# CMOS Delay-7 (H.8) Delay Model

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# References

Some Figures from the following sites

[1] http://pages.hmc.edu/harris/cmosvlsi/4e/index.html
Weste & Harris Book Site

[2] en.wikipedia.org

$\beta$ : Device Transconductance Parameter
$k$ : Process Transconductance Parameter
$\mu$ : Electron / Hole Mobility

pMOS $\quad \beta_p = k'_p \left(\frac{W}{L}\right)_p \qquad k'_p = \mu_p C_{ox} \qquad C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$

nMOS $\quad \beta_n = k'_n \left(\frac{W}{L}\right)_n \qquad k'_n = \mu_n C_{ox} \qquad C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$

pMOS $\quad \beta_p = \mu_p \frac{\varepsilon_{ox}}{t_{ox}} \left(\frac{W}{L}\right)_p$

nMOS $\quad \beta_n = \mu_n \frac{\varepsilon_{ox}}{t_{ox}} \left(\frac{W}{L}\right)_n$

Saturation Current

$$I_{dp} = \frac{\beta_p}{2}\left(V_{GSn} - |V_{Tp}|\right)^2 \qquad V_{Tp} < 0$$

$$I_{dn} = \frac{\beta_n}{2}\left(V_{GSn} - V_{Tn}\right)^2 \qquad V_{Tn} > 0$$

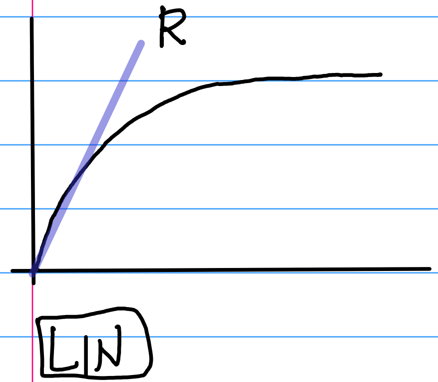
MODEL

$$\frac{\beta_n}{\beta_p} \simeq \frac{k'_n \left(\frac{W}{L}\right)_n}{k'_p \left(\frac{W}{L}\right)_p}$$

$$\frac{k'_n}{k'_p} = 2 \sim 3$$

$$\frac{k'_n}{k'_p} = \frac{\mu_n}{\mu_p} = r$$

$$\frac{\beta_n}{\beta_p} = \frac{k'_n \left(\frac{W}{L}\right)_n}{k'_p \left(\frac{W}{L}\right)_p}$$




$$R_n = \frac{1}{\beta_n (V_{DD} - V_{Tn})}$$

$$R_p = \frac{1}{\beta_n (V_{DD} - V_{Tn})}$$

fall time $t_f$ $\quad \tau_n = R_n C_{out}$

rise time $t_r$ $\quad \tau_p = R_p C_{out}$

$$C_{out} = C_{para} + C_L$$

MODEL

| | | |
|---|---|---|
| fall time | $t_f = 2.2\,\tau_n = \ln 9\,\tau_n$ | $0.9\,V_{DD} \rightarrow 0.1\,V_{DD}$ |
| rise time | $t_r = 2.2\,\tau_p = \ln 9\,\tau_p$ | $0.1\,V_{DD} \rightarrow 0.9\,V_{DD}$ |
| propagation delay time | $t_p = \frac{1}{2}(t_{pf} + t_{pr})$ <br> $= 0.35(t_{pf} + t_{pr})$ | $0.5\,V_{DD} \rightarrow 0.5\,V_{DD}$ |
| propagation fall time | $t_{pf} = 0.7\,\tau_n = \ln 2\,\tau_n$ | $V_{DD} \rightarrow 0.5\,V_{DD}$ |
| propagation rise time | $t_{pr} = 0.7\,\tau_p = \ln 2\,\tau_p$ | $0 \rightarrow 0.5\,V_{DD}$ |

$$\tau_n = R_n(C_{para} + C_L)$$

$$\tau_p = R_p(C_{para} + C_L)$$

$$C_{out} = C_{para} + C_L$$

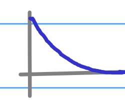
MODEL

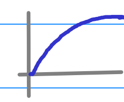
$$\left(\frac{W}{L}\right)_p = r\left(\frac{W}{L}\right)_n$$

$$r = \frac{\mu_n}{\mu_p} = \frac{k'_n}{k'_p} > 1$$

$$R_n = R_p = R = \frac{1}{\beta(V_{DD} - V_T)}$$

$$\begin{cases} V_{out}(t) = V_{DD}\left(1 - e^{-t/\tau}\right) \\ V_{out}(t) = V_{DD}\, e^{-t/\tau} \end{cases}$$





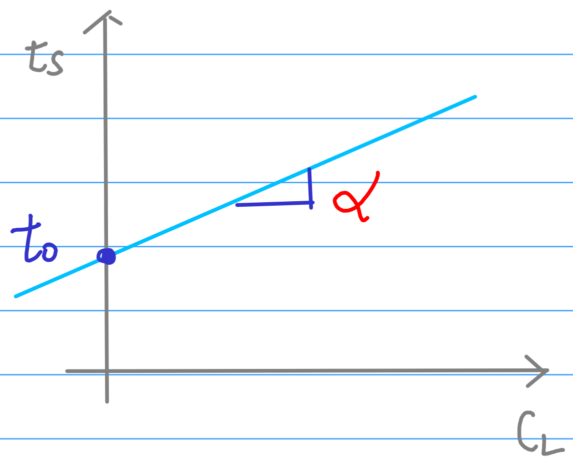
$$\tau = R\,C_{out} = R\,(C_{par} + C_L)$$

## Generic Switching Delay

$$t_s = t_0 + \alpha C_L \quad \Rightarrow \quad t_s = t_r = t_f$$

## Generic Switching Delay

$$t_s = t_0 + \alpha C_L$$




$t_0$ : zero delay

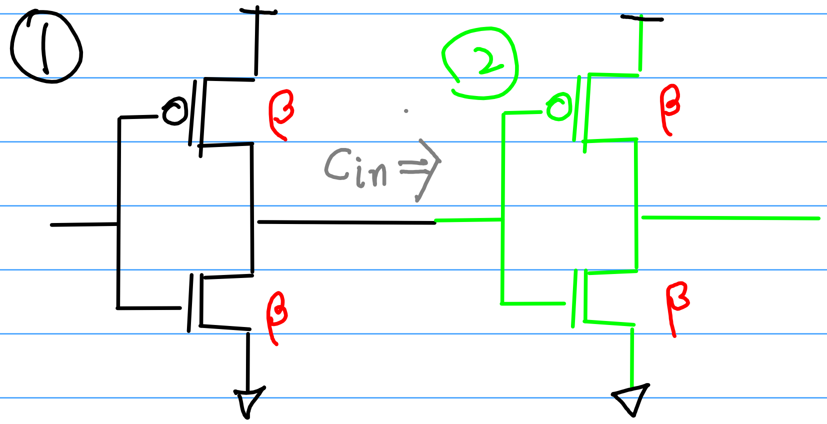
$\alpha$ : slope

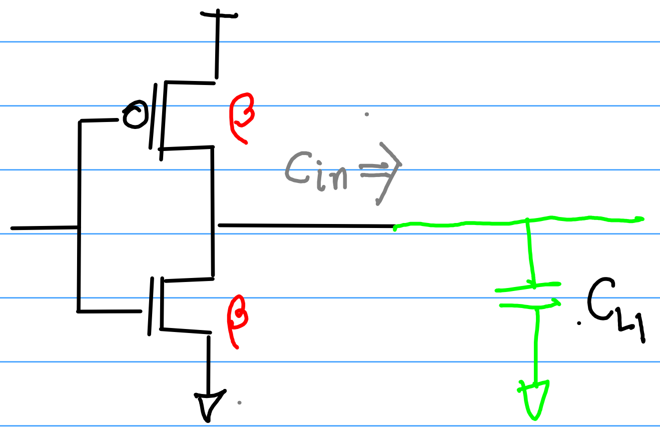
$$t \approx R\,C$$

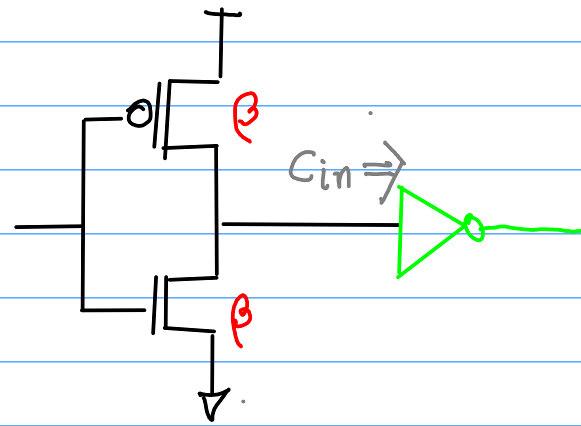
$$\alpha \propto \frac{1}{\beta(V_{DD} - V_T)} \quad (\to R)$$

$$C_L \quad (\to C)$$

MODEL




reference case $\boxed{C_{in} = C_{L1}}$

Generic Switching Delay of (1)

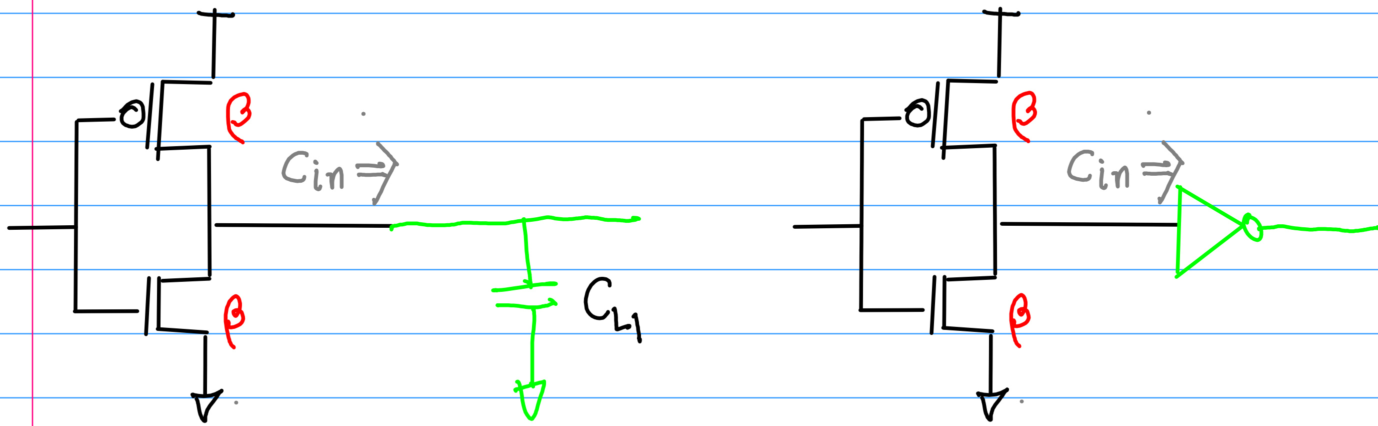
$$t_{s1} = t_0 + \alpha C_{L1}$$
$$= t_0 + \alpha C_{in}$$

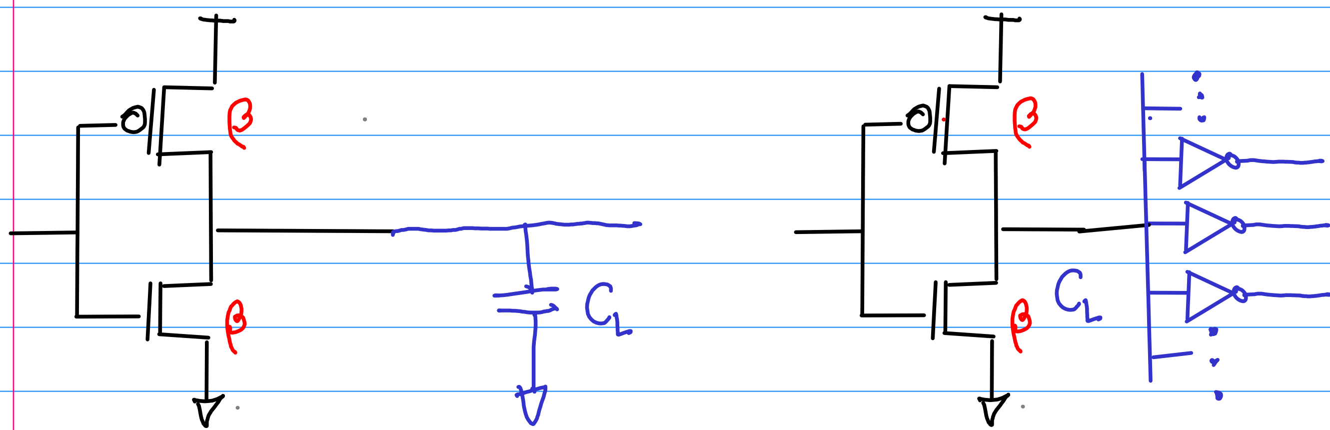
$$C_{in} = C_{Gn} + C_{Gp}$$
$$= C_{ox}(A_{Gn} + A_{Gp})$$
A: gate area

the channel length L assumed

$$C_{in} = C_{ox} L (W_n + W_p)$$
$$= C_{ox} L (W_n + r W_p)$$
$$= C_{ox} L W_n \cdot (1 + r)$$
$$= C_{Gn} (1+r)$$

When $C_L \gg C_{in}$




to minimize $t_s$

$\alpha \downarrow \Rightarrow R \downarrow \Rightarrow \beta \uparrow \Rightarrow$ bigger size

speed v.s. area tradeoff

$t_s = t_0 + \alpha C_L \quad t \approx RC$

$\alpha \propto \frac{1}{\beta(V_{DD} - V_T)}$ $\quad C_L$

to minimize $t_s$

$\alpha \downarrow \Rightarrow R \downarrow \Rightarrow \beta \uparrow \Rightarrow$ bigger size

speed v.s. area **tradeoff**

Scaling Factor $S$

$$\beta' = S\beta$$

$$R' = \frac{R}{S}$$

$$\alpha' = \frac{\alpha}{S}$$

$$t_s = t_0 + \underbrace{\frac{\alpha}{S}}_{R}\underbrace{C_L}_{C}$$

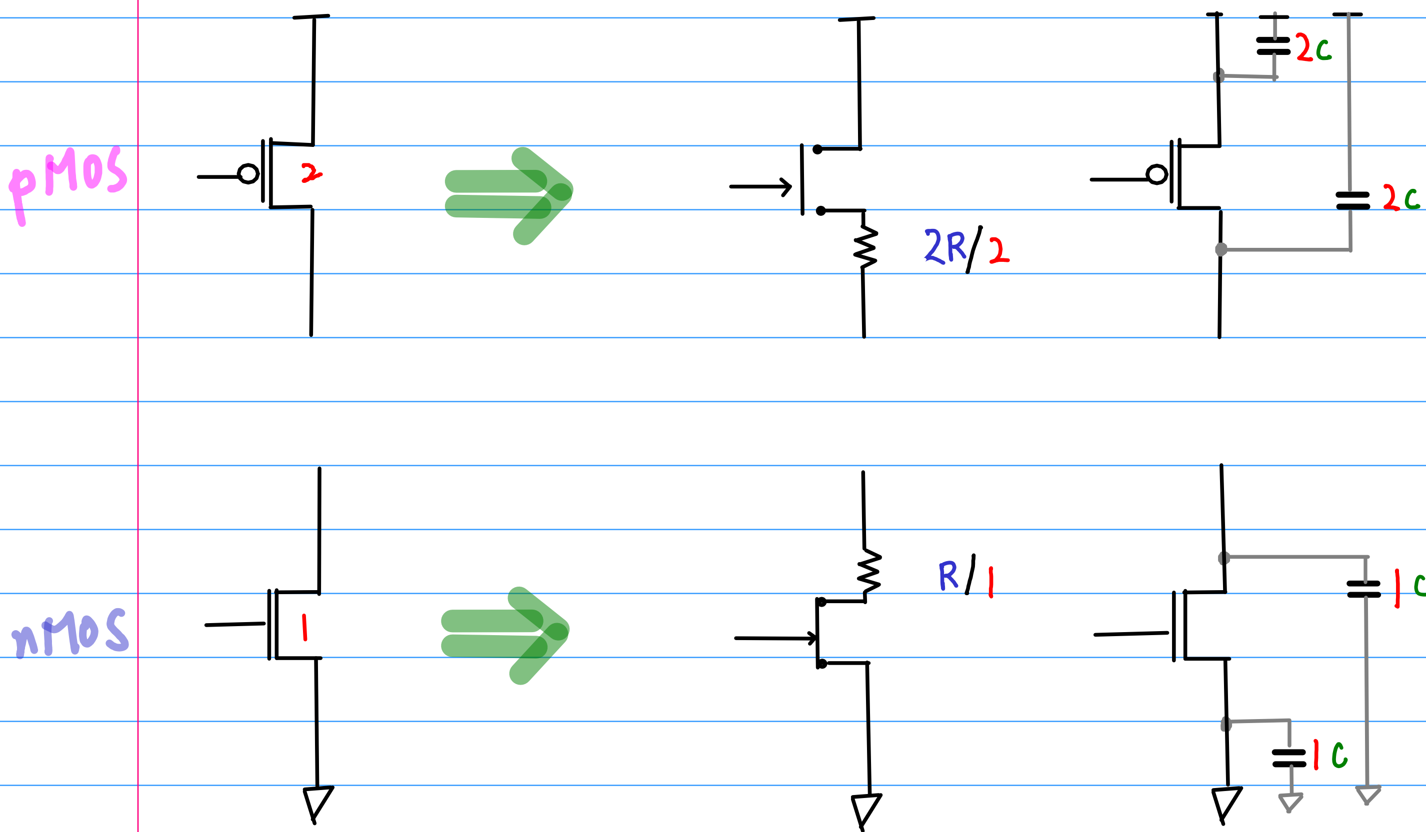
Compensation Factor $\frac{1}{S}$

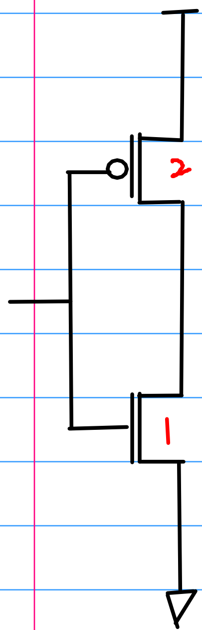
enables a NOT gate drive larger values of $C_L$

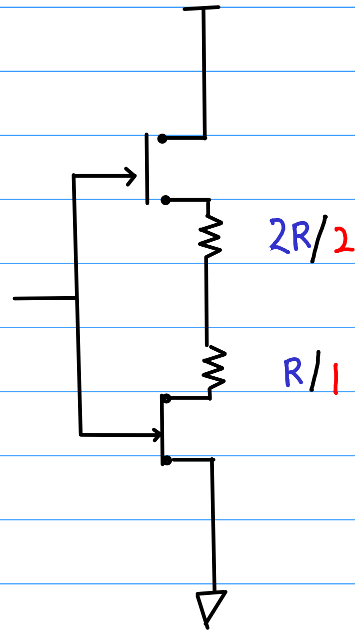
If $C_L = S\,C_{in}$ (increased by the scaling factor $S$)

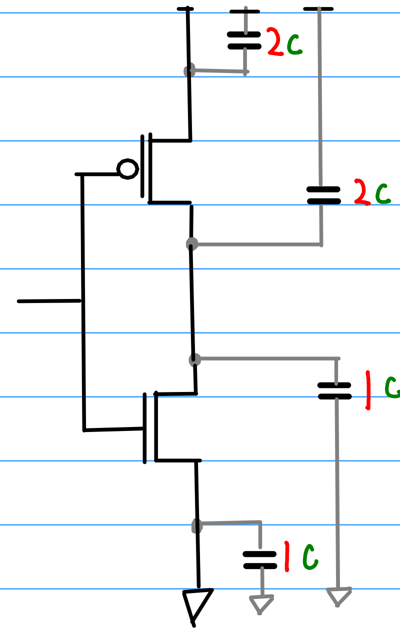
then the switching time is the same

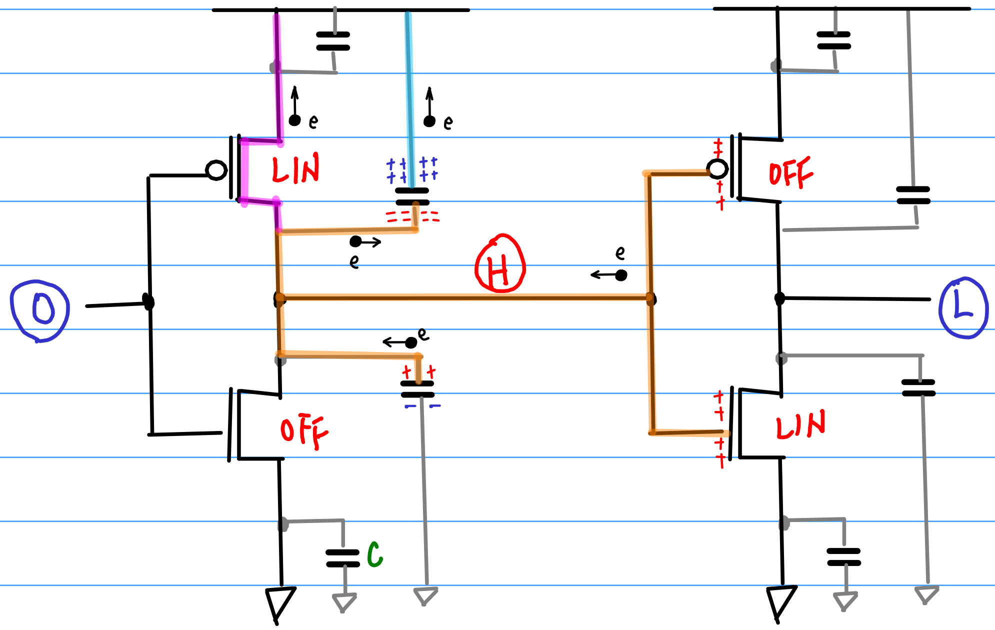
# RC Delay Model



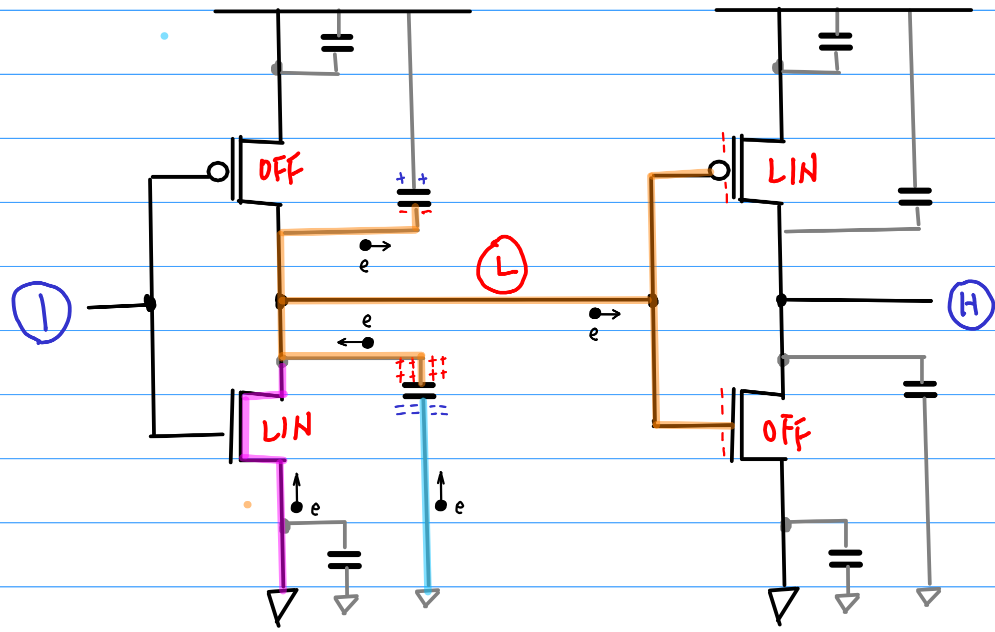

2
1


2R/2
R/1

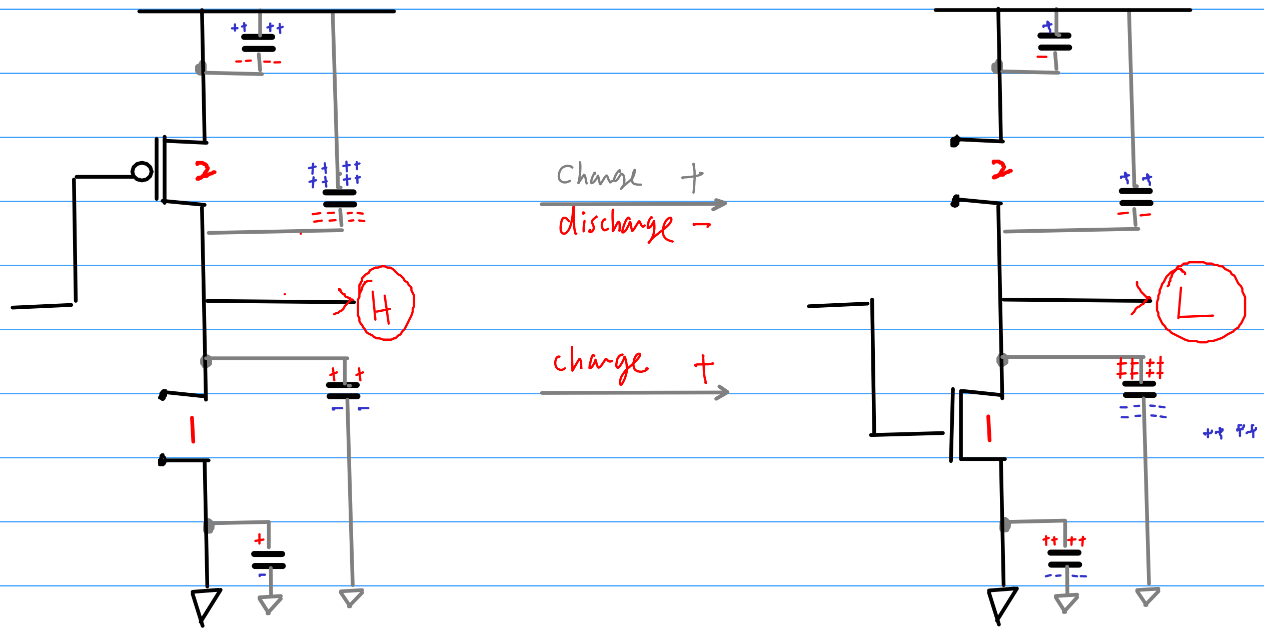

2C
2C
1C
1C



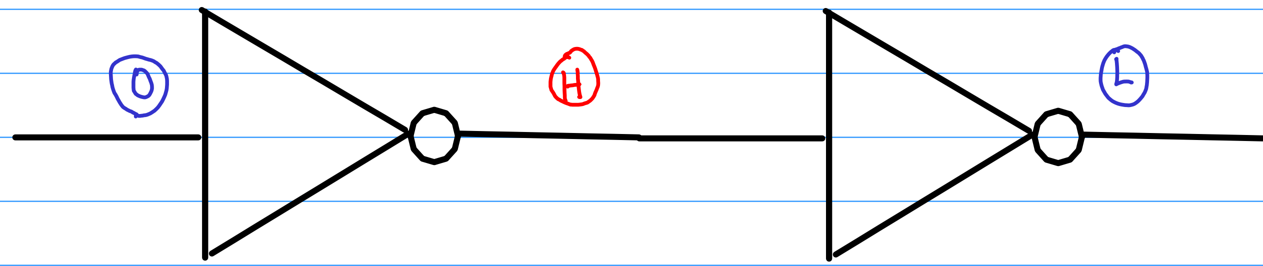
O
LIN
OFF
H
C
OFF
LIN
L
e

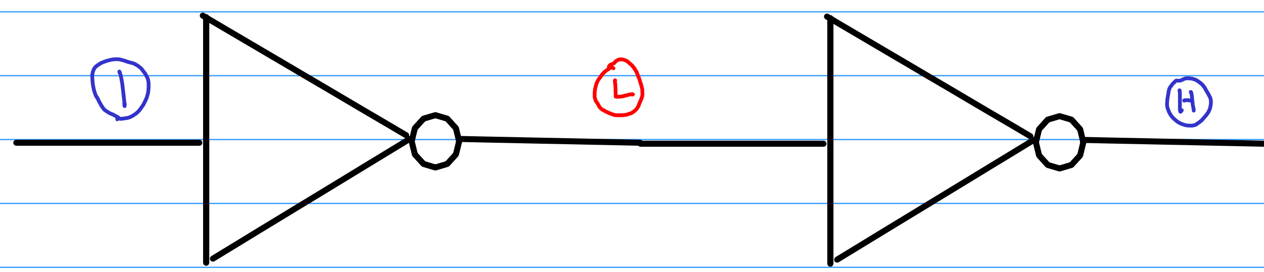



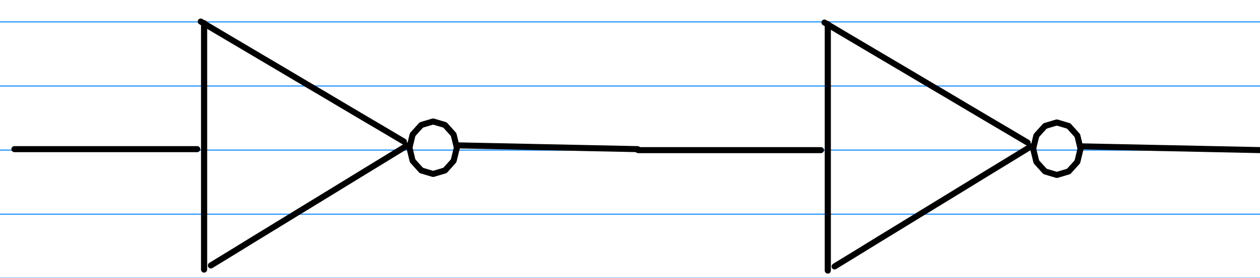
I
OFF
LIN
L
LIN
OFF
H
e

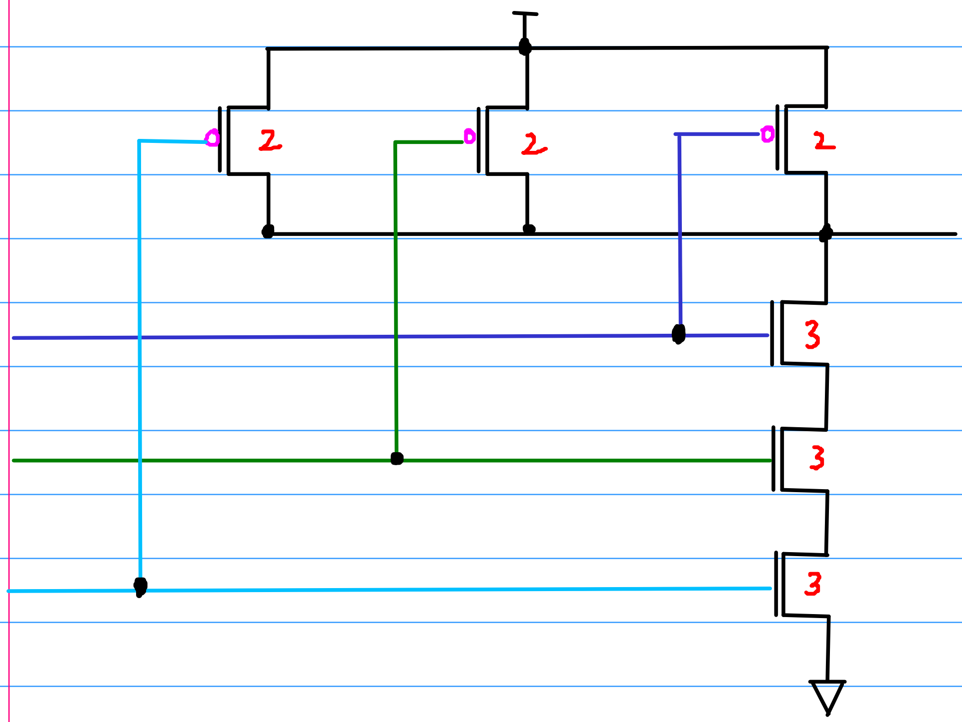


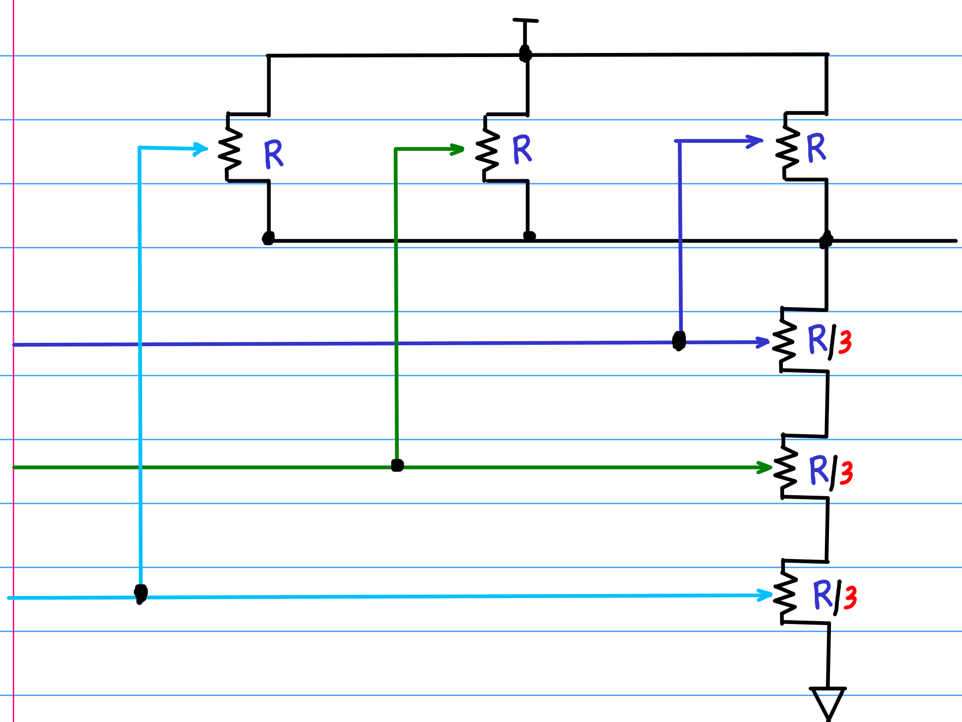
2
H
Charge +
discharge −
charge +
1
2
L
1

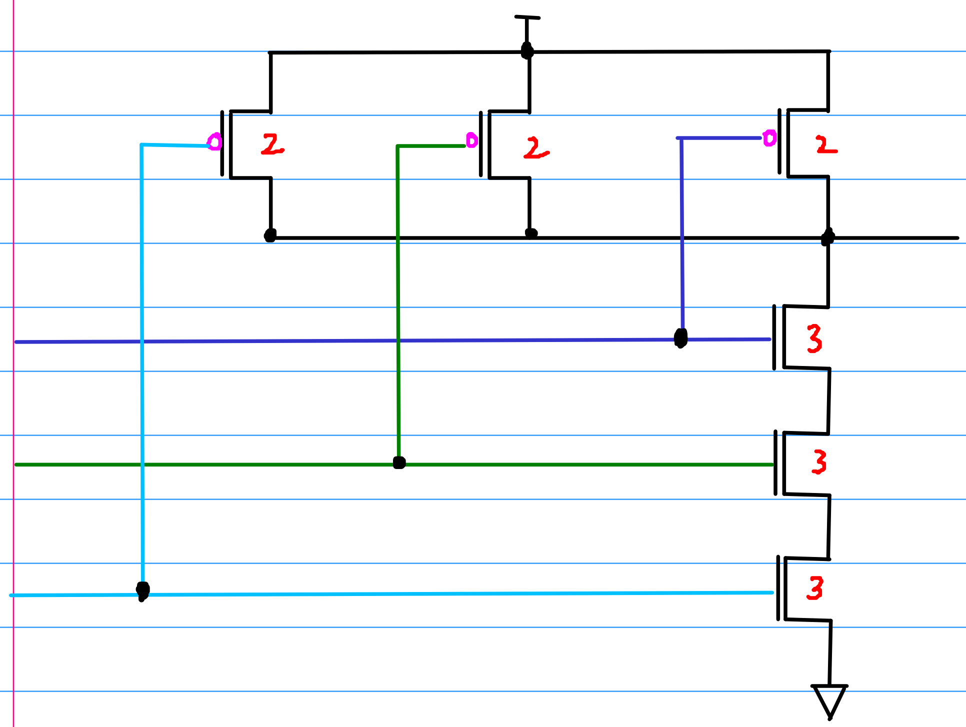

0
H
L


1
L
H

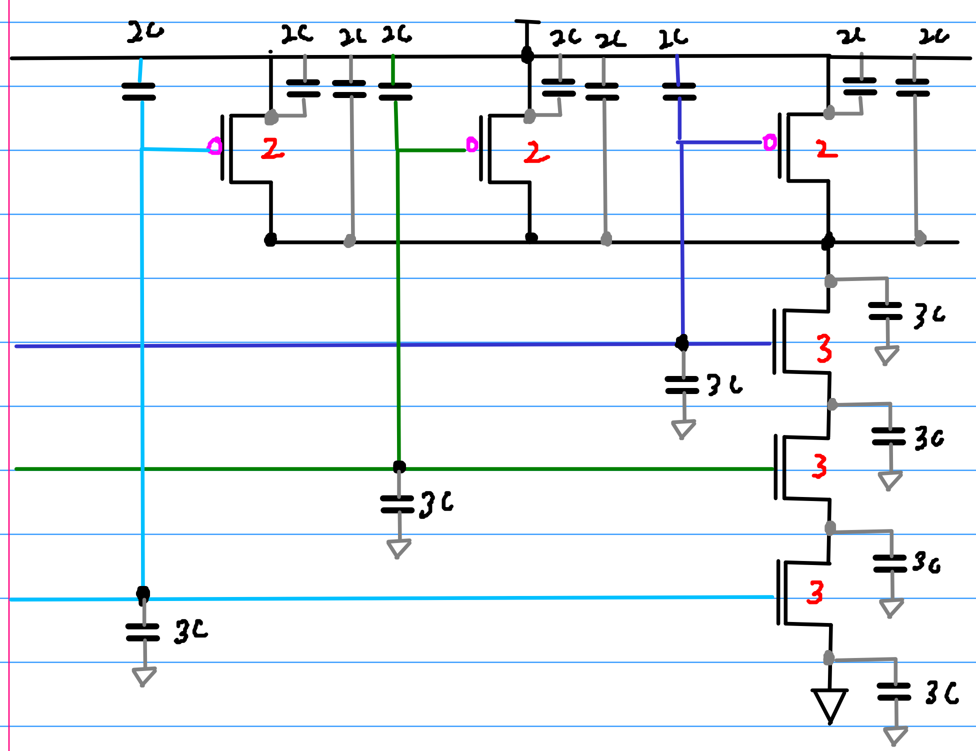



2
2
2
3
3
3


R
R
R
R/3
R/3
R/3



2
2
2
3
3
3

2C
2C
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2C
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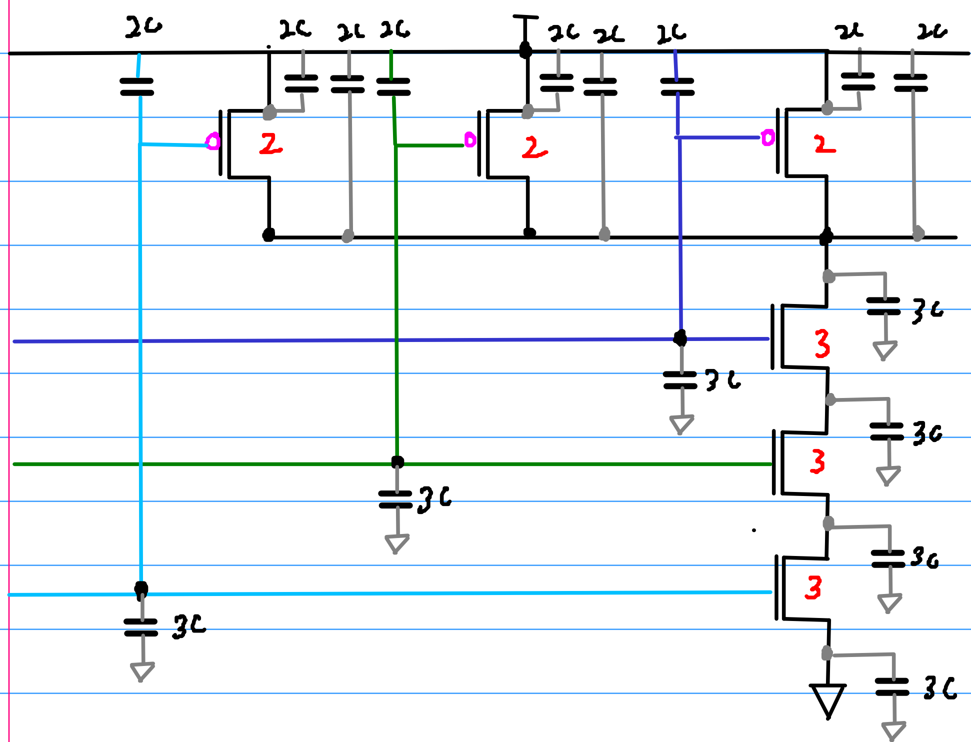
3C



2c
2c 2c 2c
2c 2c 2c
2c 2c
2
2
2
3c
3
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3c
3
3c
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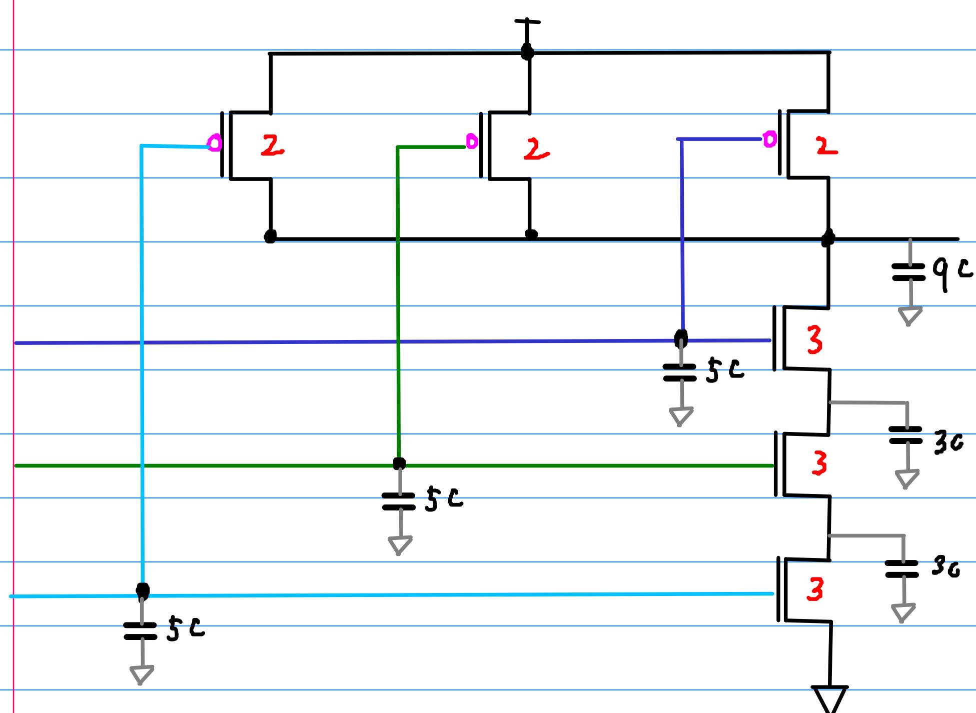
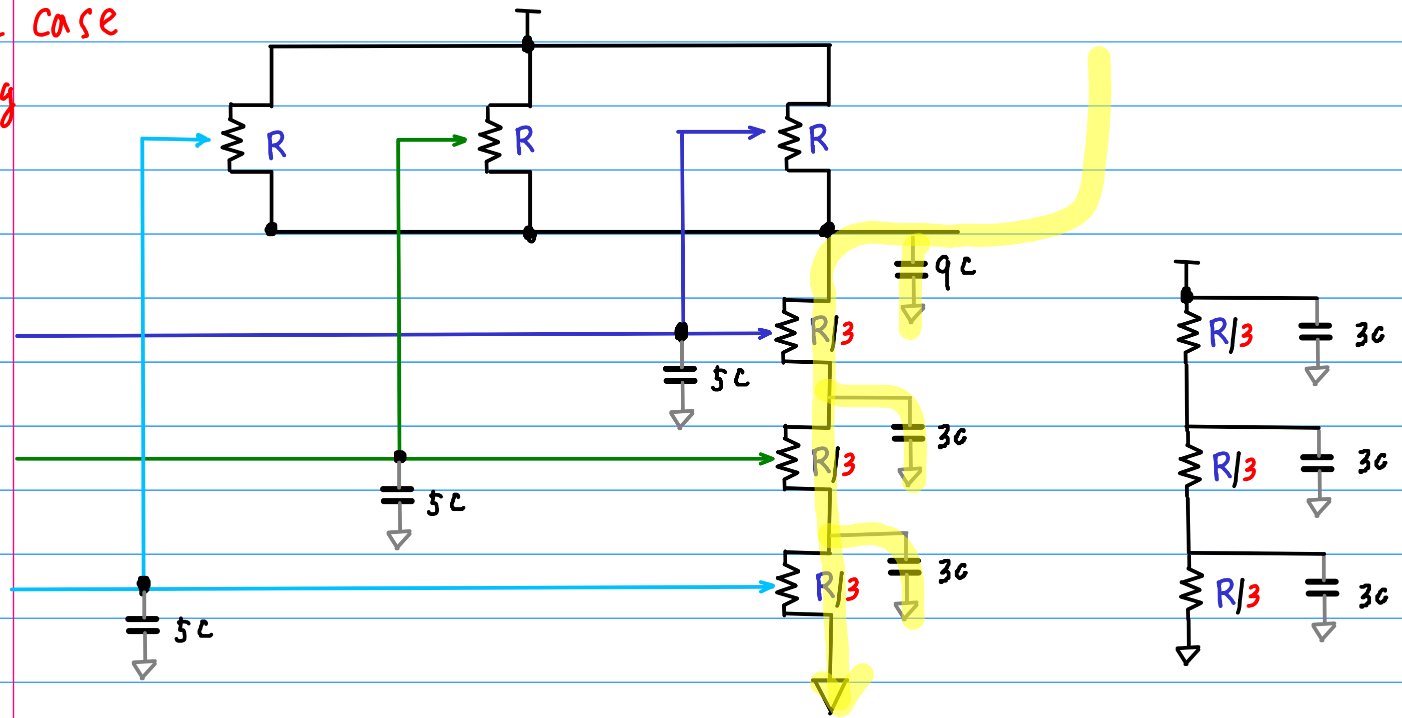
3
3c
3c
3c

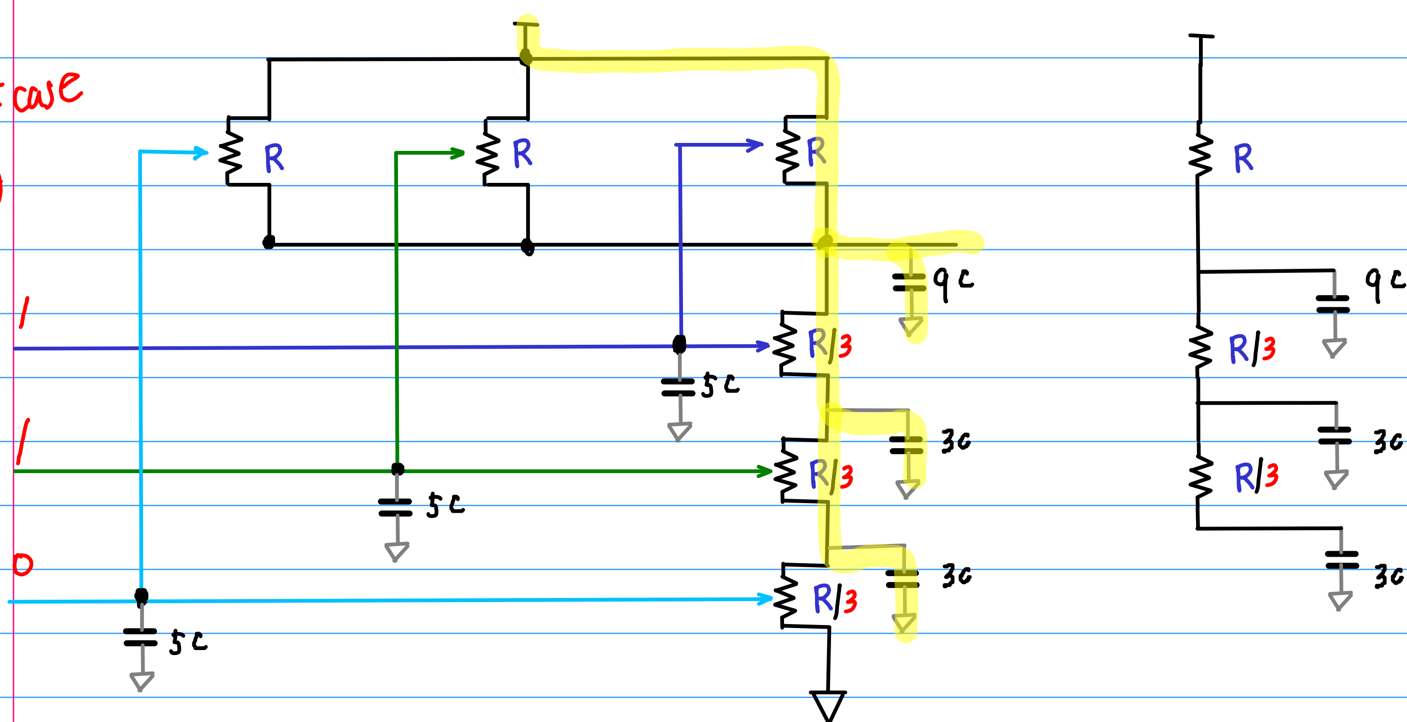



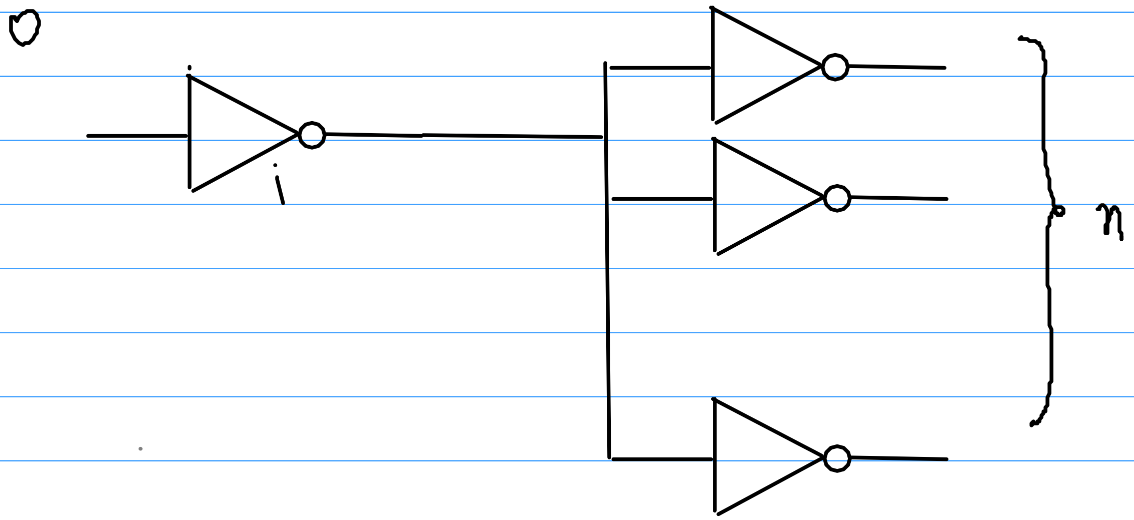
2
2
2
9c
3
5c
3
3c
5c
3
3c
5c

Worst case falling



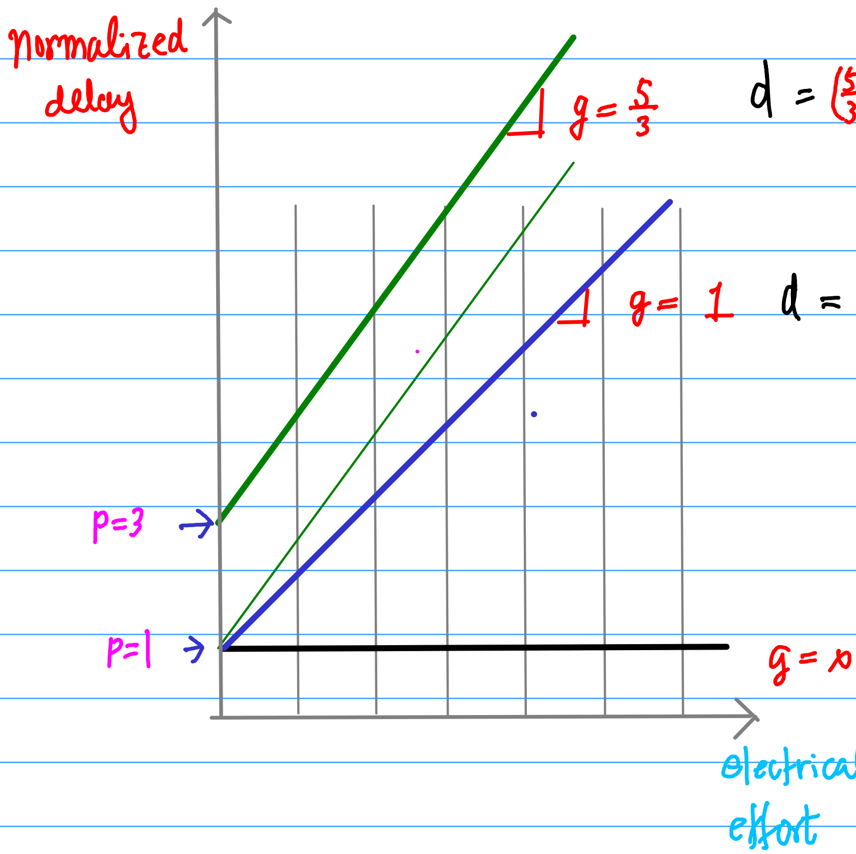

Worst case rising





n

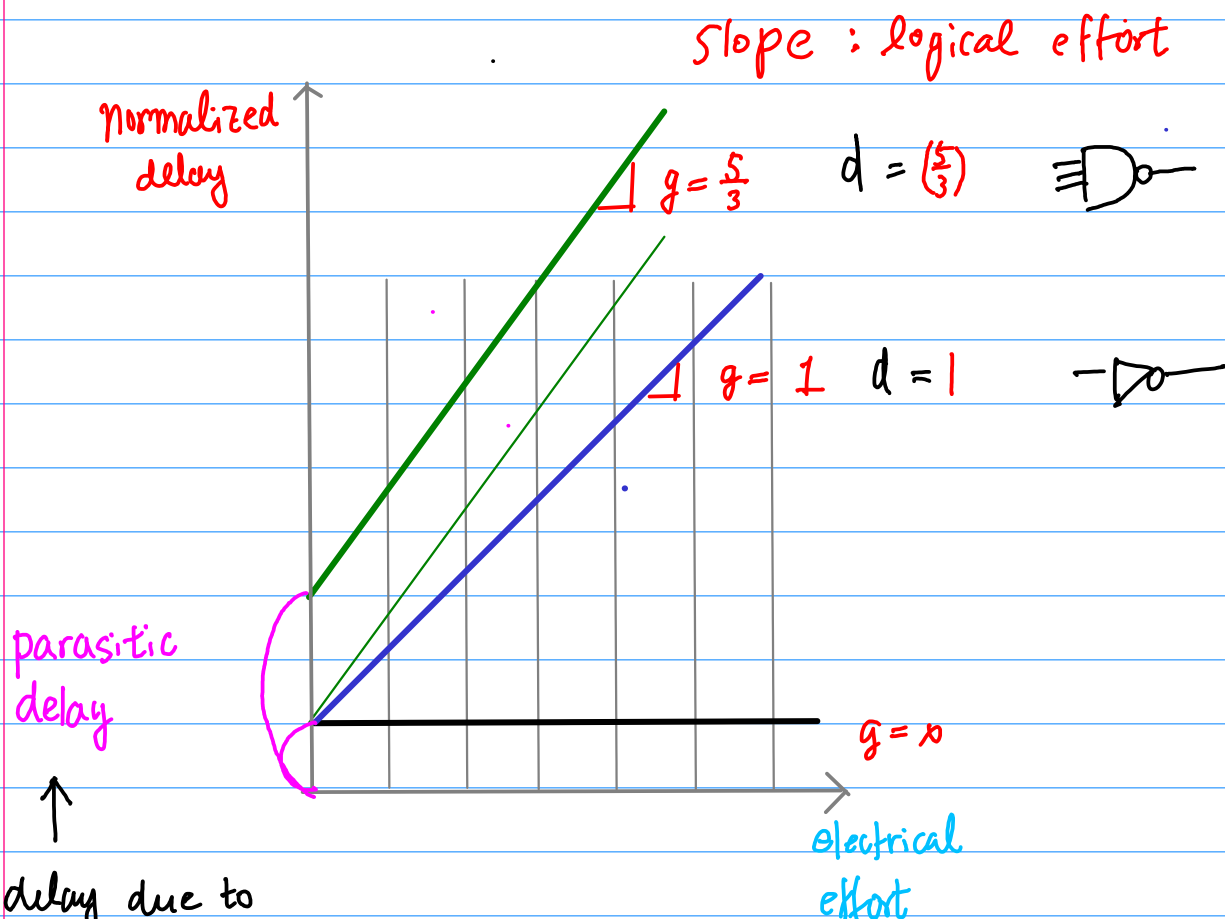
# Linear Delay Model




$d = \left(\frac{5}{3}\right) h + 3$

$d = 1 \cdot h + 1$

$h = \frac{C_{out}}{C_{in}}$




↑

delay due to only internal cap without external load cap

$$d = g \cdot h + p$$

(k) (c) (c)

## Fall Time Calculation

$$i = -C_{out}\frac{dV_{out}}{dt} = \frac{V_{out}}{R_n}$$

$$V_{out}(t) = V_{DD}\, e^{-t/\tau_n}$$

$$\tau_n = R_n C_{out}$$

$$t = \tau_n \ln\left(\frac{V_{DD}}{V_{out}}\right)$$

$$t_f = t_y - t_x = \tau_n \ln\left(\frac{V_{DD}}{0.1\,V_{DD}}\right) - \tau_n \ln\left(\frac{V_{DD}}{0.9\,V_{DD}}\right)$$
$$= \tau_n \ln(9)$$

$$t_{HL} = t_f \cong 2.2\,\tau_n$$

# Rise Time Calculation

$$i = -C_{out}\frac{dV_{out}}{dt} = \frac{V_{DD} - V_{out}}{R_p}$$

$$V_{out}(t) = V_{DD}\left[1 - e^{-t/\tau_p}\right]$$

$$\tau_p = R_p C_{out}$$

$$t = \tau_p \ln\left(\frac{V_{DD}}{V_{out}}\right)$$

$$t_f = t_v - t_u = \tau_p \ln\left(\frac{V_{DD}}{0.1\,V_{DD}}\right) - \tau_n \ln\left(\frac{V_{DD}}{0.9\,V_{DD}}\right)$$

$$= \tau_p \ln(9)$$

$$t_{LH} = t_r \cong 2.2\,\tau_p$$

# Propagation Delay

$$t_p = \frac{(t_{pf} + t_{pr})}{2}$$

$$t_{pf} = \ln(2) \cdot \tau_n$$
$$t_{pr} = \ln(2) \cdot \tau_p$$

$$t_p \cong 0.35(\tau_n + \tau_p)$$

$$C_{out} = C_{FET} + C_L$$

$$t_r = 2.2\, R_p\, (C_{FET} + C_L)$$
$$t_f = 2.2\, R_n\, (C_{FET} + C_L)$$

$$t_r = t_{ro} + \alpha_p C_L$$
$$t_f = t_{ro} + \alpha_n C_L$$

$C_L = 0 \rightarrow$ $\quad t_r = t_{ro} \cong 2.2\, R_p\, C_{FET}$

$C_L = 0$ $\quad t_f = t_{ro} \cong 2.2\, R_n\, C_{FET}$

$$\alpha_p = 2.2 R_p = \frac{2.2}{\beta_p (V_{DD} - |V_{Tp}|)}$$
$$\alpha_n = 2.2 R_n = \frac{2.2}{\beta_n (V_{DD} - |V_{Tn}|)}$$

$$\beta_p = k'_p \left(\frac{W}{L}\right)_p$$

$$\beta_n = k'_n \left(\frac{W}{L}\right)_n$$