j)

value(p[i][j][k]) ≠ &p[i][j][k] = value(p[i][j] + k)

&p[i][j][k] = value(p) + i *M·N·4 + j *N·4 + k * 4

p[i][j][k] = *(value(p) + i *M·N·4 + j *N·4 + k * 4)

pointer data	int [3][4]	value(p[i])	= &p[i]	= value(p + i)	int (*)[3][4]	virtual pointer
abstract data	int [4]	value(p[i][j])	= &p[i][j]	= value(p[i] + j)	int (*)[4]	virtual pointer
primitive data	int	value(p[i][j][k])	≠ &p[i][j][k]	= value(p[i][j] + k)	int (*)	virtual pointer

p[i][j][k] = *value(*value(*value(p+i)+j)+k) $\xrightarrow[\text{address replications}]{}$ $= \text{value}(\text{value}(\text{value}(p + i)_{3\cdot4\cdot4}) + j)_{4\cdot4} + k)_{4}$

= value(p + i *3·4·4 + j *4·4 + k *4)

Subscript [ ] and dereference * notations (2)

p[i] ≡ *(p+i)
p[i][j] ≡ *(*(p+i)+j)
p[i][j][k] ≡ *(*(*(p+i)+j)+k)

C Expressions

&p[i] ≡ value(p+i)
&p[i][j] ≡ value(*(p+i)+j)
&p[i][j][k] ≡ value(*(*(p+i)+j)+k)

C Expressions

int p [L][M][N] ;

value(&X) = value(X) (address replication)

$p[i] \longrightarrow *(p+i)_{M \cdot N \cdot 4}$
$p[i][j] \longrightarrow *(*(p+i)_{M \cdot N \cdot 4}+j)_{N \cdot 4}$
$p[i][j][k] \longrightarrow *(*(*(p+i)_{M \cdot N \cdot 4}+j)_{N \cdot 4}+k)_{1 \cdot 4}$

Math Expressions

$\&p[i] \longrightarrow \mathrm{value}(p+i)_{M \cdot N \cdot 4}$
$\&p[i][j] \longrightarrow \mathrm{value}((p+i)_{M \cdot N \cdot 4}+j)_{N \cdot 4}$
$\&p[i][j][k] \longrightarrow \mathrm{value}(((p+i)_{M \cdot N \cdot 4}+j)_{N \cdot 4}+k)_{1 \cdot 4}$

Math Expressions

int ** p[L], * q[L·M], r[L·M·N] ;

*value(X) = *X

$p[i] \longrightarrow *(p+i)_{1 \cdot 4}$
$p[i][j] \longrightarrow *(*(p+i)_{1 \cdot 4}+j)_{1 \cdot 4}$
$p[i][j][k] \longrightarrow *(*(*(p+i)_{1 \cdot 4}+j)_{1 \cdot 4}+k)_{1 \cdot 4}$

Math Expressions

$\&p[i] \longrightarrow \mathrm{value}(p+i)_{1 \cdot 4}$
$\&p[i][j] \longrightarrow \mathrm{value}(*(p+i)_{1 \cdot 4}+j)_{1 \cdot 4}$
$\&p[i][j][k] \longrightarrow \mathrm{value}(*(*(p+i)_{1 \cdot 4}+j)_{1 \cdot 4}+k)_{1 \cdot 4}$

Math Expressions

Subscript [ ] and dereference * notations (3)

int p [L][M][N] ;

value(&X) = value(X) (address replication)

&p[i] = value($(p + i)_{M \cdot N \cdot 4}$) = value(p + i * M·N·4)

&p[i][j] = value($(p[i] + j)_{N \cdot 4}$) = value(p[i] + j * N·4)

&p[i][j][k] = value($(p[i][j]+k)_{1 \cdot 4}$) = value(p[i][j] + k * 1·4)

= value(p + i * M·N·4 + j * N·4 + k * 4)

&p[i] → value$(p+i)_{M \cdot N \cdot 4}$

&p[i][j] → value$((p+i)_{M \cdot N \cdot 4}+j)_{N \cdot 4}$

&p[i][j][k] → value$(((p+i)_{M \cdot N \cdot 4}+j)_{N \cdot 4}+k)_{1 \cdot 4}$

Math Expressions

int ** p[L], * q[L·M], r[L·M·N] ;

*value(X) = *X

&p[i] = value$(p + I)_{1 \cdot 4}$ = value(p + i * 1·4)

*

&p[i][j] = value$(p[i] + j)_{1 \cdot 4}$ = value(p[i] + j * 1·4)

*

&p[i][j][k] = value$(p[i][j] + k)_{1 \cdot 4}$ = value(p[i][j] + k * 1·4)

= value(*value(*value(p +i*4) +j*4) +k*4)

&p[i] → value$(p+i)_{1 \cdot 4}$

&p[i][j] → value$(*(p+i)_{1 \cdot 4}+j)_{1 \cdot 4}$

&p[i][j][k] → value$(*(*(p+i)_{1 \cdot 4}+j)_{1 \cdot 4}+k)_{1 \cdot 4}$

Math Expressions

Operator Precedence

Precedence	Operator	Description	Associativity
1	++ -- () [] . -> (type){list}	Suffix/postfix increment and decrement Function call Array subscripting Structure and union member access member access through pointer Compound literal(C99)	Left-to-right (((x[m])[n])[p]) →
2	++ -- + - ! ~ (type) * & sizeof _Alignof	Prefix increment and decrement Unary plus and minus Logical NOT and bitwise NOT Type cast Indirection (dereference) Address-of Size-of Alignment requirement(C11)	Right-to-left *(*(*(*x))) ←

https://en.cppreference.com/w/c/language/operator_precedence

Limitations

No index Range Checking

Array Size must be a constant expression

    Variable Array Size

Arrays cannot be Copied or Compared

Aggregate Initialization and Global Arrays

Precedence Rule

Index Type Must be Integral

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