

# Laurent Series and z-Transform

## - Geometric Series

## Permutations B

20240730 Tue

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$a^n$  $\times$  $R(n)$ 

$a^n$	$a^{-n}$

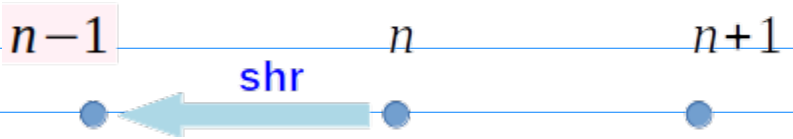
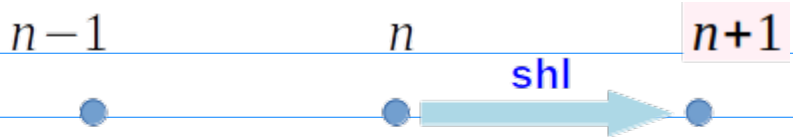
 $\times$ 

$u(n)$	$u(-n-1)$
$u(-n)$	$u(n-1)$

(1)	$a^n u(n)$	$a^{-n} u(n)$	(2)
(3)	$a^n u(-n)$	$a^{-n} u(-n)$	(4)
(5)	$a^n u(-n-1)$	$a^{-n} u(-n-1)$	(6)
(7)	$a^n u(n-1)$	$a^{-n} u(n-1)$	(8)

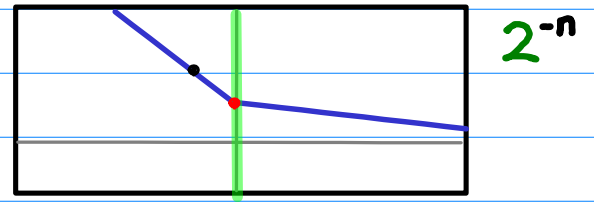
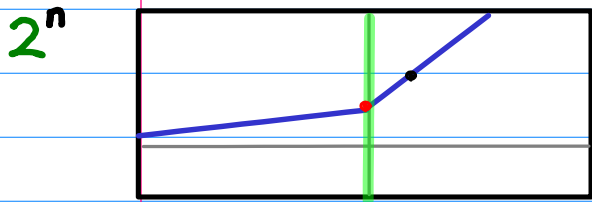
$b^n$	$shl(b^n)$	$b^n$	$shr(b^n)$
$a^n$	$a^{(n+1)}$	$a^n$	$a^{(n-1)}$
$a^{-n}$	$a^{-(n+1)}$	$a^{-n}$	$a^{-(n-1)}$

$R(n)$	$shl(R(n))$	$R(n)$	$shr(R(n))$
$u(n-1)$	$u(n)$	$u(n)$	$u(n-1)$
$u(-n)$	$u(-(n+1))$	$u(-(n+1))$	$u(-n)$



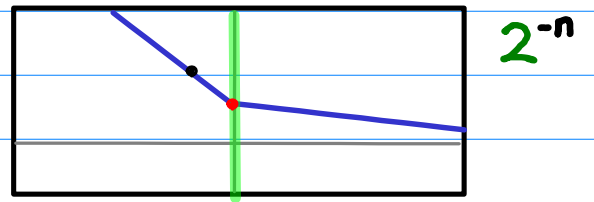
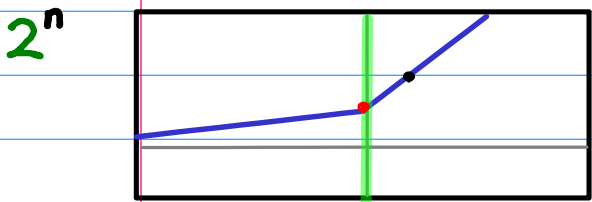
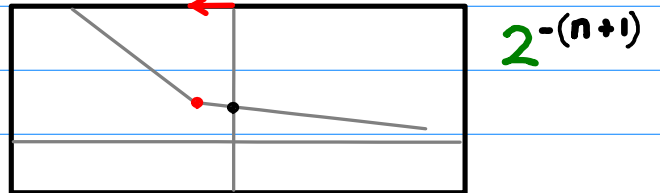
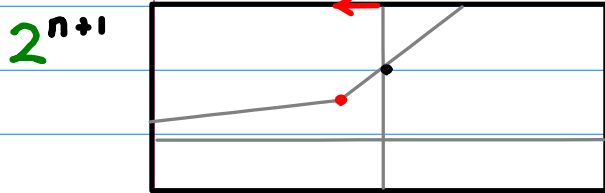
shl( $b^n$ )  
shr( $b^n$ )

shl( $n$ ) =  $n + 1$   
shr( $n$ ) =  $n - 1$



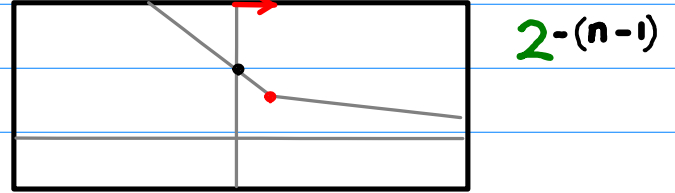
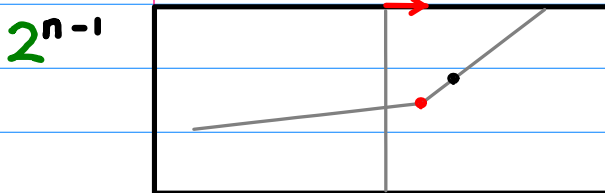
shift left  
 $n \leftarrow n+1$

shift left  
 $n \leftarrow n+1$



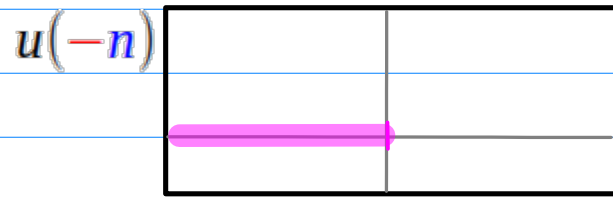
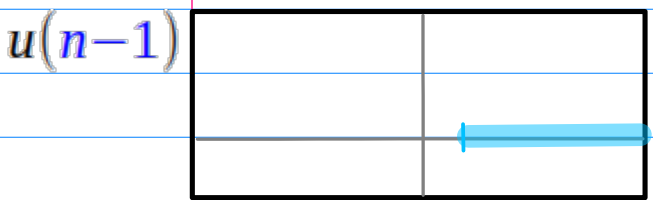
shift right  
 $n \leftarrow n-1$

shift right  
 $n \leftarrow n-1$



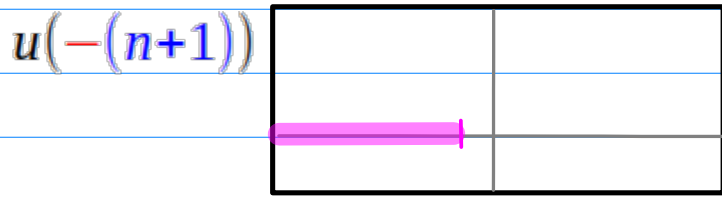
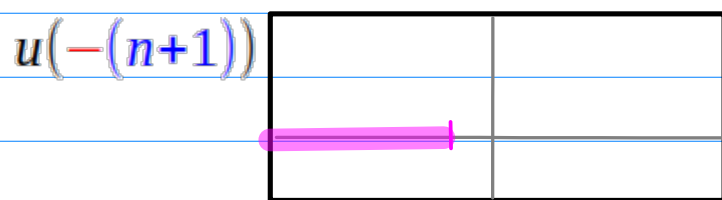
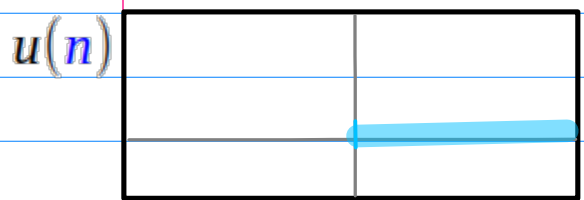
$\text{shl}(R(n))$   
 $\text{shr}(R(n))$

$\text{shl}(n) = n + 1$   
 $\text{shr}(n) = n - 1$



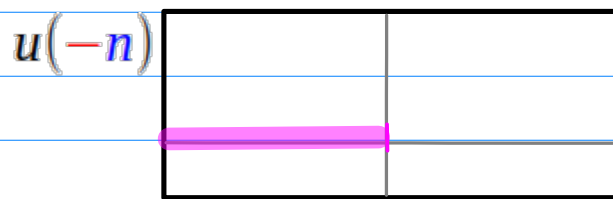
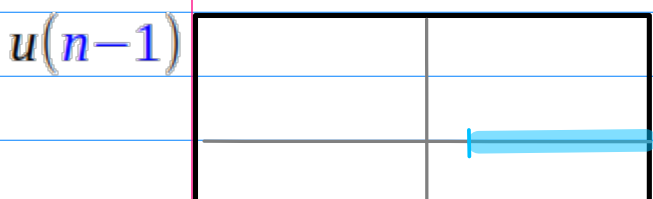
shift left  
 $n \leftarrow n+1$

shift left  
 $n \leftarrow n+1$



shift right  
 $n \leftarrow n-1$

shift right  
 $n \leftarrow n-1$



$a^n$  $\times$  $R(n)$ 

$a^{n+1}$	$a^{-n-1}$
$a^{n-1}$	$a^{-n+1}$

 $\times$ 

$u(n)$	$u(-n-1)$
$u(-n)$	$u(n-1)$

(1')	$a^{n+1}u(n)$	$a^{-n-1}u(n)$	(2')
(3')	$a^{n-1}u(-n)$	$a^{-n+1}u(-n)$	(4')
(5')	$a^{n+1}u(-n-1)$	$a^{-n-1}u(-n-1)$	(6')
(7')	$a^{n-1}u(n-1)$	$a^{-n+1}u(n-1)$	(8')

(1'')	$a^{n-1}u(n)$	$a^{-n+1}u(n)$	(2'')
(3'')	$a^{n+1}u(-n)$	$a^{-n-1}u(-n)$	(4'')
(5'')	$a^{n-1}u(-n-1)$	$a^{-n+1}u(-n-1)$	(6'')
(7'')	$a^{n+1}u(n-1)$	$a^{-n-1}u(n-1)$	(8'')

$a^n$  $\times$  $R(n)$ 

$a^{n+1}$	$a^{-n-1}$

 $\times$ 

$u(n)$	$u(-n-1)$
$u(-n)$	$u(n-1)$

$$(1') \quad a^{n+1} u(n) \quad a^{-n-1} u(n) \quad (2')$$

$$(3') \quad a^{n-1} u(-n) \quad a^{-n+1} u(-n) \quad (4')$$

$$(5') \quad a^{n+1} u(-n-1) \quad a^{-n-1} u(-n-1) \quad (6')$$

$$(7') \quad a^{n-1} u(n-1) \quad a^{-n+1} u(n-1) \quad (8')$$



$a^n$  $\times$  $R(n)$ 

$a^{n+1}$	$a^{-n-1}$
$a^{n-1}$	$a^{-n+1}$

 $\times$ 

$u(n)$	$u(-n-1)$
$u(-n)$	$u(n-1)$

(1'')	$a^{n-1} u(n)$	$a^{-n+1} u(n)$	(2'')
(3'')	$a^{n+1} u(-n)$	$a^{-n-1} u(-n)$	(4'')
(5'')	$a^{n-1} u(-n-1)$	$a^{-n+1} u(-n-1)$	(6'')
(7'')	$a^{n+1} u(n-1)$	$a^{-n-1} u(n-1)$	(8'')

**Unshifted  
Sequence  $x$**

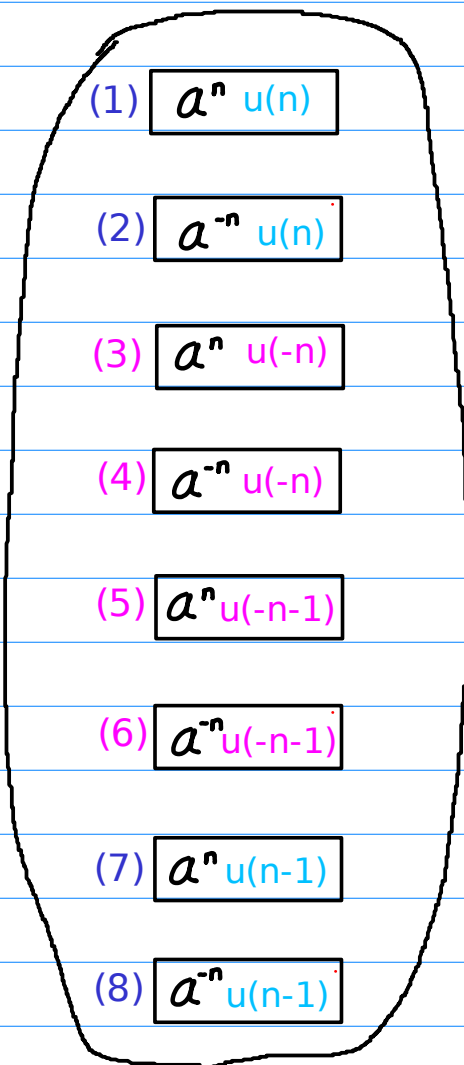
**Shifted  
Sequence 1  $x'$**

**Shifted  
Sequence 2  $x''$**

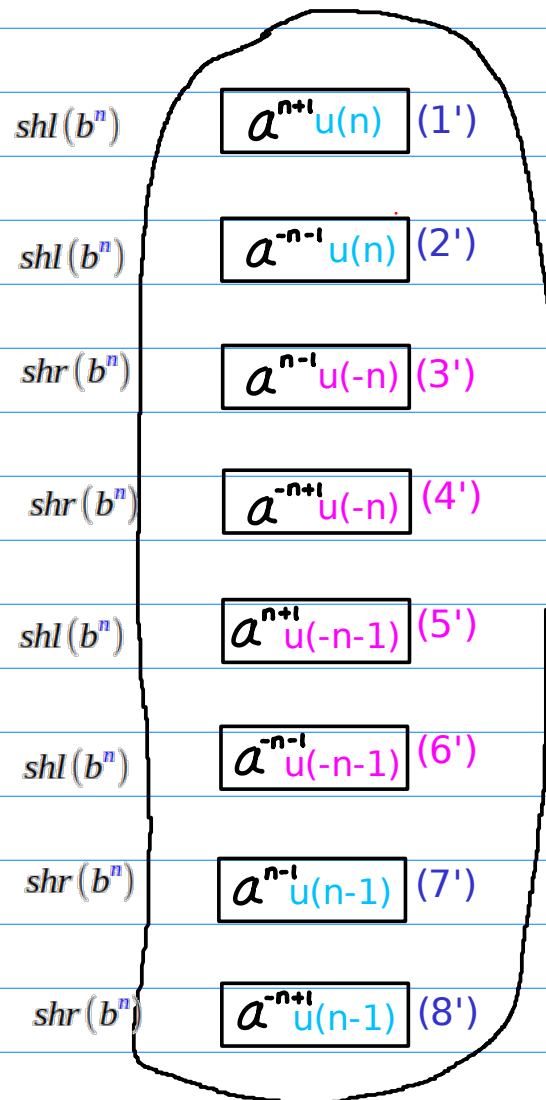
(1) $a^n u(n)$	$shl(b^n)$	$a^{n+1} u(n)$ (1')	$shl(b^{-n})$	$a^{-n-1} u(n)$ (1'')	$n \leftarrow n+1$
(2) $a^{-n} u(n)$	$shl(b^n)$	$a^{-n-1} u(n)$ (2')	$shl(b^{-n})$	$a^{n+1} u(n)$ (2'')	$n \leftarrow n+1$
(3) $a^n u(-n)$	$shr(b^n)$	$a^{n-1} u(-n)$ (3')	$shr(b^{-n})$	$a^{-n+1} u(-n)$ (3'')	$n \leftarrow n-1$
(4) $a^{-n} u(-n)$	$shr(b^n)$	$a^{-n+1} u(-n)$ (4')	$shr(b^{-n})$	$a^{n-1} u(-n)$ (4'')	$n \leftarrow n-1$
(5) $a^n u(-n-1)$	$shl(b^n)$	$a^{n+1} u(-n-1)$ (5')	$shl(b^{-n})$	$a^{-n-1} u(-n-1)$ (5'')	$n \leftarrow n+1$
(6) $a^{-n} u(-n-1)$	$shl(b^n)$	$a^{-n-1} u(-n-1)$ (6')	$shl(b^{-n})$	$a^{n+1} u(-n-1)$ (6'')	$n \leftarrow n+1$
(7) $a^n u(n-1)$	$shr(b^n)$	$a^{n-1} u(n-1)$ (7')	$shr(b^{-n})$	$a^{-n+1} u(n-1)$ (7'')	$n \leftarrow n-1$
(8) $a^{-n} u(n-1)$	$shr(b^n)$	$a^{-n+1} u(n-1)$ (8')	$shr(b^{-n})$	$a^{n-1} u(n-1)$ (8'')	$n \leftarrow n-1$

many possible permutations are possible  
but consider these two

## Unshifted Sequence $x$



## Shifted Sequence $1 x'$



**Inter-permutations over unshifted sequence and shifted sequence**

**Intra-permutations over unshifted sequence**

**Intra-permutations over shifted sequence**

## Unshifted Sequence $x$

(1)  $a^n u(n)$

(2)  $a^{-n} u(n)$

(3)  $a^n u(-n)$

(4)  $a^{-n} u(-n)$

(5)  $a^n u(-n-1)$

(6)  $a^{-n} u(-n-1)$

(7)  $a^n u(n-1)$

(8)  $a^{-n} u(n-1)$

## Shifted Sequence 2 $x''$

$shl(b^{-n})$   $a^{-n-i} u(n)$  (1'')

$shl(b^{-n})$   $a^{n+i} u(n)$  (2'')

$shr(b^{-n})$   $a^{-n+i} u(-n)$  (3'')

$shr(b^{-n})$   $a^{n-i} u(-n)$  (4'')

$shl(b^{-n})$   $a^{-n-i} u(-n-1)$  (5'')

$shl(b^{-n})$   $a^{n+i} u(-n-1)$  (6'')

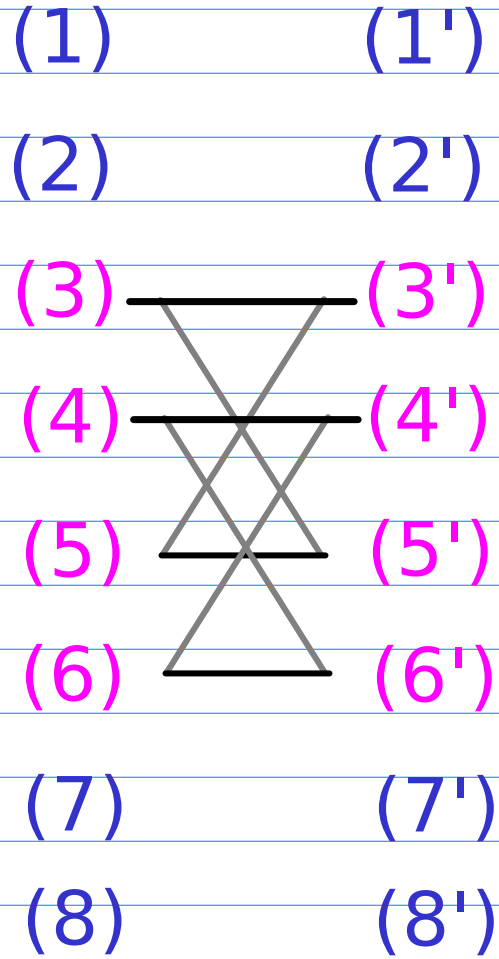
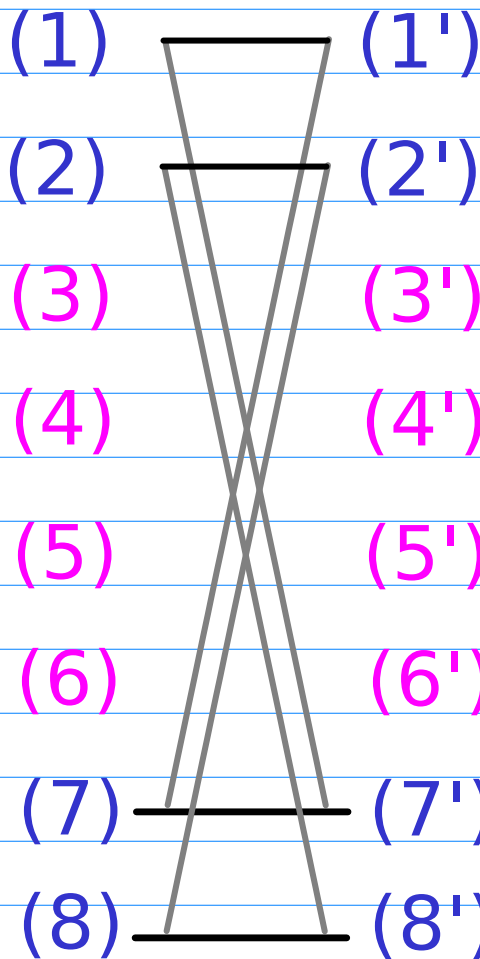
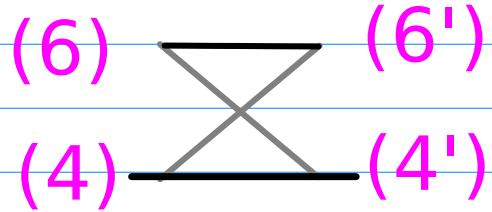
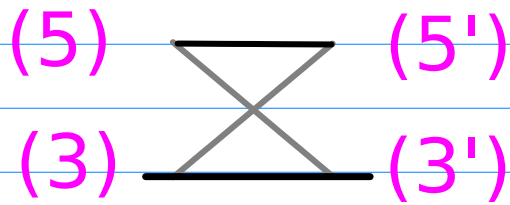
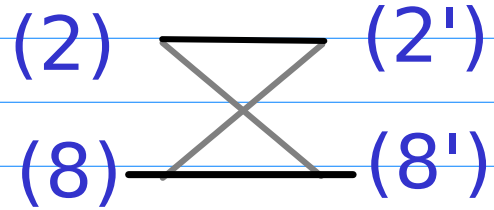
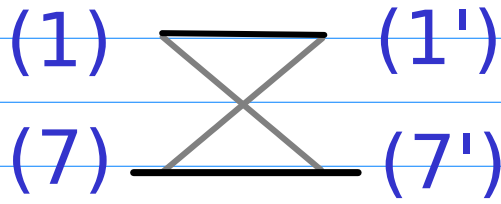
$shr(b^{-n})$   $a^{-n+i} u(n-1)$  (7'')

$shr(b^{-n})$   $a^{n-i} u(n-1)$  (8'')

(1) - (1')  
(7) - (7')  
(5) - (5')  
(3) - (3')

(2) - (2')  
(8) - (8')  
(6) - (6')  
(4) - (4')

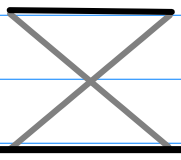

**(x)**  **(x')**  
(1)~(8) (1')~(8')

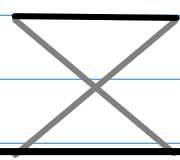



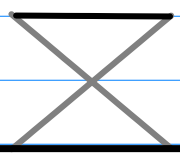

(1) - (1')  
(7) - (7')  
(5) - (5')  
(3) - (3')

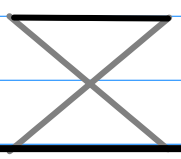

(2) - (2')  
(8) - (8')  
(6) - (6')  
(4) - (4')

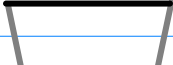







**(x)**  **(x'')**  
(1)~(8) (1')~(8')





(1)  (1'')  
(7)  (7'')

(2)  (2'')  
(8)  (8'')

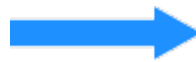
(5)  (5'')  
(3)  (3'')

(6)  (6'')  
(4)  (4'')

(1)  (1'')  
(2)  (2'')  
(3)  (3'')  
(4)  (4'')  
(5)  (5'')  
(6)  (6'')  
(7)  (7'')  
(8)  (8'')

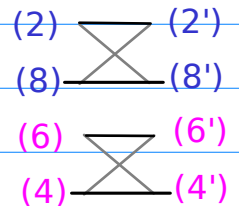
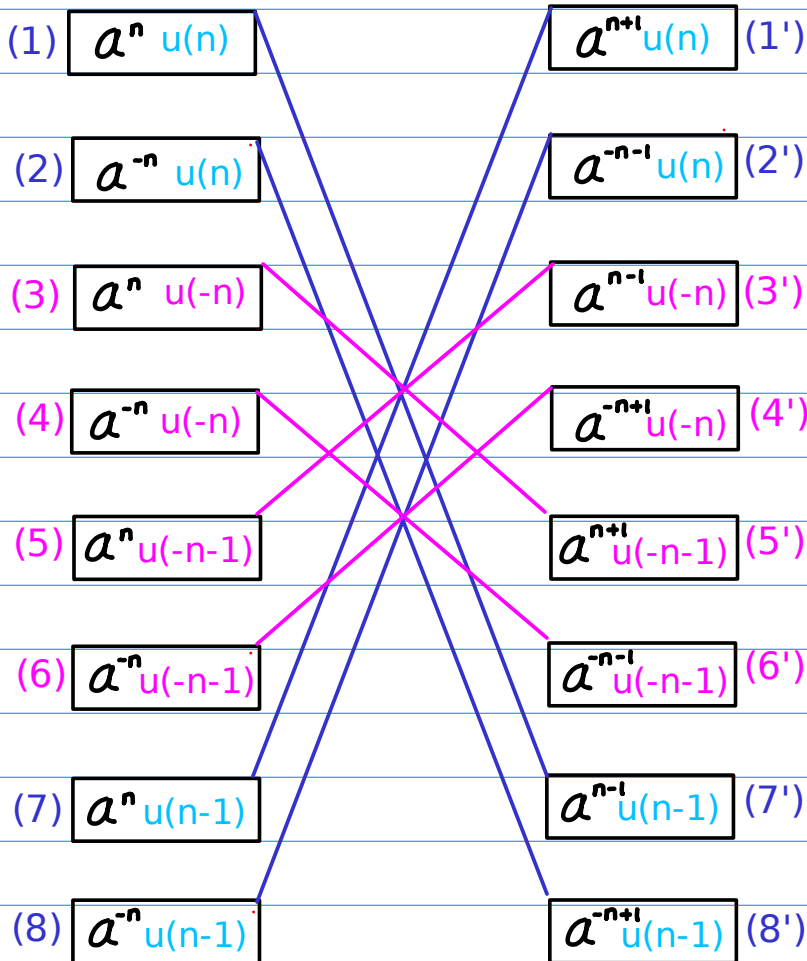
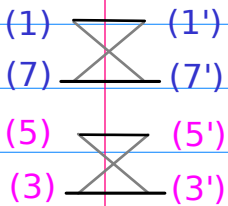
(1) (1'')  
(2) (2'')  
(3)  (3'')  
(4)  (4'')  
(5)  (5'')  
(6)  (6'')  
(7) (7'')  
(8) (8'')

# Unshifted Sequence $x$



# Shifted Sequence 1 $x'$

(1)	$\frac{1}{1-2z^{-1}} 2^n u(n)$	(2)	$\frac{1}{1-0.5z^{-1}} 2^{-n} u(n)$	(1')	$\frac{2}{1-2z^{-1}} 2^{n+1} u(n)$	(2')	$\frac{0.5}{1-0.5z^{-1}} 2^{-n-1} u(n)$
(3)	$\frac{1}{1-0.5z^{-1}} 2^n u(-n)$	(4)	$\frac{1}{1-2z^{-1}} 2^{-n} u(-n)$	(3')	$\frac{0.5}{1-0.5z^{-1}} 2^{n-1} u(-n)$	(4')	$\frac{2}{1-2z^{-1}} 2^{-n+1} u(-n)$
(5)	$\frac{0.5z^{-1}}{1-0.5z^{-1}} 2^n u(-n-1)$	(6)	$\frac{2z^{-1}}{1-2z^{-1}} 2^{-n} u(-n-1)$	(5')	$\frac{z^{-1}}{-0.5z^{-1}} 2^{n+1} u(-n-1)$	(6')	$\frac{z^{-1}}{1-2z^{-1}} 2^{-n-1} u(-n-1)$
(7)	$\frac{2z}{1-2z} 2^n u(n-1)$	(8)	$\frac{0.5z}{1-0.5z} 2^{-n} u(n-1)$	(7')	$\frac{z}{1-2z} 2^{n-1} u(n-1)$	(8')	$\frac{z}{1-0.5z} 2^{-n+1} u(n-1)$

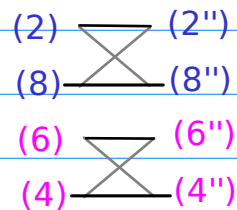
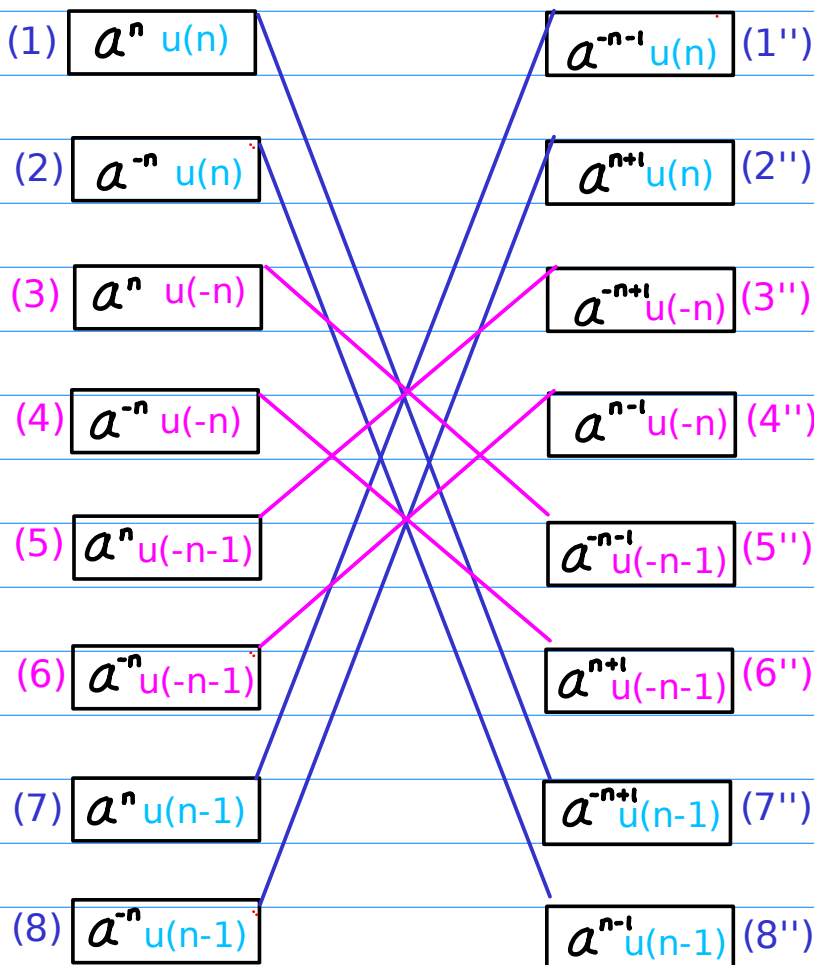
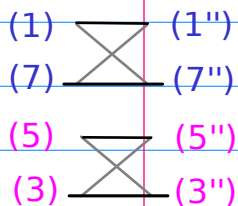


# Unshifted Sequence $x$



# Shifted Sequence $2x''$

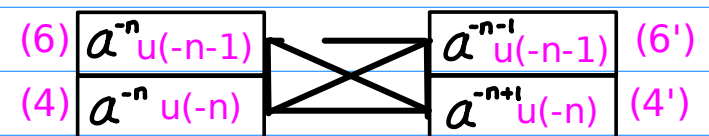
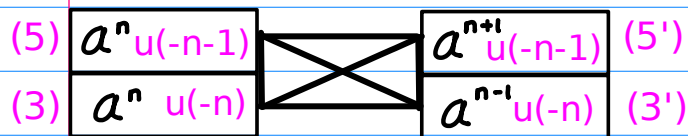
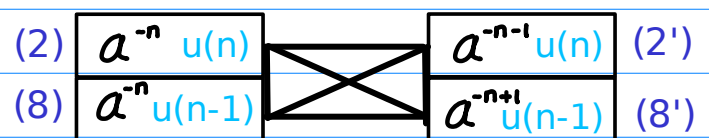
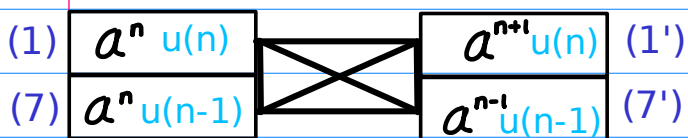
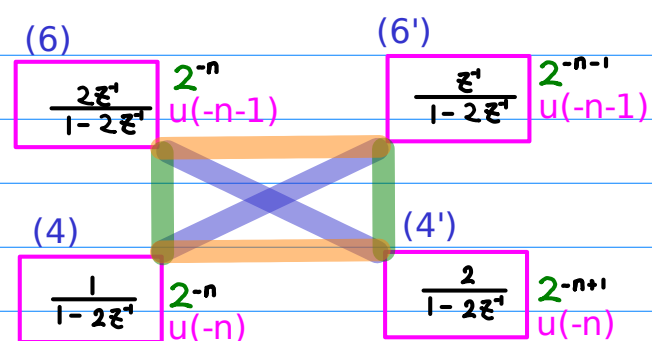
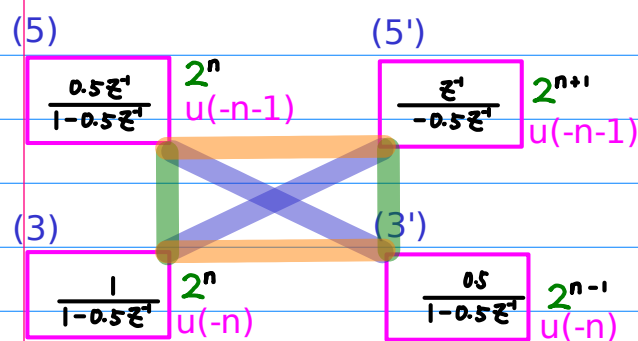
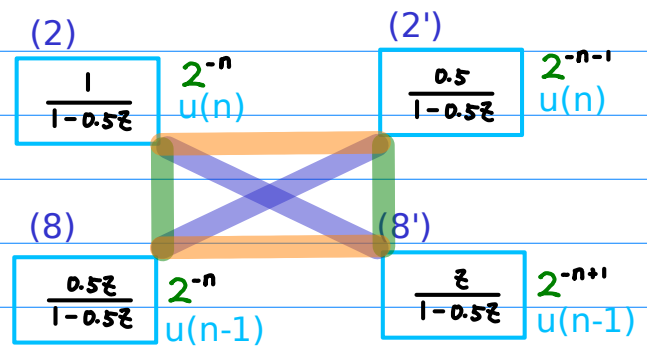
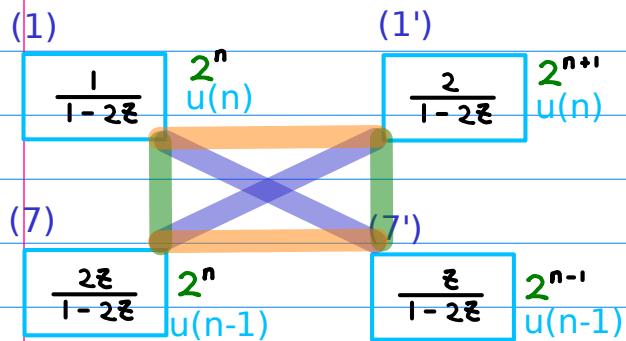
(1)	$\frac{1}{1-2z^{-1}} 2^n u(n)$	(2)	$\frac{1}{1-0.5z^{-1}} 2^{-n} u(n)$	(1'')	$\frac{0.5}{1-0.5z^{-1}} 2^{-n-1} u(n)$	(2'')	$\frac{2}{1-2z^{-1}} 2^{n+1} u(n)$
(3)	$\frac{1}{1-0.5z^{-1}} 2^n u(-n)$	(4)	$\frac{1}{1-2z^{-1}} 2^{-n} u(-n)$	(3'')	$\frac{2}{1-2z^{-1}} 2^{-n+1} u(-n)$	(4'')	$\frac{0.5}{1-0.5z^{-1}} 2^{n-1} u(-n)$
(5)	$\frac{0.5z^{-1}}{1-0.5z^{-1}} 2^n u(-n-1)$	(6)	$\frac{2z^{-1}}{1-2z^{-1}} 2^{-n} u(-n-1)$	(5'')	$\frac{z^{-1}}{1-2z^{-1}} 2^{-n-1} u(-n-1)$	(6'')	$\frac{z^{-1}}{-0.5z^{-1}} 2^{n+1} u(-n-1)$
(7)	$\frac{2z^{-1}}{1-2z^{-1}} 2^n u(n-1)$	(8)	$\frac{0.5z^{-1}}{1-0.5z^{-1}} 2^{-n} u(n-1)$	(7'')	$\frac{z^{-1}}{1-0.5z^{-1}} 2^{-n+1} u(n-1)$	(8'')	$\frac{z^{-1}}{1-2z^{-1}} 2^{n-1} u(n-1)$





# Inter-permutation

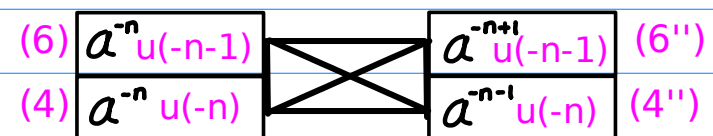
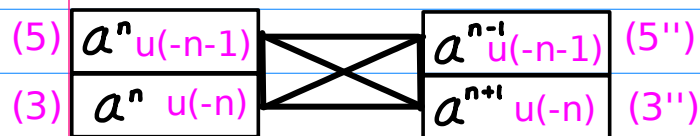
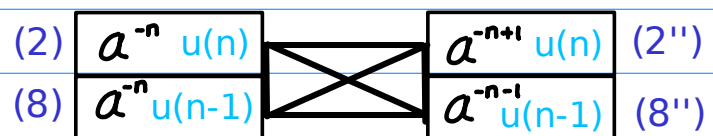
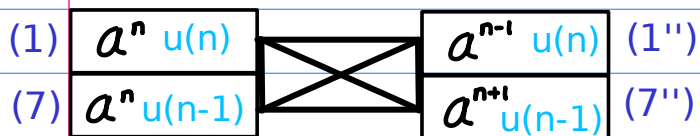
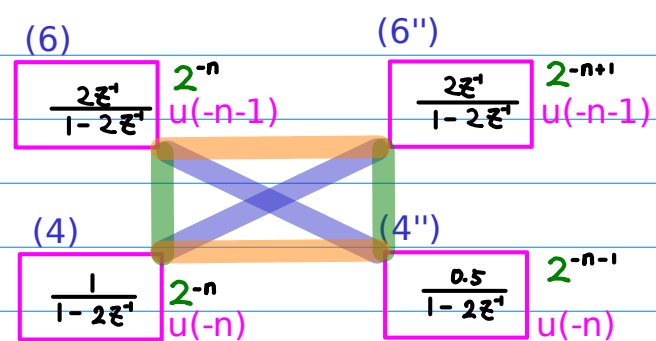
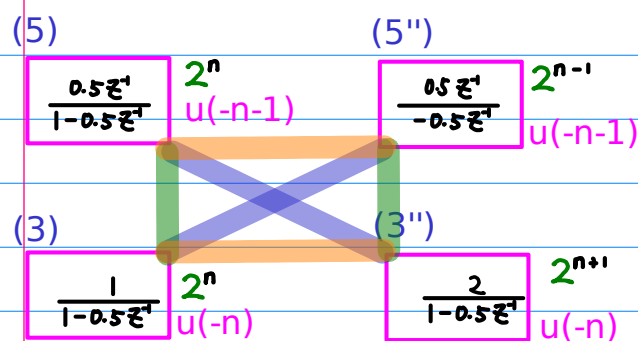
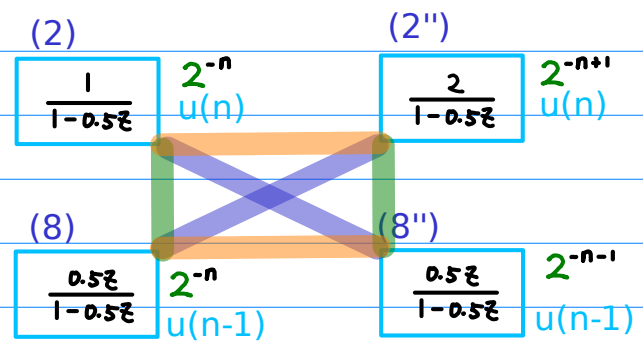
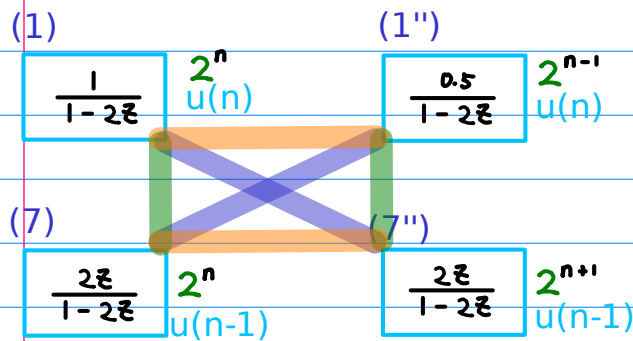
$(x) \longrightarrow (x')$   
 (1)~(8) (1')~(8')



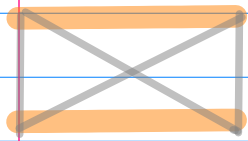
# Inter-permutation

(x)  $\longrightarrow$  (x'')

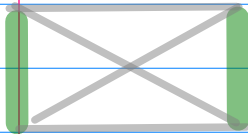
(1)~(8) (1'')~(8'')



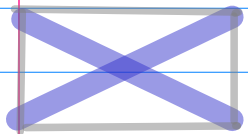
# Decomposing Shift Operations



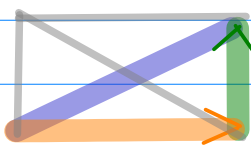
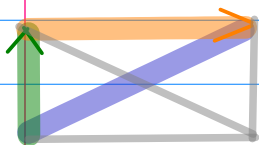
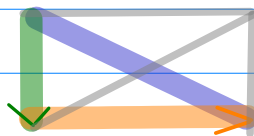
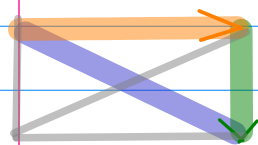
( exponent shift, identity )



( identity, range shift )



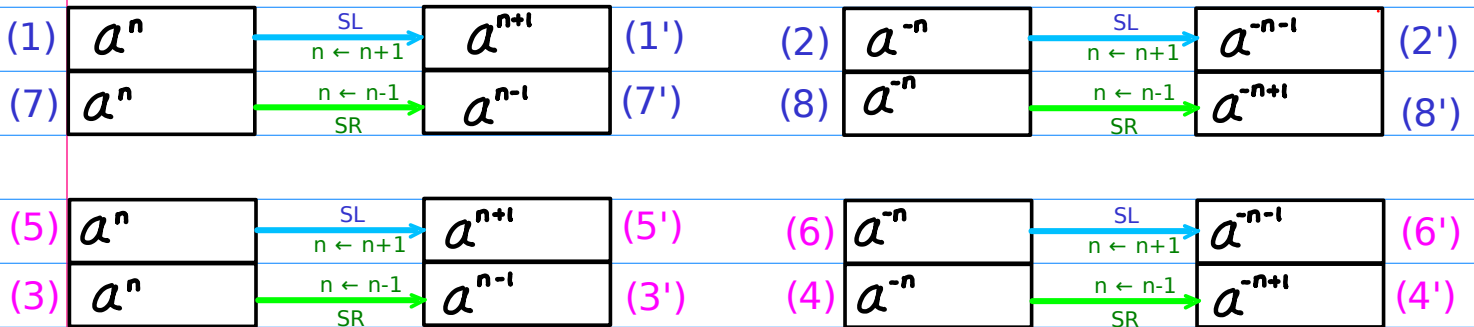
( exponent shift, range shift )  
= ( exponent shift, identity )  
+ ( identity, range shift )



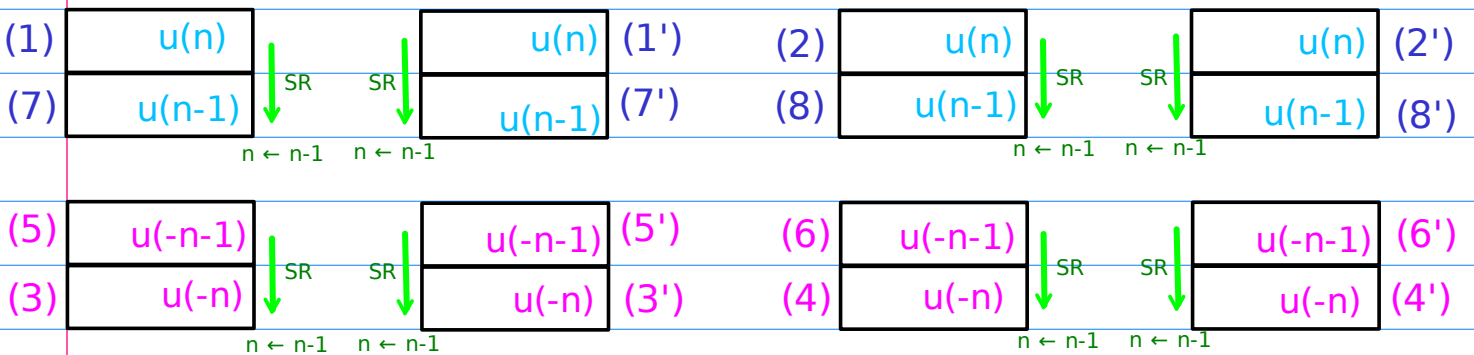
$(id, SR) + (SR, id) = (SR, SR)$   
 $(id, SL) + (SL, id) = (SL, SL)$

$(\mathbf{x}) \longrightarrow (\mathbf{x}')$   
 $(1) \sim (8) \qquad (1') \sim (8')$

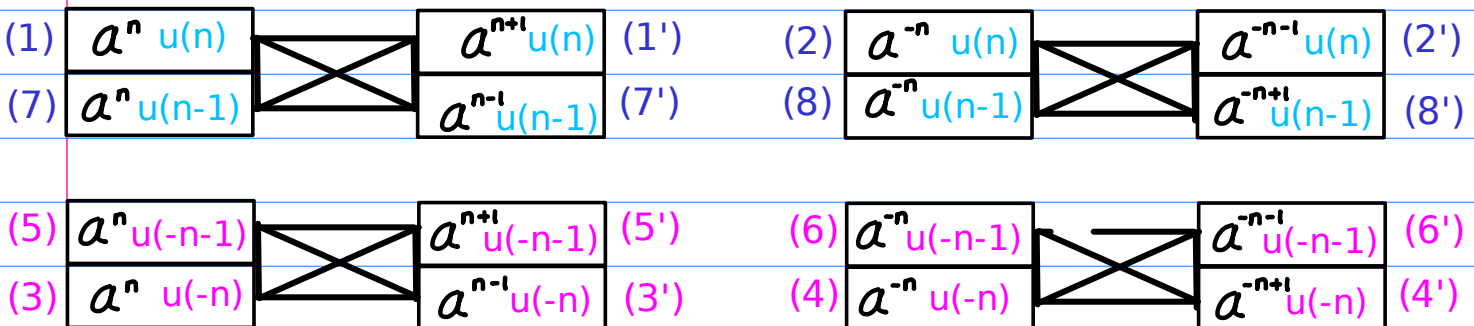
**Exponent Shifts : (SR, id) or (SL, id)**



**Range Shifts : (id, SR) or (id, SL)**



**Exponent & Range Permutations**



**Decomposition**

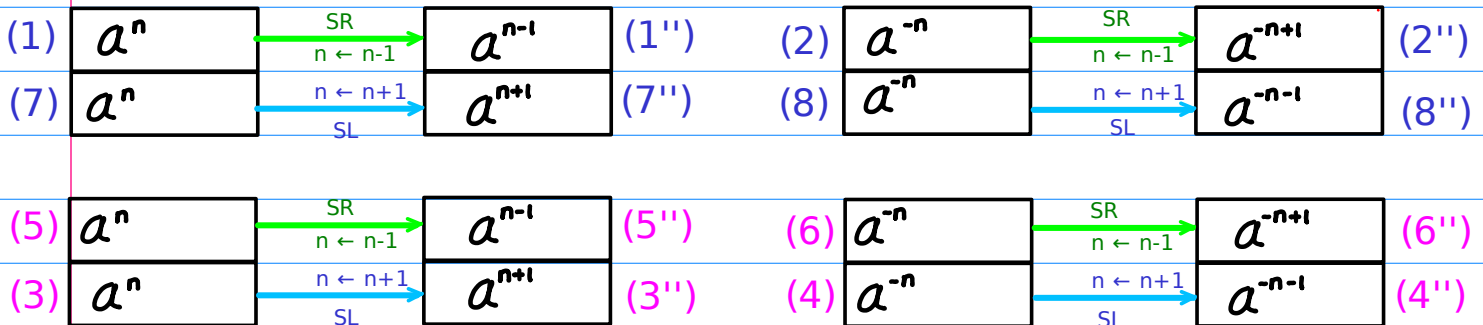
$(EP, RP) = (EP, id) + (id, RP)$

**EP : Exponent Permutations**  
**RP : Range Permutations**

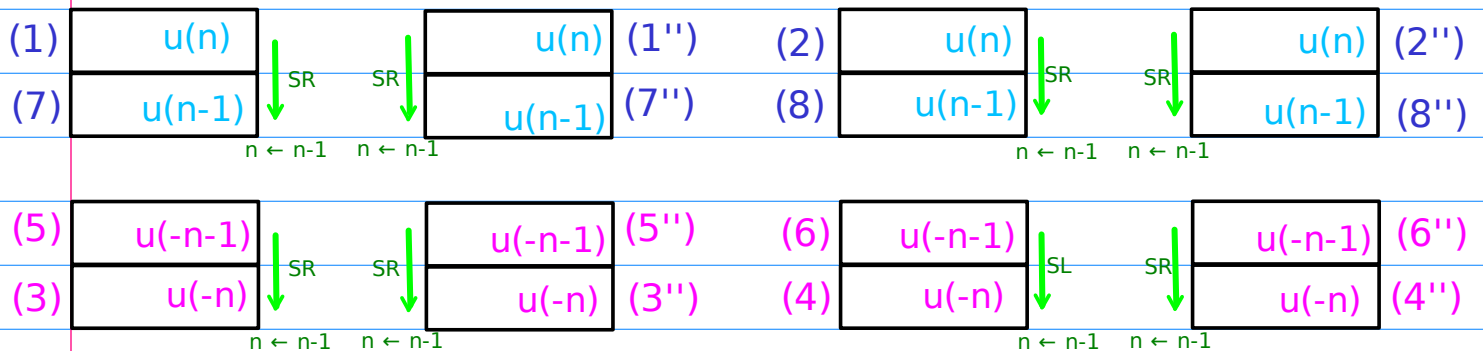
$$\begin{aligned}
 (\text{id}, \text{SR}) + (\text{SL}, \text{id}) &= (\text{SL}, \text{SR}) \\
 (\text{id}, \text{SL}) + (\text{SR}, \text{id}) &= (\text{SR}, \text{SL})
 \end{aligned}$$

$$\begin{aligned}
 (\mathbf{x}) &\longrightarrow (\mathbf{x}'') \\
 (1) \sim (8) &\qquad\qquad (1'') \sim (8'')
 \end{aligned}$$

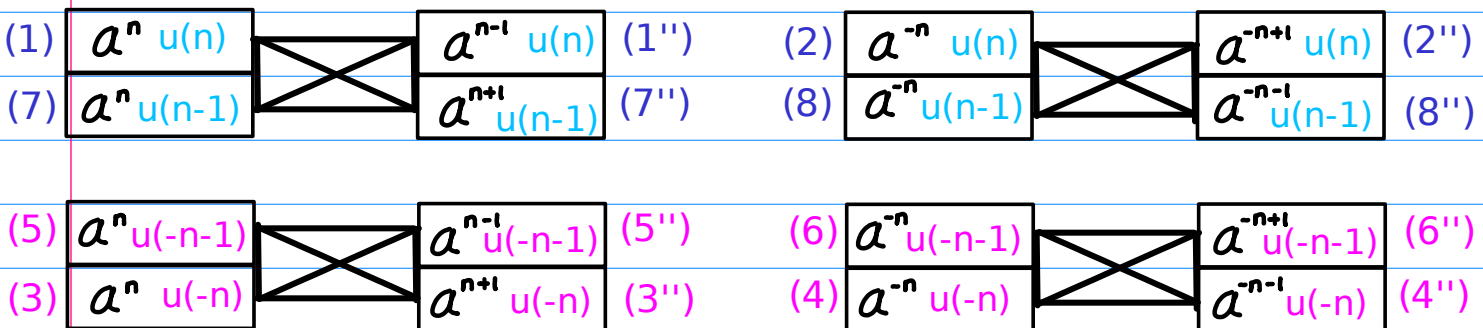
### Exponent Shifts : (SR, id) or (SL, id)



### Range Shifts : (id, SR) or (id, SL)



### Exponent & Range Permutations



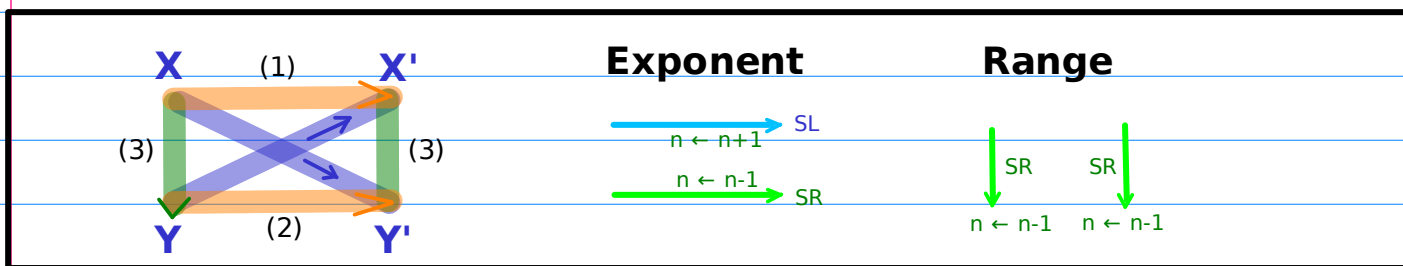
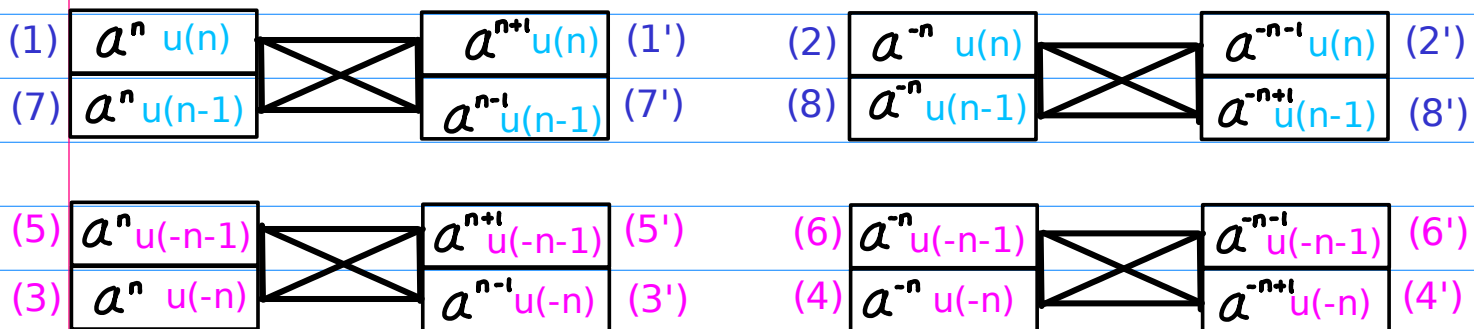
### Decomposition

$$(\text{EP}, \text{RP}) = (\text{EP}, \text{id}) + (\text{id}, \text{RP})$$

**EP : Exponent Permutations**  
**RP : Range Permutations**

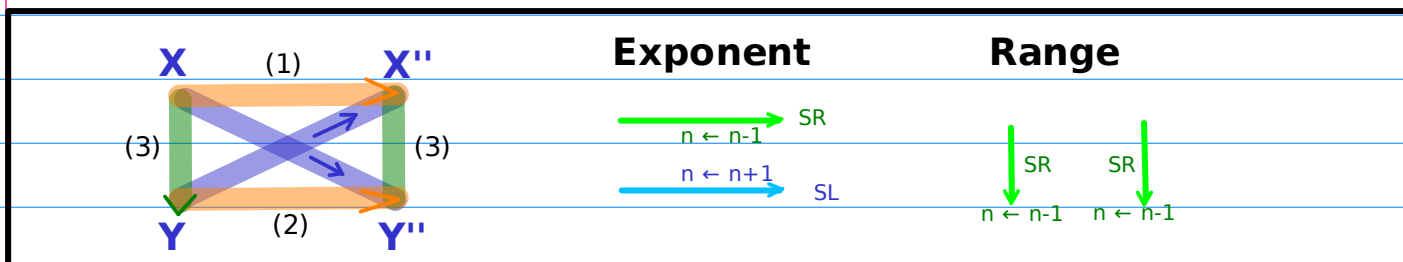
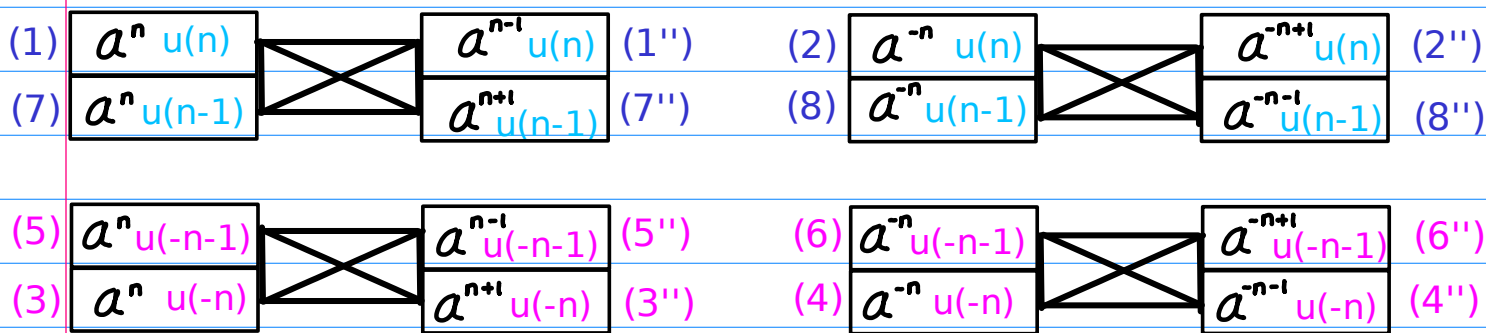
(id, SR) + (SR, id) = (SR, SR)  
 (id, SL) + (SL, id) = (SL, SL)

(X) → (X')  
 (1)~(8) (1')~(8')



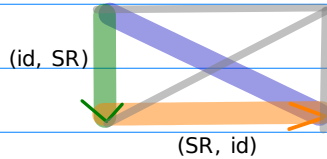
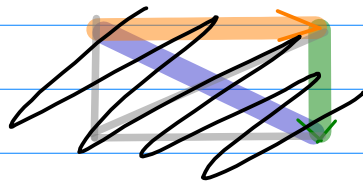
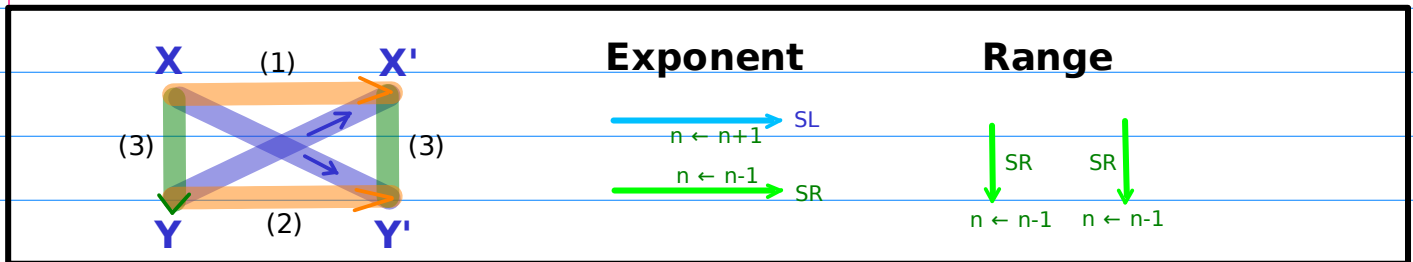
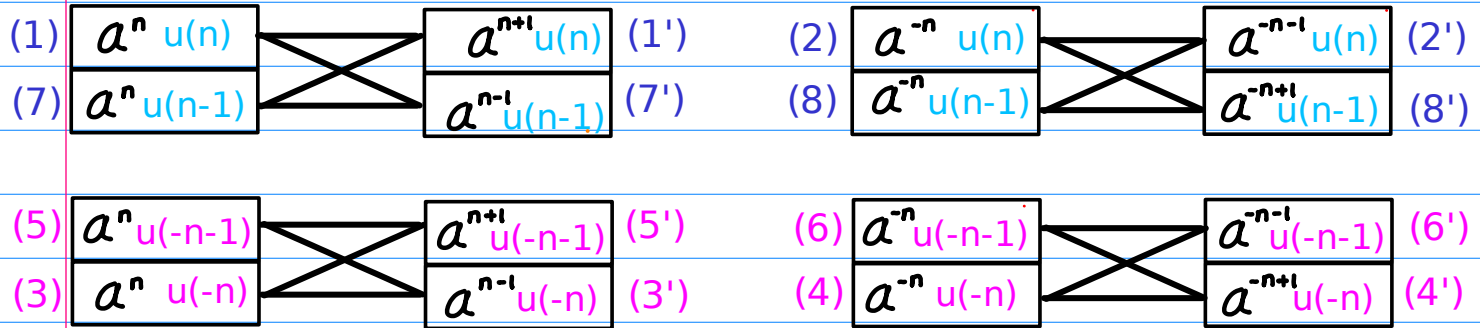
(id, SR) + (SL, id) = (SL, SR)  
 (id, SL) + (SR, id) = (SR, SL)

(X) → (X'')  
 (1)~(8) (1'')~(8'')

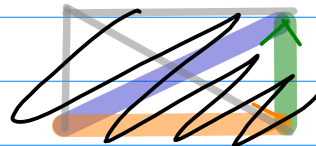
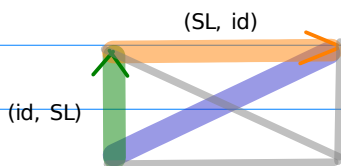


# Inter-permutation (x) $\longrightarrow$ (x')

(1)~(8) (1')~(8')



(SR, id) shift right exponent  
 (id, SR) shift right range  
 (SR, SR)

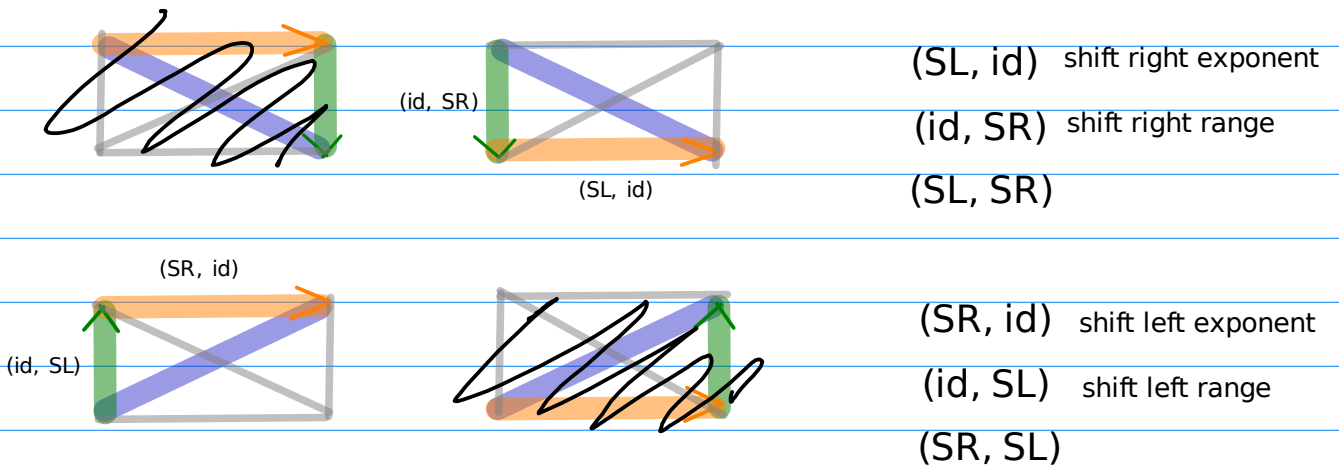
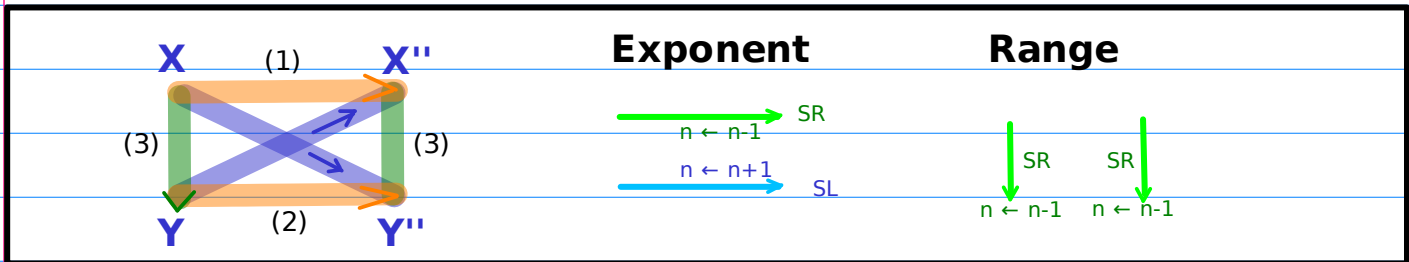
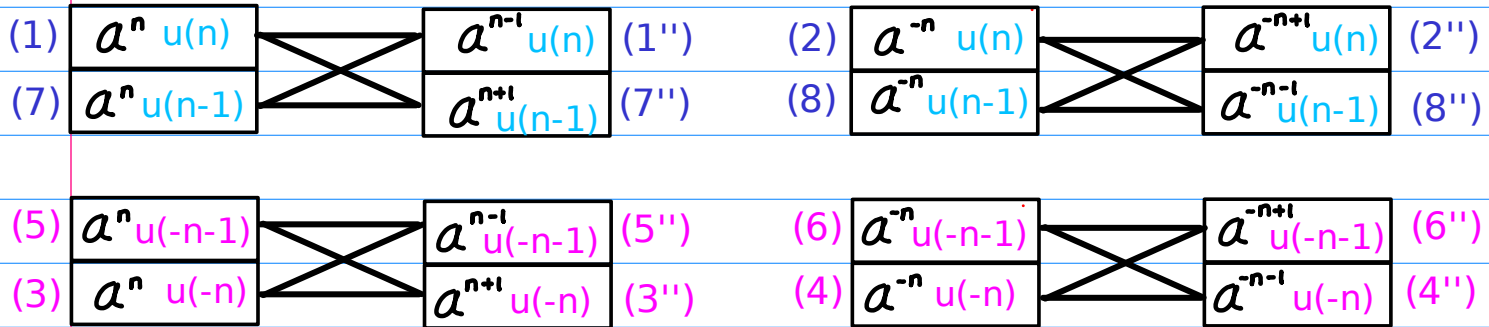


(SL, id) shift left exponent  
 (id, SL) shift left range  
 (SL, SL)

**(SR, id) + (id, SR) = (SR, SR)**  
**(SL, id) + (id, SL) = (SL, SL)**

# Inter-permutation (x) $\longrightarrow$ (x'')

Summary (1)~(8) (1'')~(8'')



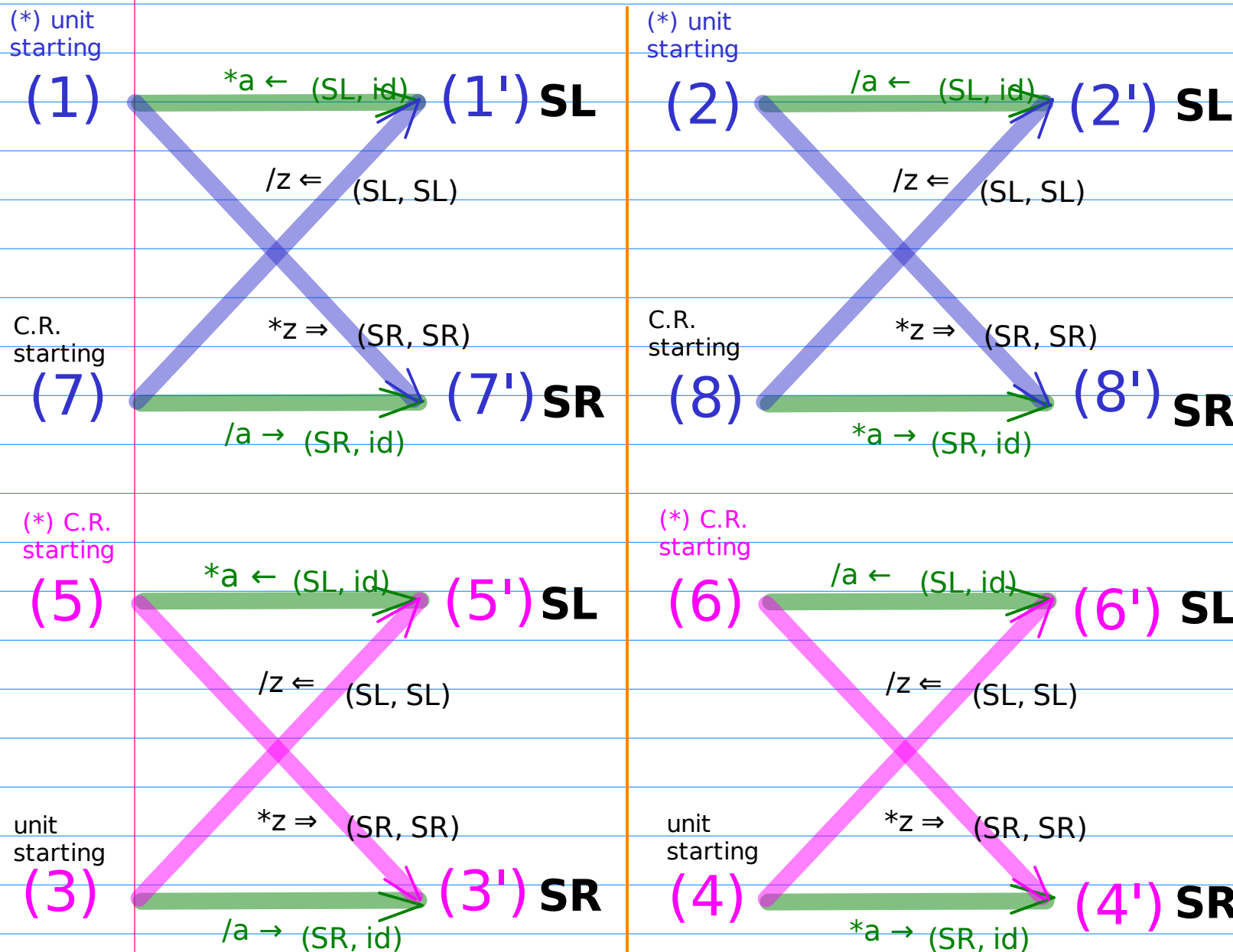
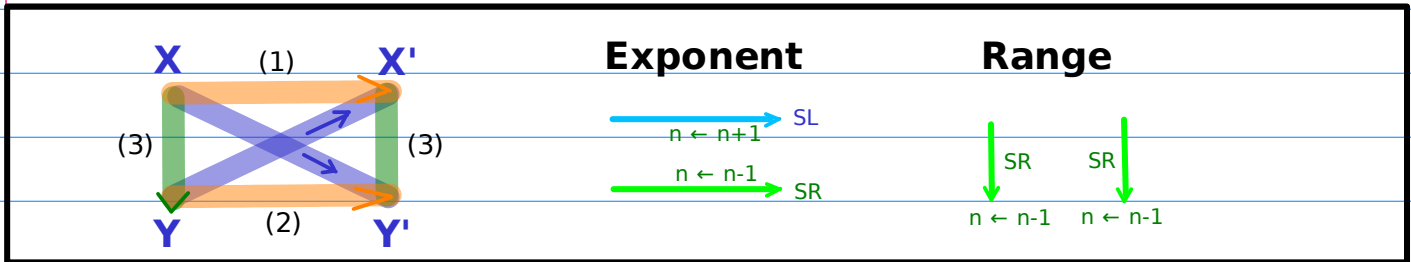
**(SL, id) + (id, SR) = (SL, SR)**  
**(SR, id) + (id, SL) = (SR, SL)**





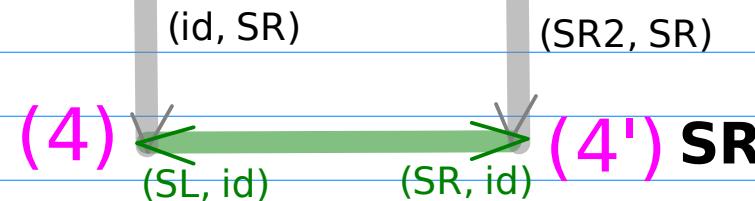
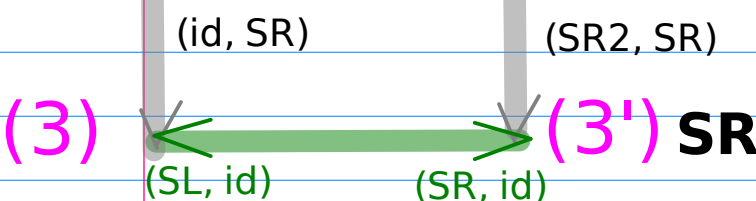
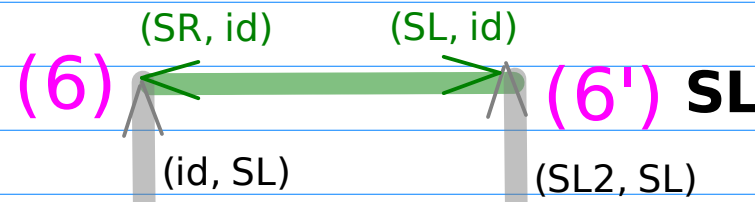
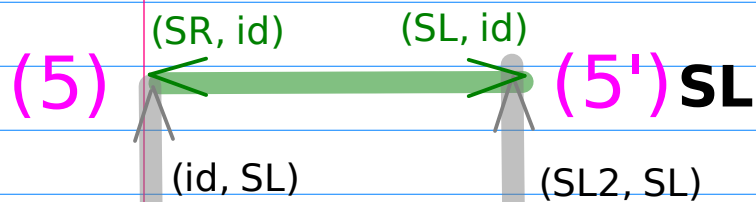
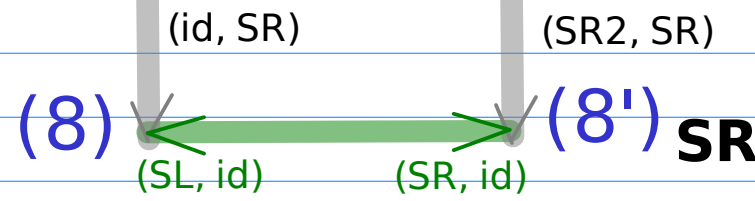
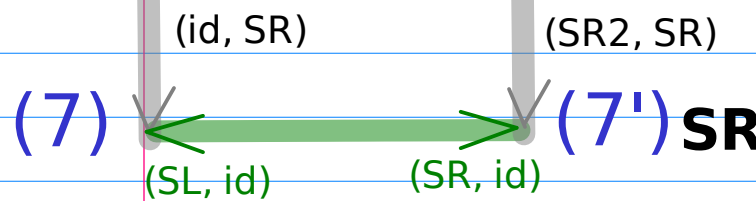
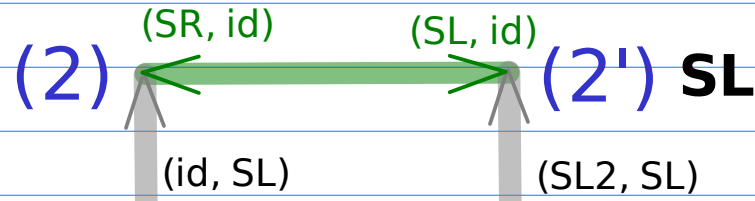
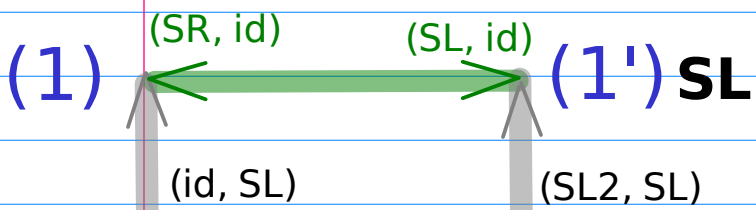
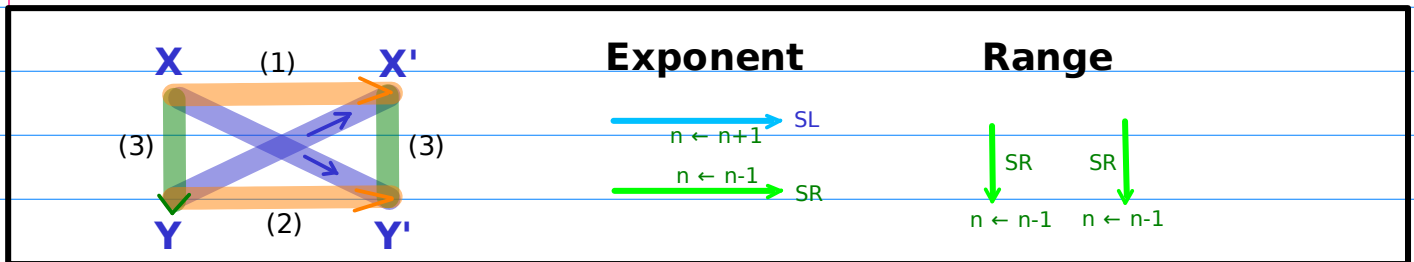
# Butterfly Relations

$(x)$   $\longrightarrow$   $(x')$   
 $(1) \sim (8)$   $\longrightarrow$   $(1') \sim (8')$



# Butterfly Relations

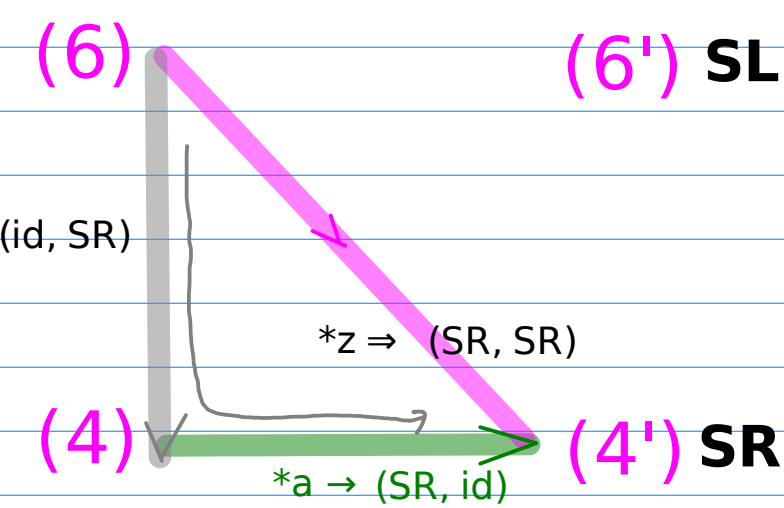
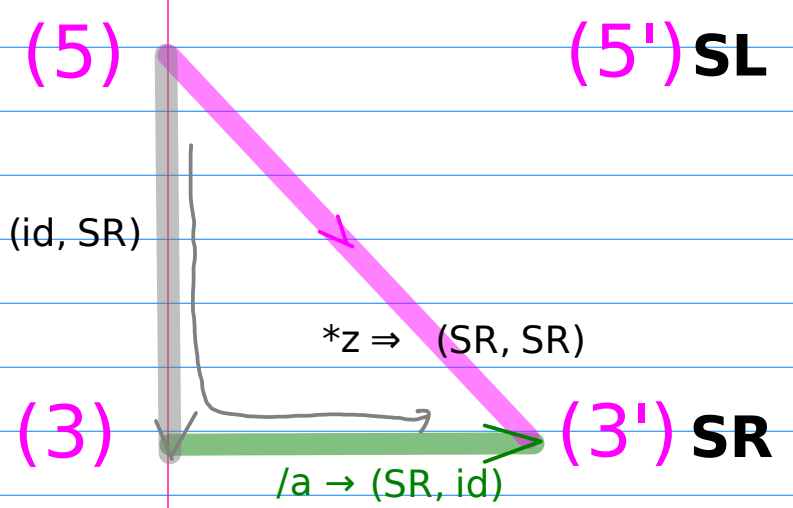
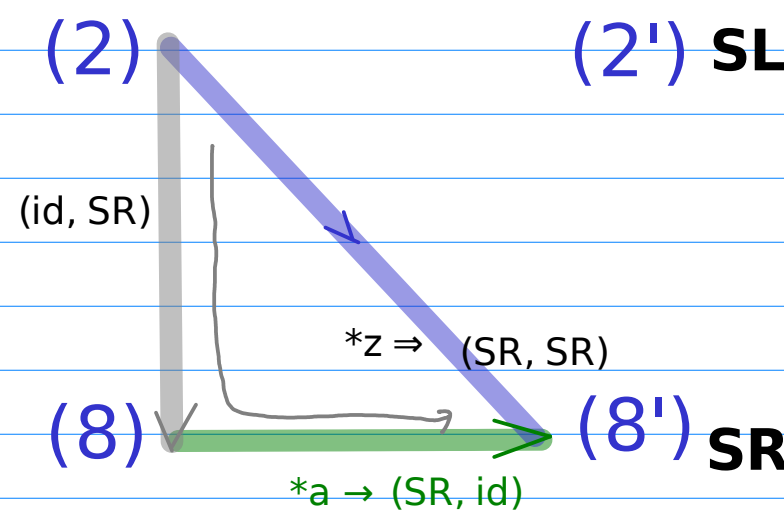
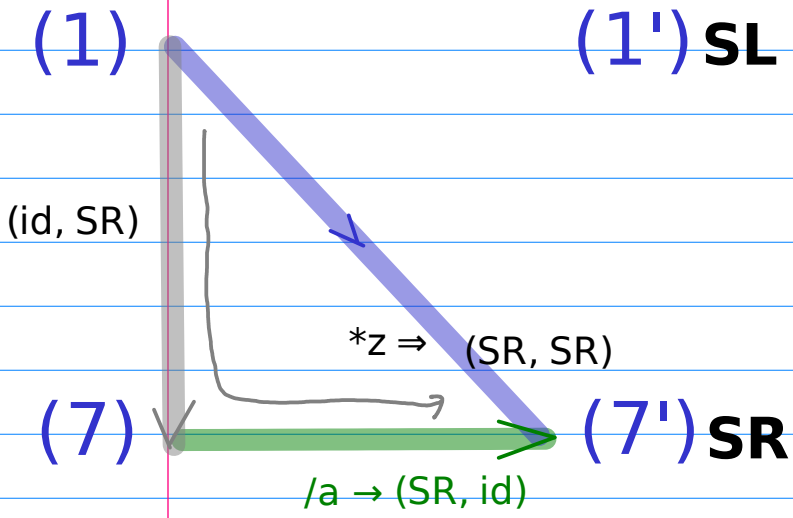
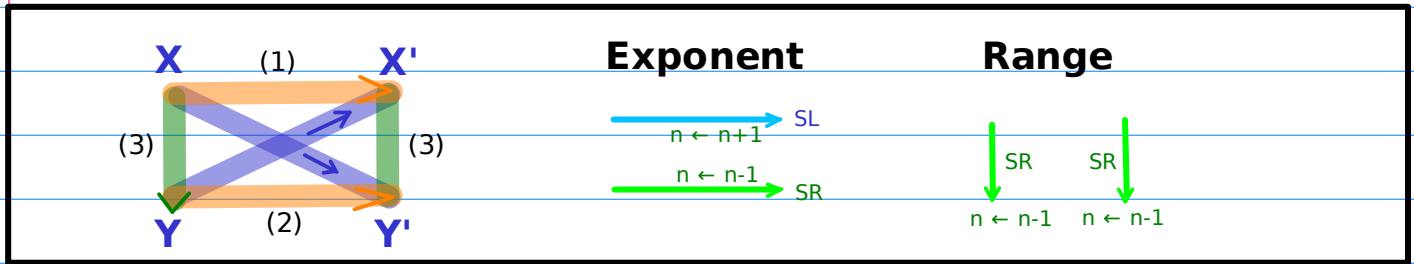
$(x)$   $\longrightarrow$   $(x')$   
 $(1) \sim (8)$   $\longrightarrow$   $(1') \sim (8')$



# Decomposition of Exp and Rng Shifts (1)

$(x) \xrightarrow{\quad} (x')$   
 $(1) \sim (8) \quad (1') \sim (8')$

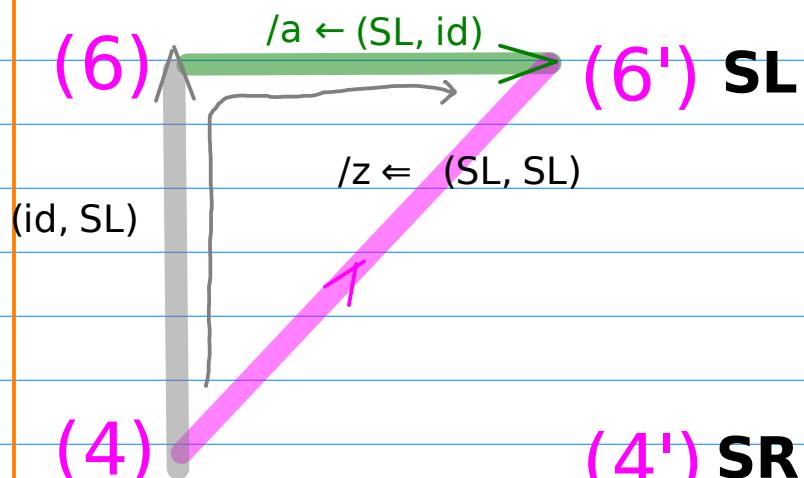
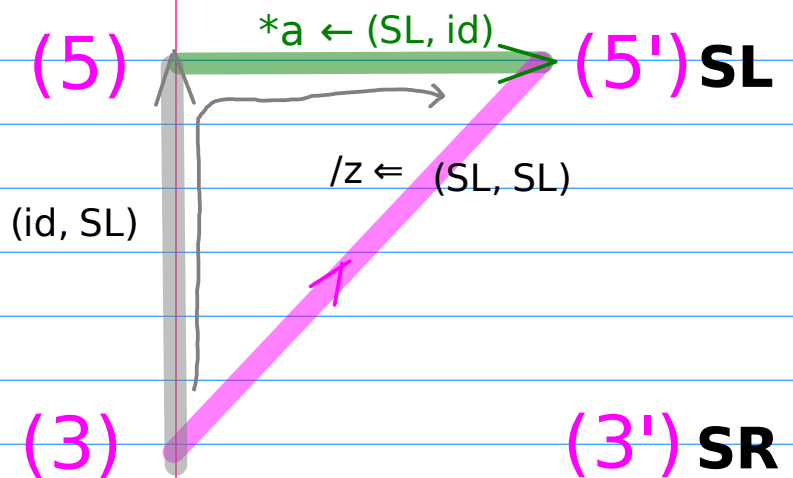
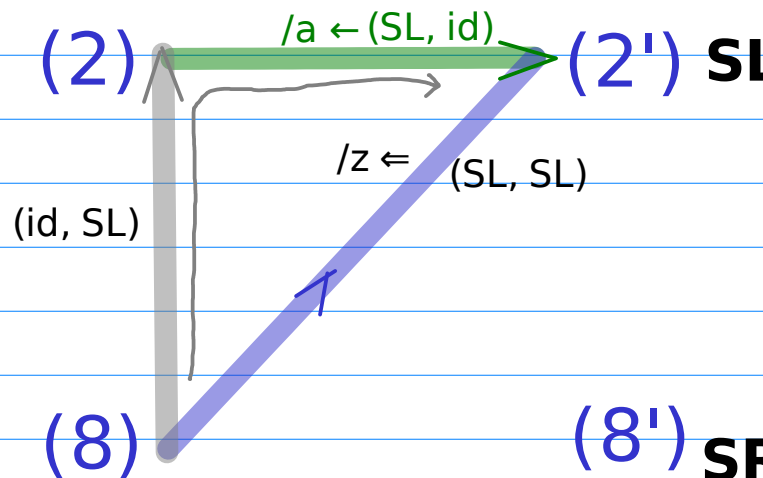
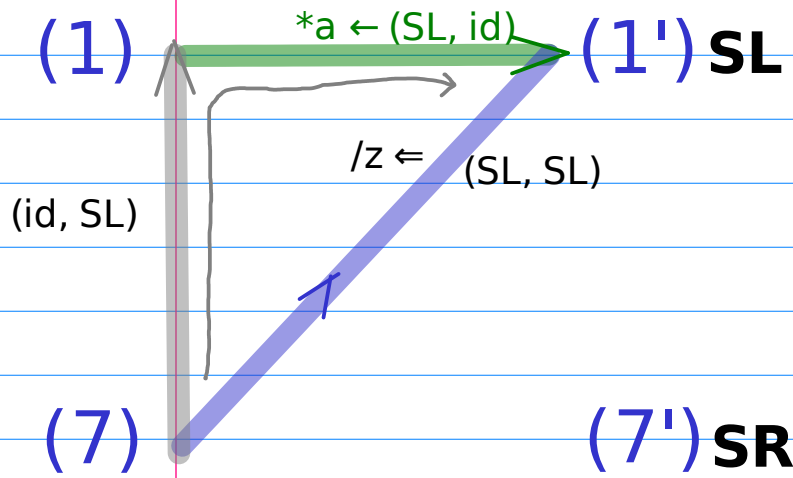
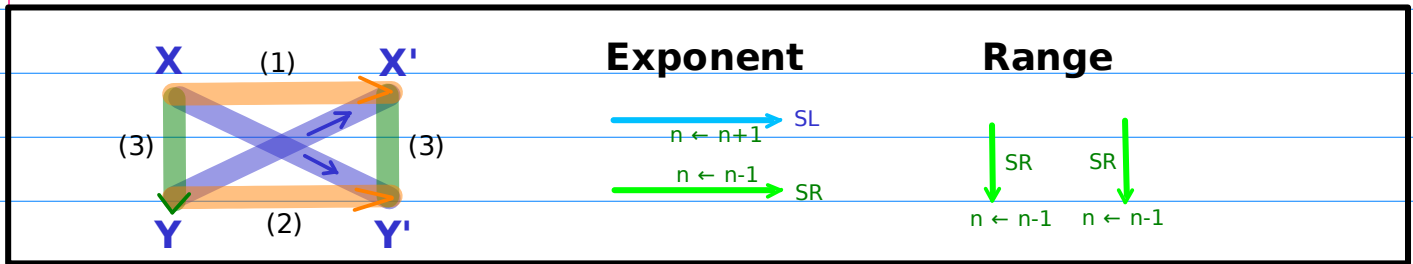
$(id, SR) + (SR, id) = (SR, SR)$



# Decomposition of Exp and Rng Shifts (2)

$(x) \xrightarrow{\quad} (x')$   
 (1)~(8) (1')~(8')

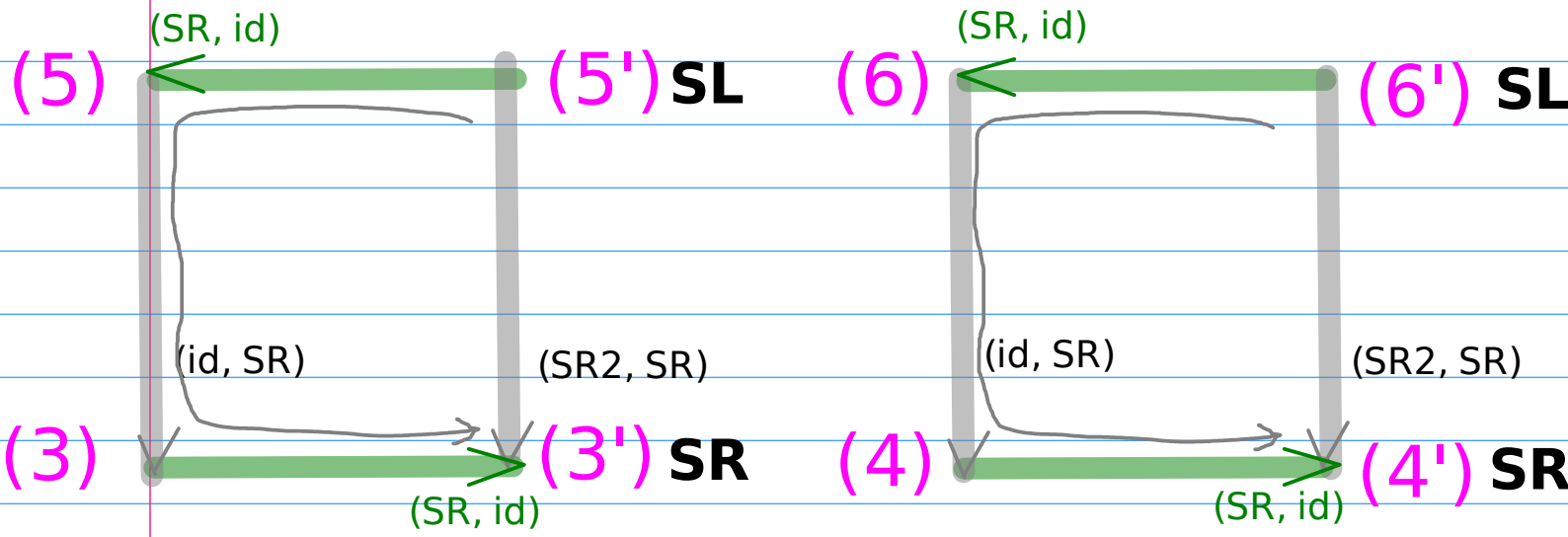
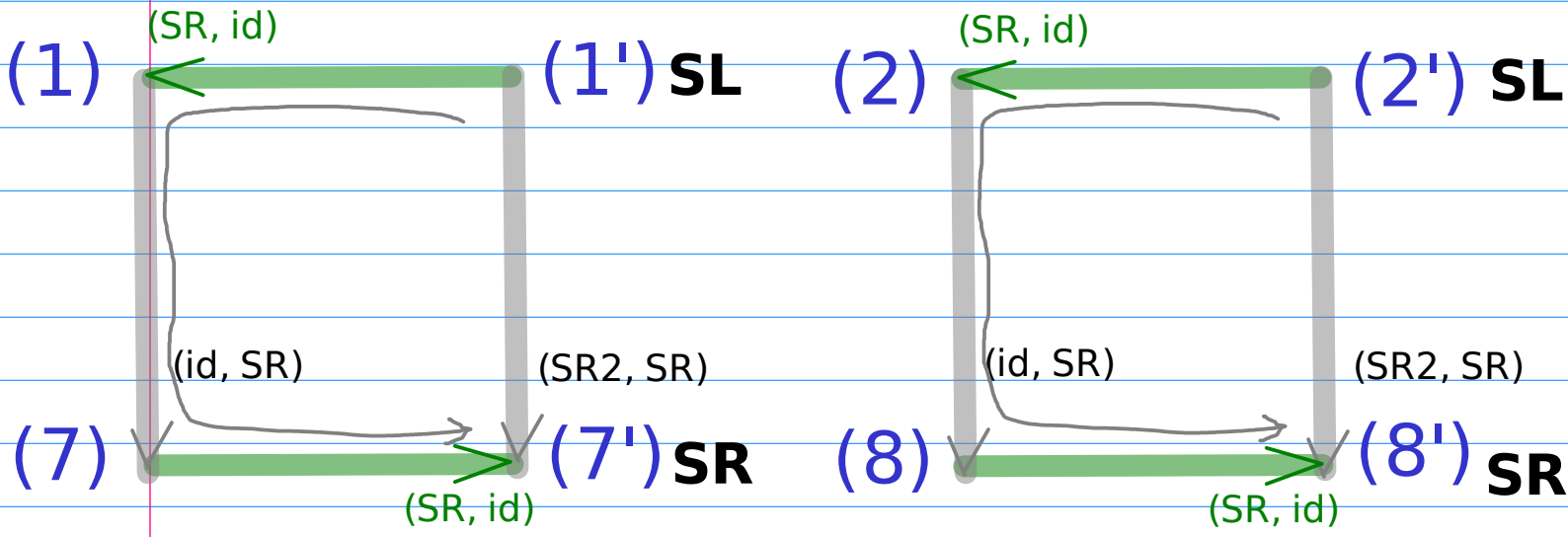
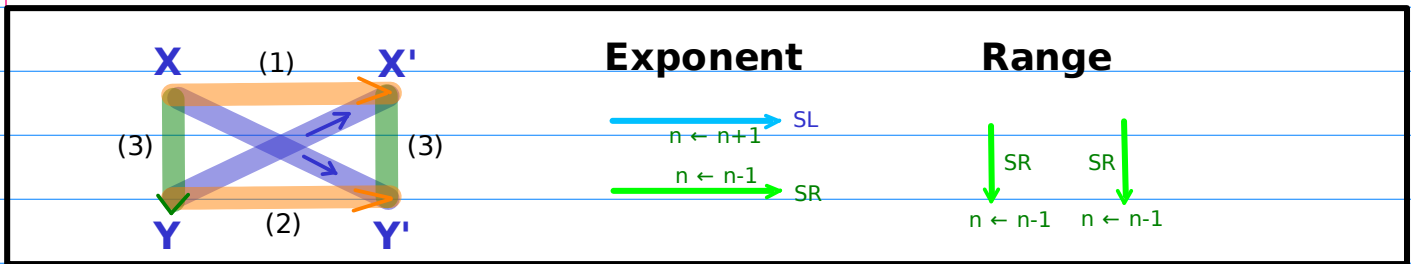
$(id, SL) + (SL, id) = (SL, SL)$



# Decomposition of Exp and Rng Shifts (3)

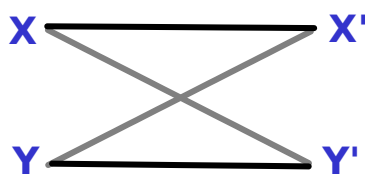
$(X) \rightarrow (X')$   
 $(1) \sim (8) \rightarrow (1') \sim (8')$

$$(SR, id) + (id, SR) + (SR, id) = (SR2, SR)$$



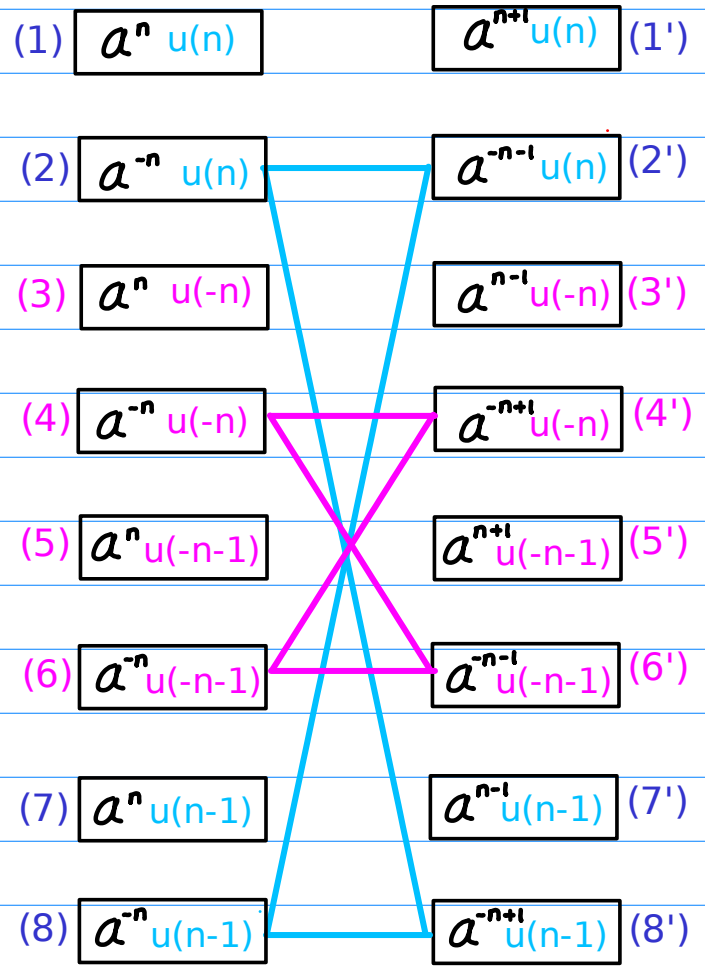
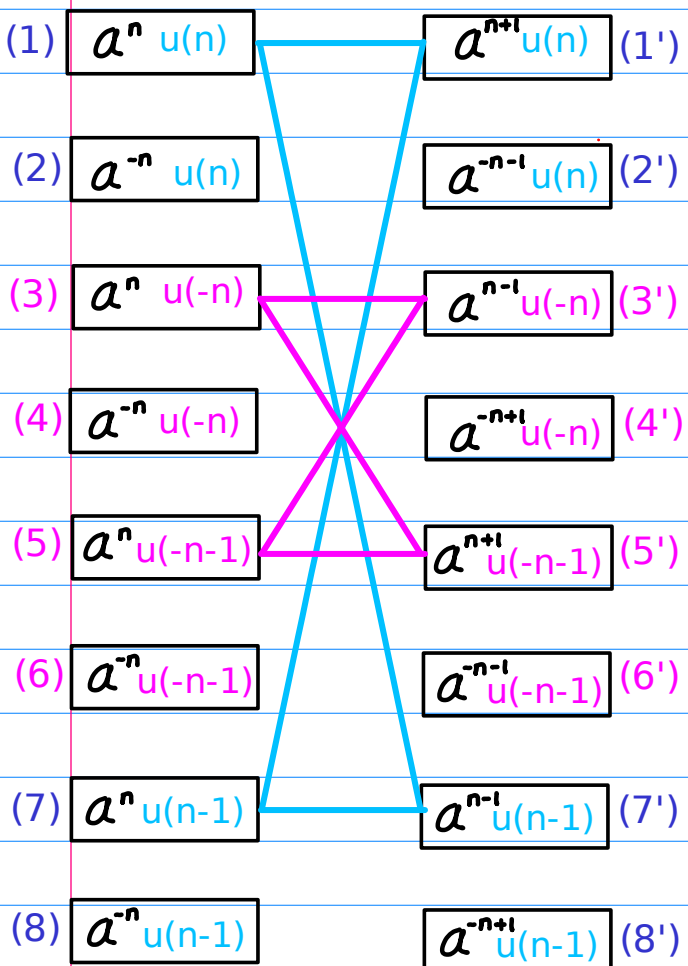
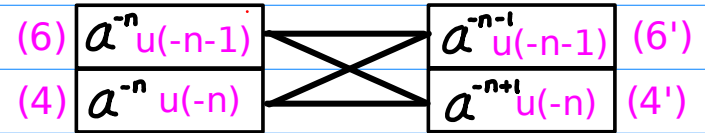
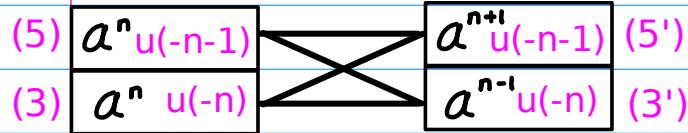
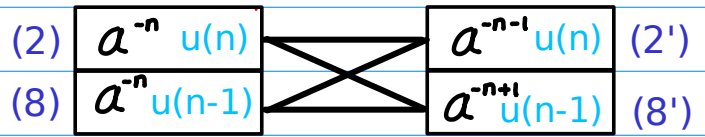
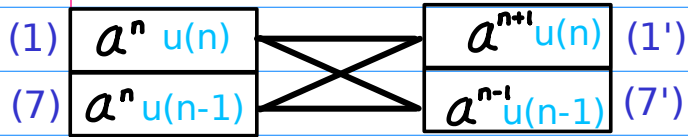


Unshifted Sequence



SL  
SR

Shifted Sequence







# Intra-Permutations over (1) ~ (8)

(1) $a^n u(n)$	$a^{n+i} u(n)$ (1')
(2) $a^{-n} u(n)$	$a^{-n-i} u(n)$ (2')
(3) $a^n u(-n)$	$a^{-i} u(-n)$ (3')
(4) $a^{-n} u(-n)$	$a^{-n+i} u(-n)$ (4')
(5) $a^n u(-n-1)$	$a^{n+i} u(-n-1)$ (5')
(6) $a^{-n} u(-n-1)$	$a^{-n-i} u(-n-1)$ (6')
(7) $a^n u(n-1)$	$a^{-i} u(n-1)$ (7')
(8) $a^{-n} u(n-1)$	$a^{-n+i} u(n-1)$ (8')

## A. Flipping

Base Inverting  
Range Flipping

## B. Range Shifting = Range Flipping + Range Complementing

Range Flipping  
Range Complementing

## C. Complementary Inverting

Base Inverting  
Range Complementing

# Intra-Permutations over (1') ~ (8')

$$(1) \quad a^n u(n)$$

$$(2) \quad a^{-n} u(n)$$

$$(3) \quad a^n u(-n)$$

$$(4) \quad a^{-n} u(-n)$$

$$(5) \quad a^n u(-n-1)$$

$$(6) \quad a^{-n} u(-n-1)$$

$$(7) \quad a^n u(n-1)$$

$$(8) \quad a^{-n} u(n-1)$$

$$a^{n+i} u(n) \quad (1')$$

$$a^{-n-i} u(n) \quad (2')$$

$$a^{n-i} u(-n) \quad (3')$$

$$a^{-n+i} u(-n) \quad (4')$$

$$a^{n+i} u(-n-1) \quad (5')$$

$$a^{-n-i} u(-n-1) \quad (6')$$

$$a^{n-i} u(n-1) \quad (7')$$

$$a^{-n+i} u(n-1) \quad (8')$$

## D. Flipping2

Base Inverting

Shifted Range Flipping = Exponent Shifting2 + Range Flipping

## E. Shifting2 = Exponent Shifting2 + Range Shifting

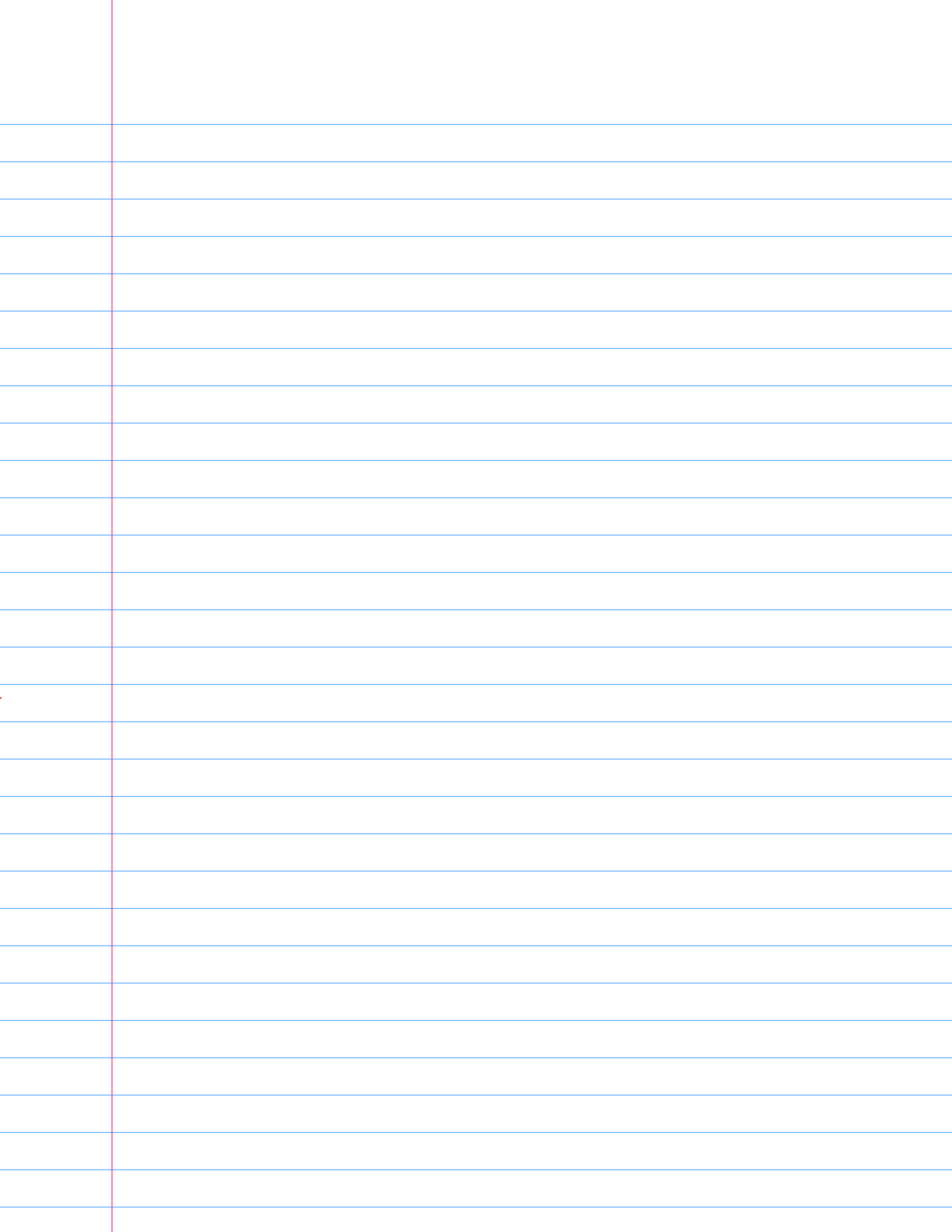
Shifted Range Flipping = Exponent Shifting2 + Range Flipping

Range Complementing

## F. Complementary Inverting

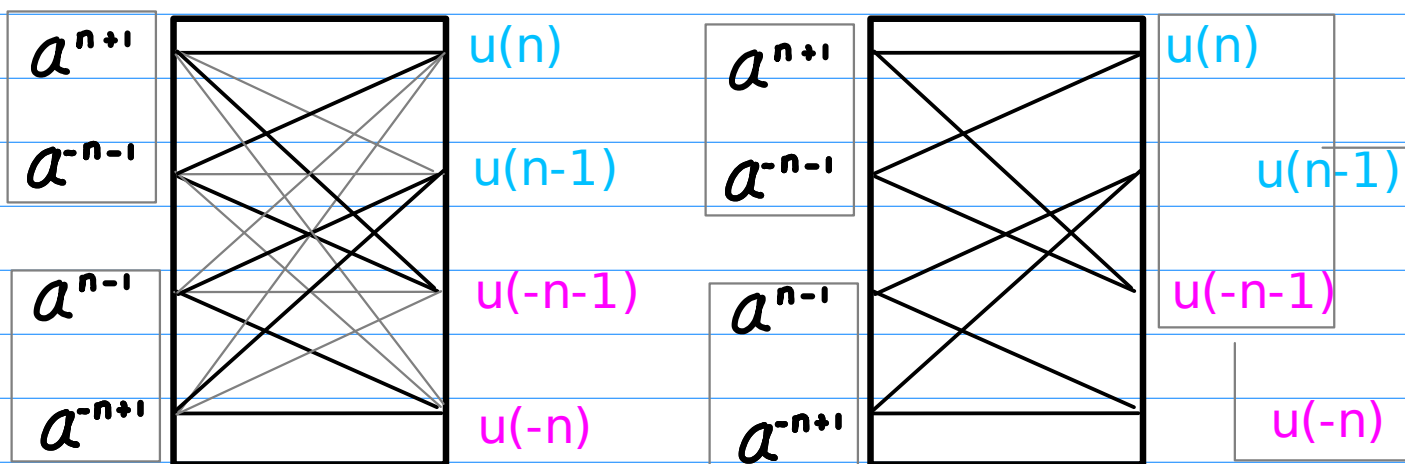
Base Inverting

Range Complementing



$$a^n \times R(n)$$

$a^{n+1}$	$a^{-n-1}$	$\times$	$u(n)$	$u(-n-1)$
$a^{n-1}$	$a^{-n+1}$	$\times$	$u(n-1)$	$u(-n)$



$n \leftarrow n+2$ or
$n \leftarrow n-2$

$(1')$	$a^{n+1}u(n)$	$a^{-n-1}u(n)$	$(2')$
$(3')$	$a^{n-1}u(-n)$	$a^{-n+1}u(-n)$	$(4')$
$(5')$	$a^{n+1}u(-n-1)$	$a^{-n-1}u(-n-1)$	$(6')$
$(7')$	$a^{n-1}u(n-1)$	$a^{-n+1}u(n-1)$	$(8')$

$(1'')$	$a^{n-1}u(n)$	$a^{-n+1}u(n)$	$(2'')$
$(3'')$	$a^{n+1}u(-n)$	$a^{-n-1}u(-n)$	$(4'')$
$(5'')$	$a^{n-1}u(-n-1)$	$a^{-n+1}u(-n-1)$	$(6'')$
$(7'')$	$a^{n+1}u(n-1)$	$a^{-n-1}u(n-1)$	$(8'')$



## permutation over (1) ~ (8)

### A. Flipping

Base Inverting  
Range Flipping

### B. Range Shifting = Range Flipping + Range Complementing

Range Flipping  
Range Complementing

### C. Complementary Inverting

Base Inverting  
Range Complementing

## permutation over (1') ~ (8')

### D. Flipping<sub>2</sub>

Base Inverting  
Shifted Range Flipping = Exponent Shifting<sub>2</sub> + Range Flipping

### E. Shifting<sub>2</sub> = Exponent Shifting<sub>2</sub> + Range Shifting

Shifted Range Flipping = Exponent Shifting<sub>2</sub> + Range Flipping  
Range Complementing

### F. Complementary Inverting

Base Inverting  
Range Complementing

**Range Shifting = Range Flipping + Range Complementing**

**Shifted Range Flipping = Exponent Shifting<sub>2</sub> + Range Flipping**

**Shifting<sub>2</sub> = Shifted Range Flipping + Range Complementing**

**= Exponent Shifting<sub>2</sub> + Range (Flipping+Complementing)**

**= Exponent Shifting<sub>2</sub> + Range Shifting**

## permutation over (1) ~ (8)

<b>A. Flipping</b>	(1) - (4)	(5) - (8)
<b>Base Inverting</b>	(2) - (3)	(6) - (7)
<b>Range Flipping</b>	(3) - (2)	(7) - (6)
	(4) - (1)	(8) - (5)
<b>B. Range Shifting</b>	(1) - (7)	(5) - (3)
<b>Range Flipping</b>	(2) - (8)	(6) - (4)
<b>Range Complementing</b>	(3) - (5)	(7) - (1)
	(4) - (6)	(8) - (2)
<b>C. Complementary Inverting</b>	(1) - (6)	(5) - (2)
<b>Base Inverting</b>	(6) - (1)	(6) - (1)
<b>Range Complementing</b>	(2) - (5)	(7) - (4)
	(5) - (2)	(8) - (3)

## permutation over (1') ~ (8')

<b>D. Flipping2</b>	(1') - (4')	(5') - (8')
<b>Base Inverting</b>	(2') - (3')	(6') - (7')
<b>Shifted Range Flipping</b>	(3') - (2')	(7') - (6')
	(4') - (1')	(8') - (5')
<b>E. Shifting2</b>	(1') - (7')	(5') - (3')
<b>Shifted Range Flipping</b>	(2') - (8')	(6') - (4')
<b>Range Complementing</b>	(3') - (5')	(7') - (1')
	(4') - (6')	(8') - (2')
<b>F. Complementary Inverting</b>	(1') - (6')	(5') - (2')
<b>Base Inverting</b>	(6') - (1')	(6') - (1')
<b>Range Complementing</b>	(2') - (5')	(7') - (4')
	(5') - (2')	(8') - (3')





## permutation over (1) ~ (8)

	A	B	C
Base Inverting	X		X
Range Flipping	X	X	
Range Complementing		X	X

## permutation over (1') ~ (8')

	D	E	F
Base Inverting	X		X
Shifted Range Flipping	X	X	
Range Complementing		X	X

**Range Shifting = Range Flipping + Range Complementing**  
**Shifted Range Flipping = Exponent Shifting2 + Range Flipping**

**Shifting2 = Shifted Range Flipping + Range Complementing**  
**= Exponent Shifting2 + Range (Flipping+Complementing)**  
**= Exponent Shifting2 + Range Shifting**



**Over (1) ~ (8)**

**Base Inverting**

$$a^n \longleftrightarrow a^{-n}$$

**Range Flipping**

$$R(n) \longleftrightarrow R(-n)$$

**Range Complementing**

$$R(n) \longleftrightarrow \overline{R(n)}$$

**Over (1') ~ (8')**

**Base Inverting**

$$a^n \longleftrightarrow a^{-n}$$

**Shifted Range Flipping**

$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

**Range Complementing**

$$R(n) \longleftrightarrow \overline{R(n)}$$

**A.I Flipping**  
**Base Inverting**  
**Range Flipping**

$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow R(-n)$$

$$a^n R(n) \longleftrightarrow a^{-n} R(-n)$$

**D.I Flipping2**  
**Base Inverting**  
**Shifted Range Flipping**

$$a^n \longleftrightarrow a^{-n}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

$$a^n R(n) \longleftrightarrow a^{-sh2(n)} R(-n)$$

**B.I Range Shifting**  
**Range Flipping**  
**Range Complementing**

$$R(n) \longleftrightarrow R(-n)$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$R(n) \longleftrightarrow \overline{R(-n)}$$

**E.I Shifting2**  
**Shifted Range Flipping**  
**Range Complementing**

$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

**C.I Complementary Inverting**  
**Base Inverting**  
**Range Complementing**

$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

**F.I Complementary Inverting**  
**Base Inverting**  
**Range Complementing**

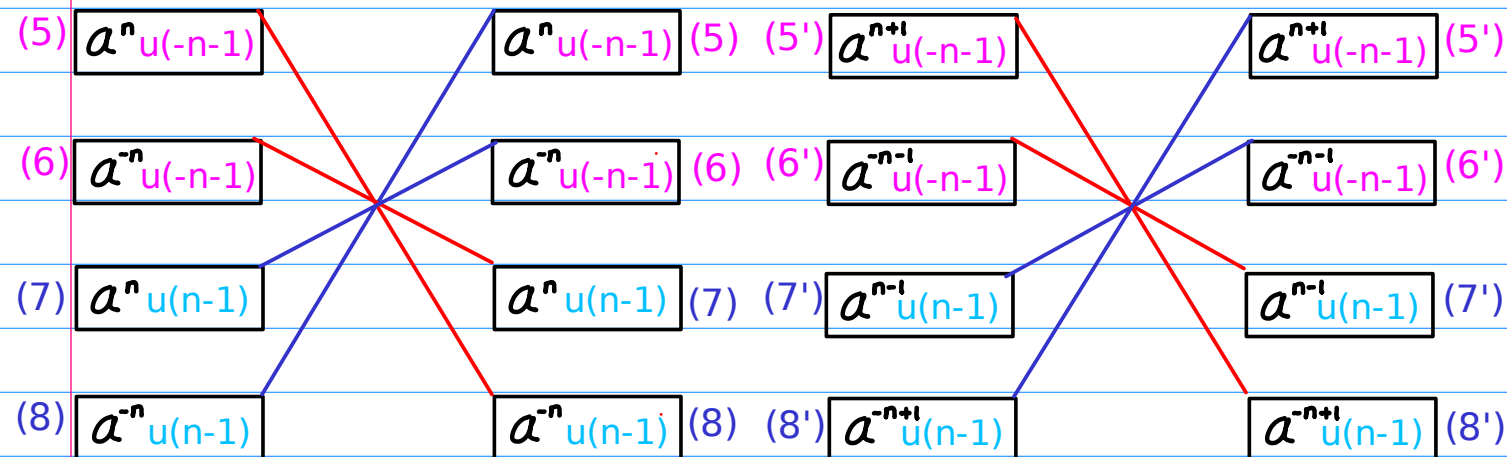
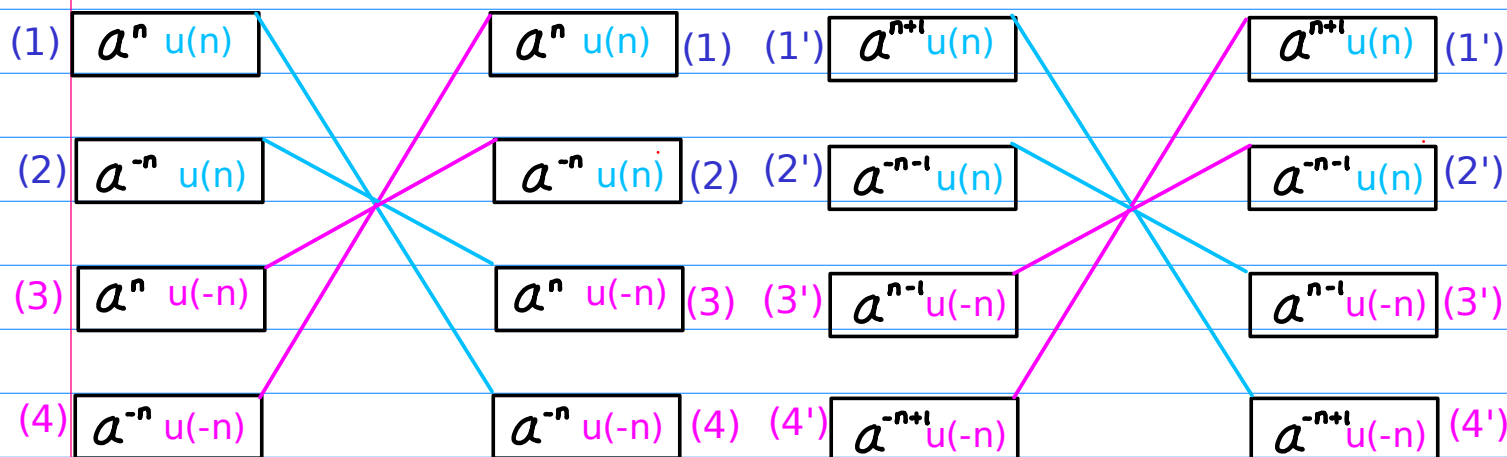
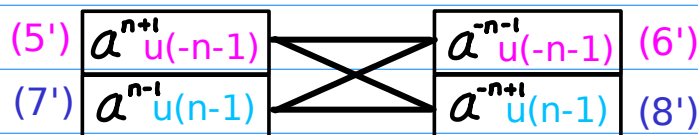
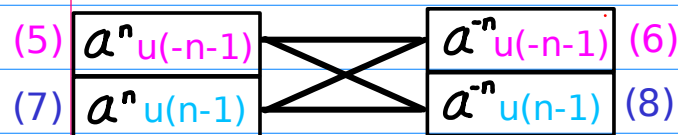
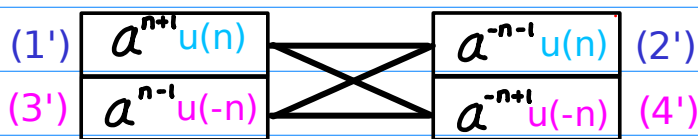
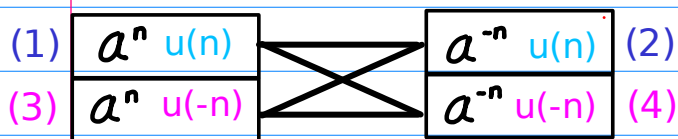
$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

**A.I Flipping**  
**Base Inverting**  
**Range Flipping**

**D.I Flipping2**  
**Base Inverting**  
**Shifted Range Flipping**

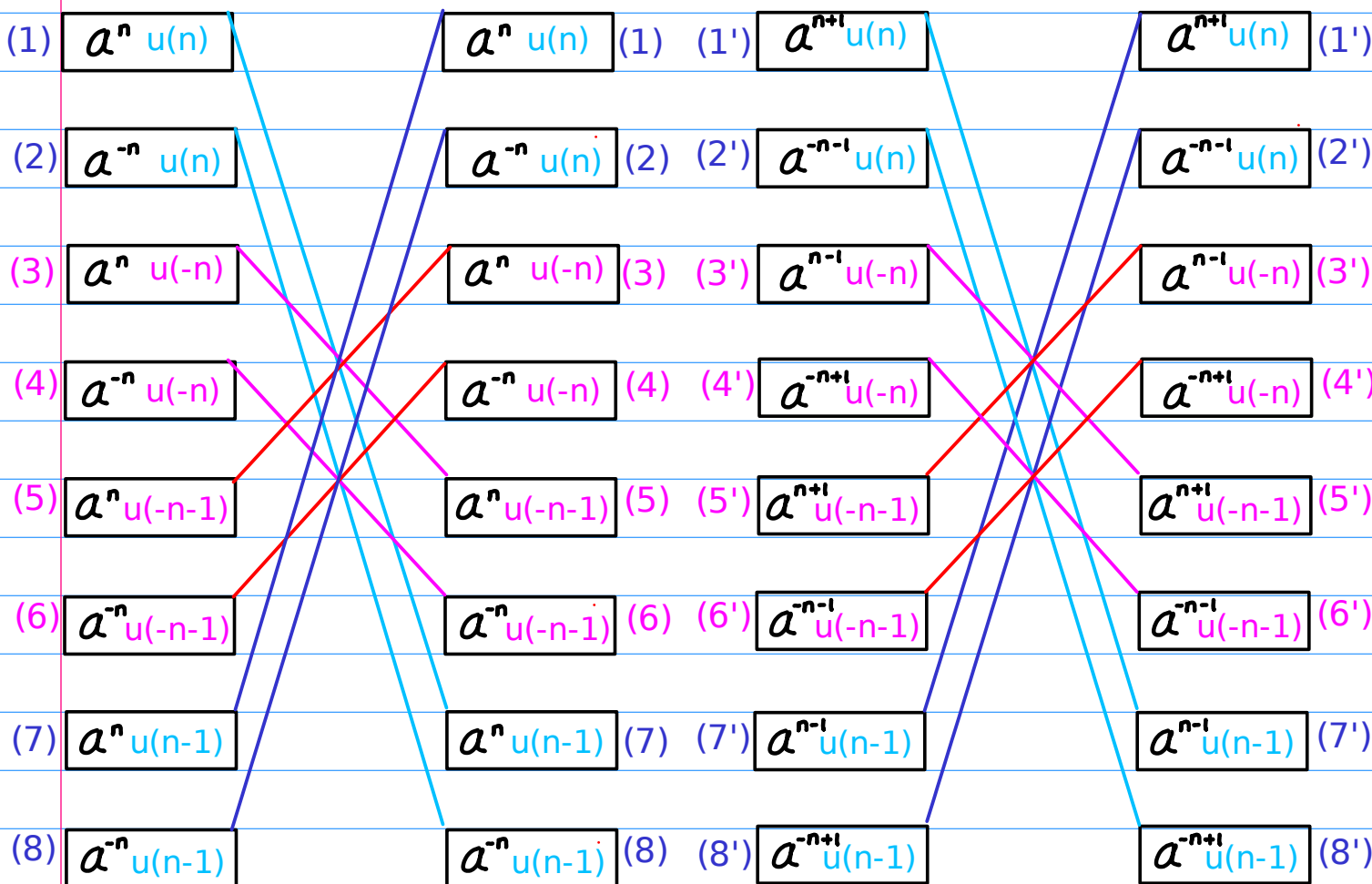
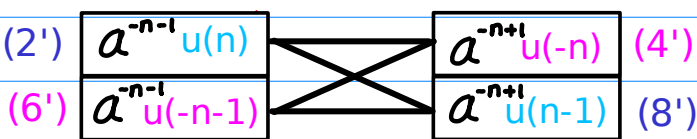
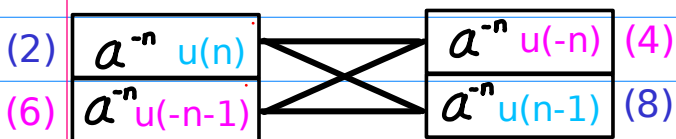
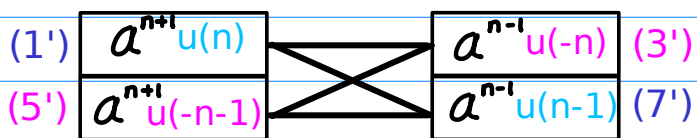
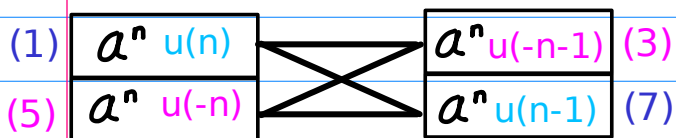


(1) - (4)      (5) - (8)  
 (2) - (3)      (6) - (7)  
 (3) - (2)      (7) - (6)  
 (4) - (1)      (8) - (5)

(1') - (4')      (5') - (8')  
 (2') - (3')      (6') - (7')  
 (3') - (2')      (7') - (6')  
 (4') - (1')      (8') - (5')

**B.I Range Shifting  
Range Flipping  
Range Complementing**

**E.I Shifting2  
Shifted Range Flipping  
Range Complementing**

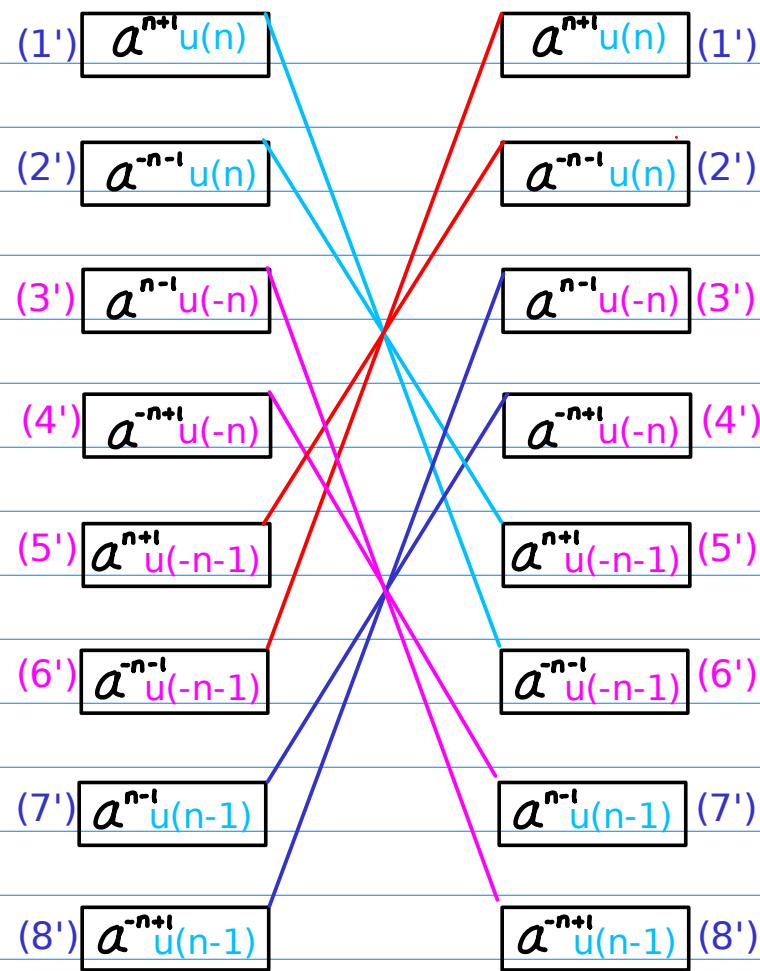
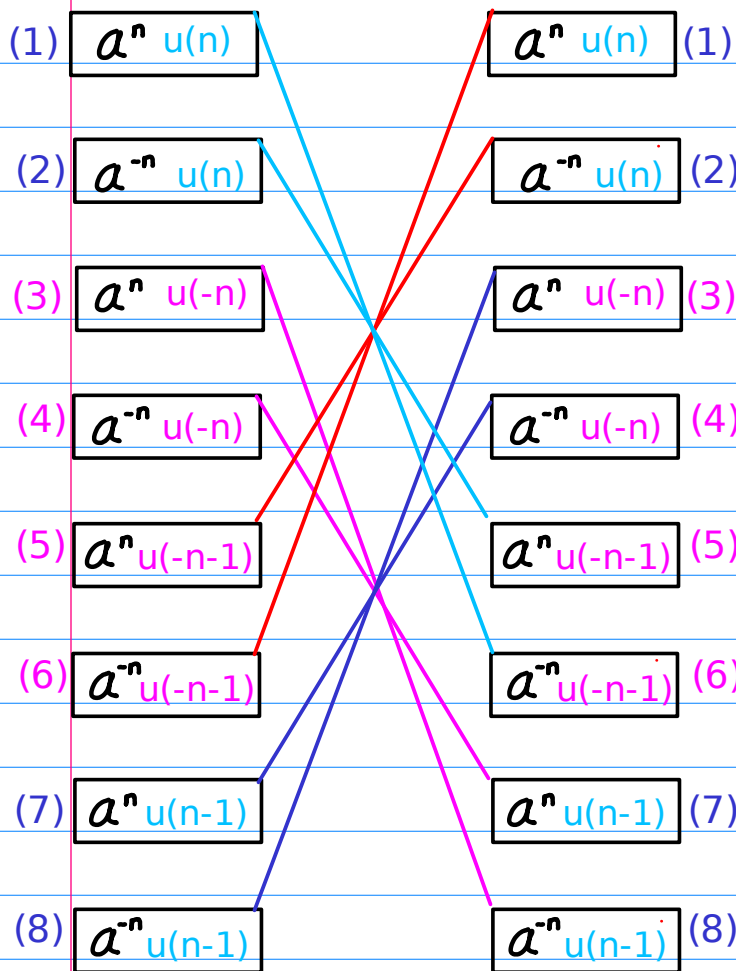
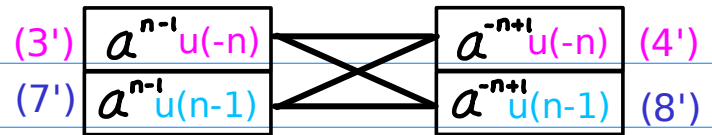
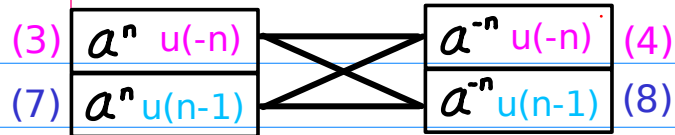
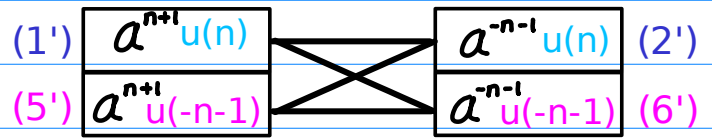
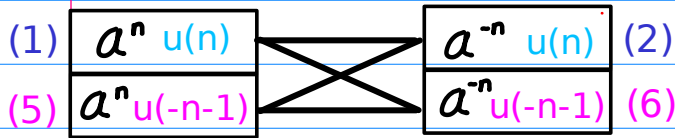


(1) - (7)      (5) - (3)  
 (2) - (8)      (6) - (4)  
 (3) - (5)      (7) - (1)  
 (4) - (6)      (8) - (2)

(1') - (7')      (5') - (3')  
 (2') - (8')      (6') - (4')  
 (3') - (5')      (7') - (1')  
 (4') - (6')      (8') - (2')

### C.I Complementary Inverting Base Inverting Range Complementing

### F.I Complementary Inverting Base Inverting Range Complementing



- (1) - (6)
- (6) - (1)
- (2) - (5)
- (5) - (2)
- (5) - (2)
- (3) - (4)
- (4) - (3)
- (7) - (8)
- (8) - (7)

- (1') - (6')
- (6') - (1')
- (2') - (5')
- (5') - (2')
- (3') - (4')
- (4') - (3')
- (7') - (8')
- (8') - (7')



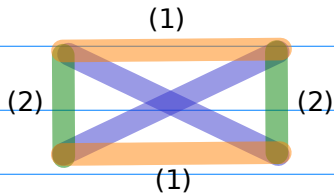


## A.I Flipping

- (1) Base Inverting
- (2) Range Flipping

$$\begin{array}{|c|} \hline (1) \ a^n u(n) \\ \hline (3) \ a^n u(-n) \\ \hline \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{|c|} \hline a^{-n} u(n) \quad (2) \\ \hline a^{-n} u(-n) \quad (4) \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline (5) \ a^n u(-n-1) \\ \hline (7) \ a^n u(n-1) \\ \hline \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{|c|} \hline a^{-n} u(-n-1) \quad (6) \\ \hline a^{-n} u(n-1) \quad (8) \\ \hline \end{array}$$

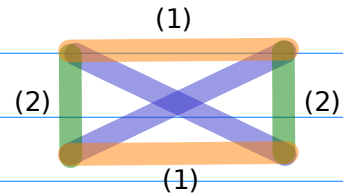


## D.I Flipping2

- (1) Base Inverting
- (2) Shifted Range Flipping

$$\begin{array}{|c|} \hline (1') \ a^{n+1} u(n) \\ \hline (3') \ a^{n-1} u(-n) \\ \hline \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{|c|} \hline a^{-n-1} u(n) \quad (2') \\ \hline a^{-n+1} u(-n) \quad (4') \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline (5') \ a^{n+1} u(-n-1) \\ \hline (7') \ a^{n-1} u(n-1) \\ \hline \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{|c|} \hline a^{-n-1} u(-n-1) \quad (6') \\ \hline a^{-n+1} u(n-1) \quad (8') \\ \hline \end{array}$$



$$\begin{array}{ccc} a^n & \xleftrightarrow{(1)} & a^{-n} \\ R(n) & \xleftrightarrow{(2)} & R(-n) \\ a^n R(n) & \longleftrightarrow & a^{-n} R(-n) \end{array}$$

$$\begin{array}{ccc} a^n & \xleftrightarrow{(1)} & a^{-n} \\ a^n R(n) & \xleftrightarrow{(2)} & a^{sh2(n)} R(-n) \\ a^n R(n) & \longleftrightarrow & a^{-sh2(n)} R(-n) \end{array}$$

$b^n$	$b^{-n}$
$a^n$	$a^{-n}$
$a^{-n}$	$a^n$

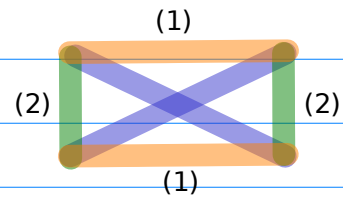
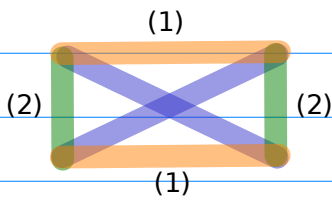
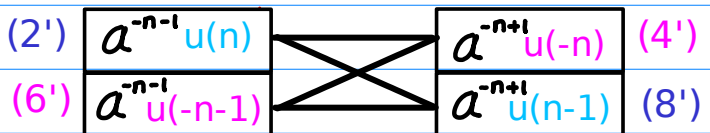
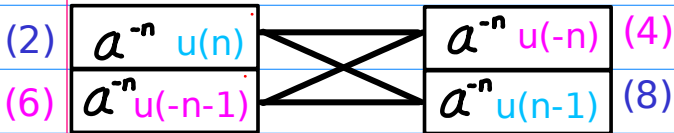
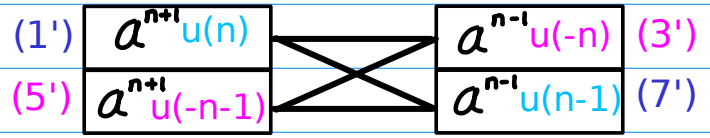
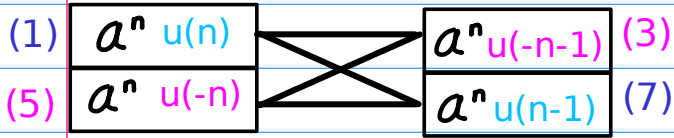
$b^n$	$b^{-sh2(n)}$
$a^{(n+1)}$	$a^{-(n-1)}$
$a^{-(n+1)}$	$a^{(n-1)}$
$a^{(n-1)}$	$a^{-(n+1)}$
$a^{-(n-1)}$	$a^{(n+1)}$

$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

**B.I Range Shifting**  
**(1) Range Complementing**  
**(2) Range Flipping**

**E.I Shifting2**  
**(1) Shifted Range Flipping**  
**(2) Range Complementing**



$$R(n) \xleftrightarrow{(1)} R(-n)$$

$$R(n) \xleftrightarrow{(2)} \overline{R(n)}$$

$$R(n) \longleftrightarrow \overline{R(-n)}$$

$$a^n R(n) \xleftrightarrow{(1)} a^{sh2(n)} \overline{R(-n)}$$

$$R(n) \xleftrightarrow{(2)} \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

$b^n$	$b^{sh2(n)}$
$a^{(n+1)}$	$a^{(n-1)}$
$a^{-(n+1)}$	$a^{-(n-1)}$
$a^{(n-1)}$	$a^{(n+1)}$
$a^{-(n-1)}$	$a^{-(n+1)}$

$R(n)$	$\overline{R(-n)}$
$u(n)$	$u(n-1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-n-1)$
$u(-n-1)$	$u(-n)$

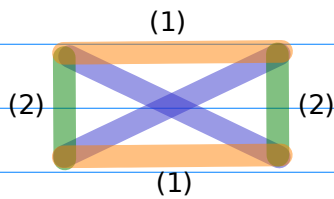
$R(n)$	$\overline{R(-n)}$
$u(n)$	$u(n-1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-n-1)$
$u(-n-1)$	$u(-n)$

## C.I Complementary Inverting

(1) Base Inverting  
(2) Range Complementing

$$\begin{array}{|c|} \hline (1) \ a^n u(n) \\ \hline (5) \ a^n u(-n-1) \\ \hline \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{|c|} \hline a^{-n} u(n) \quad (2) \\ \hline a^{-n} u(-n-1) \quad (6) \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline (3) \ a^n u(-n) \\ \hline (7) \ a^n u(n-1) \\ \hline \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{|c|} \hline a^{-n} u(-n) \quad (4) \\ \hline a^{-n} u(n-1) \quad (8) \\ \hline \end{array}$$



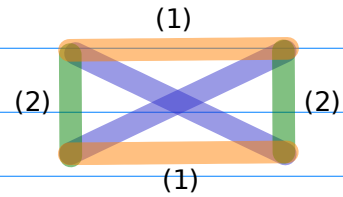
$$\begin{array}{ccc} a^n & \xleftrightarrow{(1)} & a^{-n} \\ R(n) & \xleftrightarrow{(2)} & \overline{R(n)} \\ a^n R(n) & \longleftrightarrow & a^{-n} \overline{R(n)} \end{array}$$

## F.I Complementary Inverting

(1) Base Inverting  
(2) Range Complementing

$$\begin{array}{|c|} \hline (1') \ a^{n+1} u(n) \\ \hline (5') \ a^{n+1} u(-n-1) \\ \hline \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{|c|} \hline a^{-n-1} u(n) \quad (2') \\ \hline a^{-n-1} u(-n-1) \quad (6') \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline (3') \ a^{n-1} u(-n) \\ \hline (7') \ a^{n-1} u(n-1) \\ \hline \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{|c|} \hline a^{-n+1} u(-n) \quad (4') \\ \hline a^{-n+1} u(n-1) \quad (8') \\ \hline \end{array}$$



$$\begin{array}{ccc} a^n & \xleftrightarrow{(1)} & a^{-n} \\ R(n) & \xleftrightarrow{(2)} & \overline{R(n)} \\ a^n R(n) & \longleftrightarrow & a^{-n} \overline{R(n)} \end{array}$$

$b^n$	$b^{-n}$
$a^n$	$a^{-n}$
$a^{-n}$	$a^n$

$b^n$	$b^{-n}$
$a^{(n+1)}$	$a^{-(n+1)}$
$a^{-(n+1)}$	$a^{(n+1)}$
$a^{(n-1)}$	$a^{-(n-1)}$
$a^{-(n-1)}$	$a^{(n-1)}$

$R(n)$	$\overline{R(n)}$
$u(n)$	$u(-n-1)$
$u(n-1)$	$u(-n)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

$R(n)$	$\overline{R(n)}$
$u(n)$	$u(-n-1)$
$u(n-1)$	$u(-n)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

## A.I Flipping

- (1) Base Inverting
- (2) Range Flipping

$$\begin{array}{ccc}
 a^n & \xleftrightarrow{(1)} & a^{-n} \\
 R(n) & \xleftrightarrow{(2)} & R(-n) \\
 a^n R(n) & \longleftrightarrow & a^{-n} R(-n)
 \end{array}$$

## D.I Flipping2

- (1) Base Inverting
- (2) Shifted Range Flipping

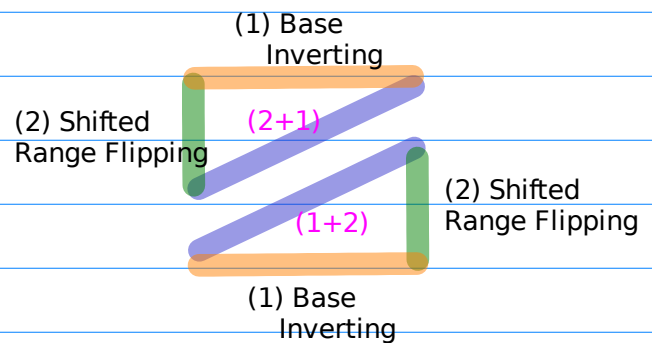
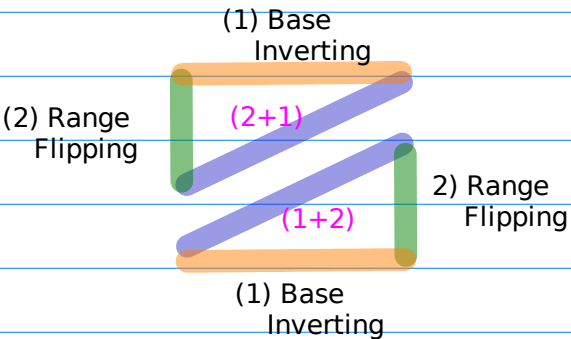
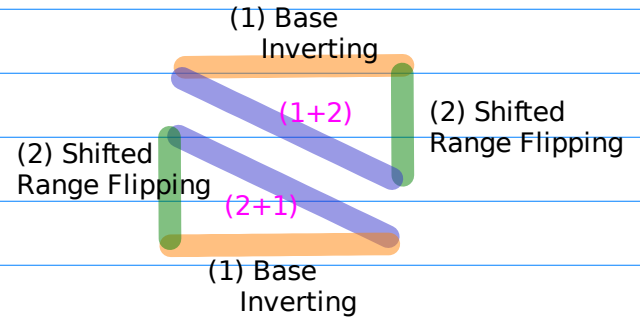
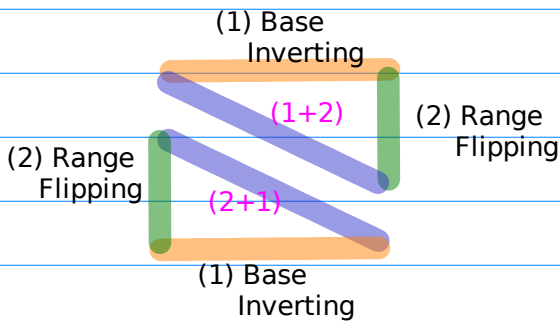
$$\begin{array}{ccc}
 a^n & \xleftrightarrow{(1)} & a^{-n} \\
 a^n R(n) & \xleftrightarrow{(2)} & a^{sh2(n)} R(-n) \\
 a^n R(n) & \longleftrightarrow & a^{-sh2(n)} R(-n)
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(1)} & a^{-n} R(n) \\
 & \xrightarrow{(2)} & a^{-n} R(-n)
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(1)} & a^{-n} R(n) \\
 & \xrightarrow{(2)} & a^{-sh2(n)} R(-n)
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(2)} & a^n R(-n) \\
 & \xrightarrow{(1)} & a^{-n} R(-n)
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(2)} & a^{sh2(n)} R(-n) \\
 & \xrightarrow{(1)} & a^{-sh2(n)} R(-n)
 \end{array}$$



**B.I Range Shifting**  
**(1) Range Complementing**  
**(2) Range Flipping**

**E.I Shifting2**  
**(1) Shifted Range Flipping**  
**(2) Range Complementing**

$$R(n) \xleftrightarrow{(1)} R(-n)$$

$$R(n) \xleftrightarrow{(2)} \overline{R(n)}$$

$$R(n) \longleftrightarrow \overline{R(-n)}$$

$$a^n R(n) \xleftrightarrow{(1)} a^{sh2(n)} R(-n)$$

$$R(n) \xleftrightarrow{(2)} \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

$$a^n R(n) \xrightarrow{(1)} a^{-n} R(n)$$

$$\xrightarrow{(2)} a^{-n} \overline{R(n)}$$

$$a^n R(n) \xrightarrow{(1)} a^{-n} R(n)$$

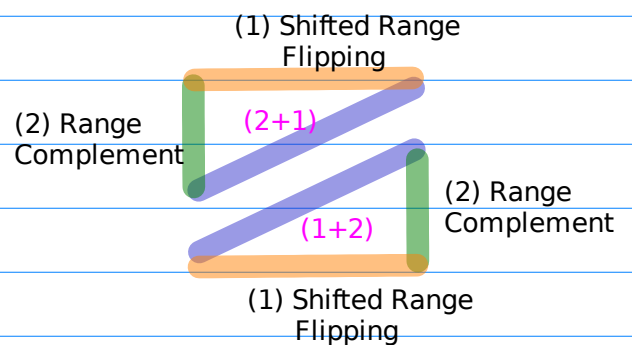
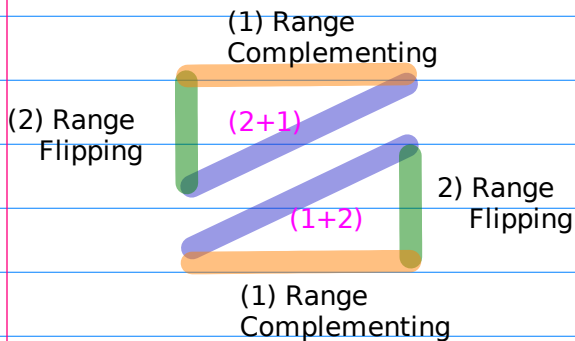
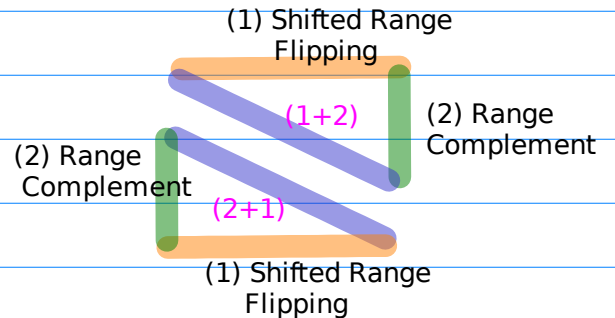
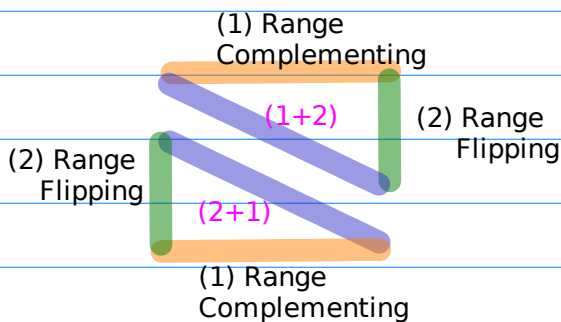
$$\xrightarrow{(2)} a^{-sh2(n)} R(-n)$$

$$a^n rng(n) \xrightarrow{(2)} a^n \overline{R(n)}$$

$$\xrightarrow{(1)} a^{-n} \overline{R(n)}$$

$$a^n R(n) \xrightarrow{(2)} a^{sh2(n)} R(-n)$$

$$\xrightarrow{(1)} a^{-sh2(n)} R(-n)$$



## C.I Complementary Inverting

(1) Base Inverting  
(2) Range Complementing

## F.I Complementary Inverting

(1) Base Inverting  
(2) Range Complementing

$$\begin{array}{ccc}
 a^n & \xleftrightarrow{(1)} & a^{-n} \\
 R(n) & \xleftrightarrow{(2)} & \overline{R(n)} \\
 a^n R(n) & \xleftrightarrow{\quad} & a^{-n} \overline{R(n)}
 \end{array}$$

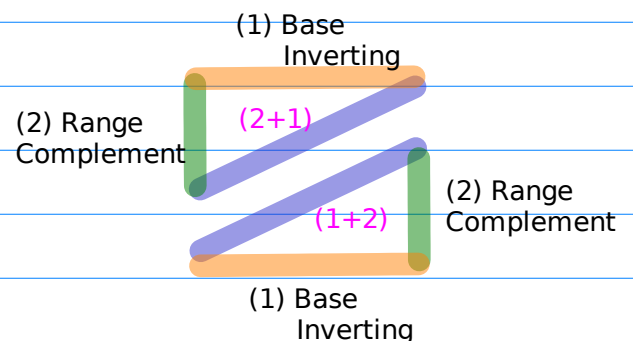
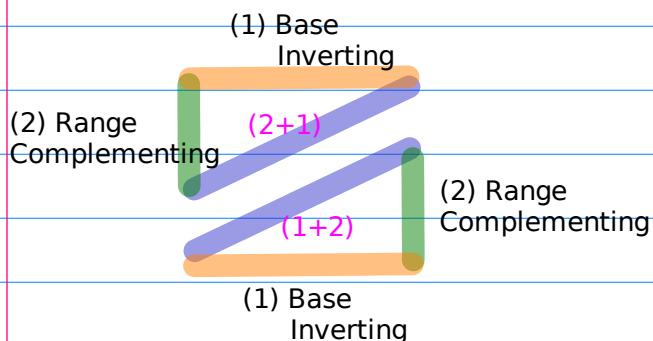
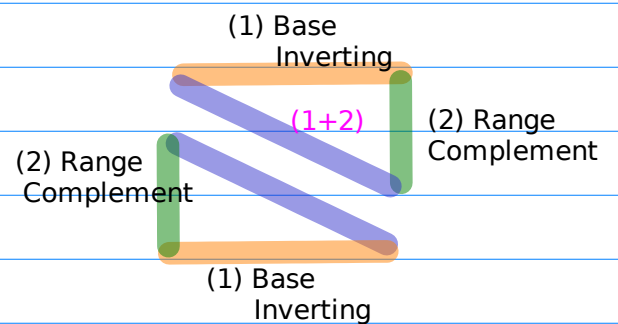
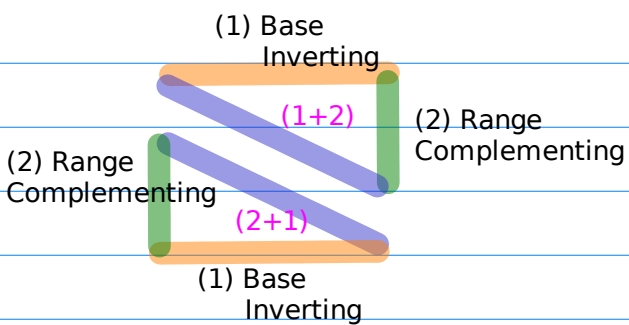
$$\begin{array}{ccc}
 a^n & \xleftrightarrow{(1)} & a^{-n} \\
 R(n) & \xleftrightarrow{(2)} & \overline{R(n)} \\
 a^n R(n) & \xleftrightarrow{\quad} & a^{-n} \overline{R(n)}
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(1)} & a^{-n} R(n) \\
 & \xrightarrow{(2)} & a^{-n} \overline{R(n)}
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(1)} & a^{-n} R(n) \\
 & \xrightarrow{(2)} & a^{-n} \overline{R(n)}
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(2)} & a^n \overline{R(n)} \\
 & \xrightarrow{(1)} & a^{-n} \overline{R(n)}
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(2)} & a^n \overline{R(n)} \\
 & \xrightarrow{(1)} & a^{-n} \overline{R(n)}
 \end{array}$$

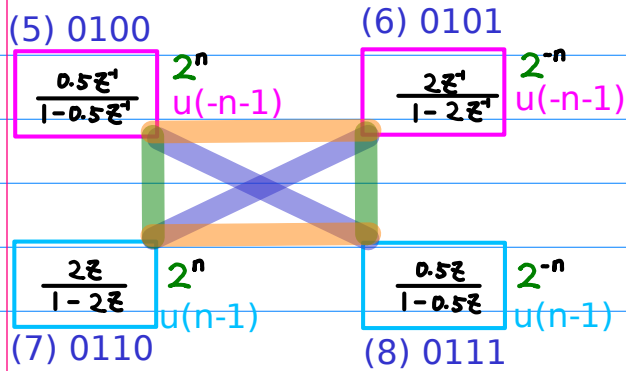
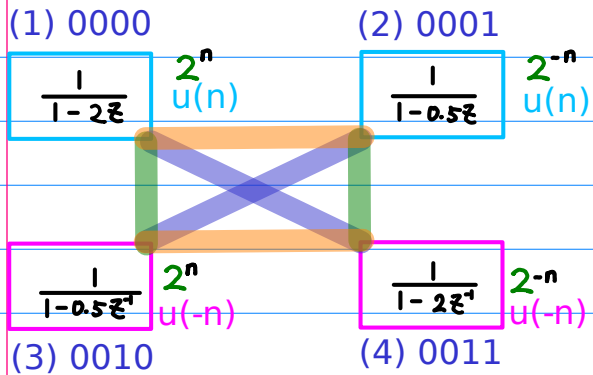




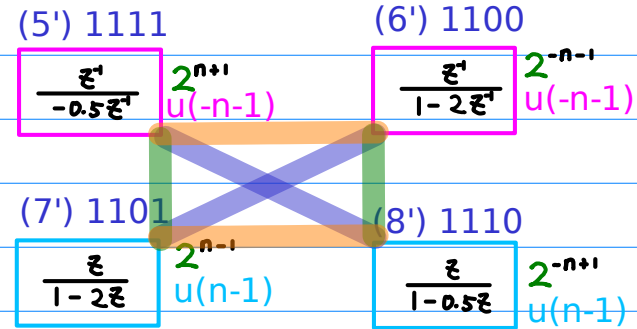
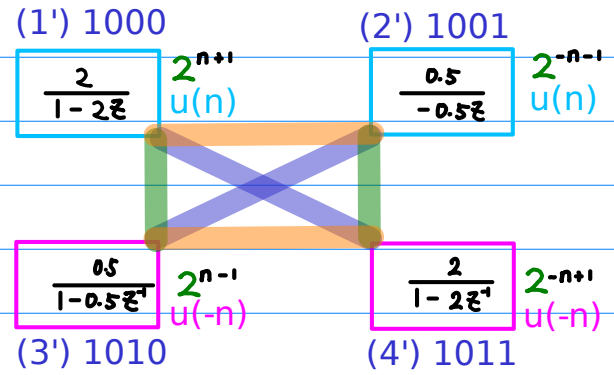




## A.I Flipping Base Inverting Range Flipping



## D.I Flipping2 Base Inverting Shifted Range Flipping



Shifted Range Flipping  
= Exponent Shifting2  
+ Range Flipping

$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow R(-n)$$

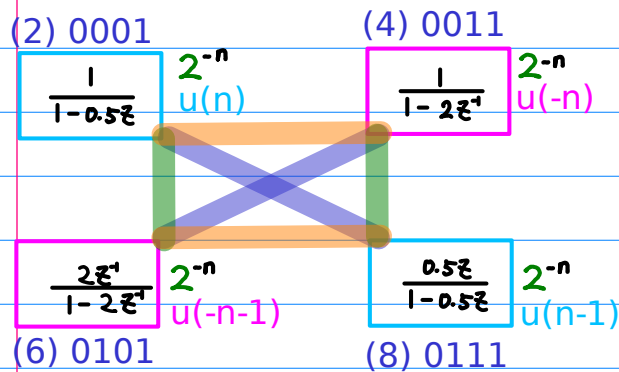
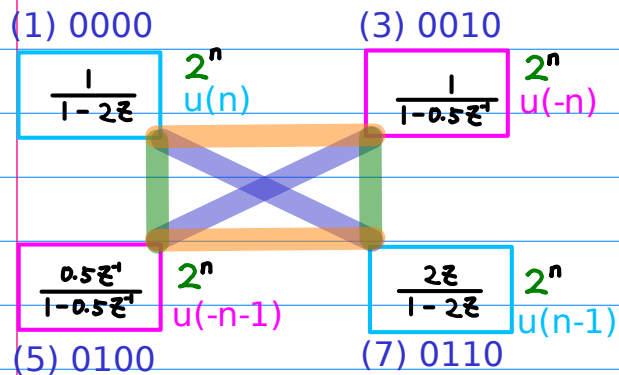
$$a^n R(n) \longleftrightarrow a^{-n} R(-n)$$

$$a^n \longleftrightarrow a^{-n}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

$$a^n R(n) \longleftrightarrow a^{-sh2(n)} R(-n)$$

## B.I Range Shifting Range Flipping Range Complementing



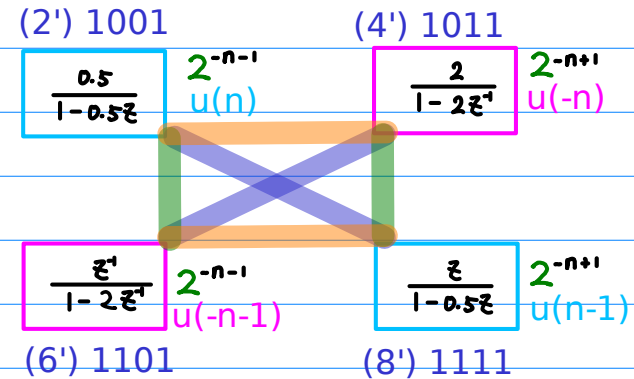
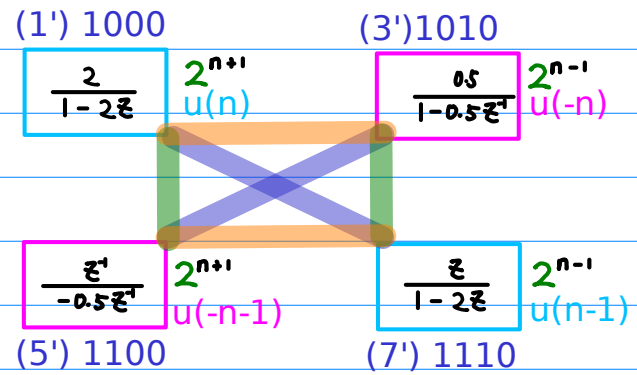
Range Shifting  
= Range Flipping  
+ Range Complementing

$$R(n) \longleftrightarrow R(-n)$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$R(n) \longleftrightarrow \overline{R(-n)}$$

## E.I Shifting2 Shifted Range Flipping Range Complementing



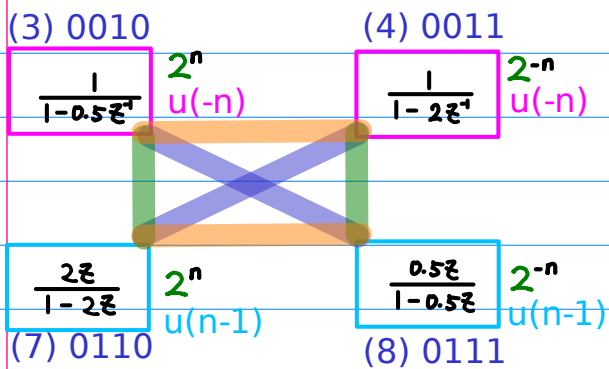
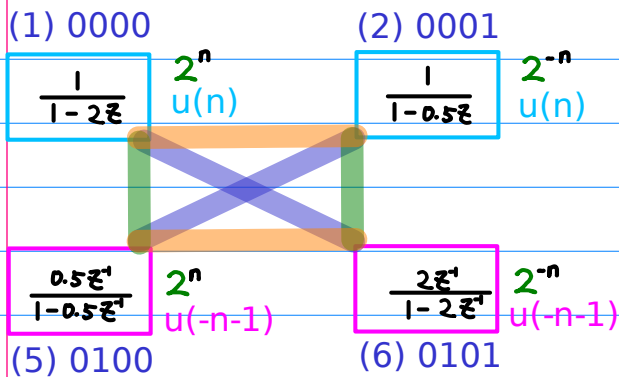
Shifted Range Flipping  
= Exponent Shifting2  
+ Range Flipping

$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

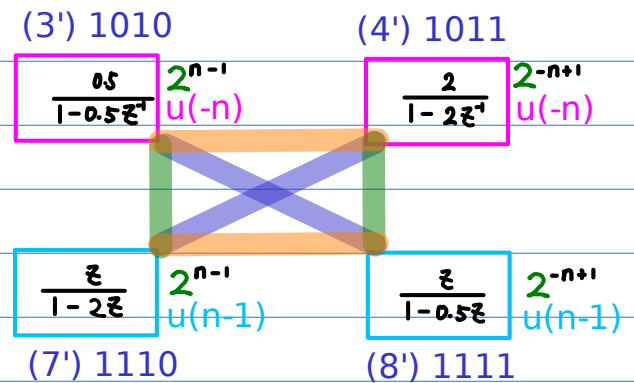
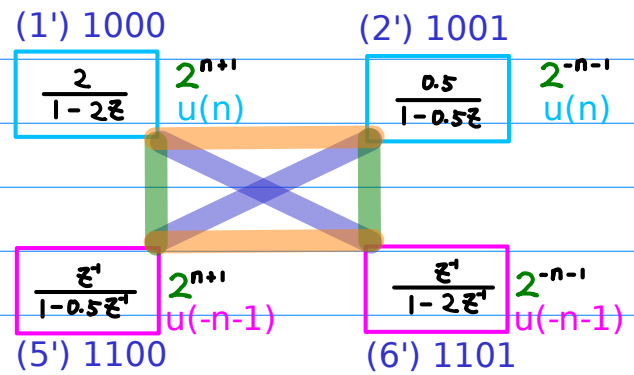
$$R(n) \longleftrightarrow \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

## C.I Complementary Inverting Base Inverting Range Complementing



## F.I Complementary Inverting Base Inverting Range Complementing



$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

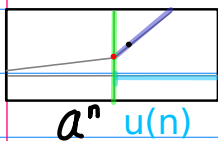
$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

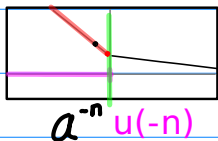
$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

## A.II Flipping Base Inverting Range Flipping

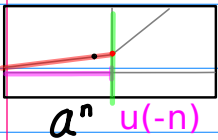
(1) 0000



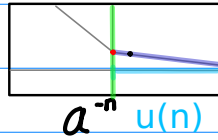
(4) 0011



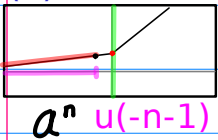
(3) 0010



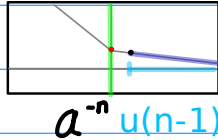
(2) 0001



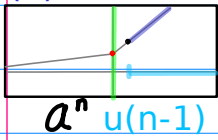
(5) 0100



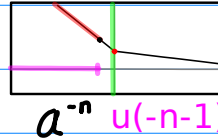
(8) 0111



(7) 0110

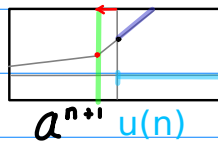


(6) 0101

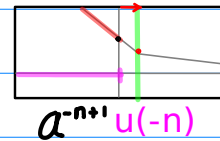


## D.II Flipping2 Base Inverting Shifted Range Flipping

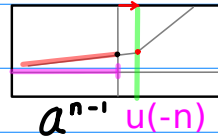
(1') 1000



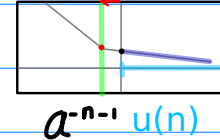
(4') 1011



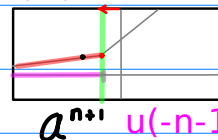
(3') 1010



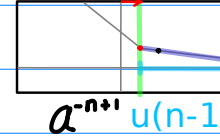
(2') 1001



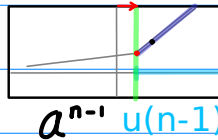
(5') 1100



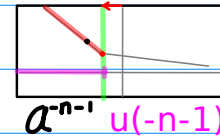
(8') 1111



(7') 1110



(6') 1101



Shifted Range Flipping  
= Exponent Shifting2  
+ Range Flipping

$$a^n R(n) \longleftrightarrow a^{-n} R(-n)$$

$$a^n R(n) \longleftrightarrow a^{-sh2(n)} R(-n)$$

$$a^n \longleftrightarrow a^{-n}$$

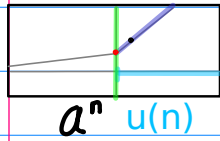
$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow R(-n)$$

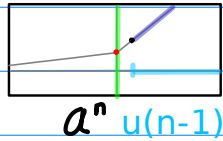
$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

## B.II Range Shifting Range Flipping Range Complementing

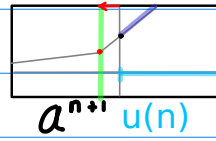
(1) 0000



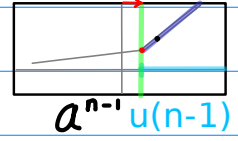
(7) 0110



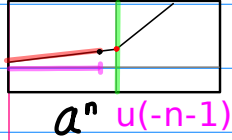
(1') 1000



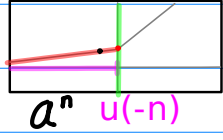
(7') 1110



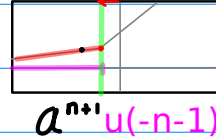
(5) 0100



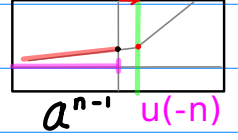
(3) 0010



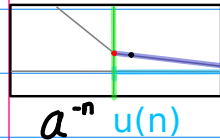
(5') 1100



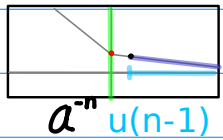
(3') 1010



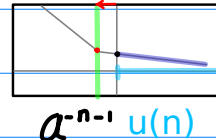
(2) 0001



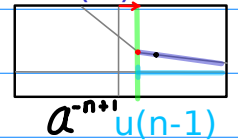
(8) 0111



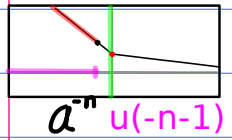
(2') 1001



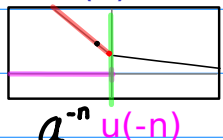
(8') 1111



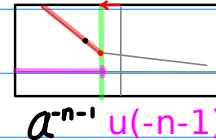
(6) 0101



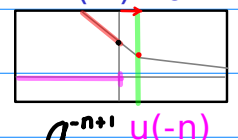
(4) 0011



(6') 1101



(4') 1011



Range Shifting  
= Range Flipping  
+ Range Complementing

Shifted Range Flipping  
= Exponent Shifting2  
+ Range Flipping

$$R(n) \longleftrightarrow \overline{R(-n)}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

$$R(n) \longleftrightarrow \overline{R(-n)}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

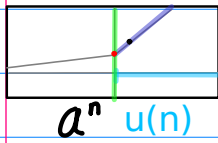
$$R(n) \longleftrightarrow \overline{R(n)}$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

## C.II Complementary Inverting Base Inverting Range Complementing

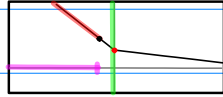
## F.II Complementary Inverting Base Inverting Range Complementing

(1) 0000



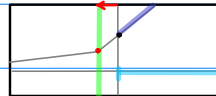
$a^n u(n)$

$a^{-n} u(-n-1)$



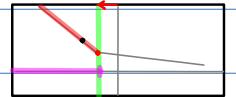
(6) 0101

(1') 1000



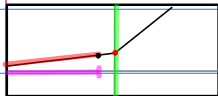
$a^{n+1} u(n)$

$a^{-n-1} u(-n-1)$



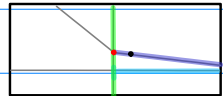
(6') 1101

(5) 0100



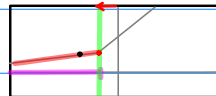
$a^n u(-n-1)$

(2) 0001



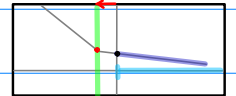
$a^{-n} u(n)$

(5') 1100



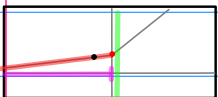
$a^{n+1} u(-n-1)$

(2') 1001



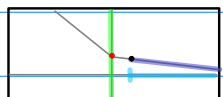
$a^{-n-1} u(n)$

(3) 0010



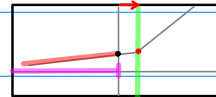
$a^n u(-n)$

(8) 0111



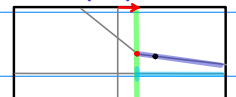
$a^{-n} u(n-1)$

(3') 1010



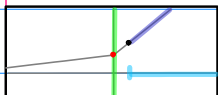
$a^{n-1} u(-n)$

(8') 1111



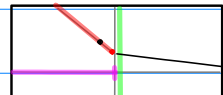
$a^{-n+1} u(n-1)$

(7) 0110



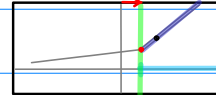
$a^n u(n-1)$

(4) 0011



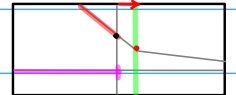
$a^{-n} u(-n)$

(7') 1110



$a^{n-1} u(n-1)$

(4') 1011



$a^{-n+1} u(-n)$

$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

$$a^n \longleftrightarrow a^{-n}$$

$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$R(n) \longleftrightarrow \overline{R(n)}$$





