HW#7 FIR Filter

#1 Square Wave

(a) The Fourier Series is defined as below:

$$x(t) = \sum_{k=-\infty}^{+\infty} c_k \, e^{+j(2\pi/T_0)kt}$$

$$c_k = \frac{1}{T_o}\int_0^{T_0} x(t)\, e^{-j(2\pi/T_0)kt}\, dt$$

Given a square wave:

$$s(t) = \begin{cases} 1 & \text{for } 0 \le t \le 0.5T_0 \\ 0 & \text{for } 0.5T_0 \le t \le T_0 \end{cases}$$

Show the FS coefficients are given by

$$c_k = \begin{cases} \dfrac{1}{j\pi k} & k = \pm 1, \pm 3, \pm 5, \cdots \\ 0 & k = \pm 2, \pm 4, \pm 6, \cdots \\ \dfrac{1}{2} & k = 0 \end{cases}$$

> Hint:
>
> $$c_k = \frac{1}{T_0}\int_0^{.5T_0} 1 \cdot e^{-j(2\pi/T_0)kt}\, dt$$
>
> $$e^{-j\pi} = -1 \qquad e^{-j\pi k} = (-1)^k$$

(b) Draw the spectrum plot ($|c_k|^2$, kf_0), where $f_0 = 1/T_0$. (matlab / octave)

(c) Let $x_N(t) = \sum_{k=-N}^{+N} c_k \, e^{+j(2\pi/T_0)kt}$.

Plot $x_5(t) = \sum_{k=-5}^{+5} c_k \, e^{+j(2\pi/T_0)kt}$, and $x_5(t) + n(t)$, where $n(t)$ is AWGN (additive white Gaussian noise) with a certain SNR. (matlab / octave)

(d) Let $x[n]$ denote the properly sampled signal of $x_5(t) + n(t)$,

and $h[n] = \frac{1}{(M+1)} \sum_{k=0}^{M} \delta[n-k]$ *(M=10)*,

show that the convolution result $y[n] = x[n] * h[n]$ is equal to

$$y[n] = \frac{1}{11}\sum_{k=0}^{10} x[n-k] \; .$$

And using matlab / octave, compute and plot the $y[n]$, $x[n]$, $h[n]$.

(i) $y[n] = \frac{1}{11}\sum_{k=0}^{10} x[n-k]$

(ii) `ones(11, 1) / 11` and `conv(hh, xx)`