t-Testing (Single)

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Outline

1. Based on
2. t Testing

Based on

"Understanding Statistics in the Behavioral Sciences" R. R. Pagano

Student t-Test (1)

- The t-test is any statistical hypothesis test
  in which the test statistic follows
  a Student's t-distribution
  under the null hypothesis.

Student t-Test (2)

- A t-test is most commonly applied
  when the test statistic would
  follow a normal distribution
  if the value of a scaling term
  in the test statistic were known.
- When the scaling term is unknown
  and is replaced by an estimate based on the data,
  the test statistics (under certain conditions)
  follow a Student's t distribution.

Student t-Test (2)

- The t-test can be used, for example, to determine if the means of two sets of data are significantly +different_ from each other.

Assumptions (1)

- Most test statistics have the form $t = Z/s$,
  where Z and s are functions of the data.
- Z may be sensitive to the alternative hypothesis
  (i.e., its magnitude tends to be larger
  when the alternative hypothesis is true),
  whereas s is a scaling parameter
  that allows the distribution of t to be determined.

Assumptions (2)

- As an example, in the one-sample t-test

$$t = \frac{Z}{s} = \frac{\bar{X} - \mu}{\widehat{\sigma}/\sqrt{n}}$$

- where $\overline{X}$ is the sample mean
from a sample $X_1, X_2, ..., X_n$, of size n,
s is the standard error of the mean,
$\widehat{\sigma}$ is the estimate of the standard deviation of the population,
and μ is the population mean.

Assumptions (3)

The assumptions underlying a t-test in its simplest form are that

1. $\overline{X}$ follows a normal distribution
   with mean *mu* and variance σ^2/n
2. s^2 follows a χ^2 distribution
   with $n-1$ degrees of freedom
3. Z and s are independent.

Assumptions (4)

- Mean of the two populations being compared should follow a normal distribution.
- This can be tested using a normality test or it can be assessed graphically using a normal quantile plot.

Assumptions (5)

- If using Student's original definition of the t-test, the two populations being compared should have the same variance
- If the sample sizes in the two groups being compared are equal, Student's original t-test is highly robust to the presence of unequal variances.

Assumptions (6)

- The data used to carry out the test
  should be sampled independently
  from the two populations being compared.

- This is in general not testable from the data,
  but if the data are known to be dependently sampled
  (that is, if they were sampled in clusters),
  then the classical t-tests discussed here
  may give misleading results.

Unpaired and paired two-sample t-tests (1)

- Two-sample t-tests for a difference in mean involve
  - independent samples (unpaired samples) or
  - paired samples.

Unpaired and paired two-sample t-tests (2)

- Paired t-tests are a form of blocking
  and have greater power than unpaired tests
  when the paired units are similar
  with respect to "noise factors"
  that are independent of membership
  in the two groups being compared.

Unpaired and paired two-sample t-tests (3)

- In a different context, paired t-tests
  can be used to reduce the effects of confounding factors
  in an observational study.

Unpaired (independent) samples

- The independent samples t-test is used
- when two separate sets of independent and identically distributed samples are obtained, one from each of the two populations being compared.

Paired (dependent) samples

- Paired samples t-tests typically consist of a sample of matched pairs of similar units, or one group of units that has been tested twice (a repeated measures t-test).

One-sample t-test (1)

- In testing the null hypothesis
  that the population mean is equal to
  a specified value μ_0 one uses the statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

- where $\bar{x}$ is the sample mean,
  s is the sample standard deviation and
  n is the sample size.

One-sample t-test (2)

- The degrees of freedom used in this test are $n - 1$
  Although the parent population does not
  need to be normally distributed,
  the distribution of the population
  of sample means $\bar{x}$ is
  assumed to be normal.

One-sample t-test (3)

- By the central limit theorem,
  if the observations are independent
  and the second moment exists,
  then t will be approximately normal $N(0; 1)$.

Independent two-sample t-test

- Equal sample sizes, equal variance
- Equal or Unequal sample sizes, equal variance
- Equal or Unequal sample sizes, unequal variance

Equal sample sizes, equal variance (1)

- Given two groups (1, 2), this test is only applicable when:
  - the two sample sizes are equal
    the number n of participants of each group are equal;
  - it can be assumed that
    the two distributions have the same variance

Equal sample sizes, equal variance (2)

- The t statistic to test
  ether the means are different
  can be calculated as follows:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{s_p \sqrt{\frac{2}{n}}}$$

$$s_p = \sqrt{\frac{s_{X_1}^2 + s_{X_2}^2}{2}}$$

Equal or unequal sample sizes, equal variance (1)

- This test is used only when it can be assumed
  that the two distributions have the same variance.

- Note that the previous formulae are a special case of
  the formulae below,
  one recovers them when both samples are equal in size:
  $n = n1 = n2$.

Equal or unequal sample sizes, equal variance (2)

- The t statistic to test
  whether the means are different
  can be calculated as follows:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{s_p \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$s_p = \sqrt{\frac{(n_1 - 1)\, s_{X_1}^2 + (n_2 - 1)\, s_{X_2}^2}{n_1 + n_2 - 2}}.$$

Equal or unequal sample sizes, unequal variance (1)

- This test, also known as Welch's t-test, is used only when the two population variances are not assumed to be equal the two sample sizes may or may not be equal and hence must be estimated separately

Equal or unequal sample sizes, unequal variance (2)

- The t statistic to test
  whether the population means are different
  is calculated as:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{s_{\bar{\Delta}}}$$

$$s_{\bar{\Delta}} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}.$$

Dependent t-test for paired samples

- This test is used when the samples are dependent;
  - when there is only one sample that has been tested twice (repeated measures) or
  - when there are two samples that have been matched or paired.
    an example of a paired difference test.

$$t = \frac{\bar{X}_D - \mu_0}{\frac{s_D}{\sqrt{n}}}.$$

Sampling distribution of t

- a probability distribution of the t values
  that would occur if all possible different samples
  of a fixed size N were drawn
  form the null-hypothesis population

- it gives
  (1) all the possible different t values for samples of size N
  (2) the probability of getting each value if sampling

estimated standard error of the mean

estimated standard error the mean

$$s_{\overline{X}} = \frac{s}{\sqrt{N}}$$

z-test vs. t-test

z Test	t Test
$z_{obt} = \frac{\overline{X}_{obt} - \mu}{\sigma/\sqrt{N}}$	$t_{obt} = \frac{\overline{X}_{obt} - \mu}{s/\sqrt{N}}$
$z_{obt} = \frac{\overline{X}_{obt} - \mu}{\sigma_{\overline{X}}}$	$t_{obt} = \frac{\overline{X}_{obt} - \mu}{s_{\overline{X}}}$

s	estimator of σ
$s_{\overline{X}}$	estimator of $\sigma_{\overline{X}}$

degree of freedom (df)

- the degree of freedom for any statistic is
  the number of scores that are free to vary in calculating static