t-Testing (Single)

Young W. Lim

2019-09-20 Fri

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2019-09-20 Fri 1/32

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"Understanding Statistics in the Behavioral Sciences" R. R. Pagano

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• The t-test is any <u>statistical hypothesis test</u> in which the test statistic follows a Student's t-distribution under the null hypothesis.

- A t-test is most commonly applied when the test statistic would follow a normal distribution if the value of a scaling term in the test statistic were known.
- When the scaling term is <u>unknown</u> and is replaced by an <u>estimate</u> based on the data, the test statistics (under certain conditions) follow a Student's t distribution.

• The t-test can be used, for example, to determine if the means of two sets of data are significantly +different_ from each other.

- Most test statistics have the form t = Z/s, where Z and s are functions of the data.
- Z may be sensitive to the alternative hypothesis (i.e., its magnitude tends to be larger when the alternative hypothesis is true), whereas s is a scaling parameter that allows the distribution of t to be determined.

• As an example, in the one-sample t-test

$$t = \frac{Z}{s} = \frac{\bar{X} - \mu}{\hat{\sigma} / \sqrt{n}}$$

The assumptions underlying a t-test in its simplest form are that

- \overline{X} follows a normal distribution with mean *mu* and variance σ^2/n
- **2** s^2 follows a χ^2 distribution with n-1 degrees of freedom
- 3 Z and s are independent.

- Mean of the two populations being compared should follow a normal distribution.
- This can be tested using a normality test or it can be assessed graphically using a normal quantile plot.

- If using Student's <u>original definition</u> of the t-test, the two populations being compared should have the same variance
- If the <u>sample sizes</u> in the two groups being compared are <u>equal</u>, Student's original t-test is highly robust to the presence of <u>unequal variances</u>.

- The data used to carry out the test should be <u>sampled</u> independently from the two populations being compared.
- This is in general <u>not testable</u> from the data, but if the data are known to be <u>dependently sampled</u> (that is, if they were sampled in clusters), then the classical t-tests discussed here may give <u>misleading</u> results.

- Two-sample t-tests for a difference in mean involve
 - independent samples (unpaired samples) or
 - paired samples.

 Paired <u>t-tests</u> are a form of blocking and have greater power than unpaired tests when the paired units are similar with respect to "noise factors" that are independent of membership in the two groups being compared. • In a different context, paired t-tests can be used to reduce the effects of confounding factors in an observational study.

- The independent samples t-test is used
- when two separate sets of <u>independent</u> and <u>identically distributed</u> samples are obtained, one from each of the two populations being compared.

 Paired samples t-tests typically consist of a sample of <u>matched pairs</u> of <u>similar</u> units, or <u>one group</u> of units that has been tested <u>twice</u> (a repeated measures t-test). In testing the null hypothesis that the population mean is equal to a specified value µ₀ one uses the statistic

$$t=\frac{\bar{x}-\mu_0}{s/\sqrt{n}}$$

where x̄ is the sample mean,
s is the sample standard deviation and
n is the sample size.

 The degrees of freedom used in this test are n - 1 Although the parent population does not need to be normally distributed, the distribution of the population of sample means x̄ is assumed to be normal. By the central limit theorem, if the observations are independent and the second moment exists, then t will be approximately normal N(0; 1).

- Equal sample sizes, equal variance
- Equal or Unequal sample sizes, equal variance
- Equal or Unequal sample sizes, unequal variance

- Given two groups (1, 2), this test is only applicable when:
 - the two sample sizes are equal the number *n* of participants of each group are equal;
 - it can be assumed that the two distributions have the same variance

• The t statistic to test ether the means are different can be calculated as follows:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{s_p \sqrt{\frac{2}{n}}}$$
$$s_p = \sqrt{\frac{s_{X_1}^2 + s_{X_2}^2}{2}}$$

- This test is used only when it can be assumed that the two distributions have the same variance.
- Note that the previous formulae are a special case of the formulae below, one recovers them when both samples are equal in size: n = n1 = n2.

• The t statistic to test whether the means are different can be calculated as follows:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{s_p \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$
$$s_p = \sqrt{\frac{(n_1 - 1)s_{X_1}^2 + (n_2 - 1)s_{X_2}^2}{n_1 + n_2 - 2}}.$$

• This test, also known as Welch's t-test, is used only when the two population variances are <u>not</u> assumed to be <u>equal</u> the two <u>sample sizes</u> may or may not be equal and hence must be estimated separately • The t statistic to test whether the population means are different is calculated as:

$$t=rac{ar{X}_1-ar{X}_2}{s_{ar{\Delta}}}$$
 $s_{ar{\Delta}}=\sqrt{rac{s_1^2}{n_1}+rac{s_2^2}{n_2}}.$

Young W. Lim

2019-09-20 Fri 27 / 32

• This test is used when the samples are dependent;

- when there is <u>only one</u> sample that has been <u>tested</u> <u>twice</u> (repeated measures) or
- when there are two samples that have been matched or paired.

an example of a paired difference test.

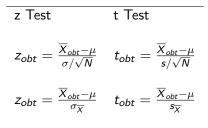
$$t=\frac{\bar{X}_D-\mu_0}{\frac{s_D}{\sqrt{n}}}.$$

- a probability distribution of the t values that would occur if all possible different samples of a fixed size N were drawn form the null-hypothesis population
- it gives

(1) all the possible different t values for samples of size N (2) the probability of getting each value if sampling

estimated standard error the mean

$$s_{\overline{X}} = \frac{s}{\sqrt{N}}$$



 $\begin{array}{ll} s & \text{estimator of } \sigma \\ s_{\overline{X}} & \text{estimator of } \sigma_{\overline{X}} \end{array}$

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• the degree of freedom for any statistic is the number of scores that are free to vary in calculating static