Monad Background (3A)

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Based on

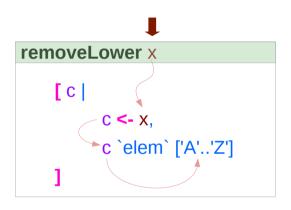
Haskell in 5 steps

https://wiki.haskell.org/Haskell_in_5_steps

A List Comprehension Function

```
let removeLower x = [c | c <- x, c `elem` ['A'..'Z']]</pre>
```

a list comprehension

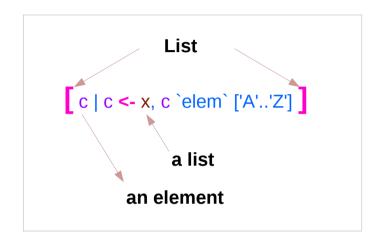


```
do { x1 <- action1
   ; x2 <- action2
   ; mk_action3 x1 x2 }</pre>
```

x1 : Return value of action1x2: Return value of action2

Pattern and Predicate

```
let removeLower x = [c \mid c \leftarrow x, c \in ['A'..'Z']]
a list comprehension
[c | c <- x, c `elem` ['A'..'Z']]
     c <- x is a generator
           (x : argument of the function removeLower)
     c is a pattern
           matching from the elements of the list x
           successive binding of c to the elements of the list x
     c `elem` ['A'..'Z']
           is a predicate which is applied to each successive binding of c
```



Only c which passes this predicate will appear in the output list

Assignment in Haskell

Assignment in Haskell: <u>declaration</u> with <u>initialization</u>:

- no uninitialized variables,
- must declare with an initial value
- no mutation
- a variable keeps its initial value throughout its scope.

Generator

```
[c| c <- x, c `elem` ['A'..'Z']]
filter ('elem' ['A' .. 'Z']) x
                                            c: an element
[c \mid c \leftarrow x]
                                            x: a list
                                                                                            action1 >>= (\ x1 ->
                                                                                             action2 >>= (\ x2 ->
                                                                                               mk_action3 x1 x2 ))
do c <- x
  return c
                                            c: an element
                                                                       c: a list
                                                                or
                                            x: an element
                                                                       x: a list
x >>= ( \c -> return c )
x >>= return
```

Anonymous Functions

```
(\x -> x + 1) 4
5 :: Integer

(\x y -> x + y) 3 5
8 :: Integer

inc1 = \x -> x + 1
```

```
incListA lst = map inc2 lst
where inc2 x = x + 1
```

```
incListB lst = map (\x -> x + 1) lst
```

```
incListC = map (+1)
```

 $https://wiki.haskell.org/Anonymous_function$

Then Operator (>>) and do Statements

a chain of actions

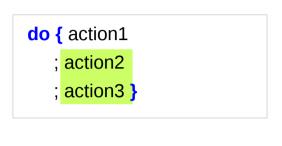
to <u>sequence</u> input / output operations

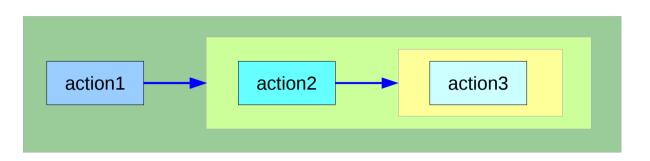
the (>>) (then) operator works almost identically in do notation

```
putStr "Hello" >>
putStr " " >>
putStr "world!" >>
putStr "\n"
```

```
do { putStr "Hello"
   ; putStr " "
   ; putStr "world!"
   ; putStr "\n" }
```

Chaining in **do** and >> notations





can **chain** any actions all of which are in **the same monad**

```
do { action1
  ; do { action2
     ; do { action3 } } }
     action3
action2
action3
```

Bind Operator (>==) and **do** statements

The bind operator (>>=)

passes a value ->

(the result of an action or function), downstream in the binding sequence.

action1 >>= (\ x1 ->
action2 >>= (\ x2 ->
mk_action3 x1 x2))

anonymous function (lambda expression) is used **do** notation <u>assigns</u> a variable name to the passed value using the <-

```
do { x1 <- action1
   ; x2 <- action2
   ; mk_action3 x1 x2 }</pre>
```

Chaining >>= and **do** notations

```
->
                                                                                                        <-
                                                                                              do { x1 <- action1</pre>
action1 >>= (\ \times1 -> action2 >>= (\ \times2 -> mk action3 \times1 \times2 ))
                                                                                                   ; x2 <- action2
                                                                                                   ; mk_action3 x1 x2 }
action1
 >>=
  (\ x1 -> action2
     >>=
      (\ x2 \rightarrow mk \ action3 \ x1 \ x2))
                                                          action1
                                                                            ×1 ·
action1 >>= (\ x1 ->
 action2 >>= (\ x2 ->
  mk_action3 x1 x2 ))
                                                                                      action2
                                                                                                        x2 <del>|</del>
                                                                                                               mk_action3
```

fail method

O.K. when action1 returns Just x1

when action1 returns **Nothing** crash with an non-exhaustive patterns error

Handling failure with fail method

-- A compiler-generated message.

do { x1 <- action1</pre>

: x2 <- action2

; mk action3 x1 x2 }

Example

```
nameDo :: IO ()
nameDo = do { putStr "What is your first name? "
    ; first <- getLine
    ; putStr "And your last name? "
    ; last <- getLine
    ; let full = first ++ " " ++ last
    ; putStrLn ("Pleased to meet you, " ++ full ++ "!") }</pre>
```

```
do { x1 <- action1
   ; x2 <- action2
   ; mk_action3 x1 x2 }</pre>
```

using the **do** statement

A possible translation into vanilla monadic code:

using then (>>) and Bind (>>=) operators

return method

```
nameReturn :: IO String
nameReturn = do putStr "What is your first name? "

first <- getLine
putStr "And your last name? "

last <- getLine
let full = first ++ " " ++ last
putStrLn ("Pleased to meet you, " ++ full ++ "!")
return full
```

Without a return method

explicit return statement returns **IO String** monad

```
nameDo :: IO ()
nameDo = do { putStr "What is your first name? "
    ; first <- getLine
    ; putStr "And your last name? "
    ; last <- getLine
    ; let full = first ++ " " ++ last
    ; putStrLn ("Pleased to meet you, " ++ full ++ "!") }</pre>
```

no return statement returns **empty IO** monad

return method – not a final statement

```
nameReturnAndCarryOn :: IO ()
nameReturnAndCarryOn = do putStr "What is your first name? "
first <- getLine
putStr "And your last name? "
last <- getLine
let full = first++" "++last
putStrLn ("Pleased to meet you, "++full++"!")
return full
putStrLn "I am not finished yet!"
```

the return statement does <u>not</u> interrupt the flow the last statements of the sequence returns a value

Data Constructor

data Color = Red | Green | Blue

is a type

Color

| | .e a 3, pe |
|-------|---|
| Red | is a <i>constructor</i> that contains a <i>value</i> of type Color . |
| Green | is a <i>constructor</i> that contains a <i>value</i> of type Color . |

Blue is a *constructor* that contains a *value* of type Color.

Data Constructor with Parameters

data Color = RGB Int Int Int

Color is a type

RGB is not a value but a <u>function</u> taking three Int's and <u>returning</u> <u>a value</u>

RGB :: Int -> Int -> Color

RGB is a **data constructor** that is a <u>function</u> taking three Int <u>values</u> as its arguments, and then uses them to <u>construct a new value</u>.

Type Constructor

Consider a binary tree to store Strings

```
data SBTree = Leaf String | Branch String SBTree SBTree
a type
     SBTree
               is a type
     Leaf
                is a data constructor (a function)
     Branch
                is a data constructor (a function)
```

Leaf :: String -> SBTree

Branch :: String -> SBTree -> SBTree

Similar Type Constructors

Consider a binary tree to store Strings

data SBTree = Leaf String | Branch String SBTree SBTree

Consider a binary tree to store Bool

data BBTree = Leaf Bool | Branch Bool BBTree BBTree

Consider a binary tree to store a parameter type

data BTree a = Leaf a | Branch a (BTree a) (BTree a)

Type Constructor with a Parameter

Type constructors

Both **SBTree** and **BBTree** are type constructors

```
data SBTree = Leaf String | Branch String SBTree SBTree
data BBTree = Leaf Bool | Branch Bool BBTree BBTree
```

data BTree a = Leaf a | Branch a (BTree a) (BTree a)

Now we introduce a type variable a as a parameter to the type constructor.

BTree has become a function.

It takes a type as its argument and it returns a new type.

Type Constructors and Data Constructors

A type constructor

- a "function" that takes 0 or more types
- gives you back a new type.

Type constructors with <u>parameters</u> allows slight variations in <u>types</u>

A data constructor

- a "function" that takes 0 or more values
- gives you back a new value.

Data constructors with <u>parameters</u> allows slight variations in <u>values</u>

type SBTree = BTree String
type BBTree = BTree Bool

RGB 12 92 27

#0c5c1b

RGB 255 0 0

RGB 0 255 0

RGB 0 0 255



```
() is both a type and a value.
() is a special type, pronounced "unit", has one value (), sometimes pronounced "void"
the unit type has only one value which is called unit.
() :: ()
Type :: Expression
```

It is the same as the void type void in Java or C/C++.

https://stackoverflow.com/questions/20380465/what-do-parentheses-used-on-their-own-mean

Unit Type

a **unit type** is a type that allows *only one value* (and thus can hold *no information*).

It is the same as the void type void in Java or C/C++.

```
:t
Expression :: Type
```

```
data Unit = Unit

Prelude> :t Unit
Unit :: Unit
```

```
Prelude> :t ()
() :: ()
```

https://stackoverflow.com/questions/20380465/what-do-parentheses-used-on-their-own-mean

Type Language and Expression Language

```
data Tconst Tvar ... Tvar = Vconst type ... type | ...
Vconst type ... type
```

A new datatype declaration

Tconst (Type Constructor)

Vconst (Value Constructor)

is added to the type language is added to the expression language and its pattern sublanguage must <u>not</u> appear in types

Argument types in Vconst type ... type

are the types given to the arguments (Tconst Tvar ... Tvar)

are used in expressions

https://stackoverflow.com/questions/16892570/what-is-in-haskell-exactly

Datatype Declaration Examples

```
data Tree a = Leaf | Node (Tree a) (Tree a)

Tree (Type Constructor)

Leaf or Node (Value Constructor)
```

data Type = Value

```
data () = ()() (Type Constructor)() (Value Constructor)
```

```
the type (), often pronounced "Unit" the value (), sometimes pronounced "void"
```

the type () containing only one value ()

https://stackoverflow.com/questions/16892570/what-is-in-haskell-exactly

Monadic Effect

class Monad m where

return :: a -> m a

(>>=) :: m a -> (a -> m b) -> m b

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/IO
https://stackoverflow.com/questions/2488646/why-are-side-effects-modeled-as-monads-in-haskell
https://stackoverflow.com/questions/7840126/why-monads-how-does-it-resolve-side-effects
https://stackoverflow.com/questions/2488646/why-are-side-effects-modeled-as-monads-in-haskell

https://www.cs.hmc.edu/~adavidso/monads.pdf



Monadic operations tend to have types which look like

val-in-type-1 -> ... -> val-in-type-n -> effect-monad val-out-type

where the **return type** is a type application: the function tells you which **effects** are possible and the argument tells you what sort of value is produced by the operation

https://stackoverflow.com/questions/16892570/what-is-in-haskell-exactly

IO()

```
put :: s -> State s ()

put :: s -> (State s) ()

one value input type s
the effect-monad State s
the value output type ()

the operation is used only for its effect;
the value delivered is uninteresting
```

```
putStr :: String -> IO ()
```

delivers a string to stdout but does not return anything exciting.

https://stackoverflow.com/questions/16892570/what-is-in-haskell-exactly

Variable definition in a file

Var1.hs Var2.hs r = 5

r = 55

definition with initialization

```
young@Sys ~ $ ghci
GHCi, version 7.10.3: http://www.haskell.org/ghc/ :? for help
Prelude> :load Var1.hs
[1 of 1] Compiling Main (var.hs, interpreted)
Ok, modules loaded: Main.
*Main> r

*Main> :t r
r:: Integer
*Main>
*Main> :load Var2.hs
[1 of 1] Compiling Main (var2.hs, interpreted)
Ok, modules loaded: Main.
*Main> r

55
```

No Mutation

Var1.hs

Var2.hs

r = 5

r = 55

No mutation

```
*Main> r = 33

<interactive>:12:3: parse error on input '='

young@Sys ~ $ ghci
GHCi, version 7.10.3: http://www.haskell.org/ghc/ :? for help
Prelude> r = 333

<interactive>:2:3: parse error on input '='
Prelude>
```

Functional & Imperative Languages

Imperative programming:

- variables as changeable locations in a computer's memory
- imperative programs explicitly commands the computer what to do

functional programming

- a way to think in higher-level mathematical terms
- defining how variables relate to one another
- leaving the compiler to translate these

to the step-by-step instructions that the computer can process.

Redefinition: not allowed

r = 5

r = 2

imperative programming:

after setting r = 5 and then changing it to r = 2.

Hakell programming:

an error: "multiple declarations of r".

Within a given scope, a variable in Haskell

gets defined only once and cannot change.

like variables in mathematics.

Immutable: They vary only based on the data we enter into a program.

We can't define r two ways in the same code,

but we could change the value by changing the file

Recursion

r = r + 1

imperative programming:

incrementing the variable r (updating the value in memory)

Hakell programming:

a recursive definition of r(defining it in terms of itself)

if r had been defined with any value beforehand,

then r = r + 1 in Haskell would bring an error message.

Data Dependence

$$y = x * 2$$

$$x = 3$$

$$x = 3$$

$$y = x * 3$$

Hakell programming:

because their values of variables do not change within a program

variables can be defined in any order

there is no notion of "x being declared before y" or the other way around.

Evaluation

```
area 5

=> { replace the LHS area r = ... by the RHS ... = pi * r^2 }

pi * 5 ^ 2

=> { replace pi by its numerical value }

3.141592653589793 * 5 ^ 2

=> { apply exponentiation (^) }

3.141592653589793 * 25

=> { apply multiplication (*) }

78.53981633974483
```

area $r = pi * r^2$

replace each function with its definition

calculate the results until a single value remains.

to apply or call a function means

to replace the LHS of its definition by its RHS.

https://en.wikibooks.org/wiki/Haskell/Variables and functions

Type Synonyms

```
type String = [Char]

phoneBook :: [(String,String)]
```

```
type PhoneBook = [(String,String)]

phoneBook :: PhoneBook
```

```
phoneBook =

[("betty","555-2938")

,("bonnie","452-2928")

,("patsy","493-2928")

,("lucille","205-2928")

,("wendy","939-8282")

,("penny","853-2492")

]
```

```
type PhoneNumber = String
type Name = String
type PhoneBook = [(Name,PhoneNumber)]
phoneBook :: PhoneBook
```

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

Record Syntax (named field)

```
data Configuration = Configuration
  { username
                    :: String
  . localHost :: String
  . currentDir :: String
                    :: String
  , homeDir
  , timeConnected :: Integer
username :: Configuration -> String
                                                         -- accessor function (automatic)
localHost:: Configuration -> String
-- etc.
changeDir :: Configuration -> String -> Configuration
                                                        -- update function
changeDir cfg newDir =
  if directoryExists newDir -- make sure the directory exists
    then cfg { currentDir = newDir }
    else error "Directory does not exist"
```

newtype and data

data



newtype

Data can only be replaced with newtype if the type has exactly <u>one constructor</u> with exactly <u>one field</u> inside it.

It ensures that the trivial **wrapping** and **unwrapping** of the single field is eliminated by the **compiler**.

simple wrapper types such as **State** are usually defined with **newtype**.

type: used for type synonyms

newtype State s a = State { runState :: s -> (s, a) }

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

newtype examples

```
newtype Fd = Fd CInt
-- data Ed = Ed CInt would also be valid
-- newtypes can have deriving clauses just like normal types
newtype Identity a = Identity a
 deriving (Eq. Ord, Read, Show)
-- record syntax is still allowed, but only for one field
newtype State s a = State { runState :: s -> (s, a) }
-- this is *not* allowed:
-- newtype Pair a b = Pair { pairFst :: a, pairSnd :: b }
-- but this is:
data Pair a b = Pair { pairFst :: a, pairSnd :: b }
-- and so is this:
newtype NPair a b = NPair (a, b)
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

Generally, a monad <u>cannot</u> perform side effects in Haskell.

there is one exception: IO monad

Suppose there is a type called World, which contains all the <u>state</u> of the external universe

A way of thinking what IO monad does

type IO t = World -> (t, World)

type synonym

World -> (t, World)

World (t, World)

IO t is a <u>function</u>

input: a World

output: the t and a new, updated World

obtained by modifying the given World

in the process of computing the t.

World → (t, World)

IO x world0 (x, world1)

IO t is a function

input: a World

output: the t and a new, updated World

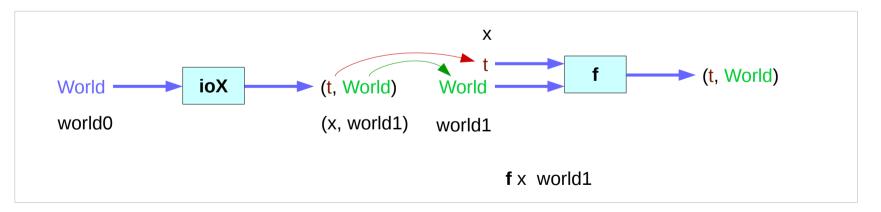
obtained by modifying the given World

in the process of computing the t.

It is <u>impossible</u> to store the extra copies of the contents of your hard drive that each of the Worlds contains

World → World

```
type IO t = World -> (t, World) type synonym
```



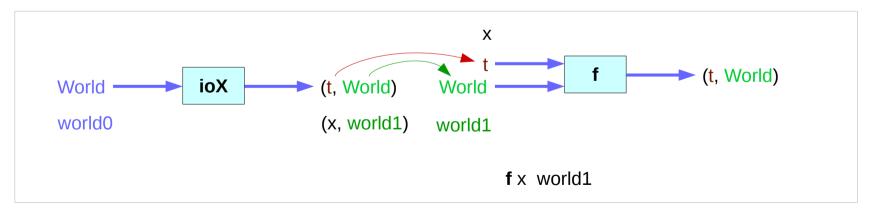
```
instance Monad IO where
return x world = (x, world)

(ioX >>= f) world0 =
let (x, world1) = ioX world0
in f x world1 -- has type (t, World)
```

```
instance Monad ST where
-- return :: a -> ST a
return x = \s -> (x,s)

-- (>>=) :: ST a -> (a -> ST b) -> ST b
st >>= f = \s -> let (x,s') = st s
in f x s'
```

```
type IO t = World -> (t, World) type synonym
```



State Transformers ST

instance Monad ST where

```
-- return :: a -> ST a

return x = \s -> (x,s)

-- (>>=) :: ST a -> (a -> ST b) -> ST b

st >>= f = \s -> let (x,s') = st s in f x s'
```

to give a second state transformer (f x),

>>= provides a means of sequencing state transformers:

st >>= f applies the state transformer st to an initial state s,
then applies the function f to the resulting value x

which is then applied to the modified state s' to give the final result:

$$st >>= f = \slash s -> f x s'$$

where $(x,s') = st s$

st >>= f = \s -> (y,s')
where
$$(x,s') = st s$$

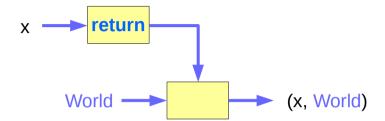
 $(y,s') = f x s'$

$$(x,s') = st s$$

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

The return function takes x
and gives back a function
that takes a World
and returns x along with the "new, updated" World
formed by not modifying the World it was given

return x world = (x, world)



```
type World -> (t, World)

a function ioX that takes world0 of the type World,
which is used to extract x from its IO monad.

x gets passed to f, resulting in another IO monad,
which again is a function that takes a World
and returns a t and a new, updated World.

We give it the World we got back from getting x out of its monad,
and the thing it gives back to us is the t with a final version of the World
```

the implementation of bind

Which World was given initially? Which World was updated?

In GHC, a main must be defined somewhere with type IO ()

a program execution <u>starts</u> from the **main**the <u>initial World</u> is contained in the **main** to start everything off
the **main** passes the updated World from each **IO**to the next **IO** as its <u>initial World</u>

an **IO** that is <u>not reachable</u> from **main** will <u>never be executed</u> an <u>initial</u> / updated World is not passed to such an **IO**

The modification of the World



when using GHCI,

everything is wrapped in **an implicit IO**, since the results get printed out to the screen.

Every time a new command is given to GHCI,

GHCI passes the current World,

GHCI gets the result of the command back,

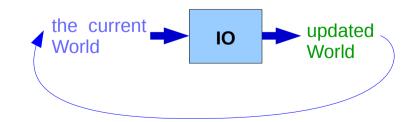
GHCI request to display the result

(which updates the World by modifying

- · the contents of the screen or
- the list of defined variables or
- the list of loaded modules or whatever),

and then saves the new World to give to the next command.

the implementation of bind



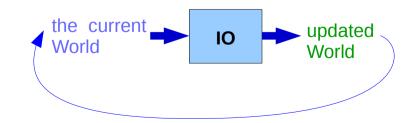
when using **GHCI**, everything is wrapped in **an implicit IO**, since the results get printed out to the screen.

there's only 1 World in existence at any given moment.

Each IO takes that one and only World, consumes it,
and gives back a single new World.

Consequently, there's no way to accidentally run out of Worlds,
or have multiple ones running around.

the implementation of bind

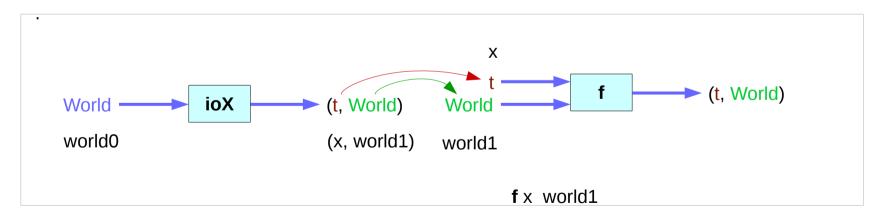


the expression (ioX >>= f) has type World -> (t, World)
a function that takes a World, called world0,
which is used to extract x from its IO monad.

This gets passed to f, resulting in another IO monad,
which again is a function that takes a World
and returns a x and a new, updated World.

the implementation of bind

We give it the World we got back from getting x out of its monad, and the thing it gives back to us is the t with a final version of the World



References

- [1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
- [2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf