Laurent Series and z-Transform Examples case 0.A

20171101

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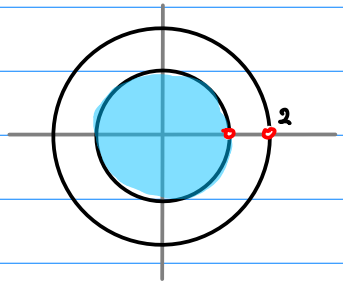
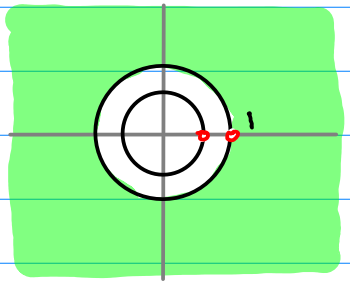
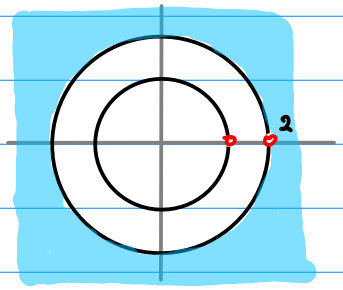
1. A

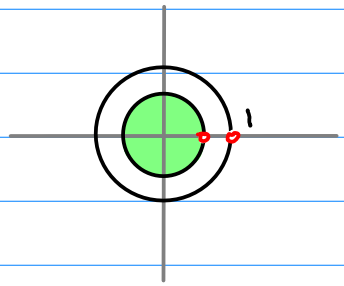
$$f(z) = \frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} X(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$



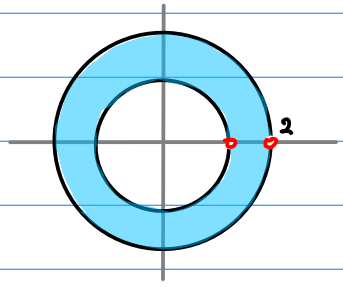


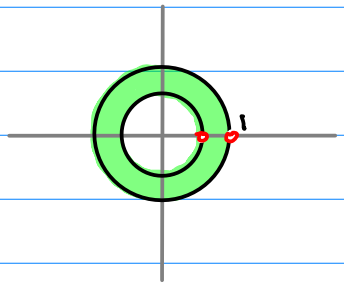
$$\sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1\right] z^n \quad = \quad \sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1\right] z^n$$





$$\sum_{n=-1}^{-\infty} \left(1 - \left(\tfrac{1}{2}\right)^{n+1}\right) z^n \quad = \quad \sum_{n=-1}^{-\infty} \left(1 - \left(\tfrac{1}{2}\right)^{n+1}\right) z^n$$

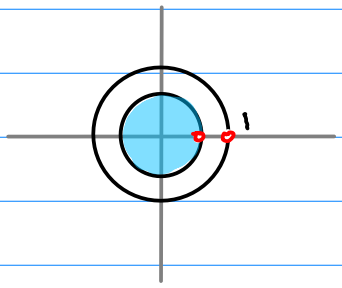




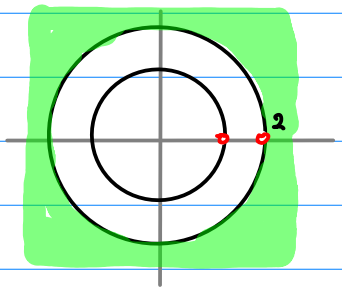
$$\sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} \cdot z^n \quad = \quad \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} \cdot z^n$$

2.A

$$f(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} X(z) = \frac{-1}{(z-1)(z-2)}$$



$$\sum_{n=1}^{\infty} \left[1 - 2^{n-1}\right] z^n$$



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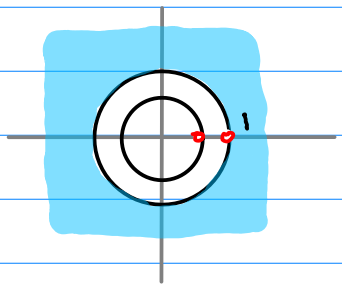
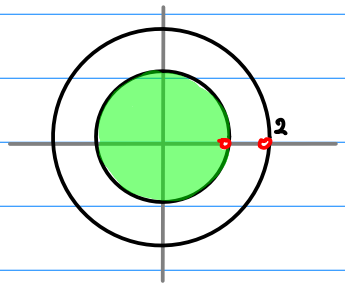


$$\sum_{n=0}^{-\infty} \left[2^{n-1} - 1\right] z^n$$



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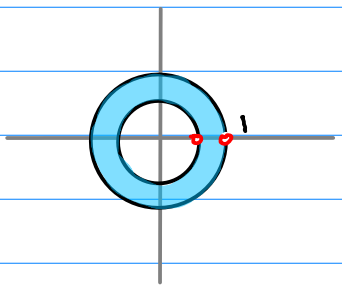
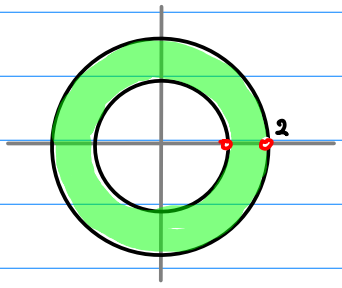


$$\sum_{n=1}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$



$$\sum_{n=1}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

3. A

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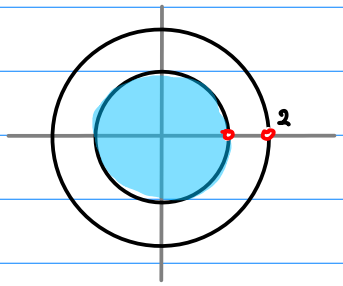
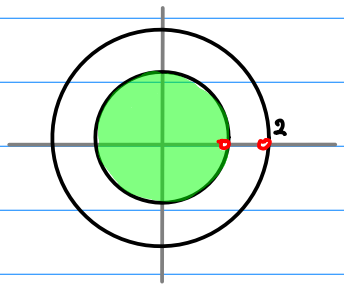
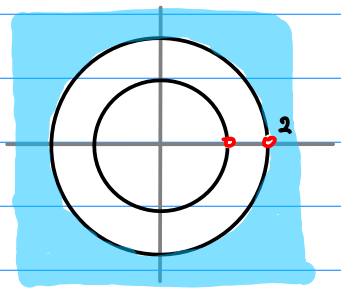


$$\sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1 \right] z^n$$

$\xrightarrow{-n}$

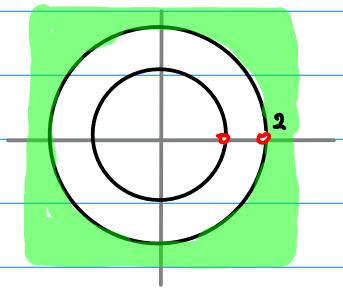


$$\sum_{n=0}^{-\infty} \left[2^{n-1} - 1 \right] z^{-n}$$

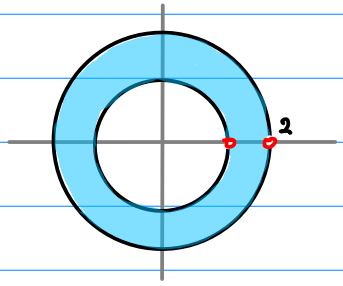


$$\sum_{n=-1}^{-\infty} \left[1 - \left(\tfrac{1}{2}\right)^{n+1} \right] z^n$$

$\xrightarrow{-n}$

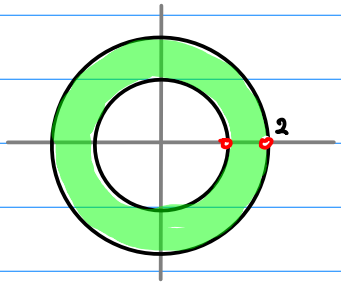


$$\sum_{n=1}^{\infty} \left[1 - 2^{n-1} \right] z^{-n}$$



$$\sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^n$$

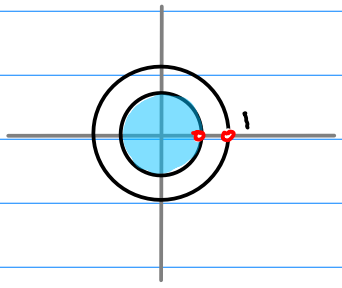
$\xrightarrow{-n}$



$$+ \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

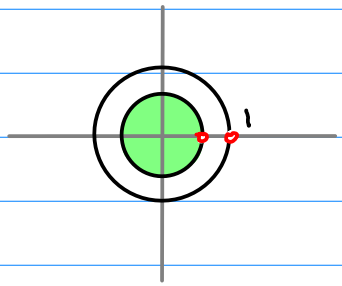
4.A

$$f(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)} \quad = \quad X(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$

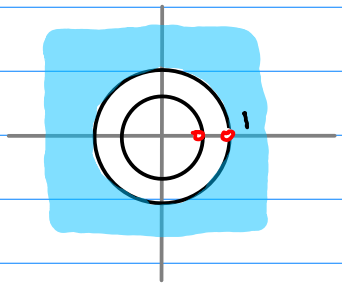


$$\sum_{n=1}^{\infty} \left[1 - 2^{n-1}\right] z^{n}$$

$-n$

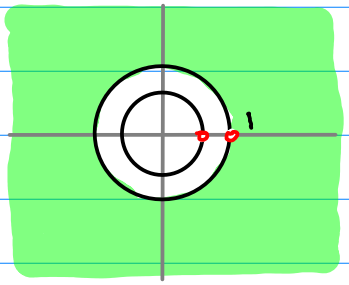


$$\sum_{n=-1}^{\infty} \left[1 - \left(\tfrac{1}{2}\right)^{n+1}\right] z^{-n}$$

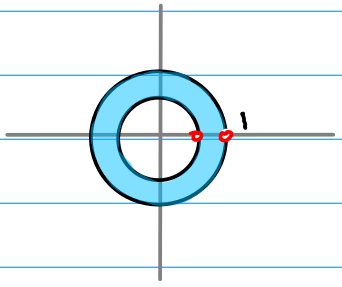


$$\sum_{n=-1}^{-\infty} \left[2^{n-1} - 1\right] z^{n}$$

$-n$

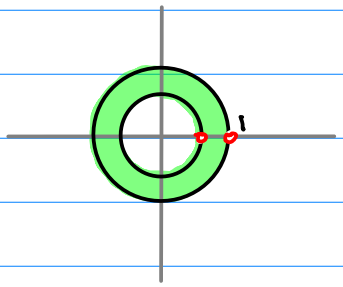


$$\sum_{n=1}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1\right] z^{-n}$$



$$+\sum_{n=1}^{\infty} z^{n} + \sum_{n=0}^{-\infty} 2^{n-1} z^{n}$$

$-n$

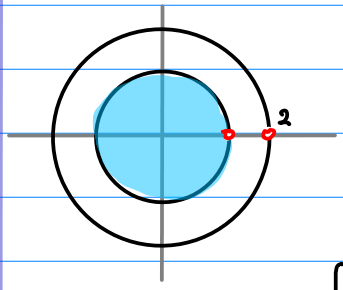


$$+\sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^{-n}$$

1. A

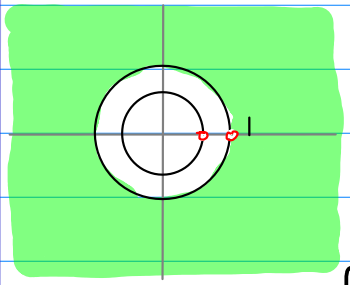
$$f(z)=\frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} X(z)=\frac{-0.5\,z^2}{(z-1)(z-0.5)}$$

①



$$a_n=\begin{cases}\left(\frac{1}{2}\right)^{n+1}-1 & (n\geq 0)\\ 0 & (n<0)\end{cases}$$

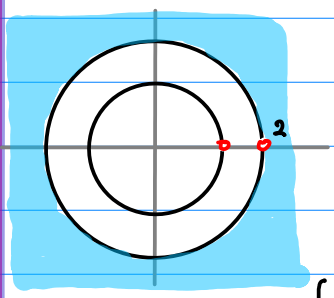
$$f(z)=\sum_{n=0}^{\infty}\left[\left(\frac{1}{2}\right)^{n+1}-1\right]z^n$$



$$x_n=\begin{cases}0 & (n>0)\\ \left(\frac{1}{2}\right)^{n+1}-1 & (n\leq 0)\end{cases}$$

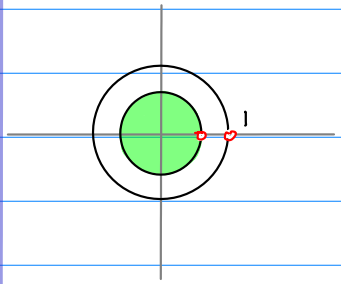
$$X(z)=\sum_{n=0}^{\infty}\left[\left(\frac{1}{2}\right)^{n+1}-1\right]z^n$$

②



$$a_n=\begin{cases}0 & (n\geq 0)\\ \left(1-\left(\frac{1}{2}\right)^{n+1}\right) & (n<0)\end{cases}$$

$$f(z)=\sum_{n=-1}^{-\infty}\left(1-\left(\frac{1}{2}\right)^{n+1}\right)z^n$$



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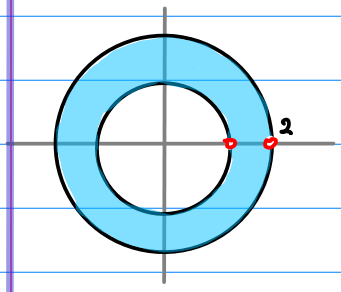
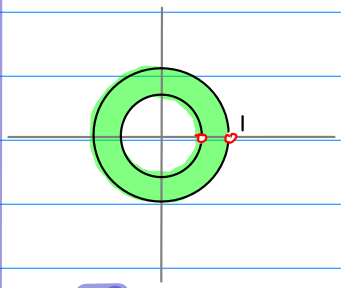


$$a_n=\begin{cases}\left(\frac{1}{2}\right)^{n+1} & (n\geq 0)\\ 1 & (n<0)\end{cases}$$

$$f(z)=\sum_{n=-1}^{-\infty}z^n+\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}\cdot z^n$$



$$x_n=\begin{cases}1 & (n>0)\\ \left(\frac{1}{2}\right)^{n+1} & (n\leq 0)\end{cases}$$

$$X(z)=\sum_{n=-1}^{-\infty}z^n+\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}\cdot z^n$$

2.A

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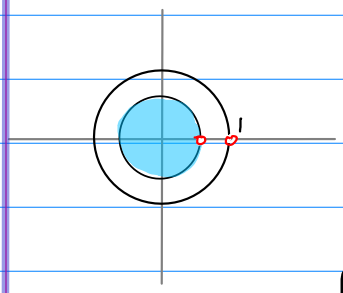
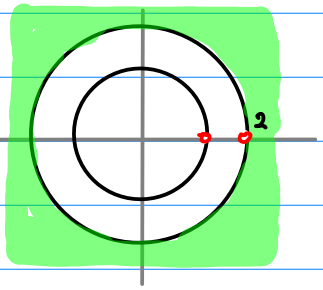


$$a_n = \begin{cases} [1-2^{n-1}] & (n>0) \\ 0 & (n\le 0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} [1-2^{n-1}]\, z^n$$



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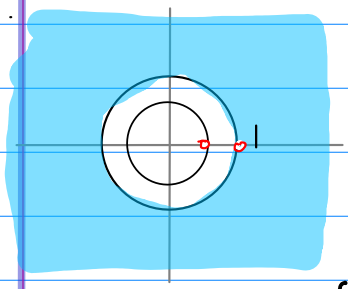

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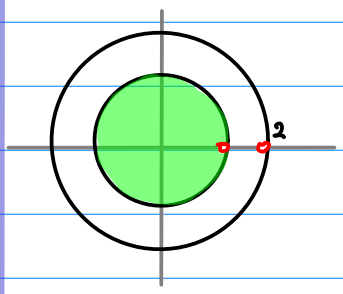

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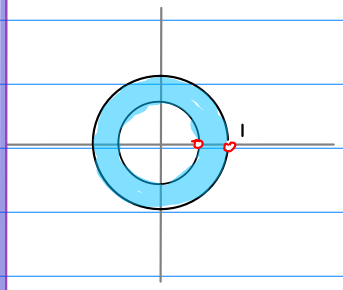

$$a_n = \begin{cases} 1 & (n>0) \\ 2^{n-1} & (n\le 0) \end{cases}$$

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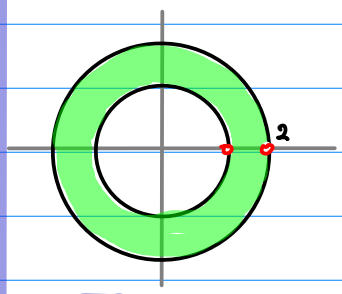

$$x_n = \begin{cases} 2^{n-1} & (n\ge 0) \\ 1 & (n<0) \end{cases}$$

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3.A

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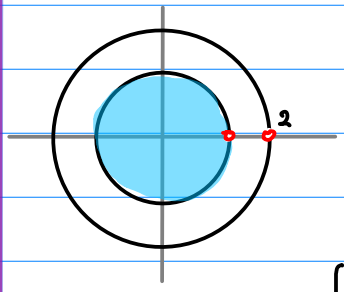

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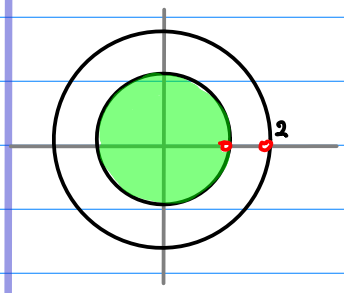

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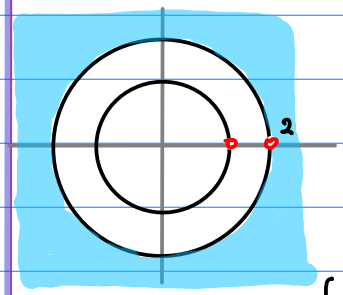
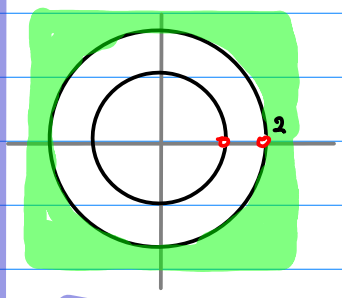


$$a_n=\begin{cases}0 & (n\geq 0)\\ 1-\left(\frac{1}{2}\right)^{n+1} & (n<0)\end{cases}$$

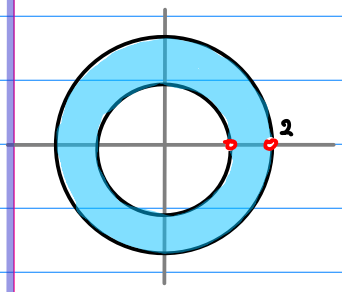
$$f(z)=\sum_{n=-1}^{-\infty}\left[1-\left(\frac{1}{2}\right)^{n+1}\right]z^n$$



$$x_n=\begin{cases}1-2^{n-1} & (n>0)\\ 0 & (n\leq 0)\end{cases}$$

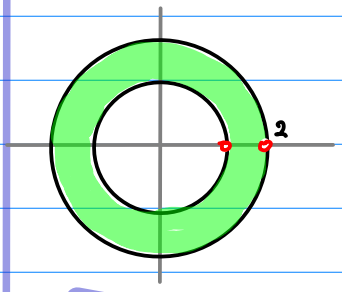
$$X(z)=\sum_{n=1}^{\infty}\left[1-2^{n-1}\right]z^{-n}$$

③



$$a_n=\begin{cases}\left(\frac{1}{2}\right)^{n+1} & (n\geq 0)\\ 1 & (n<0)\end{cases}$$

$$f(z)=\sum_{n=-1}^{-\infty}z^n+\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}z^n$$



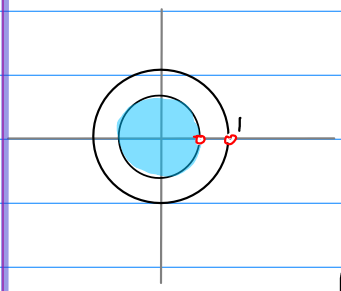
$$x_n=\begin{cases}1 & (n>0)\\ 2^{n-1} & (n\leq 0)\end{cases}$$

$$X(z)=+\sum_{n=1}^{\infty}z^{-n}+\sum_{n=0}^{\infty}2^{n-1}z^{-n}$$

4.A

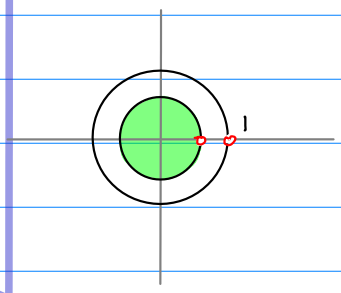
$$f(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)} \quad = \quad X(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$

(I)



$$a_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \le 0) \end{cases}$$

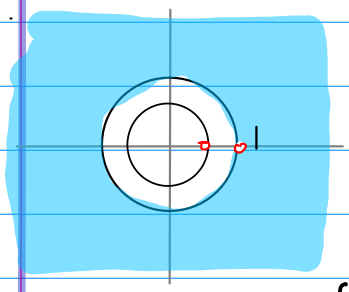
$$f(z) = \sum_{n=1}^{\infty} \left[1 - 2^{n-1} \right] z^n$$



$$x_n = \begin{cases} 0 & (n \ge 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

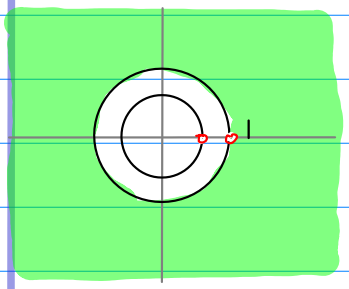
$$X(z) = \sum_{n=-1}^{\infty} \left[1 - \left(\frac{1}{2}\right)^{n+1} \right] z^{-n}$$

(II)



$$a_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \le 0) \end{cases}$$

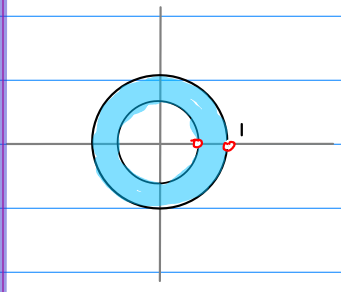
$$f(z) = \sum_{n=-1}^{-\infty} \left[2^{n-1} - 1 \right] z^n$$



$$x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \ge 0) \\ 0 & (n < 0) \end{cases}$$

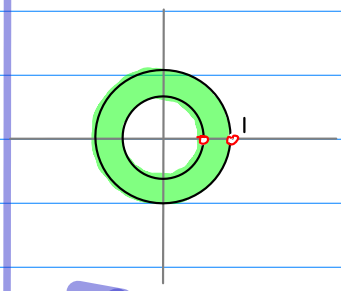
$$X(z) = \sum_{n=1}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^{-n}$$

(III)



$$a_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \le 0) \end{cases}$$

$$f(z) = +\sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1} z^n$$



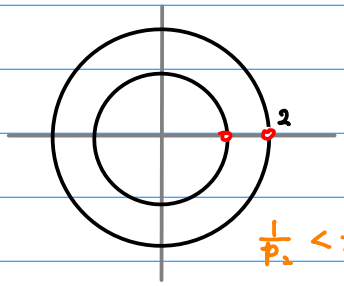
$$x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \ge 0) \\ 1 & (n < 0) \end{cases}$$

$$X(z) = +\sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n}$$

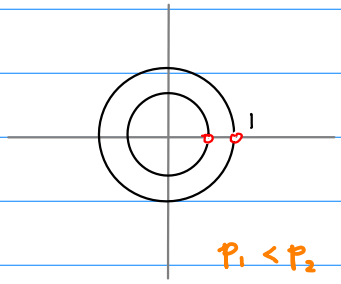
$$f(z) \xrightarrow{z^{-1}} X(z)$$

1.A

$$\frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$



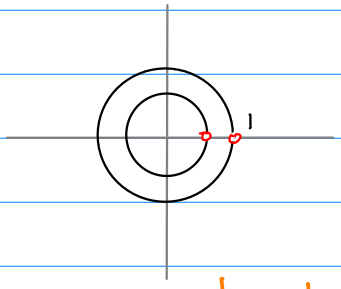
$\frac{1}{p_2} < \frac{1}{p_1}$



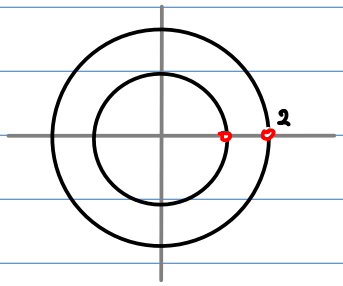
$p_1 < p_2$

1.B

$$\frac{-0.5\,z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} \frac{-1}{(z-1)(z-2)}$$



$\frac{1}{p_2} < \frac{1}{p_1}$

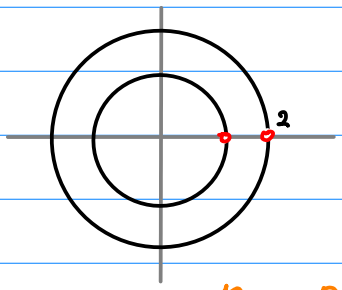


$p_1 < p_2$

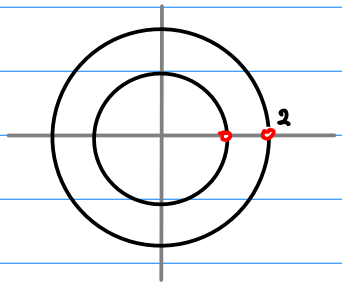
$$f(z) = X(z)$$

3.A

$$\frac{-1}{(z-1)(z-2)} = \frac{-1}{(z-1)(z-2)}$$



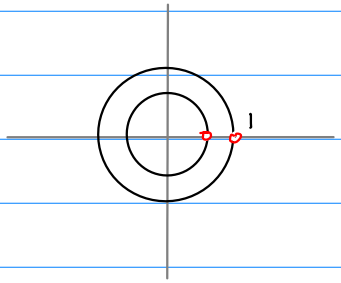
$p_1 < p_2$



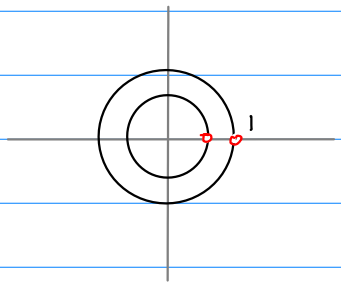
$p_1 < p_2$

4. B

$$\frac{-0.5\,z^2}{(z-1)(z-0.5)} = \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$



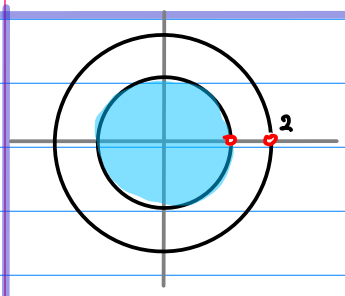
$p_1 < p_2$



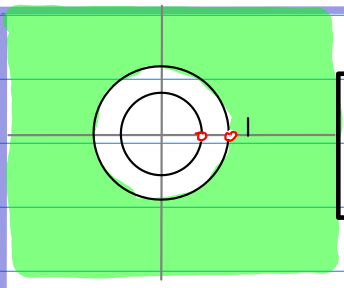
$p_1 < p_2$

$f(z) \xrightarrow{Z^{-1}} X(z)$

I

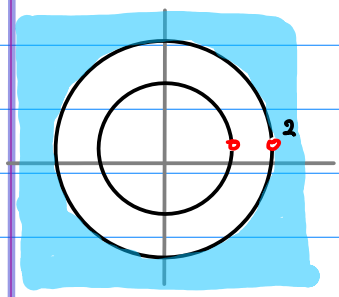


$$\left(\frac{1}{2}\right)^{n+1} - 1 \quad (n \geq 0)$$
$$0 \quad (n < 0)$$

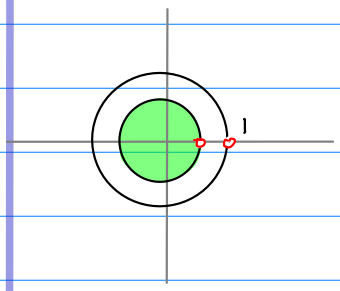


$$0 \quad (n > 0)$$
$$\left(\frac{1}{2}\right)^{n+1} - 1 \quad (n \leq 0)$$

II

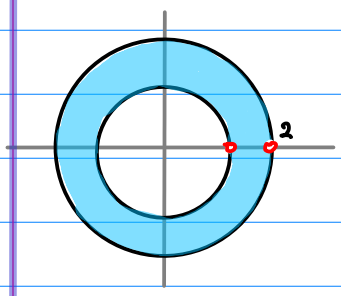


$$0 \quad (n \geq 0)$$
$$1 - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

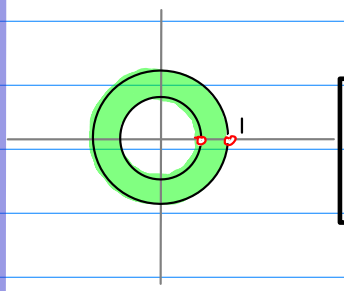


$$1 - \left(\frac{1}{2}\right)^{n+1} \quad (n > 0)$$
$$0 \quad (n \leq 0)$$

III

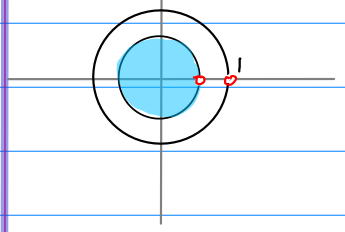


$$\left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$
$$1 \quad (n < 0)$$

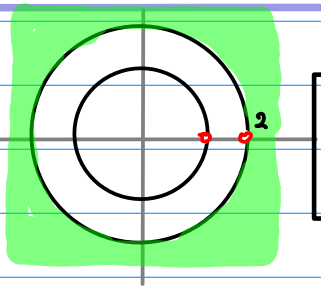


$$1 \quad (n > 0)$$
$$\left(\frac{1}{2}\right)^{n+1} \quad (n \leq 0)$$

I

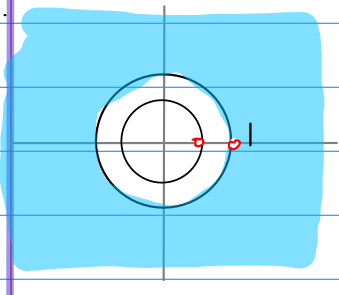


$$1 - 2^{n-1} \quad (n > 0)$$
$$0 \quad (n \leq 0)$$

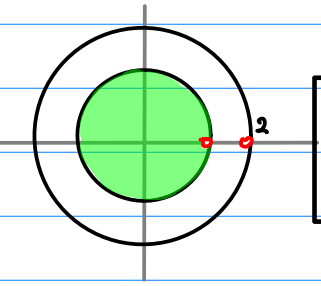


$$0 \quad (n \geq 0)$$
$$1 - 2^{n-1} \quad (n < 0)$$

II

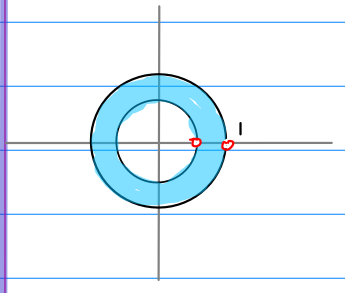


$$0 \quad (n > 0)$$
$$2^{n-1} - 1 \quad (n \leq 0)$$

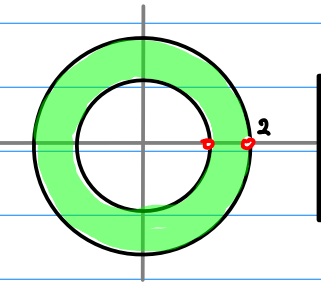


$$2^{n-1} - 1 \quad (n \geq 0)$$
$$0 \quad (n < 0)$$

III



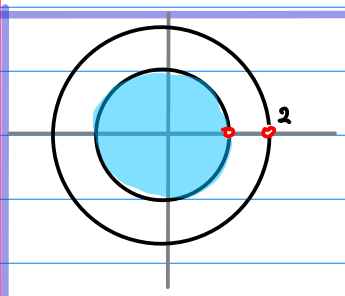
$$1 \quad (n > 0)$$
$$2^{n-1} \quad (n \leq 0)$$



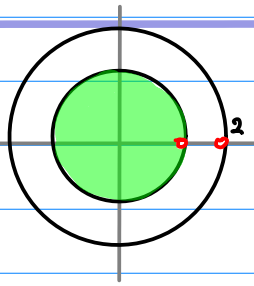
$$2^{n-1} \quad (n \geq 0)$$
$$1 \quad (n < 0)$$

$f(z) \rightleftharpoons X(z)$

I

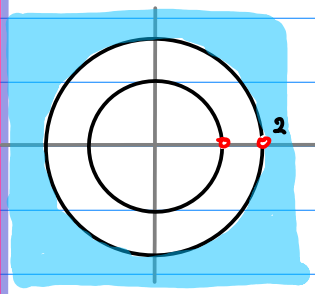


$$\left(\tfrac{1}{2}\right)^{n+1} - 1 \quad (n \geq 0)$$
$$0 \quad (n < 0)$$

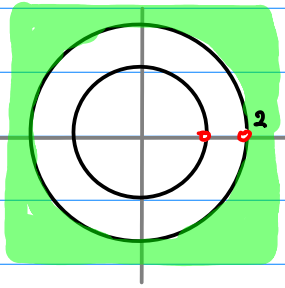


$$0 \quad (n > 0)$$
$$2^{n-1} - 1 \quad (n \leq 0)$$

II

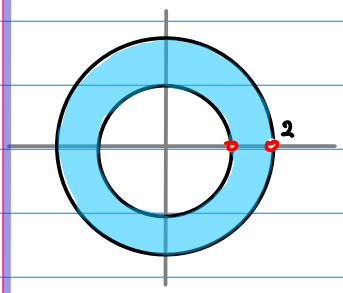


$$0 \quad (n \geq 0)$$
$$1 - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)$$

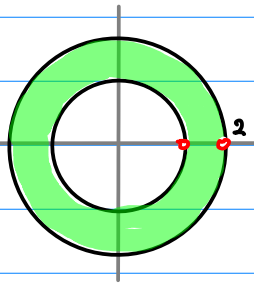


$$1 - 2^{n-1} \quad (n > 0)$$
$$0 \quad (n \leq 0)$$

III

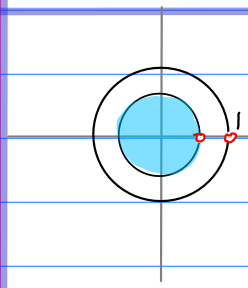


$$\left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)$$
$$1 \quad (n < 0)$$

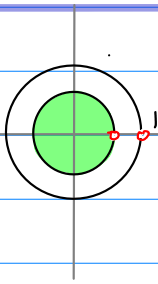


$$1 \quad (n > 0)$$
$$2^{n-1} \quad (n \leq 0)$$

I

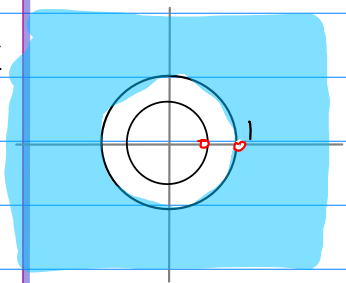


$$1 - 2^{n-1} \quad (n > 0)$$
$$0 \quad (n \leq 0)$$

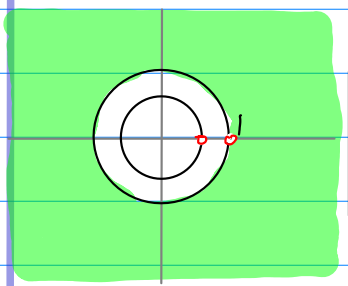


$$0 \quad (n \geq 0)$$
$$1 - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)$$

II

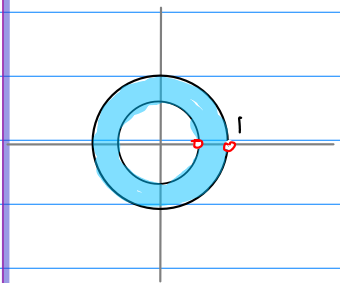


$$0 \quad (n > 0)$$
$$2^{n-1} - 1 \quad (n \leq 0)$$

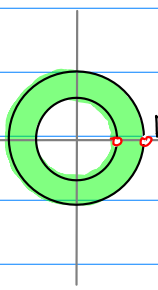


$$\left(\tfrac{1}{2}\right)^{n+1} - 1 \quad (n \geq 0)$$
$$0 \quad (n < 0)$$

III



$$1 \quad (n > 0)$$
$$2^{n-1} \quad (n \leq 0)$$



$$\left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)$$
$$1 \quad (n < 0)$$

$f(z) \xrightarrow{z^{-1}} X(z)$

Ⅰ

$\left(\frac{1}{2}\right)^{n+1} - 1 \quad (n \geq 0)$

$0 \quad (n < 0)$

$\times -1$

$0 \quad (n > 0)$

$\left(\frac{1}{2}\right)^{n+1} - 1 \quad (n \leq 0)$

$\times -1$

Ⅱ

$0 \quad (n \geq 0)$

$1 - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$

$1 - \left(\frac{1}{2}\right)^{n+1} \quad (n > 0)$

$0 \quad (n \leq 0)$

Ⅲ

$\left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$

$1 \quad (n < 0)$

$1 \quad (n > 0)$

$\left(\frac{1}{2}\right)^{n+1} \quad (n \leq 0)$

Ⅰ

$1 - 2^{n-1} \quad (n > 0)$

$0 \quad (n \leq 0)$

$\times -1$

$0 \quad (n \geq 0)$

$1 - 2^{n-1} \quad (n < 0)$

$\times -1$

Ⅱ

$0 \quad (n > 0)$

$2^{n-1} - 1 \quad (n \leq 0)$

$2^{n-1} - 1 \quad (n \geq 0)$

$0 \quad (n < 0)$

Ⅲ

$1 \quad (n > 0)$

$2^{n-1} \quad (n \leq 0)$

$2^{n-1} \quad (n \geq 0)$

$1 \quad (n < 0)$

$f(z) = X(z)$

I

$\left(\frac{1}{2}\right)^{n+1} - 1 \quad (n \geq 0)$

$0 \quad (n < 0)$

$\times -1$

$0 \quad (n > 0)$

$2^{n-1} - 1 \quad (n \leq 0)$

II

$0 \quad (n \geq 0)$

$1 - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$

$\times -1$

$1 - 2^{n-1} \quad (n > 0)$

$0 \quad (n \leq 0)$

III

$\left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$

$1 \quad (n < 0)$

$1 \quad (n > 0)$

$2^{n-1} \quad (n \leq 0)$

I

$1 - 2^{n-1} \quad (n > 0)$

$0 \quad (n \leq 0)$

$\times -1$

$0 \quad (n \geq 0)$

$1 - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$

II

$0 \quad (n > 0)$

$2^{n-1} - 1 \quad (n \leq 0)$

$\times -1$

$\left(\frac{1}{2}\right)^{n+1} - 1 \quad (n \geq 0)$

$0 \quad (n < 0)$

III

$1 \quad (n > 0)$

$2^{n-1} \quad (n \leq 0)$

$\left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$

$1 \quad (n < 0)$

$f(z)$ $\quad p_1 < p_2$

$\frac{1}{2}$ $\quad$ $\frac{1}{1}$

$|z| < p_1$

$\left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1}$ $\quad (n \geq 0)$

0 $\quad (n < 0)$

$p_1 = 1 \quad p_2 = 2$

$* -1$

$|z| > p_2$

0 $\quad (n \geq 0)$

$\left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1}$ $\quad (n < 0)$

$p_1 = 1 \quad p_2 = 2$

$p_1 < |z| < p_2$

$\left(\frac{1}{p_2}\right)^{n+1}$ $\quad (n \geq 0)$

$\left(\frac{1}{p_1}\right)^{n+1}$ $\quad (n < 0)$

$p_1 = 1 \quad p_2 = 2$

$X(z)$ $\quad p_1 < p_2$

$|z| < p_1$

$$0$$

$$(p_1)^{n+1} - (p_2)^{n+1}$$

$(n > 0)$

$(n \leq 0)$

$p_1 = 1/2 \quad p_2 = 1$

$*-1$

$|z| > p_2$

$$(p_2)^{n+1} - (p_1)^{n+1}$$

$$0$$

$n > 0)$

$n \leq 0)$

$p_1 = 1/2 \quad p_2 = 1$

$p_1 < |z| < p_2$

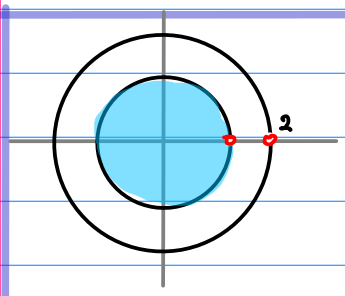
$$(p_2)^{n+1}$$

$$(p_1)^{n+1}$$

$(n > 0)$

$(n \leq 0)$

I

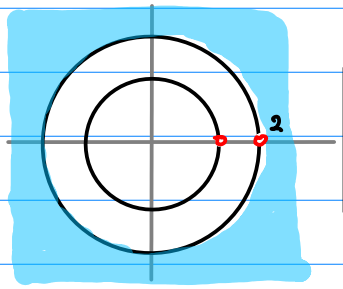


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$\begin{cases} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} \\ 0 \end{cases}$$

$p_1 = 1 \quad p_2 = 2$

II

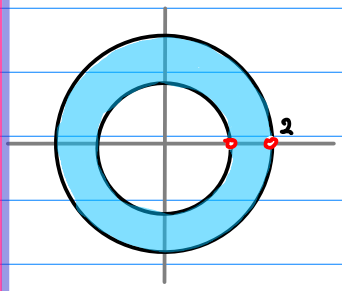


$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

$$\begin{cases} 0 \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} \end{cases}$$

$p_1 = 1 \quad p_2 = 2$

III

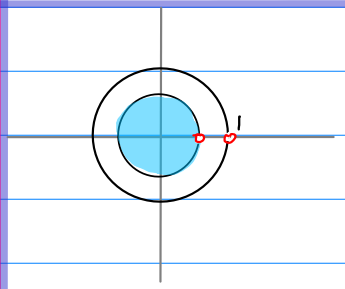


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$\begin{cases} \left(\frac{1}{p_2}\right)^{n+1} \\ \left(\frac{1}{p_1}\right)^{n+1} \end{cases}$$

$p_1 = 1 \quad p_2 = 2$

I

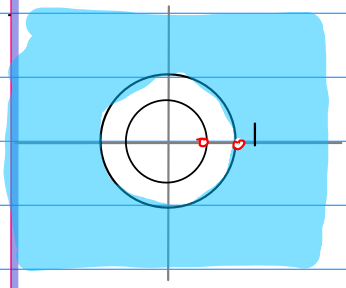


$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$\begin{cases} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} \\ 0 \end{cases}$$

$p_1 = \frac{1}{2} \quad p_2 = 1$

II

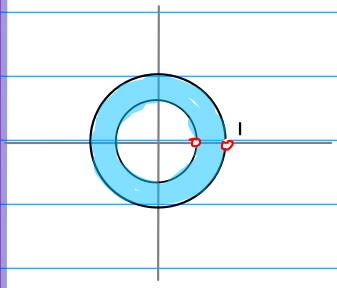


$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

$$\begin{cases} 0 \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} \end{cases}$$

$p_1 = \frac{1}{2} \quad p_2 = 1$

III



$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

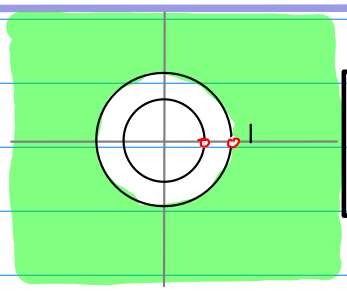
$$\begin{cases} \left(\frac{1}{p_2}\right)^{n+1} \\ \left(\frac{1}{p_1}\right)^{n+1} \end{cases}$$

$p_1 = \frac{1}{2} \quad p_2 = 1$

Ⅰ

$$\begin{matrix} 0 \\ (p_1)^{n+1} - (p_2)^{n+1} \end{matrix}$$

$p_1 = 1/2 \quad p_2 = 1$

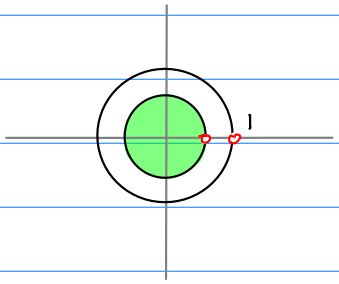


$$\begin{matrix} 0 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} - 1 & (n \leq 0) \end{matrix}$$

Ⅱ

$$\begin{matrix} (p_2)^{n+1} - (p_1)^{n+1} \\ 0 \end{matrix}$$

$p_1 = 1/2 \quad p_2 = 1$

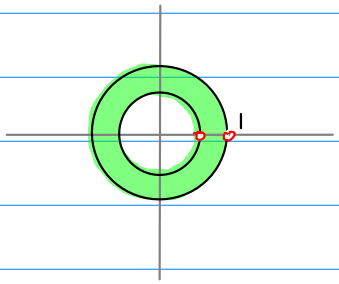


$$\begin{matrix} 1 - \left(\frac{1}{2}\right)^{n+1} & (n > 0) \\ 0 & (n \leq 0) \end{matrix}$$

Ⅲ

$$\begin{matrix} (p_2)^{n+1} \\ (p_1)^{n+1} \end{matrix}$$

$p_1 = 1/2 \quad p_2 = 1$

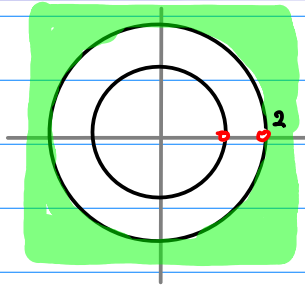


$$\begin{matrix} 1 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} & (n \leq 0) \end{matrix}$$

Ⅰ

$$\begin{matrix} 0 \\ (p_1)^{n+1} - (p_2)^{n+1} \end{matrix}$$

$p_1 = 1 \quad p_2 = 2$

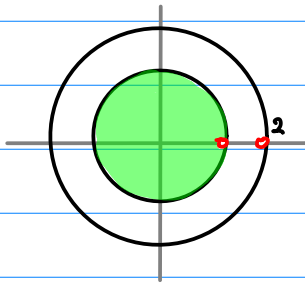


$$\begin{matrix} 0 & (n \geq 0) \\ 1 - 2^{n-1} & (n < 0) \end{matrix}$$

Ⅱ

$$\begin{matrix} (p_2)^{n+1} - (p_1)^{n+1} \\ 0 \end{matrix}$$

$p_1 = 1 \quad p_2 = 2$

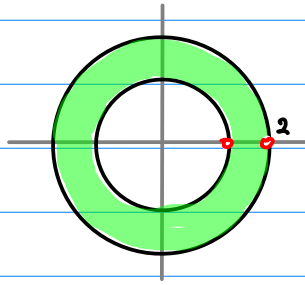


$$\begin{matrix} 2^{n-1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{matrix}$$

Ⅲ

$$\begin{matrix} (p_2)^{n+1} \\ (p_1)^{n+1} \end{matrix}$$

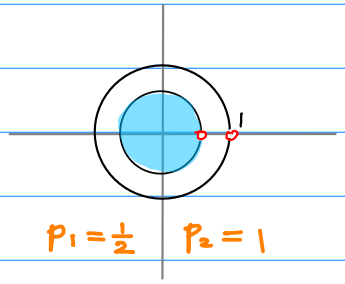
$p_1 = 1 \quad p_2 = 2$

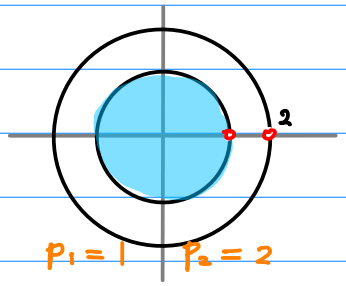


$$\begin{matrix} 2^{n-1} & (n \geq 0) \\ 1 & (n < 0) \end{matrix}$$

$\left(\frac{1}{\rho_2}\right)^{n+1} - \left(\frac{1}{\rho_1}\right)^{n+1}$

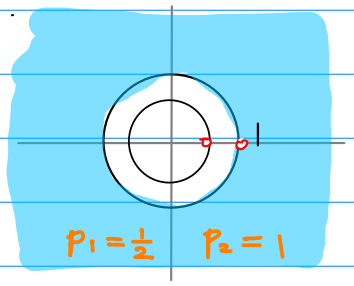
0

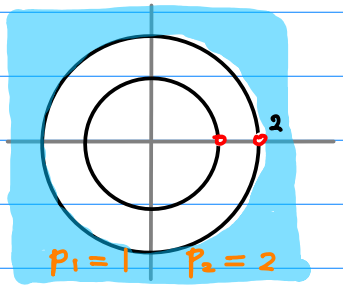




0

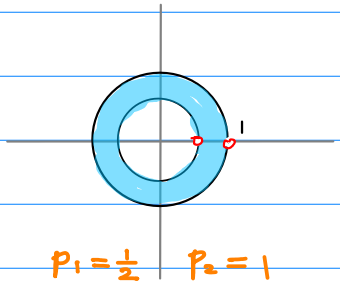
$\left(\frac{1}{\rho_1}\right)^{n+1} - \left(\frac{1}{\rho_2}\right)^{n+1}$

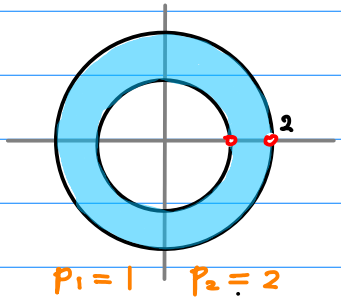




$\left(\frac{1}{\rho_2}\right)^{n+1}$

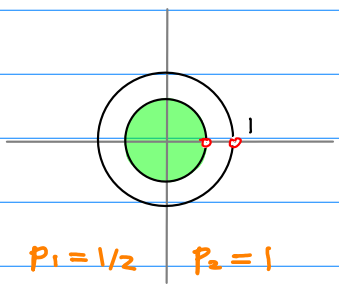
$\left(\frac{1}{\rho_1}\right)^{n+1}$

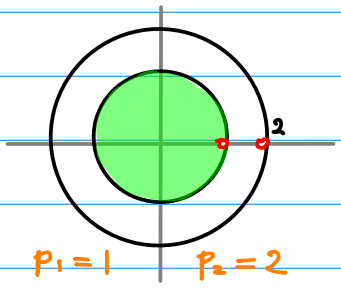




$\left(\rho_2\right)^{n+1} - \left(\rho_1\right)^{n+1}$

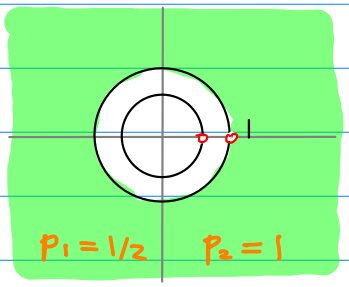
0

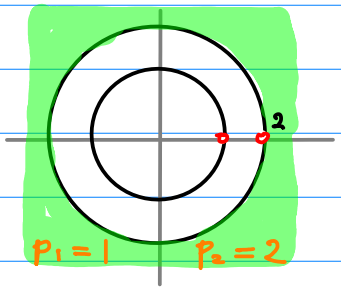


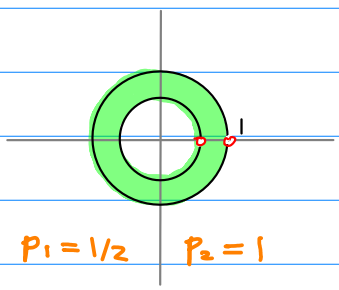
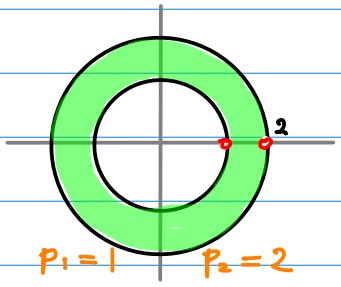


0

$\left(\rho_1\right)^{n+1} - \left(\rho_2\right)^{n+1}$





$\left(\rho_2\right)^{n+1}$

$\left(\rho_1\right)^{n+1}$