Laurent Series and z-Transform - Properties of a Geometric Series Examples A

20180209

Causal signal a_n $n \geq 0$

anti-causal signal a_n $n < 0$

Laurent Series $f(z)$

z - Transform $X(z)$

① Causal $(n \geq 0)$ $a_n = \left(\frac{1}{2}\right)^n$

$a_n : \left(\frac{1}{2}\right)^0, \quad \left(\frac{1}{2}\right)^1, \quad \left(\frac{1}{2}\right)^2, \quad \cdots \qquad (n \geq 0)$

$\quad\ \ n=0 \qquad\quad n=1 \qquad\quad n=2$

$$f(z) = \left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \cdots = \frac{1}{1-\frac{z}{2}} = \frac{2}{2-z}$$

$$\frac{|z|}{2} < 1 \qquad |z| < 2$$

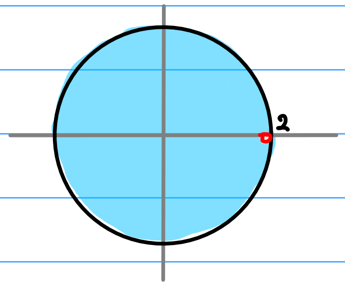
$$X(z) = \left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \cdots = \frac{1}{1-\frac{1}{2z}} = \frac{z}{z-0.5}$$

$$\frac{1}{2|z|} < 1 \qquad |z| > 0.5$$

$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$

$$f(z) = \frac{1}{1-\frac{z}{2}} \qquad |z| < 2$$

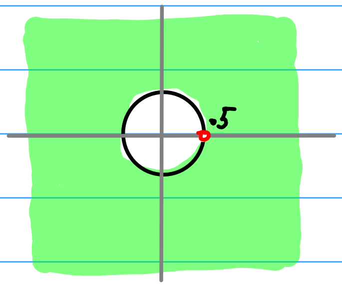
$$= \frac{z^{-1}}{z^{-1}-0.5} = \frac{2}{2-z}$$



$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$

$$X(z) = \frac{1}{1-\frac{1}{2z}} \qquad |z| > 0.5$$

$$= \frac{z}{z-0.5}$$



② **Causal** $(n \geq 0)$ $a_n = (2)^n$

$a_n : \quad (2)^0, \quad (2)^1, \quad (2)^2, \quad \cdots \qquad (n \geq 0)$

$n=0 \qquad n=1 \qquad n=2$

$$f(z) = (2)^0 z^0 + (2)^1 z^1 + (2)^2 z^2 + \cdots = \frac{1}{1-2z} = \frac{0.5}{0.5-z}$$

$2|z| < 1 \qquad |z| < 0.5$

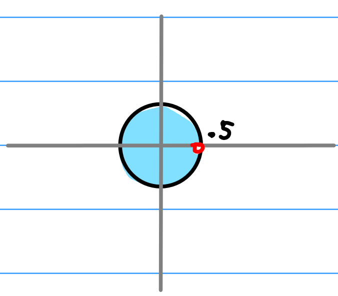
$$X(z) = (2)^0 z^0 + (2)^1 z^{-1} + (2)^2 z^{-2} + \cdots = \frac{1}{1-\frac{2}{z}} = \frac{z}{z-2}$$

$\frac{2}{|z|} < 1 \qquad |z| > 2$

$a_n = (2)^n \quad (n \geq 0)$

$f(z) = \frac{1}{1-2z} \quad |z| < 0.5$

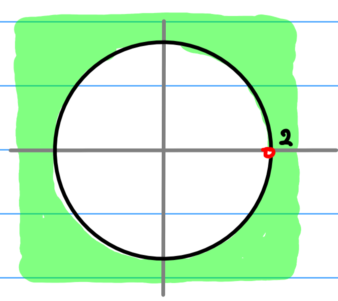
$= \frac{0.5}{0.5-z}$



$a_n = (2)^n \quad (n \geq 0)$

$X(z) = \frac{1}{1-\frac{2}{z}} \quad |z| > 2$

$= \frac{z}{z-2}$



③ Anti-causal $(n<0)$ $a_n = \left(\frac{1}{2}\right)^n$

a_n : $\left(\frac{1}{2}\right)^{-1}$, $\left(\frac{1}{2}\right)^{-2}$, $\left(\frac{1}{2}\right)^{-3}$, $\cdots$ $(n<0)$

$n=-1$ $\quad n=-2$ $\quad n=-3$

$$f(z) = (2)^1 z^{-1} + (2)^2 z^{-2} + (2)^3 z^{-3} + \cdots = \frac{\frac{2}{z}}{1-\frac{2}{z}} = \frac{2}{z-2}$$

$\frac{2}{|z|} < 1$ $\quad |z| > 2$

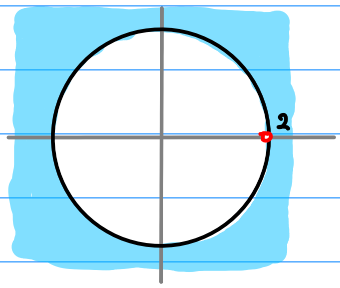
$$X(z) = (2)^1 z^1 + (2)^2 z^2 + (2)^3 z^3 + \cdots = \frac{2z}{1-2z} = \frac{z}{0.5-z}$$

$2|z| < 1$ $\quad |z| < 0.5$

$a_n = \left(\frac{1}{2}\right)^n$ $(n<0)$

$$f(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \quad |z|>2$$

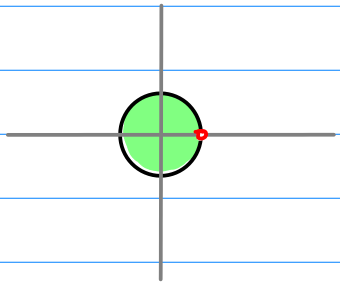
$$= -\frac{2}{2-z}$$



$a_n = \left(\frac{1}{2}\right)^n$ $(n<0)$

$$X(z) = \frac{2z}{1-2z} \quad |z| < 0.5$$

$$= -\frac{z}{z-0.5}$$



④ **Anti-causal** $(n<0)$ $a_n = (2)^n$

$a_n: \quad (2)^{-1}, \quad (2)^{-2}, \quad (2)^{-3}, \quad \cdots \qquad (n<0)$

$n=-1 \qquad n=-2 \qquad n=-3$

$$f(z) = \left(\frac{1}{2}\right)^1 z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \left(\frac{1}{2}\right)^3 z^{-3} + \cdots = \frac{\frac{1}{2z}}{1-\frac{1}{2z}} = \frac{0.5}{z-0.5}$$

$\frac{1}{2|z|} < 1 \qquad |z| > 0.5$

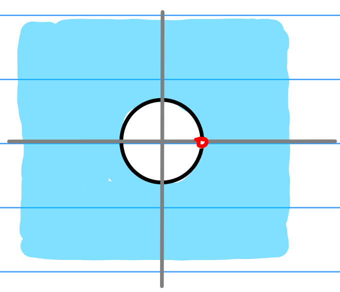
$$X(z) = \left(\frac{1}{2}\right)^1 z^{1} + \left(\frac{1}{2}\right)^2 z^{2} + \left(\frac{1}{2}\right)^3 z^{3} + \cdots = \frac{\frac{z}{2}}{1-\frac{z}{2}} = \frac{z}{2-z}$$

$\frac{|z|}{2} < 1 \qquad |z| < 2$

$a_n = (2)^n \quad (n<0)$

$f(z) = \dfrac{\frac{1}{2z}}{1-\frac{1}{2z}} \qquad |z| > 0.5$

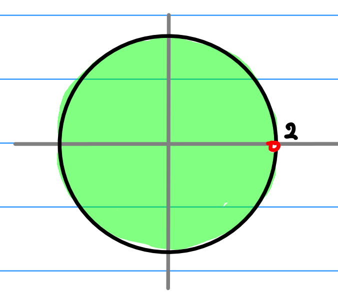
$= -\dfrac{0.5}{0.5-z}$



$a_n = (2)^n \quad (n<0)$

$X(z) = \dfrac{\frac{z}{2}}{1-\frac{z}{2}} \qquad |z| < 2$

$= -\dfrac{z}{z-2}$



⑤ Causal $(n > 0)$ $a_n = \left(\frac{1}{2}\right)^{n-1}$

$a_n : \cancel{\left(\frac{1}{2}\right)^{-1}}, \left(\frac{1}{2}\right)^{0}, \left(\frac{1}{2}\right)^{1}, \cdots \quad (n > 0)$

$n=0 \quad n=1 \quad n=2$

$$f(z) = \left(\frac{1}{2}\right)^0 z^1 + \left(\frac{1}{2}\right)^1 z^2 + \left(\frac{1}{2}\right)^2 z^3 + \cdots = \frac{z}{1-\frac{z}{2}} = \frac{2z}{2-z}$$

$\frac{|z|}{2} < 1 \qquad |z| < 2$

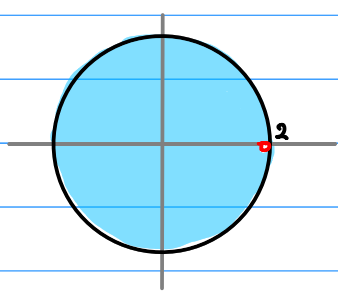
$$X(z) = \left(\frac{1}{2}\right)^0 z^{-1} + \left(\frac{1}{2}\right)^1 z^{-2} + \left(\frac{1}{2}\right)^2 z^{-3} + \cdots = \frac{\frac{1}{z}}{1-\frac{1}{2z}} = \frac{1}{z-0.5}$$

$\frac{1}{2|z|} < 1 \qquad |z| > 0.5$

$a_n = \left(\frac{1}{2}\right)^{n-1} \quad (n > 0)$

$f(z) = \frac{z}{1-\frac{z}{2}} \qquad |z| < 2$

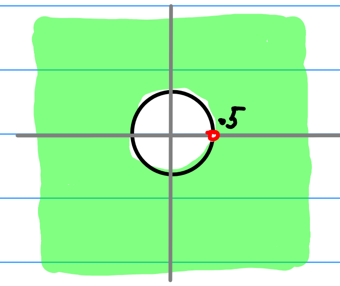
$= \frac{1}{z^{-1}-0.5} = \frac{2z}{2-z}$



$a_n = \left(\frac{1}{2}\right)^{n-1} \quad (n > 0)$

$X(z) = \frac{\frac{1}{z}}{1-\frac{1}{2z}} \qquad |z| > 0.5$

$= \frac{1}{z-0.5}$



⑥ Causal $(n > 0)$ $a_n = (2)^{n-1}$

$a_n : \cancel{(2)^{-1}}, \quad (2)^0, \quad (2)^1, \quad \cdots \qquad (n > 0)$

$\quad\quad\; n=0 \qquad\quad n=1 \qquad\quad n=2$

$$f(z) = (2)^0 z^1 + (2)^1 z^2 + (2)^2 z^3 + \cdots = \frac{z}{1-2z} = \frac{0.5z}{0.5-z}$$

$$2|z| < 1 \qquad\qquad |z| < 0.5$$

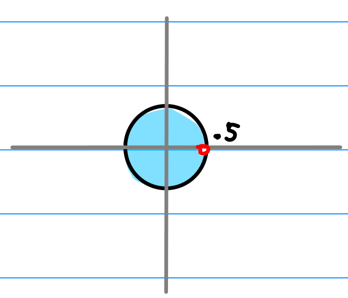
$$X(z) = (2)^0 z^{-1} + (2)^1 z^{-2} + (2)^2 z^{-3} + \cdots = \frac{\frac{1}{z}}{1-\frac{2}{z}} = \frac{1}{z-2}$$

$$\frac{2}{|z|} < 1 \qquad\qquad |z| > 2$$

$a_n = (2)^{n-1} \quad (n > 0)$

$f(z) = \dfrac{z}{1-2z} \qquad |z| < 0.5$

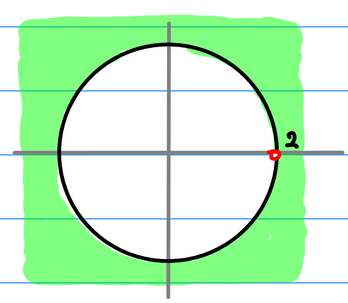
$= \dfrac{0.5z}{0.5-z}$



$a_n = (2)^{n-1} \quad (n > 0)$

$X(z) = \dfrac{\frac{1}{z}}{1-\frac{2}{z}} \qquad |z| > 2$

$= \dfrac{1}{z-2}$



⑦ Anti-causal $(n \leq 0)$ $a_n = \left(\frac{1}{2}\right)^{n-1}$

$a_n : \left(\frac{1}{2}\right)^{-1}, \quad \left(\frac{1}{2}\right)^{-2}, \quad \left(\frac{1}{2}\right)^{-3}, \quad \cdots \qquad (n \leq 0)$

$n=0 \qquad n=-1 \qquad n=-2$

$$f(z) = (2)^1 z^0 + (2)^2 z^{-1} + (2)^3 z^{-2} + \cdots = \frac{\frac{2}{1}}{1-\frac{2}{z}} = \frac{2z}{z-2}$$

$\frac{2}{|z|} < 1 \qquad |z| > 2$

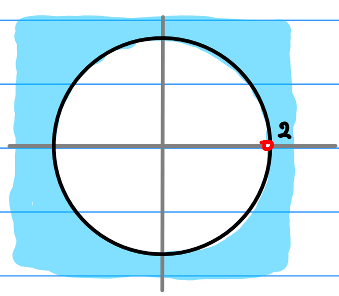
$$X(z) = (2)^1 z^0 + (2)^2 z^1 + (2)^3 z^2 + \cdots = \frac{2}{1-2z} = \frac{1}{0.5-z}$$

$2|z| < 1 \qquad |z| < 0.5$

$a_n = \left(\frac{1}{2}\right)^{n-1} \quad (n \leq 0)$

$$f(z) = \frac{\frac{2}{1}}{1-\frac{2}{z}} \qquad |z| > 2$$

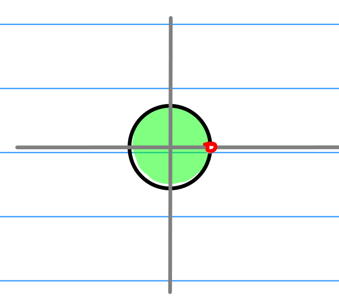
$$= -\frac{2z}{2-z}$$



$a_n = \left(\frac{1}{2}\right)^{n-1} \quad (n \leq 0)$

$$X(z) = \frac{2}{1-2z} \qquad |z| < 0.5$$

$$= -\frac{1}{z-0.5}$$



⑧ Anti-causal $(n \le 0)$ $a_n = (2)^{n-1}$

$a_n :$ $(2)^{-1},$ $(2)^{-2},$ $(2)^{-3},$ $\cdots$ $(n \le 0)$

$n=0$ $n=-1$ $n=-2$

$$f(z) = \left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots = \frac{\frac{1}{2}}{1-\frac{1}{2z}} = \frac{0.5z}{z-0.5}$$

$\frac{1}{2|z|} < 1$ $|z| > 0.5$

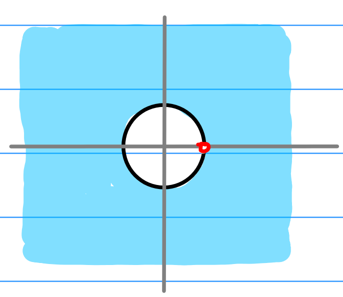
$$X(z) = \left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots = \frac{\frac{1}{2}}{1-\frac{z}{2}} = \frac{1}{2-z}$$

$\frac{|z|}{2} < 1$ $|z| < 2$

$a_n = (2)^{n-1}$ $(n \le 0)$

$$f(z) = \frac{\frac{1}{2}}{1-\frac{1}{2z}} \quad |z| > 0.5$$

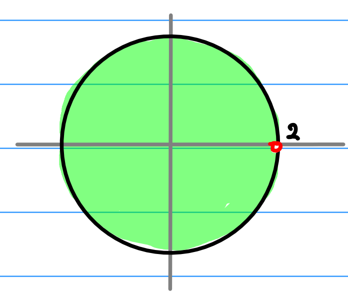
$$= -\frac{0.5z}{0.5-z}$$



$a_n = (2)^{n-1}$ $(n \le 0)$

$$X(z) = \frac{\frac{1}{2}}{1-\frac{z}{2}} \quad |z| < 2$$

$$= -\frac{1}{z-2}$$



$2 \longleftrightarrow \frac{1}{2}$

1. $(n \geq 0)$ $a_n = \left(\frac{1}{2}\right)^n$ $f(z) = \frac{2}{2-z}$ $X(z) = \frac{z}{z-0.5}$

2. $(n \geq 0)$ $a_n = (2)^n$ $f(z) = \frac{0.5}{0.5-z}$ $X(z) = \frac{z}{z-2}$

3. $(n < 0)$ $a_n = \left(\frac{1}{2}\right)^n$ $f(z) = -\frac{2}{2-z}$ $X(z) = -\frac{z}{z-0.5}$

4. $(n < 0)$ $a_n = (2)^n$ $f(z) = -\frac{0.5}{0.5-z}$ $X(z) = -\frac{z}{z-2}$

5. $(n > 0)$ $a_n = \left(\frac{1}{2}\right)^{n-1}$ $f(z) = \frac{2z}{2-z}$ $X(z) = \frac{1}{z-0.5}$

6. $(n > 0)$ $a_n = (2)^{n-1}$ $f(z) = \frac{0.5z}{0.5-z}$ $X(z) = \frac{1}{z-2}$

7. $(n \leq 0)$ $a_n = \left(\frac{1}{2}\right)^{n-1}$ $f(z) = -\frac{2z}{2-z}$ $X(z) = -\frac{1}{z-0.5}$

8. $(n \leq 0)$ $a_n = (2)^{n-1}$ $f(z) = -\frac{0.5z}{0.5-z}$ $X(z) = -\frac{1}{z-2}$

1 $(n \geqslant 0) \quad a_n = \left(\frac{1}{2}\right)^n \qquad f(z) = \frac{2}{2-z} \qquad X(z) = \frac{z}{z-0.5}$

2 $(n \geqslant 0) \quad a_n = (2)^n \qquad f(z) = \frac{0.5}{0.5-z} \qquad X(z) = \frac{z}{z-2}$

shift to the right $\rightarrow$ $\times z$ $\times z^{-1}$

delete a_0

5 $(n > 0) \quad a_n = \left(\frac{1}{2}\right)^{n-1} \qquad f(z) = \frac{2z}{2-z} \qquad X(z) = \frac{1}{z-0.5}$

6 $(n > 0) \quad a_n = (2)^{n-1} \qquad f(z) = \frac{0.5z}{0.5-z} \qquad X(z) = \frac{1}{z-2}$

3 $(n < 0) \quad a_n = \left(\frac{1}{2}\right)^n \qquad f(z) = -\frac{2}{2-z} \qquad X(z) = -\frac{z}{z-0.5}$

4 $(n < 0) \quad a_n = (2)^n \qquad f(z) = -\frac{0.5}{0.5-z} \qquad X(z) = -\frac{z}{z-2}$

shift to the right $\rightarrow$ $\times z$ $\times z^{-1}$

insert a_0

7 $(n \leqslant 0) \quad a_n = \left(\frac{1}{2}\right)^{n-1} \qquad f(z) = -\frac{2z}{2-z} \qquad X(z) = -\frac{1}{z-0.5}$

8 $(n \leqslant 0) \quad a_n = (2)^{n-1} \qquad f(z) = -\frac{0.5z}{0.5-z} \qquad X(z) = -\frac{1}{z-2}$

$n=-3$	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$	$n=3$	
b^{-3}	b^{-2}	b^{-1}	b^{0}	b^{1}	b^{2}	b^{3}	

b^n $n=-1,-2,-3,\cdots$

b^n $n=0,1,2,\cdots$

$n=-3$	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$	$n=3$		
	b^{-3}	b^{-2}	b^{-1}	b^{0}	b^{1}	b^{2}	b^{3}	

b^{n-1} $n=0,-1,-2,\cdots$

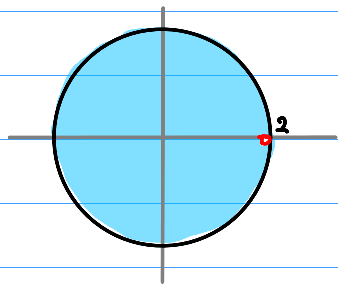
b^{n-1} $n=1,2,3,\cdots$

Causal $(n \geq 0)$ $\quad (\frac{1}{2})^n, \; (2)^n$

$a_n = (\frac{1}{2})^n \quad (n \geq 0)$

$$f(z) = \frac{1}{1-\frac{z}{2}} \qquad |z| < 2$$

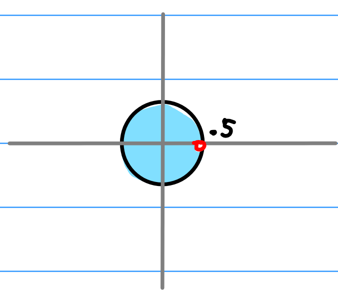
$$= \frac{z^{-1}}{z^{-1}-0.5} = \frac{2}{2-z}$$



$a_n = (2)^n \quad (n \geq 0)$

$$f(z) = \frac{1}{1-2z} \qquad |z| < 0.5$$

$$= \frac{z^{-1}}{z^{-1}-2} = \frac{0.5}{0.5-z}$$

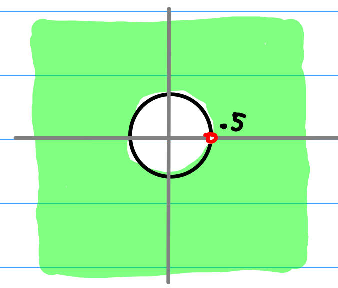


$f(z)$

$a_n = (\frac{1}{2})^n \quad (n \geq 0)$

$$X(z) = \frac{1}{1-\frac{1}{2z}} \qquad |z| > 0.5$$

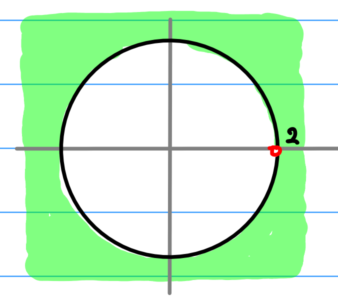
$$= \frac{z}{z-0.5}$$



$a_n = (2)^n \quad (n \geq 0)$

$$X(z) = \frac{1}{1-\frac{2}{z}} \qquad |z| > 2$$

$$= \frac{z}{z-2}$$



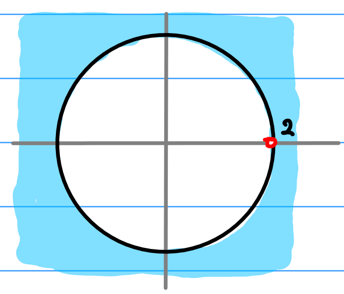
$X(z)$

Anti-causal $(n<0)$ $\left(\frac{1}{2}\right)^n$, $(2)^n$

$$a_n = \left(\frac{1}{2}\right)^n \quad (n<0)$$

$$f(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \qquad |z| > 2$$

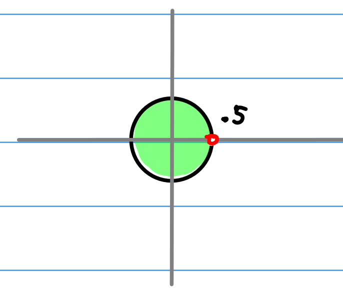
$$= -\frac{2}{2-z}$$



$$a_n = \left(\frac{1}{2}\right)^n \quad (n<0)$$

$$X(z) = \frac{2z}{1-2z} \qquad |z| < 0.5$$

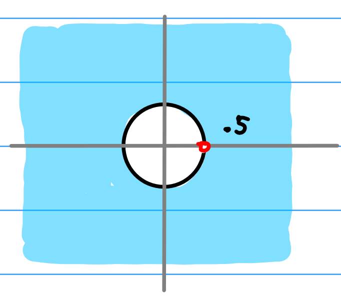
$$= -\frac{z}{z-0.5}$$



$$a_n = (2)^n \quad (n<0)$$

$$f(z) = \frac{\frac{1}{2z}}{1-\frac{1}{2z}} \qquad |z| > 0.5$$

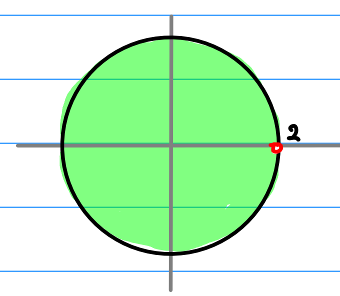
$$= -\frac{0.5}{0.5-z}$$



$$a_n = (2)^n \quad (n<0)$$

$$X(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \qquad |z| < 2$$

$$= -\frac{z}{z-2}$$



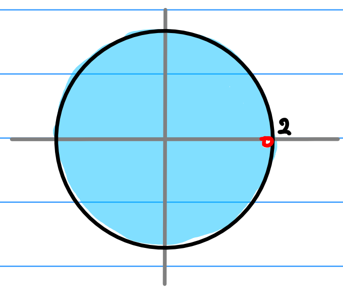
$f(z)$ $X(z)$

Causal $f(z)$

$$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$$

$$f(z) = \frac{1}{1-\frac{z}{2}} \quad |z| < 2$$

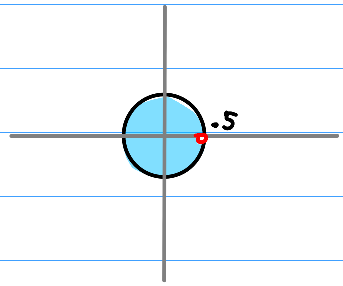
$$= \frac{z^{-1}}{z^{-1}-0.5} = \frac{2}{2-z}$$



$$a_n = (2)^n \quad (n \geq 0)$$

$$f(z) = \frac{1}{1-2z} \quad |z| < 0.5$$

$$= \frac{z^{-1}}{z^{-1}-2} = \frac{0.5}{0.5-z}$$

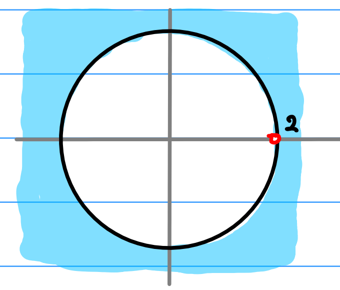


Anti-causal $f(z)$

$$a_n = \left(\frac{1}{2}\right)^n \quad (n < 0)$$

$$f(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \quad |z| > 2$$

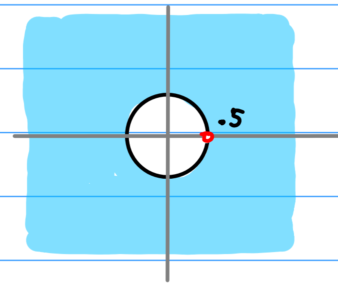
$$= -\frac{2}{2-z}$$



$$a_n = (2)^n \quad (n < 0)$$

$$f(z) = \frac{\frac{1}{2z}}{1-\frac{1}{2z}} \quad |z| > 0.5$$

$$= -\frac{0.5}{0.5-z}$$

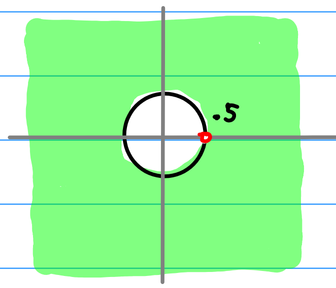


Causal X(z)

$$a_n = \left(\tfrac{1}{2}\right)^n \quad (n \geq 0)$$

$$X(z) = \frac{1}{1-\frac{1}{2z}} \qquad |z| > 0.5$$

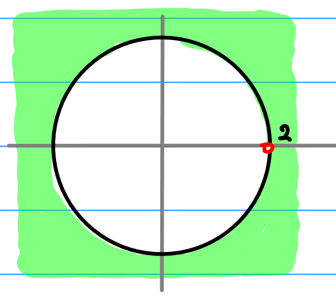
$$= \frac{z}{z-0.5}$$



$$a_n = (2)^n \quad (n \geq 0)$$

$$X(z) = \frac{1}{1-\frac{2}{z}} \qquad |z| > 2$$

$$= \frac{z}{z-2}$$

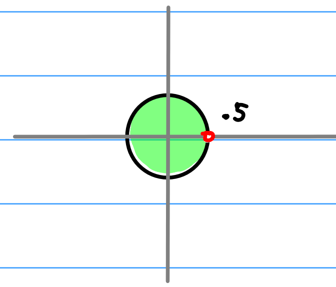


Anti-causal X(z)

$$a_n = \left(\tfrac{1}{2}\right)^n \quad (n < 0)$$

$$X(z) = \frac{2z}{1-2z} \qquad |z| < 0.5$$

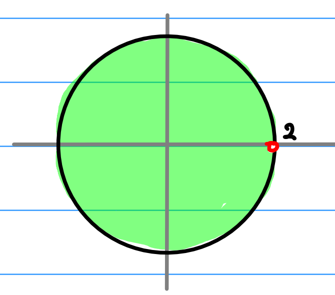
$$= -\frac{z}{z-0.5}$$



$$a_n = (2)^n \quad (n < 0)$$

$$X(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \qquad |z| < 2$$

$$= -\frac{z}{z-2}$$



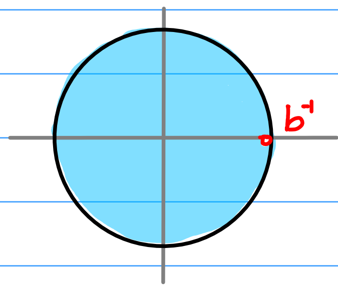
Causal b^n | Anti-causal b^n

$f(z)$

$$a_n = (b)^n \quad (n \geq 0)$$

$$f(z) = \frac{1}{1-bz} \qquad |z| < b^{-1}$$

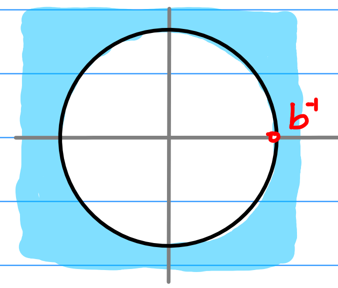
$$= \frac{b^{-1}}{b^{-1}-z}$$



$$a_n = (b)^n \quad (n < 0)$$

$$f(z) = \frac{b^{-1}z^{-1}}{1-b^{-1}z^{-1}} \qquad |z| > b^{-1}$$

$$= -\frac{b^{-1}}{b^{-1}-z}$$

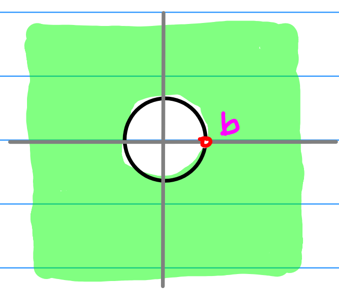


$X(z)$

$$a_n = (b)^n \quad (n \geq 0)$$

$$X(z) = \frac{1}{1-bz^{-1}} \qquad |z| > b$$

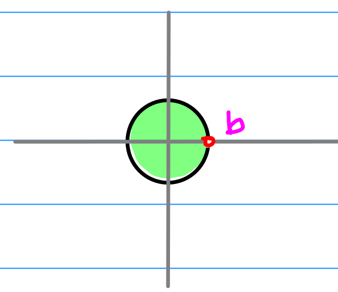
$$= \frac{z}{z-b}$$



$$a_n = (b)^n \quad (n < 0)$$

$$X(z) = \frac{b^{-1}z}{1-b^{-1}z} \qquad |z| < b$$

$$= -\frac{z}{z-b}$$



2 formulas of z

① $\dfrac{-1}{(z-1)(z-2)} = \Big(\qquad\qquad \Big)$

$\updownarrow \; z^{-1}$

② $\dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(\dfrac{0.5z}{(z-0.5)} - \dfrac{2z}{(z-2)} \right)$

case Ⅰ $f(z) = \dfrac{-1}{(z-1)(z-2)}$ $X(z) = \dfrac{-0.5z^2}{(z-1)(z-0.5)}$

case Ⅱ $f(z) = \dfrac{-0.5z^2}{(z-1)(z-0.5)}$ $X(z) = \dfrac{-1}{(z-1)(z-2)}$

case Ⅲ $f(z) = \dfrac{-1}{(z-1)(z-2)}$ $X(z) = \dfrac{-1}{(z-1)(z-2)}$

case Ⅳ $f(z) = \dfrac{-0.5z^2}{(z-1)(z-0.5)}$ $X(z) = \dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}} \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right).$$

$$= \left(\frac{1}{z-0.5}-\frac{1}{z-2}\right)$$

$$\Big\downarrow z^{-1}$$

$$\frac{3}{2}\frac{-1}{(z^{-1}-0.5)(z^{-1}-2)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{1}{z^{-1}-0.5}-\frac{1}{z^{-1}-2}\right)$$

$$= \left(\frac{2}{2z^{-1}-1}-\frac{0.5}{0.5z^{-1}-1}\right)$$

$$= \left(\frac{2z}{2-z}-\frac{0.5z}{0.5-z}\right)$$

$$= \left(\frac{-2z}{z-2}+\frac{0.5z}{z-0.5}\right)$$

$$= z\left(\frac{-2}{z-2}+\frac{0.5}{z-0.5}\right)$$

$$= z\left(\frac{-\frac{3}{2}z}{(z-2)(z-0.5)}\right)$$

$$= \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

① − Ⓐ

$|z| < 0.5$ $\quad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad \boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $\quad (n \geq 0)$

$|z| > 2$ $\quad f(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$ $\quad \boxed{+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}}$ $\quad (n < 0)$

① − Ⓑ

$|z| < 0.5$ $\quad X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n \leq 0)$

$|z| > 2$ $\quad X(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$ $\quad \boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}}$ $\quad (n > 0)$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

② − Ⓐ

$|z| < 0.5$ $\quad f(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$ $\quad \boxed{-2^{n-1} + \left(\frac{1}{2}\right)^{n-1}}$ $\quad (n > 0)$

$|z| > 2$ $\quad f(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$ $\quad \boxed{+2^{n-1} - \left(\frac{1}{2}\right)^{n-1}}$ $\quad (n \leq 0)$

② − Ⓑ

$|z| < 0.5$ $\quad X(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$ $\quad \boxed{-\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}}$ $\quad (n < 0)$

$|z| > 2$ $\quad X(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$ $\quad \boxed{+\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}}$ $\quad (n \geq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
Ⓐ $f(z)$	$\lvert z\rvert < \frac{1}{2}$	$-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$-2^{n-1}+\left(\frac{1}{2}\right)^{n-1}$ $(n>0)$
	$\lvert z\rvert > 2$	$+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$+2^{n-1}-\left(\frac{1}{2}\right)^{n-1}$ $(n \leq 0)$
Ⓑ $X(z)$	$\lvert z\rvert < \frac{1}{2}$	$-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}$ $(n \leq 0)$	$-\left(\frac{1}{2}\right)^{n+1}+2^{n+1}$ $(n<0)$
	$\lvert z\rvert > 2$	$+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}$ $(n>0)$	$+\left(\frac{1}{2}\right)^{n+1}-2^{n+1}$ $(n \geq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$-2^{n-1}+\left(\frac{1}{2}\right)^{n-1}$ $(n>0)$
	$X(z)$	$-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}$ $(n \leq 0)$	$-\left(\frac{1}{2}\right)^{n+1}+2^{n+1}$ $(n<0)$
$\lvert z\rvert > 2$	$f(z)$	$+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$+2^{n-1}-\left(\frac{1}{2}\right)^{n-1}$ $(n \leq 0)$
	$X(z)$	$+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}$ $(n>0)$	$+\left(\frac{1}{2}\right)^{n+1}-2^{n+1}$ $(n \geq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	causal $(n \geq 0)$	causal $(n > 0)$
$\lvert z\rvert > 2$	$f(z)$	anticausal $(n < 0)$	anticausal $(n \leq 0)$
$\lvert z\rvert < \frac{1}{2}$	$X(z)$	anticausal $(n \leq 0)$	anticausal $(n < 0)$
$\lvert z\rvert > 2$	$X(z)$	causal $(n > 0)$	causal $(n \geq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	causal $(n \geq 0)$	causal $(n > 0)$
$\lvert z\rvert < \frac{1}{2}$	$X(z)$	anticausal $(n \leq 0)$	anticausal $(n < 0)$
$\lvert z\rvert > 2$	$f(z)$	anticausal $(n < 0)$	anticausal $(n \leq 0)$
$\lvert z\rvert > 2$	$X(z)$	causal $(n > 0)$	causal $(n \geq 0)$

① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}}$ ② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

$$\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right) \qquad \left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad -\frac{z}{1-2z}+\frac{z}{1-0.5z}$$

$\boxed{|z|<0.5}$ $|2z|<1$ $|0.5z|<1$ $\boxed{|z|<0.5}$ $|2z|<1$ $|0.5z|<1$

$$\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad \frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$$

$\boxed{|z|>2}$ $|0.5z^{-1}|<1$ $|2z^{-1}|<1$ $\boxed{|z|>2}$ $|0.5z^{-1}|<1$ $|2z^{-1}|<1$

$$① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \quad \overset{z^{-1}}{\longleftrightarrow} \quad ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$$\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right) \qquad \left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad -\frac{z}{1-2z}+\frac{z}{1-0.5z}$$

$\boxed{|z|<0.5}$ f(z) causal $(n \geq 0)$ — X(z) anticausal $(n \leq 0)$

$\boxed{|z|<0.5}$ f(z) causal $(n > 0)$ — X(z) anticausal $(n < 0)$

$$\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad \frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$$

$\boxed{|z|>2}$ f(z) anticausal $(n < 0)$ — X(z) causal $(n > 0)$

$\boxed{|z|>2}$ f(z) anticausal $(n \leq 0)$ — X(z) causal $(n \geq 0)$

① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$ $\overset{z^{-1}}{\longleftrightarrow}$ ② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

$|z|<0.5$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$$

$$f(z) = -\left[2+2^2z^1+2^3z^2+\cdots\right] \quad -2^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right)+\left(\tfrac{1}{2}\right)^2z^1+\left(\tfrac{1}{2}\right)^3z^2+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^{-1}+\left(\tfrac{1}{2}\right)^{-2}z^1+\left(\tfrac{1}{2}\right)^{-3}z^2+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n-1}$$
$$+\left[2^{-1}+2^{-2}z^1+2^{-3}z^2+\cdots\right] \quad +2^{n-1}$$

$|z|<0.5$

$$-\frac{z}{1-2z}+\frac{z}{1-0.5z}$$

$$f(z) = -\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right] \quad -2^{n-1}$$
$$+\left[\left(\tfrac{1}{2}\right)^0z^1+\left(\tfrac{1}{2}\right)^1z^2+\left(\tfrac{1}{2}\right)^2z^3+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n-1}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^0z^1+\left(\tfrac{1}{2}\right)^{-1}z^2+\left(\tfrac{1}{2}\right)^{-2}z^3+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$
$$+\left[2^0z^1+2^{-1}z^2+2^{-2}z^3+\cdots\right] \quad +2^{n+1}$$

$|z|>2$

$$\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$$f(z) = +\left[2^0z^{-1}+2^{-1}z^{-2}+2^{-2}z^{-3}+\cdots\right] \quad +2^{n+1}$$
$$-\left[\left(\tfrac{1}{2}\right)^0z^{-1}+\left(\tfrac{1}{2}\right)^{-1}z^{-2}+\left(\tfrac{1}{2}\right)^{-2}z^{-3}+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0z^{-1}+\left(\tfrac{1}{2}\right)^1z^{-2}+\left(\tfrac{1}{2}\right)^2z^{-3}+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n-1}$$
$$-\left[2^0z^{-1}+2^1z^{-2}+2^2z^{-3}+\cdots\right] \quad -2^{n-1}$$

$|z|>2$

$$\frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$$

$$f(z) = +\left[2^{-1}z^0+2^{-2}z^{-1}+2^{-3}z^{-2}+\cdots\right] \quad +2^{n-1}$$
$$-\left[\left(\tfrac{1}{2}\right)^{-1}z^0+\left(\tfrac{1}{2}\right)^{-2}z^{-1}+\left(\tfrac{1}{2}\right)^{-3}z^{-2}+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n-1}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^1z^0+\left(\tfrac{1}{2}\right)^2z^{-1}+\left(\tfrac{1}{2}\right)^3z^{-2}+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$
$$-\left[2^1z^0+2^2z^{-1}+2^3z^{-2}+\cdots\right] \quad -2^{n+1}$$

$$① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$|z| < 0.5$

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -\left[2 + 2^2 z^1 + 2^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$a_n = -2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^{-1} + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right] + \left[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right]$$

$$a_n = -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1} \quad (n \leq 0)$$

$|z| > 2$

$$\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$f(z) = +\left[2^0 z^{-1} + 2^{-1} z^{-2} + 2^{-2} z^{-3} + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right]$$

$$a_n = +2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right] - \left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right]$$

$$a_n = +\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1} \quad (n > 0)$$

$|z| < 0.5$

$$-\frac{z}{1-2z} + \frac{z}{1-0.5z}$$

$$f(z) = -\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right] + \left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^1 z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right]$$

$$a_n = -2^{n-1} + \left(\tfrac{1}{2}\right)^{n-1} \quad (n > 0)$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right] + \left[2^0 z^1 + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right]$$

$$a_n = -\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1} \quad (n < 0)$$

$|z| > 2$

$$\frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$

$$f(z) = +\left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^{-1} + \left(\tfrac{1}{2}\right)^{-3} z^{-2} + \cdots\right]$$

$$a_n = +2^{n-1} - \left(\tfrac{1}{2}\right)^{n-1} \quad (n \leq 0)$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right] - \left[2^1 z^0 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right]$$

$$a_n = +\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1} \quad (n \geq 0)$$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$-2^{n-1} + \left(\frac{1}{2}\right)^{n-1}$ $(n > 0)$
		$\updownarrow$ -1	$\updownarrow$ -1
$\lvert z\rvert > 2$	$f(z)$	$+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n < 0)$	$+2^{n-1} - \left(\frac{1}{2}\right)^{n-1}$ $(n \leq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$			
	$X(z)$	$-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$ $(n \leq 0)$	$-\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}$ $(n < 0)$
$\lvert z\rvert > 2$		$\updownarrow$ -1	$\updownarrow$ -1
	$X(z)$	$+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}$ $(n > 0)$	$+\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}$ $(n \geq 0)$

$2^{-n+1} = \left(\frac{1}{2}\right)^{n} \cdot 2 = \left(\frac{1}{2}\right)^{n-1}$ $\qquad$ $\left(\frac{1}{2}\right)^{-n-1} = 2^{n} \cdot 2 = 2^{n+1}$

$\left(\frac{1}{2}\right)^{-n+1} = 2^{n} \cdot \frac{1}{2} = 2^{n-1}$ $\qquad$ $2^{-n-1} = \left(\frac{1}{2}\right)^{n} \cdot \frac{1}{2} = \left(\frac{1}{2}\right)^{n+1}$

z^{-1}

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$-2^{n-1} + \left(\frac{1}{2}\right)^{n-1}$ $(n>0)$
		$-n$	$-n$
$\lvert z\rvert > 2$	$f(z)$	$+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$+2^{n-1} - \left(\frac{1}{2}\right)^{n-1}$ $(n \leq 0)$

z^{-1}

z^{-1}

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$			
	$X(z)$	$-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$ $(n \leq 0)$	$-\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}$ $(n<0)$
$\lvert z\rvert > 2$		$-n$	$-n$
	$X(z)$	$+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}$ $(n>0)$	$+\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}$ $(n \geq 0)$

z^{-1}

z^{-1}

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n\geq 0)$	$-2^{n-1}+\left(\frac{1}{2}\right)^{n-1}$ $(n>0)$
		$-n,$	-1
$\lvert z\rvert > 2$	$f(z)$	$+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$+2^{n-1}-\left(\frac{1}{2}\right)^{n-1}$ $(n\leq 0)$
		$-n,$	-1

z^{-1}

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$		$-n,$	-1
	$X(z)$	$-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}$ $(n\leq 0)$	$-\left(\frac{1}{2}\right)^{n+1}+2^{n+1}$ $(n<0)$
$\lvert z\rvert > 2$		$-n,$	-1
	$X(z)$	$+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}$ $(n>0)$	$+\left(\frac{1}{2}\right)^{n+1}-2^{n+1}$ $(n\geq 0)$

		① $\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$-n$
	$X(z)$	$-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}$ $(n \leq 0)$	
$\lvert z\rvert > 2$	$f(z)$	$+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$-n$
	$X(z)$	$+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}$ $(n>0)$	

		① $\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-n$	$-2^{n-1}+\left(\frac{1}{2}\right)^{n-1}$ $(n>0)$
	$X(z)$		$-\left(\frac{1}{2}\right)^{n+1}+2^{n+1}$ $(n<0)$
$\lvert z\rvert > 2$	$f(z)$	$-n$	$+2^{n-1}-\left(\frac{1}{2}\right)^{n-1}$ $(n \leq 0)$
	$X(z)$		$+\left(\frac{1}{2}\right)^{n+1}-2^{n+1}$ $(n \geq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	case Ⅰ $(n \geq 0)$	case Ⅱ $(n > 0)$
	$X(z)$	case Ⅱ $(n \leq 0)$	case Ⅰ $(n < 0)$
$\lvert z\rvert > 2$	$f(z)$	case Ⅰ $(n < 0)$	case Ⅱ $(n \leq 0)$
	$X(z)$	case Ⅱ $(n > 0)$	case Ⅰ $(n \geq 0)$

case Ⅰ $\quad f(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \qquad X(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

case Ⅱ $\quad f(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} \qquad X(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$

case Ⅲ $\quad f(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \qquad X(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$

case Ⅳ $\quad f(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} \qquad X(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	case Ⅰ $(n \geq 0)$	$-n$
	$X(z)$		$-n$ case Ⅰ $(n<0)$
$\lvert z\rvert > 2$	$f(z)$	case Ⅰ $(n<0)$	$-n$
	$X(z)$		$-n$ case Ⅰ $(n \geq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-n$ $-n$	case Ⅱ $(n>0)$
	$X(z)$	case Ⅱ $(n \leq 0)$	
$\lvert z\rvert > 2$	$f(z)$	$-n$	case Ⅱ $(n \leq 0)$
	$X(z)$	$-n$ case Ⅱ $(n>0)$	

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	case Ⅲ	case Ⅳ
	$X(z)$	case Ⅲ	case Ⅳ
$\lvert z\rvert > 2$	$f(z)$	case Ⅲ	case Ⅳ
	$X(z)$	case Ⅲ	case Ⅳ

case Ⅰ $\quad f(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \qquad X(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

case Ⅱ $\quad f(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} \qquad X(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$

case Ⅲ $\quad f(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \qquad X(z) = \frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$

case Ⅳ $\quad f(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} \qquad X(z) = \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	case Ⅲ	$-n$
	$X(z)$	case Ⅲ	
$\lvert z\rvert > 2$	$f(z)$	case Ⅲ	$-n$
	$X(z)$	case Ⅲ	

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-n$	case Ⅳ
	$X(z)$		case Ⅳ
$\lvert z\rvert > 2$	$f(z)$	$-n$	case Ⅳ
	$X(z)$		case Ⅳ

$f(z)$ $\quad |z| < 0.5 \quad |z| > 2$

causal $\quad$ anticausal

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$

$$f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$\frac{a}{1-az} \qquad \frac{z^{-1}}{a^{-1}z^{-1}-1}$$

$$-\left(\underset{n=0}{2} + \underset{n=1}{2^2 z} + \underset{n=2}{2^3 z^2} + \cdots\right) + \left(\underset{n=0}{\left(\tfrac{1}{2}\right)} + \underset{n=1}{\left(\tfrac{1}{2}\right)^2 z} + \underset{n=2}{\left(\tfrac{1}{2}\right)^3 z^2} + \cdots\right)$$

$|z| > 2$

$$f(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$\left(z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(z^{-1} + 2z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$$\left(\underset{n=-1}{2^0 z^{-1}} + \underset{n=-2}{2^{-1} z^{-2}} + \underset{n=-3}{2^{-2} z^{-3}} + \cdots\right) - \left(\underset{n=-1}{\left(\tfrac{1}{2}\right)^0 z^{-1}} + \underset{n=-2}{\left(\tfrac{1}{2}\right)^{-1} z^{-2}} + \underset{n=-3}{\left(\tfrac{1}{2}\right)^{-2} z^{-3}} + \cdots\right)$$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$|z| < 0.5$

$$f(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z} \qquad \boxed{-2^{n-1} + \left(\tfrac{1}{2}\right)^{n-1}} \quad (n > 0)$$

$$-\left(\underset{n=1}{z} + \underset{n=2}{2z^2} + \underset{n=3}{2^2 z^3} + \cdots\right) + \left(\underset{n=1}{z} + \underset{n=2}{\left(\tfrac{1}{2}\right) z^2} + \underset{n=3}{\left(\tfrac{1}{2}\right)^2 z^3} + \cdots\right)$$

$|z| > 2$

$$f(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}} \qquad \boxed{+2^{n-1} - \left(\tfrac{1}{2}\right)^{n-1}} \quad (n \leq 0)$$

$$\left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right) + \left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right)$$

$$\left(\underset{n=0}{2^{-1}} + \underset{n=-1}{2^{-2} z^{-1}} + \underset{n=-2}{2^{-3} z^{-2}} + \cdots\right) + \left(\underset{n=0}{\left(\tfrac{1}{2}\right)^{-1}} + \underset{n=-1}{\left(\tfrac{1}{2}\right)^{-2} z^{-1}} + \underset{n=-2}{\left(\tfrac{1}{2}\right)^{-3} z^{-2}} + \cdots\right)$$

$$X(z) \qquad |z| < 0.5 \qquad |z| > 2$$

anticausal causal

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$

$$X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \qquad (n \leq 0)$$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad n=0 \quad n=-1 \quad n=-2$

$|z| > 2$

$$X(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \qquad (n > 0)$$

$$\left(\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$|z| < 0.5$

$$X(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z} \qquad \boxed{-\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1}} \qquad (n < 0)$$

$$-\left(z + 2z^2 + 2^2 z^3 + \cdots\right) + \left(z + \left(\tfrac{1}{2}\right) z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^0 z + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right) + \left(2^0 z + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right)$$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

$|z| > 2$

$$X(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}} \qquad \boxed{+\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}} \qquad (n \geq 0)$$

$$\left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right) + \left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5}-\frac{1}{z-2}\right)$$

$|z|<0.5 \quad f(z) \qquad a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n \leq 0)$

$|z|>2 \quad f(z) \qquad b_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n > 0)$

$\{\,|z|<0.5\,\} \cap \{\,|z|>2\,\} = \phi \Longrightarrow a_n + b_n = 0$

$a_n = -b_n$

$|z|<a \quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n>0 \quad n \leq 0 \quad n<0$

$\| \qquad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n \qquad \big\downarrow -n$

$|z|>a \quad f(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k} \qquad a^{-n+1} \qquad n \leq 0 \quad n<0 \quad n \geq 0 \quad n>0$

$= \left(\frac{1}{a}\right)^{n-1}$

$a^{n+1} z^n$

$a\,(az)^n$

$a\left(\frac{1}{az}\right)^{-n}$

$\frac{a}{1-az} \qquad \sum_{n=0}^{\infty} a^{n+1} z^n \qquad \frac{z}{1-az} \qquad \sum_{n=1}^{\infty} a^{n-1} z^n$

$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \qquad -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad -\frac{a^{-1}}{1-a^{-1}z^{-1}} \qquad -\sum_{n=1}^{\infty} a^{-n+1} z^{-n}$

$\sum_{n=-1}^{-\infty} a^{n+1} z^n \qquad -\sum_{n=0}^{-\infty} a^{n-1} z^n$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$|z|<0.5 \qquad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \qquad (n \geq 0)$$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad\qquad n=0 \quad n=1 \quad n=2$

$$|z|<0.5 \qquad X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \qquad (n \leq 0)$$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad\qquad n=0 \quad n=-1 \quad n=-2$

$$\text{ROC} \qquad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n > 0 \quad n \leq 0 \quad n < 0$$

$$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n \qquad -n$$

$$\text{ROC} \qquad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k} \qquad a^{-n+1} = \left(\frac{1}{a}\right)^{n-1} \qquad n \leq 0 \quad n < 0 \quad n \geq 0 \quad n > 0$$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^{n} \qquad a^{n+1} \qquad n \geq 0 \quad n > 0 \quad n \leq 0 \quad n < 0$

$z^{-1} \qquad z^{-1} \qquad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^{n} \qquad -n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} (a)^{k-1} z^{-k} \qquad \left(\frac{1}{a}\right)^{-n+1} \qquad n \leq 0 \quad n < 0 \quad n \geq 0 \quad n > 0$

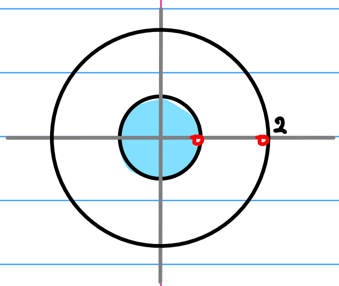
$= a^{n-1}$

Causal $\quad f(z) \quad X(z)$

$|z| < 0.5 \quad |z| > 2$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z|<2 \quad |z|<0.5$



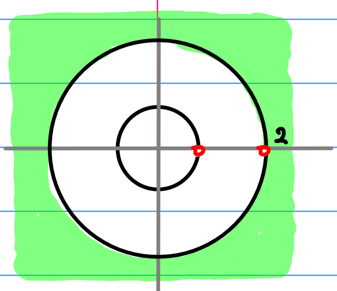
$$f(z) = (-2)\frac{0.5}{0.5-z} + (0.5)\frac{2}{2-z} \qquad (|z|<0.5)$$

$$a_n = (-2)\ 2^n + (0.5)\ \left(\tfrac{1}{2}\right)^n \qquad (n \geq 0)$$

$$-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}$$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) \qquad |z|>2$$

$|z|>2 \quad |z|>0.5$



$$X(z) = 0.5\frac{z}{z-0.5} - 2\frac{z}{z-2} \qquad (|z|>2)$$

$$a_n = (0.5)\left(\tfrac{1}{2}\right)^n - 2 \cdot 2^n \qquad (n \geq 0)$$

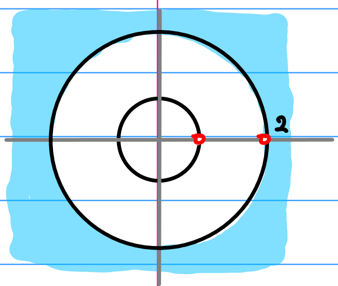
$$\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}$$

Anti-causal $\quad f(z) \quad X(z)$

$|z| > 2 \quad |z| < 0.5$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| > 2 \qquad |z| > 0.5$



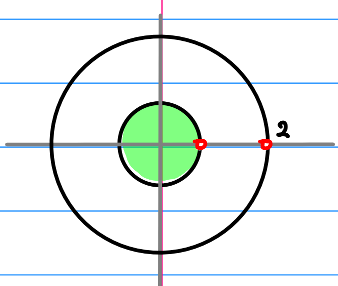
$$f(z) = (-2)\,\frac{-0.5}{0.5-z} + (0.5)\,\frac{-2}{2-z} \qquad (|z| > 0.5)$$

$$a_n = (+2)\;2^n - (0.5)\left(\tfrac{1}{2}\right)^n \qquad (n < 0)$$

$$+2^{n+1} \quad - \quad \left(\tfrac{1}{2}\right)^{n+1}$$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) \qquad |z| < 2$$

$|z| < 2 \qquad |z| < 0.5$



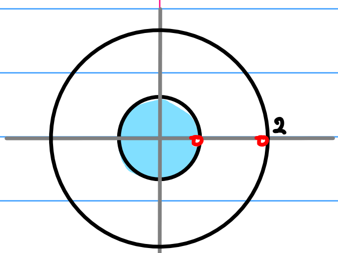
$$X(z) = 0.5\,\frac{-z}{z-0.5} - 2\,\frac{-z}{z-2} \qquad (|z| < 2)$$

$$a_n = -(0.5)\left(\tfrac{1}{2}\right)^n + 2 \cdot 2^n \qquad (n < 0)$$

$$-\left(\tfrac{1}{2}\right)^{n+1} + \quad 2^{n+1}$$

①-Ⓐ $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \boxed{f(z)}$ $|z| < 0.5$ causal $|z| > 2$ anticausal

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{-2}{1-2z} + \frac{0.5}{1-0.5z}$$

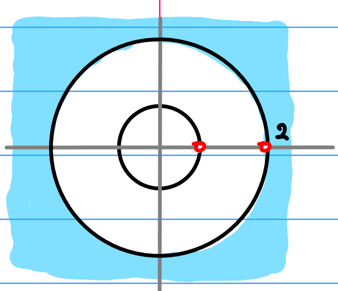


$|z| < 0.5$

$$f(z) = \boxed{\begin{aligned} &\frac{(-2)}{1-(2z)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)} \\ &= -\sum_{n=0}^{\infty}(2)^{n+1}(z)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}(z)^n \\ &= -\sum_{n=0}^{\infty}(2)^{n+1} z^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^n \end{aligned}}$$

$(n \geq 0)$ $a_n = -2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



$|z| > 2$

$$f(z) = \boxed{\begin{aligned} &\frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{2z}\right)} - \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{2}{z}\right)} \\ &= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n\left(\frac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty}(2)^n\left(\frac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1} z^{-n} - \sum_{n=1}^{\infty}(2)^{n-1} z^{-n} \end{aligned}} \quad \neq$$

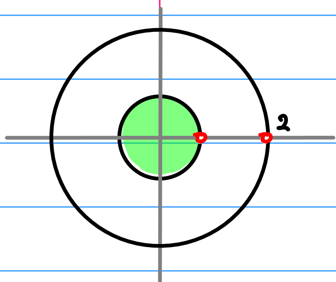
$$= \sum_{n=-1}^{-\infty}(2)^{n+1} z^n - \sum_{n=-1}^{-\infty}\left(\frac{1}{2}\right)^{n+1} z^n$$

$(n < 0)$ $a_n = 2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$

① - Ⓑ $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \boxed{X(z)}$

$|z| < 0.5$ anticausal $\qquad |z| > 2$ causal

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{-2}{1-2z} + \frac{0.5}{1-0.5z}$$



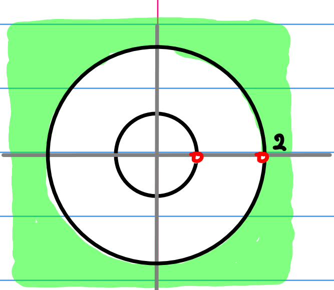
$|z|<0.5$

$$X(z) = \boxed{\begin{aligned} &\frac{(-2)}{1-(2z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(2)^{n+1}(z)^n + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}(z)^n \\ &= -\sum_{n=0}^{\infty}(2)^{n+1} z^n + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1} z^n \end{aligned}}$$

$$= -\sum_{n=0}^{-\infty}\left(\tfrac{1}{2}\right)^{n-1} z^{-n} + \sum_{n=0}^{-\infty}(2)^{n-1} z^{-n}$$

$(n \leq 0) \qquad a_n = -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



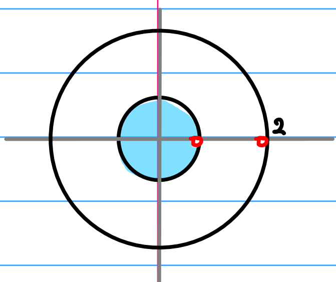
$|z|>2$

$$X(z) = \boxed{\begin{aligned} &\frac{(\frac{1}{z})}{1-(\frac{1}{2z})} - \frac{(\frac{1}{z})}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^n\left(\tfrac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty}(2)^n\left(\tfrac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty}\left(\tfrac{1}{2}\right)^{n-1} z^{-n} - \sum_{n=1}^{\infty}(2)^{n-1} z^{-n} \end{aligned}} \qquad \neq$$

$(n > 0) \qquad a_n = \left(\tfrac{1}{2}\right)^{n-1} - (2)^{n-1}$

② - Ⓐ $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \boxed{f(z)}$ $|z| < 0.5$ causal $|z| > 2$ anticausal

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$$



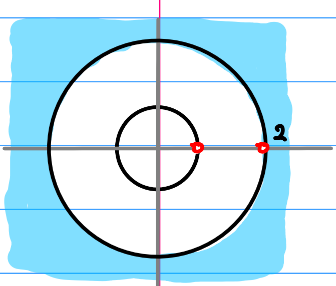
$|z| < 0.5$

$$\boxed{\begin{aligned} f(z) &= -\frac{(z)}{1-(2z)} + \frac{(z)}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(2)^n(z)^{n+1} + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^n(z)^{n+1} \\ &= -\sum_{n=1}^{\infty}(2)^{n-1}z^n + \sum_{n=1}^{\infty}\left(\tfrac{1}{2}\right)^{n-1}z^n \end{aligned}}$$

$\neq$

$(n > 0)$ $a_n = -2^{n-1} + \left(\frac{1}{2}\right)^{n-1}$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$



$|z| > 2$

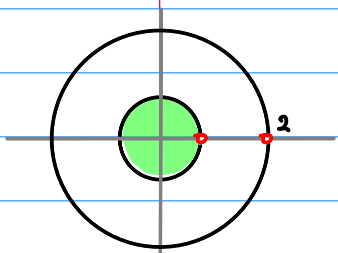
$$\boxed{\begin{aligned} f(z) &= \frac{(\frac{1}{2})}{1-(\frac{1}{2z})} - \frac{(2)}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}\left(\tfrac{1}{z}\right)^n - \sum_{n=0}^{\infty}(2)^{n+1}\left(\tfrac{1}{z}\right)^n \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}z^{-n} - \sum_{n=0}^{\infty}(2)^{n+1}z^{-n} \end{aligned}}$$

$$= \sum_{n=0}^{-\infty}(2)^{n-1}z^n - \sum_{n=0}^{-\infty}\left(\tfrac{1}{2}\right)^{n-1}z^n$$

$(n \leq 0)$ $a_n = 2^{n-1} - \left(\frac{1}{2}\right)^{n-1}$

② - Ⓑ $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \boxed{X(z)}$ $|z| < 0.5$ anticausal $|z| > 2$ causal

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$$



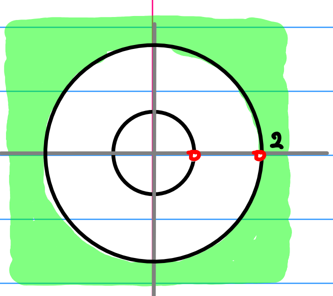
$|z| < 0.5$

$$\begin{aligned} X(z) &= -\frac{(z)}{1-(2z)} + \frac{(z)}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(2)^n(z)^{n+1} + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^n(z)^{n+1} \\ &= -\sum_{n=1}^{\infty}(2)^{n-1}z^n + \sum_{n=1}^{\infty}\left(\tfrac{1}{2}\right)^{n-1}z^n \\ &= -\sum_{n=-1}^{-\infty}\left(\tfrac{1}{2}\right)^{n+1}z^{-n} + \sum_{n=-1}^{-\infty}(2)^{n+1}z^{-n} \end{aligned}$$

$\neq$

$(n < 0)$ $a_n = -\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$



$|z| > 2$

$$\begin{aligned} X(z) &= \frac{(\frac{1}{2})}{1-(\frac{1}{2z})} - \frac{(2)}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}\left(\tfrac{1}{z}\right)^n - \sum_{n=0}^{\infty}(2)^{n+1}\left(\tfrac{1}{z}\right)^n \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}z^{-n} - \sum_{n=0}^{\infty}(2)^{n+1}z^{-n} \end{aligned}$$

$(n \geq 0)$ $a_n = \left(\frac{1}{2}\right)^{n+1} - 2^{n+1}$