Multiple Linear Regression I

Lecture 7

Survey Research & Design in Psychology

James Neill, 2018

Readings

1. Howitt & Cramer (2014):
   - Regression: Prediction with precision [Ch 9] [Textbook/UCLearn Reading List]
   - Multiple regression & multiple correlation [Ch 32] [Textbook/UCLearn Reading List]
2. StatSoft (2016). *How to find relationship between variables, multiple regression*. StatSoft Electronic Statistics Handbook. [Online]
3. Tabachnick & Fidell (2013). Multiple regression (Ch 5) (includes example write-ups) [UCLearn Reading List]

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Overview

1 Correlation (Review)
2 Simple linear regression
3 Multiple linear regression

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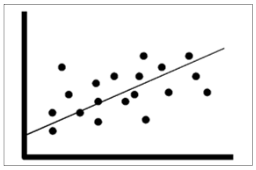
Correlation (Review)



Linear relation between two variables

Purposes of correlational statistics

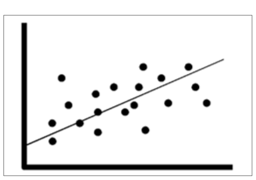
Purpose	Correlation	Factor analysis	Regression
Exploratory	√	√	
Descriptive	√	√	
Explanatory	√		√
Predictive			√

Explanatory - Regression
e.g., cross-sectional study (all data collected at same time)

Predictive - Regression
e.g., longitudinal study (predictors collected prior to outcome measures)

Linear correlation

- Linear relations between interval or ratio variables
- Best fitting straight-line on a scatterplot



Correlation - Key points

- Covariance = sum of cross-products (unstandardised)
- Correlation = sum of cross-products (standardised), ranging from -1 to 1 (sign indicates direction, value indicates size)
- Coefficient of determination (r^2) indicates % of shared variance
- Correlation does not necessarily equal causality

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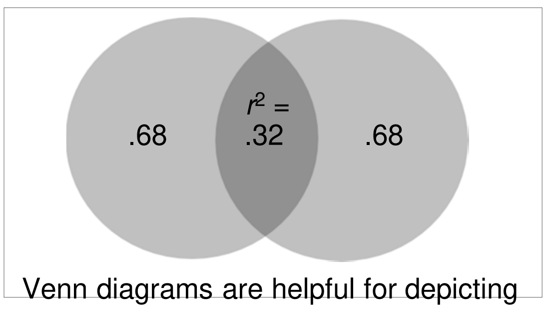
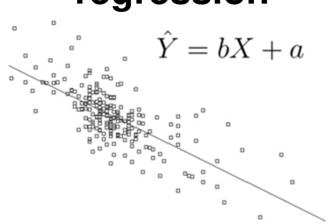
Correlation is shared variance



Venn diagrams are helpful for depicting relations between variables.

Simple linear regression

$$\hat{Y} = bX + a$$



Explains and predicts a Dependent Variable (DV) based on a linear relation with an Independent Variable (IV)

Linear regression

- Extension of correlation
- Best-fitting straight line for a scatterplot between two variables:
  - **predictor (X)** – or independent variable (IV)
  - **outcome (Y)** - or dependent variable (DV) or criterion variable
- IV is used to explain a DV
- Helps to understand relationships and possible causal effects of one variable on another.

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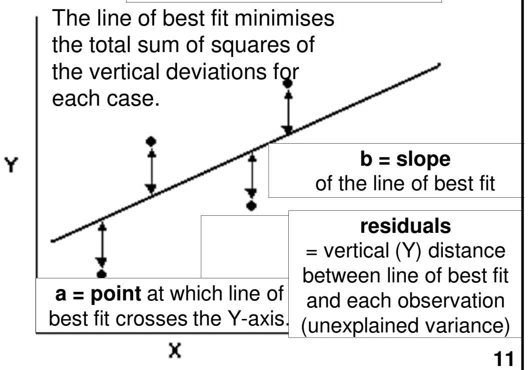
Least squares criterion



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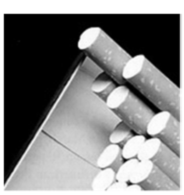
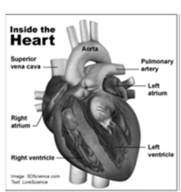
Linear regression - Example:
Cigarettes & coronary heart disease

Landwehr & Watkins (1987, cited in Howell, 2004, pp. 216-218)





IV = Cigarette consumption

DV = Coronary Heart Disease

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Linear regression - Example:
Cigarettes & coronary heart disease
(Howell, 2004)

Research question:
How fast does CHD mortality rise with a one unit increase in smoking?
- **IV** = Av. # of cigs per adult per day
- **DV** = CHD mortality rate (deaths per 10,000 per year due to CHD)
- **Unit of analysis** = Country

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Linear regression - Example:
Cigarettes & coronary heart disease
(Howell, 2004)

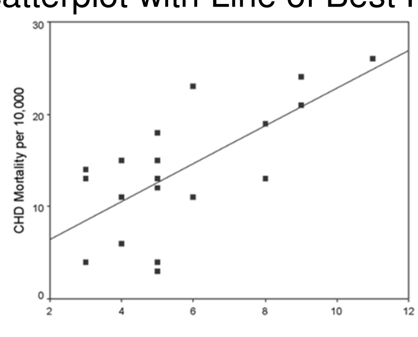
Cigarette Consumption and Coronary Heart Disease Mortality for 21 Countries

Cig.	11	9	9	9	8	8	8	6	6	5	5
CHD	26	21	24	21	19	13	19	11	23	15	13

Cig.	5	5	5	5	4	4	4	3	3	3
CHD	4	18	12	3	11	15	6	13	4	14

Cig. = Cigarettes per adult per day
CHD = Cornary Heart Disease Mortality per 10,000 population

Linear regression - Example:
Scatterplot with Line of Best Fit



Linear regression - Equation

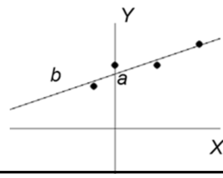
(without error)

$$\hat{Y} = bX + a$$

predicted values of *Y*

b = slope = rate of predicted ↑/↓ for Y scores for each unit increase in *X*

Y-intercept = level of *Y* when X is 0



Linear regression equation (with error)

$$Y = bX + a + e$$

X = IV values

Y = DV values

a = *Y*-axis intercept

b = slope of line of best fit

(regression coefficient)

e = error

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Linear regression - Example Equation

$$\hat{Y} = bX + a$$

Variables:

- (DV) = predicted rate of CHD mortality
- *X* (IV) = mean # of cigarettes per adult per day per country

Regression co-efficients:

- *b* = rate of ↑/↓ of CHD mortality for each extra cigarette smoked per day
- *a* = baseline level of CHD (i.e., CHD when no cigarettes are smoked)

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Linear regression - Example
Explained variance

- $r = .71$
- $r^2 = .71^2 = .51$
- $p < .05$
- Approximately 50% in variability of incidence of CHD mortality is associated with variability in countries' smoking rates.

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Linear regression - Example:
Test for overall significance

$r = .71$, $r^2 = .51$, $p < .05$

ANOVA[b]

	Sum of Squares	df	Mean Square	F	Sig.
Regression	454.482	1	454.48	19.59	.00[a]
Residual	440.757	19	23.198		
Total	895.238	20			

a. Predictors: (Constant), Cigarette Consumption per Adult per Day

b. Dependent Variable: CHD Mortality per 10,000

Linear regression - Example:
Regression coefficients - SPSS

Coefficients[a]

		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
(Constant)	*a*	2.37	2.941		.80	.43
Cigarette Consumption per Adult per Day	*b*	2.04	.461	.713	4.4	.00

a. Dependent Variable: CHD Mortality per 10,000

Linear regression - Example:
Making a prediction

- What if we want to predict CHD mortality when cigarette consumption is 6?

$$\hat{Y} = bX + a = 2.04X + 2.37$$

$$\hat{Y} = 2.04 * 6 + 2.37 = 14.61$$

- We predict that 14.61 / 10,000 people in a country with an average cigarette consumption of 6 per person will die of CHD per annum.

Linear regression - Example
Accuracy of prediction - Residual

- Finnish smokers smoke 6 cigarettes per adult per day
- We predict 14.61 deaths / 10,000
- But Finland actually has 23 deaths / 10,000
- Therefore, the error ("residual") for this case is 23 - 14.61 = 8.39

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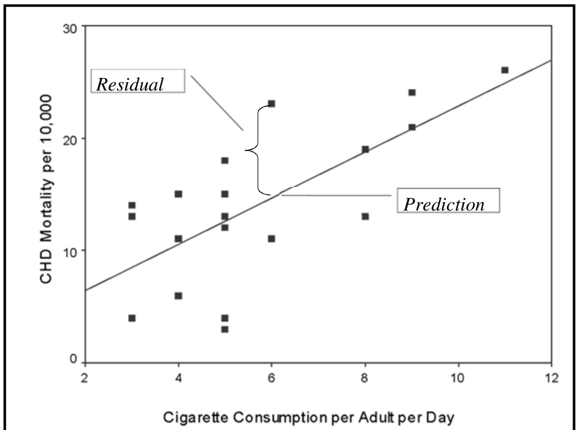

Hypothesis testing

Null hypotheses (H_0):

- *a* (Y-intercept) = 0
  Unless the DV is ratio (meaningful 0), we are not usually very interested in the *a* value (starting value of Y when X is 0).
- *b* (slope of line of best fit) = 0

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Linear regression - Example: Testing slope and intercept

Coefficients[a]

a is not significant - baseline CHD may be negligible *b* is significant (+ve) - smoking is positively associated with CHD		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
a	(Constant)	2.37	2.941		.80	.43
b	Cigarette Consumption per Adult per Day	2.04	.461	.713	4.4	.00

a. Dependent Variable: CHD Mortality per 10,000

Linear regression - Example

Does a tendency to "ignore problems" (IV) predict "psychological distress" (DV)?

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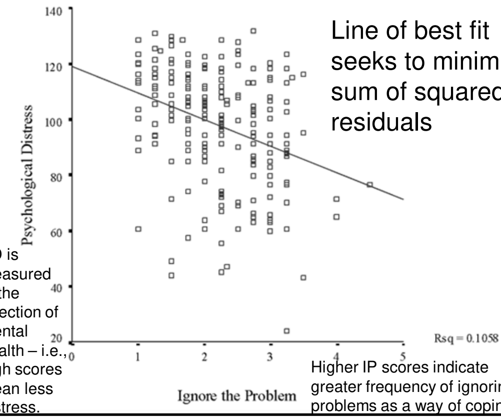


Line of best fit seeks to minimise sum of squared residuals

PD is measured in the direction of mental health – i.e., high scores mean less distress.

Higher IP scores indicate greater frequency of ignoring problems as a way of coping.

Linear regression - Example: Model summary

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.325[a]	.106	.102	19.4851

a. Predictors: (Constant), IGNO2 ACS Time 2 - 11. Ignore

R = .32, R^2 = .11, Adjusted R^2 = .10

Ignoring Problems accounts for ~10% of the variation in Psychological Distress

The predictor (Ignore the Problem) explains approximately 10% of the variance in the dependent variable (Psychological Distress).

Linear regression - Example: Overall significance

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	9789.888	1	9789.888	25.785	.000[a]
	Residual	82767.884	218	379.669		
	Total	92557.772	219			

a. Predictors: (Constant), IGNO2 ACS Time 2 - 11. Ignore

b. Dependent Variable: GWB2NEG

The population relationship between Ignoring Problems and Psychological Distress is unlikely to be 0% because p = .000
(i.e., reject the null hypothesis that there is no relationship)

Linear regression - Example:

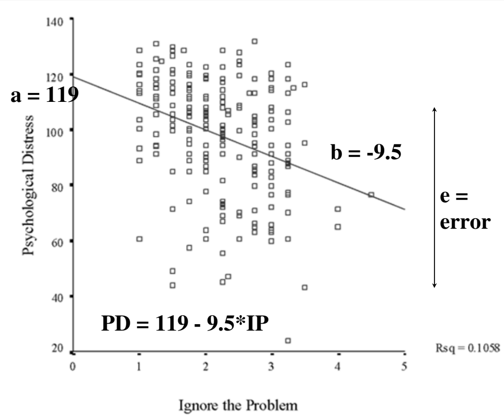
Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	118.897	4.351		27.327	.000
	IGNO2 ACS Time 2 - 11. Ignore	-9.505	1.872	-.325	-5.078	.000

a. Dependent Variable: GWB2NEG

There is a sig. *a* or constant (Y-intercept) - this is the baseline level of Psychological Distress. In addition, Ignore Problems (IP) is a significant predictor of Psychological Distress (PD).

PD = 119 - 9.5*IP



Linear regression - Summary

- Linear regression is for *explaining* or *predicting* the linear relationship between two variables
- Y = *b*x + a + e
- = *b*x + a

(*b* is the slope; *a* is the *Y*-intercept)

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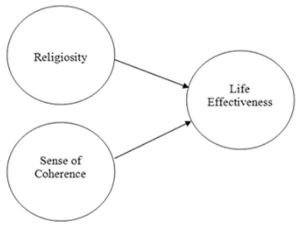
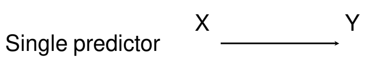
Multiple linear regression



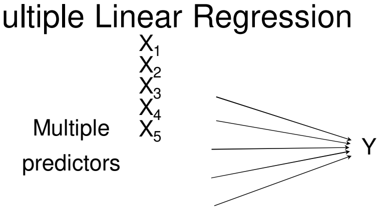
Linear relations between two or more IVs and a single DV

Multiple linear regression
Visual model

Linear Regression



Multiple Linear Regression



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What is MLR?

- Use of several IVs to predict a DV
- Weights each predictor (IV) according to the strength of its linear relationship with the DV
- Makes adjustments for inter-relationships among predictors
- Provides a measure of overall fit (*R*)

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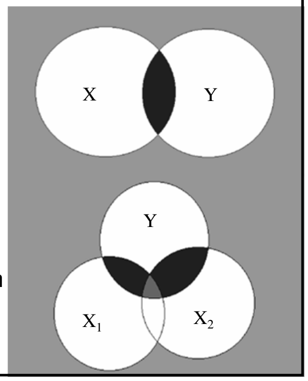
What is MLR?

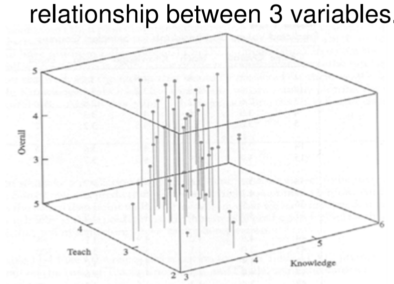
Correlation
Regression

Correlation
Partial correlation
Multiple linear regression



Multiple linear regression

A 3-way scatterplot can depict the correlational relationship between 3 variables.



However, it is difficult to graph/visualise 4+-way relationships via scatterplot.

MLR - General steps

1 Develop a visual model (path or Venn diagram) and state a research question and/or hypotheses
2 Check assumptions
3 Choose type of MLR
4 Interpret output
5 Develop a regression equation (if needed)

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LR → MLR example:
Cigarettes & coronary heart disease

- Using linear regression, ~50% of the variance in CHD mortality could be explained by cigarette smoking
- Strong effect - but what about the other 50% (unexplained variance)?
- What about other predictors?
  – e.g., exercise and cholesterol?

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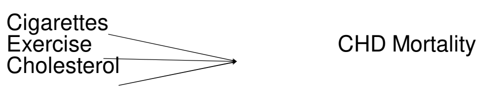
MLR example:
Research question 1

How well do these three IVs:

- # of cigarettes / day (IV_1)
- exercise (IV_2) and
- cholesterol (IV_3)

predict

- CHD mortality (DV)?



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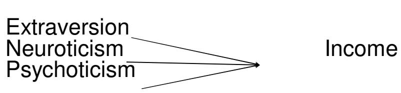
MLR example:
Research question 2

To what extent do personality factors (IVs) predict annual income (DV)?



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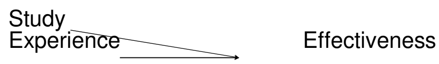
MLR example:
Research question 3

Does the # of years of formal study of psychology (IV1) and the no. of years of experience as a psychologist (IV2) predict clinical psychologists' effectiveness in treating mental illness (DV)?



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MLR example:
Choose your own research question

Generate your own MLR research question

(e.g., based on some of the following variables):

- Gender & Age
- Enrolment Type
- Hours
- Stress
- Time management
  - Planning
  - Procrastination
  - Effective actions
- Time perspective
  - Past-Negative
  - Past-Positive
  - Present-Hedonistic
  - Present-Fatalistic
  - Future-Positive
  - Future-Negative

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MLR - Assumptions

- Level of measurement
- Sample size
- Normality (univariate, bivariate, and multivariate)
- Linearity: Linear relations between IVs & DVs
- Homoscedasticity
- Multicollinearity
  - IVs are not overly correlated with one another (e.g., not over .7)
- Residuals are normally distributed

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MLR - Level of measurement

- **DV = Continuous**
  (Interval or Ratio)
- **IV = Continuous or Dichotomous**
  (if neither, may need to recode
  into a dichotomous variable
  or create dummy variables)

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Dummy coding

- Dummy coding converts a complex variable into a series of dichotomous variables (i.e., 0 or 1)
- i.e., several dummy variables are created to represent a variable with a higher level of measurement.

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Dummy coding - Example

- Religion
  (1 = Christian; 2 = Muslim; 3 = Atheist)
  in this format, can't be an IV in regression
  (a linear correlation with a categorical variable doesn't make sense)
- However, it can be dummy coded into dichotomous variables:
  - **Christian (0 = no; 1 = yes)**
  - **Muslim (0 = no; 1 = yes)**
  - ~~**Atheist (0 = no; 1 = yes)**~~ **(redundant)**
- These variables can then be used as IVs.
- More information (Dummy variable (statistics), Wikiversity)

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Sample size - Rule of thumb

- Enough data is needed to provide reliable estimates of the correlations.
- *N* >= 50 cases + *N* >= 10 to 20 cases x no. of IVs, otherwise the estimates of the regression line are probably unstable and are unlikely to replicate if the study is repeated.
- Green (1991) and Tabachnick & Fidell (2013) suggest:
  - 50 + 8(*k*) for testing an overall regression model and
  - 104 + *k* when testing individual predictors (where k is the number of IVs)
  - Based on detecting a medium effect size (β >= .20), with critical α <= .05, with power of 80%.

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Sample size - Rule of thumb

Q: Does a researcher have enough data to conduct an MLR with 4 predictors and 200 cases?

A: Yes; satisfies all rules of thumb:
- *N* > 50 cases + 4 x 20 = 130 cases
- *N* > 50 + 8 x 4 = 82 cases
- *N* > 104 + 4 = 108 cases

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Dealing with outliers

Extreme cases should be deleted or modified if they are overly influential.
- Univariate outliers - detect via initial data screening (e.g., min. and max.)
- Bivariate outliers - detect via scatterplots
- Multivariate outliers - unusual combination of predictors – detect via Mahalanobis' distance

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Multivariate outliers

- A case may be within normal range for each variable individually, but be a multivariate outlier because of an unusual combination of responses which unduly influences multivariate test results.
- e.g., a person who:
  - Is 18 years old
  - Has 3 children
  - Has a post-graduate degree

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Multivariate outliers

- Identify & check for unusual cases using Mahalanobis' distance or Cook's *D*

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Multivariate outliers

- Mahalanobis' distance (MD)
  - Distributed as χ^2 with *df* equal to the number of predictors (with critical α = .001)
  - Cases with a MD greater than the critical value could be influential multivariate outliers.
- Cook's *D*
  - Cases with CD values > 1 could be influential multivariate outliers.
- Use either MD or CD
- Examine cases with extreme MD or CD scores - if in doubt, remove & re-run.

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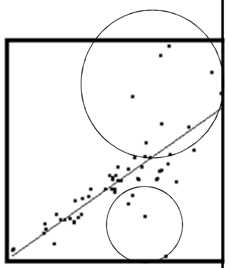
Homoscedasticity & normality

Homoscedasticity
- Variance around the regression line should be the same throughout the distribution
- Even spread in residual plots

Normality
- If variables are non-normal, this will create heteroscedasticity



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Multicollinearity

- IVs shouldn't be overly correlated (e.g., over .7) - leads to unstable regression
- If IVs are overly correlated, consider combining them into a single variable or removing one
- Singularity - perfect correlations among IVs
- Leads to unstable regression coefficients

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Multicollinearity

Detect via:
- **Correlation matrix** - are there large correlations among IVs?
- **Tolerance statistics** - if < .3 then exclude that variable.
- **Variance Inflation Factor (VIF)** – if > 3, then exclude that variable.
- VIF is the reciprocal of Tolerance (so use TOL or VIF – not both)

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Causality

- Like correlation, regression does not tell us about the causal relationship between variables.
- In many analyses, the IVs and DVs could be swapped around – therefore, it is important to:
  - Adopt a theoretical position
  - Acknowledge alternative explanations

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Multiple correlation coefficient

- "Big R" (capitalised)
- Equivalent of *r*, but takes into account that there are multiple predictors (IVs)
- Always positive, between 0 and 1
- Interpretation is similar to that for *r* (correlation coefficient)

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Coefficient of determination

- "Big R squared"
- Squared multiple correlation coefficient
- Always report R^2
- Indicates the % of variance in DV explained by combined effects of the IVs
- Analogous to r^2

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CoD - Rule of thumb

0.00 = no linear relationship
0.10 = small (R ~ .3)
0.25 = moderate (R ~ .5)
0.50 = strong (R ~ .7)
1.00 = perfect linear relationship

R^2 > .30
is "good" in social sciences

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Adjusted R^2

- R^2 = explained variance in a sample.
- Adjusted R^2 = explained variance in a population.
- Report both R^2 and adjusted R^2.
- Take more note of adjusted R^2, particularly for small *N* and where results are to be generalised.

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MLR - Overall significance

- Tests whether there is a significant linear relationship between the X variables (taken together) and Y
- Indicated by *F* and *p* in the ANOVA table.
- *p* is the likelihood that the explained variance in Y could have occurred by chance.

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MLR - Equation

$Y = b_1x_1 + b_2x_2 + \ldots + b_ix_i + a + e$

- Y = observed DV scores
- b_i = unstandardised regression coefficients (the *B*s in SPSS) - slopes
- x_1 to x_i = IV scores
- a = *Y* axis intercept
- e = error (residual)

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MLR - Coefficients

- Y-intercept (*a*)
- Slopes (*b*):
  - Unstandardised
- Slopes are the weighted loading of each IV on the DV, adjusted for the other IVs in the model.

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Unstandardised regression coefficients

- *B* = unstandardised regression coefficient
- Used for regression equations
- Used for predicting Y scores
- But can't be compared with other *B*s unless all IVs are measured on the same scale

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Standardised regression coefficients

- Beta (β) = standardised regression coefficient
- Useful for comparing the relative strength of predictors
- $\beta = r$ in LR but this is only true in MLR when the IVs are uncorrelated.

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MLR - IV significance

Indicates the likelihood of a linear relationship between each IV (X_i) and Y occurring by chance.

Hypotheses:

H_0: $\beta_i = 0$ (No linear relationship)
H_1: $\beta_i \neq 0$ (Linear relationship between X_i and Y)

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Relative importance of IVs

- Which IVs are the most important?
- To answer this, compare the standardised regression coefficients (βs)

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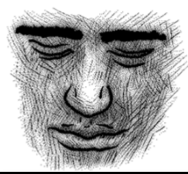
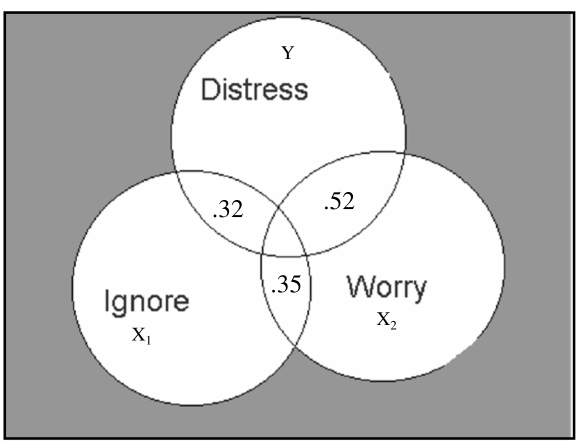
Multiple linear regression - Example

Does 'ignoring problems' (IV_1) and 'worrying' (IV_2) predict 'psychological distress' (DV)?



Correlations

	Psychological Distress	Worry	Ignore the Problem
Psychological Distress	1.000	-.521	-.325
Worry	-.521	1.000	.352
Ignore the Problem	-.325	.352	1.000
Psychological Distress	.	.000	.000
Worry	.000	.	.000
Ignore the Problem	.000	.000	.
Psychological Distress	220	220	220
Worry	220	220	220
Ignore the Problem	220	220	220



MLR - Example:
Model summary

Model Summary[b]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.543[a]	.295	.288	17.34399

a. Predictors: (Constant), Ignore the Problem, Worry

b. Dependent Variable: Psychological Distress

Together, Ignoring Problems and Worrying explain 30% of the variance in Psychological Distress in the Australian adolescent population (R^2 = .30, Adjusted R^2 = .29).

MLR - Example:
Overall significance

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	27281.12	2	13640.558	45.345	.000[a]
	Residual	65276.66	217	300.814		
	Total	92557.77	219			

a. Predictors: (Constant), Ignore the Problem, Worry

b. Dependent Variable: Psychological Distress

The explained variance in the population is unlikely to be 0 (p = .00).

MLR - Example:
Coefficients

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	138.932	4.680		29.687	.000
	Worry	-11.511	1.510	-.464	-7.625	.000
	Ignore the Problem	-4.735	1.780	-.162	-2.660	.008

a. Dependent Variable: Psychological Distress

Worry predicts about three times more variance in Psychological Distress than Ignoring the Problem, although both are significant, negative predictors of mental health.

MLR example - Equations

Linear Regression

PD (hat) = 119 – 9.50*Ignore

R^2 = .11

Multiple Linear Regression

PD (hat) = 139 - 4.7*Ignore - 11.5*Worry

R^2 = .30

	B
(Constant)	138.932
Worry	-11.511
Ignore the Problem	-4.735

MLR - Example:

Confidence interval for the slope

Coefficients[a]

Model		Standardized Coefficients	95% Confidence Interval for B	
		Beta	Lower Bound	Upper Bound
1	(Constant)		129.708	148.156
	Worry	-.464	-14.486	-8.536
	Ignore the Problem	-.162	-8.242	-1.227

a. Dependent Variable: Psychological Distress

Mental Health (PD) is reduced by between 8.5 and 14.5 units per increase of Worry units.
Mental Health (PD) is reduced by between 1.2 and 8.2 units per increase in Ignore the Problem units.

Multiple linear regression - Example

Effect of violence, stress, social support on internalising behaviour problems

Kliewer, Lepore, Oskin, & Johnson, (1998)

Image source: http://cloudking.com/artists/noa-terliuc/family-violence.php

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MLR example - Violence study - Design

- Participants were children:
  - 8 - 12 years
  - Living in high-violence areas, USA
- **Hypotheses**:
  - Stress → ↑ internalising behaviour
  - Violence → ↑ internalising behaviour
  - Social support → ↓ internalising behaviour

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MLR example - Violence study - Variables

- **Predictors**
  - Witnessing violence
  - Life stress
  - Social support
- **Outcome**
  - Internalising behaviour (e.g., depression, anxiety, withdrawal symptoms) – measured using the Child Behavior Checklist (CBCL)

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Correlations

Pearson Correlation

	Amount violenced witnessed	Current stress	Social support	Internalizing symptoms on CBCL
Amount violenced witnessed				
Current stress	.050			
Social support	.080	-.080		
Internalizing symptoms on CBCL	.200*	.270*	-.170	

*. Correlation is significant at the 0.05 level (2-tailed).

**. Correlation is significant at the 0.01 level (2-tailed).

Correlations amongst the IVs

Correlations between the IVs and the DV

R^2

13.5% of the variance in children's internalising symptoms can be explained by the 3 predictors.

Model Summary

R	R Square	Adjusted R Square	Std. Error of the Estimate
.37[a]	.135	.108	2.2198

a. Predictors: (Constant), Social support, Current stress, Amount violenced witnessed

Coefficients[a]

2 predictors have $p < .05$

	Unstandardized Coefficients		Standardized Coefficients		
	B	Std. Error	Beta	t	Sig.
(Constant)	.477	1.289		.37	.712
Amount violenced witnessed	.038	.018	.201	2.1	.039
Current stress	.273	.106	.247	2.6	.012
Social support	-.074	.043	-.166	-2	.087

a. Dependent Variable: Internalizing symptoms on CB

MLR example - Violence study - Equation

$$\hat{Y} = b_1X_1 + b_2X_2 + b_3X_3 + b_0$$
$$= 0.038Wit + 0.273Stress - 0.074SocSupp + 0.477$$

- A separate coefficient or slope for each variable
- An intercept (here called b_0)

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MLR example - Violence study - Equation

$$\hat{Y} = b_1X_1 + b_2X_2 + b_3X_3 + b_0$$
$$= 0.038 Wit + 0.273 Stress - 0.074 SocSupp + 0.477$$

- Slopes for Witness and Stress are +ve; slope for Social Support is -ve.
- Ignoring Stress and Social Support, a one unit increase in Witness would produce .038 unit increase in Internalising symptoms.

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MLR example - Violence study - Prediction

Q: If Witness = 20, Stress = 5, and SocSupp = 35, what we would predict internalising symptoms to be?
A: .012

$$\hat{Y} = .038 * Wit + .273 * Stress - .074 * SocSupp + 0.477$$
$$= .038(20) + .273(5) - .074(35) + 0.477$$
$$= .012$$

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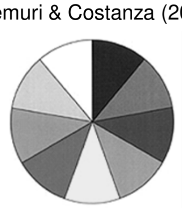
MLR - Example:

The role of human, social, built, and natural capital in explaining life satisfaction at the country level:
Towards a National Well-Being Index (NWI)

Vemuri & Costanza (2006)



MLR example - Life satisfaction - Design

- IVs:
  - Human & Built Capital (Human Development Index)
  - Natural Capital (Ecosystem services per km^2)
  - Social Capital (Press Freedom)
- DV = Life satisfaction
- Units of analysis: Countries (*N* = 57; mostly developed countries, e.g., in Europe and America)

Table 1
Bivariate correlations between variables

		Average life satisfaction	HDI	Log ESP/km² in…
Average life satisfaction	Pearson cor.	1		
	Significance			
HDI	Pearson cor.	.463	1	
	Significance	.000		
Log ESP/km² index	Pearson cor.	.358	.071	1
	Significance	.007	.353	
Press freedom	Pearson cor.	.502	.502	.295
	Significance	.000	.000	.000

- There are moderately strong positive and statistically significant linear relations between the IVs and the DV
- The IVs have small to moderate positive inter-correlations.

Table 2
Basic regression model coefficients for national-level analysis

	Unstandardized coefficients		Standardized coefficients	*t*-value	Significance
	B	Std. error	Beta		
Constant	1.857	.900		2.063	.044
HDI	3.524	.832	.470	4.234	.000
Log ESP/km² Index	3.498	1.021	.380	3.427	.001

Sample size of the regression model was 56.

- R^2 = .35
- Two sig. IVs (not Social Capital - dropped)

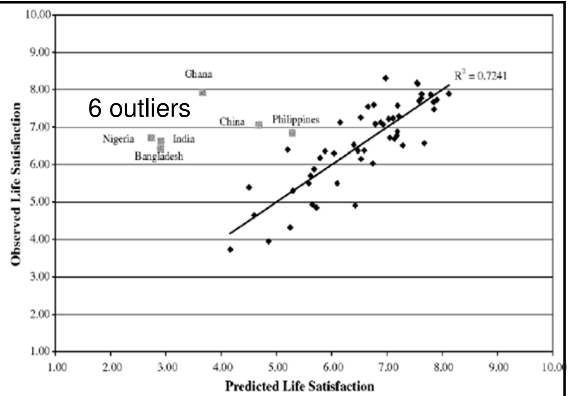


Fig. 2. Observed versus predicted life satisfaction.

Table 4
Revised regression model coefficients for national-level analysis

	Unstandardized coefficients		Standardized coefficients	*t*-value	Significance
	B	Std. error	Beta		
Constant	−2.220	.799		−2.781	.008
HDI	8.875	.884	.777	10.038	.000
Log ESP/km² index	2.453	.739	.257	3.319	.002

Sample size of the regression model was 50.

- R^2 = .72
  (after dropping 6 outliers)

Types of MLR

- Standard or direct (simultaneous)
- Hierarchical or sequential
- Stepwise (forward & backward)

Image source: https://commons.wikimedia.org/wiki/File:IStumbler.png

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Direct (Standard)

- All predictor variables are entered together, at the same time.
- Assesses relationship between all predictor variables and the outcome (*Y*) variable simultaneously.
- Manual technique & commonly used.
- If you're not sure what type of MLR to use, start with this approach.

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Hierarchical (Sequential)

- IVs are entered in blocks or stages.
  - Researcher defines order of entry for the variables, based on theory.
  - e.g., enter "nuisance" variables first to "control" for them, then test "purer" effect of next block of important variables.
- R^2 change - change in variance of *Y* explained at each stage of the regression.
  - *F* test of R^2 change.

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Hierarchical (Sequential)

Example: Research question: To what extent does Drug B reduce AIDS symptoms *above and beyond* the effect of Drug A?

- **Drug A** is a cheap, well-proven drug which reduces AIDS symptoms
- **Drug B** is an expensive, experimental drug which could help to cure AIDS
- Hierarchical linear regression:
  - **Step 1: Drug A (IV1)**
  - **Step 2: Drug B (IV2)**
  - DV = AIDS symptoms
  - Examine change in R^2 between Step 1 & Step 2

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Forward selection

- Computer-driven – controversial.
- Starts with 0 predictors, then the strongest predictor is entered into the model, then the next strongest etc. if they reach a criteria (e.g., $p < .05$)

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Backward elimination

- Computer-driven – controversial.
- All predictor variables are entered, then the weakest predictors are removed, one by one, if they meet a criteria (e.g., $p > .05$)

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Stepwise

- Computer-driven – controversial.
- Combines forward & backward.
- At each step, variables may be entered or removed if they meet certain criteria.
- **Useful for developing the best prediction equation** from a large number of variables.
- Redundant predictors are removed.

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Types of MLR - Summary

- Standard: To assess impact of all IVs simultaneously
- Hierarchical: To test IVs in a specific order (based on hypotheses derived from theory)
- Stepwise: If the goal is accurate statistical prediction from a large # of variables - computer driven

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Summary

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Summary: General steps

1 Develop model and hypotheses
2 Check assumptions
3 Choose type
4 Interpret output
5 Develop a regression equation (if needed)

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Summary: Linear regression

1 Best-fitting straight line for a scatterplot of two variables
2 Y = bX + a + e
  1 Predictor (X; IV)
  2 Outcome (Y; DV)
3 Least squares criterion
4 Residuals are the vertical distance between actual and predicted values

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Summary: Assumptions

1. Level of measurement
2. Sample size
3. Normality
4. Linearity
5. Homoscedasticity
6. Collinearity
7. Multivariate outliers
8. Residuals should be normally distributed

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Summary: LoM & dummy coding

1 Level of measurement
  1 DV = Interval or ratio
  2 IV = Interval or ratio or dichotomous
2 Dummy coding
  1 Convert complex variables into series of dichotomous IVs

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Summary: MLR output

1 Overall fit
  1. R, R^2, Adjusted R^2
  2. F, p
2 Coefficients
  1. Relation between each IV and the DV, adjusted for the other IVs
  2. B, β, t, p, and r_p
3 Regression equation (if useful)

$$Y = b_1x_1 + b_2x_2 + \ldots + b_ix_i + a + e$$

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Summary: MLR types

1. Standard
2. Hierarchical
3. Stepwise / Forward / Backward

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Practice quiz

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Practice quiz question:
MLR type of analysis

Multiple linear regression is a _________ type of statistical analysis.
a) univariate
b) bivariate
c) multivariate

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Practice quiz question:
MLR LoM

The following types of data can be used in MLR (choose all that apply):
a) Interval or higher DV
b) Interval or higher IVs
c) Dichotomous Ivs
d) All of the above
e) None of the above

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Practice quiz question:
R^2

In MLR, the square of the multiple correlation coefficient, R^2, is called the:
a) Coefficient of determination
b) Variance
c) Covariance
d) Cross-product
e) Big R

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Practice quiz question:
MLR equation

A linear regression analysis produces the equation Y = 0.4X + 3. This indicates that:
a) When Y = 0.4, X = 3
b) When Y = 0, X = 3
c) When X = 3, Y = 0.4
d) When X = 0, Y = 3
e) None of the above

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Practice quiz question:
MLR residuals

In MLR, a residual is the difference between the predicted Y and actual Y values.
a) True
b) False

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Next lecture

Multiple linear regression II
- Review of MLR I
- Semi-partial correlations
- Residual analysis
- Interactions
- Analysis of change

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