# Capacitors in an AC circuit

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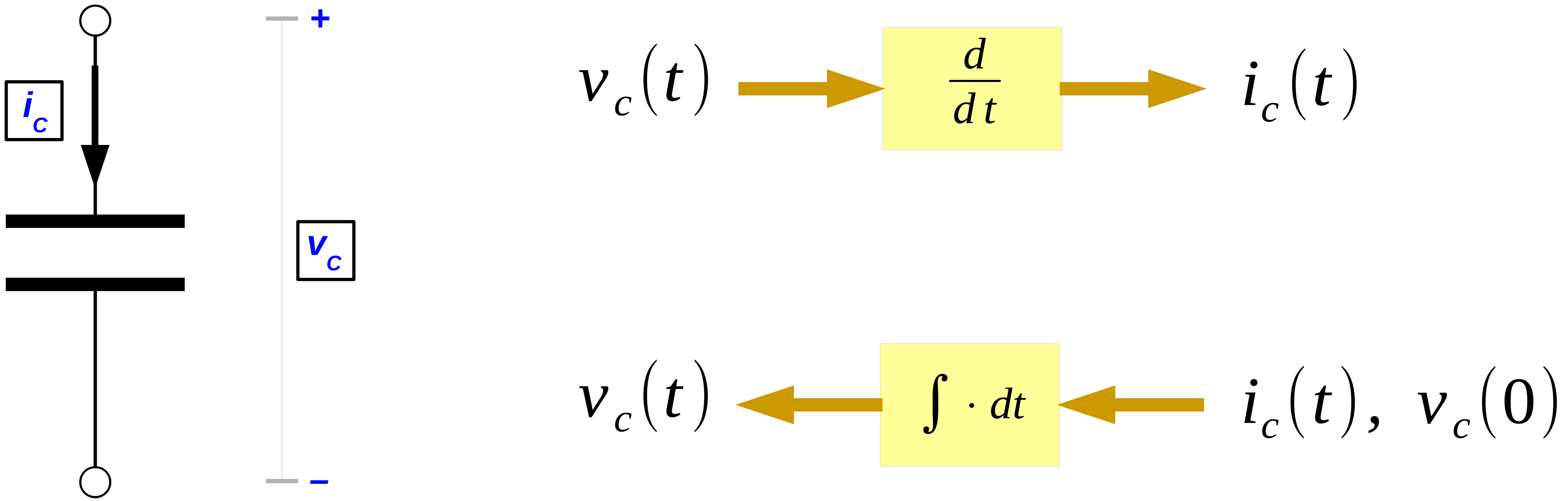
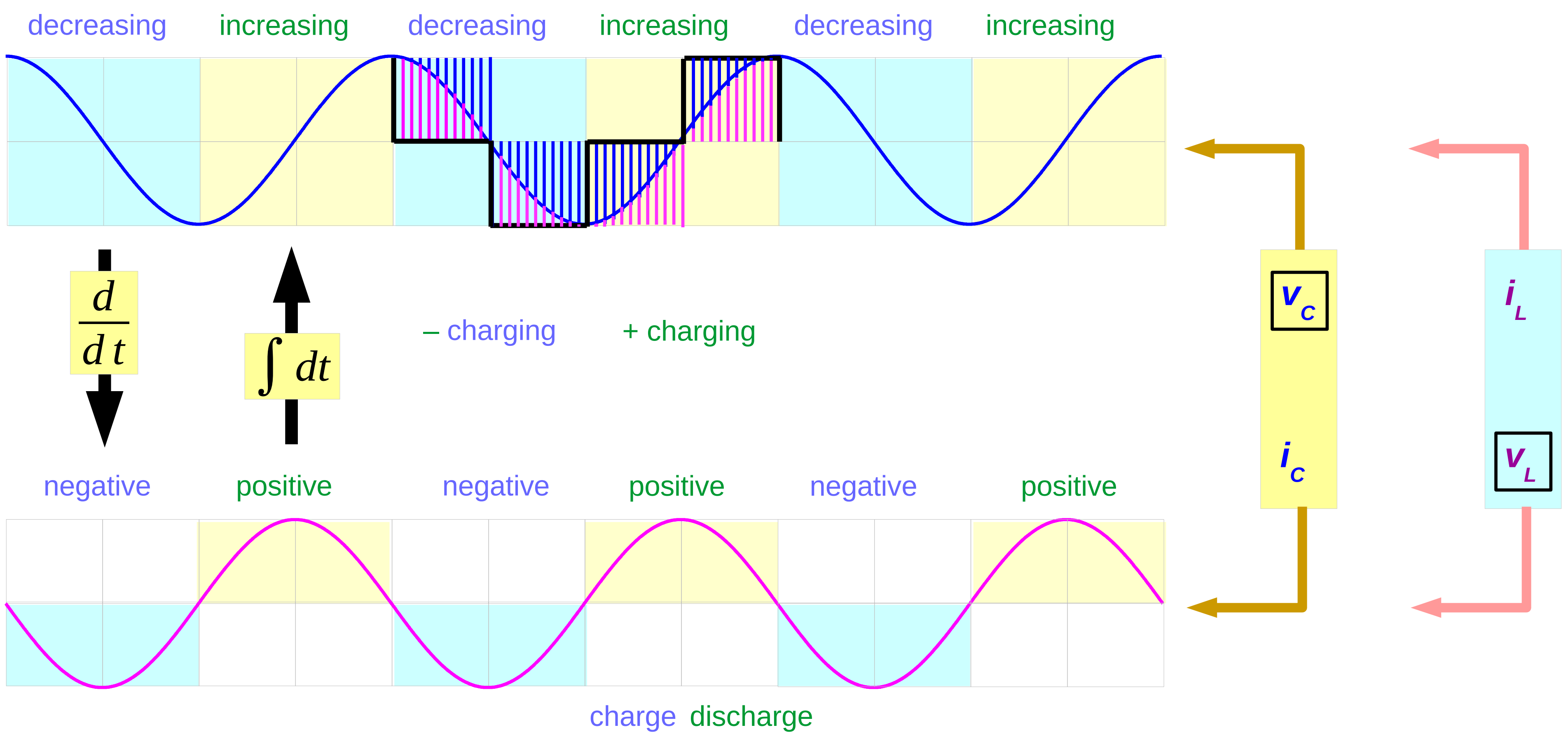
Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

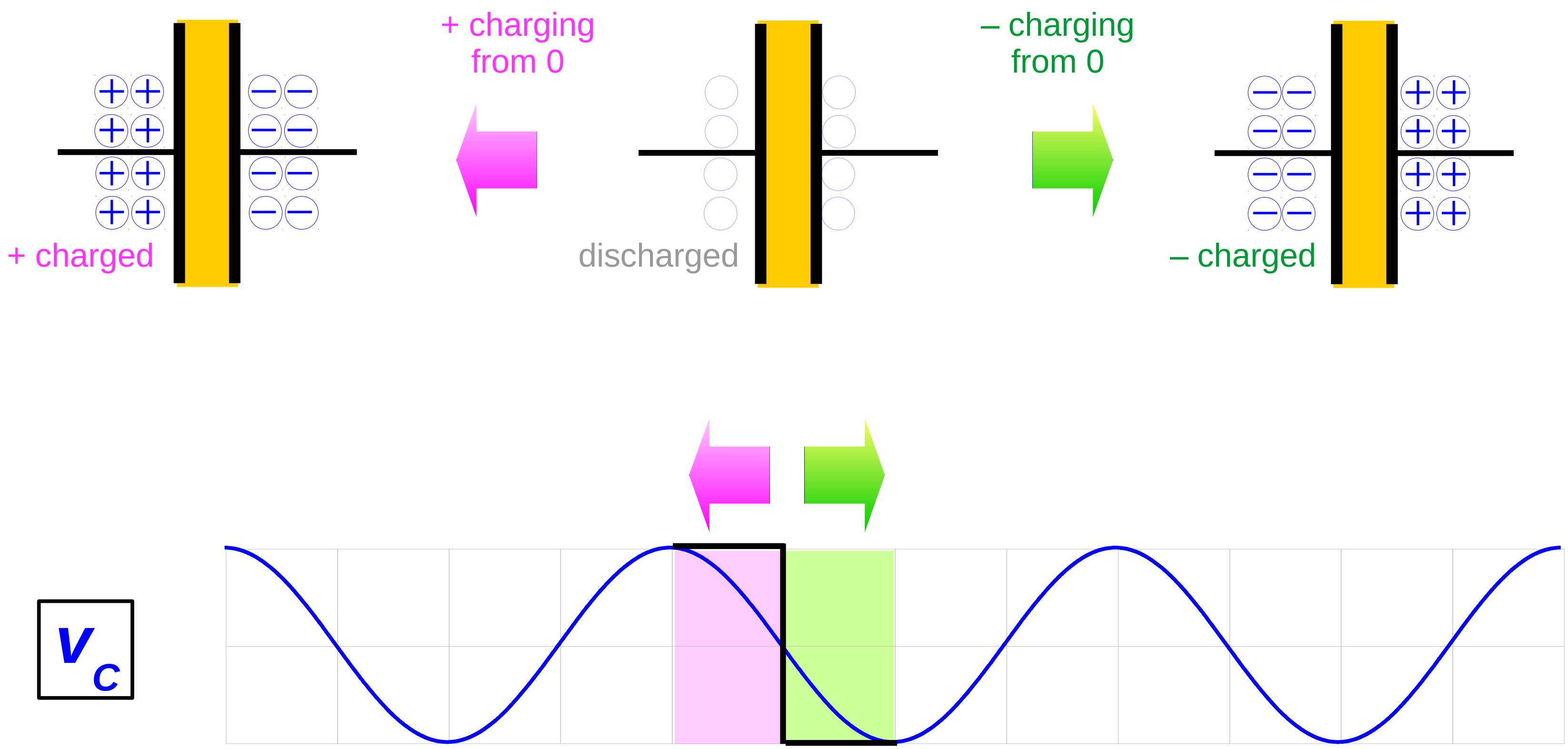
# Invertible Functions



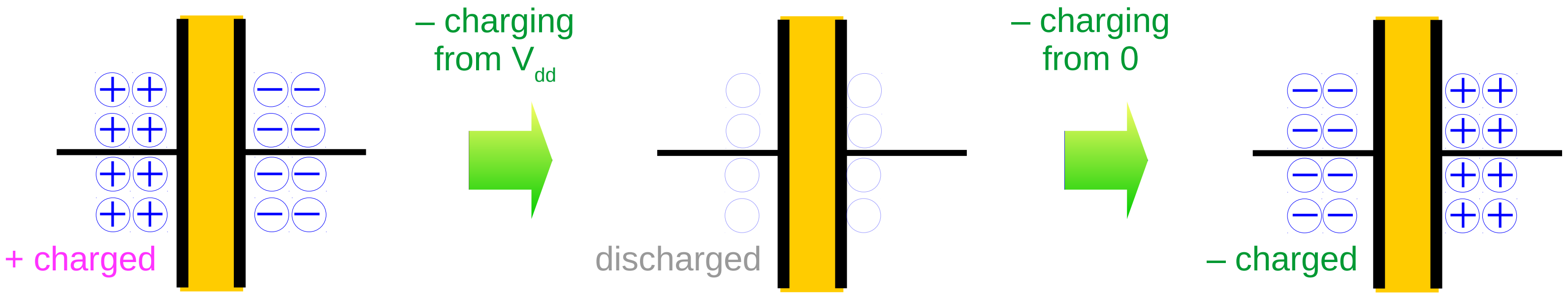
# Ever-charging signal pairs

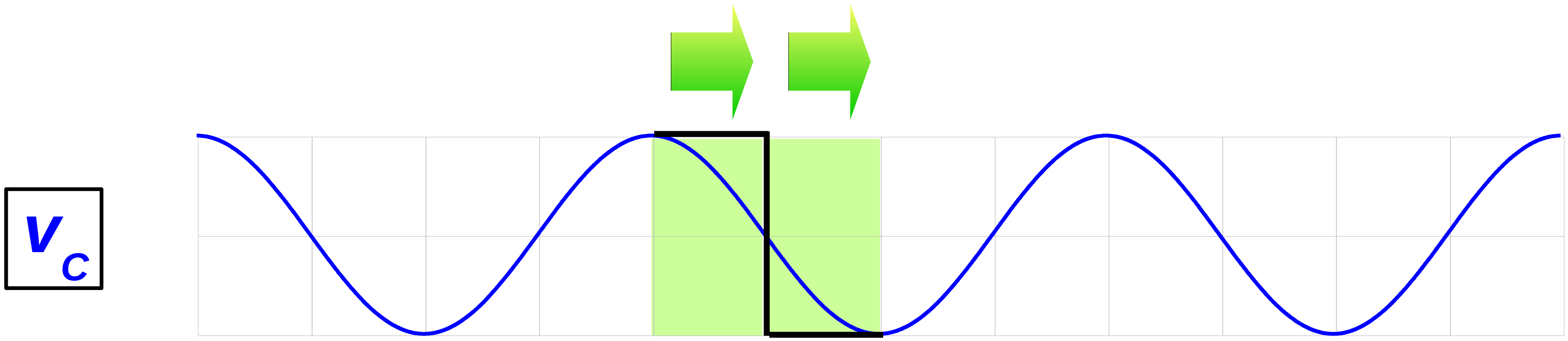


# Positive and Negative Charging

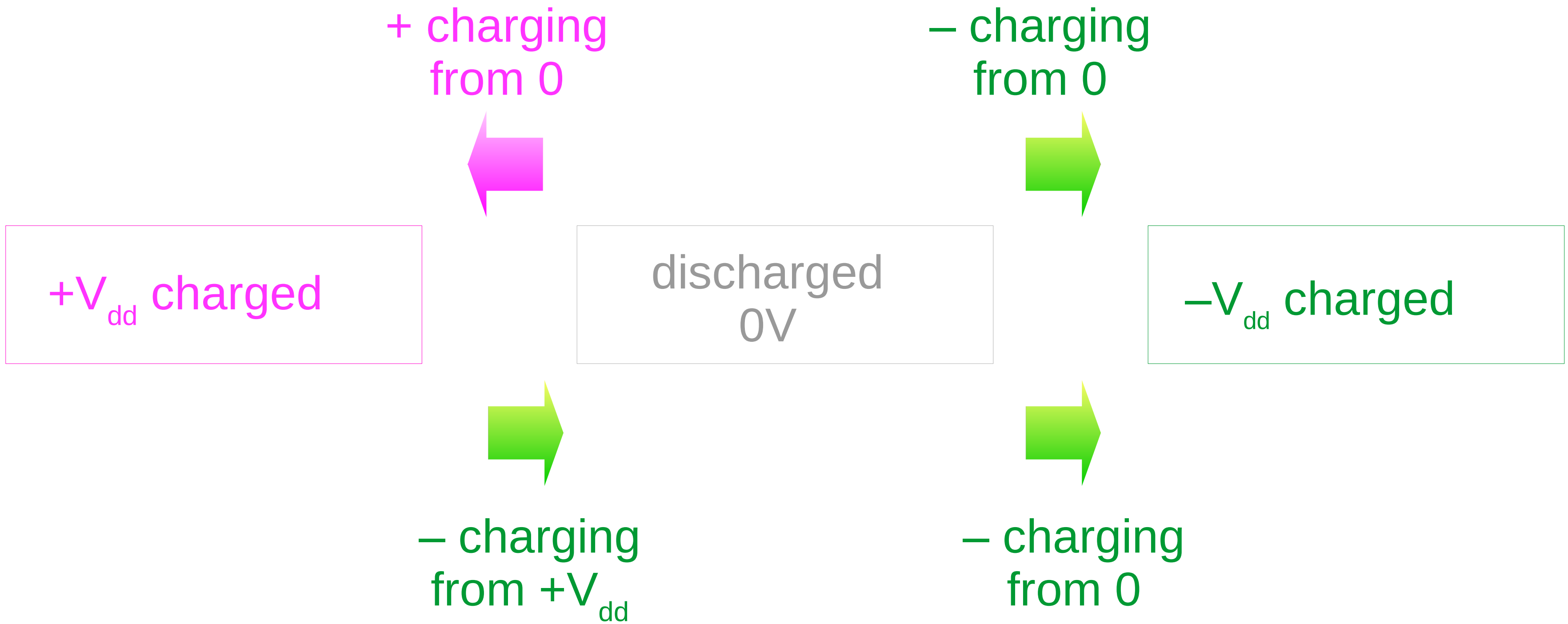


# Negative Charging from $V_{dd}$ and 0



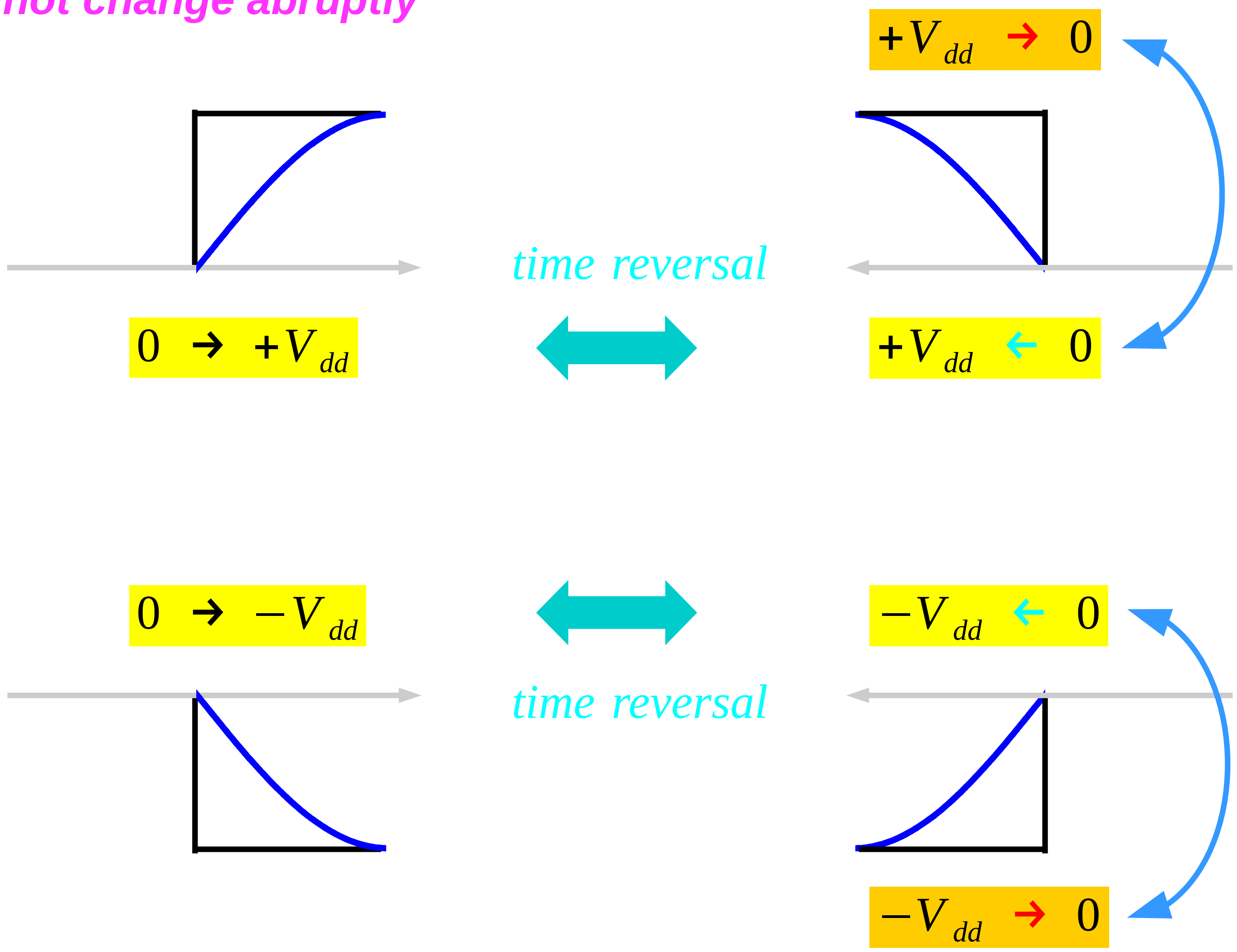



# Positive and Negative Charging

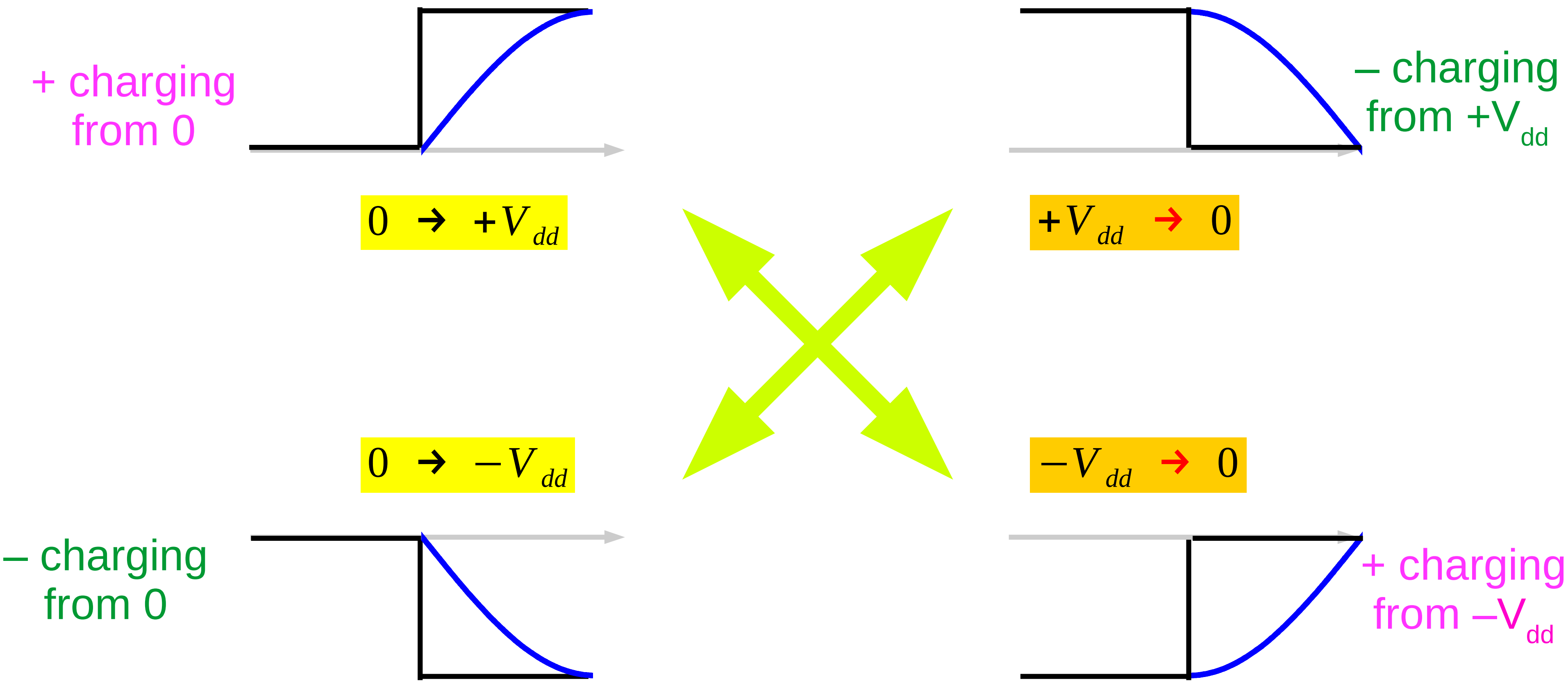


# Time Reversal

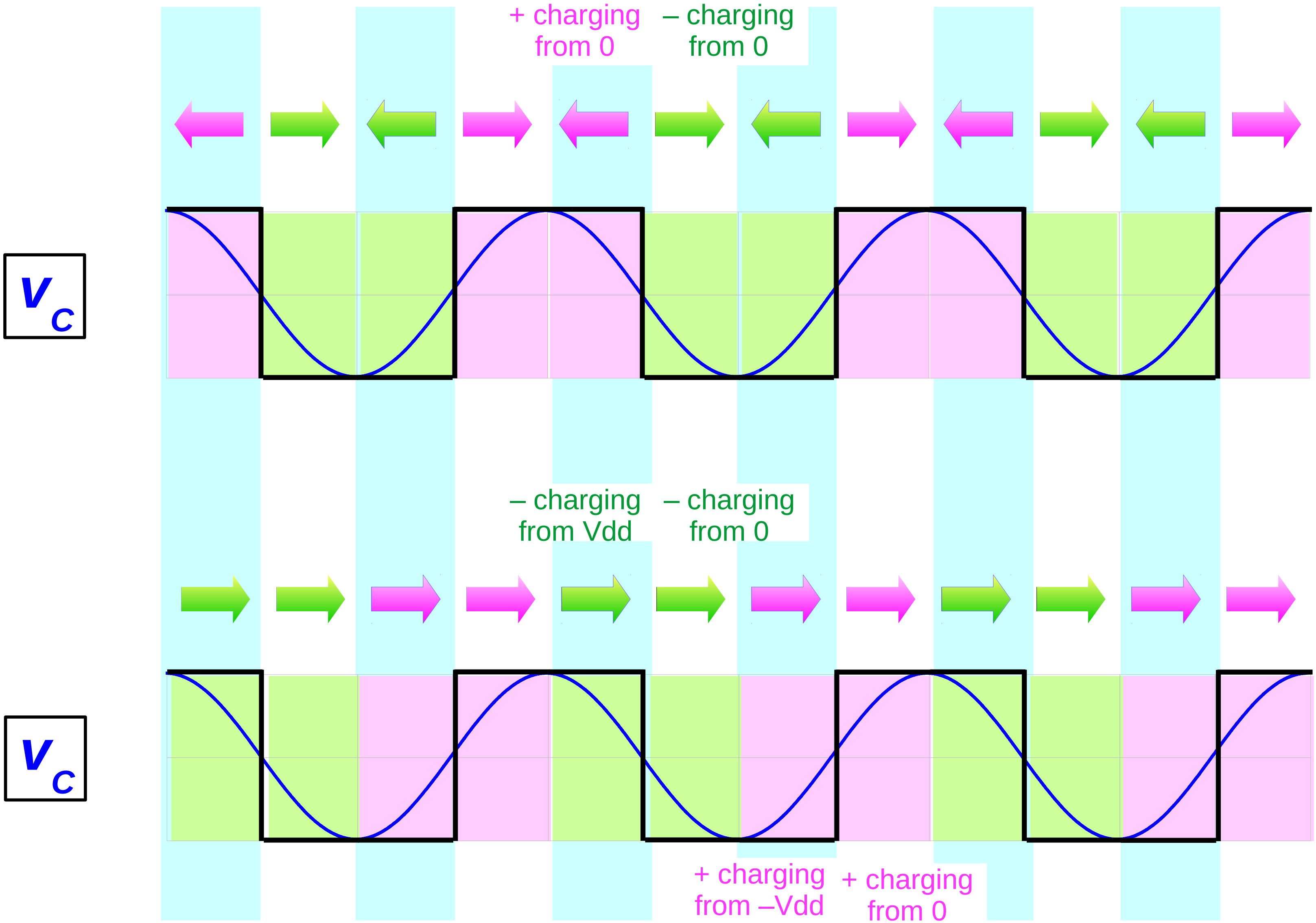
*$V_c$ cannot change abruptly*



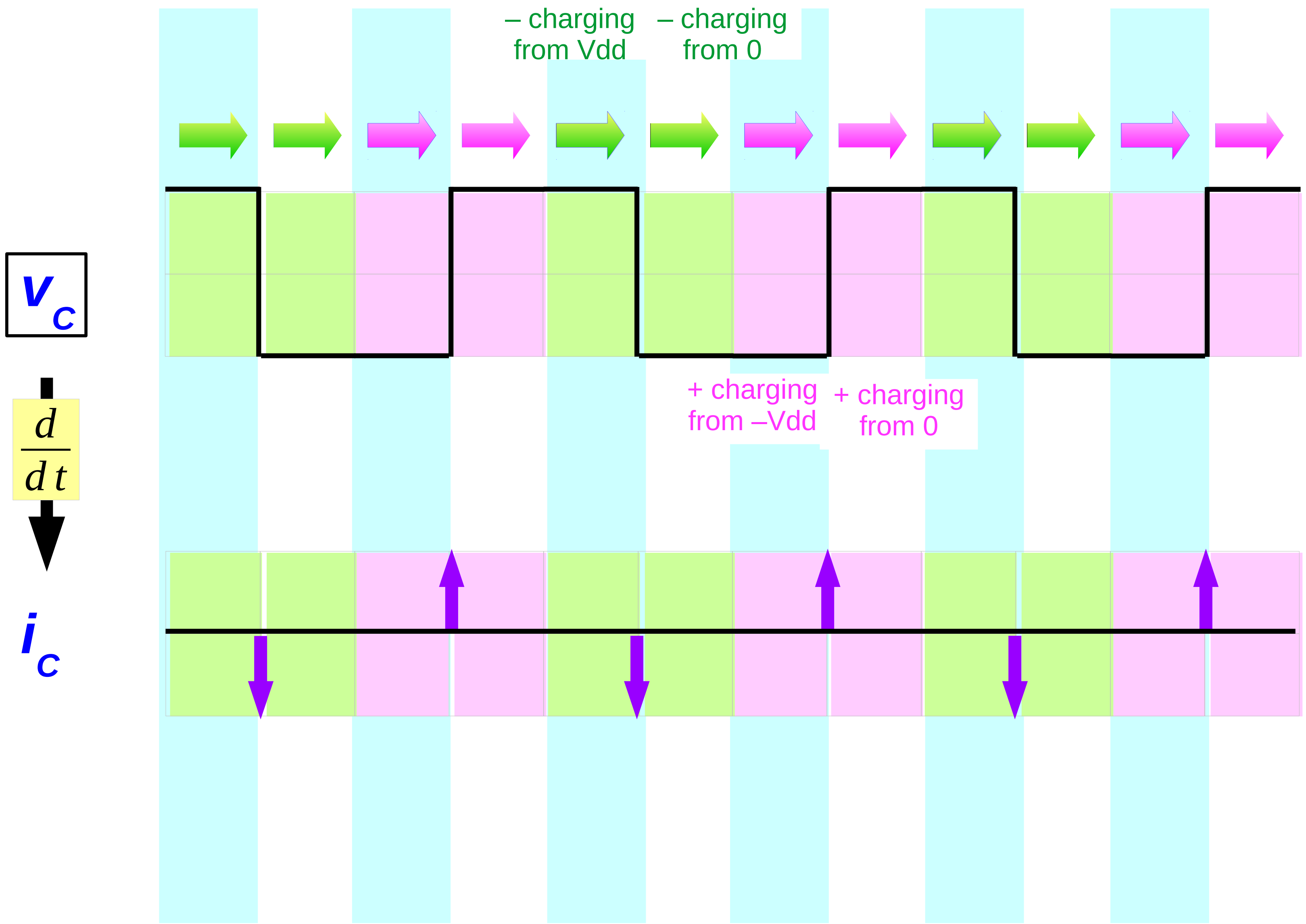
# (+) and (–) Charging



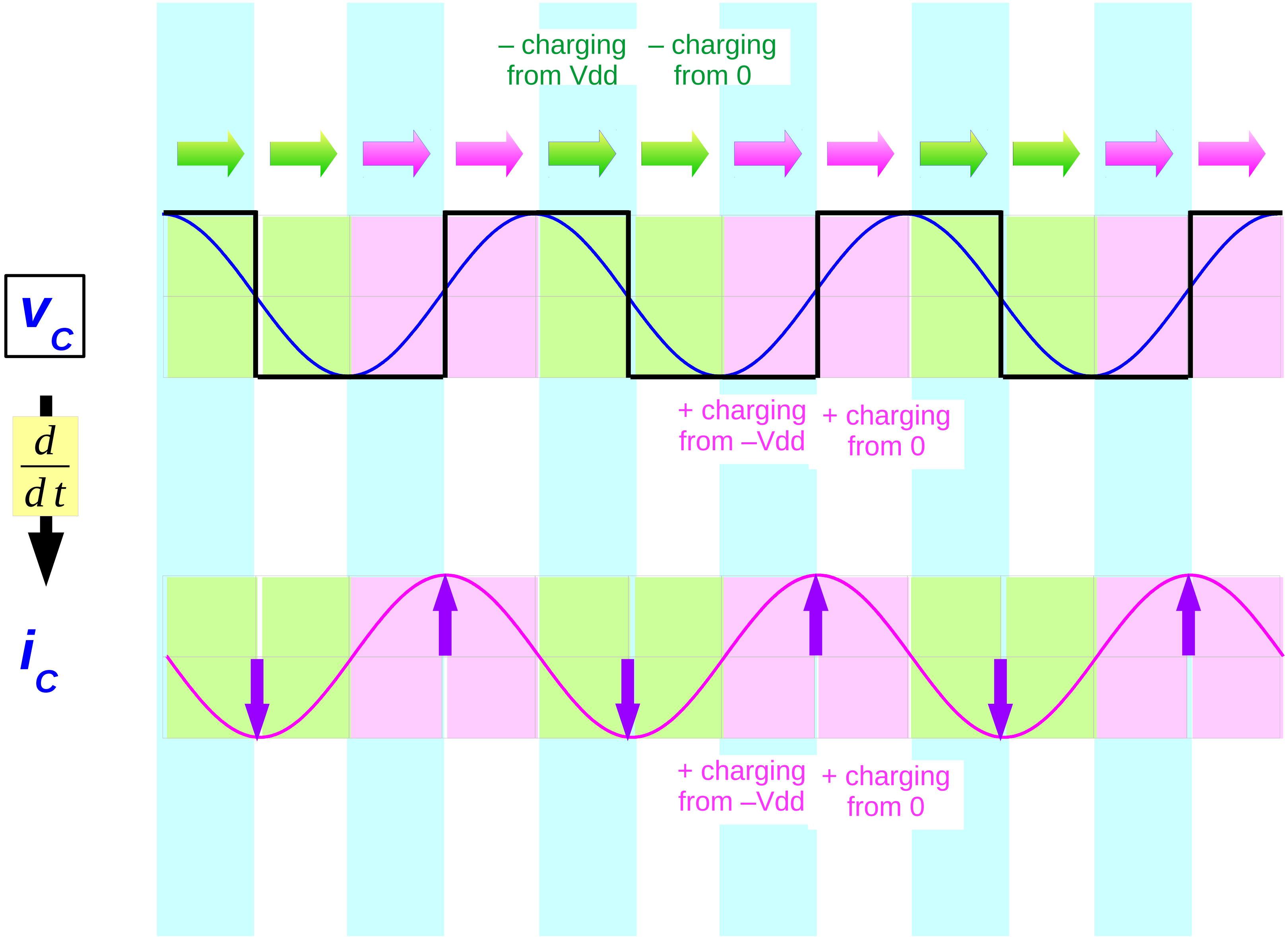
# (+) charging / (–) charging



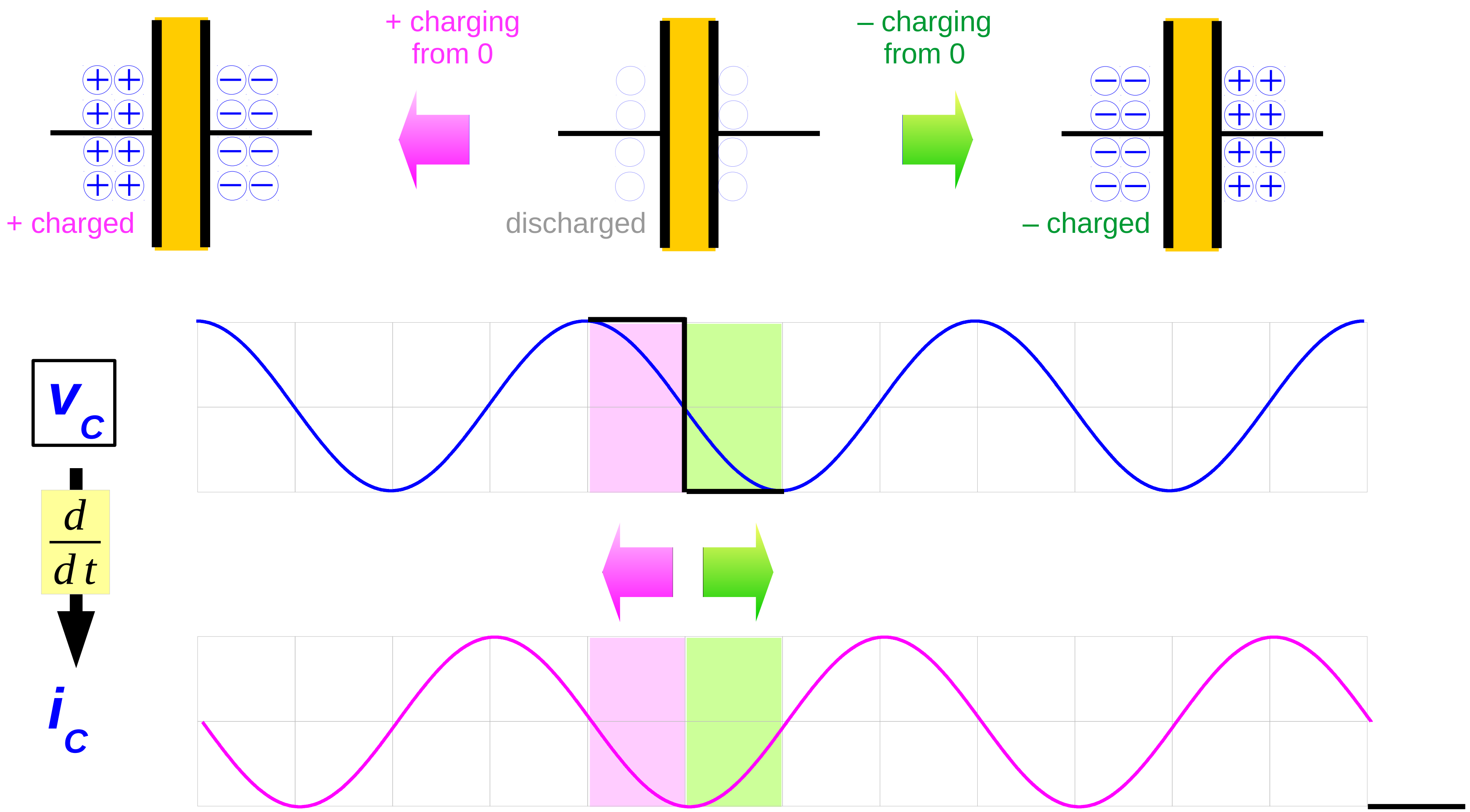
# Ideal Voltage $V_c$ and $I_c$



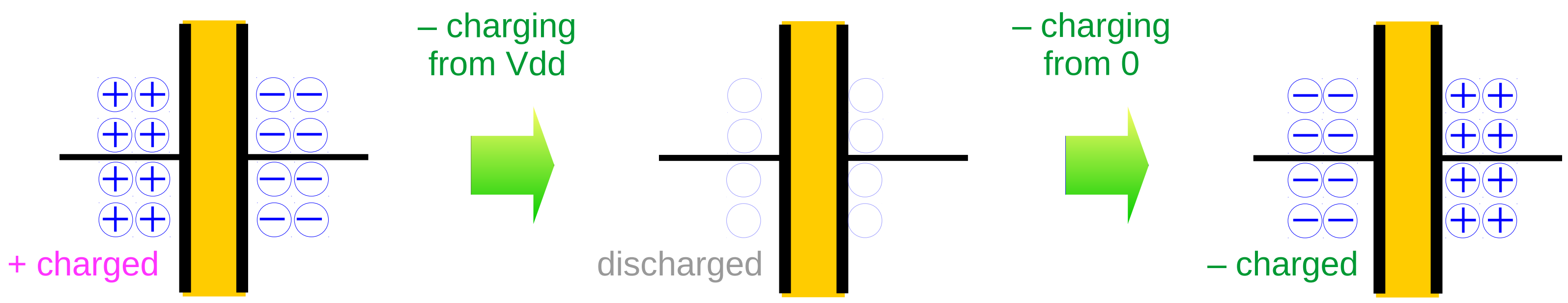
# Sinusoidal $V_C$ and $I_C$

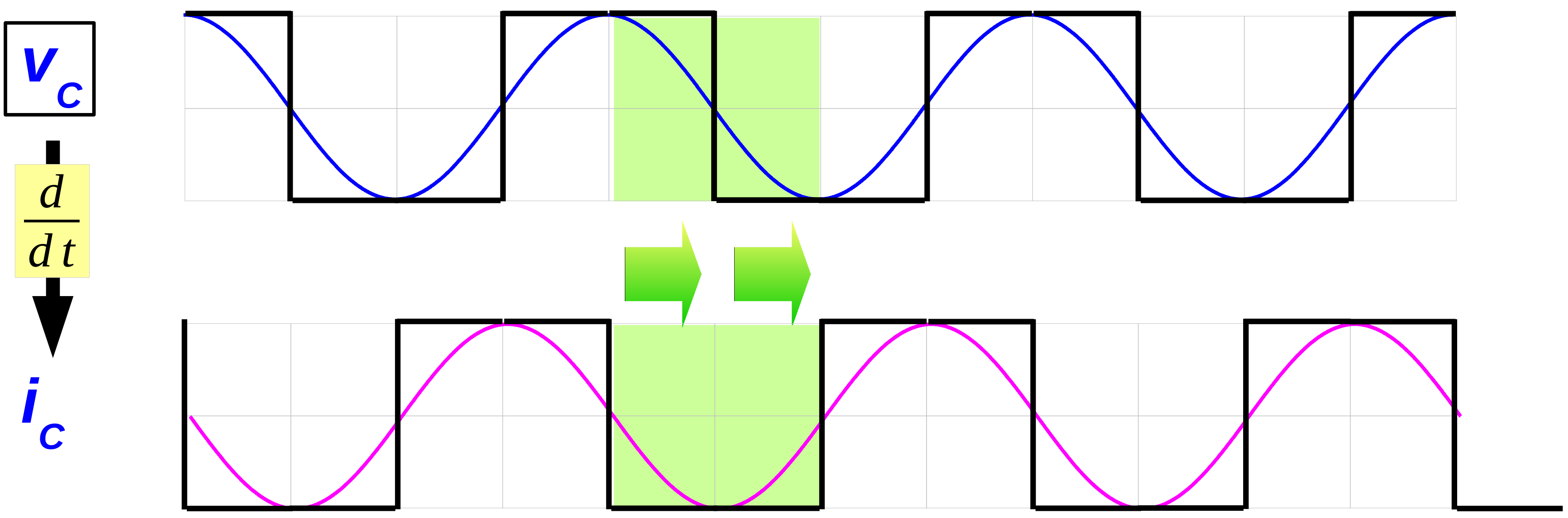


# Sinusoidal $V_C$ and $I_C$



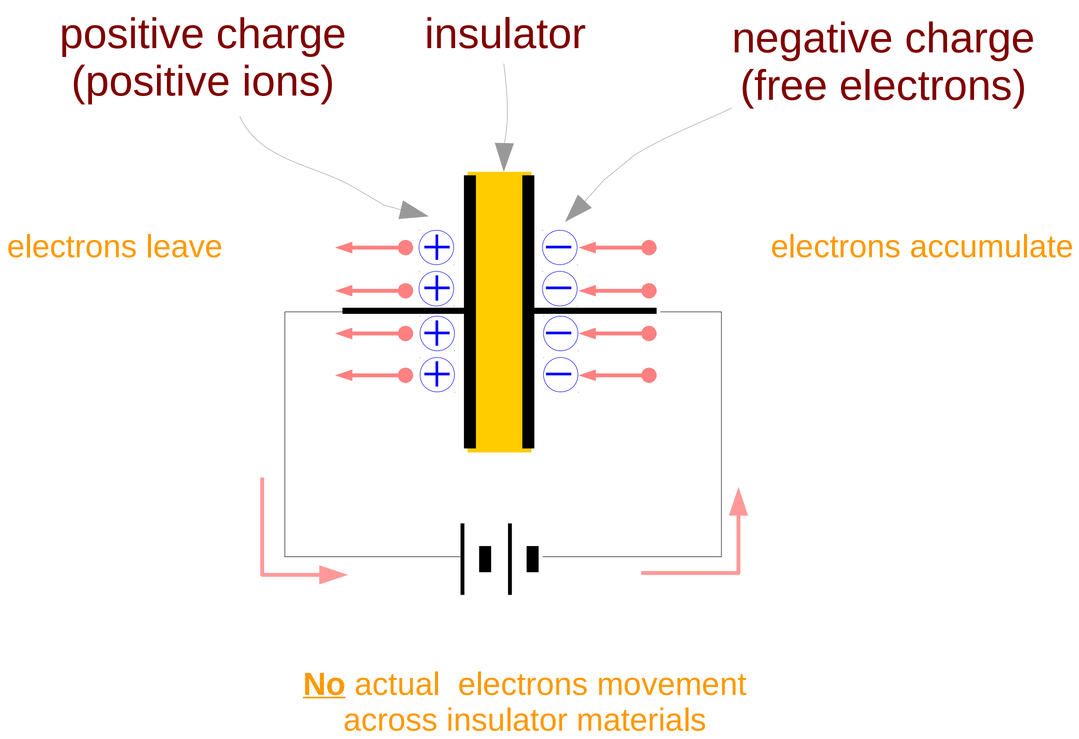
# Sinusoidal $V_C$ and $I_C$

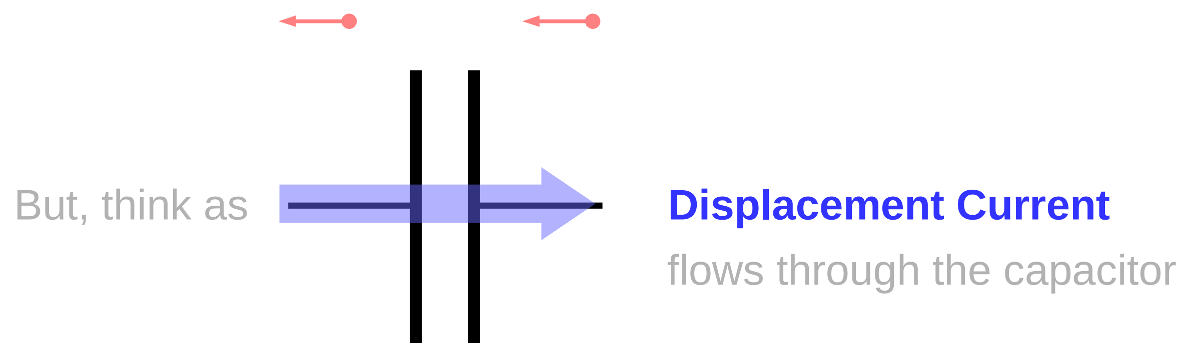


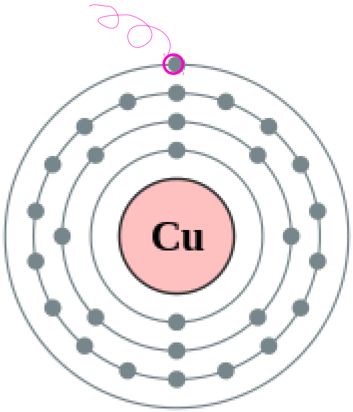
# Three States

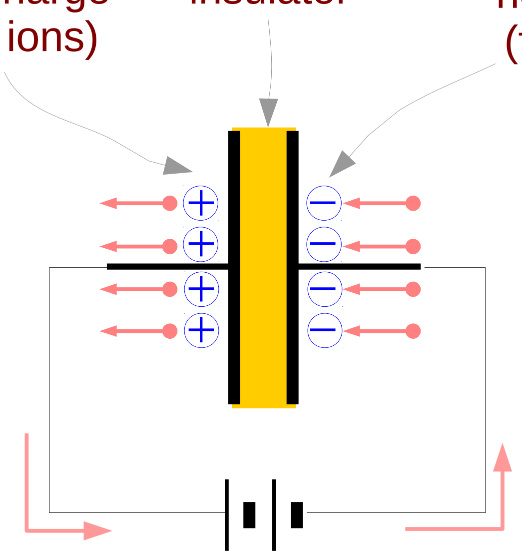
# Capacitor Current



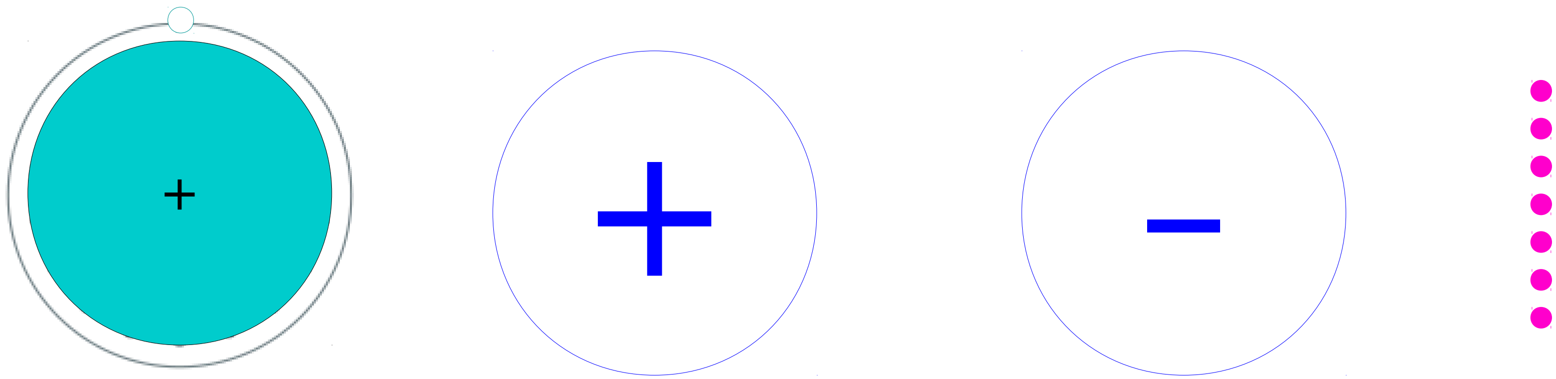



# Positive ions and free electrons




[[commons:User crap ]] (original work by commons:User:Greg Robson)
https://upload.wikimedia.org/wikipedia/commons/thumb/f/f7/Electron_shell_029_Copper_-_no_label.svg/200px-Electron_shell_029_Copper_-_no_label.svg.png

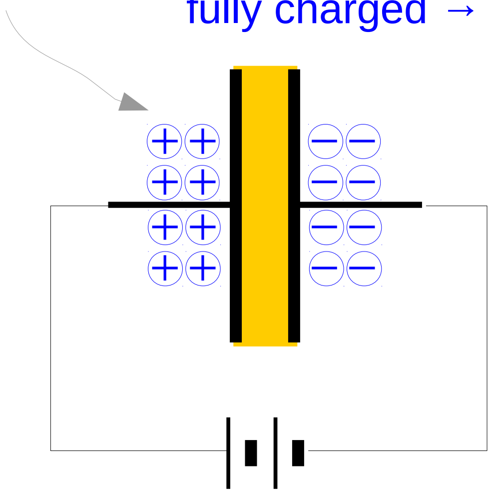


# Three States

positive charge
(positive ions)

**Positively Charged State**

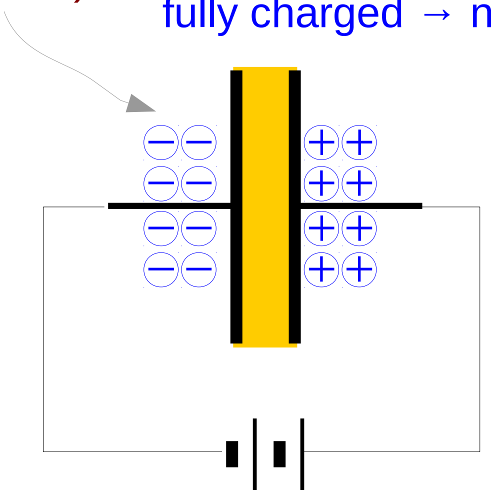
fully charged → no current

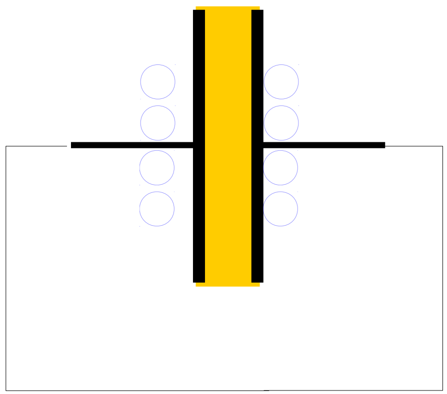


negative charge
(free electrons)

**Negatively Charged State**

fully charged → no current

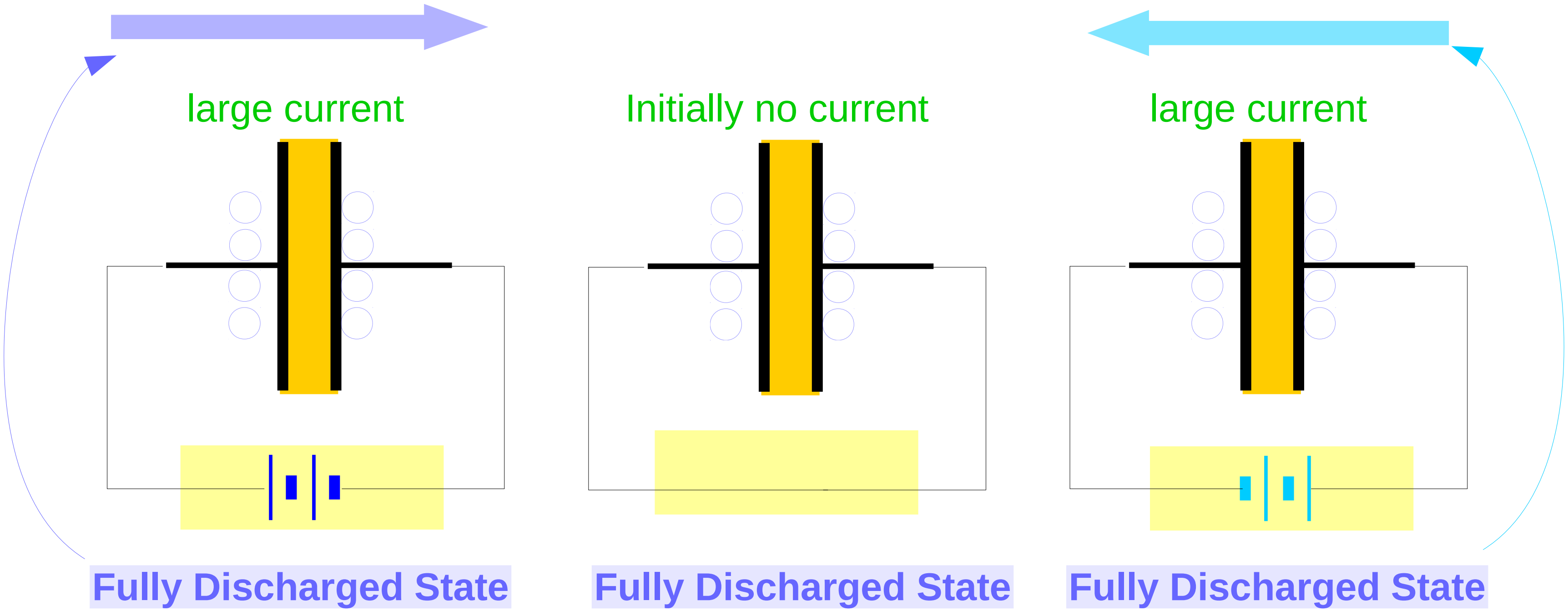




**Fully Discharged State**

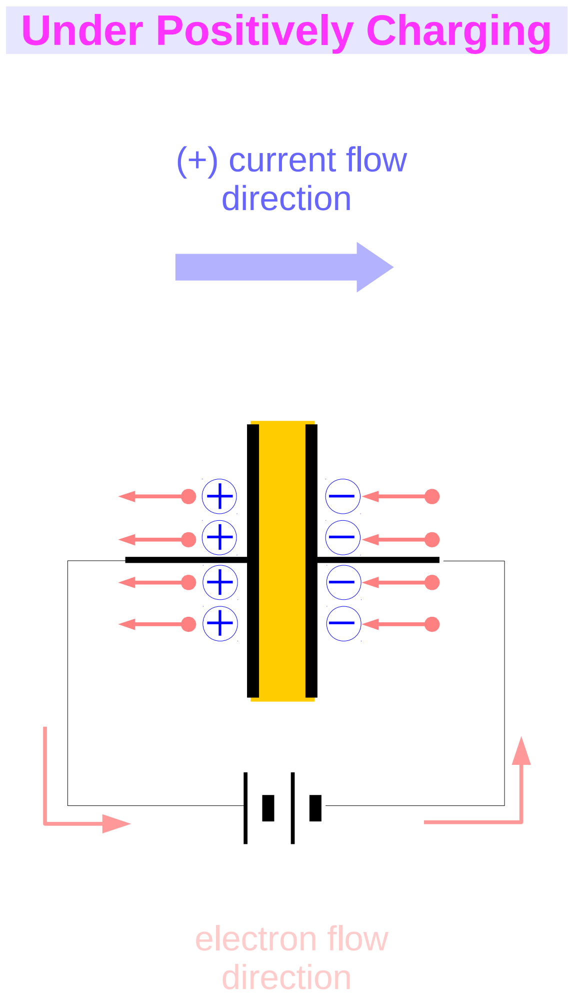
possible large current

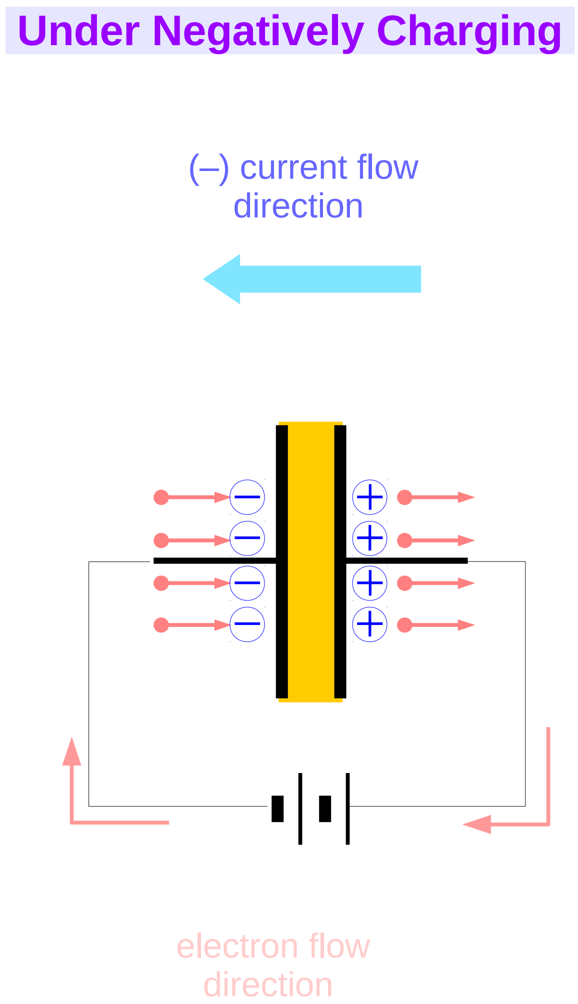
# Currents in the Fully Discharged State




This state can flow
large current
in either direction
depending on the
voltage change

# Inter-State Current Flowing

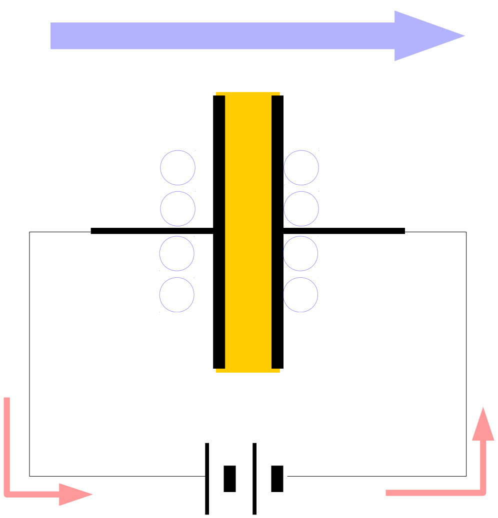


# Inter-State Current Flowing

## Fully Discharged State

(+) current flow direction

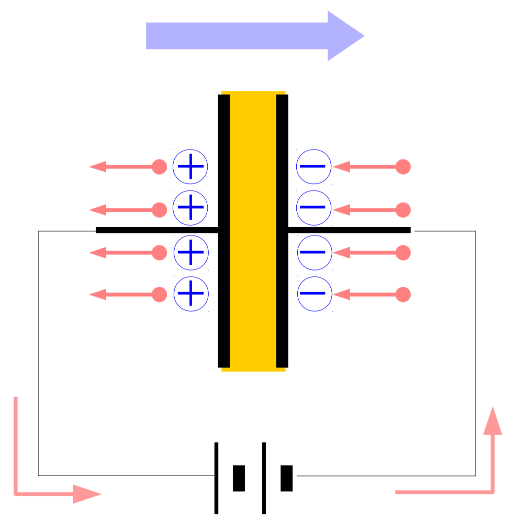


electron flow direction

large current

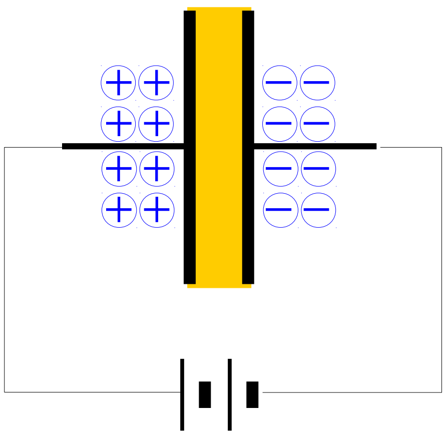
## Under Positively Charging

(+) current flow direction



electron flow direction

## Positively Charged State



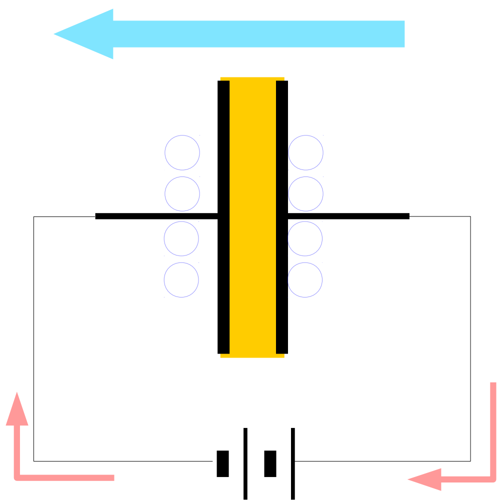
Crowded →
No more space

no current

# Inter-State Current Flowing

**Fully Discharged State**

(–) current flow direction

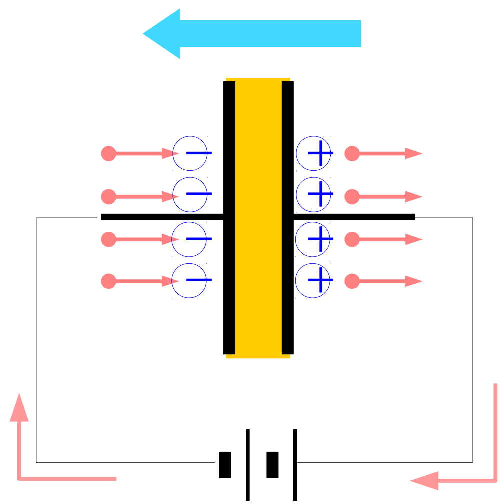


electron flow direction

Initial large current

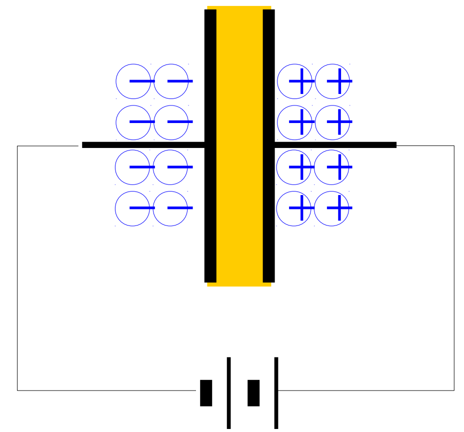
**Under Negatively Charging**

(–) current flow direction



electron flow direction

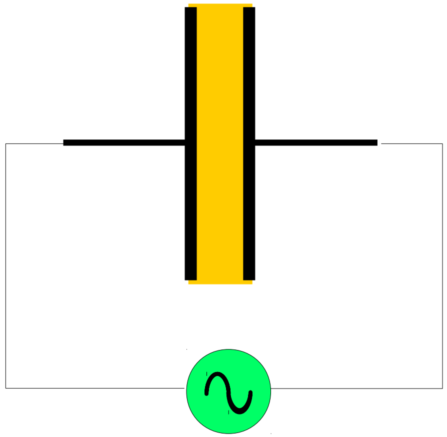
**Negatively Charged State**

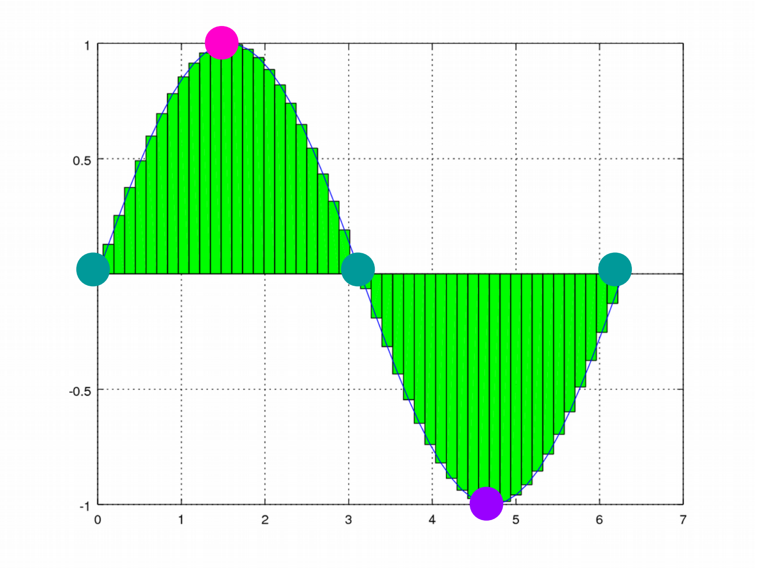


Crowded →
No more space

no current

# An AC Voltage Source

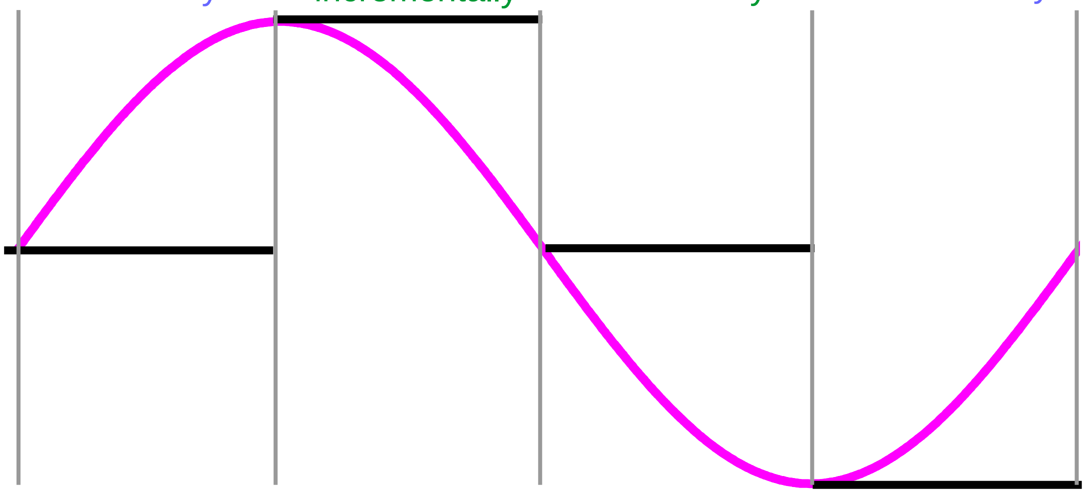




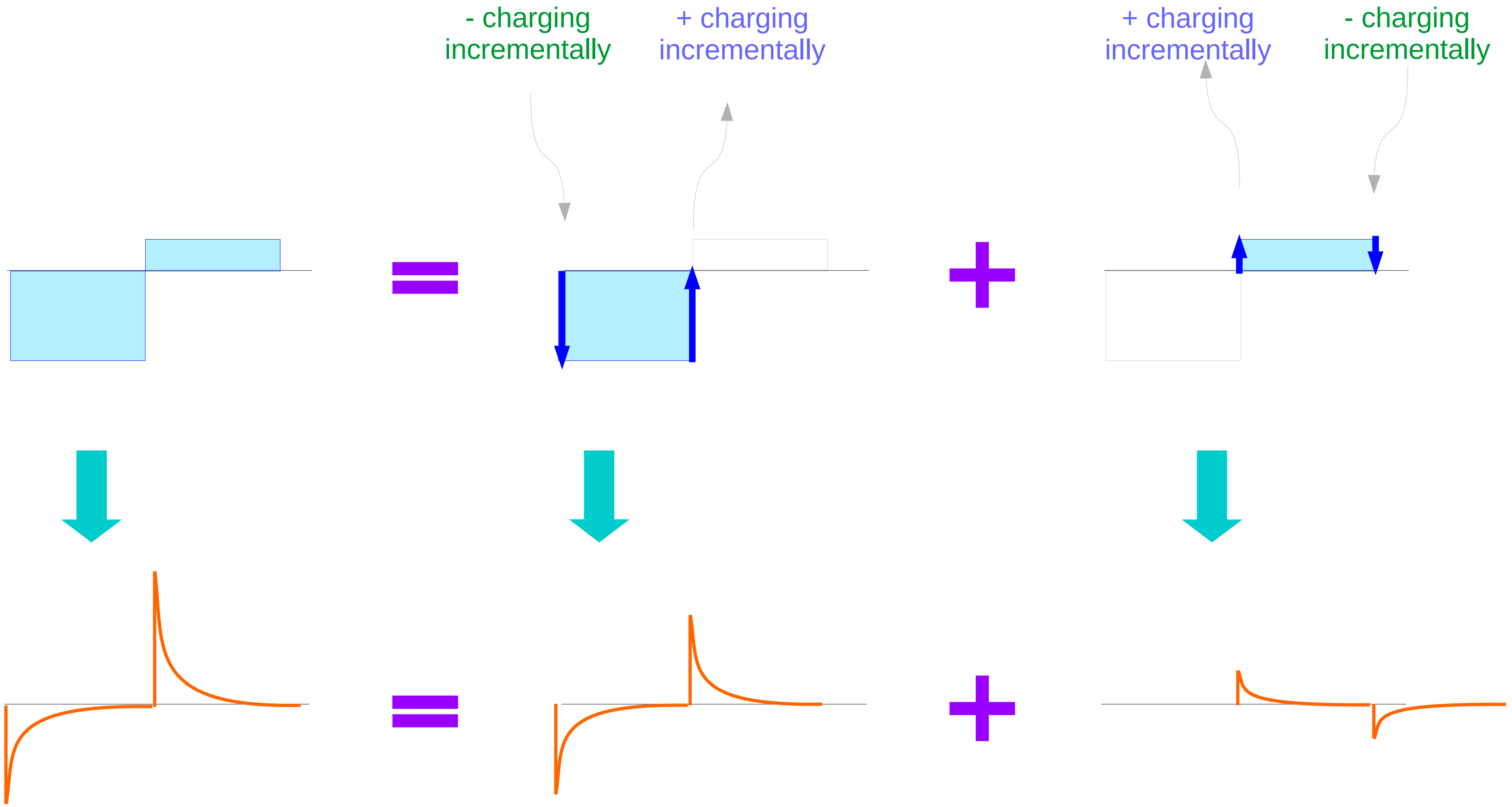
# Continuous (Ever-) Charing Operations

**Incremental Voltage Increment** ➡ **+ Charging incrementally**

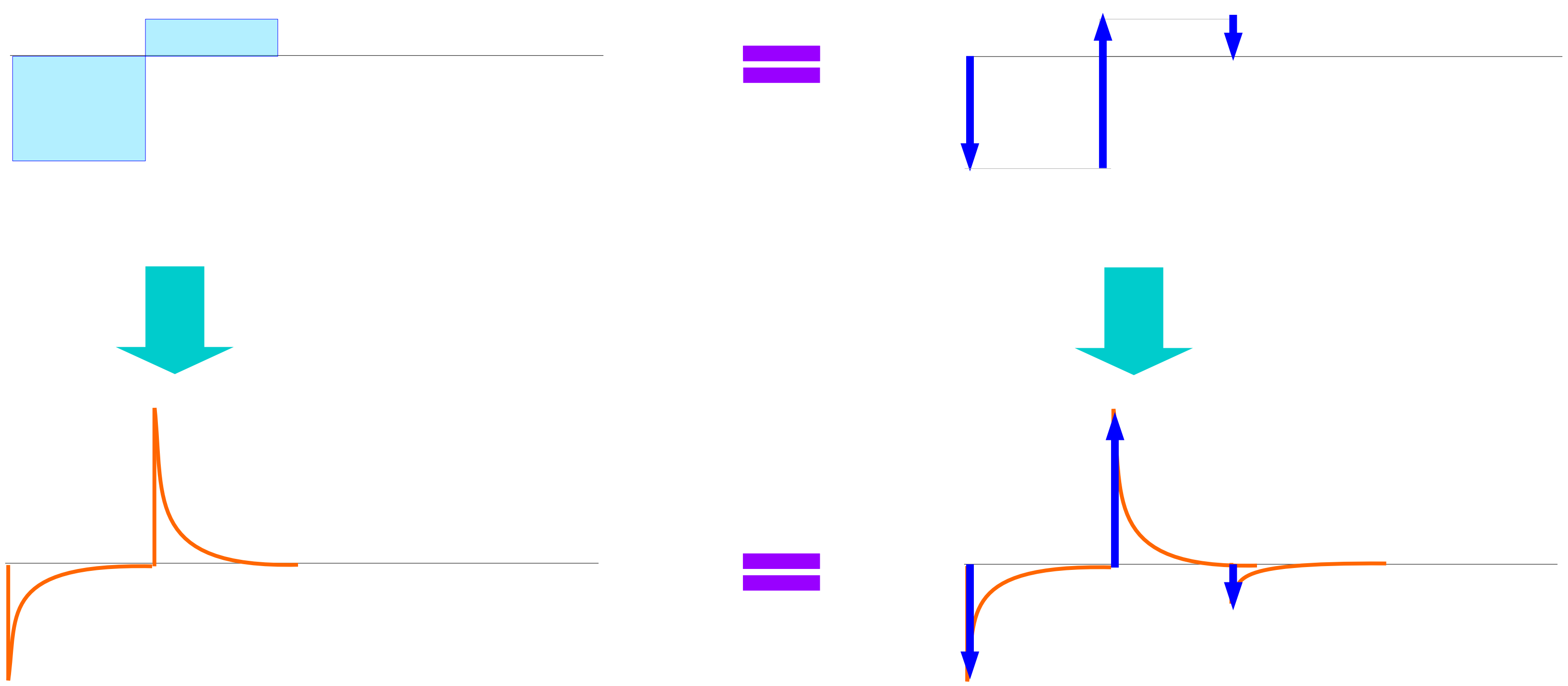
**Incremental Voltage Decrement** ➡ **– Charging incrementally**



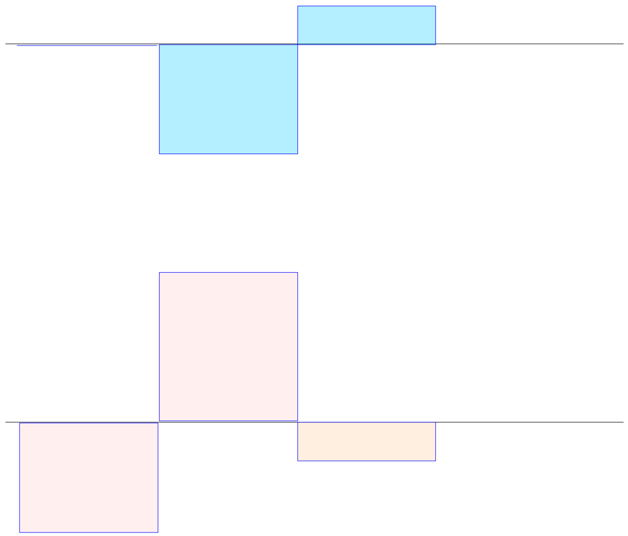
# Superposition



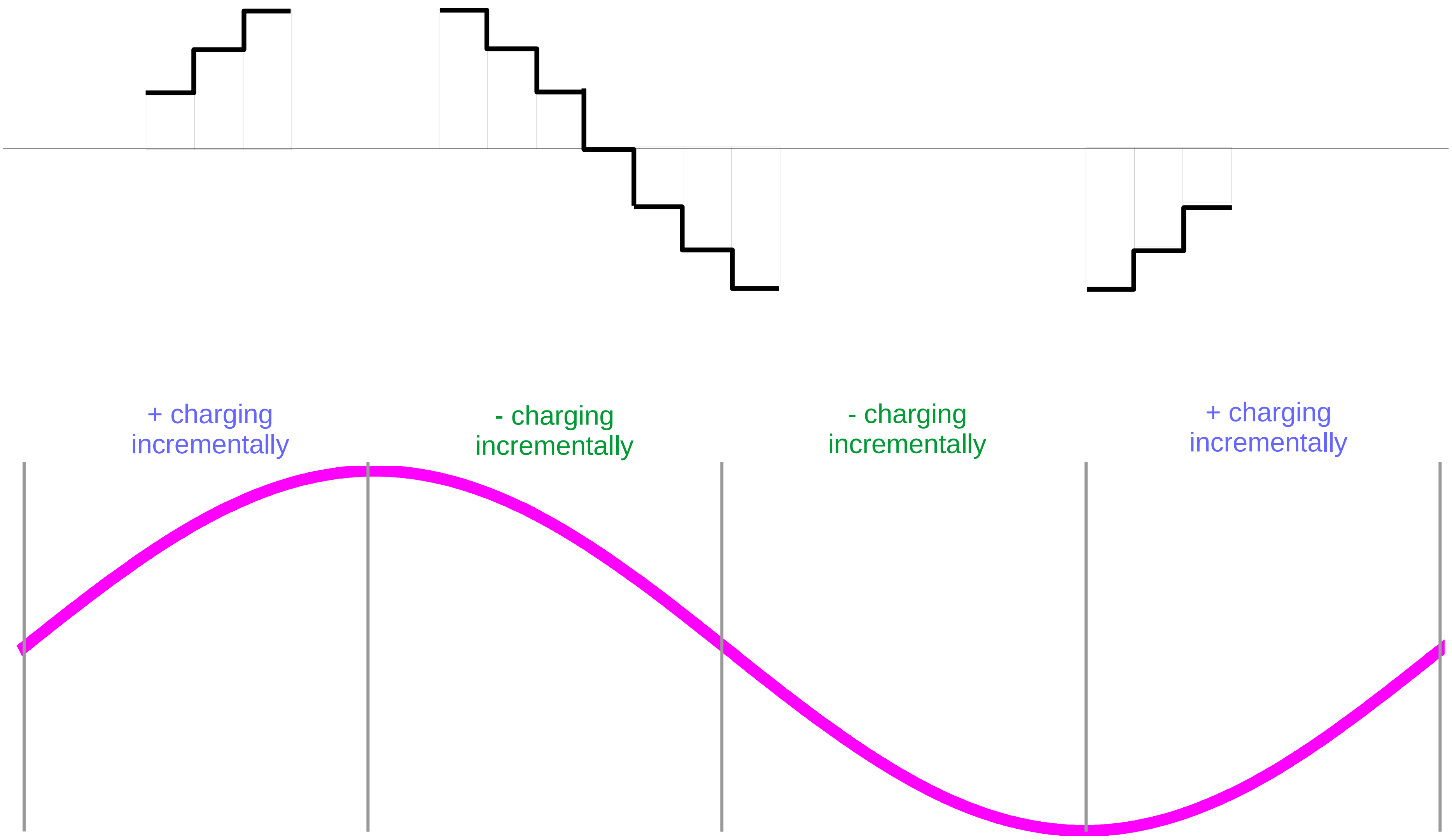
# Superposition - Small Time Constant



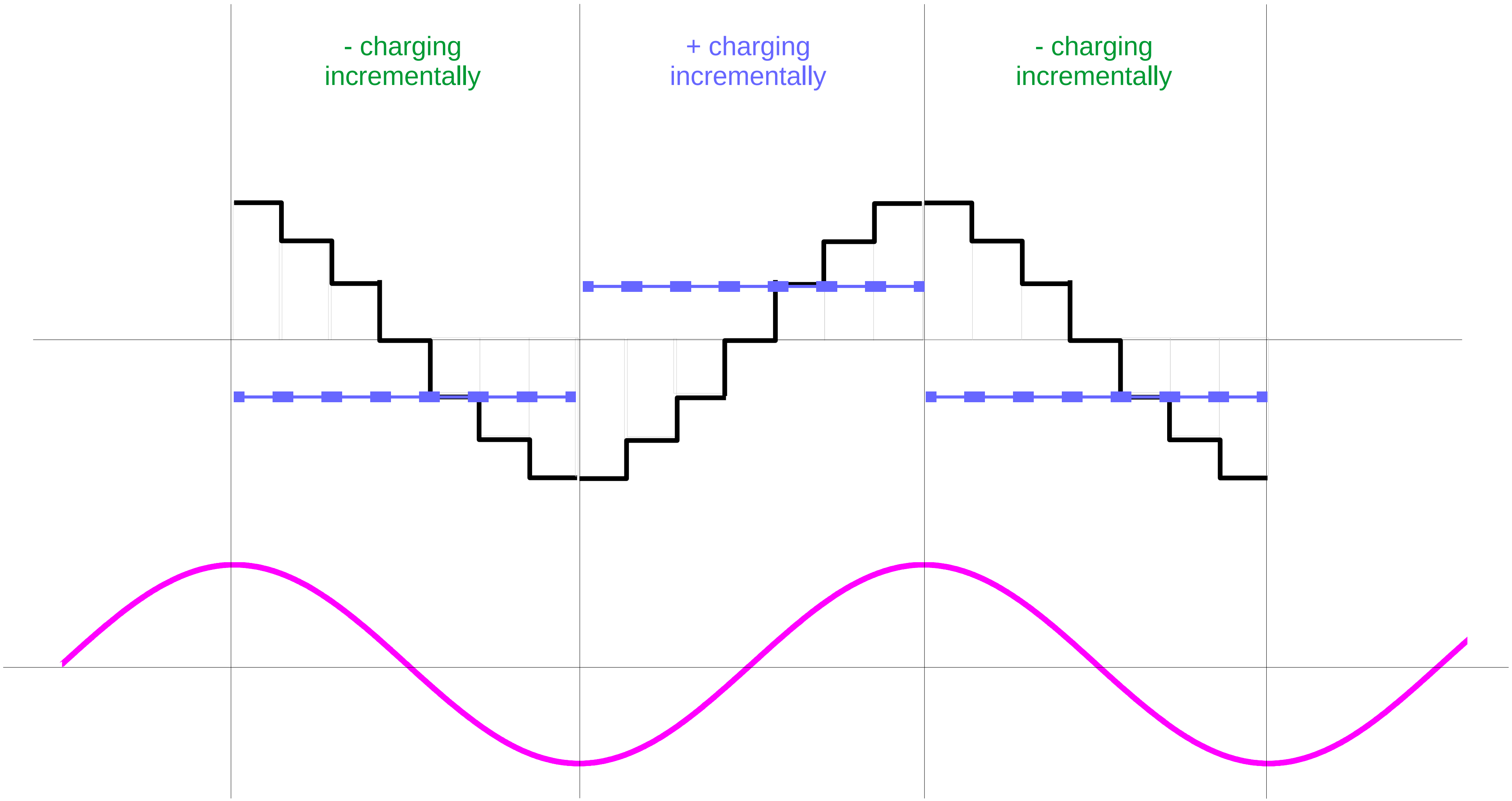
# Difference, Differentiation



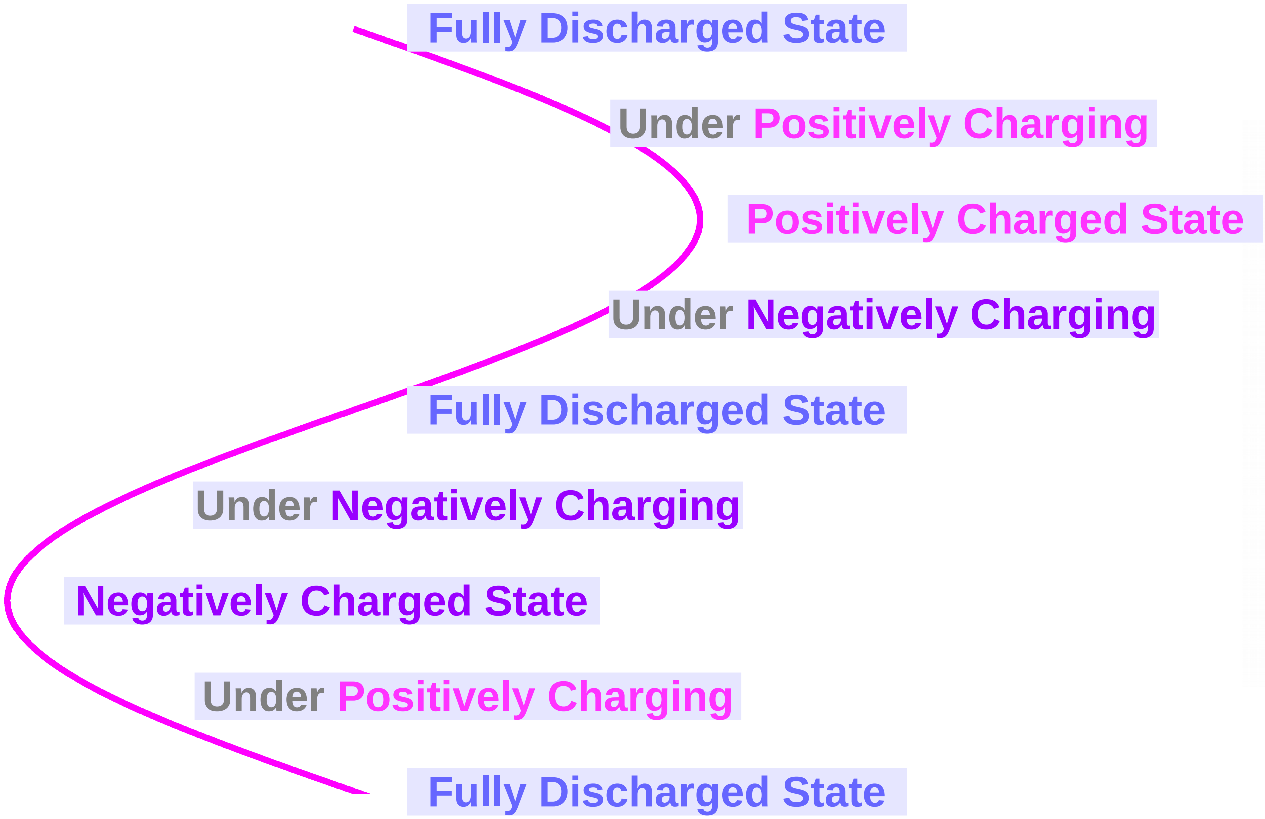
# Continuous Charing and Discharging Operations

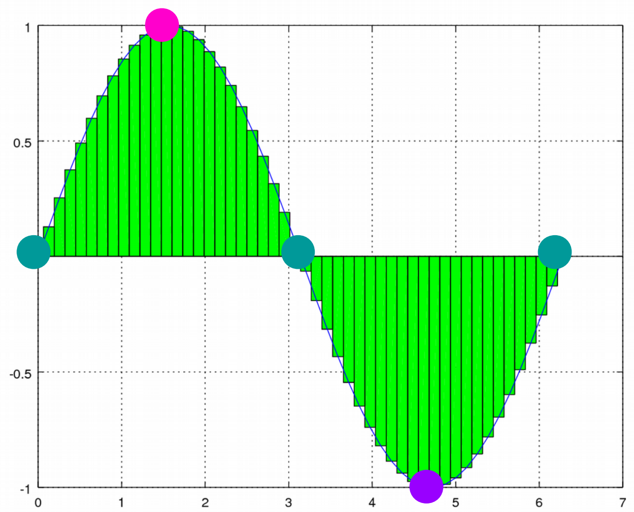


# Incrementally Charging

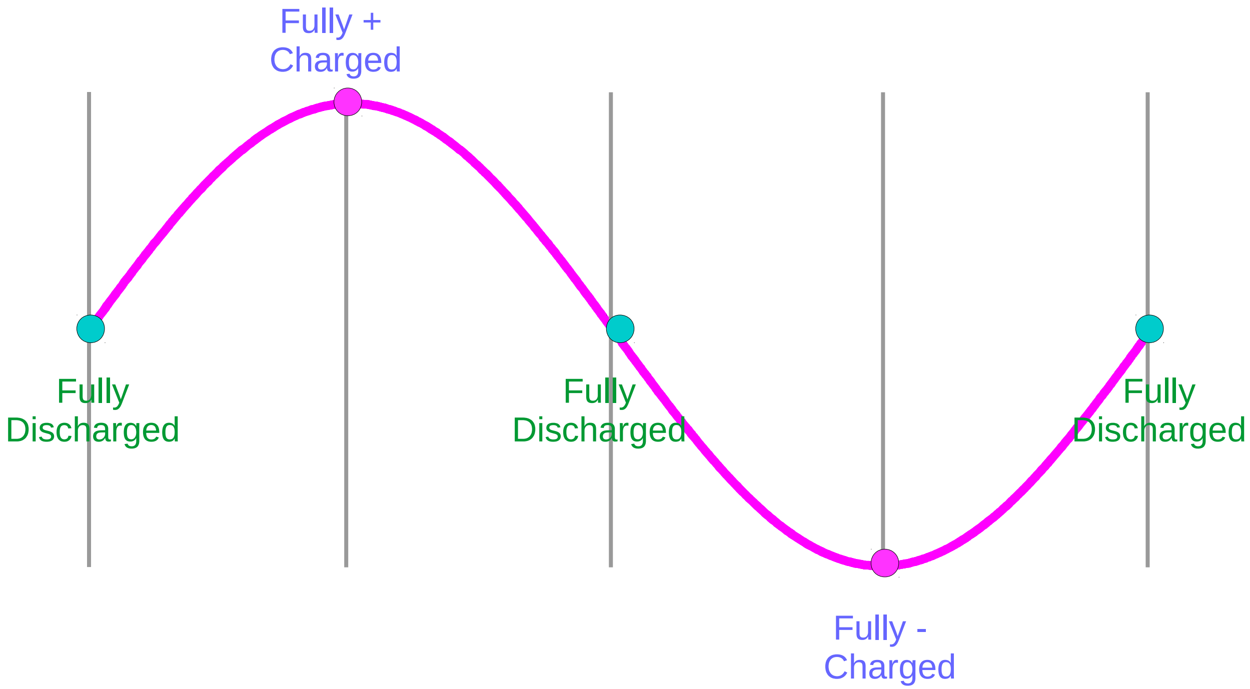


# An AC Voltage Source



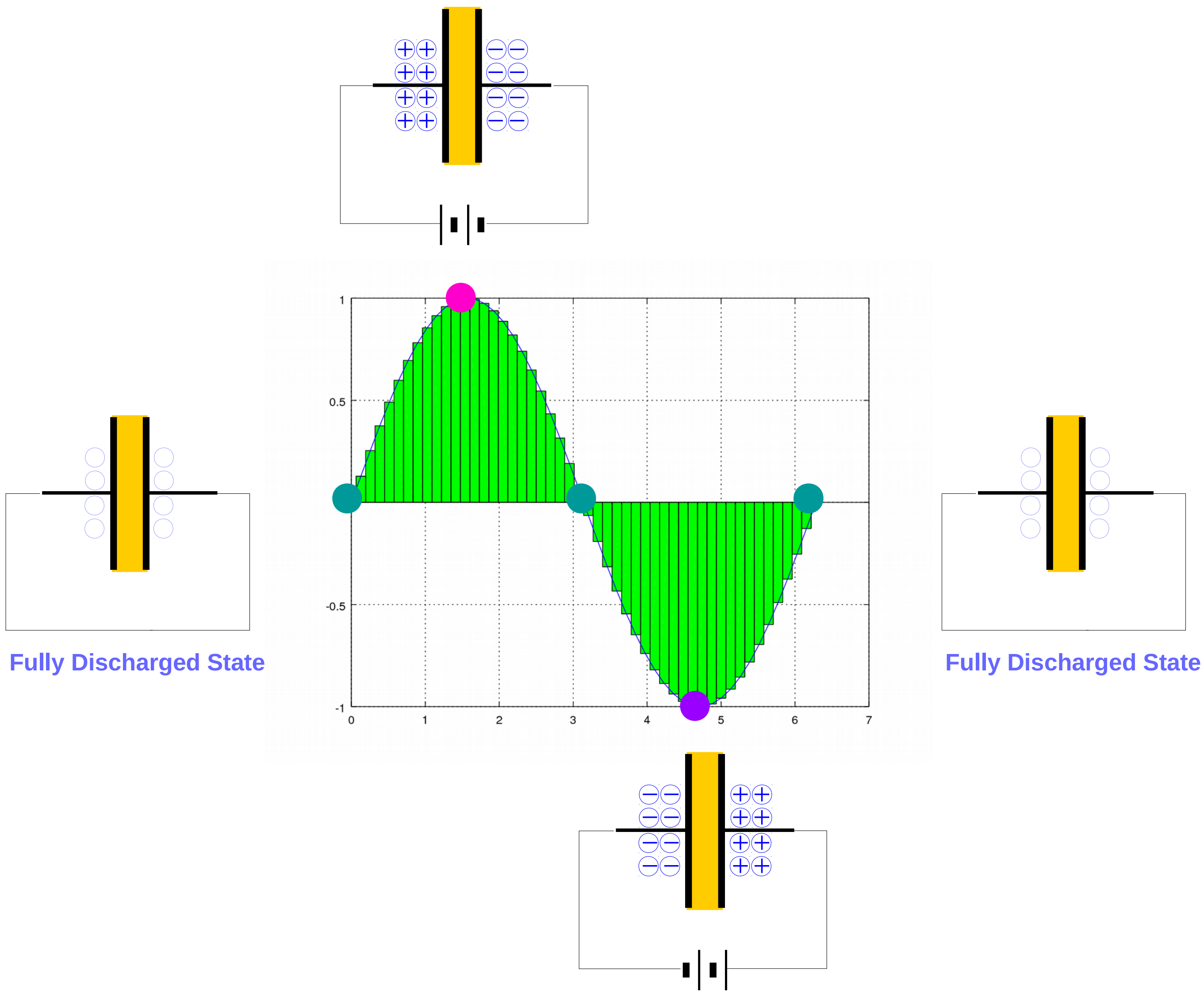



# Fully Charged and Fully Discharged

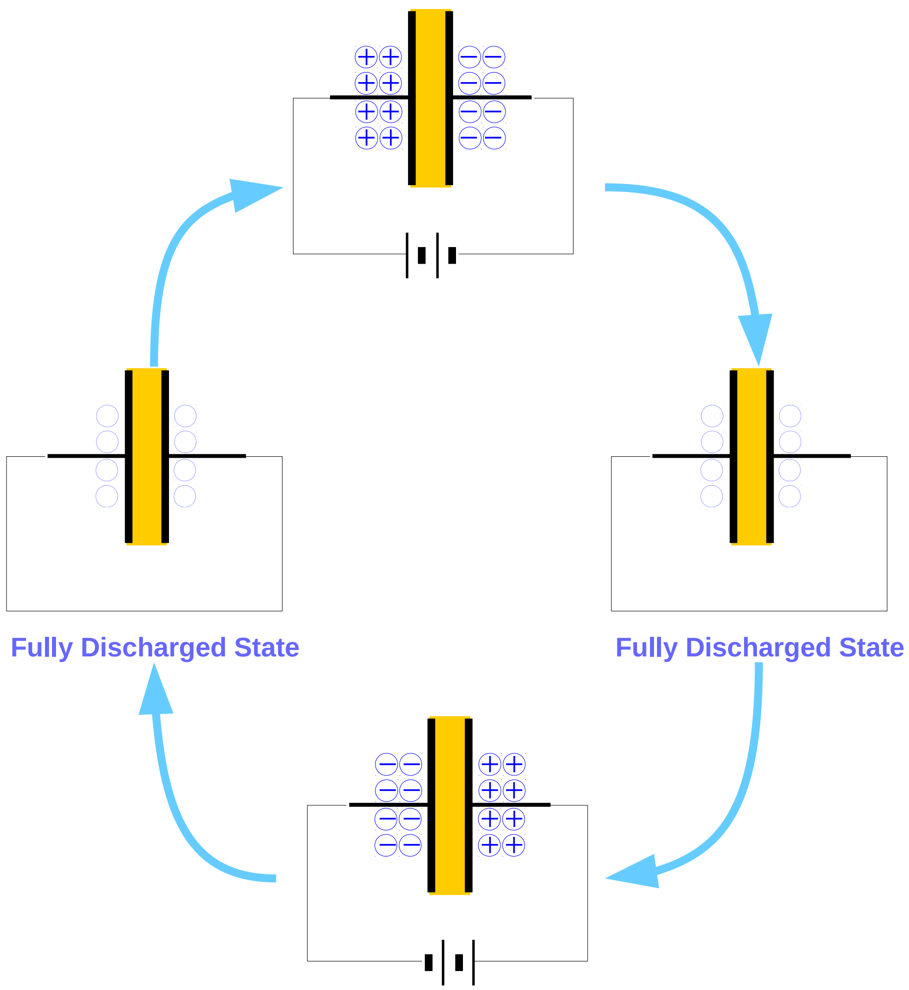


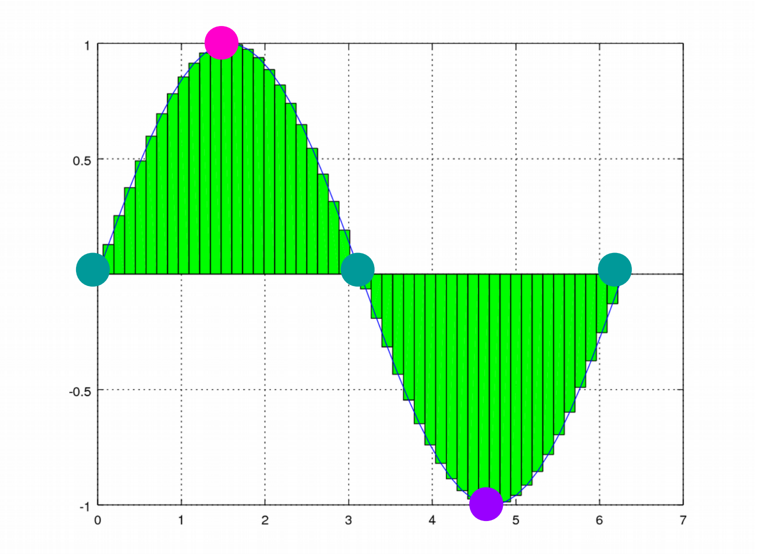

| | | | |
|---|---|---|---|
| (+) Charging | (–) Charging | (–) Charging | (+) Charging |
| (+) Current | (–) Current | (–) Current | (+) Current |
| (+) Charging | (+) **Dis**charging | (–) Charging | (–) **Dis**charging |

# A Cycle

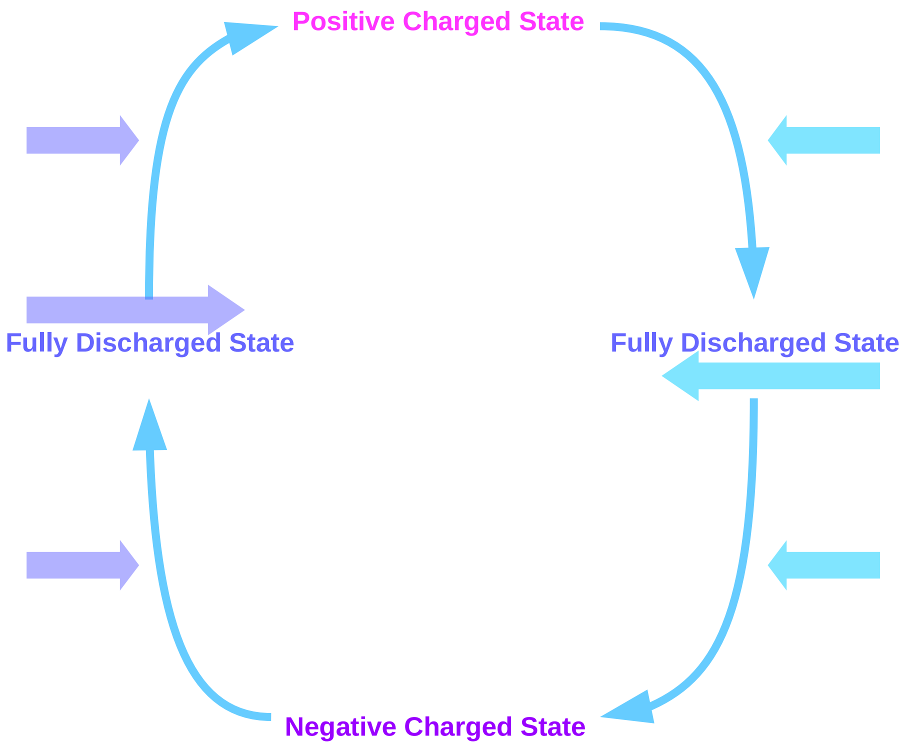


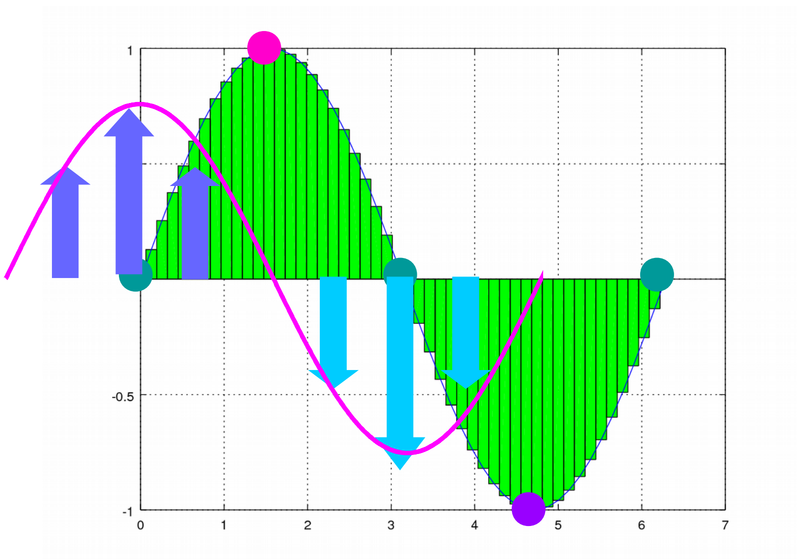
# State Transition Diagram



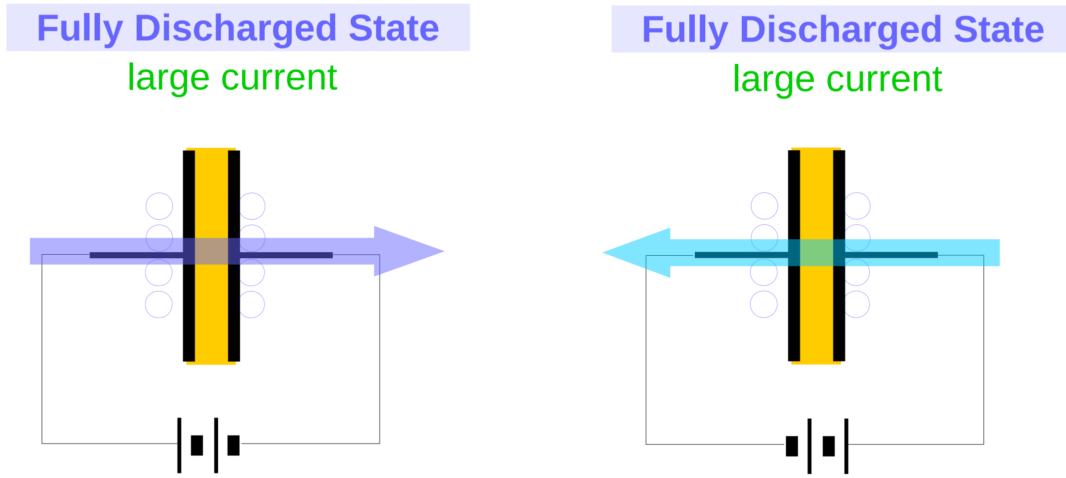



# Current Flow



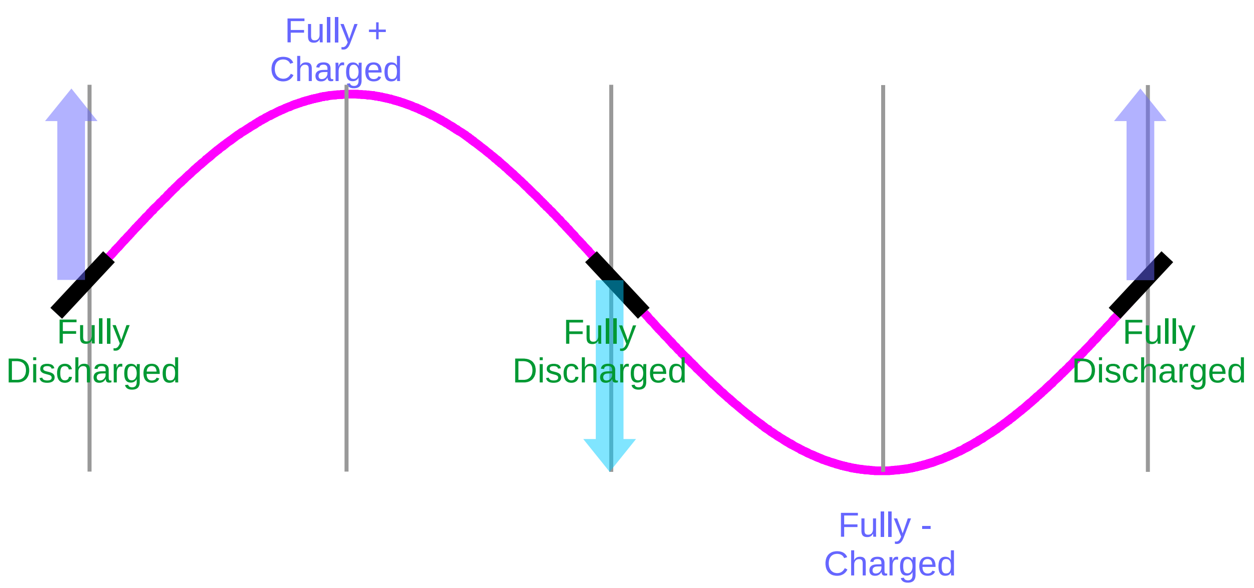



# Fully Discharged : Large Current




Enough space for
large movement of charges

This state can flow
large current
in either direction

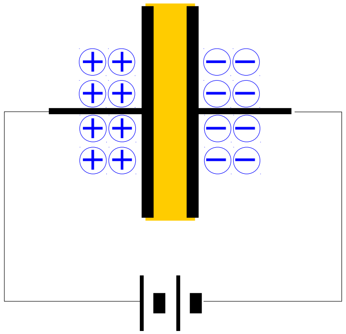



$$\left|\frac{dv_c}{dt}\right| = 1 \quad (max\ value)$$

# Fully Charged : Zero Current

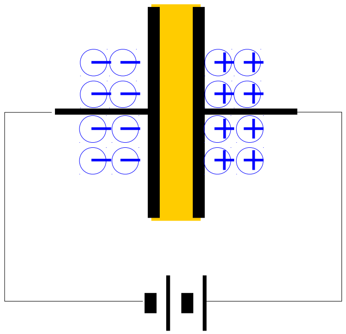
**Positively Charged State**

fully charged → no current



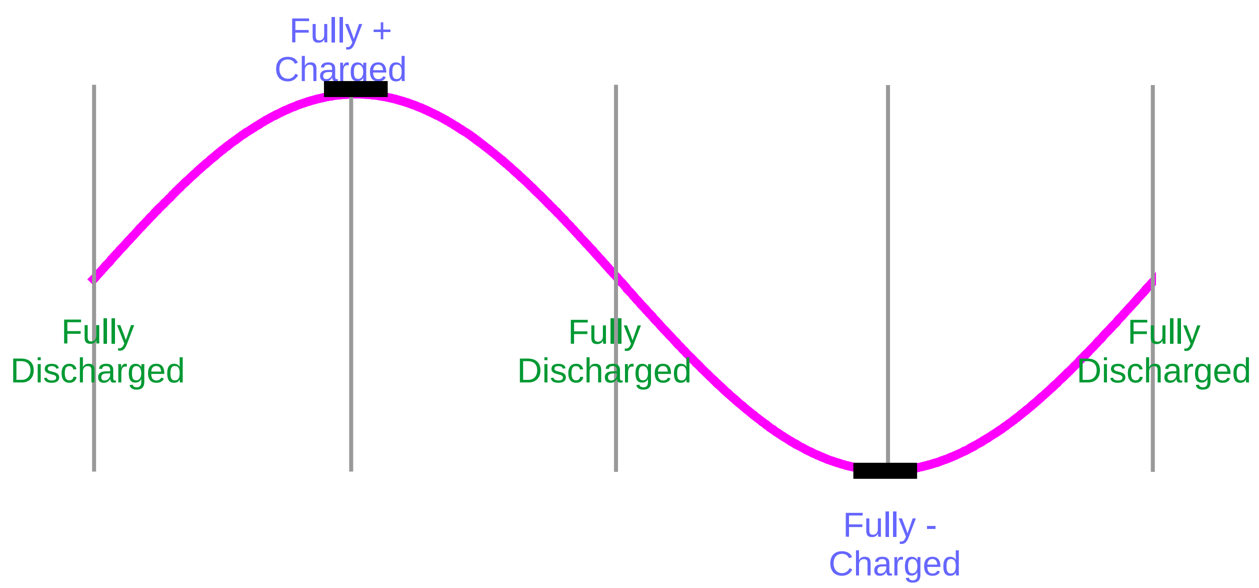
**Negatively Charged State**

fully charged → no current



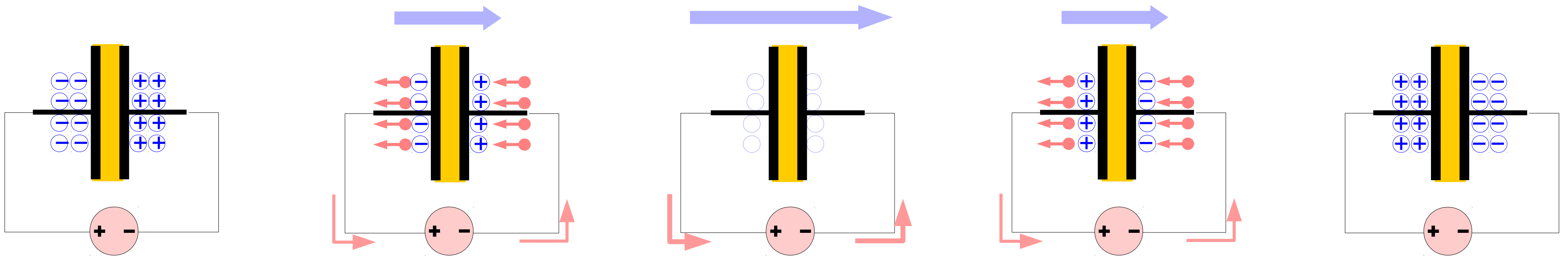
Crowded →
No more space

no current




$$\left|\frac{dv_c}{dt}\right| = 0 \quad (min\ value)$$

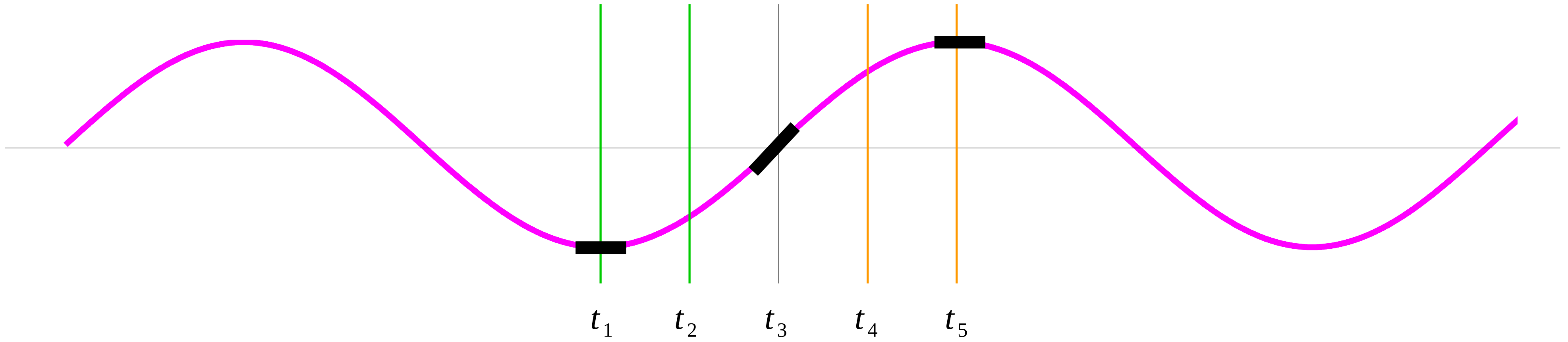
# Incrementally, Charging Positively



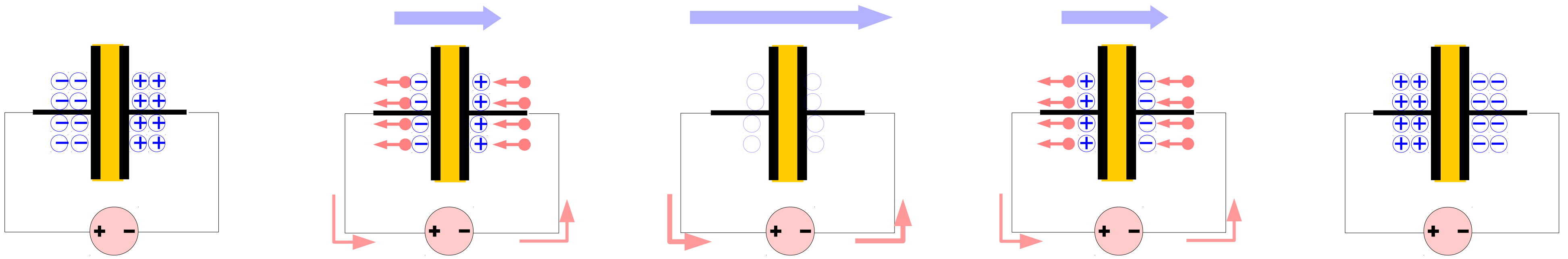
$v(t_1) < 0$ | $v(t_2) < 0$ | $v(t_3) < 0$ | $v(t_4) > 0$ | $v(t_5) > 0$

$v'(t_1) > 0$ | $v'(t_2) > 0$ | $v'(t_3) > 0$ | $v'(t_4) > 0$ | $v'(t_5) > 0$

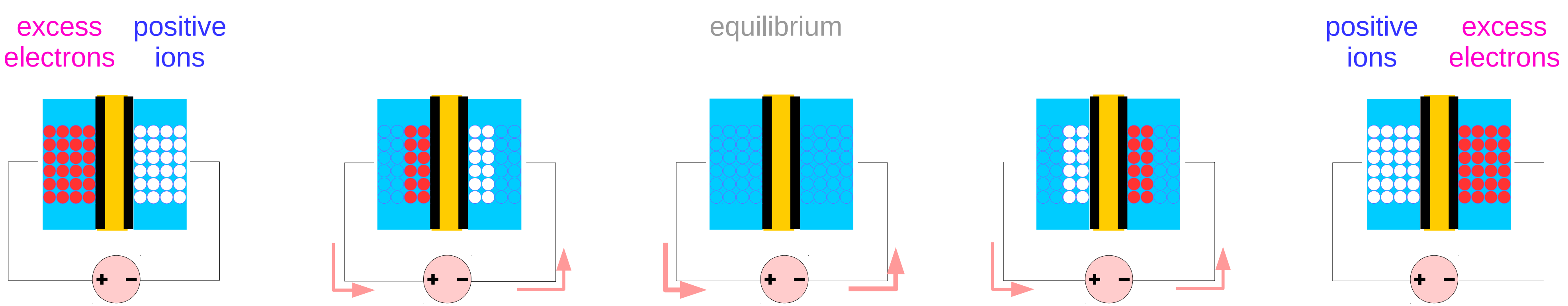
$\Delta v(t_1) > 0$ | $\Delta v(t_2) > 0$ | $\Delta v(t_3) > 0$ | $\Delta v(t_4) > 0$ | $\Delta v(t_5) > 0$



# Incrementally, Charging Positively

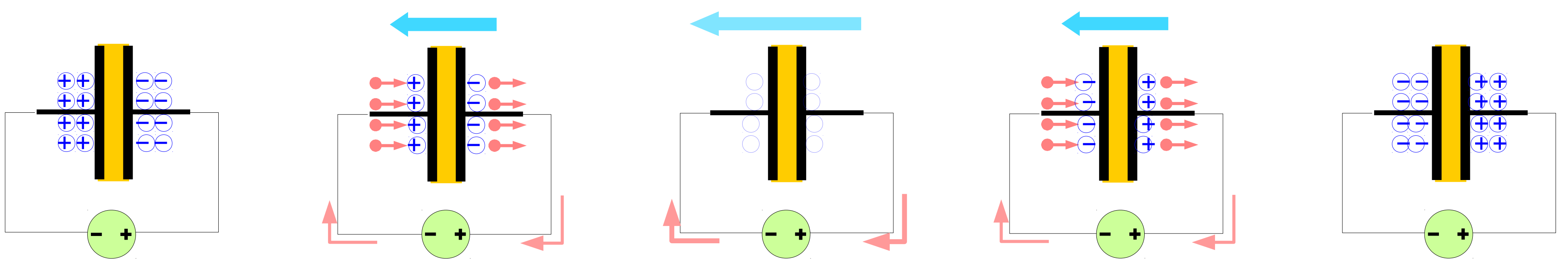



| | | | | |
|---|---|---|---|---|
| $v(t_1) < 0$ | $v(t_2) < 0$ | $v(t_3) < 0$ | $v(t_4) > 0$ | $v(t_5) > 0$ |
| $v'(t_1) > 0$ | $v'(t_2) > 0$ | $v'(t_3) > 0$ | $v'(t_4) > 0$ | $v'(t_5) > 0$ |
| $\Delta v(t_1) > 0$ | $\Delta v(t_2) > 0$ | $\Delta v(t_3) > 0$ | $\Delta v(t_4) > 0$ | $\Delta v(t_5) > 0$ |




| | | | | |
|---|---|---|---|---|
| $\Delta v(t_1) > 0$ | $\Delta v(t_2) > 0$ | $\Delta v(t_3) > 0$ | $\Delta v(t_4) > 0$ | $\Delta v(t_5) > 0$ |

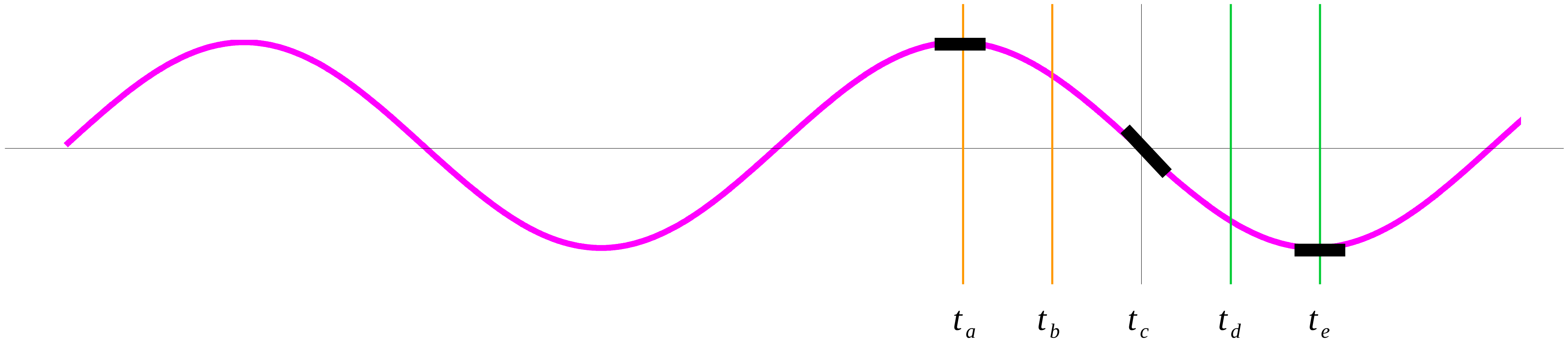
# Incrementally, Charging Negatively



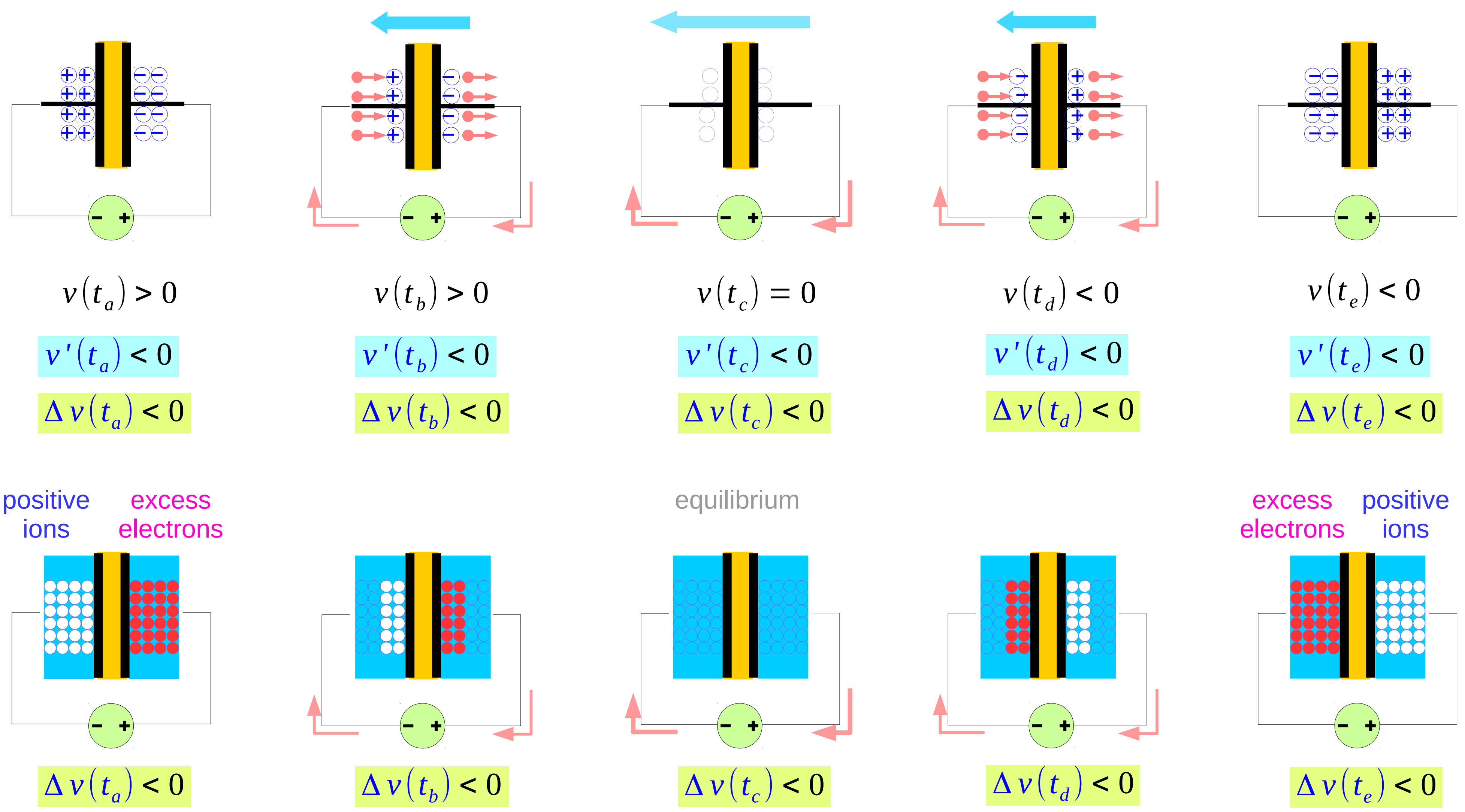

$v(t_a) > 0$ $v(t_b) > 0$ $v(t_c) = 0$ $v(t_d) < 0$ $v(t_e) < 0$

$v'(t_a) < 0$ $v'(t_b) < 0$ $v'(t_c) < 0$ $v'(t_d) < 0$ $v'(t_e) < 0$

$\Delta v(t_a) < 0$ $\Delta v(t_b) < 0$ $\Delta v(t_c) < 0$ $\Delta v(t_d) < 0$ $\Delta v(t_e) < 0$



# Incrementally, Charging Negatively



# Difference of Samples

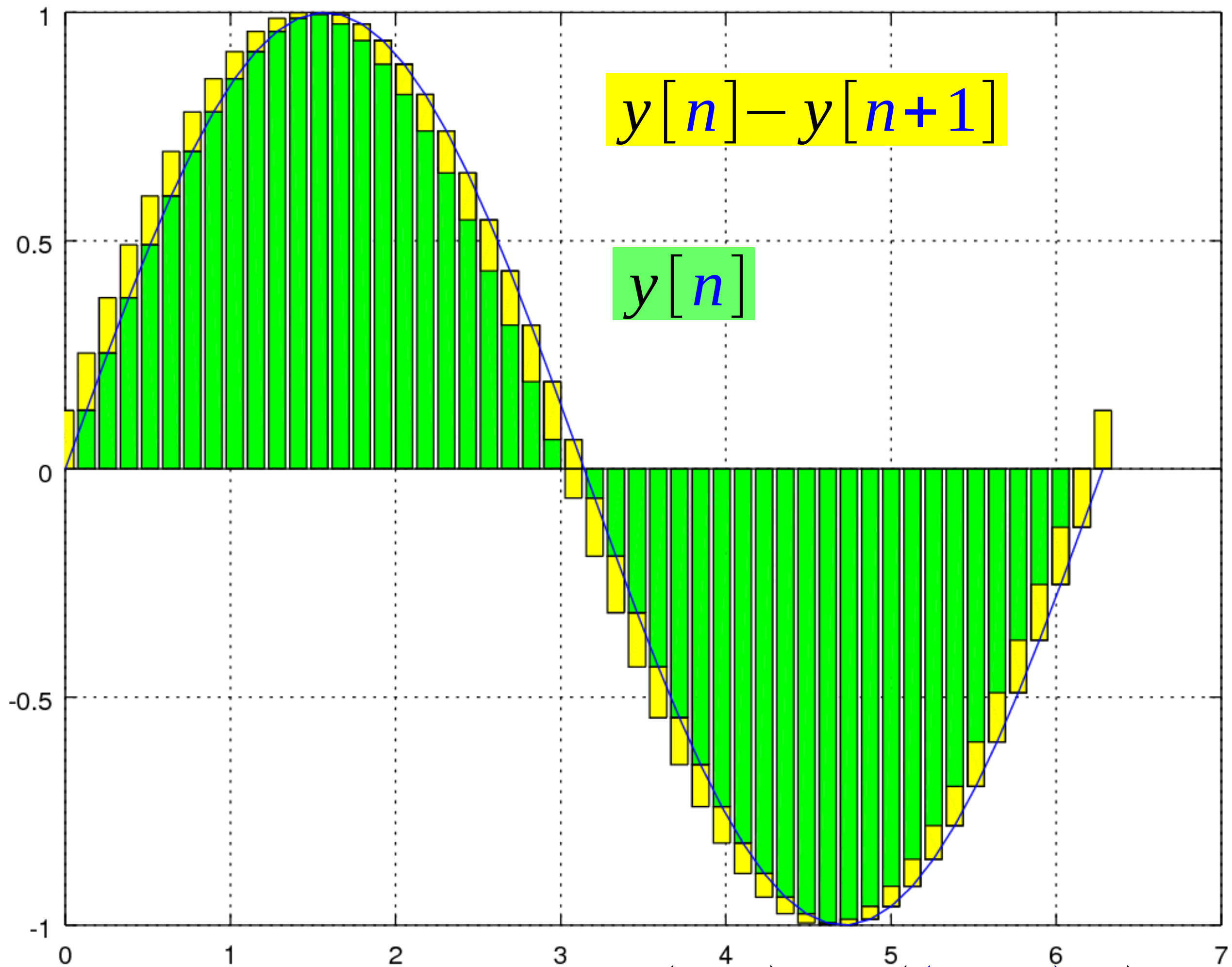
$$y(t) = \sin(t)$$

$$y[n] = \sin(nT)$$

$$y[n] - y[n+1] = \sin(nT) - \sin((n+1)T)$$

$$\frac{y[n] - y[n+1]}{T} \propto \frac{dy}{dt}$$

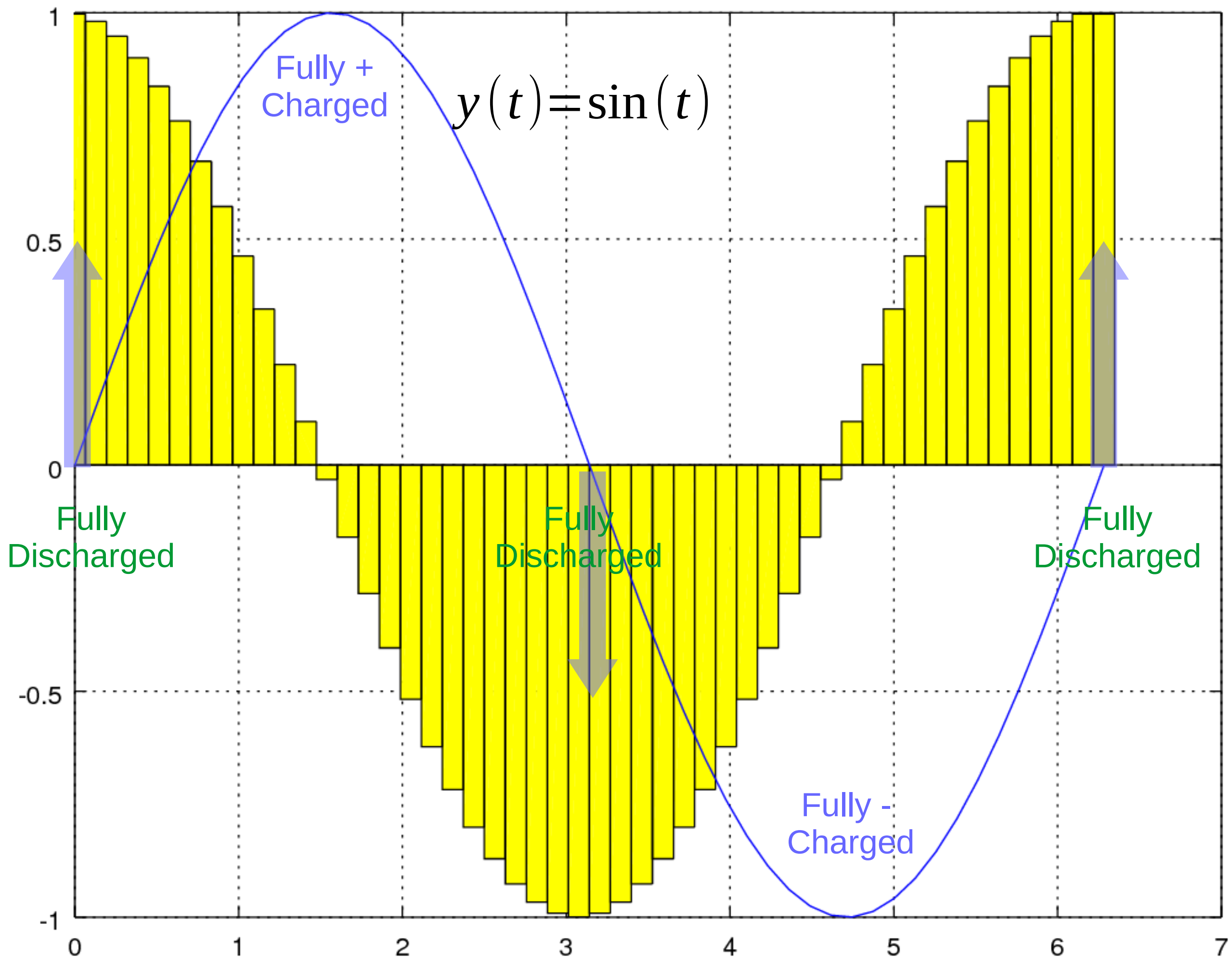
# Fully Charged and Fully Discharged




```
h = bar(t1, [y1' y2'],
"stacked")
set(h(1), "facecolor", "g");
set(h(2), "facecolor", "y");
hold on
plot(t1, y1)
axis([0 pi]);
```

$$y[n]-y[n+1] = y(nT)-y((n+1)T)=\sin(nT)-\sin((n+1)T)$$

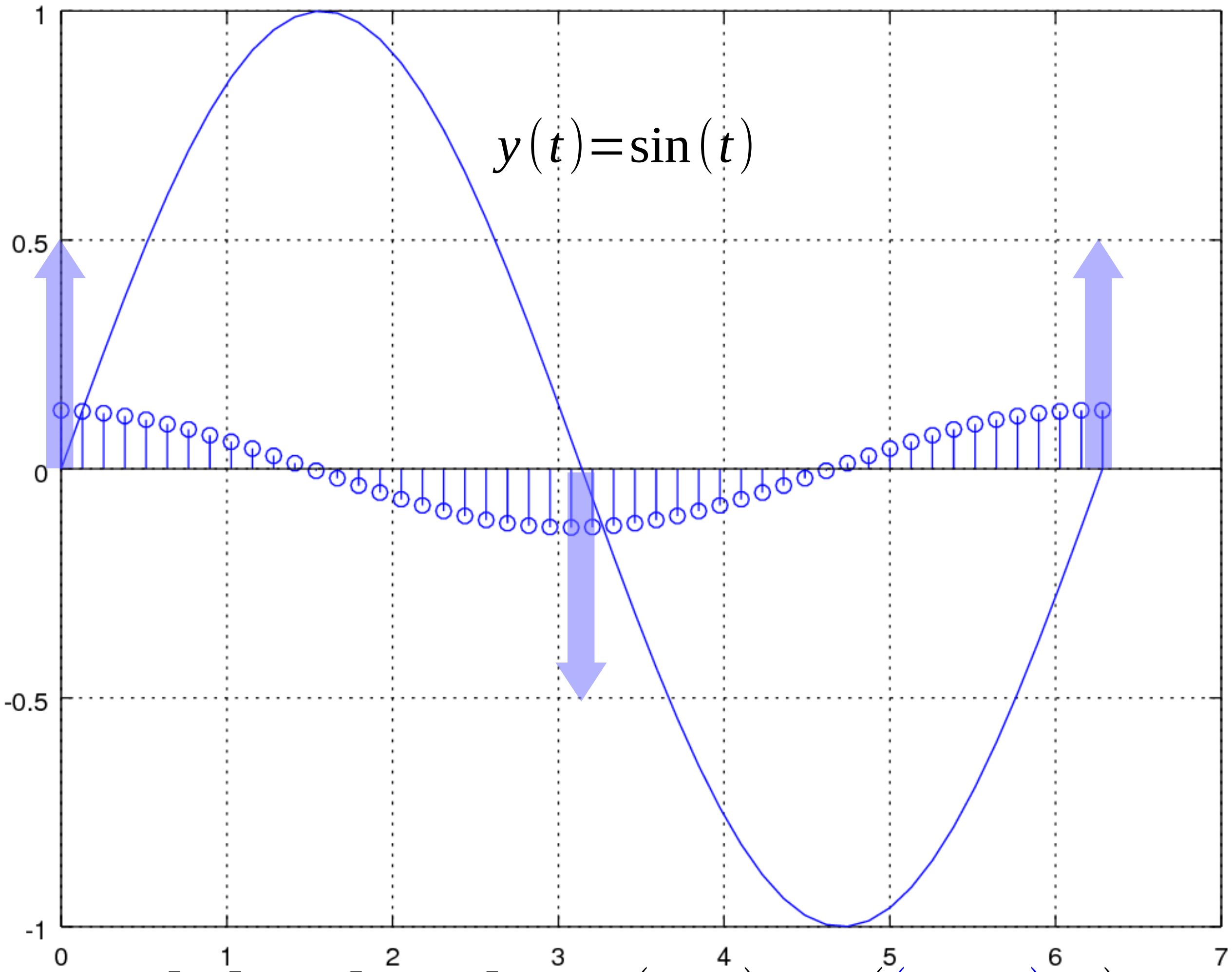
# Fully Charged and Fully Discharged




```
h = bar(t1, y2/t(2), "hist")
set(h(1), "facecolor", "y");
hold on
plot(t1, y1)
axis([0 7 -1 1]);
```

$$\frac{y[n] - y[n+1]}{T}$$

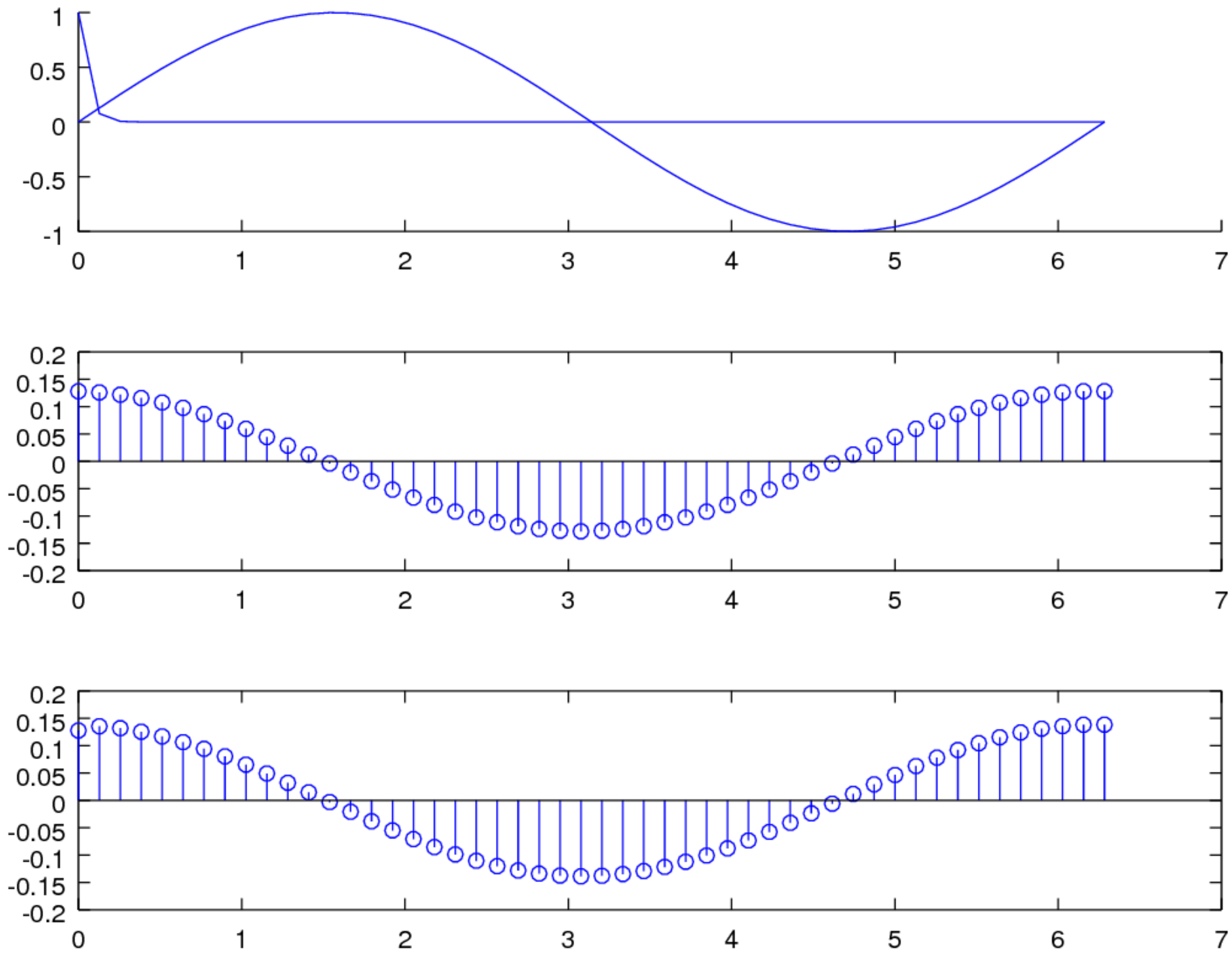
$$\propto \frac{dy}{dt}$$




```
t = linspace(0, pi*2, 50);
t1 = t;
t2 = t + t(2);
y1 = sin(t1);
y2 = sin(t2) - sin(t1);
stem(t1, y2)
hold on
plot(t1, y1)
```

$$y[n]-y[n+1] = y(nT)-y((n+1)T)=\sin(nT)-\sin((n+1)T)$$

# Fully Charged and Fully Discharged

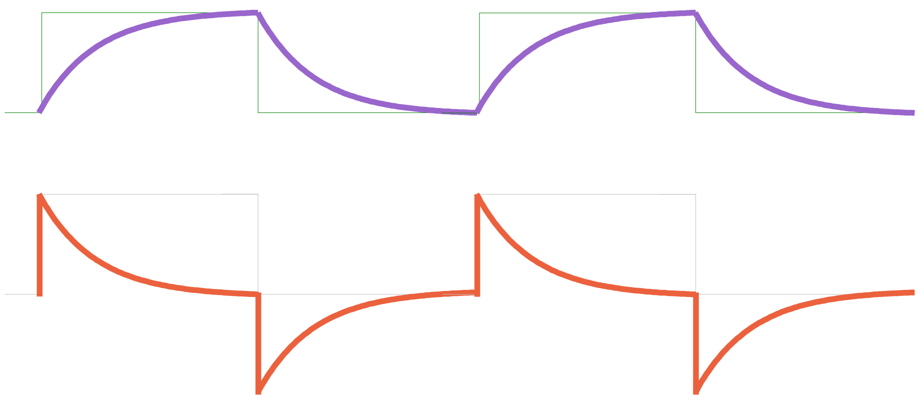



```
clf
t = linspace(0, pi*2, 50);
t1 = t;
t2 = t + t(2);
y1 = sin(t1);
y2 = sin(t2) - sin(t1);
y3 = e.^(-20*t);
y4 = conv(y2, y3);
y5 = y4([1:length(t1)]);

subplot(3, 1, 2);
stem(t1, y2)
subplot(3, 1, 1);
hold on
plot(t1, y1);
plot(t1, y3);
subplot(3, 1, 3);
stem(t1, y5);
```

# Pulse

$v_C$

$i_C$

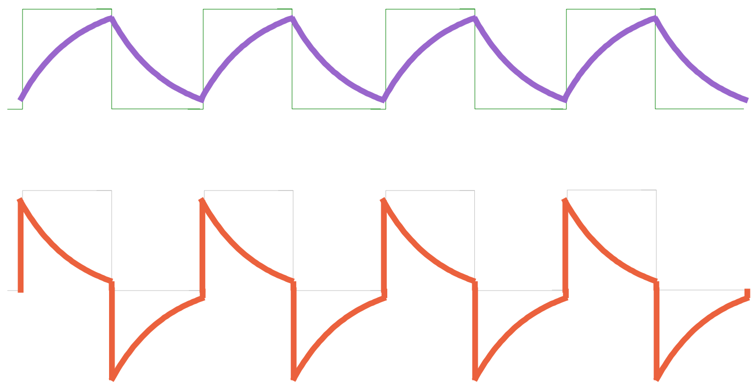


$$i_C = C \frac{d v_C}{d t}$$

$\omega \uparrow$ $\quad i_C \uparrow$ $\quad X_C \downarrow$

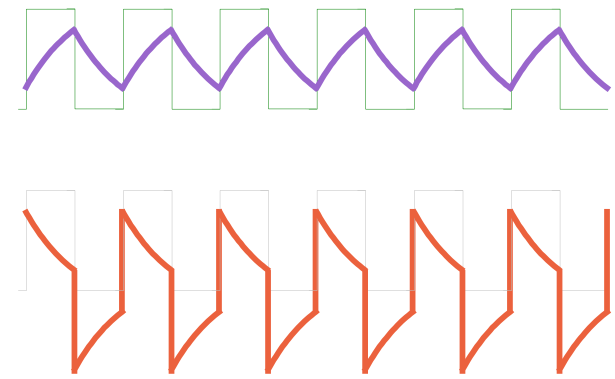
$v_C$

$i_C$

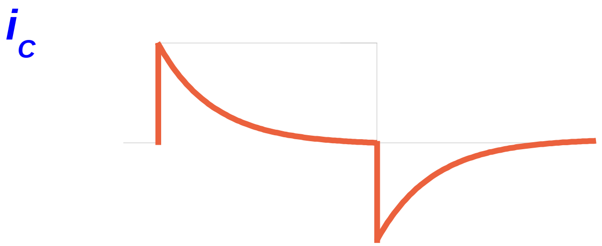


$v_C$

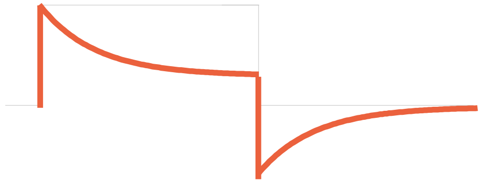
$i_C$



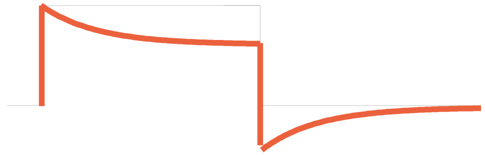
# Time Constants




$\tau = RC$ *small time constant*

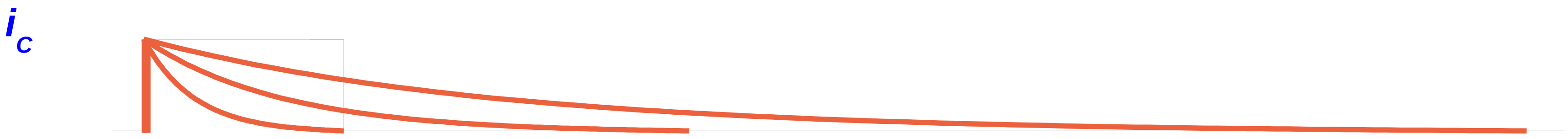


$\tau = RC$ *medium time constant*



$\tau = RC$ *large time constant*

# Time Constants




$$\tau_1 < \tau_2 < \tau_3$$

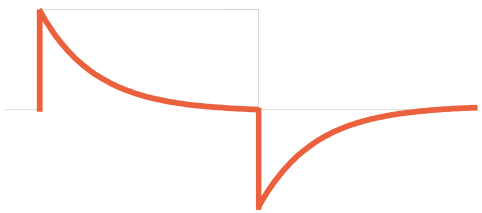
$$a_1 > a_2 > a_3$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}} = e^{-at}$$

$$\tau = RC = \frac{1}{a}$$

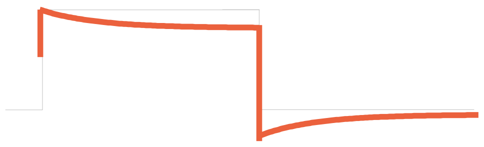
# Time Constants

$i_C$



$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$



$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

*small* $\tau$

*small* $C$

*large* $\frac{1}{\omega C} \gg R$

Fully Capacitative

*large* $\tau$

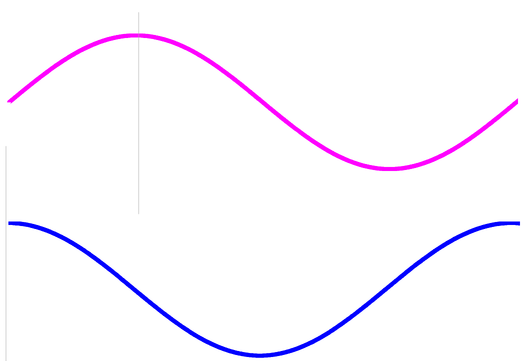
*large* $C$

*small* $\frac{1}{\omega C} \ll R$

Fully Resistive

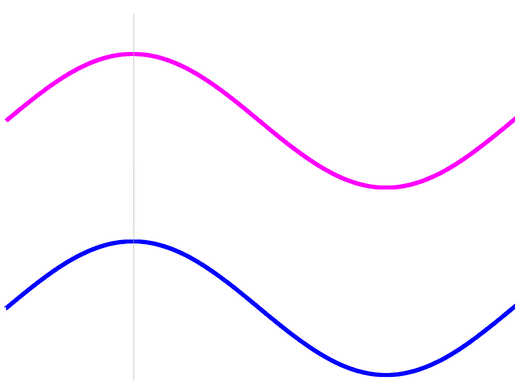
$v_C(t)$

$i_C(t)$

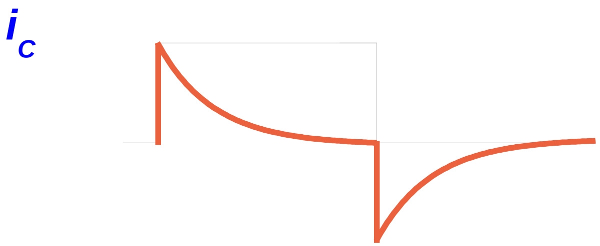


$v_C(t)$

$i_C(t)$



# Time Constants




$$\tau = RC$$

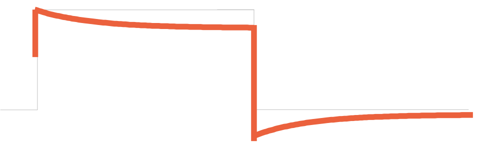
$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

$small\ \tau$

$small\ C$

$$large\ \frac{1}{\omega C} \gg R$$

Fully Capacitative



$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

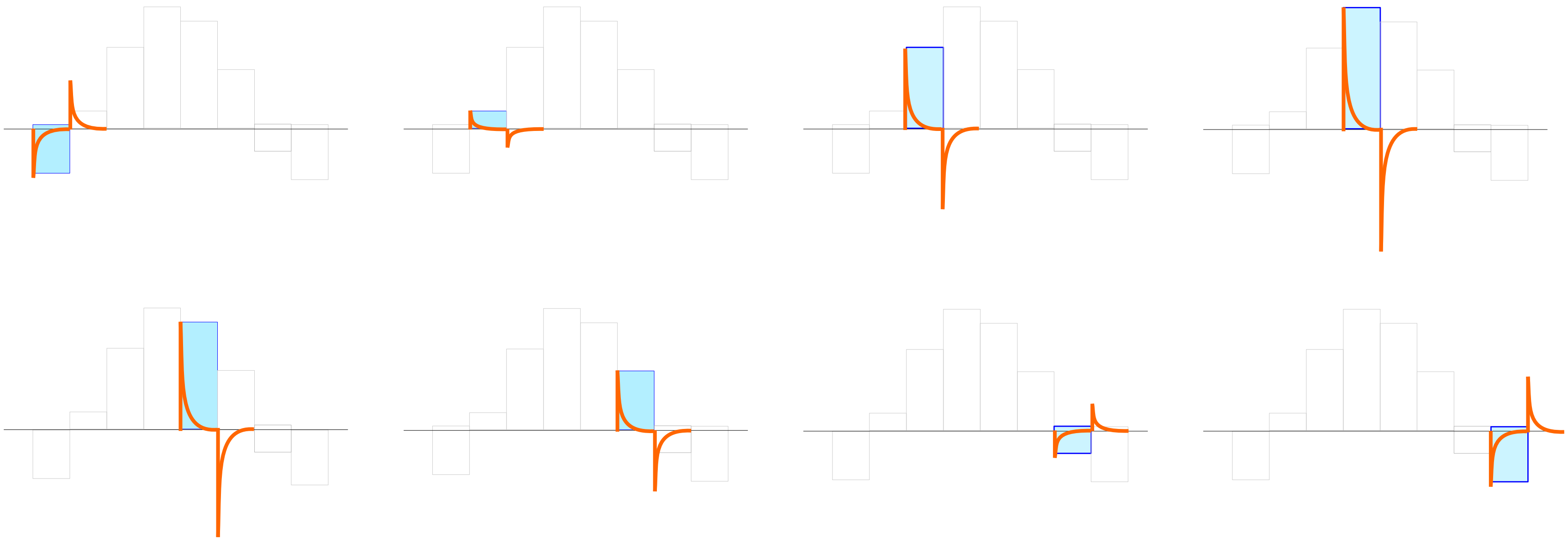
$large\ \tau$

$large\ C$

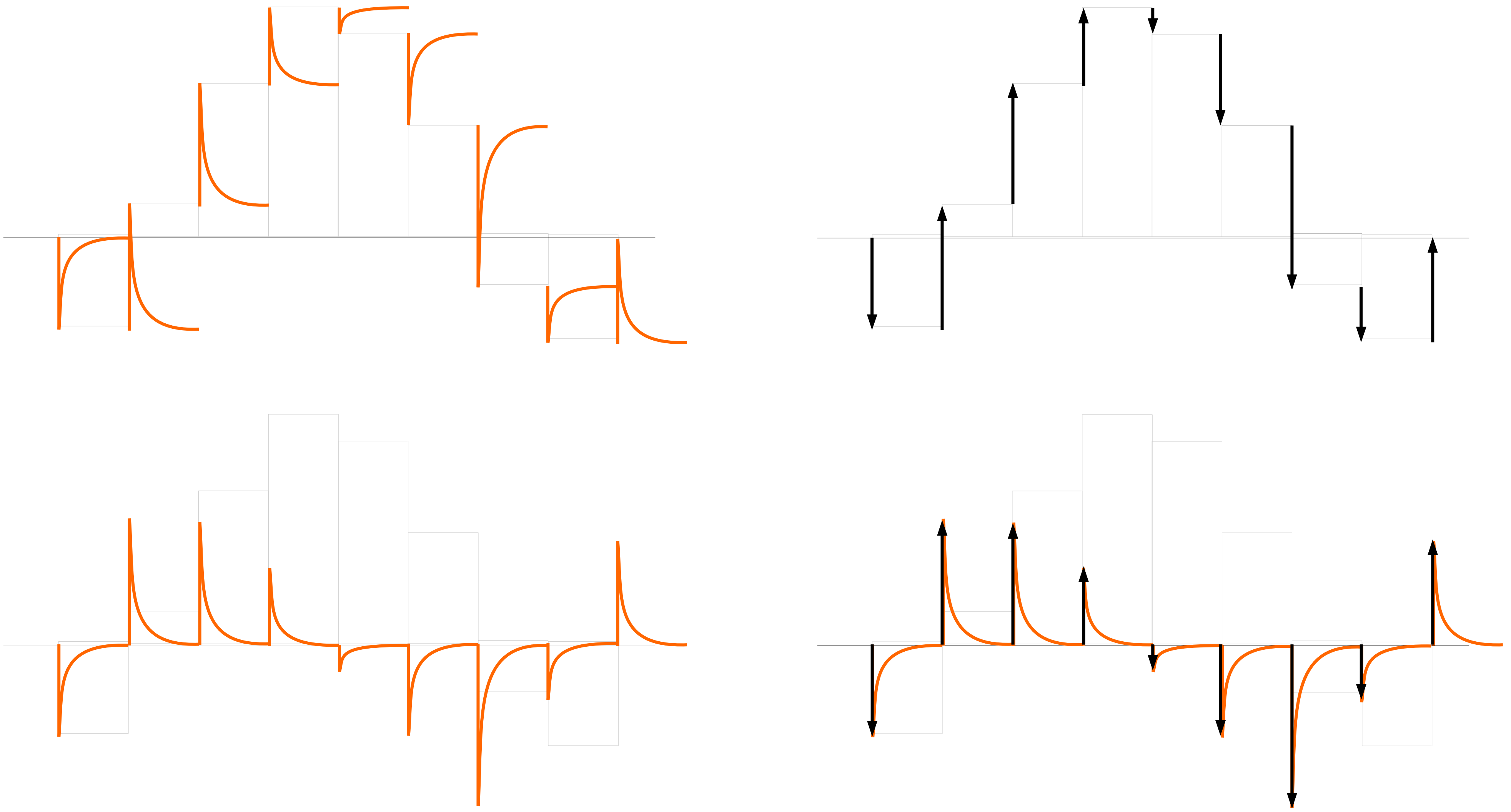
$$small\ \frac{1}{\omega C} \ll R$$

Fully Resistive

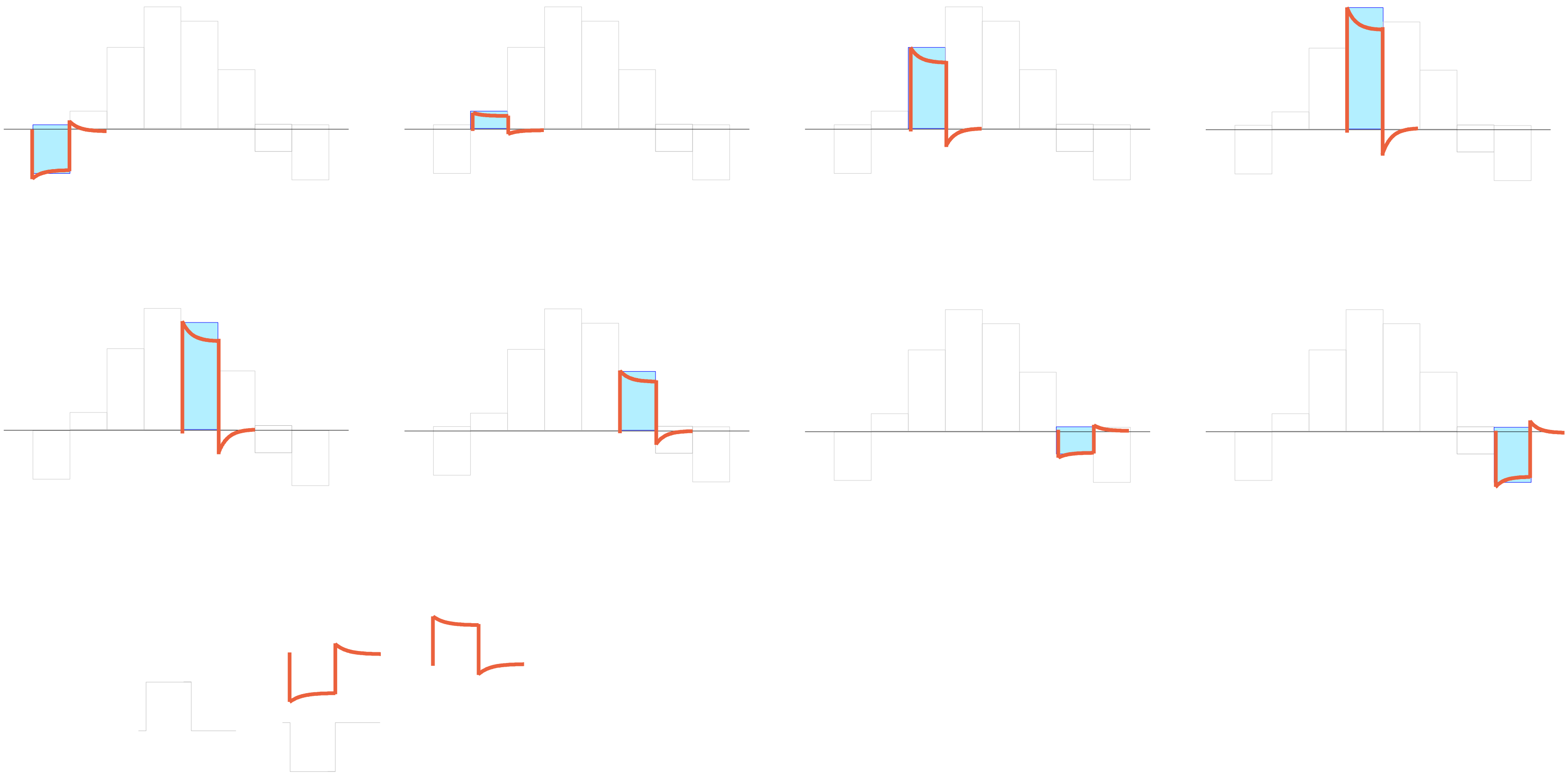
# Superposition - Small Time Constant



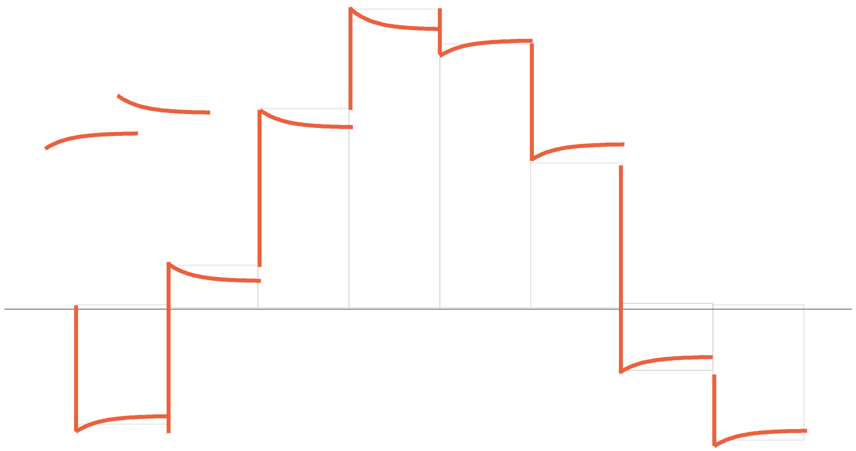
# Small Time Constants

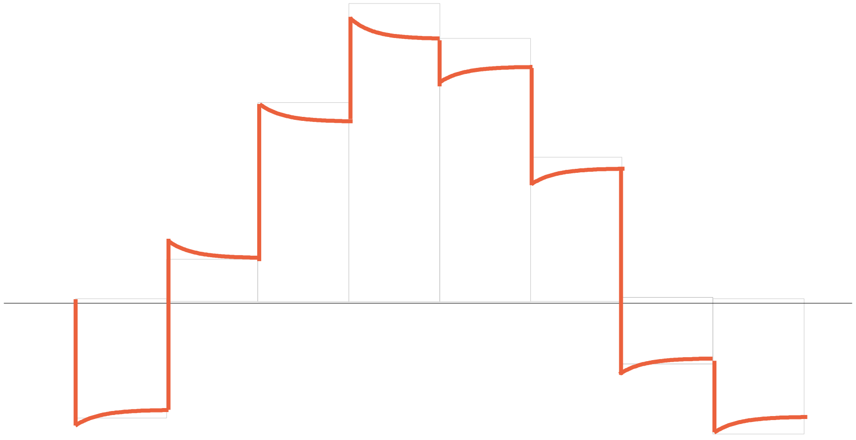


# Superposition – Large Time Constant

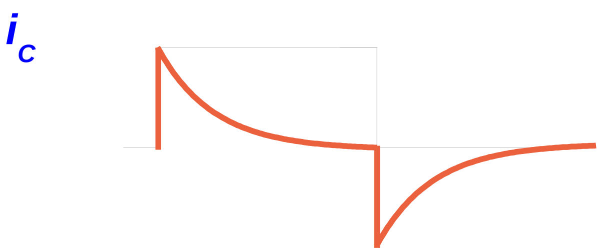


# Large Time Constants





# Time Constants



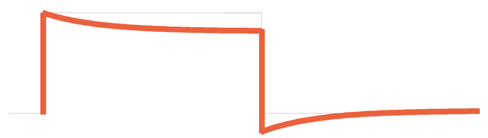

$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

*small* $\tau$

*small* $C$

$$large \; \frac{1}{\omega C} \gg R$$



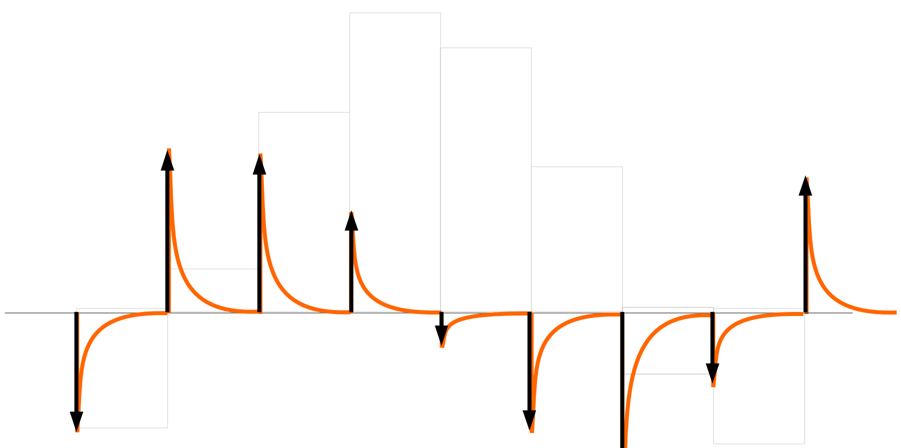
$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

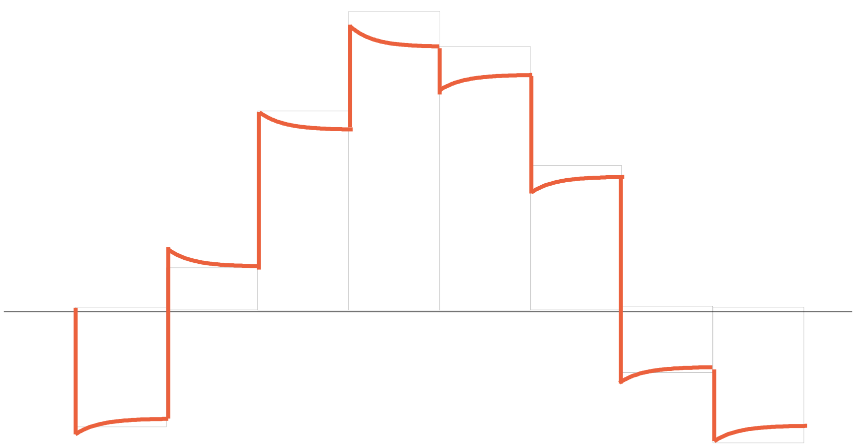
*large* $\tau$

*large* $C$

$$small \; \frac{1}{\omega C} \ll R$$



Fully Capacitative



Fully Resistive

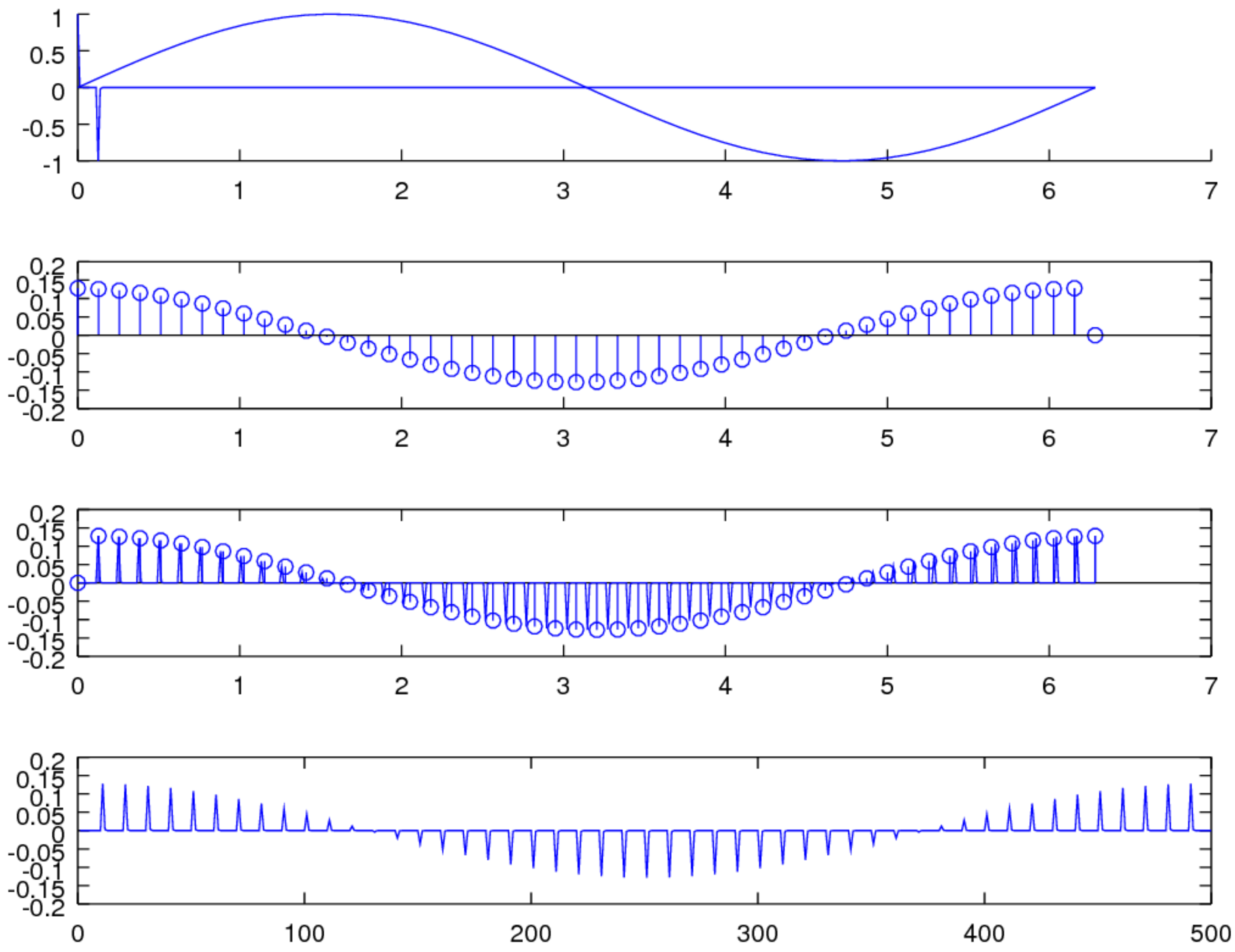
# Plotting superposition results

```
clf
t = linspace(0, pi*2, 50);
tt= linspace(0, pi*2, 500);
N = length(t);
NN= length(tt);

t1 = t;
t2 = [t(2:N), t(N)];
y1 = sin(t1);
y2 = sin(t2) - sin(t1);

yy = [y1; zeros(NN/N-1, N)];
yy2= yy(:)';
a = 1/300;
yy3= e.^(-a*tt);
yy3 =yy3 - [zeros(1, NN/N),
e.^(-a*tt)](1:NN);
```

```
svec = zeros(1, NN);
for i = 1:NN;
  tvec = zeros(1, NN);
  tvec = [zeros(1, i-1), yy3];
  tvec = yy2(i) * tvec(1:NN);
  svec = svec + tvec;
endfor
yy4 = svec;
% yy4= conv(yy2, yy3);
y5 = yy4([1:NN/N:NN]);
yy5= yy4([1:NN]);
```

```
subplot(4, 1, 2);
stem(t1, y2)
subplot(4, 1, 1);
hold on
plot(t1, y1);
plot(tt, yy3);
subplot(4, 1, 3);
stem(t1, y5); hold on
plot(tt, yy5)
subplot(4, 1, 4);
plot(yy4);
```

# Small Time Constant




yy = [y1; zeros(NN/N-1, N)];
yy2= yy(:)';
a = 300;
yy3= e.^(-a*tt);
yy3 =yy3 - [zeros(1, NN/N), e.^(-a*tt)](1:NN);

$$\tau \ = \ RC$$

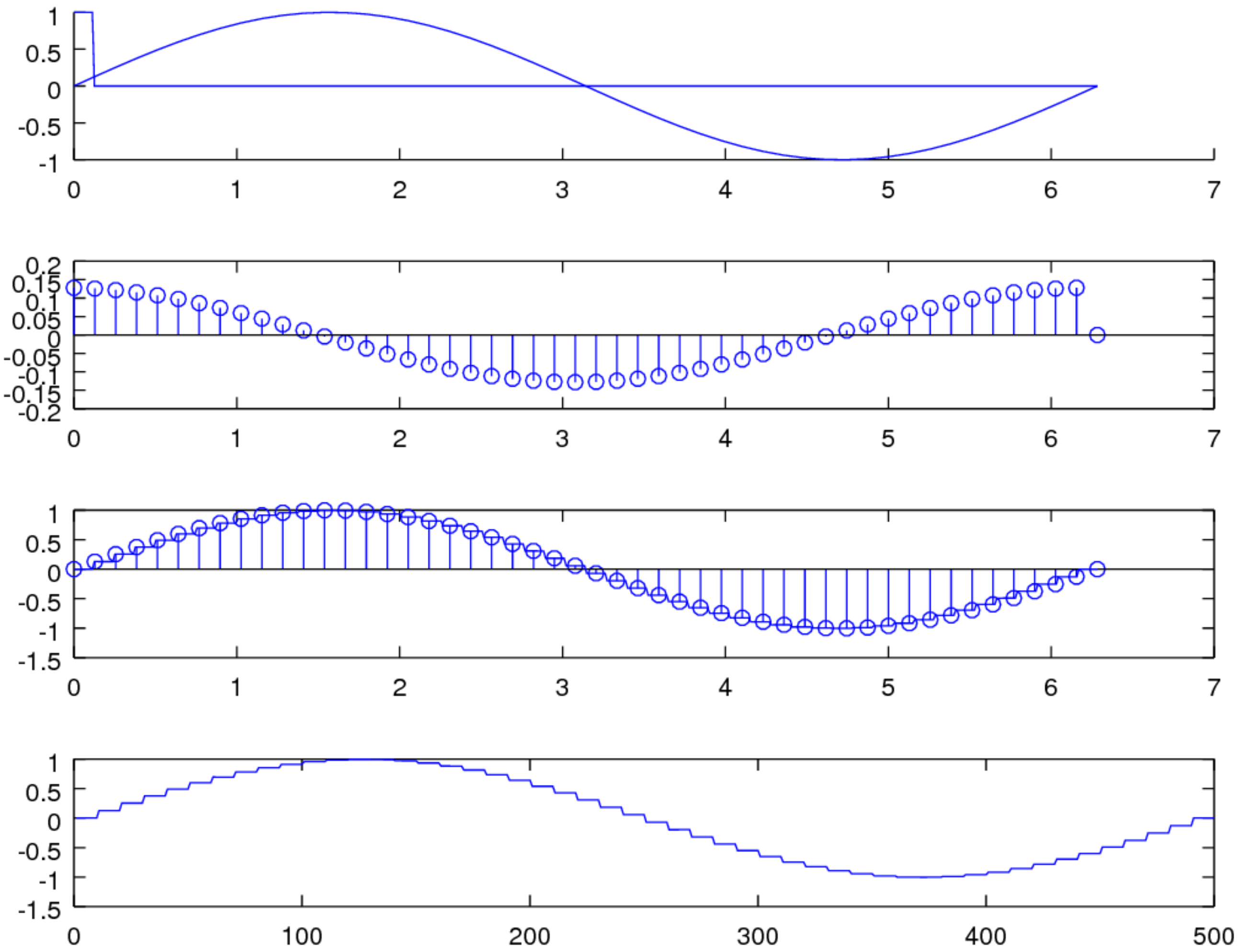
$$e^{-\frac{t}{\tau}} \ = \ e^{-\frac{t}{RC}}$$

$small \ \tau$

$small \ C$

$$large \ \frac{1}{\omega C}$$

# Large Time Constant




```
yy = [y1;
zeros(NN/N-1, N)];
yy2= yy(:)';
a = 1/300;
yy3= e.^(-a*tt);
yy3 =yy3 -
[zeros(1, NN/N),
e.^(-a*tt)](1:NN);
```

$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

*large* $\tau$

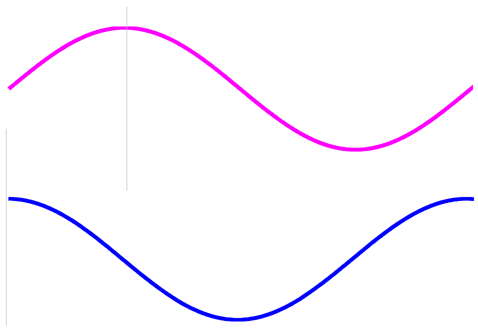
*large* $C$

*small* $\frac{1}{\omega C}$

# Envelope of the samples

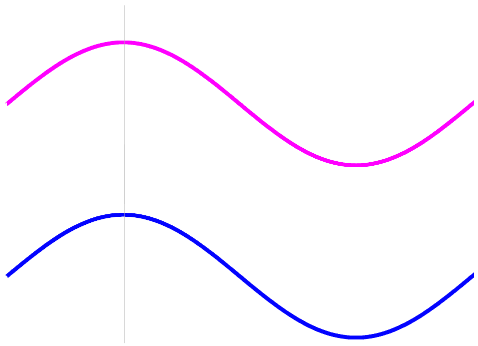
$v_C(t)$

$i_C(t)$



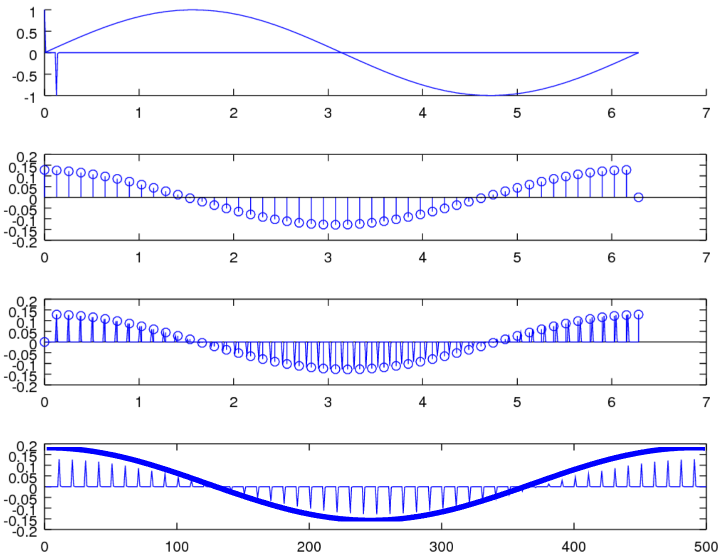
$v_C(t)$

$i_C(t)$



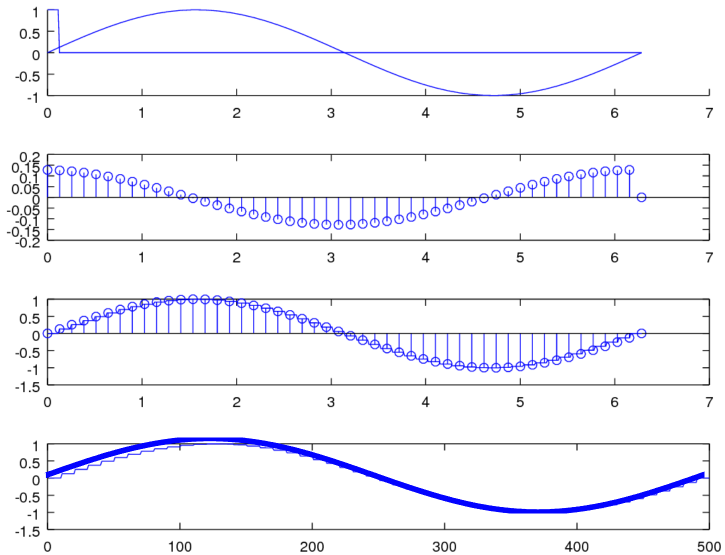
$v_C(t)$

$i_C(t)$

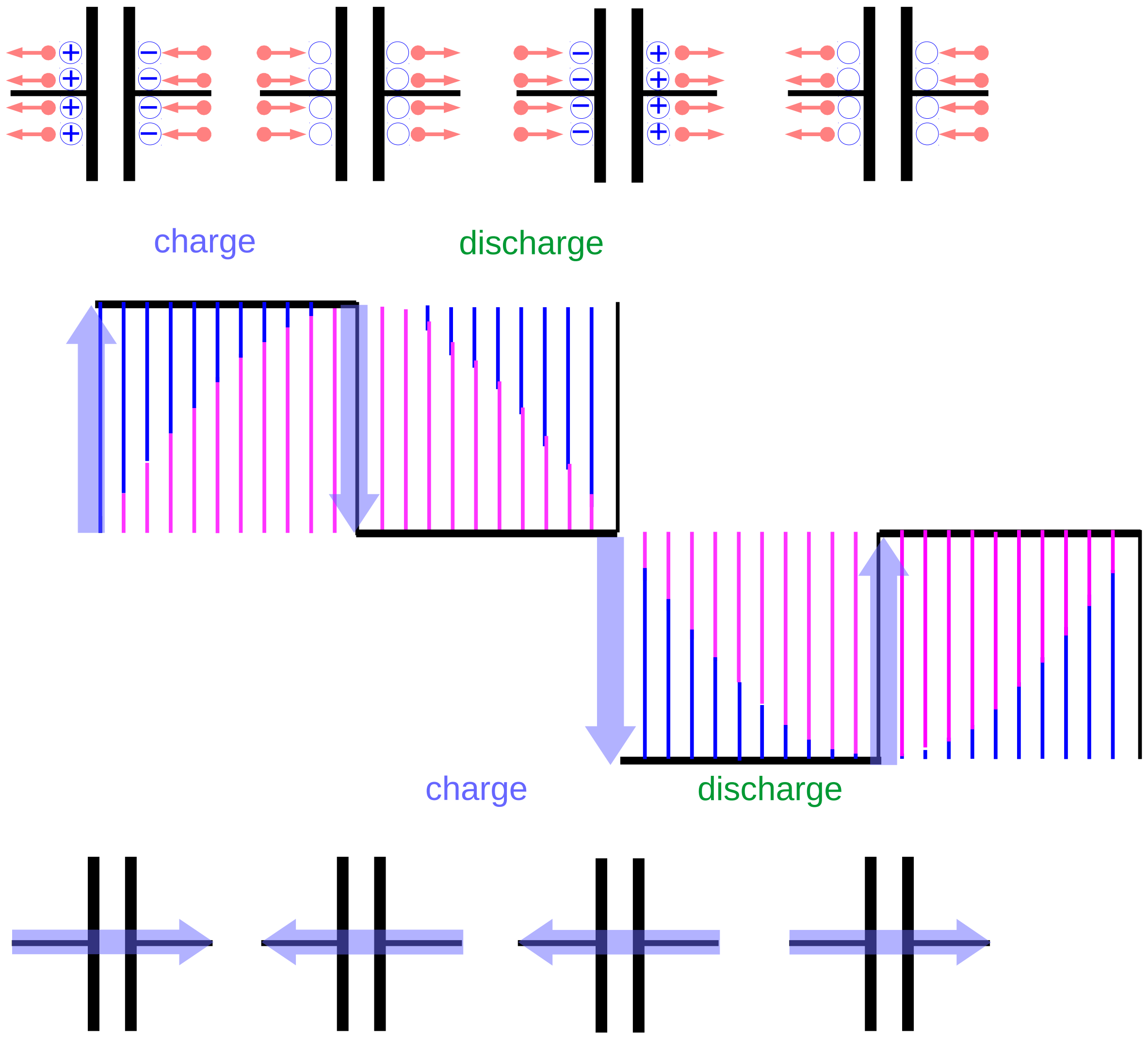



$v_C(t)$

$i_C(t)$

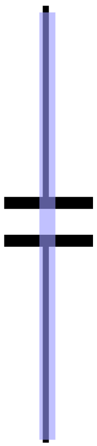


# Evercharging signal pairs



# I leads V by 90°

***Initial charge***

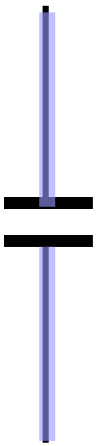


***SHORT***

***V = 0***

***I : peak***

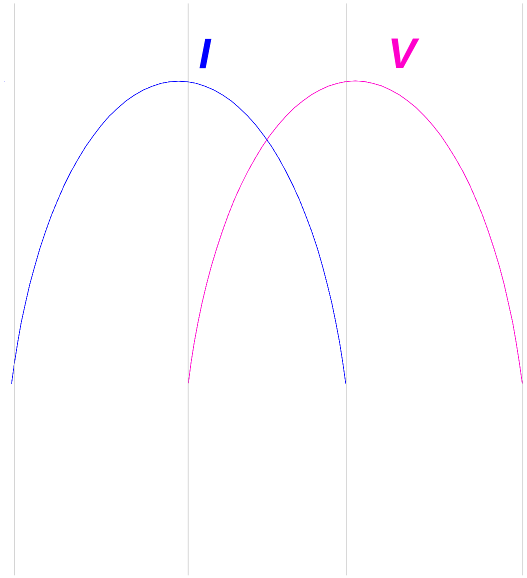
***Full charge***



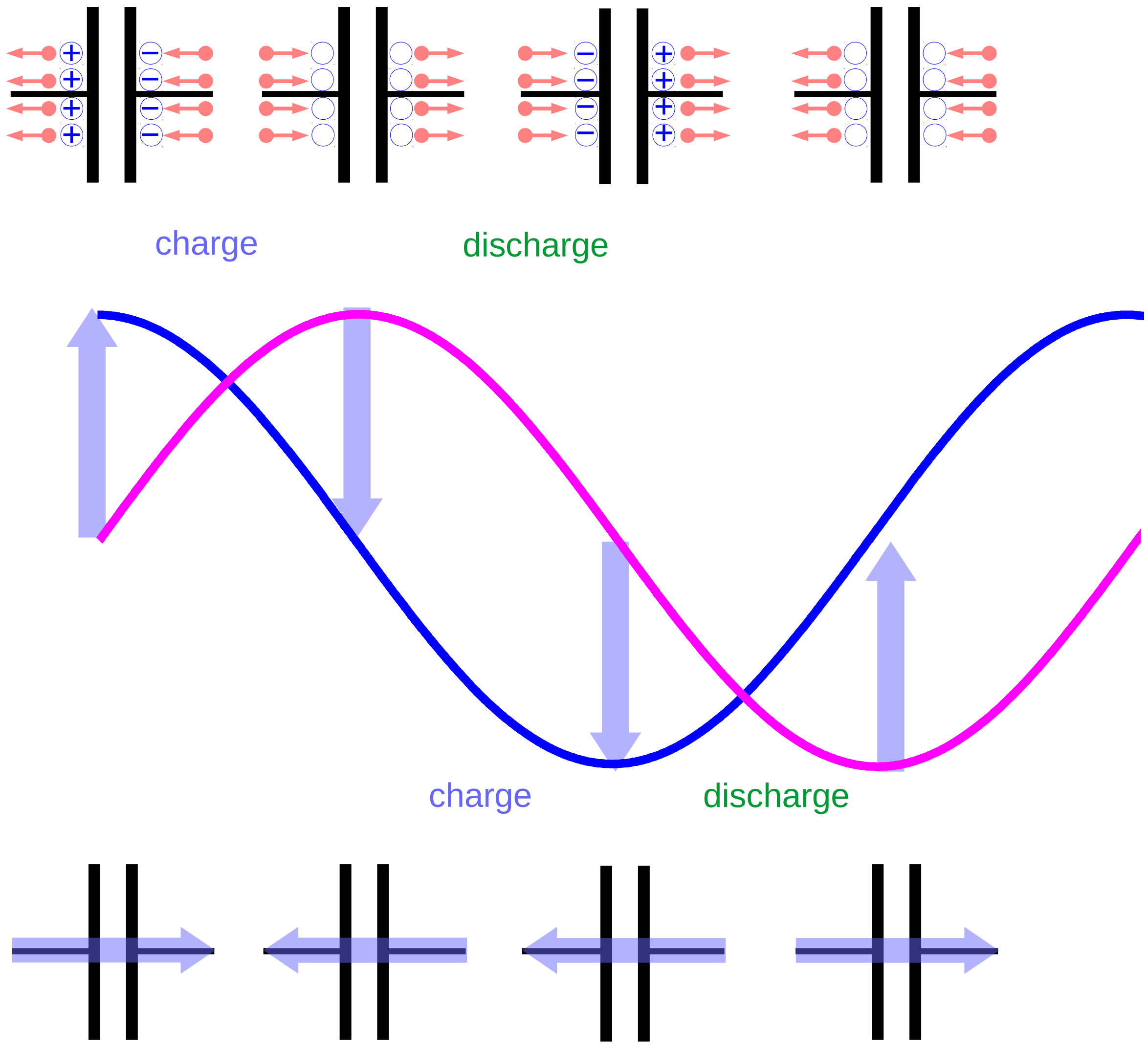
***OPEN***

***I = 0***

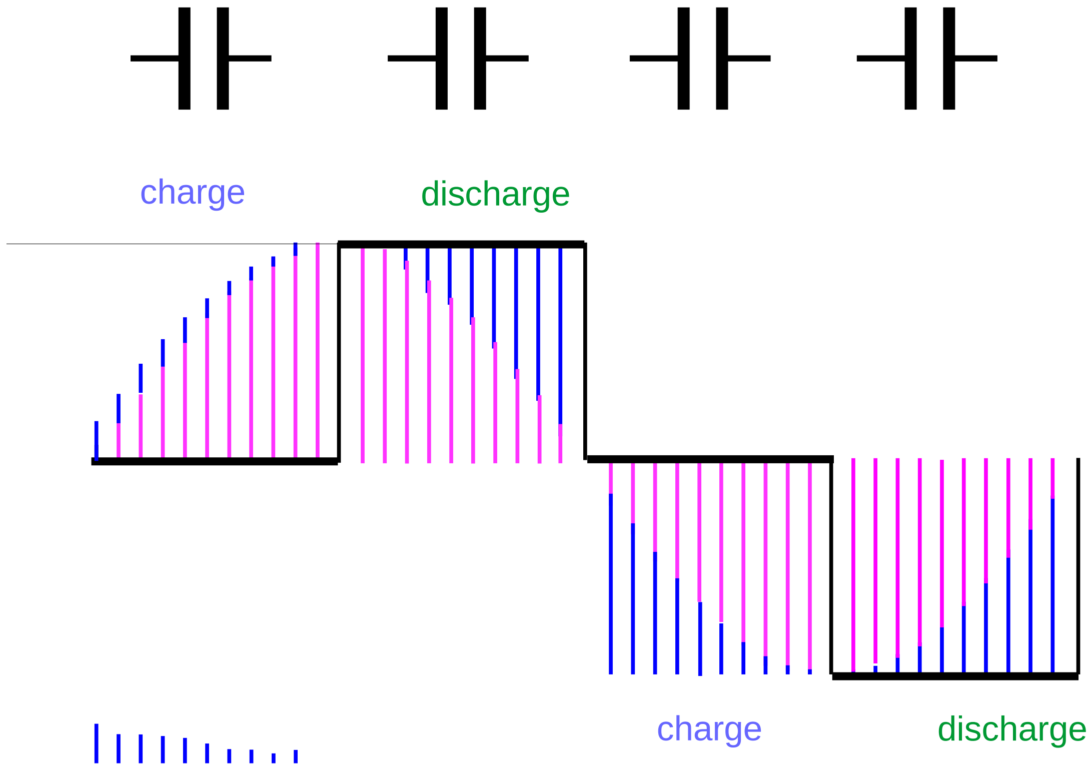
***V : peak***



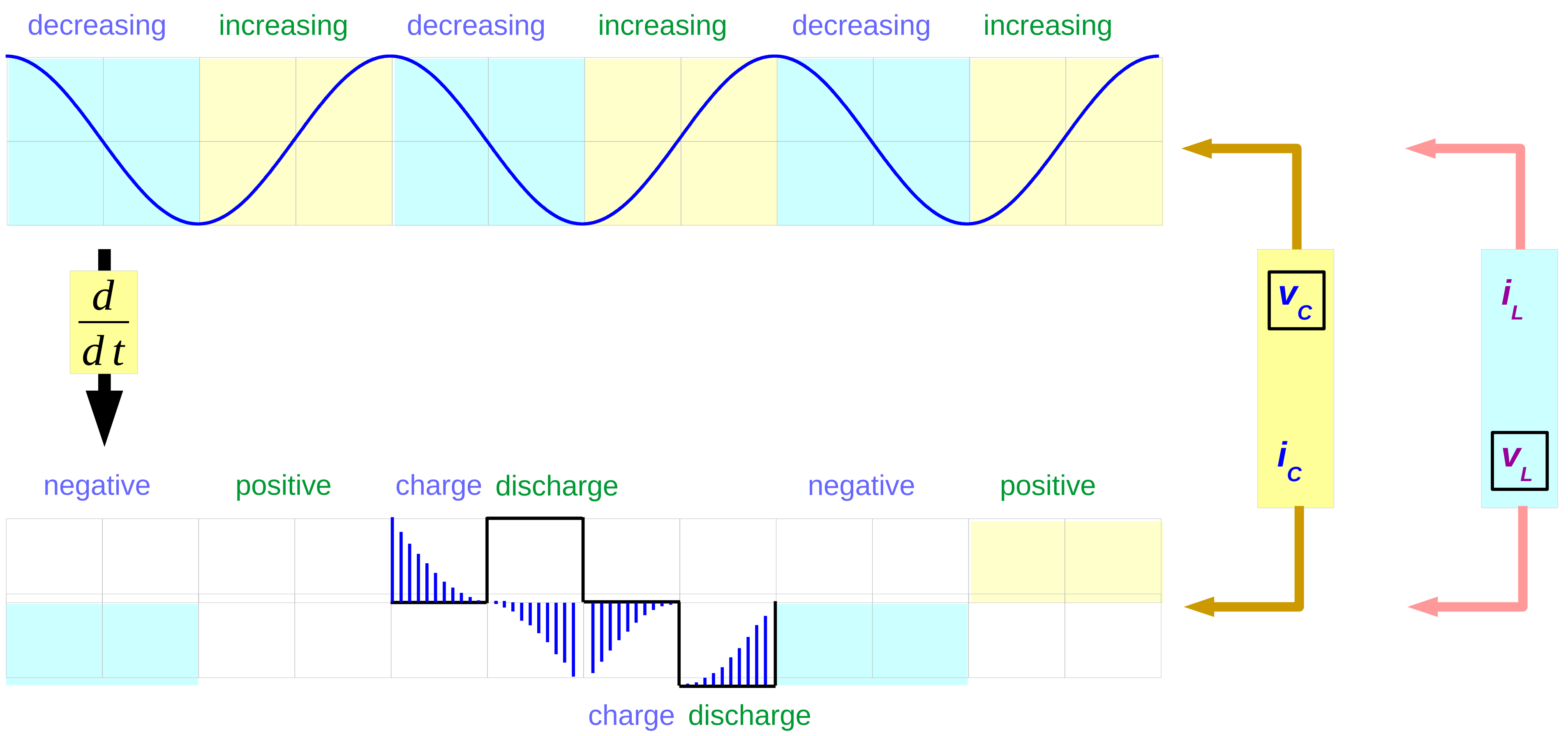
# Evercharging signal pairs



# Evercharging signal pairs



# Evercharging signal pairs



## References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003