Applicative Sequencing (3C)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using LibreOffice.

Based on

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Haskell in 5 steps
https://wiki.haskell.org/Haskell_in_5_steps

<$> related operators

Functor map <$>

```
(<$>)  :: Functor f =>  (a -> b) -> f a   -> f b
(<$)   :: Functor f =>  a        -> f b   -> f a
($>)   :: Functor f => f a       -> b     -> f b
```

replace b in f b with a … f a

replace a in f a with b … f b

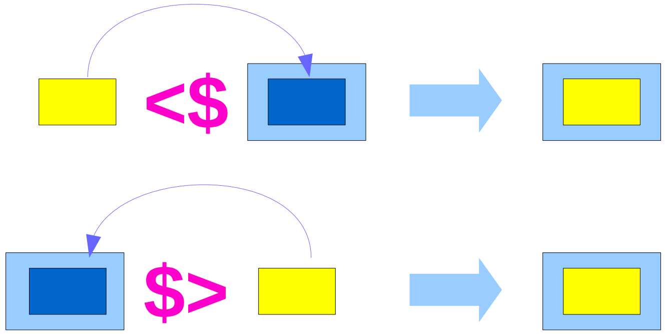
The <$> operator is just a synonym

for the **fmap** function in the Functor typeclass.

fmap generalizes **map** for **lists**

to other data types : **Maybe**, **IO**, **Map**.

Replacing the core



https://haskell-lang.org/tutorial/operators

<$ / <$> / $> operators

there are two additional operators provided

which replace a **value** inside a Functor

instead of applying a function.

This can be both more convenient in some cases,

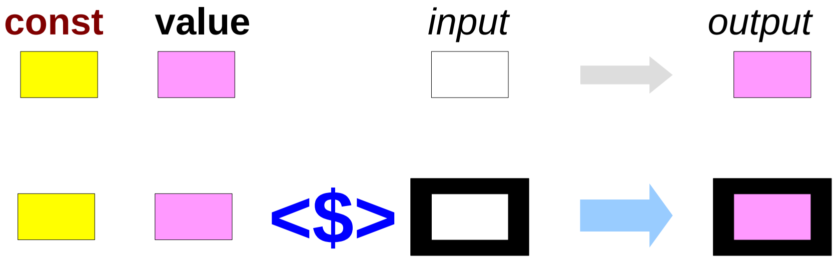
as well as for some Functors be more efficient.

value <$ functor = const value <$> functor

functor $> value = const value <$> functor

x <$ y = y $> x **y :: functor**

x $> y = y <$ x **x :: functor**



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<$ / <$> / $> operators examples

```
import Data.Functor

Prelude> Just 1 $> 2
Just 2
Prelude> Just 2 $> 1
Just 1

Prelude>  1 <$ Just 3
Just 1
Prelude>  3 <$ Just 1
Just 3

Prelude>  1 <$ Just 3
Just 1
Prelude>  3 <$ Just 1
Just 3
```

```
import Data.Functor

Prelude> (+1) <$> Just 2
Just 3
Prelude> (+1) <$> Just 3
Just 4

Prelude> (+1) <$> Nothing
Nothing

Prelude> const 2 <$> Just 111
Just 2
```

https://www.schoolofhaskell.com/school/to-infinity-and-beyond/pick-of-the-week/Simple%20examples

<$> examples

```
#!/usr/bin/env stack
-- stack --resolver ghc-7.10.3 runghc
import Data.Monoid ((<>))

main :: IO ()
main = do
   putStrLn "Enter your year of birth"
   year <- read <$> getLine
   let age :: Int
      age = 2020 - year
   putStrLn $ "Age in 2020: " <> show age
```

```
getLine :: IO String
```

Input: read "12"::Double

Output: 12.0

```
-- this infix synonym for mappend is found in Data.Monoid
x <> y = mappend x y
infixr 6 <>
```

https://haskell-lang.org/tutorial/operators

<*> related operators

Applicative function application <*>

```
(<*>)  :: Applicative f => f (a -> b) -> f a -> f b
(*>)   :: Applicative f => f a        -> f b -> f b
(<*)   :: Applicative f => f a        -> f b -> f a
```

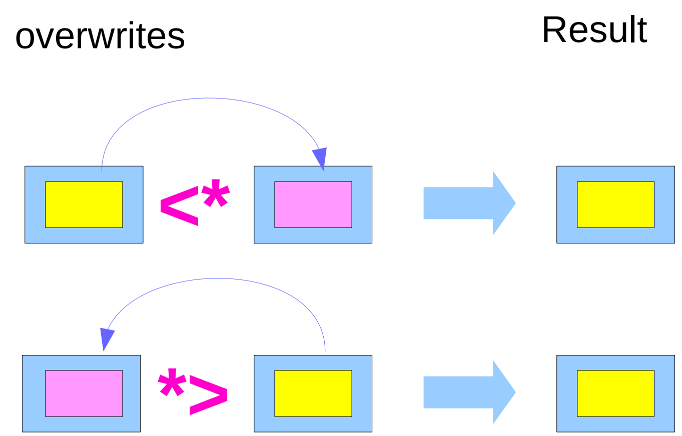
<*> is an operator that applies

a wrapped function to a wrapped value.

<*> is a part of the Applicative typeclass,

<*>is very often used as follows

```
foo <$> bar <*> baz
faa <*> bar <*> baz
```



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*> operator

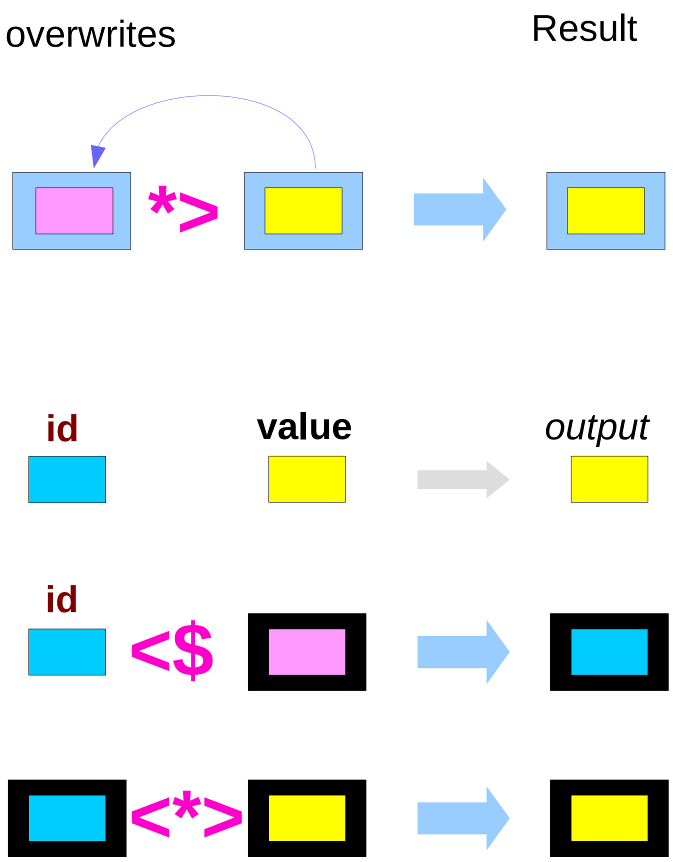
two helper operators

*> ignores the value from the first argument.

*> is completely equivalent to >> in Monad

a1 *> a2 = (id <$ a1) <*> a2

a1 *> a2 = do
_ <- a1
a2



(id <$ a1)

(id <$ a1) <*> a2

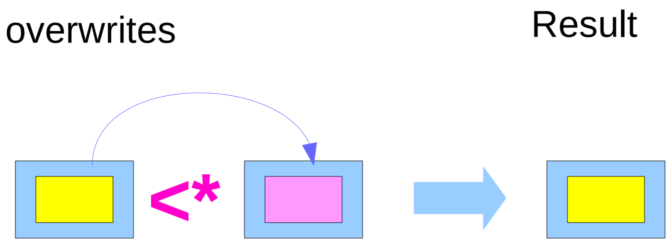
https://haskell-lang.org/tutorial/operators

<* operator

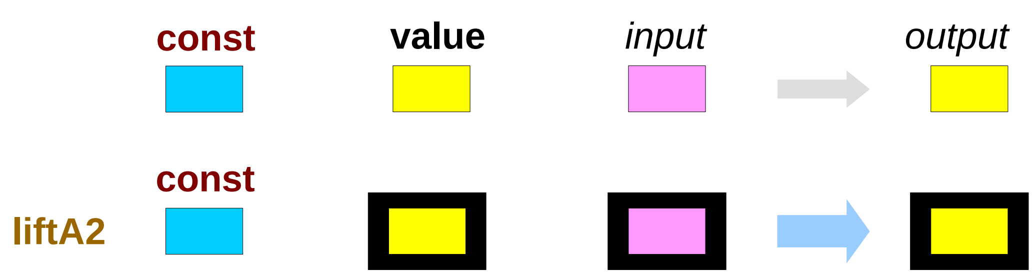
<* is the same thing in reverse: perform the first action then the second,

but only take the value from the first action.

(<*) = liftA2 const




```
a1 <* a2 = do
   res <- a1
     _ <- a2
   return res
```



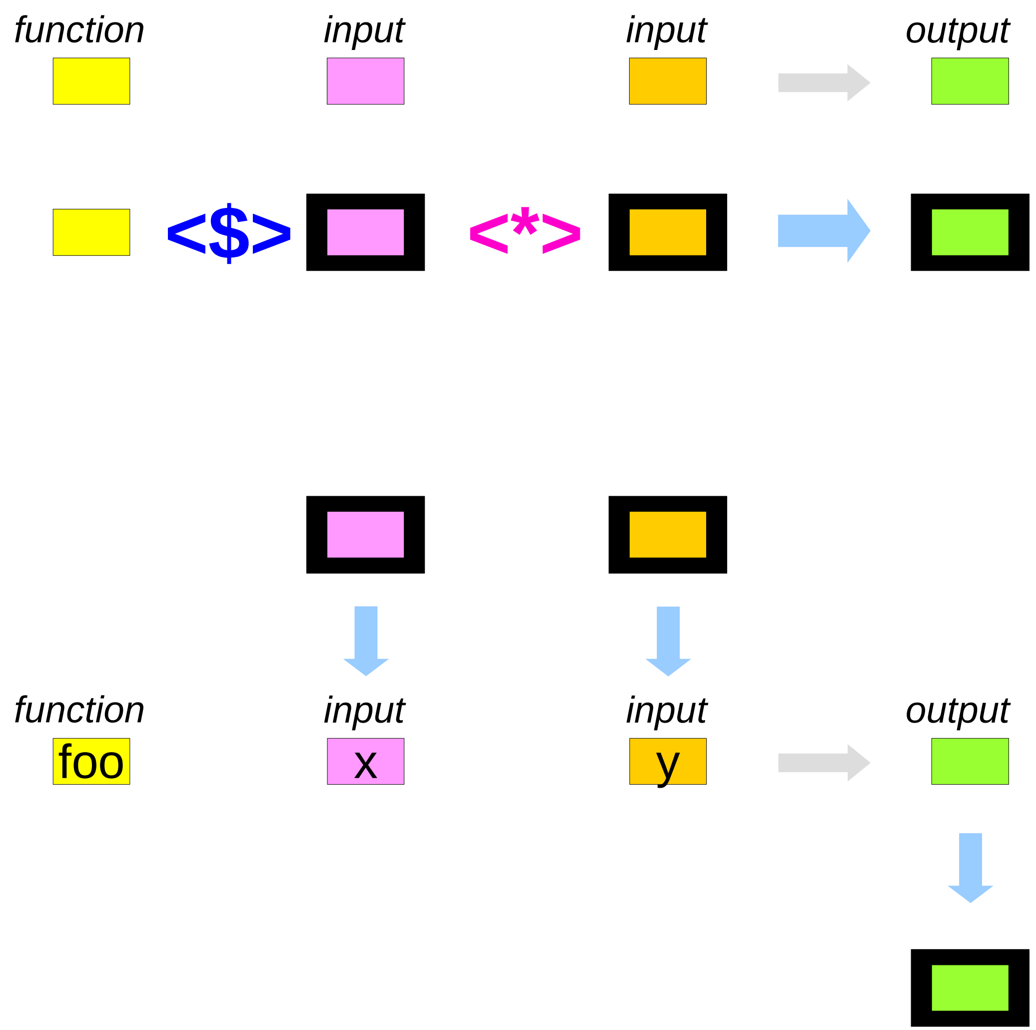
https://haskell-lang.org/tutorial/operators

<*> examples

foo <$> bar <*> baz

With a Monad, this is equivalent to:

```
do x <- bar
   y <- baz
   return (foo x y)
```



https://haskell-lang.org/tutorial/operators

<*> examples

examples including parsers and serialization libraries.

using the **aeson** package: (handling **JSON** data)

```
data Person = Person { name :: Text, age :: Int } deriving Show
```

-- We expect a JSON object, so we fail at any non-Object value.

```
instance FromJSON Person where
  parseJSON (Object v) = Person <$> v .: "name" <*> v .: "age"
  parseJSON _ = empty
```

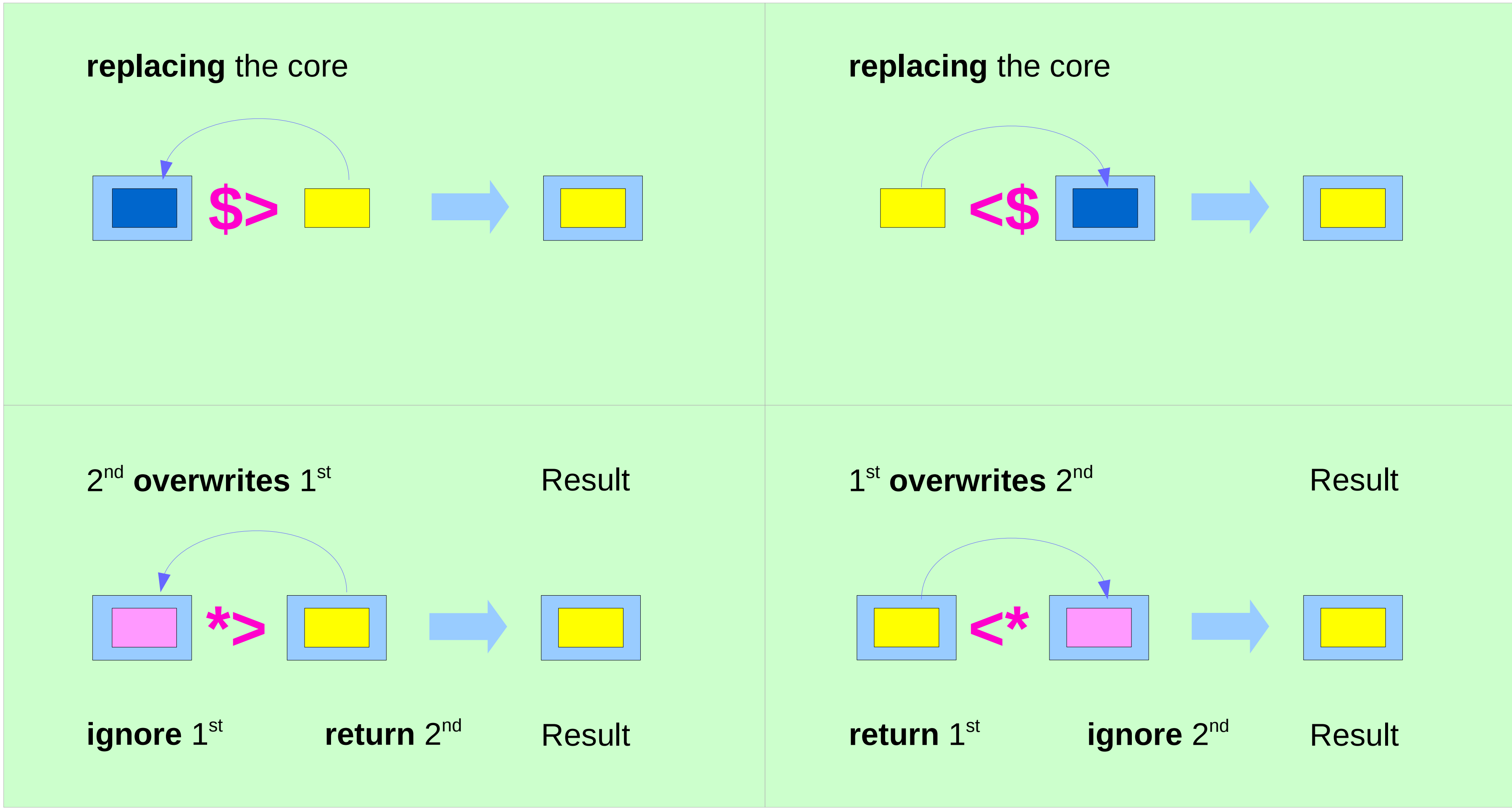
: append-head operator (cons)

. function composition operators

. name qualifier

https://haskell-lang.org/tutorial/operators

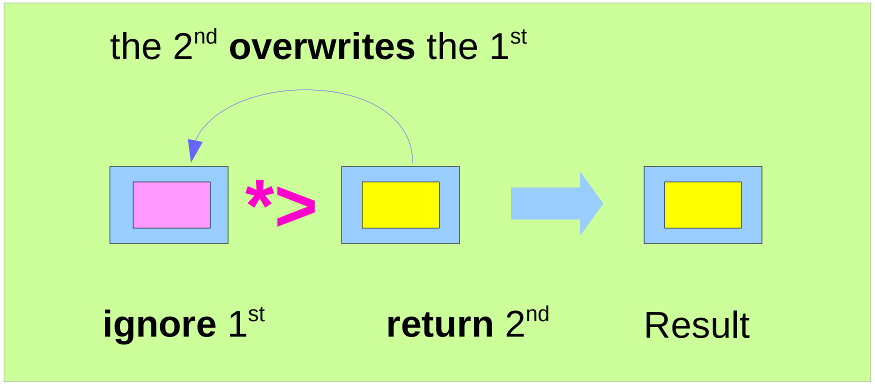
($> v.s. <$) and (*> v.s. <*)



https://en.wikibooks.org/wiki/Haskell/Applicative_functors

(*> v.s. >>) and (pure v.s. return)

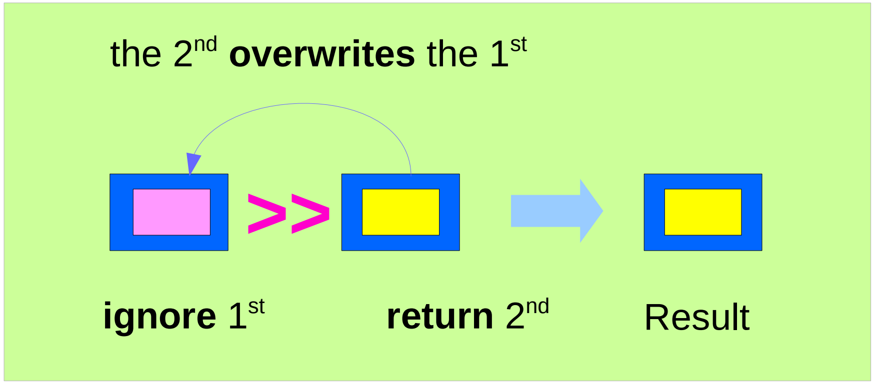
```
(*>) ::     Applicative f => f a -> f b -> f b
(>>) ::     Monad m       => m a -> m b -> m b

pure ::     Applicative f => a ->        f a
return ::   Monad m       => a ->        m a
```



the constraint changes from **Applicative** to **Monad**.

(*>) in [**Applicative**] ⟺ **(>>)** in [**Monad**]

pure in [**Applicative**] ⟺ **return** in [**Monad**]



https://en.wikibooks.org/wiki/Haskell/Applicative_functors

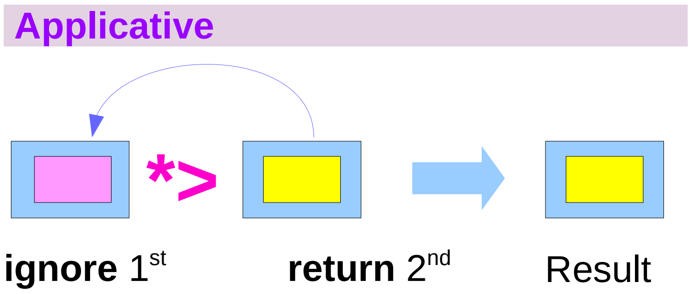
Commutativity

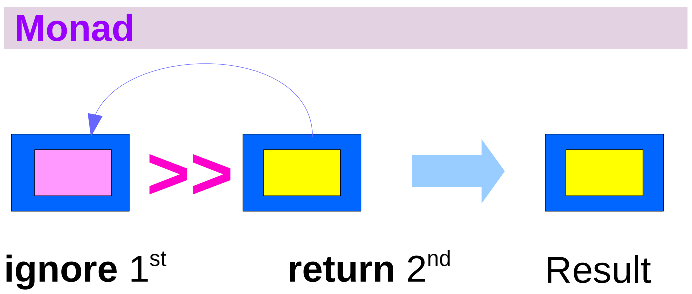
the concept involved in **commutative monads**,
is the same as the one in **commutative applicatives**,
only specialised to **Monad**.

commutativity (or the lack thereof) affects
other functions which are derived from **(<*>)** as well.
(*>) is a clear example:

(*>) :: Applicative f => f a -> f b -> f b

(*>) combines **effects**
preserves only the **values** of
its second argument.
is equivalent to **(>>)**, for **monads**





https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Commutativity examples (1)

Prelude> **Just 2** ***>** **Just 3**
Just 3 **– with value**
Prelude> **Just 3** ***>** **Just 2**
Just 2 **– with value**

Prelude> **Just 2** ***>** **Nothing**
Nothing **– non-value**
Prelude> **Nothing** ***>** **Just 2**
Nothing **– non-value**

Maybe is **commutative**

swapping the arguments does not affect the **effects** (the being and nothingness of wrapped values).

for **IO**, however, swapping the arguments does reorder the **effects**:

(*>) combines **effects**
preserves only the **values** of
its second argument.
is equivalent to **(>>)**, for **monads**

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Commutativity examples (2)

```
Prelude> (print "foo" *> pure 2) *> (print "bar" *> pure 3)
"foo"
"bar"
3
Prelude> (print "bar" *> pure 3) *> (print "foo" *> pure 2)
"bar"
"foo"
2
```

```
Prelude> (print "foo" *> pure 2) <* (print "bar" *> pure 3)
"foo"
"bar"
2
```

IO is **non-commutative**

swapping the arguments
does reorder the effects:

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Sequencing of Effects

```
Prelude> [(2*),(3*)] <*> [4,5]
1)  [8,10,12,15]        -- correct answer
2)  [8,12,10,15]
```

The difference is that for the first (and correct) answer
the result is obtained
by taking the skeleton of the first list
and replacing each element
by all possible combinations
with elements of the second list,

[(2*),(3*)] <*> [4,5]

[**(2*)** <*> 4, **(2*)** <*> 5, **(3*)** <*> 4, **(3*)** <*> 5]

while for the other possibility
the starting point is the second list.

[(2*),(3*)] <*> [4,5]

[(2*) <*> **4**, (3*) <*> **4**, (2*) <*> **5**, (3*) <*> **5**]

sequencing of effects

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Non-commutative Functors

by **effects** we mean the functorial **context**,
as opposed to the **values** within the functor

some **effects** examples:

- the <u>skeleton</u> of a **list**,
- <u>actions</u> performed in the real world in **IO**,
- the <u>existence</u> of a value in **Maybe**

The existence of two legal implementations of **(<*>)** for lists
only differ in the **sequencing** of **effects**
[] is a **non-commutative** applicative functor.

Prelude> **[(2*),(3*)] <*> [4,5]**

1) [8,10,12,15]
2) [8,12,10,15]

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Commutative Functors

a commutative applicative functor is
one for which the following holds:

swapping the arguments does not affect
the **effects** as well as the **value**

```
liftA2 fn u v = liftA2 (flip fn) v u
```

-- Commutativity

Or, equivalently,

```
fn <$> u <*> v = flip fn <$> v <*> u
```

```
liftA2  :: (a -> b -> c) -> f a -> f b -> f c
fn      :: (a -> b -> c)
flip fn :: (b -> a -> c)
u       :: f a
v       :: f b
```

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Commutative Monads (1)

```
do
  a <- actA
  b <- actB
  return (a + b)
```

```
do
  b <- actB
  a <- actA
  return (a + b)
```

commutative if the order of **side effects** is not important.

there are many monads that commute (e.g. **Maybe**, **Random**).
If the monad is **commutative**,
then the operations captured within it
can be computed in parallel.

No good syntax for monads that commute
still an open research problem

https://stackoverflow.com/questions/5897845/relax-ordering-constraints-in-monadic-computation

Commutative Monads (2)

Commutative monads are monads
for which the order of actions makes no difference
(they commute), that is when following code:

```
do
  a <- actA
  b <- actB
  m a b
```

```
do
  b <- actB
  a <- actA
  m a b
```

commutative if the order of **side effects** is not important.

Examples of commutative include:

Reader monad
Maybe monad

https://wiki.haskell.org/Monad#Commutative_monads
https://stackoverflow.com/questions/6089997/how-do-i-find-out-whether-a-monad-is-commutative

Left-to-right sequencing

The convention in Haskell is to always implement **`(<*>)`**
and other applicative operators using **left-to-right sequencing**.

Even though this convention helps reducing confusion,
it also means appearances sometimes are misleading.

For instance, the **`(<*)`** function is not flip **`(*>)`**,
as it sequences **effects** from left to right just like **`(*>)`**:

```
(<*>)  :: Applicative f =>  f (a -> b)  -> f a          -> f b
(*>)   :: Applicative f =>  f a         -> f b          -> f b
(<*)   :: Applicative f =>  f a         -> f b          -> f a
(<**>) :: Applicative f =>  f a         -> f (a -> b)   -> f b
```

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

<**> operators

(<**>) :: Applicative f => f a -> f (a -> b) -> f b

(<*>) :: Applicative f => f (a -> b) -> f a -> f b

from **Control.Applicative**

not flip **(<*>)**

a way of inverting the sequencing

[[(2*),(3*)] <*> [4, 5]

[8,10,12,15]

[4, 5] <> [(2*),(3*)]**

[8,*12*,*10*,15]

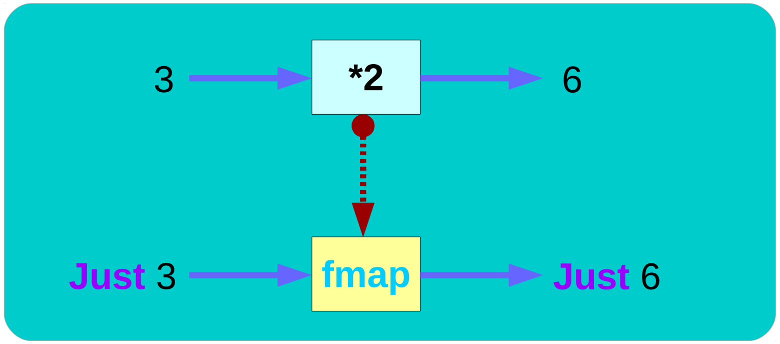
[4, 5] <> [(2*),(3*)]**

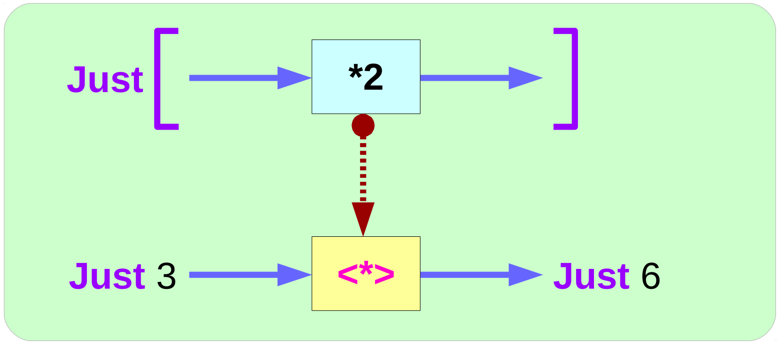
[8,10,12,15]

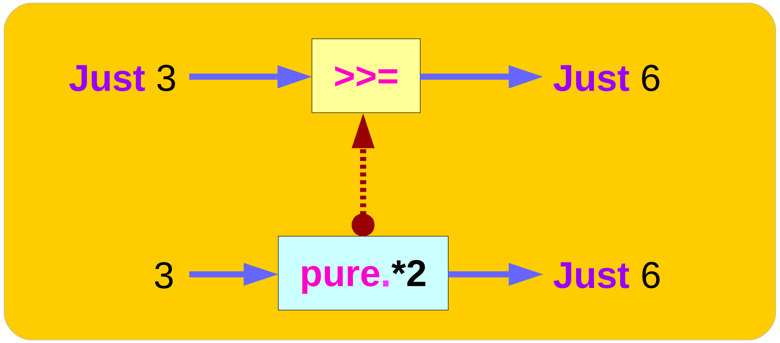
https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Functors, Applicative, and Monad

```
fmap  :: Functor      f => (a -> b)   -> f a              -> f b
(<*>) :: Applicative  f => f (a -> b) -> f a              -> f b
(>>=) :: Monad        m => m a        -> (a -> m b)       -> m b
```







https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Functors, Applicative, and Monad Examples

fmap :: Functor f => (a -> b) -> f a -> f b

```
Prelude> fmap (*2) (Just 3)
Just 6
```

(<*>) :: Applicative f => f (a -> b) -> f a -> f b

```
Prelude> (Just (*2)) <*>  (Just 3)
Just 6
```

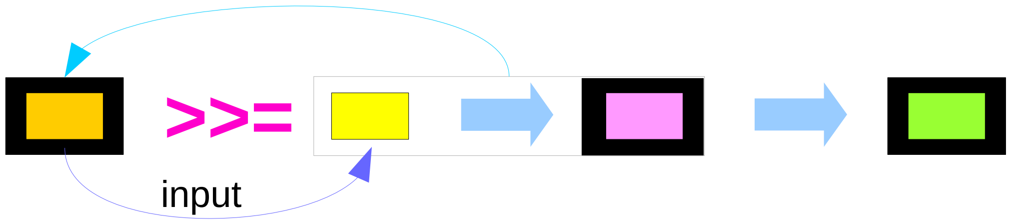
(>>=) :: Monad m => m a -> (a -> m b) -> m b

```
Prelude> (Just 3) >>= (pure . (*2))
Just 6
Prelude> (Just 3) >>= (return . (*2))
Just 6
```

(=<<) : the flipped version of (>>=)

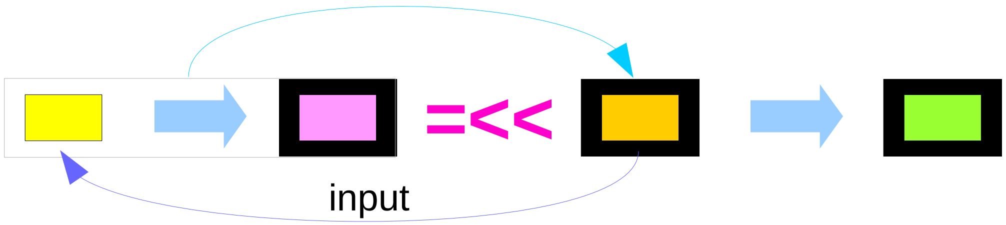
```
(>>=) ::    Monad    m    => m a    -> (a -> m b)    -> m b
```

<u>maps</u> **a -> m b** function at the right
<u>over</u> monadic **ma** <u>functors</u> at the left




```
(=<<) ::    Monad    m    => (a -> m b) -> m a    -> m b
```

<u>maps</u> **a -> m b** function at the left
<u>over</u> monadic **ma** <u>functors</u> at the right

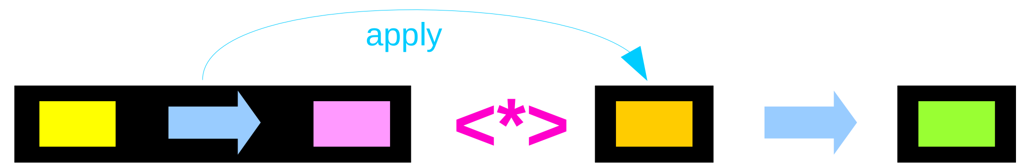


https://en.wikibooks.org/wiki/Haskell/Applicative_functors

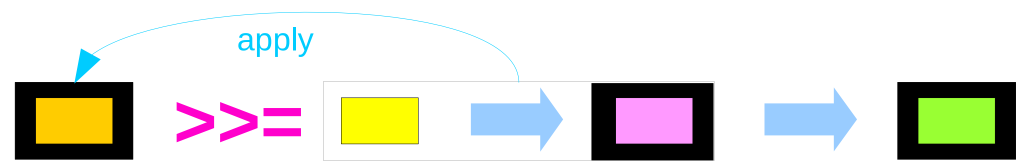
<$>, <*>, >>=, and =<< examples

Prelude> **(*2) <$> (Just 3)**
Just 6

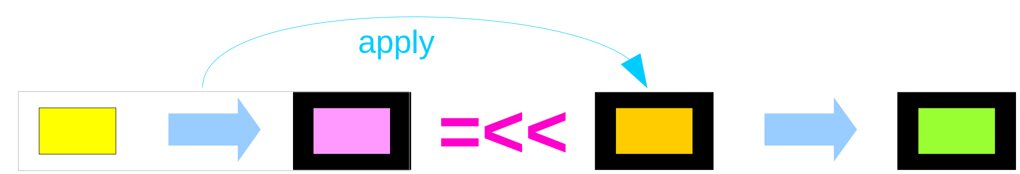
Prelude> **(Just (*2)) <*> (Just 3)**
Just 6



Prelude> **(Just 3) >>= (pure . (*2))**
Just 6



Prelude> **(pure . (*2)) =<< (Just 3)**
Just 6



Comparing the three characteristic methods

replace fmap by its infix synonym, (<$>)

replace (>>=) by its flipped version, (=<<)

```
fmap  :: Functor      f  =>   (a -> b)  -> f a          -> f b
(<*>) :: Applicative  f  => f (a -> b)  -> f a          -> f b
(>>=) :: Monad        m  => m a         -> (a -> m b)   -> m b

(<$>) :: Functor      t  =>   (a ->   b)    -> (t a -> t b)
(<*>) :: Applicative  t  => t (a ->   b)    -> (t a -> t b)
(=<<) :: Monad        t  =>   (a -> t b)    -> (t a -> t b)
```

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

All mapping functions over Functors

fmap, **(<*>)** and **(=<<)** are all **mapping functions** over **Functors**.

The differences between them are in what is being mapped (functions) over in each case:

```
(<$>) :: Functor t     =>   (a ->   b)  -> (t a -> t b)
(<*>) :: Applicative t => t (a ->   b)  -> (t a -> t b)
(=<<) :: Monad t       =>   (a -> t b)  -> (t a -> t b)
```

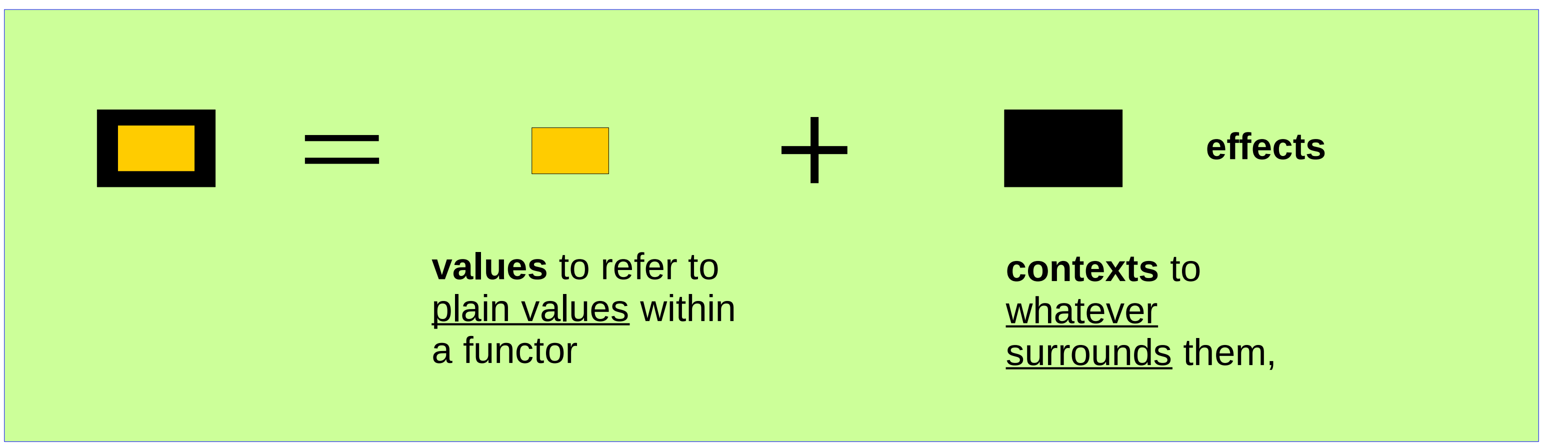
fmap maps	**(a -> b)**	arbitrary functions	over functors.
(<*>) maps	**t (a -> b)**	morphisms	over (applicative) functors.
(=<<) maps	**a -> t b**	functions	over (monadic) functors.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Power, Flexibility, Control

The differences of **Functor**, **Applicative** and **Monad** follow from
what these three **mapping functions** allow you to do.

As you move from **fmap** to **(<*>)** and then to **(>>=)**,
you gain in power, versatility and control,
at the cost of guarantees about the results.



https://en.wikibooks.org/wiki/Haskell/Applicative_functors

fmap does not change in the context

The type of **fmap** ensures that it is impossible
to use it to **change the context**,
no matter which function it is given.

In **(a -> b) -> t a -> t b**, the **(a -> b)** function
has nothing to do with the **t** context of the **t** a functorial value,
and so applying it cannot affect the **t** context.
For that reason, if you do **fmap f xs** on some list **xs**
the number of elements of the list will never change.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

(<*>) changes the context

fmap cannot change the **context**

- the **(a -> b)** function has no relation with the **t context**
- the application of this function does not affect the **context t**
- the number of elements of the list will never change

Prelude> **fmap (2*) [2,5,6]** *a list with 3 elements*

[4,10,12] *a list with 3 elements*

That could be a safety guarantee or an unfortunate restriction

depending on your purpose

(<*>) is clearly able to change the **context**:

Prelude> **[(2*),(3*)] <*> [2,5,6]** *two lists each with 3 elements*

[4,10,12,6,15,18] *a list with 6 elements*

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

(<*>) carries a context

The **t (a -> b)** morphism carries a **context** of its own,
which is combined (applied) with the **context**
of the **t** a **functorial value (a -> b)**.

(<*>), however, is subject to a more subtle restriction

while **t (a -> b)** morphisms carry **context**,
within them there are plain **(a -> b)**,
which are still unable to modify the **context**.

this means the changes to the **context** **(<*>)** performs
are fully determined by the **context** of its **arguments**,
and the **values** have no influence over the resulting context.

t (a->b) or **t b**
(a->b) or **a**

Prelude> **[(2*),(3*)] <*> [2,5,6]** *two lists each with 3 elements*
[4,10,12,6,15,18] *a list with 6 elements*

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Carrying a context examples

```
Prelude> (print "foo" *> pure (2*)) <*> (print "bar" *> pure 3)        (pure (2*)) <*> (pure 3)
"foo"
"bar"
6
Prelude> (print "foo" *> pure 2) *> (print "bar" *> pure 3)            (pure 2) *> (pure 3)
"foo"
"bar"
3
Prelude> (print "foo" *> pure undefined) *> (print "bar" *> pure 3)
"foo"                                                               (pure undefined) *> (pure 3)
"bar"
3
```

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

(>>=) creates a context

Prelude> **[(2*),(3*)] <*> [2,5,6]** *two lists each with 3 elements*

[4,10,12,6,15,18] *a list with 6 elements*

with **list (<*>)** you know that the length of the resulting list
will be the product of the lengths of the original lists,

with **IO (<*>)** you know that all real world effect
will happen as long as the evaluation terminates, and so forth.

with **Monad**, however, it is very different
(>>=) takes a (**a -> t b)** function,
and so it is able to create **context**
from **values**
which means a lot of flexibility:

creaing ***context t***
a -> t b

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Creating a context examples

```
Prelude> [1,2,5] >>= \x -> replicate x x          [ replicate 1 1, replicate 2 2, replicate 5,5 ]
[1,2,2,5,5,5,5,5]

Prelude> [0,0,0] >>= \x -> replicate x x          [ replicate 0 0, replicate 0 0, replicate 0,0 ]
[]

Prelude> return 3 >>= \x -> print $ if x < 10 then "Too small" else "OK"
"Too small"

Prelude> return 42 >>= \x -> print $ if x < 10 then "Too small" else "OK"
"OK"
```

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Deciding context (1)

(<*>) :: m (a->b) -> (m a->m b)

(=<<) :: (a->m b) -> (m a->m b)

In both cases there is **m a**, but only in the second case
m a can decide whether the function **(a->m b)** gets applied.
In its turn, the function **(a->m b)** can "decide"
whether the function bound next gets applied
by producing such **m b** that does not "contain" b
(like **[]**, **Nothing** or Left).

In **Applicative** there is no way for functions "inside" **m (a->b)**
to make such "decisions" - they always produce a **value** of type **b**.

https://stackoverflow.com/questions/23342184/difference-between-monad-and-applicative-in-haskell

Deciding context (2)

```
f 1 = Nothing         -- here f "decides" to produce Nothing
f x = Just x          -- if the argument is 1, then Nothing

Just 1 >>= f >>= g    -- g doesn't get applied, because f decided so.
                      -- f gets 1 and returns Nothing
```

In **Applicative** this is not possible, no example can be shown.

The closest is:

```
f 1 = 0
f x = x

g <$> f <$> Just 1    -- f gets 1 and produces Just 0, g
                      -- but f can't stop from getting applied
```

https://stackoverflow.com/questions/23342184/difference-between-monad-and-applicative-in-haskell

Flexibility

the extra **flexibility**

the less **guarantees** about

- whether your functions are able to unexpectedly erase parts of a data structure for pathological inputs
- whether the control flow in your application remains intelligible

performance implications

- the complex **data dependencies** of monadic codes might prevent **refactoring** and **optimizations**.

use only as much power as needed

it is often good to check
whether **Applicative** or **Functor** are sufficient
just before using **Monad**.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Monadic binding / composition operators

```
(>>=)  :: Monad m => m a         -> (a -> m b)  -> m b
(=<<)  :: Monad m => (a -> m b) -> m a          -> m b
(>>)   :: Monad m => m a         -> m b         -> m b
(>=>)  :: Monad m => (a -> m b) -> (b -> m c)  -> (a -> m c)
(<=<)  :: Monad m => (b -> m c) -> (a -> m b)  -> (a -> m c)
```

https://haskell-lang.org/tutorial/operators

Monadic binding operators (1)

```
(>>=)  :: Monad m => m a          -> (a -> m b)  -> m b
(=<<)  :: Monad m => (a -> m b) -> m a           -> m b
(>>)   :: Monad m => m a          -> m b         -> m b
```

monadic binding operators

The two most basic are **>>=** and **>>**

- **>>=**, **>>**, **=<<** can be expressed in **do-notation**
- **>>** is just a synonym for ***>** from the **Applicative** class
- **=<<** is just **>>=** with the arguments reversed

https://haskell-lang.org/tutorial/operators

Monadic binding operators (2)

```
(>>=)  :: Monad m => m a          -> (a -> m b)  -> m b
(=<<)  :: Monad m => (a -> m b) -> m a          -> m b
(>>)   :: Monad m => m a          -> m b          -> m b
```

```
m1 >>= func = do                                   m1 :: m a
    x <- m1            -- extract the value        x  :: a
    func x                                         func :: a -> m b
```

```
m1 >> m2 = do                                      m1 :: m a
    _ <- m1            -- side effect only, ignore the value   m2 :: m b
    m2
```

```
func =<< m1 = do                                   m1 :: m a
    x <- m1            -- extract the value        x  :: a
    func x                                         func :: a -> m b
```

https://haskell-lang.org/tutorial/operators

Monadic composition operators (1)

(>=>) :: Monad m => (a -> m b) -> (b -> m c) -> (a -> m c)

(<=<) :: Monad m => (b -> m c) -> (a -> m b) -> (a -> m c)

composition operators for two **monadic functions**

>=>=, **<=<** can be expressed in **do-notation**

>=> **pipes** the **result** from the left side to the right side

<=< **pipes** the **result** from the right side to the left side

https://haskell-lang.org/tutorial/operators

Monadic composition operators (2)

(>=>) :: Monad m => (a -> m b) -> (b -> m c) -> (a -> m c)

(<=<) :: Monad m => (b -> m c) -> (a -> m b) -> (a -> m c)

```
f >=> g = \x -> do                  f ::  a -> m b,    x :: a,    f x :: m b
    y <- f x                        g ::  b -> m c,    y :: b,    g y :: m c
    g y
```

```
g <=< f = \x -> do                  f ::  a -> m b,    x :: a,    f x :: m b
    y <- f x                        g ::  b -> m c,    y :: b,    g y :: m c
    g y
```

```
f >=> g = g <=< f
g >=> f = f <=< g
```

First f

Then g

https://haskell-lang.org/tutorial/operators

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf