FPGA Carry Chain Adder (1A)

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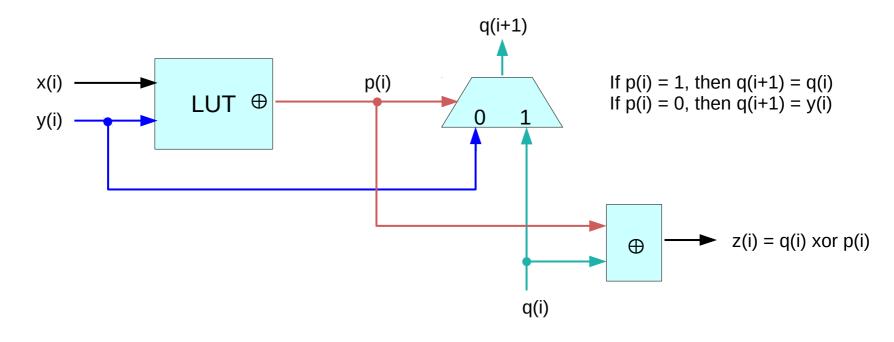
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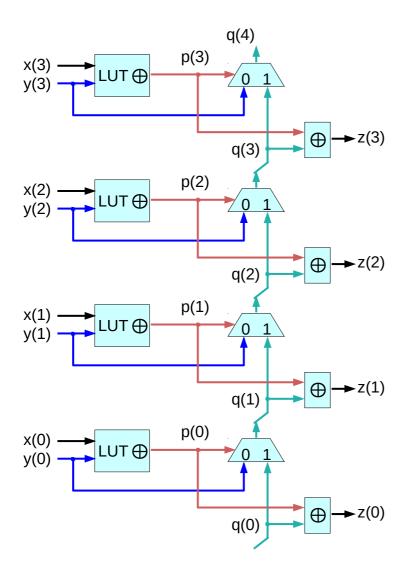
$$\begin{aligned} s_i &= (a_i \oplus b_i) \oplus c_i = p_i \oplus c_i \\ c_{i+1} &= (a_i \cdot b_i) + (a_i \oplus b_i) c_i = \overline{p_i} \cdot g_i + p_i \cdot c_i = \overline{p_i} \cdot a_i + p_i \cdot c_i = \overline{p_i} \cdot b_i + p_i \cdot c_i \end{aligned}$$

when
$$\overline{p}_i = 1$$
, then $a_i = b_i$
when $g_i = 1$, then $a_i = b_i = 1$

p(i)	0	1
0	0	1
1	1	0

g(i)	0	1
0	0	0
1	0	1

Synthesis of Arithmetic Circuits: FPGA, ASIC and Ebedded Systems, J-P Deschamps et al



Synthesis of Arithmetic Circuits: FPGA, ASIC and Ebedded Systems, J-P Deschamps et al

FPGA Carry Chain

FPGAs generally contain dedicated computation resources for generating fast adders

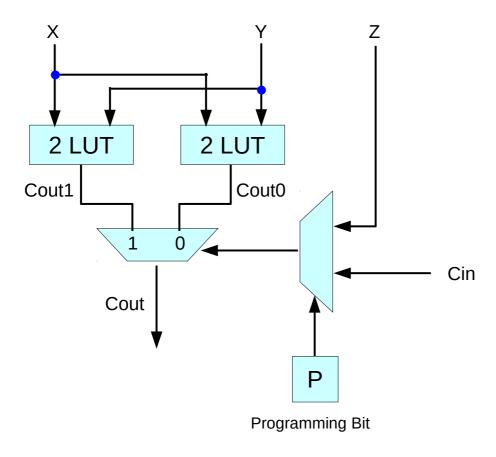
The Virtex family programmable arrays include logic gates (XOR) and multiplexers that along with the general purpose lookup tables allow one to build effective carry-chain adders

The carry chain is made up of multiplexers belonging to adjacent configurable blocks

the lookup table is used for implementing the exclusive or function

$$p(i) = x(i) xor y(i)$$

https://en.wikipedia.org/wiki/Carry-lookahead_adder



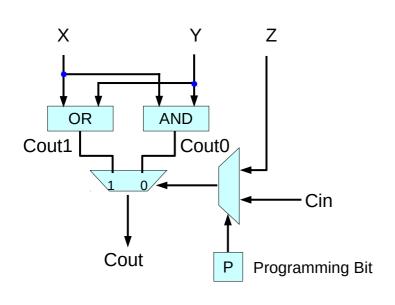
Cout1, Cout2: functions of X, Y, Cin

Cout1 = X+Y when Cin=1 Cout0 = X Y when Cin=0

Cout = $(X + Y) Cin + X Y \overline{Cin}$

Cout = P' Cin + G $\overline{\text{Cin}}$... P' = relaxed P

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	Cin	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate



		Cin	Cin	
X	Υ	Cout1	Cout0	
0	0	0	0	$\overline{X} \overline{Y}$
0	1	1	0	$\overline{X}Y$
1	0	1	0	$X\overline{Y}$
1	1	1	1	ΧY

Cout: functions of X, Y, Cin

Cout(X, Y, 1) = Cout1 = X + YCout(X, Y, 0) = Cout0 = X Y

Cout1 = X + Y when Cin=1 Cout0 = XY when Cin=0

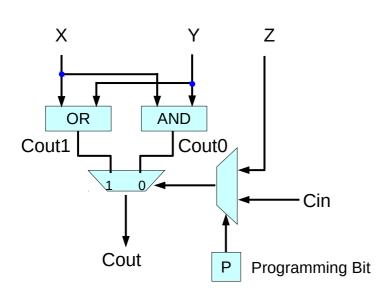
Cout1 = P' $\underline{\text{Cin}}$... P' = relaxed P Cout0 = $\underline{\text{Cin}}$

High Performance Carry Chains for FPGAs, S. Hauck, M. M. Hosler, T. W. Fry

If \underline{Cin} , then $\underline{Cout} = (\overline{X} \ Y + X \ \overline{Y} + X \ Y)$ If \underline{Cin} , then $\underline{Cout} = X \ Y$

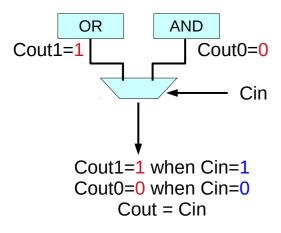
 $\begin{array}{l} \text{Cin } (X + Y) + \overline{\text{Cin}} \ X \ Y \\ \text{Cin } (\overline{X} \ Y + X \ \overline{Y} + X \ Y) + \overline{\text{Cin}} \ X \ Y \\ \text{Cin } (\overline{X} \ Y + X \ \overline{Y}) + (\text{Cin} + \overline{\text{Cin}}) \ X \ Y \\ \text{P Cin + G} \end{array}$

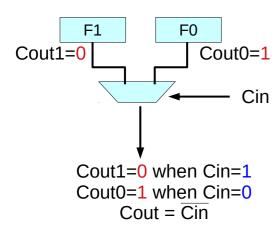
Cin $(X + \underline{Y}) + \overline{Cin} X Y$ Cin P' + $\overline{Cin} G$... P' : relaxed P



		Cin	Cin	
Χ	Υ	Cout1	Cout0	
0	0	0	0	$\overline{X} \overline{Y}$
0	1	1	0	$\overline{X}Y$
1	0	1	0	$X\overline{Y}$
1	1	1	1	ΧY

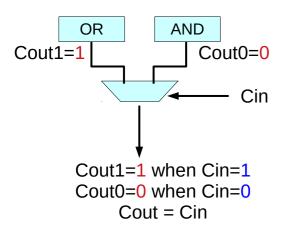
X	Υ	Cin	Cout	
0	0	0	0	Cout0
0	1	0	0	Cout0
1	0	0	0	Cout0
1	1	0	1	Cout0
0	0	1	0	Cout1
0	1	1	1	Cout1
1	0	1	1	Cout1
1	1	1	1	Cout1

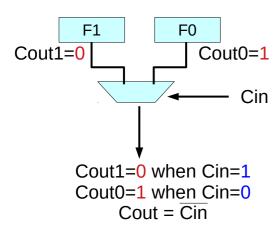




Cout0	Cout1	Cout	Name
0	0	0	Kill
0	1		Propagate
1	0	Cin	Inverse Propagate
1	1	1	Generate

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	Cin	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate



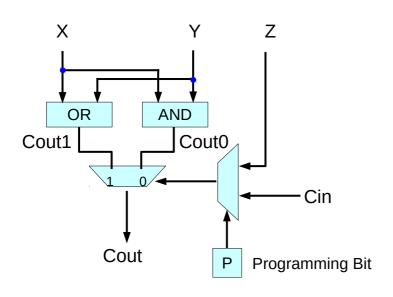


Cout0	Cout1	Cout	Name
0	0	0	Kill
0	1	Cin	Propagate
1	0	Cin	Inverse Propagate
1	1	1	Generate

Χ	Υ	Cin	Cout		Cout1	Cout0
0	0	0	0	Cout0	0	0
0	1	0	0	Cout0	1	0
1	0	0	0	Cout0	1	0
1	1	0	1	Cout0	1	1
0	0	1	0	Cout1	0	0
0	1	1	1	Cout1	1	0
1	0	1	1	Cout1	1	0
1	1	1	1	Cout1	1	1

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Carry Chain



Car	ry Ou	ıt ,		
Χ	Υ	Cin		
0	0	Cin	Cin	
0	1	Cin	Cin	
1	0	Cin	Cin	
1	1	Cin	Cin	

		Cin	Cin	
Χ	Υ	Cout1	Cout0	
0	0	0	0	$\overline{X} \overline{Y}$
0	1	1	0	$\overline{X}Y$
1	0	1	0	$X\overline{Y}$
1	1	1	1	ΧY

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	Cin	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

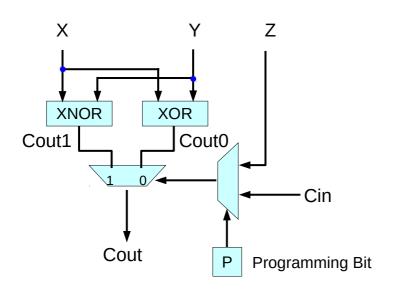
Cout1=1 when Cin=1 Cout0=0 when Cin=0

Cout = Cin propagate

Cout1=0 when Cin=1 Cout0=1 when Cin=0

Cout = $\overline{\text{Cin}}$ inverse propagate

Parity Checker



		Cin	Cin	
X	Υ	Cout1	Cout0	
0	0	1	0	$\overline{X} \overline{Y}$
0	1	0	1	$\overline{X} Y$
1	0	0	1	$X \overline{Y}$
1	1	1	0	ΧY

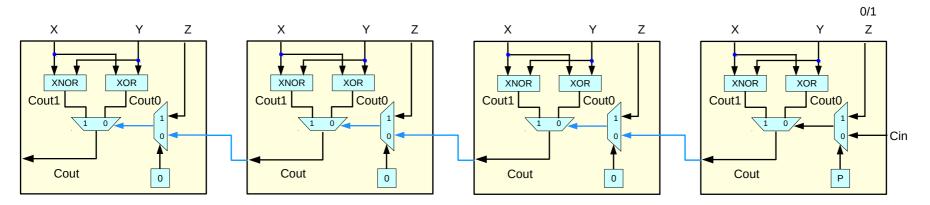
Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	Cin	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

Computing Par	ity	
X ⊕ Y ⊕ Cin		
0 ⊕ 0 ⊕ Cin	Cin	
0 ⊕ 1 ⊕ Cin	Cin	
1 ⊕ 0 ⊕ Cin	Cin	
1 ⊕ 1 ⊕ Cin	Cin	

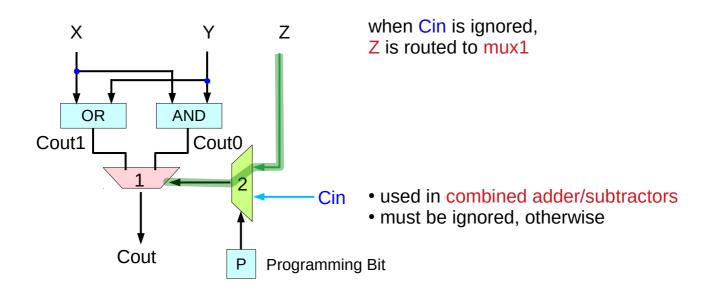
Cout1=1 when Cin=1
Cout0=0 when Cin=0
Cout = Cin propagate

Cout1=0 when Cin=1
Cout0=1 when Cin=0
Cout = Cin inverse propagate

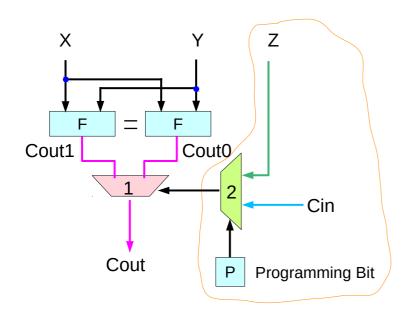
Ripple Carry Chain



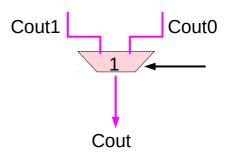
the first cell in the chain



the logic cells - resources to compute a function the exact location of logic cells depends on the user. a user can start or end a carry computation at any place in an fpga. But in many carry computations, the first cell has only 2 inputs, and forcing the carry chain to wait for the arrival of an additional, unnecessary input Z will only needlessly slow down the circuit's computation.



when Cin is ignored,
Z can also be ignored
by having the same LUTs



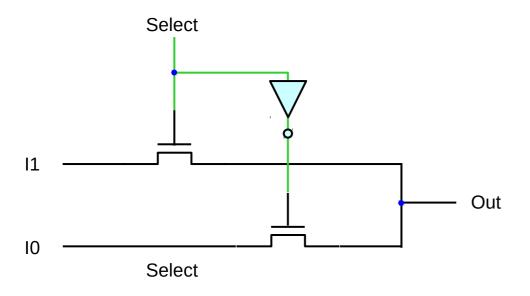
the first cell in the chain

the same LUTs

the <u>same</u> output regardless of Z and Cin

Cout1 = Cout0 = Cout regardless of the select

Ripple Carry Chain



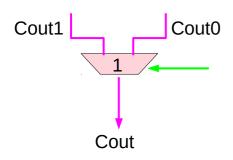


fig1b shows an implementation of a mux that does not obey this requirement

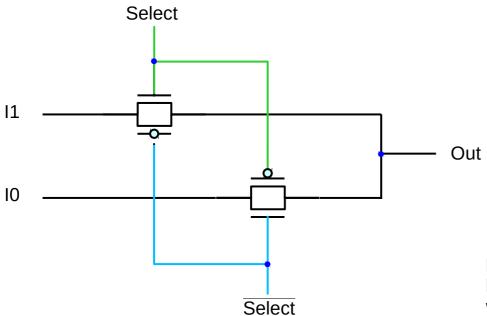
since the carry chain is part of an fpga, the input to this mux could be connected to some unused logic in another row which is generating unknown values.

if that unused logic had multiple transitions which caused the signal to change quicker than the gate could react, then it is possible that the select signal to this mux could be stuck midway between true and false (2.5V for 5V CMOS)

in this case, it will <u>not</u> be able to <u>pass a true value</u> from the input to the output and thus will not function properly for this application.

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Ripple Carry Chain



however a mux built with both n-transistor and p-transistor pass gates will operate properly for this case

assume this mux implementation will be used

tristate driver based muxes could be used, which restore signal drive and cut series RC chains

Unit Gate Delay Model

All simple gate of two or three inputs that are directly implementable in one logic level in CMOS are considered to have a delay of one.

All other gate must be implemented by such gates, and have the delay of the underlying circuit.

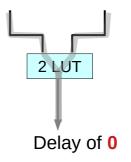
Delay of one

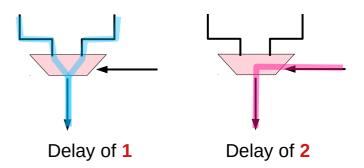
- inverters and
- 2 to 3 input NAND
- 2 to 3 input NOR gates

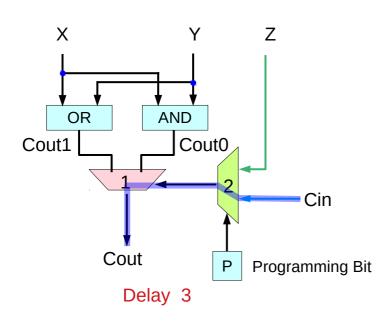
A 2:1 mux has a delay of one from the IO or I1 inputs to the output, But has a delay of two from the select input to the output due to the Inverter delay

Delay of zero (constant delay)

- the delay of the 2-LUTs,
- any routing leading to them,







Significantly slower two muxes on the carry chain in each cell

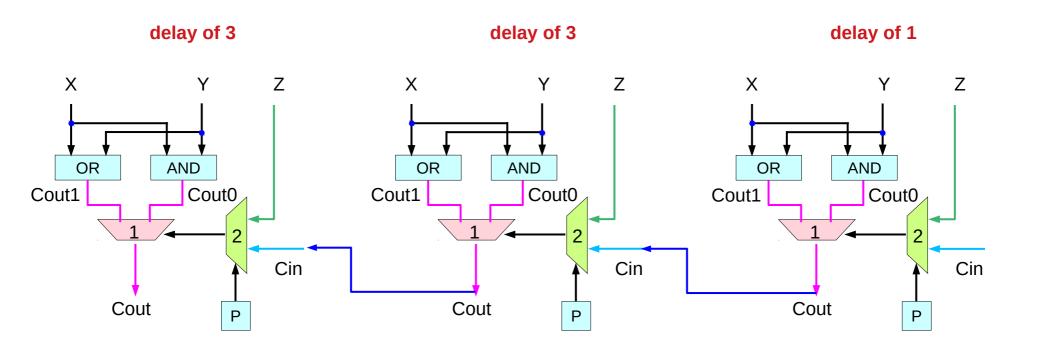
Delay 1 for first cell
Delay 3 for each additional cell in the carry chain
delay 1 for mux2
delays 2 for mux1

Overall 3n-2 for an n-cell carry chain

larger delay

X
Y
Z
Cout1
Cout0
Programming Bit
Delay 1

The critical path comes from the 2-LUTs and not from the input Z since the delay through the 2-LUTs will be larger than through mux 2 in the first cell



delay of 3n-2 for an n-bit ripple carry chain

the linear delay growth of ripple carry adders

optimize a ripple carry chain structure for use in FPGAs

while this provides some performance gain over the basis ripple carry scheme found in many current FPGAs,

still much slower than what is done in custom logic

advanced adder techniques in custom logic can be integrated into reconfigurable logic

Design A

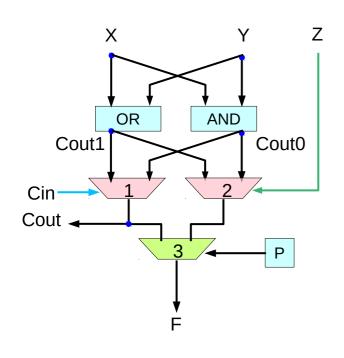
2n / 2n+2

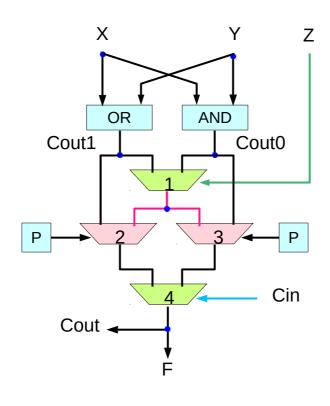
Design B

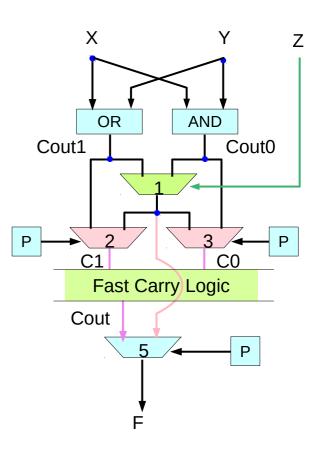
2n / 2n+1

Design C

2n+2





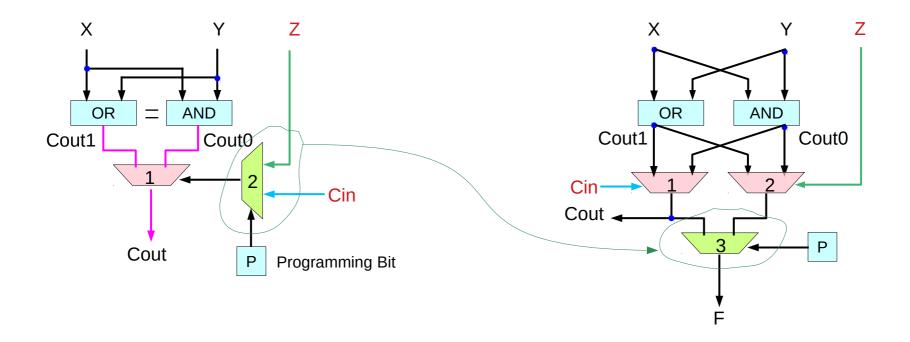


Design A (1)

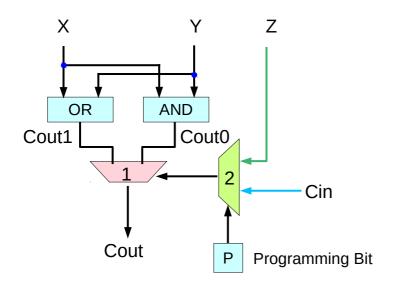
to reduce the delay of the ripple carry chain

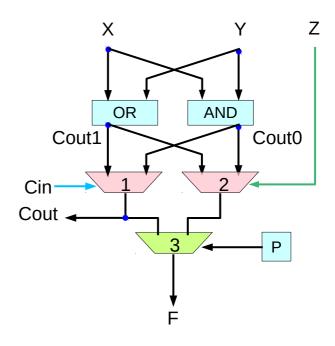
- remove mux2 from the carry path.
- no need to choose between Cin and Z for the select line to the output mux1

- two separate muxes, mux1 and mux2, controlled by Cin and Z, respectively.
- the circuit chooses between these outputs with mux3.



Design A (2)

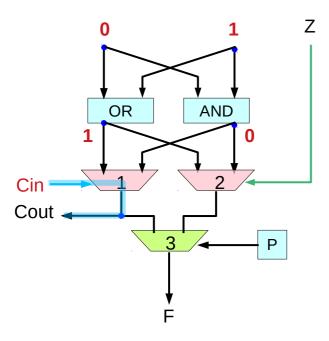




- not logically equivalent
- the Z input in the <u>first</u> cell cannot be used
 - Z is only attached to mux2
 - mux2 does not lead to the carry cells
 - not connected to Cout

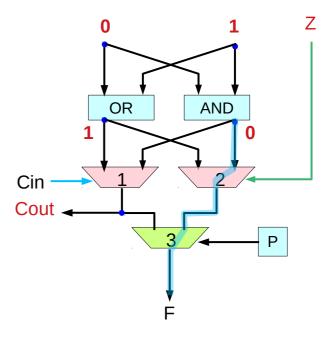
Design A (3)

delay of 2



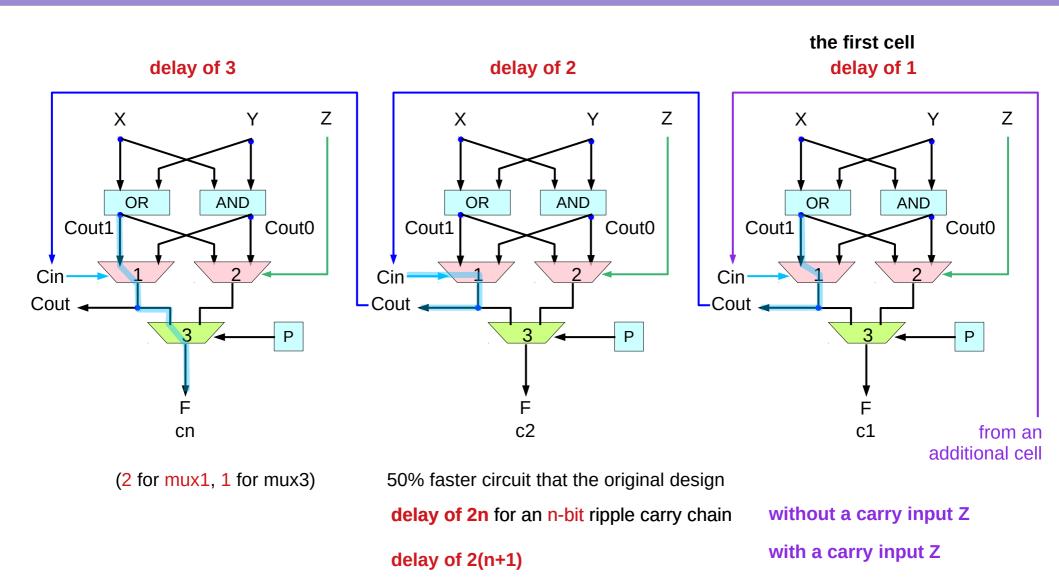
an additional cell for generating Cin

delay of 2

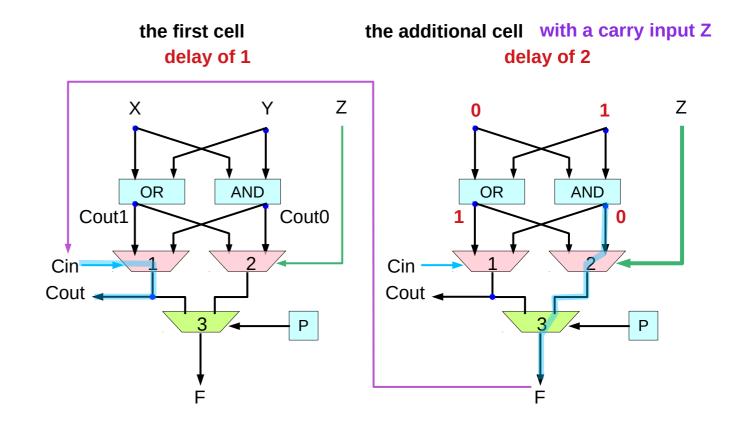


 need an <u>additional cell</u> to use Z as a carry input

Design A (4)



Design A (5)

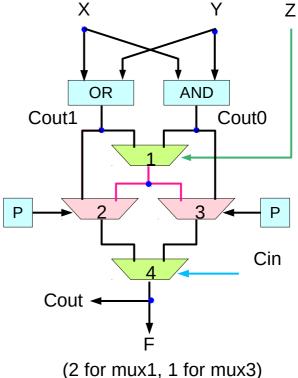


delay of 2(n+1) for an n-bit ripple carry chain with a carry input

Design B (1)

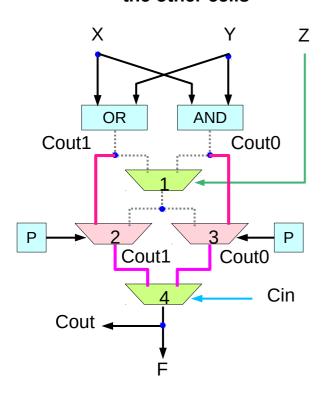
although this design is 1 gate delay slower than that of fig 2a, it provides the ability to have a carry input to the first cell in a carry chain, something that is important in many computations.

Also, for carry computations that do not need this feature, without a carry input the first cell in a carry chain built from fig 2b can be configured to bypass mux1, reducing the overall delay to 2n, which is identical to that of fig2a.



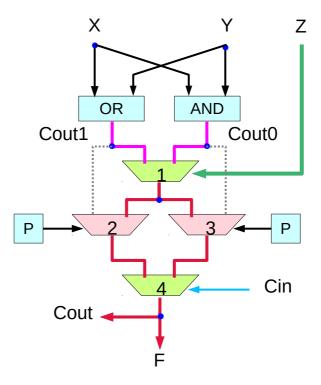
Design B (2)

the other cells



for cells in the middle of a carry chain mux2 passes Cout1 mux3 passes Cout0 mux4 receives Cout1 and Cout0 provides a standard ripple carry path.

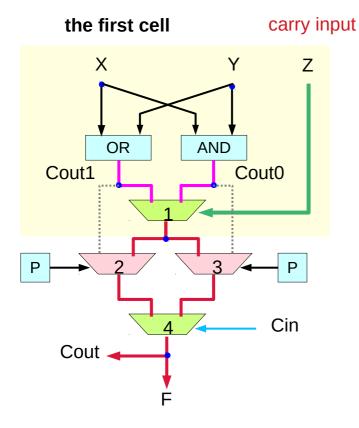
the first cell carry input



For the <u>first</u> cell in a carry chain with a <u>carry input</u> (provided by input Z), <u>mux2</u> and <u>mux3</u> both pass the value from <u>mux1</u>

the two main inputs to mux4 are identical the output of mux4 (Cout) will be the same as the output of mux1 (ignoring Cin)

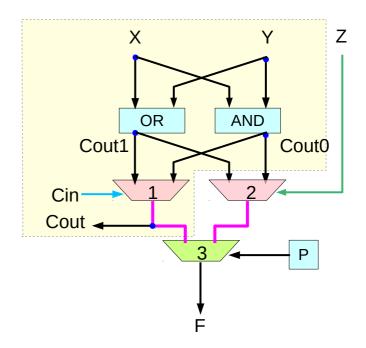
Design B (3)



mux1's main inputs are driven by two 2-LUTs (OR, AND) controlled by X and Y mux1 forms a **3-LUT** with the other 2-LUTs

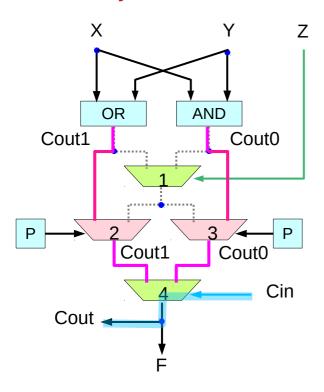
When mux2 and mux3 pass the value from mux1 (Cout1 and Cout2 respectively) the circuit is configured to continue the carry chain

Functionally equivalent



Design B (4)

delay of 2 the other cells

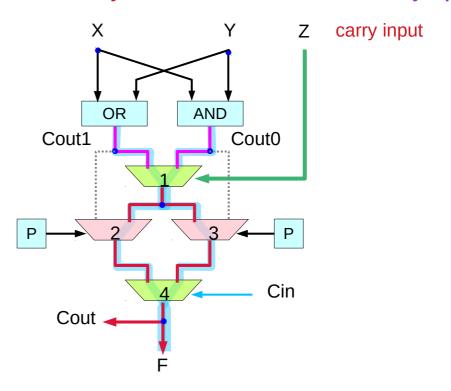


A delay of 2 in all other cells except the first cell in the carry chain

an total delay of **2n+1** for an n-bit carry chain when a carry input to the first cell is enabled

1 gate delay slower than that of fig 2a,

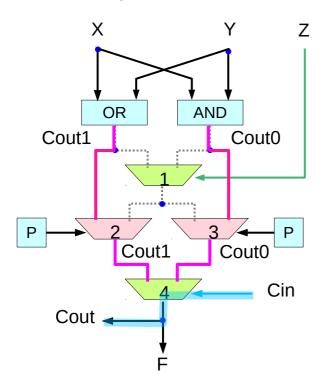
delay of 3 the first cell with a carry input



a delay of 3 in the first cell 1 in mux1, 1 in mux2, 1 in mux4

Design B (5)

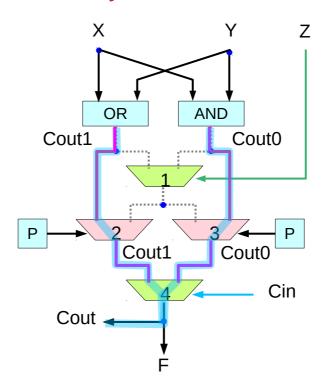
delay of 2 the other cells



A delay of 2 in all other cells except the first cell in the carry chain

an total delay of 2n for an n-bit carry chain when a carry input to the first cell is disabled

delay of 2 the first cell without a carry input



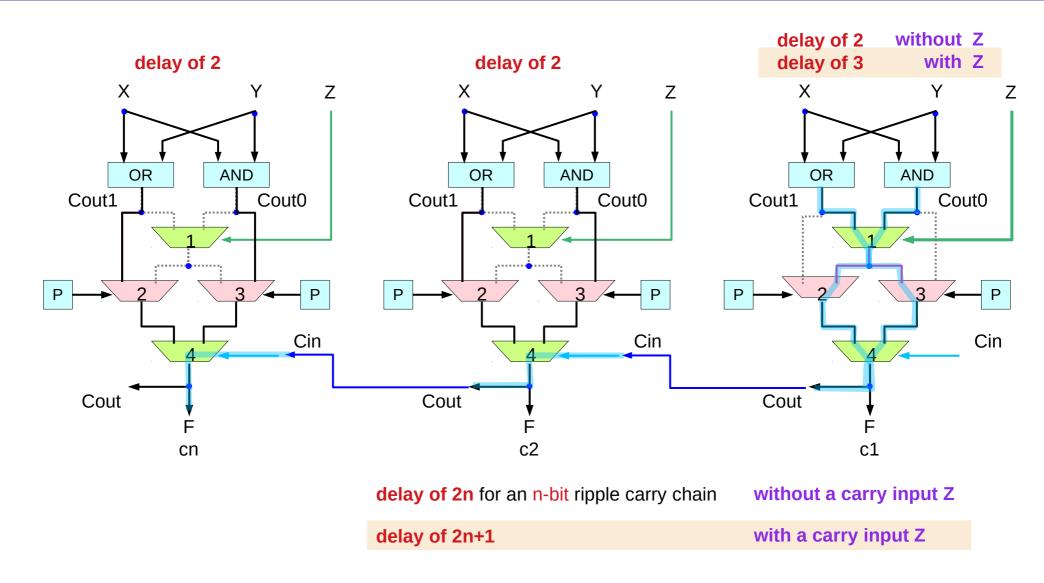
a delay of 2 in the first cell when a carry input is not used

32

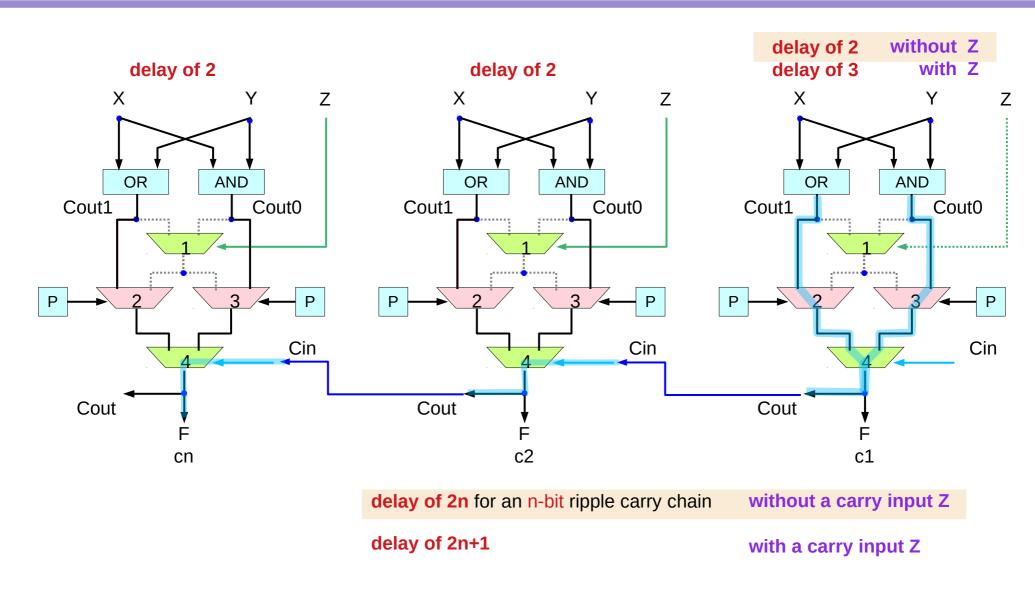
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Design B (6)



Design B (7)



Design C (1)

the actual carry chain (<u>mux4</u>) in Design B has been replaced by

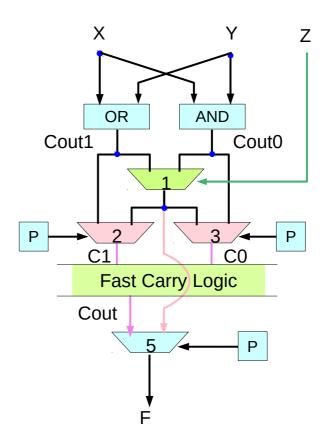
- · an abstract fast carry logic unit
- mux5 has been added

to the abstract fast carry logic units, various high performance carry chains can be applied

mux5 is present because

- significant delay for non-carry computations
- much <u>faster</u> carry propagation for long carry chains

when used as a simple normal **3 LUT**, using inputs X, Y, and Z mux5 allows us to bypass the carry chain by selecting the output of mux1



Design C (2)

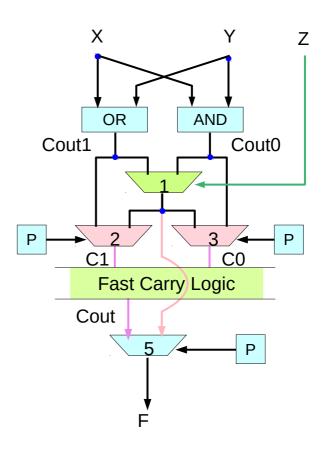
All the developed fast carry logic units in Design C that can compute the following value, can provide the functionality necessary to support the needs of FPGA carry chain computations

$$Cout_i = (Cout_{i-1} \cdot C 1_i) + (\overline{Cout_{i-1}} \cdot C 0_i)$$

where *i* is the position of the cell within the carry chain,

thus, the fast carry logic unit can contain any logic structure implementing this equation (including Brent-Kung), Variable Bit, and Ripple Carry.

Note that because of the needs and requirements of carry chains for FPGAs, new circuits are developed, by utilizing the standard adder structures, but which are more appropriate for FPGAs



Design C (3)

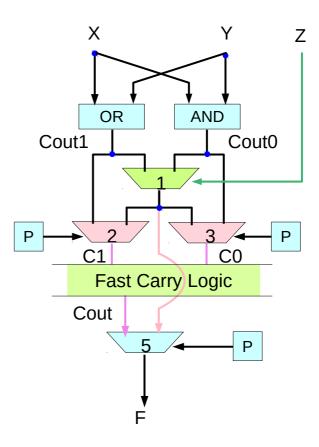
the main difference is to support all states

- Generate
- Propagate
- Kill
- Inverse Propagate

These 4 states are encoded on signals C1 and C0

Also, while standard adders are concerened only with the maximum delay through an entire n-bit adder structure, the delay concerns for FPGAs are more complicated

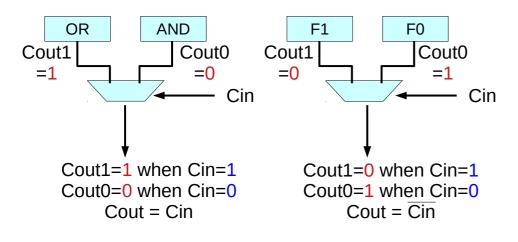
Specifically, when an n-bit carry chain is built into the architecture of an FPGA it does <u>not</u> represent an <u>actual</u> computation, but only the <u>potential</u> for a computation.



Design C (4)

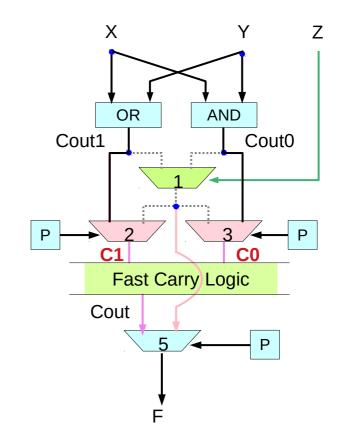
		Cin	Cin	
Χ	Υ	Cout1	Cout0	
0	0	0	0	$\overline{X} \overline{Y}$
0	1	1	0	$\overline{X}Y$
1	0	1	0	$X\overline{Y}$
1	1	1	1	ΧY

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	Cin	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate



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C1	C0	Name	
0	0	0	Kill
0	1	Cin	Inverse Propagate
1	0	Cin Propagate	
1	1	1	Generate



Design C (5)

Χ	Υ	C1	C0	
0	0	0	0	$\overline{X} \overline{Y}$
0	1	1	0	$\overline{X}Y$
1	0	1	0	$X\overline{Y}$
1	1	1	1	ΧY

$$C1_i = X_i + Y_i$$

$$C0_i = X_i \cdot Y_i$$

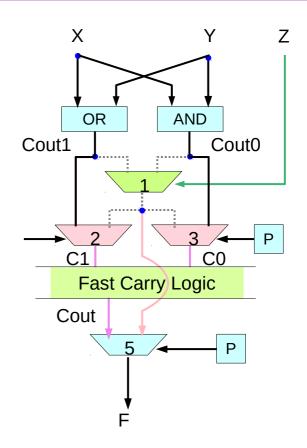
C1	C0	Name	
0	0	0	Kill
0	1	Cin	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

$$Cout_i = (Cout_{i-1} \cdot C \cdot 1_i) + (\overline{Cout_{i-1}} \cdot C \cdot 0_i)$$

$$(Cout_{i-1} \cdot C1_i) = Cout_{i-1} \cdot (\overline{X}Y + X\overline{Y} + XY)$$

$$(\overline{Cout_{i-1}} \cdot C \, 0_i) = \overline{Cout_{i-1}} \cdot X \, Y$$

X	Υ	Cout	Cout _{i+1}
0	0	0	0
0	1	0	0
1	0	0	0
1	1	0	1
0	0	1	0
0	1	1	1
1	0	1	1
1	1	1	1



Design C (6) - Complements of C0 and C1

_	-	_	
$C1=\lambda$	XY+X	Y + X	Y

Χ	Υ	C1
0	0	0
0	1	1
1	0	1
1	1	1

$$C0 = XY$$

Χ	Υ	C0
0	0	0
0	1	0
1	0	0
1	1	1

$$C1 = \bar{X}Y + X\bar{Y} + XY$$

$$C0 = XY$$

$$\overline{C1} = \overline{X} \overline{Y}$$

Χ	Υ	C 1
0	0	0
0	1	1
1	0	1
1	1	1

$$\overline{C0} = \overline{X}Y + X\overline{Y} + \overline{X}\overline{Y}$$

Χ	Υ	C0
0	0	0
0	1	1
1	0	1
1	1	1

$$\bar{C} 1 = \overline{(\bar{X}Y) + (X\bar{Y}) + (XY)} = \bar{X}\bar{Y}$$

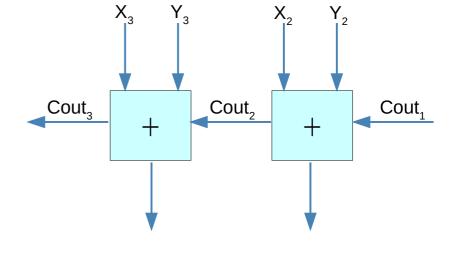
$$\bar{C}0 = \bar{X} + \bar{Y} = \bar{X}Y + X\bar{Y} + \bar{X}\bar{Y}$$

$$Cout_3 = (Cout_1 \cdot (C \cdot 1_3 \cdot C \cdot 1_2 + C \cdot 0_3 \cdot \overline{C \cdot 1_2})) + (\overline{Cout_1} \cdot (C \cdot 1_3 \cdot C \cdot 0_2 + C \cdot 0_3 \cdot \overline{C \cdot 0_2}))$$

$$= (Cout_{1} \cdot (C \, 1_{3} \cdot (\bar{X}_{2}Y_{2} + X_{2}\bar{Y}_{2} + X_{2}Y_{2}) + C \, 0_{3} \cdot \bar{X}_{2}\bar{Y}_{2})) + (\overline{Cout_{1}} \cdot (C \, 1_{3} \cdot X_{2}Y_{2} + C \, 0_{3} \cdot (\bar{X}_{2}Y_{2} + X_{2}\bar{Y}_{2} + \bar{X}_{2}\bar{Y}_{2})))$$

Design C (7) – Cout₃ in terms of Cout₁

$X_3 Y_3$	$X_2 Y_2$	Cout ₂	Cout ₃	Cout3
0 0	0 0	0	0	0
0 0	0 1	Cout ₁	0	0
0 0	1 0	Cout ₁	0	0
0 0	1 1	1	0	0
0 1	0 0	0	Cout ₃	0
0 1	0 1	Cout ₁	Cout ₃	Cout ₁
0 1	1 0	Cout ₁	Cout ₃	Cout₁
0 1	1 1	1	Cout ₃	1
1 0	0 0	0	Cout ₃	0
1 0	0 1	Cout ₁	Cout ₃	Cout ₁
1 0	1 0	Cout₁	Cout ₃	Cout ₁
1 0	1 1	1	Cout	1
1 1	0 0	0	1	1
1 1	0 1	Cout ₁	1	1
1 1	1 0	Cout ₁	1	1
1 1	1 1	1	1	1



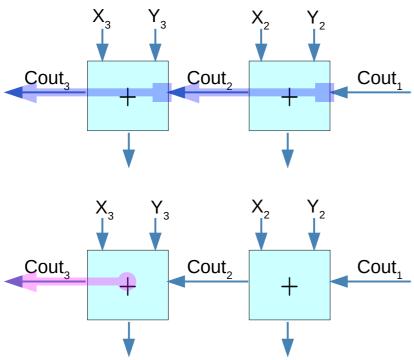
$$Cout_3 = (Cout_1 \cdot (C \, 1_3 \cdot C \, 1_2 + C \, 0_3 \cdot \overline{C} \, \overline{1_2})) + (\overline{Cout_1} \cdot (C \, 1_3 \cdot C \, 0_2 + C \, 0_3 \cdot \overline{C} \, \overline{0_2}))$$

Design C (8) - Cout₃ in terms of Cout₁

		_				Cout ₁	Cout ₁	$\overline{Cout}_{\scriptscriptstyle{\overline{1}}}$	$\overline{Cout}_{\scriptscriptstyle{\overline{1}}}$	
$X_3 Y_3$	$X_2 Y_2$	C1 ₃	C0 ₃	C1 ₂	C0 ₂	C1 ₃ C1 ₂	$C0_3\overline{C1}_2$	C1 ₃ C0 ₂	C0 ₃ C0 ₂	Cout ³
0 0	0 0	0	0	0	0	0	0	0	0	0
0 0	0 1	0	0	1	0	0	0	0	0	0
0 0	1 0	0	0	1	0	0	0	0	0	0
0 0	1 1	0	0	1	1	0	0	0	0	0
0 1	0 0	1	0	0	0	0	0	0	0	0
0 1	0 1	1	0	1	0	1	0	0	0	Cout ₁
0 1	1 0	1	0	1	0	1	0	0	0	Cout ₁
0 1	1 1	1	0	1	1	1	0	1	0	1
1 0	0 0	1	0	0	0	0	0	0	0	0
1 0	0 1	1	0	1	0	1	0	0	0	Cout ₁
1 0	1 0	1	0	1	0	1	0	0	0	Cout,
1 0	1 1	1	0	1	1	1	0	1	0	1
1 1	0 0	1	1	0	0	0	1	0	1	1
1 1	0 1	1	1	1	0	1	0	0	1	1
1 1	1 0	1	1	1	0	1	0	0	1	1
1 1	1 1	1	1	1	1	1	0	1	0	1

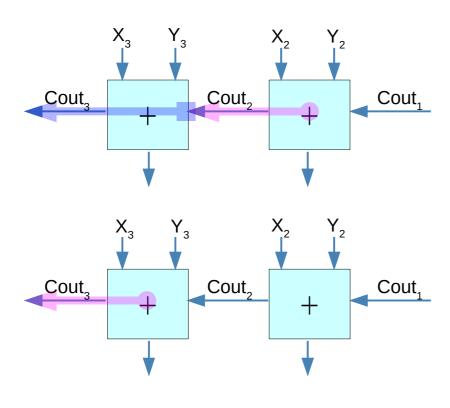
$$Cout_3 = (Cout_1 \cdot (C \, 1_3 \cdot C \, 1_2 + C \, 0_3 \cdot \overline{C} \, \overline{1_2})) + (\overline{Cout_1} \cdot (C \, 1_3 \cdot C \, 0_2 + C \, 0_3 \cdot \overline{C} \, \overline{0_2}))$$

Design C (9) – When Cout 1 = 1



$$Cout_3 = (Cout_1 \cdot (C \, 1_3 \cdot C \, 1_2 + C \, 0_3 \cdot \overline{C} \, 1_2)) + (\overline{Cout_1} \cdot (C \, 1_3 \cdot C \, 0_2 + C \, 0_3 \cdot \overline{C} \, 0_2))$$

Design C (10) – When Cout1 = 0



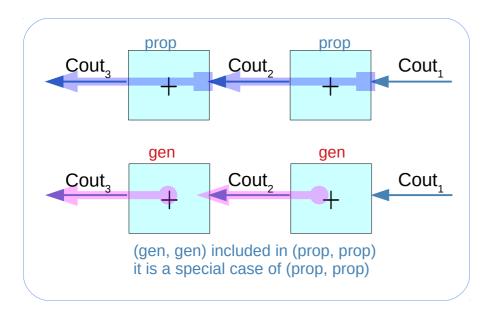
$$Cout_3 = (Cout_1 \cdot (C \, 1_3 \cdot C \, 1_2 + C \, 0_3 \cdot \overline{C} \, 1_2)) + (\overline{Cout_1} \cdot (C \, 1_3 \cdot C \, 0_2 + C \, 0_3 \cdot \overline{C} \, 0_2))$$

$$\begin{array}{ccc} C \, 1_3 \cdot C \, 0_2 \cdot \overline{Cout}_1 \\ \text{prop} & \text{gen} \\ \overline{X_3} \underline{Y_3} \\ X_3 \overline{Y_3} \\ X_3 Y_3 \end{array}$$

$$C \ 0_3 \cdot \overline{C} \ 0_2 \cdot \overline{Cout}_1$$
gen \overline{gen}
 $X_3Y_3 \quad \overline{X_2}\underline{Y_2}$
 $\underline{X_2}\overline{Y_2}$
 $\overline{X_2}\underline{Y_2}$

$$(C1_3C1_2 + C0_3\overline{C1}_2)Cout_1 + (C1_3C0_2 + C0_3\overline{C0}_2)\overline{Cout}_1$$

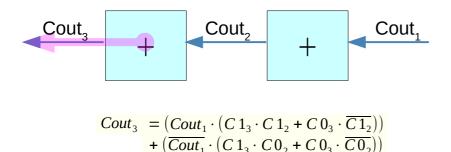
Design C (11) – When Cout1 = 1



$$C\, 1_3 \cdot C\, 1_2 \cdot Cout_1$$

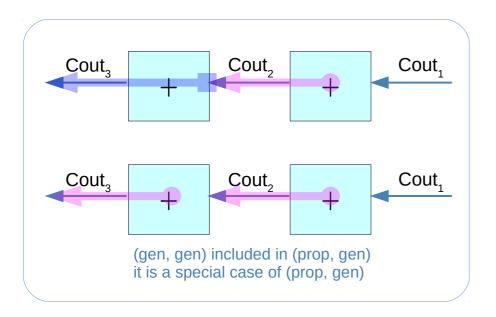
prop

 $\overline{X_3} \underline{Y_3} \qquad \overline{X_2} \underline{Y_2} \qquad X_3 \overline{Y_3} \qquad X_2 \overline{Y_2} \qquad X_3 \underline{Y_3} \qquad X_2 \underline{Y_2} \qquad X_3 \underline{Y_3} \qquad X_2 \underline{Y_2} \qquad X_3 \underline{Y_3} \qquad X_$

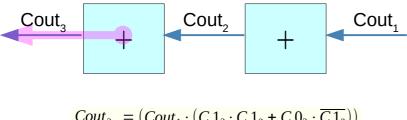


$$C \ 0_3 \cdot \overline{C} \ 1_2 \cdot Cout_1$$
gen \overline{prop}
 $X_3 Y_3 \quad \overline{X_2} \overline{Y_2}$

Design C (12) – When Cout1 = 0



$$C\,1_3\cdot C\,0_2\cdot \overline{Cout}_1$$
 prop gen X_3Y_3 X_2Y_2 X_3Y_3



$$Cout_3 = (Cout_1 \cdot (C \cdot 1_3 \cdot C \cdot 1_2 + C \cdot 0_3 \cdot \overline{C} \cdot \overline{1_2})) + (\overline{Cout_1} \cdot (C \cdot 1_3 \cdot C \cdot 0_2 + C \cdot 0_3 \cdot \overline{C} \cdot \overline{0_2}))$$

$$\begin{array}{ccc} C \, \mathbf{0}_3 \cdot \overline{C} \, \mathbf{0}_2 \cdot \overline{Cout}_1 \\ \\ \text{gen} & \overline{\mathbf{gen}} \\ & \overline{\mathbf{X}_2} \underline{\mathbf{Y}_2} \\ & \underline{\mathbf{X}_2} \overline{\underline{\mathbf{Y}_2}} \\ & \underline{\mathbf{X}_2} \overline{\mathbf{Y}_2} \\ & \underline{\mathbf{X}_2} \overline{\mathbf{Y}_2} \end{array}$$

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 $(C1_3C1_2 + C0_3\overline{C1}_2)Cout_1 + (C1_3C0_2 + C0_3\overline{C0}_2)\overline{Cout}_1$

$$C1 = \overline{X}Y + X\overline{Y} + XY$$
 $C0 = XY$

$$\overline{C1} = \overline{X}\overline{Y}$$

$$\overline{C0} = \overline{X}Y + X\overline{Y} + \overline{X}\overline{Y}$$

C1 and C0 are not mutually exclusive

C1 includes C0

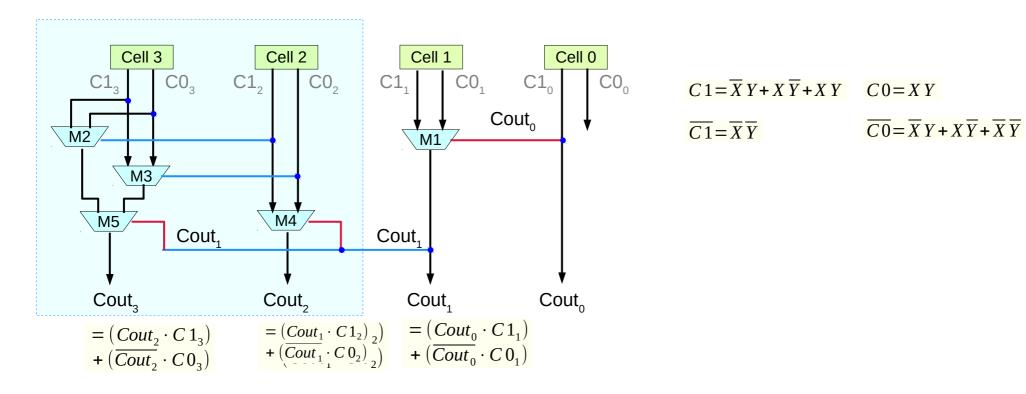
$$C1 \cdot C0 = C0$$

$$C1 + C0 = C1$$

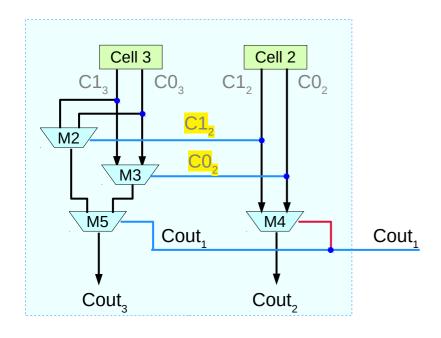
$$\overline{C1} \cdot \overline{C0} = \overline{C1}$$

$$\overline{C1} \cdot \overline{C0} = \overline{C1}$$
 $\overline{C1} + \overline{C0} = \overline{C0}$

Design C - Carry Select (1)



$$\begin{aligned} Cout_3 &= \left(Cout_1 \cdot \left(C \, \mathbf{1}_3 \cdot C \, \mathbf{1}_2 + C \, \mathbf{0}_3 \cdot \overline{C} \, \overline{\mathbf{1}_2} \right) \right) \\ &+ \left(\overline{Cout_1} \cdot \left(C \, \mathbf{1}_3 \cdot C \, \mathbf{0}_2 + C \, \mathbf{0}_3 \cdot \overline{C} \, \overline{\mathbf{0}_2} \right) \right) \end{aligned} \\ &+ \left(\overline{Cout_1} \cdot \left(C \, \mathbf{1}_3 \cdot C \, \mathbf{0}_2 + C \, \mathbf{0}_3 \cdot \overline{C} \, \overline{\mathbf{0}_2} \right) \right) \\ &+ \left(\overline{Cout_1} \cdot \left(C \, \mathbf{1}_3 \cdot X_2 Y_2 + C \, \mathbf{0}_3 \cdot (\overline{X}_2 Y_2 + X_2 \overline{Y}_2 + \overline{X}_2 \overline{Y}_2) \right) \right) \end{aligned}$$



$$(C1_3C1_2 + C0_3\overline{C1}_2)Cout_1 + (C1_3C0_2 + C0_3\overline{C0}_2)\overline{Cout}_1$$

$$C1 = \overline{X}Y + X\overline{Y} + XY$$
 $C0 = XY$

$$\overline{C1} = \overline{X} \overline{Y}$$
 $\overline{C0} = \overline{X} Y + X \overline{Y} + \overline{X} \overline{Y}$

$$\begin{aligned} Cout_2 &= (Cout_1 \cdot C \, \mathbf{1}_2) + (\overline{Cout_1} \cdot C \, \mathbf{0}_2) \\ Cout_3 &= (Cout_2 \cdot C \, \mathbf{1}_3) + (\overline{Cout_2} \cdot C \, \mathbf{0}_3) \\ &= (((Cout_1 \cdot C \, \mathbf{1}_2) + (\overline{Cout_1} \cdot C \, \mathbf{0}_2)) \cdot C \, \mathbf{1}_3) \\ &+ ((\overline{(Cout_1 \cdot C \, \mathbf{1}_2)} + (\overline{Cout_1} \cdot C \, \mathbf{0}_2)) \cdot C \, \mathbf{0}_3) \end{aligned}$$

$$(((Cout_1 \cdot C 1_2) + (\overline{Cout_1} \cdot C 0_2)) \cdot C 1_3)$$

$$= (C 1_3 C 1_2 Cout_1 + C 1_3 C 0_2 \overline{Cout_1})$$

$$\begin{split} & \left(\left(\overline{(Cout_1 \cdot C \, \mathbf{1}_2)} \cdot \overline{(\overline{Cout_1} \cdot C \, \mathbf{0}_2)} \right) \cdot C \, \mathbf{0}_3 \right) \\ &= \left(\left(\left(\overline{Cout_1} + \overline{C \, \mathbf{1}_2} \right) \cdot \left(Cout_1 + \overline{C \, \mathbf{0}_2} \right) \right) \cdot C \, \mathbf{0}_3 \right) \\ &= \left(\overline{Cout_1} Cout_1 + \overline{C \, \mathbf{1}_2} Cout_1 + \overline{Cout_1} \overline{C \, \mathbf{0}_2} + \overline{C \, \mathbf{1}_2} \overline{C \, \mathbf{0}_2} \right) \cdot C \, \mathbf{0}_3 \\ &= \left(\overline{C \, \mathbf{1}_2} Cout_1 + \overline{C \, \mathbf{0}_2} \overline{Cout_1} \right) \cdot C \, \mathbf{0}_3 \\ &= \left(C \, \mathbf{0}_3 \overline{C \, \mathbf{1}_2} Cout_1 + C \, \mathbf{0}_3 \overline{C \, \mathbf{0}_2} \overline{Cout_1} \right) \end{split}$$

$$\begin{split} &\overline{C}\, \overline{1}_2 \overline{C}\, \overline{0}_2 = (\overline{X}_2 \, \overline{Y}_2) \cdot (\overline{X}_2 \, Y_2 + X_2 \, \overline{Y}_2 + \overline{X}_2 \, \overline{Y}_2) = \overline{C}\, \overline{1}_2 \\ &\overline{C}\, \overline{1}_2 Cout_1 + \overline{C}\, \overline{1}_2 = \overline{C}\, \overline{1}_2 (Cout_1 + 1) \end{split}$$

$$= (\overline{Cout_1}Cout_1 + \overline{C1_2}Cout_1 + \overline{Cout_1}\overline{C0_2} + \overline{C1_2}\overline{C0_2}) \cdot C0_3$$

$$= (\overline{C1_2}Cout_1 + \overline{Cout_1}\overline{C0_2} + \overline{C1_2}\overline{C0_2}) \cdot C0_3$$

$$= (\overline{C1_2}\overline{C0_2}Cout_1 + \overline{Cout_1}\overline{C0_2} + \overline{C1_2}\overline{C0_2}) \cdot C0_3$$

$$C1 \cdot C0 = C0 \qquad C1 + C0 = C1$$

$$\overline{C1} \cdot \overline{C0} = \overline{C1}$$
 $\overline{C1} + \overline{C0} = \overline{C0}$

$$\begin{aligned} Cout_2 &= \left(Cout_1 \cdot C \cdot 1_2\right) + \left(\overline{Cout_1} \cdot C \cdot 0_2\right) \\ Cout_3 &= \left(Cout_2 \cdot C \cdot 1_3\right) + \left(\overline{Cout_2} \cdot C \cdot 0_3\right) \\ &= \left(\left(\left(Cout_1 \cdot C \cdot 1_2\right) + \left(\overline{Cout_1} \cdot C \cdot 0_2\right)\right) \cdot C \cdot 1_3\right) \\ &+ \left(\overline{\left(\left(Cout_1 \cdot C \cdot 1_2\right) + \left(\overline{Cout_1} \cdot C \cdot 0_2\right)\right)} \cdot C \cdot 0_3\right) \end{aligned}$$

$$(((Cout_1 \cdot C \cdot 1_2) + (\overline{Cout_1} \cdot C \cdot 0_2)) \cdot C \cdot 1_3)$$

$$= (C \cdot 1_3 C \cdot 1_2 Cout_1 + C \cdot 1_3 C \cdot 0_2 \overline{Cout_1})$$

$$\begin{split} & \left(\left(\overline{(Cout_1 \cdot C \, \mathbf{1}_2)} \cdot \overline{(\overline{Cout_1} \cdot C \, \mathbf{0}_2)} \right) \cdot C \, \mathbf{0}_3 \right) \\ &= \left(\left(\left(\overline{Cout_1} + \overline{C \, \mathbf{1}_2} \right) \cdot \left(Cout_1 + \overline{C \, \mathbf{0}_2} \right) \right) \cdot C \, \mathbf{0}_3 \right) \\ &= \left(\overline{Cout_1} Cout_1 + \overline{C \, \mathbf{1}_2} Cout_1 + \overline{Cout_1} \overline{C \, \mathbf{0}_2} + \overline{C \, \mathbf{1}_2} \overline{C \, \mathbf{0}_2} \right) \cdot C \, \mathbf{0}_3 \\ &= \left(\overline{C \, \mathbf{1}_2} Cout_1 + \overline{C \, \mathbf{0}_2} \overline{Cout_1} \right) \cdot C \, \mathbf{0}_3 \\ &= \left(C \, \mathbf{0}_3 \overline{C \, \mathbf{1}_2} Cout_1 + C \, \mathbf{0}_3 \overline{C \, \mathbf{0}_2} \overline{Cout_1} \right) \end{split}$$

$$\begin{split} &\overline{C}\,\overline{1}_2\overline{C}\,\overline{0}_2 = (\overline{X}_2\overline{Y}_2)\cdot (\overline{X}_2\overline{Y}_2 + \overline{X}_2\overline{Y}_2 + \overline{X}_2\overline{Y}_2) = \overline{C}\,\overline{1}_2 \\ &\overline{C}\,\overline{1}_2Cout_1 + \overline{C}\,\overline{1}_2 = \overline{C}\,\overline{1}_2(Cout_1 + 1) \end{split}$$

$$C1 = \overline{X} Y + X \overline{Y} + X Y \qquad C0 = X Y$$

$$\overline{C1} = \overline{X} \overline{Y} \qquad \overline{C0} = \overline{X} Y + X \overline{Y} + \overline{X} \overline{Y}$$

C1 and C0 are <u>not</u> mutually exclusive C1 includes C0

$$\begin{aligned} Cout_2 &= \left(Cout_1 \cdot C \mathbf{1}_2 \right) + \left(\overline{Cout_1} \cdot C \mathbf{0}_2 \right) \\ Cout_3 &= \left(Cout_2 \cdot C \mathbf{1}_3 \right) + \left(\overline{Cout_2} \cdot C \mathbf{0}_3 \right) \\ &= \left(\left(\left(Cout_1 \cdot C \mathbf{1}_2 \right) + \left(\overline{Cout_1} \cdot C \mathbf{0}_2 \right) \right) \cdot C \mathbf{1}_3 \right) \\ &+ \left(\overline{\left(\left(Cout_1 \cdot C \mathbf{1}_2 \right) + \left(\overline{Cout_1} \cdot C \mathbf{0}_2 \right) \right) \cdot C \mathbf{0}_3 \right) \end{aligned}$$

$$\overline{\left(\left(Cout_1 \cdot C \mathbf{1}_2 \right) + \left(\overline{Cout_1} \cdot C \mathbf{0}_2 \right) \right)}$$

$$\overline{\left(\left(Cout_1 \cdot C \mathbf{1}_2 + C \mathbf{0}_2 \right) + \left(\overline{Cout_1} \cdot C \mathbf{0}_2 \right) \right)}$$

$$\overline{\left(Cout_1 \cdot C \mathbf{1}_2 + \left(Cout_1 + \overline{Cout_1} \right) \cdot C \mathbf{0}_2 \right)}$$

$$\overline{\left(Cout_1 \cdot C \mathbf{1}_2 + C \mathbf{0}_2 \right)}$$

$$\overline{\left(\left(\overline{Cout_1} \cdot C \mathbf{1}_2 + C \mathbf{0}_2 \right) \right)}$$

$$\overline{\left(\left(\overline{Cout_1} \cdot C \mathbf{1}_2 + C \mathbf{0}_2 \right)}$$

$$\overline{\left(\left(\overline{Cout_1} \cdot C \mathbf{0}_2 + \overline{C} \mathbf{1}_2 \right) \right)}$$

$$= \left(\overline{Cout_1} \cdot Cout_1 + \overline{Cout_1} \cdot \overline{Co_2} + \overline{Cout_1} \cdot \overline{Co_2} \right)$$

$$\overline{\left(\overline{Cout_1} \cdot Cout_1 + \overline{Cout_1} \cdot \overline{Co_2} + \overline{Cout_1} \right)}$$

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 $= (\overline{C1}_2 + \overline{Cout}_1 \overline{C0}_2)$

Design C - Carry Select (1)

$O4 \overline{V}V \cdot V\overline{V} \cdot VV$	CO MM
$C1 = \bar{X}Y + X\bar{Y} + XY$	C0 = XY

$$\overline{C1} = \overline{X} \, \overline{Y}$$

$$\overline{C0} = \overline{X} \, Y + X \, \overline{Y} + \overline{X} \, \overline{Y}$$

C1	C0		Name
0	0	0	Kill
0	1	Cin	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

$$Cout_1 = (Cout_0 \cdot C1_1) + (\overline{Cout_0} \cdot C0_1)$$

$$Cout_2 = (Cout_1 \cdot C1_2) + (\overline{Cout_1} \cdot C0_2)$$

$$Cout_3 = (Cout_2 \cdot C1_3) + (\overline{Cout_2} \cdot C0_3)$$

$$Cout_3 = (Cout_1 \cdot (C \, 1_3 \cdot C \, 1_2 + C \, 0_3 \cdot \overline{C} \, \overline{1_2}))$$

+ $(\overline{Cout_1} \cdot (C \, 1_3 \cdot C \, 0_2 + C \, 0_3 \cdot \overline{C} \, \overline{0_2}))$

$$= Cout_1 \cdot \left[(\bar{X}Y + X\bar{Y} + XY)_3 \cdot (\bar{X}Y + X\bar{Y} + XY)_2 + (XY)_3 \cdot (XY)_2 \right] + Cout_1 \cdot \left[(\bar{X}Y + X\bar{Y} + XY)_3 \cdot (XY)_2 + (XY)_3 \cdot (\bar{X}Y + X\bar{Y} + \bar{X}\bar{Y})_2 \right]$$

$$= (Cout_1 \cdot (C \, 1_3 \cdot (\bar{X}_2 Y_2 + X_2 \bar{Y}_2 + X_2 Y_2) + C \, 0_3 \cdot \bar{X}_2 \bar{Y}_2)) + (\overline{Cout_1} \cdot (C \, 1_3 \cdot X_2 Y_2 + C \, 0_3 \cdot (\bar{X}_2 Y_2 + X_2 \bar{Y}_2 + \bar{X}_2 \bar{Y}_2)))$$

Design C (3)

A carry chain resource may span the entire height of a column in the FPGA, but a mapping to the logic may use only a small portion of this chain, with the carry logic in the mapping starting and ending at arbitrary points in the column

Must consider

- the carry delay from the first to the last position in a carry chain,
- the delay for a **carry computation** beginning at any point within this **column**.

For example, even though the FPGA architecture may provide support for carry chains of up to 32 bits, it must also efficiently support 8 bit carry computations placed at any point within this carry chain resource

Design C (4)

Carry Select

the problem with a ripple carry structure is that the **computation** of the **Cout** for bit position i cannot begin until after the **computation** has been completed in bit positions **0** .. i-1

A carry select structure overcomes this limitation

the main observation is that for <u>any bit position</u>, the only information it received from the previous bit positions is its **Cin** signal, which can be either **true** or **false**.

Design C (5)

In a carry select adder the **carry chain** is <u>broken</u> at a specific column, and <u>two separate additions</u> occur

one for the **true** Cin signal the other for the **false** Cin signal

These computations can take place <u>before</u> the completion of the <u>previous columns</u>, since they do <u>not</u> depend on the <u>actual value</u> of the <u>Cin signal</u>

This Cin signal is instead used to <u>determine</u> which adder's outputs should be used

if the Cin signal is **true**, the output of the following stages comes from the adder that assumed that the Cin would be **true**

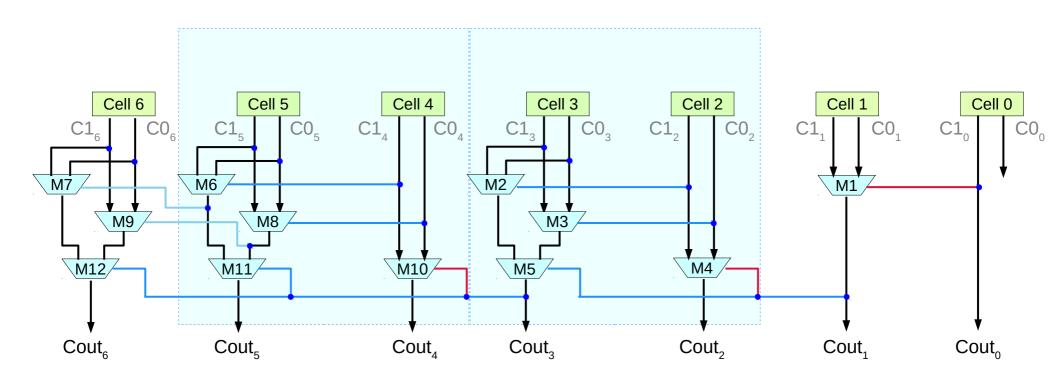
likewise, a **false** Cin chooses the other adder's output

Design C (6)

This <u>splitting</u> of the <u>carry chain</u> can be done <u>multiple times</u>, breaking the computation into <u>several pairs</u> of <u>short adders</u> with <u>output muxes</u> choosing which adder's output to <u>select</u>

the length of the adders and the breakpoint are carefully chosen such that the **small adders** finish computation just as their Cin signals become available

Short adders handle the low-order bits, and the adder length is increased further along the carry chain, since later computations have more time until their Cin signal is available



$$Cout_i = (Cout_{i-1} \cdot C 1_i) + (\overline{Cout_{i-1}} \cdot C 0_i)$$

$$Cout_1 = (Cout_0 \cdot C1_1) + (\overline{Cout_0} \cdot C0_1)$$

$$Cout_1 = (C1_0 \cdot C1_1) + (\overline{C1_0} \cdot C0_1)$$

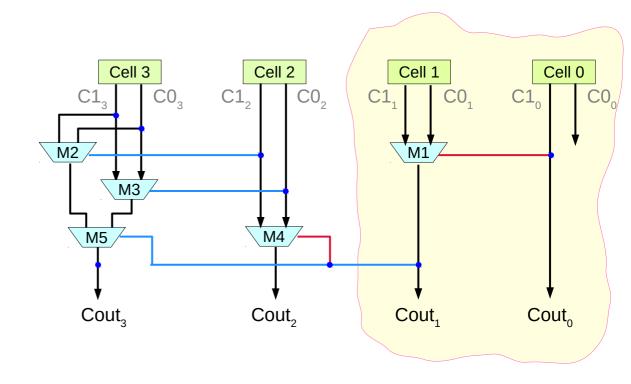
$$Cout_{i+1} = (Cout_i \cdot C1_{i+1}) + (\overline{Cout_i} \cdot C0_{i+1})$$

$$Cout_{i+1} = \left(\left[\left(Cout_{i-1} \cdot C \, \mathbf{1}_i\right) + \left(\overline{Cout_{i-1}} \cdot C \, \mathbf{0}_i\right)\right] \cdot C \, \mathbf{1}_{i+1}\right) + \left(\overline{\left[\left(Cout_{i-1} \cdot C \, \mathbf{1}_i\right) + \left(\overline{Cout_{i-1}} \cdot C \, \mathbf{0}_i\right)\right]} \cdot C \, \mathbf{0}_{i+1}\right)$$

Design C - Carry Select (1)

A **Carry Select carry chain** structure for use in FPGAs

the carry computation for the <u>first two cells</u> is performed with the simple **ripple-carry** structure implemented by <u>mux1</u>



Design C - Carry Select (2)

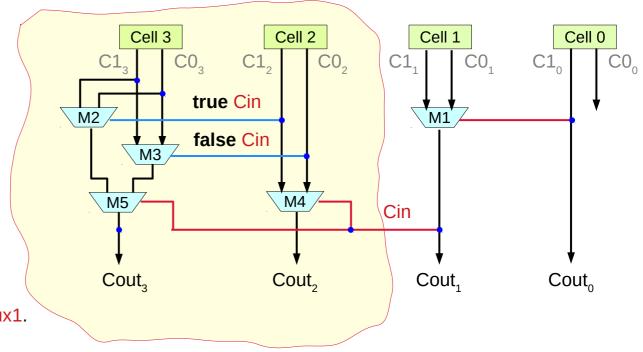
A Carry Select carry chain structure for use in FPGAs

For cell2 and cell3 we use two ripple carry adders,

with <u>one</u> adder (mux2) assuming the Cin is true,

and the other (mux3) assuming the Cin is false

Then mux4 and mux5 pick between these two adders' outputs based on the actual Cin coming from mux1.



Design C - Carry Select (3)

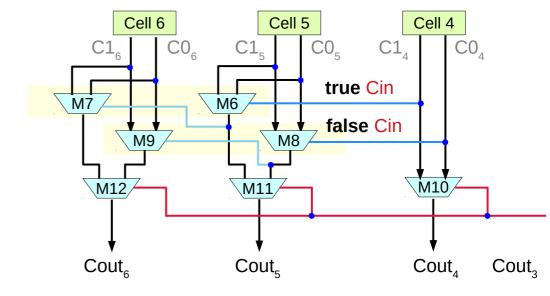
Similarly, cell4, cell5, cell6 have

two ripple carry adders

mux6 & mux7 for a Cin of 1

mux8 & mux9 for a Cin of 0

with output muxes (mux10, mux11, mux12) deciding between the two based upon the actual Cin (from mux5).



Design C - Carry Select (3)

Subsequent stages will continue to grow in length by one,

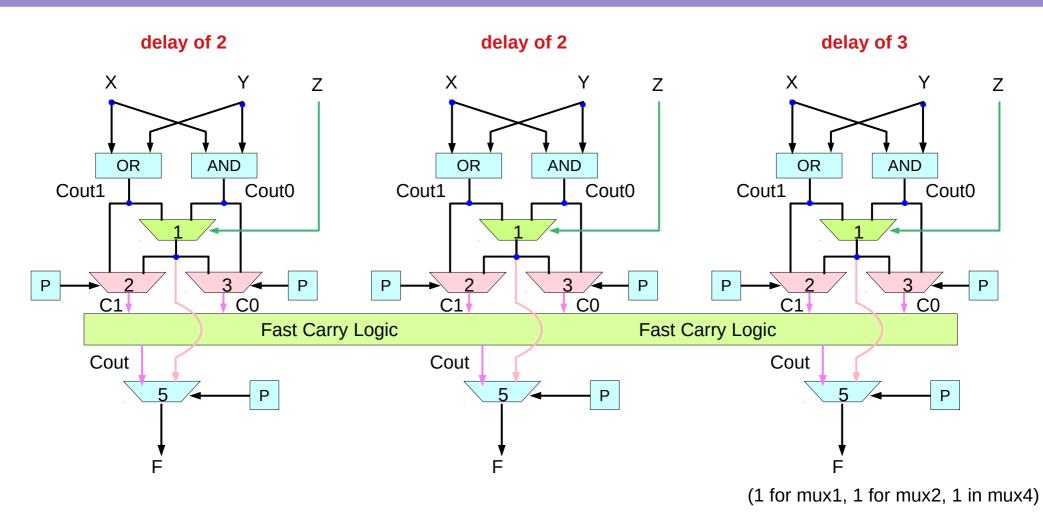
with cells7, cell8, cell9, cell10 in one block,

cell11, cell12, cell13, cell14, cell15 in another,

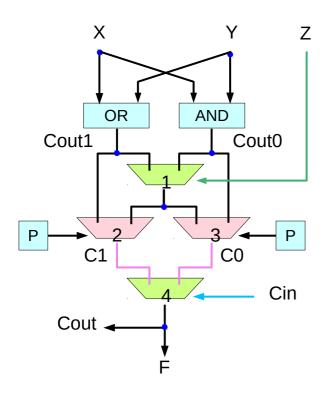
and so on.

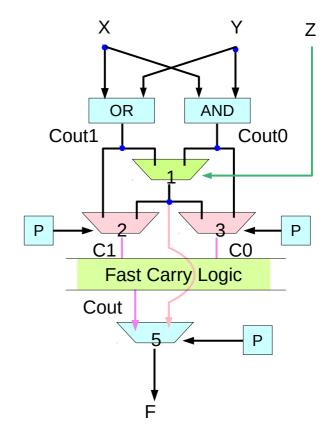
timing values showing the delay of the Carry Select carry chain relative to other carry chain will be presented later

Design C



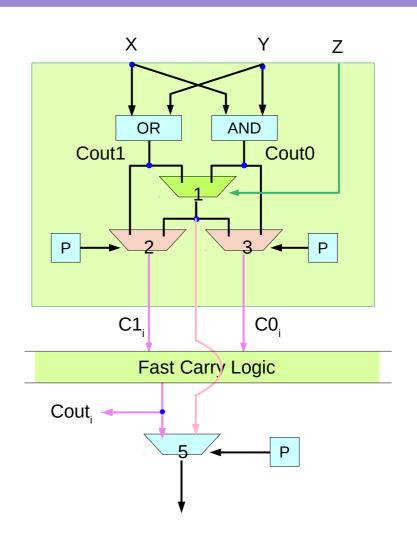
delay of 2n+2 for an n-bit ripple carry chain





$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$



Fast Carry Logc

Carry Select Adder
Carry Lookahead Adder
Brent-Kung
Variable Block
Ripple Carry Adder

https://en.wikipedia.org/wiki/Carry-lookahead_adder

Variable Block

like the carry select chain, a variable block structure consists of blocks of ripple carry element however, instead of precomputing the Cout value for each possible Cin value, it instead provides a way for the carry signal to skip over intermediate cells where appropriate.

Contiguous blocks of the computation are grouped together to form a unit with a standard ripple carry chain As part of this block, logic is to the value of the block's Cin, allowing the carry chain to bypass this block's normal carry chain on its way to later blocks.

https://en.wikipedia.org/wiki/Carry-lookahead_adder

Variable Block

The Cin still ripples through the block itself, since the intermediate carry values must also be computed If any of the cells in the carry chain are not in propagate mode, the Cout output is generated normally by the ripple carry chain. While this carry chain does start at the block's Cin signal, and leads to the block's Cout, this long path is a false path That is since there is some cell in the block that is not in propagate mode, it must be in generate or kill mode, and thus the block's Cout output does not depend on the block's Cin input

https://en.wikipedia.org/wiki/Carry-lookahead_adder