Discrete Time Linear Time Invariant System

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

Definition

$$Y[n] = -\sum_{i=1}^{N} a_i(n)Y[n-i] + \sum_{i=0}^{M} b_i(n)X[n-i]$$

$$Y[n] = -\sum_{i=1}^{N} a_i Y[n-i] + \sum_{i=0}^{M} b_i X[n-i]$$

Definition

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

$$\delta[-n] = \delta[n]$$

$$\sum_{n=-\infty}^{+\infty} \delta[n] = 1$$

Definition

$$X[n] = \sum_{m=-\infty}^{+\infty} X[m]\delta[n-m]$$

$$Y[n] = \sum_{m=-\infty}^{+\infty} X[m]h[n-m]$$

$$= \sum_{k=-\infty}^{+\infty} X[n-k]h[k]$$

Definition

$$Y[n] = \sum_{m=-\infty}^{+\infty} X[m]h[n-m] = X[n] * h[n]$$

$$= \sum_{k=-\infty}^{+\infty} X[n-k]h[k] = h[n] * X[n]$$

Definition

$$h[n] = h_1[n] * h_2[n] = h_2[n] * h_1[n]$$

Definition

$$X[n] = a_1 X_1[n] + a_2 X_2[n]$$

$$Y[n] = a_1 Y_1[n] + a_2 Y_2[n]$$

Definition

$$\sum_{n=-\infty}^{+\infty} |h[n]| < \infty$$

Definition

$$y[n] = \sum_{i=0}^{M} b_i X[n-i]$$

$$h[n] = \sum_{i=0}^{M} b_i \delta[n-i] = \begin{cases} b_n & 0 < n \leq M \\ 0 & otherwise \end{cases}$$

Definition

$$S_{X_sX_s}(\omega) = S_{X_sX_s}(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} R_{XX}[n]e^{-j\Omega}$$

$$\Omega = \omega T_s$$

Definition

$$H(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} h[n] e^{-jn\Omega}$$

$$\Omega = \omega T_s$$

$$h[n] = -\sum_{r=1}^{N} a_r h[n-r] + \sum_{r=0}^{M} b_r \delta[n-r]$$

Definition

$$\delta[n] \Longleftrightarrow 1$$

$$\delta[n-r] \Longleftrightarrow e^{-j\Omega}$$

$$h[n-r] \Longleftrightarrow H(e^{j\Omega})e^{-j\Omega}$$

$$H(e^{j\Omega}) = -\sum_{r=1}^{N} a_r H(e^{j\Omega})e^{-jr\Omega} + \sum_{r=0}^{M} b_r e^{-j\Omega}$$

$$H(e^{j\Omega}) = \frac{\sum_{r=0}^{M} b_r e^{-j\Omega}}{1+\sum_{r=1}^{N} a_r e^{-jr\Omega}} +$$