CTFT (2A)

- Continuous Time Fourier Transform

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Continuous Time Fourier **Series**

$$C_k = \frac{1}{T}\int_0^T x(t)\, e^{-j\omega_0 k t}\, dt \quad \Longleftrightarrow \quad x(t) = \sum_{k=-\infty}^{+\infty} C_k\, e^{+j\omega_0 k t}$$

Discrete Frequency – Aperiodic

Periodic Continuous Time Signal

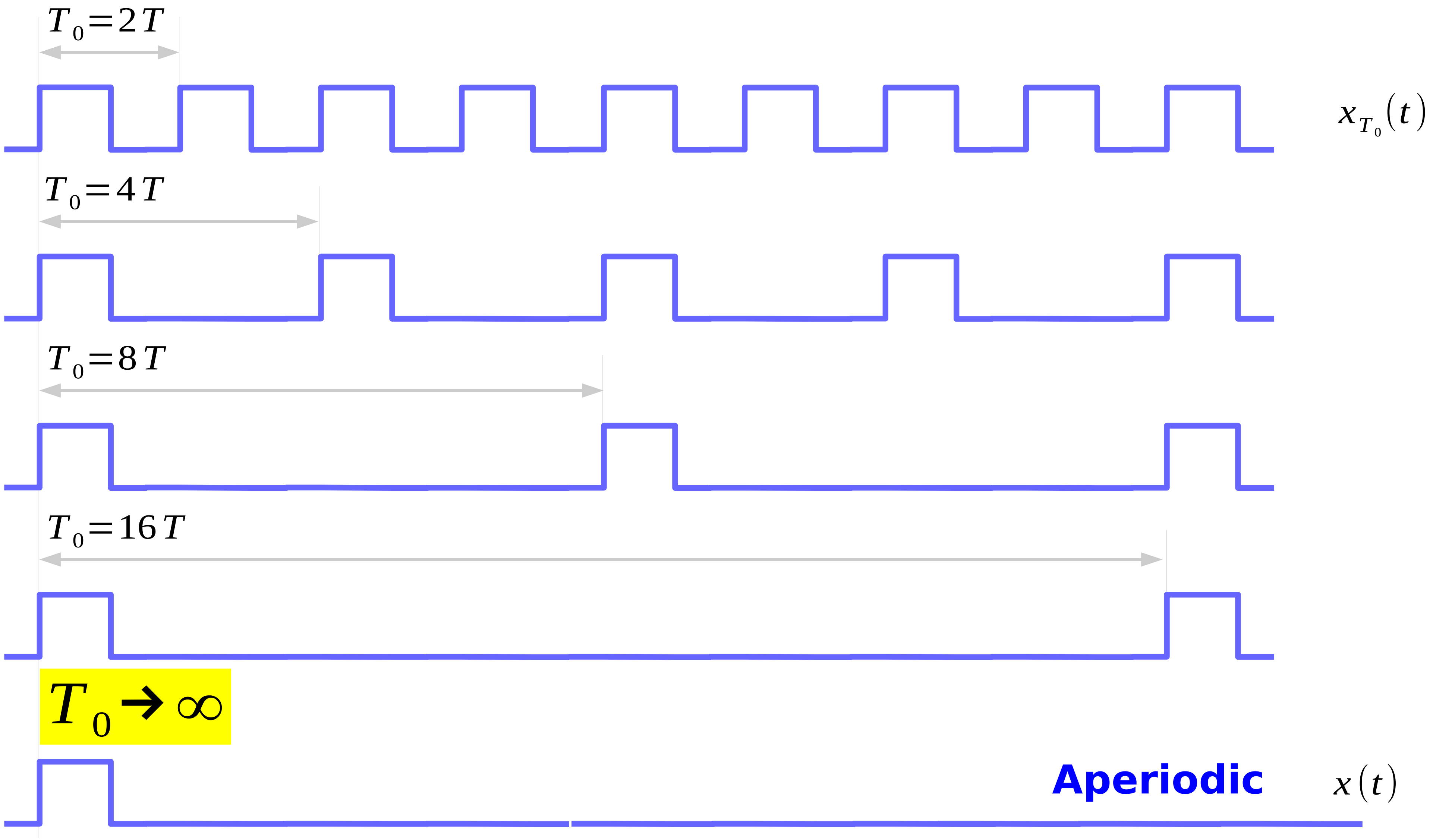
Continuous Time Fourier **Transform**

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Longleftrightarrow \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

Continuous Frequency – Aperiodic

Aperiodic Continuous Time Signal

Aperiodic Signal Conversion



From Summation to Integration

CTFS

$$C_k = \frac{1}{T}\int_0^T x(t)\, e^{-j\omega_0 k t}\, dt \quad \Leftrightarrow \quad x(t) = \sum_{k=-\infty}^{+\infty} C_k\, e^{+j\omega_0 k t}$$

$$T_0 \rightarrow \infty \qquad \omega = \left(\frac{2\pi}{T_0}\right) \rightarrow 0$$

$$\omega_0 \rightarrow d\omega, \quad k\omega_0 \rightarrow \omega$$

$$x_{T_0}(t) \rightarrow x(t), \quad C_k T_0 \rightarrow X(j\omega)$$

$$x_{T_0}(t) = \sum_{k=-\infty}^{\infty} C_k\, e^{+jk\omega_0 t} \cdot \frac{2\pi}{2\pi} \cdot \frac{T_0}{T_0}$$

$$x_{T_0}(t) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} C_k T_0\, e^{+jk\omega_0 t} \cdot \frac{2\pi}{T_0}$$

CTFT

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Leftrightarrow \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

From CTFS to CTFT

$$x_{T_0}(t) = \sum_{k=-\infty}^{+\infty} C_k \, e^{+j\omega_0 k t} \cdot 1$$

$$= \sum_{k=-\infty}^{+\infty} C_k \, e^{+j\omega_0 k t} \cdot \left(\frac{T_0}{2\pi}\right) \cdot \left(\frac{2\pi}{T_0}\right)$$

$$= \frac{1}{2\pi} \sum_{k=-\infty}^{+\infty} C_k T_0 \, e^{+j\omega_0 k t} \cdot \left(\frac{2\pi}{T_0}\right)$$

$$x_{T_0}(t) = \frac{1}{2\pi} \sum_{k=-\infty}^{+\infty} C_k T_0 \, e^{+j\omega_0 k t} \cdot \omega_0$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) \, e^{+j\omega t} \, d\omega$$

$$T_0 \rightarrow \infty \qquad \omega = \left(\frac{2\pi}{T_0}\right) \rightarrow 0$$

$$\omega_0 \rightarrow d\omega, \quad k\omega_0 \rightarrow \omega$$

$$x_{T_0}(t) \rightarrow x(t), \quad C_k T_0 \rightarrow X(j\omega)$$

CTFT of Time Domain Impulse

Continuous Time Fourier Transform

$$x(t) = A\delta(t) \iff X(j\omega) = A$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{+\infty} A\delta(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{+\infty} A\delta(t) e^{0} dt$$

$$= A\int_{-\infty}^{+\infty} \delta(t) dt = A$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} A e^{+j\omega t} d\omega$$

$$= \frac{A}{2\pi}\int_{-\infty}^{+\infty} e^{+j\omega t} d\omega = A\delta(t)$$

CTFT of Frequency Domain Impulse

Continuous Time Fourier Transform

$$X(j\omega) = 2\pi\,\delta(\omega) \Longleftrightarrow x(t) = 1$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} 2\pi\,\delta(\omega)\, e^{+j\omega t}\, d\omega$$

$$= \int_{-\infty}^{+\infty} \delta(\omega)\, e^{0}\, d\omega = 1$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{+\infty} e^{-j\omega t}\, dt = 2\pi\,\delta(\omega)$$

CTFS of Impulse Train

Continuous Time Fourier Series

$$C_n = \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega_0 t}\, dt$$

$$x(t) = \sum_{n=0}^{\infty} C_n\, e^{+jn\omega_0 t}$$

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$$

$$C_n = \frac{1}{T_s}\int_{-T_s/2}^{+T_s/2} \delta(t) e^{-jn\omega_s t}\, dt$$

$$= \frac{1}{T_s}\int_{-T_s/2}^{+T_s/2} \delta(t)\, dt = \frac{1}{T_s}$$

$$p(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega_s t}$$

$$= \sum_{n=-\infty}^{+\infty} \frac{1}{T_s} e^{jn\omega_s t}$$

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) = \sum_{n=-\infty}^{+\infty} \frac{1}{T_s} e^{jn\omega_s t}$$

CTFT of Impulse Train

Continuous Time Fourier Transform

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$$

$$P(j\omega) = \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) e^{-jn\omega t} dt = \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \frac{1}{T_s} e^{jn\omega_s t} e^{-jn\omega t} dt$$

$$= \sum_{n=-\infty}^{+\infty} \frac{1}{T_s} \int_{-\infty}^{+\infty} e^{-j(\omega - n\omega_s)t} dt$$

$$= \sum_{n=-\infty}^{+\infty} \left(\frac{2\pi}{T_s}\right) \delta(\omega - n\omega_s)$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t} dt$$

$$p(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \left(\frac{2\pi}{T_s}\right) \delta(\omega - n\omega_s)\, e^{+j\omega t} d\omega = \sum_{n=-\infty}^{+\infty} \frac{1}{T_s} \int_{-\infty}^{+\infty} \delta(\omega - n\omega_s)\, e^{+j\omega t} d\omega$$

$$= \sum_{n=-\infty}^{+\infty} \frac{1}{T_s} e^{jn\omega_s t} = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t} d\omega$$

Other Convention

Continuous Time Fourier Transform {unitary, angular frequency}

$$X(j\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Leftrightarrow$$

$$x(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

Continuous Time Fourier Transform {non-unitary, angular frequency}

$$X(j\omega) = 1 \cdot \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Leftrightarrow$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

References

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