Ray Theory (4A)

- Ray Theory

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Snell's Law

$\Delta x \qquad \Delta t$

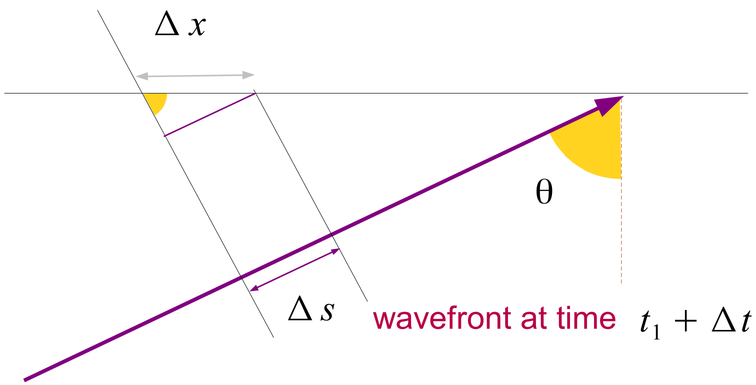
$$\Delta s = \Delta x \sin\theta$$

$$\Delta s = v\,\Delta t \qquad\qquad v = \frac{\Delta s}{\Delta t}$$

$$\frac{\Delta t}{\Delta x} = \frac{\sin\theta}{v} = \frac{1}{v}\sin\theta = u\sin\theta \equiv p$$

$\boxed{\frac{1}{v} = u}$ slowness

$\boxed{\frac{\Delta t}{\Delta x} = u\sin\theta \equiv p}$ ray parameter
horizontal slowness



wavefront at time $t_1 + \Delta t$

a plane wave incident on a horizontal surface
the incidence angle θ

Wave Number, Angular Frequency

wave number

$$k = \frac{2\pi}{\lambda}$$

How many λ in 2π *(rad / m)*

angular frequency

$$\omega = \frac{2\pi}{T}$$

How many T in 2π *(rad / sec)*

3-dimensional space

$$\omega\,\delta t - \boldsymbol{k}\cdot\delta\boldsymbol{x} = 0$$

period wavelength

$$\delta t \Rightarrow T = \frac{2\pi}{\omega}$$

$$\delta x \Rightarrow \lambda = \frac{2\pi}{\boldsymbol{k}}$$

wave number vector

spatial frequency variable

Its magnitude represents the number of cycles (in rad) per meter of length that the monochromatic plane wave exhibits *in the direction of propagation*.

Wavelength, Frequency

$$s(\boldsymbol{x}, t) = A e^{j(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})}$$

position vector

$$s(\boldsymbol{x}, t) = A e^{j(\omega(t - \boldsymbol{\alpha}\cdot\boldsymbol{x}))}$$

$$(\omega t - \boldsymbol{k}\cdot\boldsymbol{x}) = \omega\left(t - \left(\frac{\boldsymbol{k}}{\omega}\right)\cdot\boldsymbol{x}\right)$$

temporal angular frequency

3-d spatial frequency

slowness vector α

$$= [\omega(t - \boldsymbol{\alpha}\cdot\boldsymbol{x})]$$

Function of a single variable

$$s(u) = A e^{j(\omega u)}$$

$$s(t - \boldsymbol{\alpha}\cdot\boldsymbol{x}) = A e^{j(\omega(t - \boldsymbol{\alpha}\cdot\boldsymbol{x}))}$$

$$= s(\boldsymbol{x}, t)$$

Slowness Vector

$$\boldsymbol{\alpha} = \frac{\boldsymbol{k}}{\omega} \qquad \alpha = \frac{2\pi/\lambda}{2\pi/T} = \frac{T}{\lambda}$$

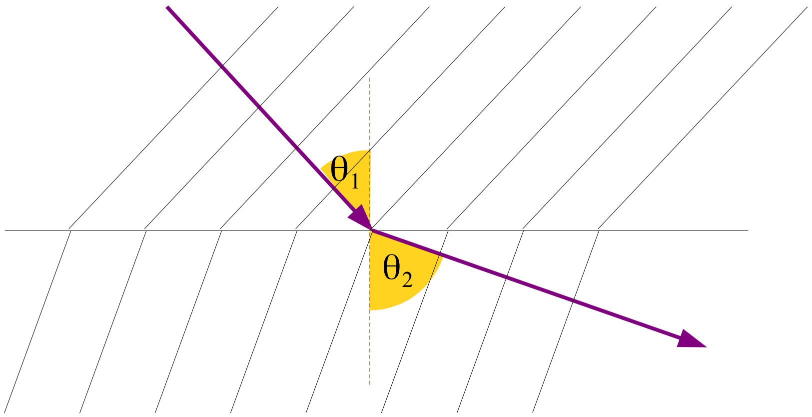
Speed Vector (Phase Velocity)

$$\boldsymbol{v}_p = \frac{\omega}{\boldsymbol{k}} = \frac{1}{\boldsymbol{\alpha}}$$

Laterally Homogeneous Models

two homogeneous layers
of different velocity
evenly spaced wavefronts
must be separated
by different distances
in the different layers

ray angles at the interface
must change the timing
of the wavefronts
across the interface



the slower velocity top layer

$$v_1 < v_2$$

the larger slowness top layer

$$u_1 < u_2$$

the ray parameter

$$p = u_1 \sin\theta_1 = u_2 \sin\theta_2$$

Dispersionless Wave

References

[1] http://en.wikipedia.org/
[2] http://www.people.fas.harvard.edu/~djmorin/book.html D Morin, “Waves”
[3] P. M. Shearer, "Introduction to Seismology: The wave equation and body waves"