

Applicative (2A)

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Based on

<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Haskell in 5 steps

https://wiki.haskell.org/Haskell_in_5_steps

Currying

Currying recursively transforms
a function that takes multiple arguments
into a function that takes just a single argument and
returns another function if any arguments are still needed.

$f :: a \rightarrow b \rightarrow c$

$f\ x\ y$

$f :: a \rightarrow b \rightarrow c$

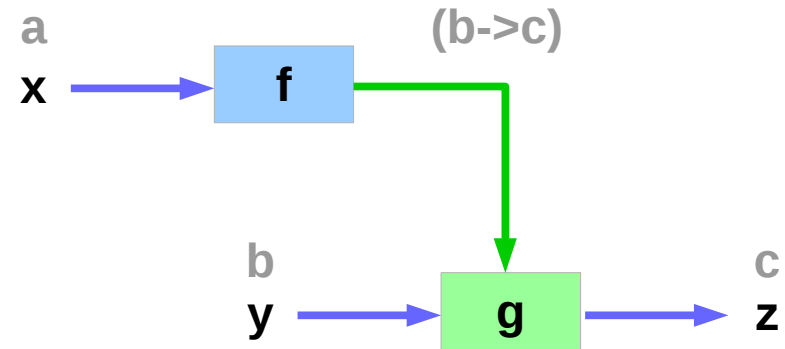
$(f\ x)\ y$

$f :: a \rightarrow (b \rightarrow c)$

$g\ y$

$g :: b \rightarrow c$

$f :: a \rightarrow b \rightarrow c$



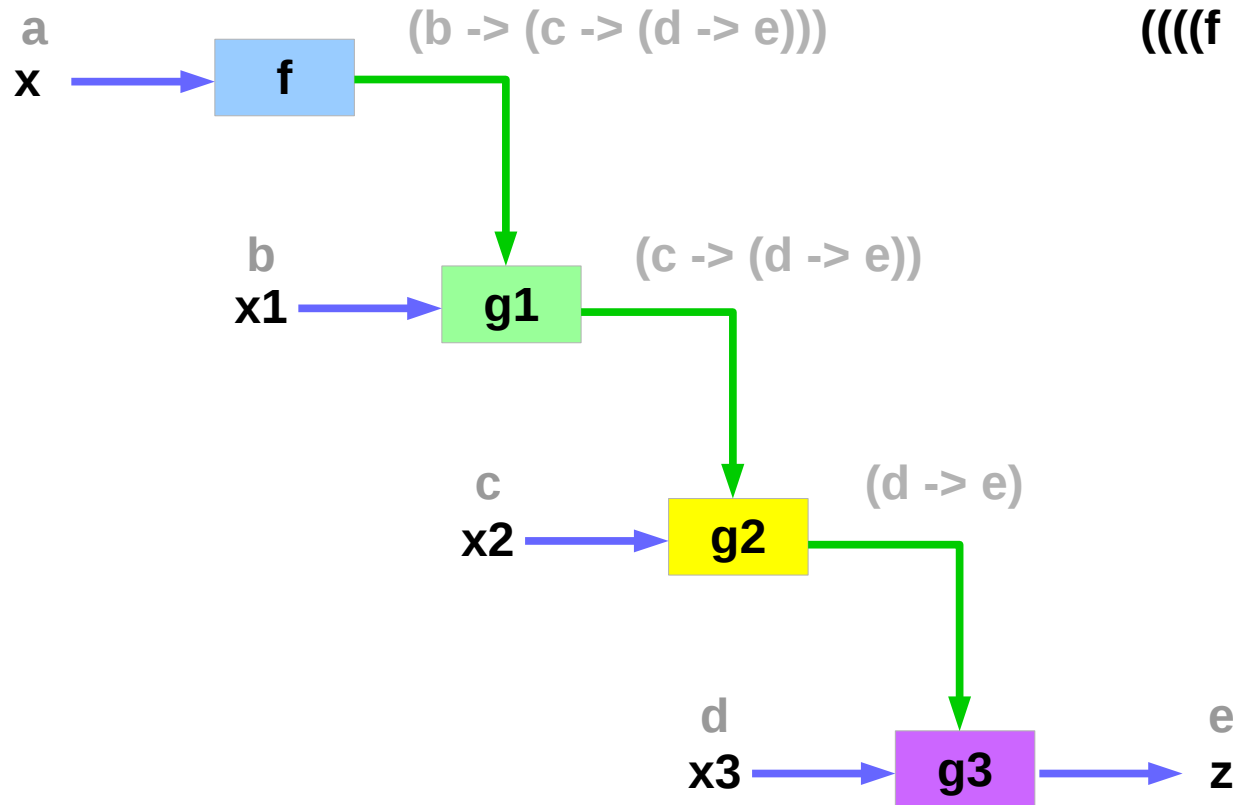
<https://wiki.haskell.org/Currying>

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Currying Examples

$f :: a \rightarrow b \rightarrow c \rightarrow d \rightarrow e$

$f :: a \rightarrow (b \rightarrow (c \rightarrow (d \rightarrow e)))$



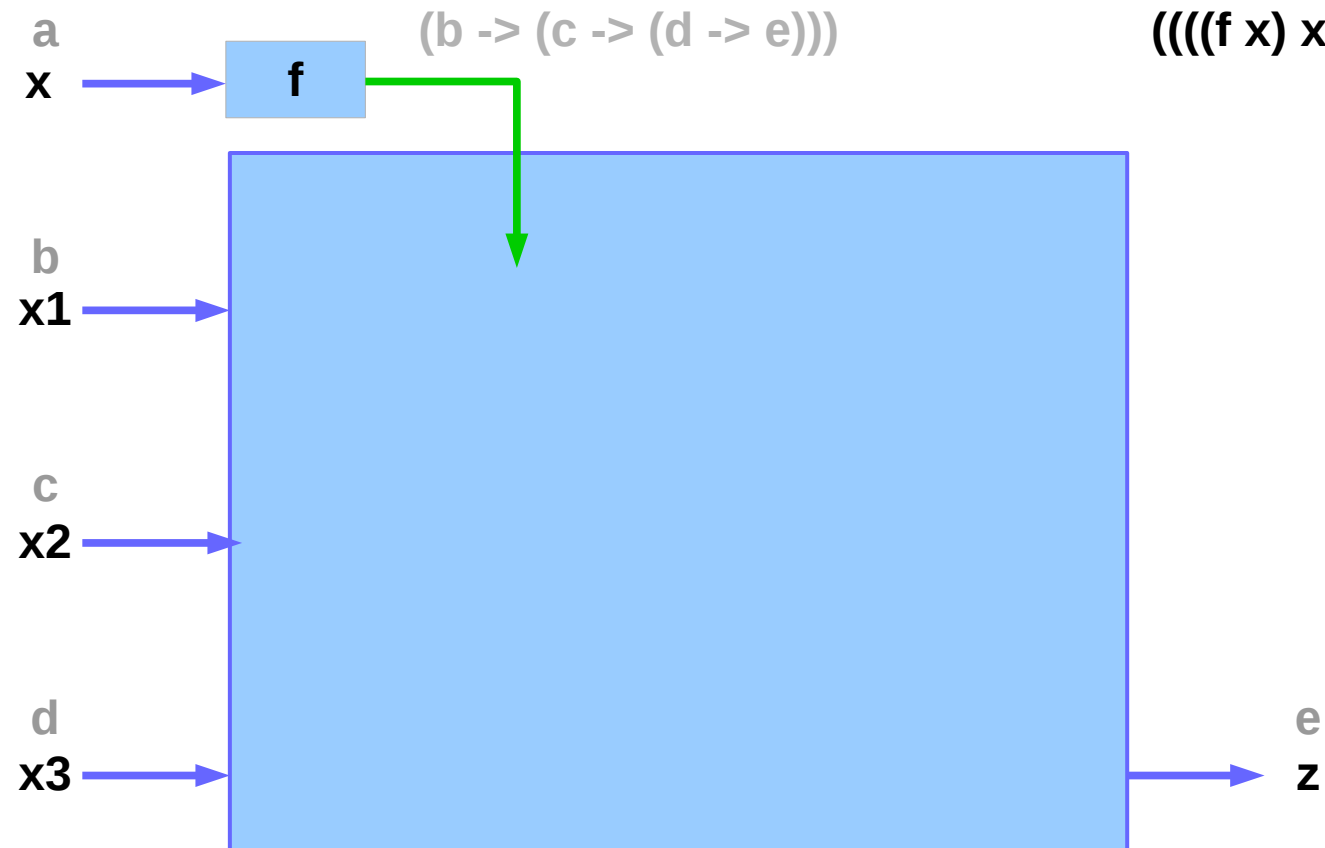
$(((((f\ x)\ x1)\ x2)\ x3))$

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Currying Examples

$f :: a \rightarrow b \rightarrow c \rightarrow d \rightarrow e$

$f :: a \rightarrow (b \rightarrow (c \rightarrow (d \rightarrow e)))$



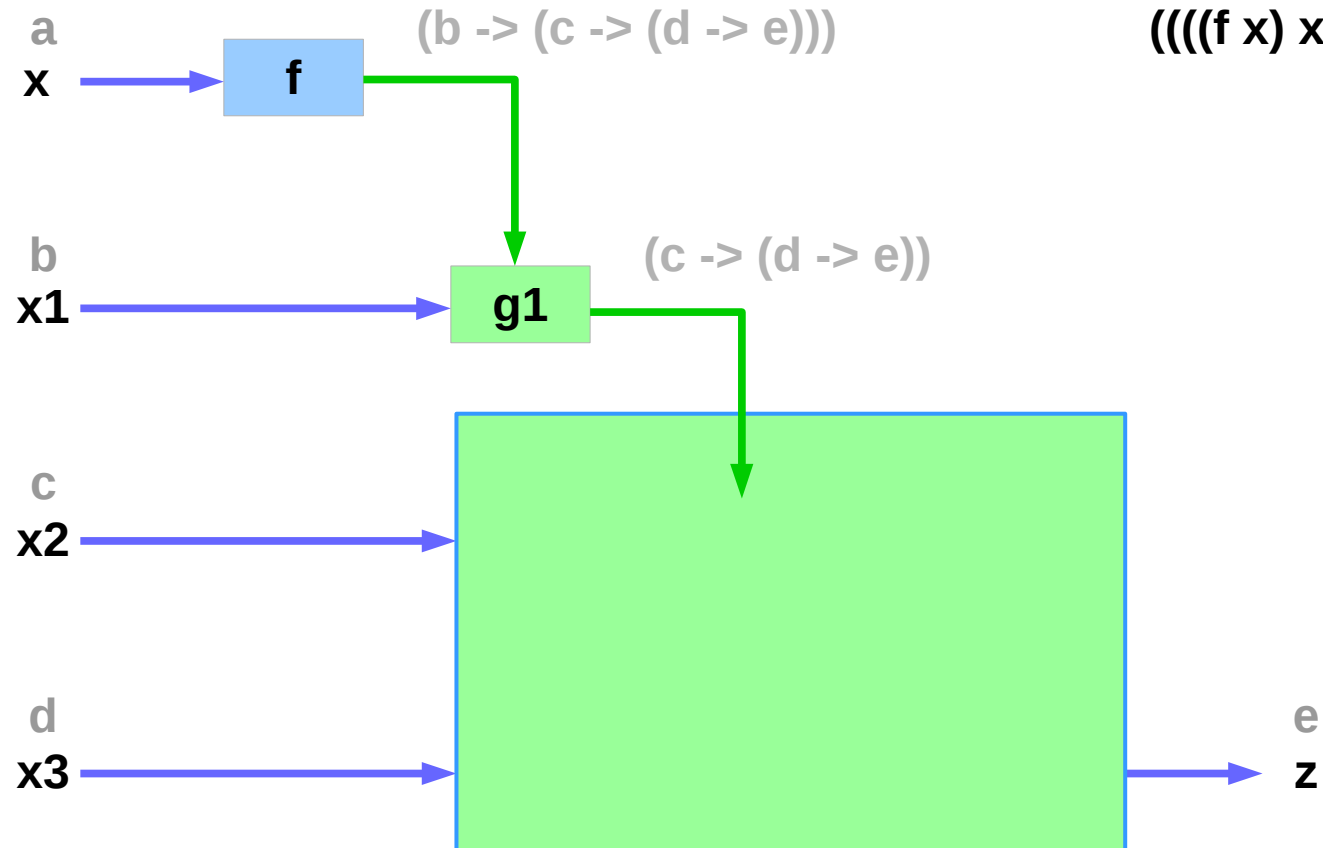
$((((f\ x)\ x_1)\ x_2)\ x_3)$

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Currying Examples

$f :: a \rightarrow b \rightarrow c \rightarrow d \rightarrow e$

$f :: a \rightarrow (b \rightarrow (c \rightarrow (d \rightarrow e)))$



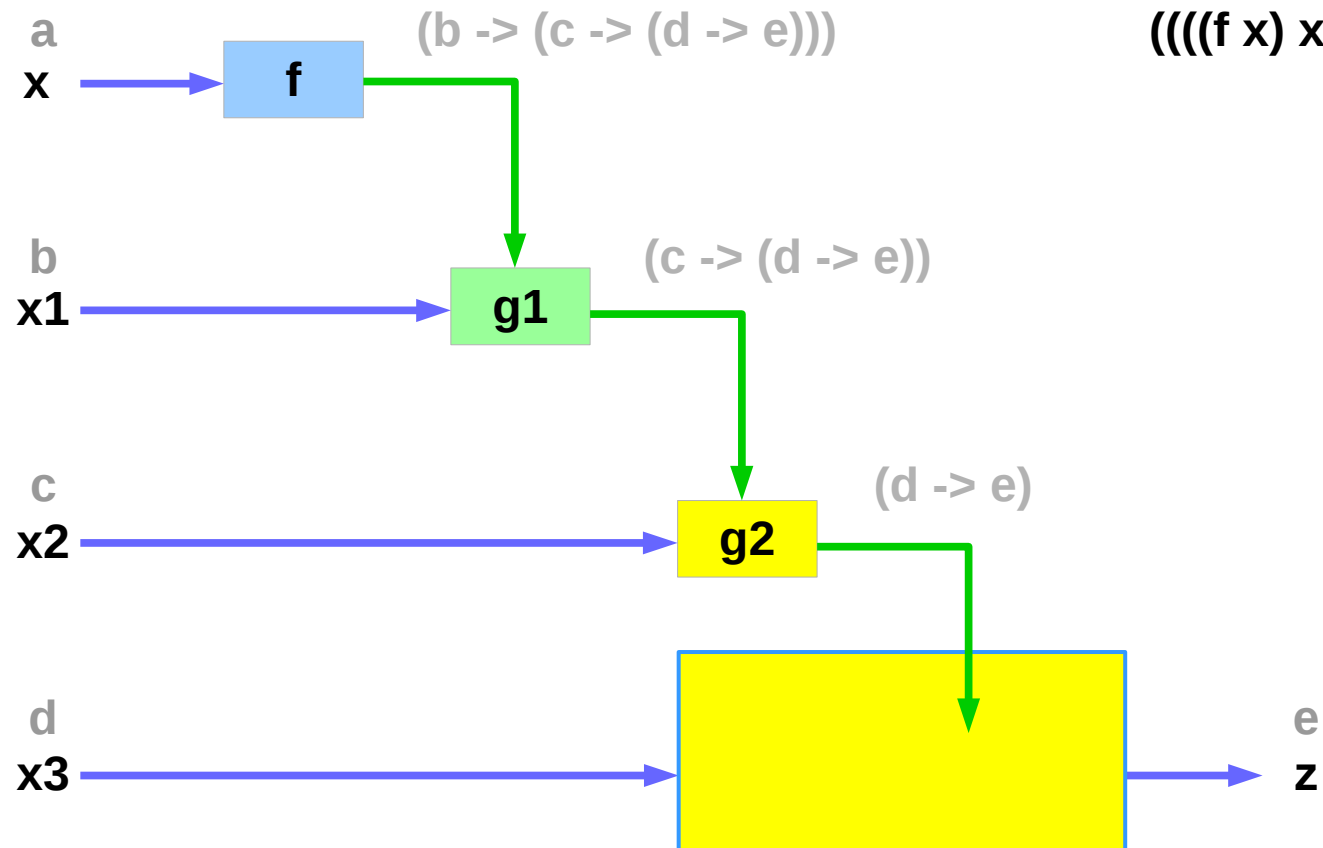
$(((((f\ x)\ x_1)\ x_2)\ x_3))$

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Currying Examples

$f :: a \rightarrow b \rightarrow c \rightarrow d \rightarrow e$

$f :: a \rightarrow (b \rightarrow (c \rightarrow (d \rightarrow e)))$



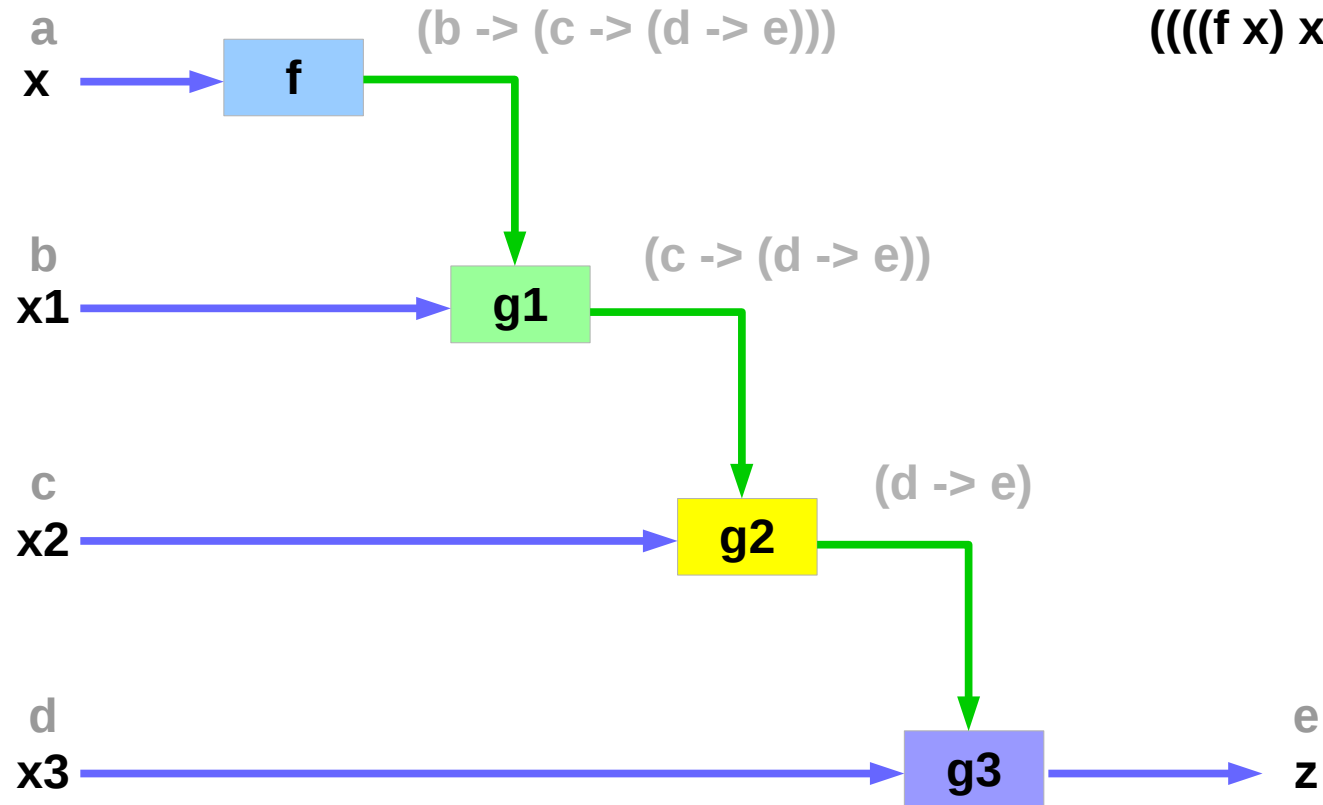
$(((((f\ x)\ x1)\ x2)\ x3))$

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Currying Examples

$f :: a \rightarrow b \rightarrow c \rightarrow d \rightarrow e$

$f :: a \rightarrow (b \rightarrow (c \rightarrow (d \rightarrow e)))$



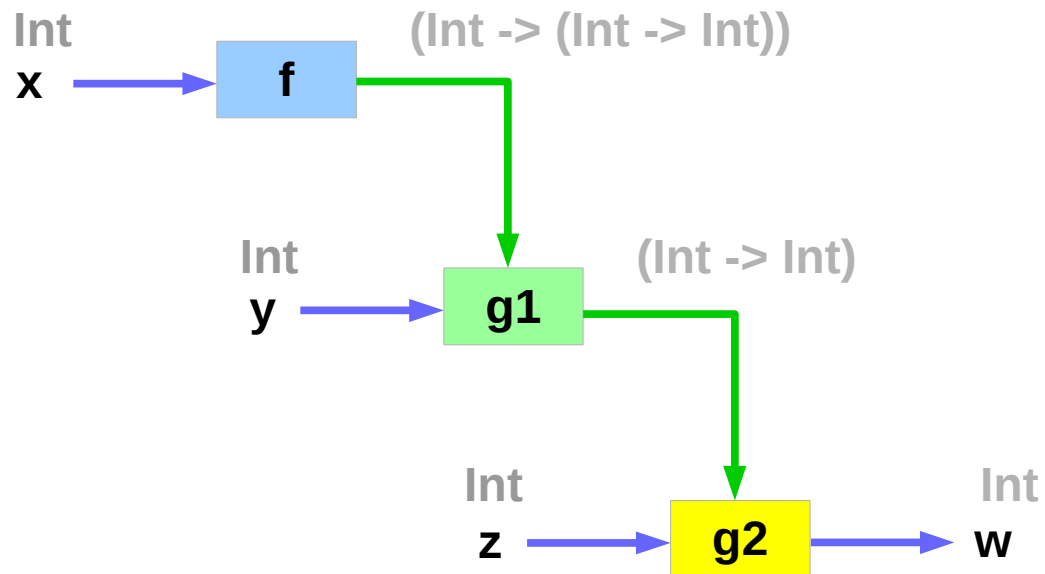
$(((((f\ x)\ x_1)\ x_2)\ x_3))$

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Currying Examples

`mult :: Int -> Int -> Int -> Int`

`f :: a -> (b -> (c -> (d -> e)))`



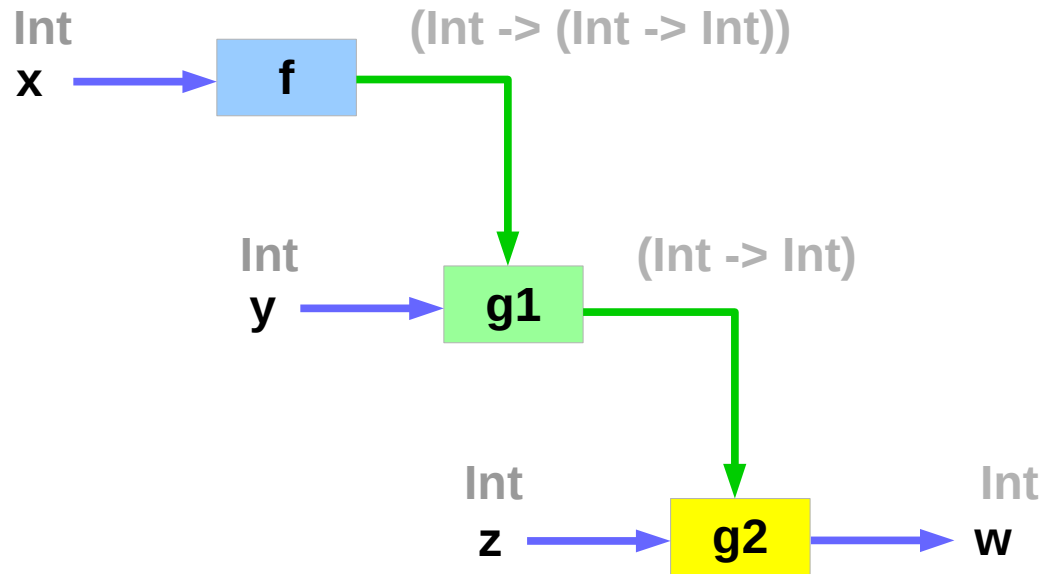
`(((mult x) y) z)`

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Currying Examples

`mult :: Int -> Int -> Int -> Int`

`f :: Int -> (Int -> (Int -> Int))`



`f x :: Int -> (Int -> Int)`

`g1 :: Int -> (Int -> Int)`

`f x y :: Int -> Int`

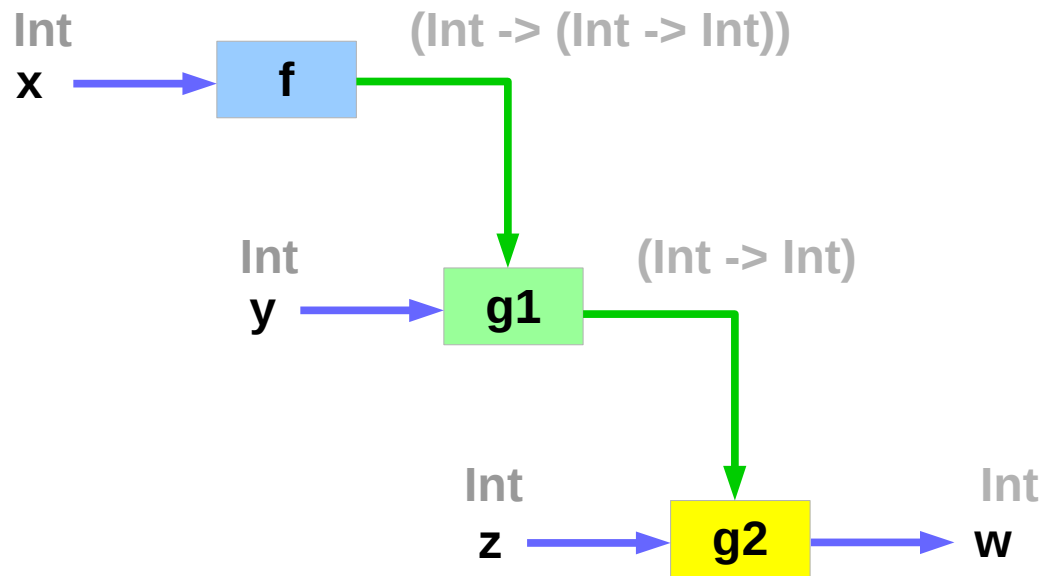
`g2 :: Int -> Int`

`f x y z :: Int`

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Currying Examples

`mult :: Int -> Int -> Int -> Int`



`mult x y z`

`mult a1 y z`

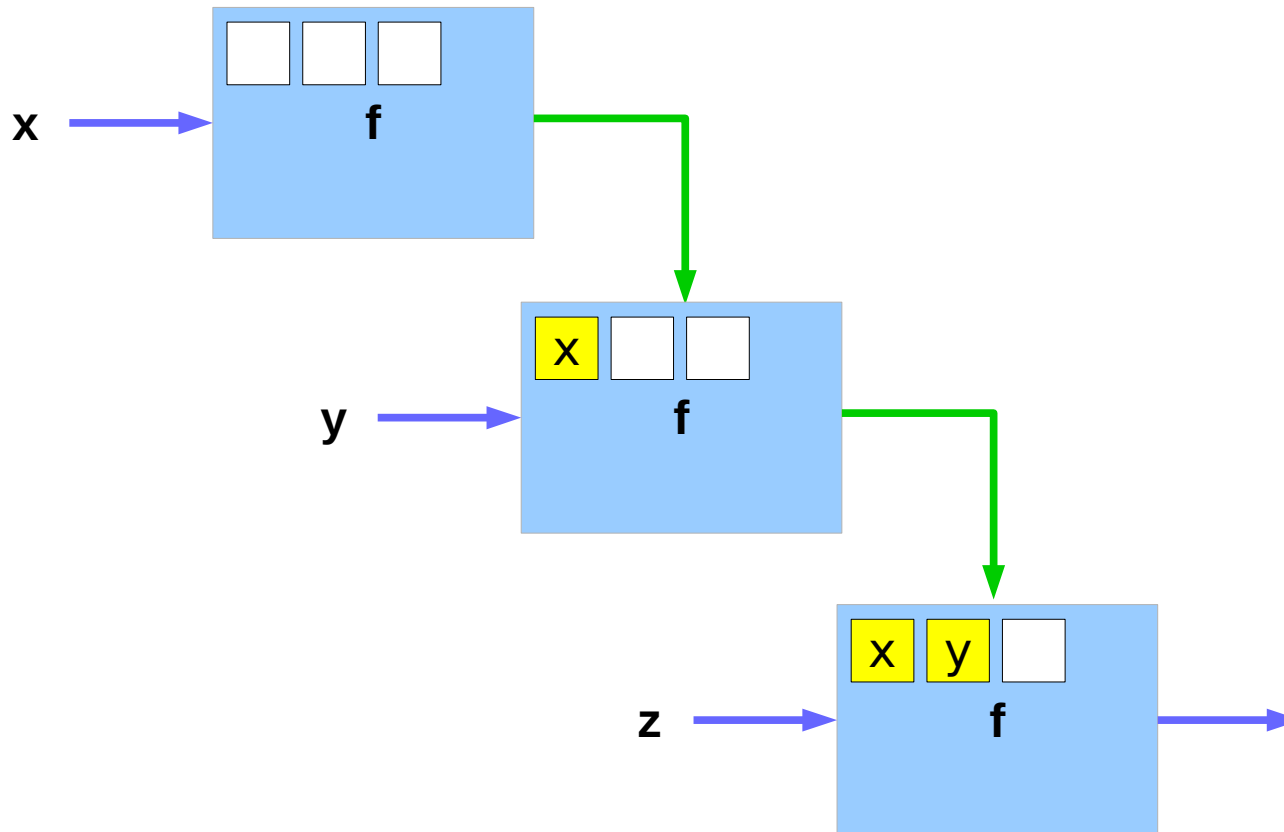
`mult a1 a2 z`

`mult a1 a2 a3`

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Currying Examples

`mult :: Int -> Int -> Int -> Int`



`mult x y z`

`mult a1 y z`

`mult a1 a2 z`

`mult a1 a2 a3`

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Curry & Uncurry

$f :: a \rightarrow b \rightarrow c$ the curried form of $g :: (a, b) \rightarrow c$

$f = \text{curry } g$
 $g = \text{uncurry } f$

$f \ x \ y = g \ (x,y)$

the curried form is usually more convenient because it allows **partial application**.

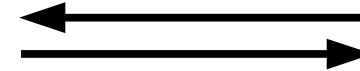
all functions are considered **curried**

all functions take **just one argument**

the curried form

$f :: a \rightarrow b \rightarrow c$

currying



$g :: (a, b) \rightarrow c$

uncurrying

$f \ x \ y$

$g \ (x,y)$

<https://wiki.haskell.org/Currying>

Mapping functions over the Functor [] (1)

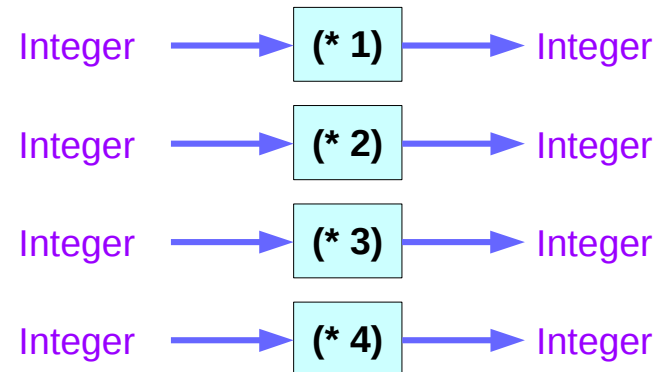
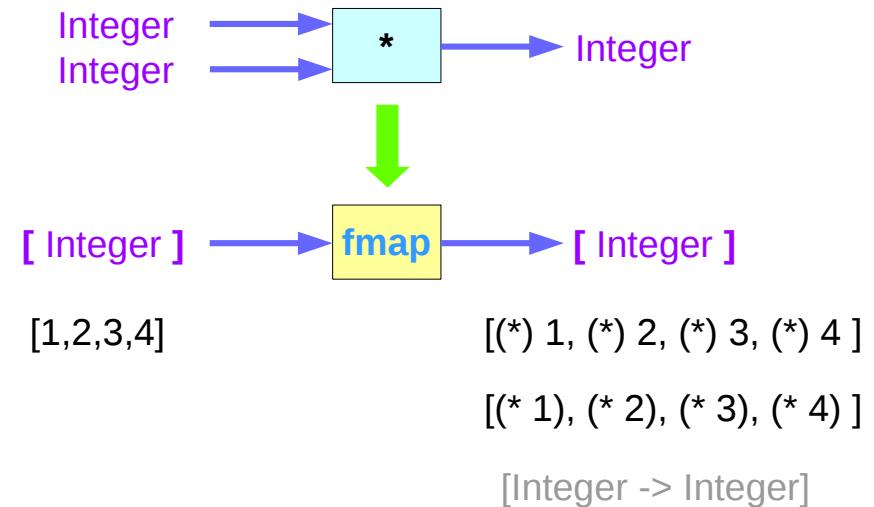
```
ghci> let a = fmap (*) [1,2,3,4]
```

```
ghci> :t a
```

```
a :: [Integer -> Integer]
```

```
ghci> fmap (\f -> f 9) a
```

```
[9,18,27,36]
```



<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Mapping functions over the Functor [] (2)

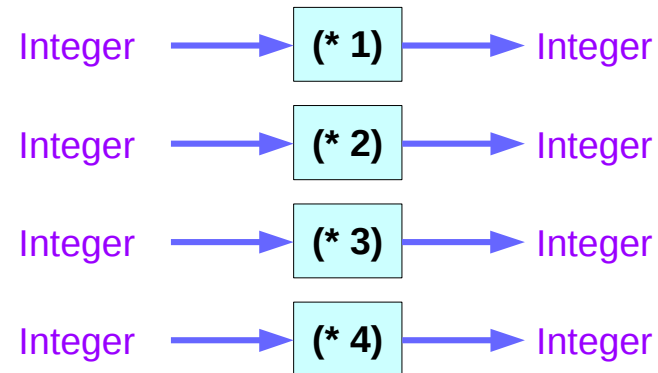
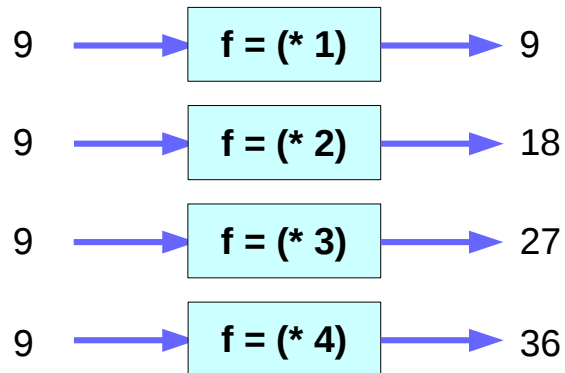
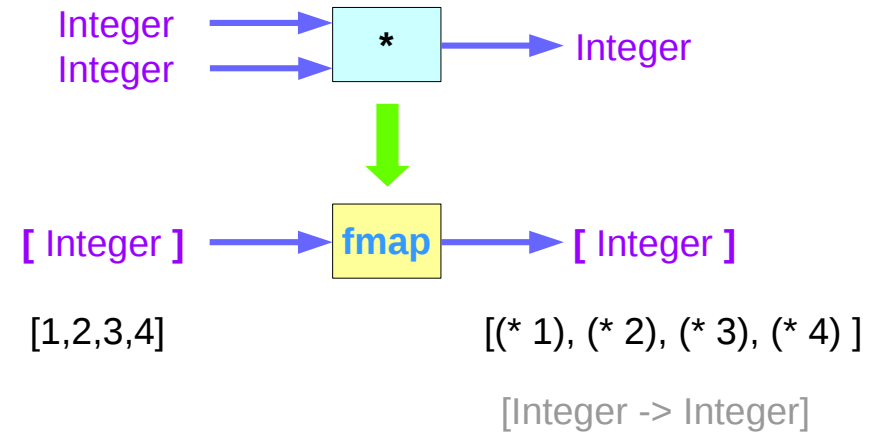
```
ghci> let a = fmap (*) [1,2,3,4]
```

```
ghci> :t a
```

```
a :: [Integer -> Integer]
```

```
ghci> fmap (\f -> f 9) a
```

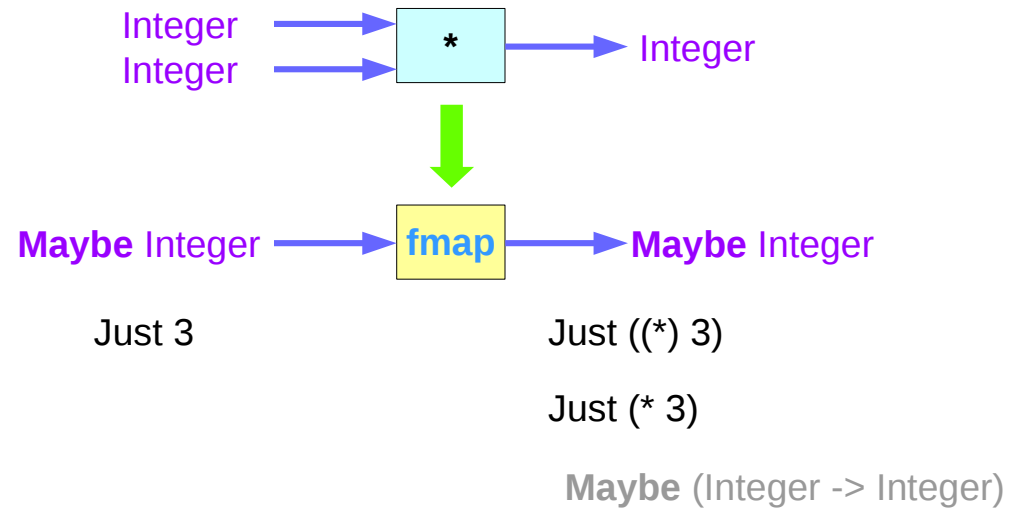
```
[9,18,27,36]
```



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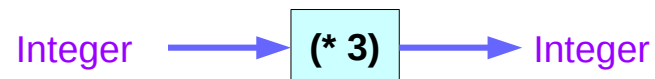
Mapping functions over the Functor Maybe (1)

fmap (*) (Just 3)



function wrapped in a **Just**

Just (* 3)



integer wrapped in a **Just**

Just 2

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Extracting and Mapping a function

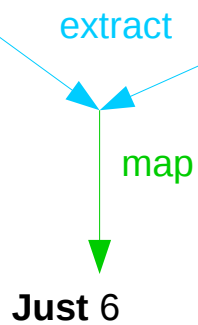
fmap (*) (Just 3)

function wrapped in a **Just**

Just (* 3)

integer wrapped in a **Just**

Just 2



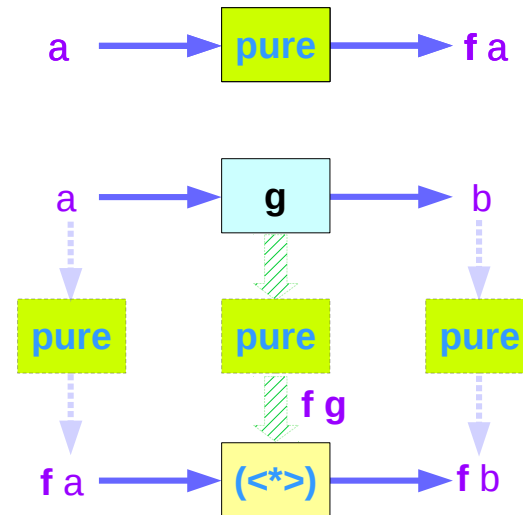
Just (* 3) <*> Just 2

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The Applicative Typeclass

```
class (Functor f) => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

```
instance Applicative Maybe where
  pure = Just
  Nothing <*> _ = Nothing
  (Just f) <*> something = fmap f something
```



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Applicative Functor Examples

ghci> Just (+3) <*> Just 9

Just 12

ghci> pure (+3) <*> Just 10

Just 13

ghci> pure (+3) <*> Just 9

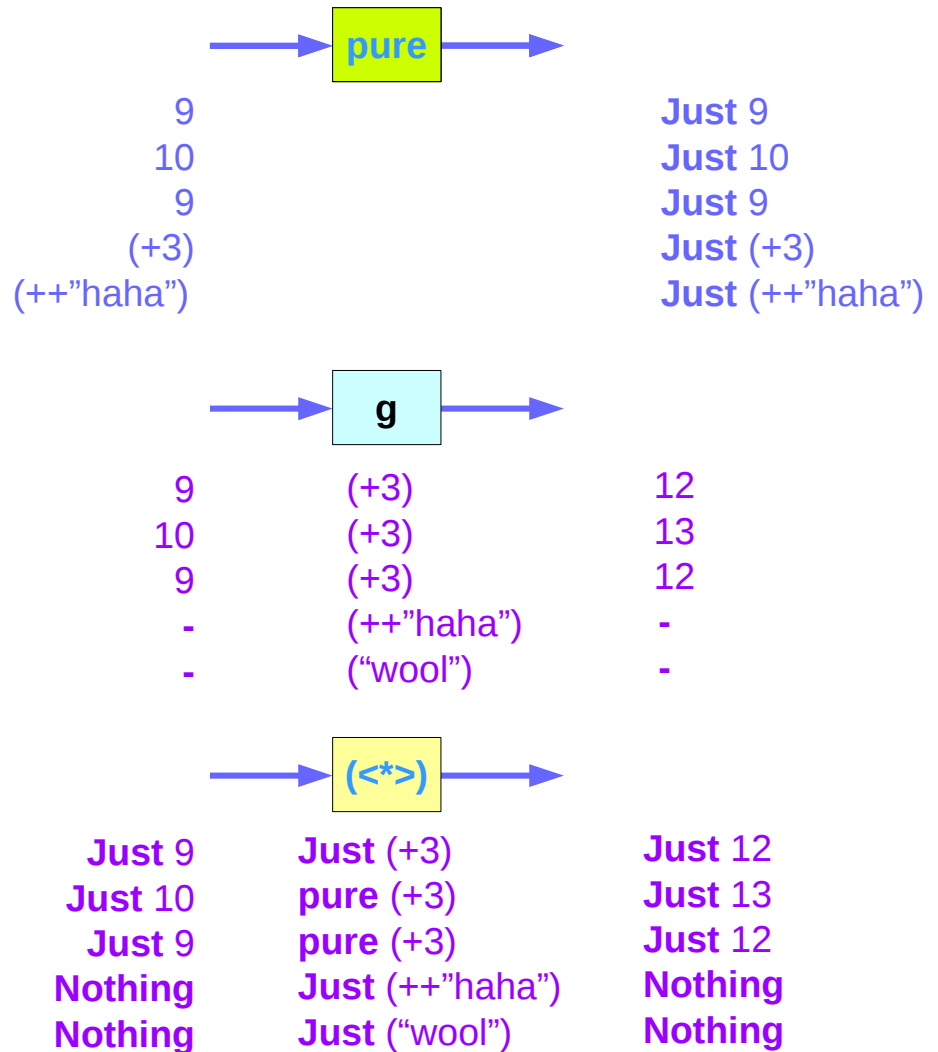
Just 12

ghci> Just (++"hahah") <*> Nothing

Nothing

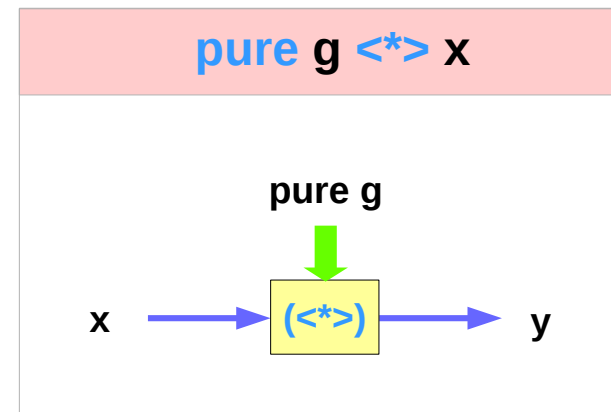
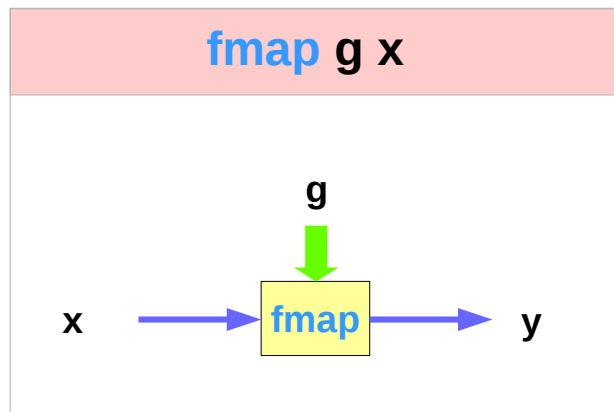
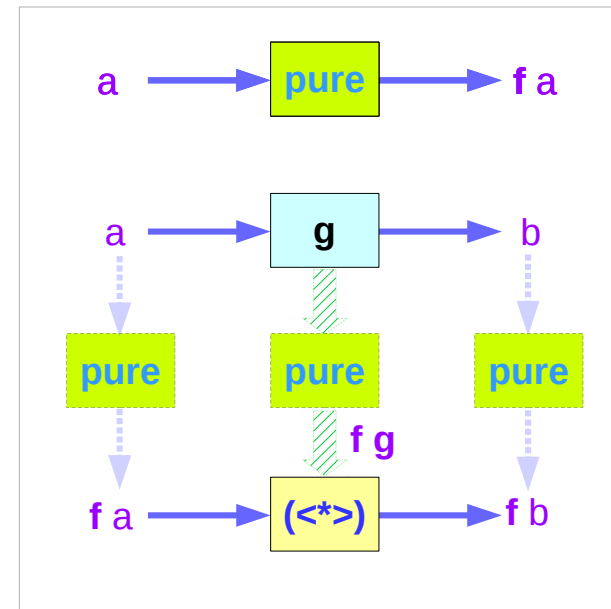
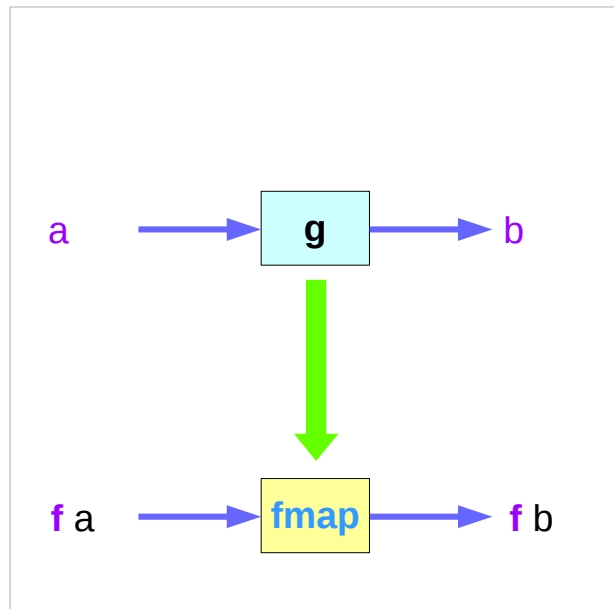
ghci> Nothing <*> Just "woot"

Nothing



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$$\text{fmap } g \ x = \text{pure } g \ \langle * \rangle \ x$$



<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Left Associative <*>

```
ghci> pure (+) <*> Just 3 <*> Just 5
```

```
Just 8
```

```
ghci> pure (+) <*> Just 3 <*> Nothing
```

```
Nothing
```

```
ghci> pure (+) <*> Nothing <*> Just 5
```

```
Nothing
```

```
pure (+) <*> Just 3 <*> Just 5
```

```
pure (+3) <*> Just 5
```

```
Just 8
```

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

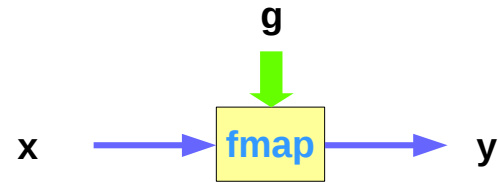
Infix Operator $\langle \$ \rangle$

`pure f` $\langle * \rangle$ `x` $\langle * \rangle$ `y` $\langle * \rangle$ `z`

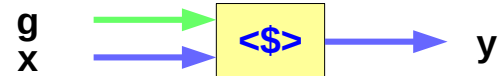
`fmap f` `x` $\langle * \rangle$ `y` $\langle * \rangle$ `z`

`f` $\langle \$ \rangle$ `x` $\langle * \rangle$ `y` $\langle * \rangle$ `z`

`fmap g x`



`g` $\langle \$ \rangle$ `x`



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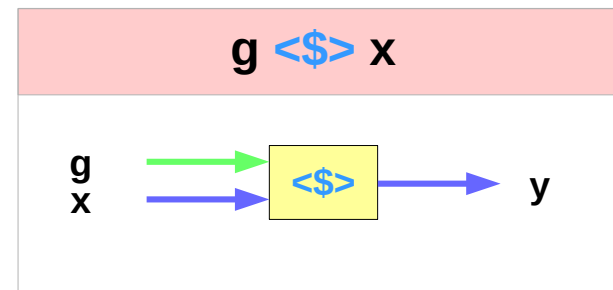
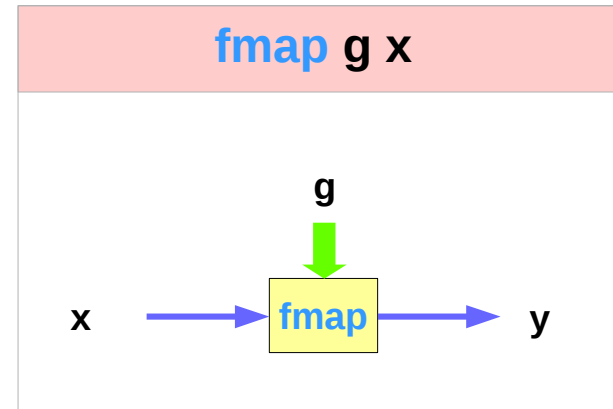
Infix Operator $\langle \$ \rangle$

```
class (Functor f) => Applicative f where  
  pure :: a -> f a  
  (<*>) :: f (a -> b) -> f a -> f b
```

not a class method

```
(<$>) :: (Functor f) => (a -> b) -> f a -> f b  
f <$> x = fmap f x
```

```
instance Applicative Maybe where  
  pure = Just  
  Nothing <*> _ = Nothing  
  (Just f) <*> something = fmap f something
```



<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

References

- [1] <ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf>
- [2] <https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf>