Monad P3 : Existential Types (1D)

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9/5/21

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using LibreOffice.

Based on

Haskell in 5 steps

https://wiki.haskell.org/Haskell_in_5_steps

Haskell quantification

- the things being quantified over are **types** (ignoring certain language extensions, at least),
- logical statements are also **types**
- a "**true**" logical statement as "can be implemented".
- technically "**false**" should correspond to an **uninhabited data type** (often called **Void**)

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Logical negation and forall

technically "**false**" should correspond to

an **uninhabited data type** (often called **Void**)

so "**not (not A)**" would be

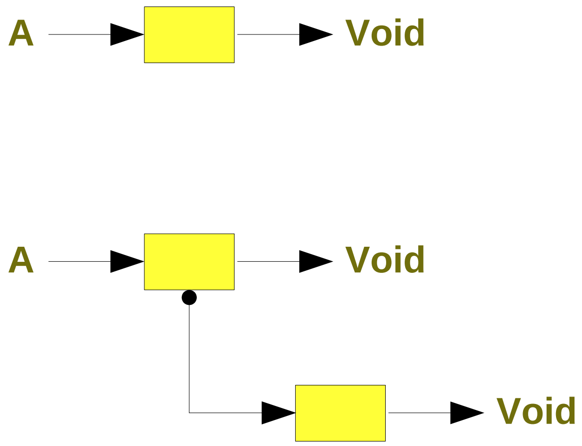
(A -> Void) -> Void -- useless

Assume **forall r. r** stands for "**false**"

forall r. (A -> r) -> r -- can extract the **A** value, i.e.

-- double-negation elimination.

using **r** instead of **Void** lets us get values back out.



https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

De Morgan's law and forall

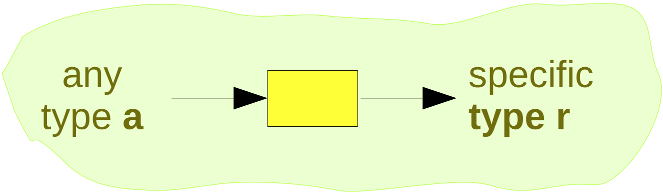
De Morgan's laws as applied to **quantifiers**;
function inputs are **negated**, logically speaking.

There's a similar equivalence between

Either a b … implicit universal quantification

forall r. (a -> r, b -> r) -> r

which corresponds to "**A or B**"
being the same as "**not (not A) and (not B)**".



(Not a) and (Not b)

(a -> r , b -> r)

Not ((Not a) and (Not b))

((a -> r , b -> r)) -> r

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Logical double negation and continuation passing style

Look up the connection between **logical double-negation** and **continuation-passing style** if you want to know more

Due to duality, **exists a. a** can be expressed as

forall r. (forall a. a -> r) -> r

Due to duality, **forall a. a** can be expressed as

exists r. (exists a. a -> r) -> r

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

CPS (Continuation Passing Style)

```
map ($ 2) [ (2*), (4*), (8*) ]

[ (2*) $ 2, (4*) $ 2, (8*) $ 2 ]

[4,8,16]

map (*2) [ 2, 4, 8 ]

[ (*2) 2, (*2) 4, (*2) 8 ]
```

https://en.wikibooks.org/wiki/Haskell/Continuation_passing_style

CPS (Continuation Passing Style)

```
map ($ 2) [ (2*), (4*), (8*) ]
[4,8,16]

map (*2) [ 2, 4, 8 ]
```

The **($) section** makes the code appear backwards,
as if we are applying a **value** to the **functions**
rather than the other way around.

such an **reversal** is at heart of
continuation passing style!

https://en.wikibooks.org/wiki/Haskell/Continuation_passing_style

CPS (Continuation Passing Style)

From a CPS perspective, **($ 2)** is a suspended computation:

a function with general type

(a -> r) -> r

given another function as **argument**,

produces a final result.

the **a -> r** argument is the **continuation**;

it specifies how the computation will be brought to a conclusion.

https://en.wikibooks.org/wiki/Haskell/Continuation_passing_style

CPS (Continuation Passing Style)

map ($ 2) [(2*), (4*), (8*)]

the **functions** in the list are supplied
as **continuations** via **map**, producing three distinct results.

note that suspended computations are largely
interchangeable with plain values:
flip ($) converts any **value**
into a suspended computation,
and passing **id** as its **continuation**
gives back the original value.

https://en.wikibooks.org/wiki/Haskell/Continuation_passing_style

CPS (Continuation Passing Style)

They make it possible
to explicitly manipulate,
and dramatically alter,
the **control flow** of a program.

For instance, returning early from a procedure
can be implemented with **continuations**.
Exceptions and failure can also
be handled with **continuations**
- pass in a **continuation** for success,
- another **continuation** for fail,
- and invoke the appropriate **continuation**.

https://en.wikibooks.org/wiki/Haskell/Continuation_passing_style

CPS (Continuation Passing Style)

Other possibilities include suspending a computation
and returning to it at another time,
and implementing simple forms of **concurrency**

(notably, one Haskell implementation, Hugs,
uses continuations to implement cooperative concurrency).

https://en.wikibooks.org/wiki/Haskell/Continuation_passing_style

CPS (Continuation Passing Style)

In Haskell, **continuations** can be used in a similar fashion,
for implementing interesting **control flow** in **monads**.

Note that there usually are alternative techniques for such use cases,
especially in tandem with **laziness**.

In some circumstances, **CPS** can be used to improve performance
by eliminating certain **construction-pattern matching sequences**
(i.e. a **function** returns a **complex structure** which the caller will
at some point deconstruct),
though a sufficiently smart compiler should be able to do the elimination

https://en.wikibooks.org/wiki/Haskell/Continuation_passing_style

CPS (Continuation Passing Style)

An elementary way to take advantage of continuations
is to modify our functions
so that they return suspended computations
rather than ordinary values.

We will illustrate how that is done with two simple examples

https://en.wikibooks.org/wiki/Haskell/Continuation_passing_style

CPS (Continuation Passing Style)

Example: A simple module, no continuations

```
-- We assume some primitives add and square for the example:

add :: Int -> Int -> Int
add x y = x + y

square :: Int -> Int
square x = x * x

pythagoras :: Int -> Int -> Int
pythagoras x y = add (square x) (square y)
```

https://en.wikibooks.org/wiki/Haskell/Continuation_passing_style

CPS (Continuation Passing Style)

Example: A simple module, using continuations

```
-- We assume CPS versions of the add and square primitives,
-- (note: the actual definitions of add_cps and square_cps are not
-- in CPS form, they just have the correct type)
```

https://en.wikibooks.org/wiki/Haskell/Continuation_passing_style

CPS (Continuation Passing Style)

```
add_cps :: Int -> Int -> ((Int -> r) -> r)
add_cps x y = \k -> k (add x y)

square_cps :: Int -> ((Int -> r) -> r)
square_cps x = \k -> k (square x)

pythagoras_cps :: Int -> Int -> ((Int -> r) -> r)
pythagoras_cps x y = \k ->
 square_cps x $ \x_squared ->
 square_cps y $ \y_squared ->
 add_cps x_squared y_squared $ k
```

https://en.wikibooks.org/wiki/Haskell/Continuation_passing_style

CPS (Continuation Passing Style)

```
fact x =
  if x <= 1 then 1 else x * fact (x - 1)

 fact 4
 4 * fact 3
 4 * (3 * fact 2)
 4 * (3 * (2 * fact 1))
 4 * (3 * (2 * 1))
 4 * (3 * 2)
 4 * 6
 24
```

Each call of fact is made with the promise that the value returned will be multiplied by the value of the parameter at the time of the call.

Thus fact is invoked with larger and larger control contexts as the calculation proceeds.

https://www.seas.upenn.edu/~cis552/13fa/lectures/FunCont.html

CPS (Continuation Passing Style)

```
fact_cps x k =
  if x <= 1 then k 1 else fact_cps (x - 1) (\v -> k (x * v))

fact_cps 4 id
fact_cps 3 (\v -> id (4 * v))
fact_cps 2 (\v' -> (\v -> id (4 * v)) (3 * v'))
fact_cps 1 (\v'' -> (\v' -> (\v -> id (4 * v)) (3 * v')) (2 * v''))
(\v'' -> (\v' -> (\v -> id (4 * v)) (3 * v')) (2 * v'')) 1
(\v' -> (\v -> id (4 * v)) (3 * v')) (2 * 1)
(\v -> id (4 * v)) (3 * (2 * 1))
id (4 * (3 * (2 * 1)))
(4 * (3 * (2 * 1)))
24
```

the **control context** is made explicit
in the continuation argument to **fact_cps**.
we never have a call to **fact_cps**
that is the argument
to some other computation.

Instead, each step remembers
what to do with the result
as a first-class function.

At the bottom of the recursion,
these continuations are evaluated.

using 'id' as the first continuation.

https://www.seas.upenn.edu/~cis552/13fa/lectures/FunCont.html

CPS (Continuation Passing Style)

When is a **function** written in continuation passing style?

No function call is allowed to return to its caller, ever.

Instead, it must always pass its result directly
to an explicit continuation.

https://www.seas.upenn.edu/~cis552/13fa/lectures/FunCont.html

CPS (Continuation Passing Style)

Every function takes an **extra argument** (a **callback**)
and passes its **return value** this callback.

When a function is ready to "return",
it invokes the "**current continuation**" **callback**
(provided by its caller) on the return value.

When calling functions written in **CPS-style**,
callers must also provide the "**continuation**", i.e.
a **function** that says what to do
with the result of the **function call**.

https://www.seas.upenn.edu/~cis552/13fa/lectures/FunCont.html

Existential types and forall

```
forall r. (a -> r) -> r

forall r. (forall a. a -> r) -> r

exists a. a
```

think a **callback function** **forall a. a -> r**

forall a. a -> Int
forall a. a -> String a caller chooses **type r**
forall a. a -> Double

The **caller** of the overall function
(a -> r) -> r
chooses any type **r**

The **body** of the overall function
(a -> r) -> r
chooses any type **a**

the **body** of the callback function
must handle for all type **a**

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

id function example

```
id :: forall a. a -> a
id x = x
```

for <u>any</u> possible type **a**, — *quantified over types*

a function whose type is **a -> a**

<u>can</u> <u>be</u> <u>implemented</u> — *a true logical statement*

id works for <u>all</u> **a**.

a will unify with (or will be fixed to) <u>any type</u>

that <u>caller</u> of **id** may <u>choose</u>.

universally quantified type variables

in a <u>type</u> <u>signature</u> are

existentially quantified

in a <u>function</u> <u>body</u>

https://markkarpov.com/post/existential-quantification.html

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

A type signature and a function body

universally quantified **type variables** in a type signature

will be fixed when the corresponding **function**

is used (called)

in a type signature, **a** is universally quantified

but in the **body** of the function

we know nothing about the **argument a**,

we cannot inspect the **argument a**

(**a** *is fixed when the function is used*)

id :: forall a. a -> a

id x = x

universally quantified type variables

existentially quantified in a function body

https://markkarpov.com/post/existential-quantification.html

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Lack of information in a function body

universally quantified **type variables** in a type signature

callers can pass (choose) anything to **id**

but due to the lack of information
about the **argument** in the body of **id**

a caller can only pass a value to **id**
without doing anything meaningful

So, **id x = x** is the only possible function of the type **a -> a**

id :: forall a. a -> a

id x = x

a **caller** chooses values for
universally quantified variables

in the **body** of a such function,
must handle any type values
which is given by a caller **:**
existentially quantified variable

https://markkarpov.com/post/existential-quantification.html

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Fictitious syntax ***exists a.***

An **existentially quantified type** could be better explained
using the fictitious ***exists a.*** syntax

exists a. **a -> a**

for a certain **type a**,
we can implement a **function** whose type is **a -> a**.

any function will do,
then the "**not**" function on **Bool** satisfies the type **a -> a**

func :: ***exists a.*** **a -> a**
func True = False
func False = True

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Function implementations and applications

the function implementation on booleans

func :: *exists a.* a -> a

func True = False

func False = True

but we cannot use (apply) it as the "**not**" function

because all we know about the **type a** is

that it exists.

Any information about which type it might be

has been discarded (i.e, is not used),

this means we can't apply **func** to any values

Existentials are always about

throwing type information away.

sometimes we want to work with **types**

that we don't know at compile time.

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Existential types and forall

in *pseudo*-Haskell:

(exists x. p x x) -> c ≅ forall x. p x x -> c

a function **p** that takes an **existential type x**
is equivalent to a **polymorphic function**
using a **universal quantifier forall x**

because the **function p** must be prepared
to handle any one of the types **x**
that may be encoded in the **existential type**. **exists x.**

Haskell does not need an existential quantifier

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Existential types and forall

a function that accepts a **sum type** must be implemented as
a **case** statement, with a **tuple of handlers**,
one for every type present in the sum.

Here, the sum type is replaced by a coend,
and a family of handlers becomes an end,
or a polymorphic function.

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

No direct existential types

This fact brings us back to **universal quantifiers**,
and the reason why Haskell doesn't have **existential types** directly
(***exists a.*** above is entirely fictitious)

since things with **existentially quantified types**
can only be used with **operations**
that have **universally quantified types**,

- for the **callers** of **myPrettyPrinter**
  b is existentially quantified
- in the **body** of **myPrettyPrinter**
  b is universally quantified

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Parametric polymorphism (1)

universal quantification is the default

any **type variables** in a **type signature** are
implicitly universally quantified,

```
id ::  a -> a
id :: forall a. a -> a
```

also known as **parametric polymorphism**
in some other languages (e.g., C#) known as **generics**.

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Parametric polymorphism (2)

Parametric polymorphism refers to
when **the type of a value** contains
one or more (unconstrained) **type variables**,
beginning with a lowercase letter
without constraints (nothing to the **left** of a =>)

so that **the value** may adopt ***any type***
that results from substituting those **type variables**
with **concrete types**.

data Maybe a = Just a | Nothing

Just 2.0 :: Maybe Double
Just 'a' :: Maybe Char
Just True :: Maybe Boolean

https://wiki.haskell.org/Polymorphism

Parametric polymorphism (3)

Polymorphic datatypes

```
data Maybe a    = Nothing | Just a
data List a     = Nil | Cons a (List a)
data Either a b = Left a  Right b
```

Polymorphic functions

```
reverse :: [a] -> [a]
fst :: (a, b) -> a
id :: a -> a
```

```
Just 2.0   :: Maybe Double
Just 'a'   :: Maybe Char
Just True  :: Maybe Boolean
```

http://sm-haskell-users-group.github.io/pdfs/Ben%20Deane%20-%20Parametric%20Polymorphism.pdf

Parametric polymorphism (4)

Since a **parametrically polymorphic value** does not know anything about the unconstrained **type variables**,

it must behave identically **for all type** (regardless of its **type**)
(related to universally quantification)

This is a somewhat limiting but extremely useful property known as **parametricity**.

```
data Maybe a    = Nothing | Just a

reverse :: [a] -> [a]
```

https://wiki.haskell.org/Polymorphism

Parametric polymorphism (5)

the function **id :: a -> a** contains

an unconstrained type variable **a** in its type,

and so can be used in a context requiring

Char -> Char or

Integer -> Integer or

(Bool -> Maybe Bool) -> (Bool -> Maybe Bool) or

any of a literally infinite list of other possibilities.

if a single **type variable** appears multiple times,

it must take the same **type** everywhere it appears

→ the **result type** of **id** must be the same as the **argument type**

https://wiki.haskell.org/Polymorphism

Quantified variable choice

A **variable** is universally quantified

when the consumer of the variable's expression

can choose what it will be.

A **variable** is existentially quantified

when the consumer of the variable's expression

has to deal with the fact that the choice was made for him.

consumers of **a function**

callers of a **function**	the **body** of such a **function**

Universally quantified variable:

the consumer chooses a value

Existentially quantified variable:

the choice is made for the consumer

https://markkarpov.com/post/existential-quantification.html

Quantified variables with forall

Both universally and existentially quantified variables

are introduced with **forall**.

There is no **exists** in Haskell.

In fact, it's not necessary.

https://markkarpov.com/post/existential-quantification.html

Making existentials – hiding type variables

```
data Something where
  Something :: forall a. a -> Something
```

one way to have **existentials** –

by putting **values** in **wrappers**

that “hide” **type variables** from **signatures**.

```
  Something  a          :: Something
```

the **type variable a** is hidden in the **type Something**

https://markkarpov.com/post/existential-quantification.html

Existential wrappers – data and type constructors

```
data Something where
  Something :: forall a.  a -> Something

  Something  a          :: Something

  Something 2.0         :: Something
  Something 'a'         :: Something
  Something True        :: Something
```

the constructor function Something return
data value of type Something

type constructor   data constructor

```
data Point a      = Pt a a
```

polymorphic type

```
Pt  2.0  3.0       :: Point Float
Pt  'a'  'b'       :: Point Char
Pt True False      :: Point Bool
```

type constructor +
bounded type parameter
: a concrete type

https://markkarpov.com/post/existential-quantification.html

Existential wrappers – pattern matching

```
data Something where
  Something :: forall a. a -> Something
```

```
findx :: Something -> Float
findx (Something x) -> x
```

X X

```
data Point a     = Pt a a
```

```
pointx :: Point Float -> Float
pointx (Pt x _) = x
```

```
pointy :: Point Float -> Float
pointy (Pt  _ y) = y
```

The **constructor** accepts any **a** we like,

but after construction we

lose the type information

and pattern matching afterwards only reveals

that there is some **a**,

but nothing regarding what it is.

https://markkarpov.com/post/existential-quantification.html

Existential wrappers – constructing and using a value

```
data Something where
  Something :: forall a. a -> Something
```

the constructor function Something return

existentially quantified data of type Something

Something a	:: Something
a data value is *constructed*	a data value is *used*
universally quantified **a**	existentially quantified **a**

a function parameter, pattern matching

```
Something 1     :: Something
Something 'a'   :: Something
Something 2.0   :: Something
```

https://markkarpov.com/post/existential-quantification.html

Returning existentially quantified data

- passing a value to **id**: (universally quantified)

  we can pass anything to **id** but we lack any information about the **argument** in the **body** of **id**.

- passing a value to **Something** (existentially quantified)

  existential wrappers

  - return **existentially quantified data** from a **function**.
  - avoid unification of **existentials** with *outer context*
  - avoid escaping of **type variables**.

id 1 :: Int
id 'a' :: Char
Id 2.0 :: Double

Something 1 :: Something
Something 'a' :: Something
Something 2.0 :: Something

findx (Something x) -> x

not possible !!!

cannot extract type variable a

https://markkarpov.com/post/existential-quantification.html

Returning existentially quantified data

- passing a value to **id**: (universally quantified)

  universally quantified variable

  the consumer chooses

  id :: forall a. a -> a

- passing a value to **Something** (existentially quantified)

  existentially quantified variable

  the choice is made for the consumer

  data Something where

  Something :: forall a. a -> Something

id Int :: Int

id Char :: Char

id Double :: Double

example consumer function

foo :: Something -> Int

foo x = …

x :: Something

type variable **a** is already chosen

could be one of these

Something 1 :: Something

Something 'a' :: Something

Something 2.0 :: Something

https://markkarpov.com/post/existential-quantification.html

Existential wrappers – similar forms

```
data Something where
  Something :: forall a.  a -> Something
```

```
data r where
  r :: forall a.  a -> r
```

```
forall r. ( forall a.  a -> r ) -> r
```

Assume the callback function name is r

the **type variable a** is <u>hidden</u> in the **type r**

• • •

```
Something 1    :: Something
Something 'a'  :: Something
Something 2.0  :: Something
  • • •
```

```
r   1     :: r
r   'a'   :: r
r   2.0   :: r
  • • •
```

```
r   1     :: Int
r   'a'   :: Int
r   2.0   :: Int
  • • •
r   1     :: Char
r   'a'   :: Char
r   2.0   :: Char
  • • •
r   1     :: Double
r   'a'   :: Double
r   2.0   :: Double
  • • •
```

https://markkarpov.com/post/existential-quantification.html

Existential wrappers – similar forms

```
data Something where
  Something :: forall a. a -> Something
```

```
data r where
  r :: forall a. a -> r
```

```
forall r. ( forall a. a -> r ) -> r
```

Assume the callback function name is r

the **type variable a** is <u>hidden</u> in the **type r**

r a	:: r
a data value is *constructed*	a data value is *used*
universally quantified **a**	existentially quantified **a**

the **type variable a** is <u>hidden</u> in the **type r**

• • •

https://markkarpov.com/post/existential-quantification.html

Existential wrappers – rank-2 type

forall r. (forall a.   a -> r) -> r

forall r.	*argument callback* *exponentially quantified **a***	-> r
	(forall a. a -> r) *universally quantified **a***	

Outer level

Inner level

Inner level	**Outer level**
callback function body	callback function as an argument
universally quantified **a**	existentially quantified **a**

the **type variable a** is <u>hidden</u> in the **type r**

https://markkarpov.com/post/existential-quantification.html

Existential types and forall

we can write the type

***exists a.* a**

as

forall r. (forall a. a -> r) -> r

for all **result types r**,

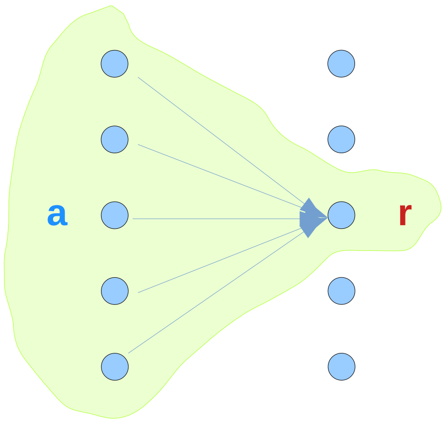
given a function **a -> r**

that takes an argument of **type a**, for all **types a**

and returns a value of **type r**,

we can get a result of **type r**

a caller supplies the callback function of the type **a -> r**



A caller supplies the callback function with the type **a -> r**

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Existential types and forall

we can write the type

***exists a.* a**

as

forall r. (forall a. a -> r) -> r

a caller supplies the callback function of the type **a -> r**

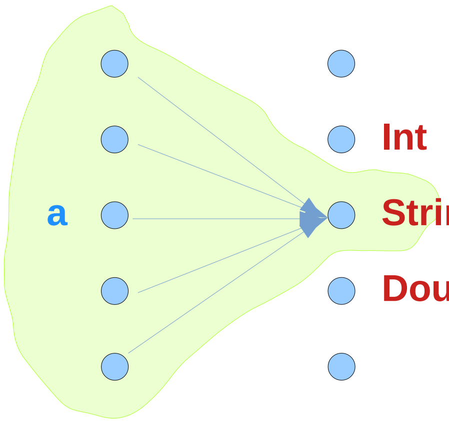
for a given **type r**

forall a. a -> Int

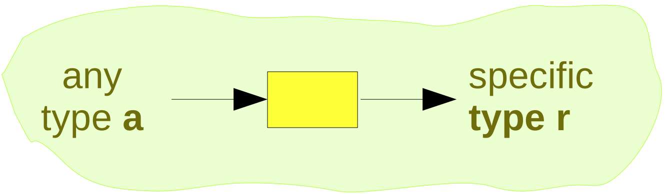
forall a. a -> String

forall a. a -> Double

a caller chooses **type r**



a caller of the overall type
determines the specific type r



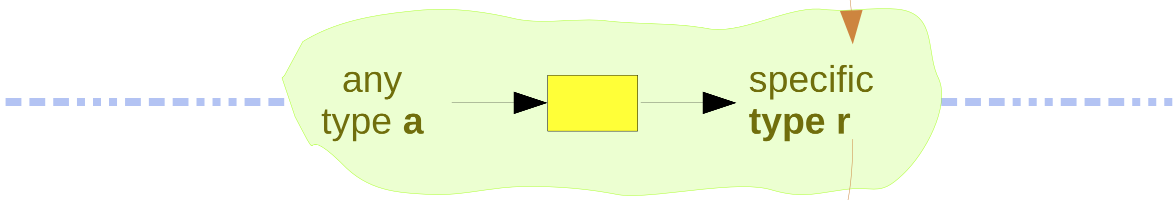
https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Existential types and forall

forall r. (forall a. a -> r) -> r

a caller of the overall type function
chooses the specific **type r**

universally quantified **r**



The body of the overall type function
must handle any **type r**

existentially quantified **r**

for the callers of the **function**	in the **body** of the **function**
universally quantified **r**	existentially quantified **r**
existentially quantified **a**	universally quantified **a**

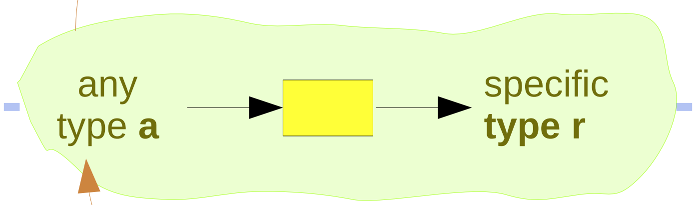
https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Existential types and forall

forall r. (forall a. a -> r) -> r

existentially quantified **a**

a caller of the overall type function
must handle any **type a**



universally quantified **a**

The body of the overall type function
chooses the specific **type a**

The body of the callback function
must also handle any **type a**

for the **callers** of the **function**	in the **body** of the **function**
universally quantified **r**	existentially quantified **r**
existentially quantified **a**	universally quantified **a**

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Existential types and forall

overall function type

forall r. (forall a. a -> r) -> r

universally quantified **r**

specific type a

existentially quantified **a**

Body

the overall type can choose whatever specific type **r**

the 1st argument of the **overall** type is a **callback** function
its 1st argument **a** is selected somehow in the **body** of the **overall** function

callback function type

Caller

forall r. (forall a. a -> r) -> r

universally quantified **a**

specific type r

existentially quantified **r**

the callback function type can choose whatever specific type **a**

the **caller** of the **overall** function supplies a **callback** function for a specific return type **r**

For the caller of the function

for the **callers** of the **function**
universally quantified **r**
existentially quantified **a**

For the body of the function

in the **body** of the **function**
existentially quantified **r**
universally quantified **a**

Existential types and forall

we can write the type

exists a. a

as

forall r. (forall a. a -> r) -> r

the overall type is _not_ **universally quantified** for **a**

only its argument **(forall a. a -> r) is universally quantified** for **a**

The overall type takes an **argument** … **(forall a. a -> r)**

that itself is **universally quantified** for **a**,

The overall type can then use

with whatever specific **type r** it chooses.

for the **callers** of the **function**		in the **body** of the **function**	
universally quantified	**r**	existentially quantified	**r**
existentially quantified	**a**	universally quantified	**a**

The overall type can choose
whatever specific type r
Universally quantified

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Existentially quantified data constructors (1)

data Foo = forall a. MkFoo a (a -> Bool) | Nil

the **data type** Foo has *two* **constructors** with types:

MkFoo :: forall a. a -> (a -> Bool) -> Foo

Nil :: Foo

Notice that the **type variable a** does not appear

in the type of **MkFoo** and

in the **data type** itself, Foo

Hidden

MkFoo 3 even :: Foo

MkFoo 'c' isUpper :: Foo

even :: Integer -> Bool

isUpper :: Char -> Bool

https://downloads.haskell.org/~ghc/6.6/docs/html/users_guide/type-extensions.html

Existentially quantified data constructors (2)

MkFoo :: forall a. a -> (a -> Bool) -> Foo

a valid expression example

[MkFoo 3 even, MkFoo 'c' isUpper] :: [Foo]

(**MkFoo 3 even**) packages an **integer** with a function

(**MkFoo 'c' isUpper**) packages a **character** with a function

Each of these are of type **Foo** and can be put in a list.

even :: Integer -> Bool

isUpper :: Char -> Bool

https://downloads.haskell.org/~ghc/6.6/docs/html/users_guide/type-extensions.html

Existentially quantified data constructors (3)

What can we do with a **value** of **type** Foo?.

In particular, what happens when we pattern-match on **MkFoo**?

```
f (MkFoo val fn) = ???
```

Since all we know about **val** and **fn** is that they are compatible,
the only (useful) thing we can do with them is
to apply **fn** to **val** to get a **boolean**.

cannot extract **val** and **fn**

```
f :: Foo -> Bool
fn ::  a -> Bool
f (MkFoo val fn) = fn val
```

https://downloads.haskell.org/~ghc/6.6/docs/html/users_guide/type-extensions.html

Existentially quantified data constructors (4)

```
data Foo   = forall a. MkFoo a (a -> Bool)  |   Nil
MkFoo :: forall a. a -> (a -> Bool) -> Foo

 [MkFoo 3 even,    MkFoo 'c' isUpper] :: [Foo]
```

What this allows us to do is
to package heterogenous **values** together
with a bunch of **functions** that manipulate them,
and then treat that collection of packages in a uniform manner.

In this way, you can express **object-oriented-like** programming

```
fn ::  a -> Bool

even ::  Integer -> Bool
isUpper ::  Char -> Bool
```

https://downloads.haskell.org/~ghc/6.6/docs/html/users_guide/type-extensions.html

Unknown types at compile time

Existentials have always to do with

throwing type information away.

sometimes we want to work with **types**

that we don’t know at compile time.

the **types** typically depend on the **state** of **external world**:

the **types** could depend on user’s input,

on contents of a file to be parsed, etc.

Haskell’s type system is powerful enough in these cases

https://markkarpov.com/post/existential-quantification.html

Preserving information about existentials

We want to work with **values** of **types**

that we don't know at compile time,

but at run time there are **no types** at all:

they have been erased!

then we have to *preserve* some information

about existentially quantified type to make use of it,

otherwise we'll be in the same position as implementers of **id**

having a **value** and only being able to pass it around

never doing anything meaningful with it.

There are various degrees of how much we might want to *preserve*:

https://markkarpov.com/post/existential-quantification.html

Parameterizing another type

We could have **a** in the type **[a]** existentially quantified.

There are still some things we could do with a **value** of this type.

we could compute length of the list.

So knowing nothing about **a** type is also an option sometimes

when it parameterizes **another type** and

we have parametrically-polymorphic functions

that work on that type.

In this case the set of possible types for **a** is open i.e. it can grow.

https://markkarpov.com/post/existential-quantification.html

Existentially quantified type with **constraints**

```
data Showable where
  Showable :: forall a. Show a => a -> Showable
```

We could assume that the existentially quantified type has *certain properties* (instances):

- pattern-matching on **Showable** will give us the corresponding dictionary back.
- can do as much as the knowledge about the attached **constraint**
- the set of possible types for **a** is open (additional new **instances** of **Show** can be defined).

```
data Something where
  Something :: forall a. a -> Something
```

simple existentially quantified type variable

https://markkarpov.com/post/existential-quantification.html

The first **forall** at the type signature

```
myPrettyPrinter
  :: forall a. Show a =>
       (forall b. Show b => b -> String)
       -> Int
       -> Bool
       -> a
       -> String
```

Only **variables** with **forall**s <u>at the beginning</u> of **type signature** will be <u>fixed</u> when the corresponding **function** is <u>used</u>

Other **forall**s deal with independent **type variables**:

forall a. * (forall b. ***)**

when **myPrettyPrinter** is used

a will be *fixed*

but not **b**

the 1st argument is

a call back function

b -> String

https://markkarpov.com/post/existential-quantification.html

Two levels of **forall**s

```
myPrettyPrinter
  :: forall a. Show a =>
       (forall b. Show b => b -> String)    -- call back function
                                  -> Int
                                  -> Bool
                                  -> a
                                  -> String
```

two levels of **forall**s (**rank-2 type**)

forall a. * (forall b. ***)**

in general such constructions are called **rank-N types**.

https://markkarpov.com/post/existential-quantification.html

For consumers of a function

Both universally and existentially quantified variables are introduced with **forall**.

for callers of **myPrettyPrinter**

- **a** is universally quantified
  we can choose what the type will be

- **b** is existentially quantified
  the callback function has to prepare to deal with any **b**
  that will be given to the callback **b -> String**

```
myPrettyPrinter
  :: forall a. Show a =>
       (forall b. Show b => b -> String)
       -> Int
       -> Bool
       -> a
       -> String
```

callers of **myPrettyPrinter** provide
the call back **b -> String**
which must handle any **b**

https://markkarpov.com/post/existential-quantification.html

For consumers of a function

```
print (myPrettyPrinter  callback 123 True )
```

Consumers of the expression 1

```
myPrettyPrinter fn i t x =

      ...  fn 0.8   ...

  Consumers of the expression 2

                        return str
```

fn :: b -> String

i :: Int

t :: Bool

x :: a

str :: String

```
myPrettyPrinter
  :: forall a. Show a =>
       (forall b. Show b => b -> String)
       -> Int
       -> Bool
       -> a
       -> String
```

https://markkarpov.com/post/existential-quantification.html

In the body of a function

- for the **callers** of **myPrettyPrinter**, **a** is universally quantified
- in the **body** of **myPrettyPrinter**, **a** is existentially quantified
  - → the caller of **myPrettyPrinter** *already has chosen* the **type**
  - → A specific return type of the callback function **b -> String**

- for the **callers** of **myPrettyPrinter**, **b** is existentially quantified
- in the **body** of **myPrettyPrinter**, **b** is universally quantified
  - → **b** is the first **argument** of the call back function **b -> String**
  - → when the call back function is applied with **b**
    the body of **myPrettyPrinter** *can choose* its **concrete type**

b -> String-> Int-> Bool-> a-> String

```
myPrettyPrinter
  :: forall a. Show a =>
       (forall b. Show b => b -> String)
       -> Int
       -> Bool
       -> a
       -> String
```

Universally quantified variable
the consumer choose

Existentially quantified variable
the choice is made for the consumer

https://markkarpov.com/post/existential-quantification.html

Existential types and forall

```
forall r.
   (forall a. a -> r)
   -> r
```

for the **callers** of the **function**	in the **body** of the **function**
universally quantified **r**	existentially quantified **r**
existentially quantified **a**	universally quantified **a**

```
myPrettyPrinter
 :: forall a. Show a =>
     (forall b. Show b => b -> String)
     -> Int
     -> Bool
     -> a
     -> String
```

for the **callers** of the **function**	in the **body** of the **function**
universally quantified **a**	existentially quantified **a**
existentially quantified **b**	universally quantified **b**

callers of **myPrettyPrinter** provide
the call back function **b -> String**
which must handle any **b**

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Subtyping

subtyping (also **subtype polymorphism**)
is a form of type polymorphism in which a subtype is a datatype
that is related to another datatype (the supertype)
by some notion of substitutability,
meaning that program elements,
typically subroutines or functions,
written to operate on elements of the supertype
can also operate on elements of the subtype.

https://en.wikipedia.org/wiki/Subtyping

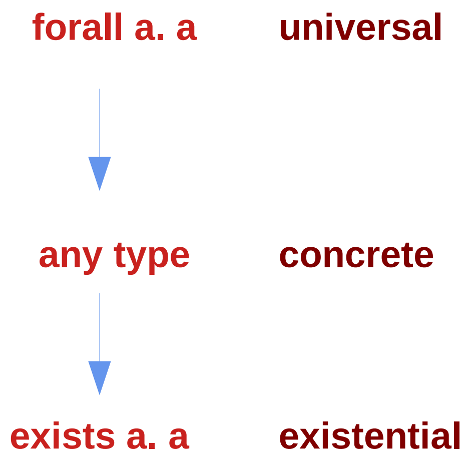
Existential types and forall

Haskell doesn't have a notion of **subtyping**

Quantifiers can be considered as a tool for **subtyping**,
with a **hierarchy** going from **universal** to **concrete** to **existential**.

type forall a. a could be converted to **any other type**,
so it could be seen as a **subtype** of everything;

any type could be converted to the **type exists a. a**,
making that a **supertype** of everything.



https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Existential types and forall

forall a. a is impossible

there are no values of type **forall a. a** except errors

exists a. a is useless

you canot do anything with the type **exists a. a**

but the analogy works on paper at least.

So, the basic idea is roughly that

universally quantified types describe

things that work the same for **any type**,

existentially quantified types describe

things that work with a **specific** but **unknown** type.

forall a. a subtype of everything

↓ impossible – no such value

any type

↓ useless – cannot do anything

exists a. a supertype of everything

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Restoring exact types

```
data EType a where
    ETypeWord8      :: EType Word8
    ETypeInt        :: EType Int
    ETypeFloat      :: EType Float
    ETypeDouble     :: EType Double
    ETypeString     :: EType String

data Something where
    Something :: EType a -> a -> Something
```

We could use GADTs to restore exact types of existentially quantified variables later:

https://markkarpov.com/post/existential-quantification.html

How to make use of existentials

Matching on one of the **data constructors** of **EType** reveals **a** and after that we are free to do anything with the **value** of corresponding **type** because we know it.

With this approach the set of possible types for **a** is limited and closed.

It can be expanded by changing the **definition** of **EType** though.

```
data EType a where
    ETypeWord8      :: EType Word8
    ETypeInt        :: EType Int
    ETypeFloat      :: EType Float
    ETypeDouble     :: EType Double
    ETypeString     :: EType String

data Something where
    Something
      :: EType a -> a -> Something
```

https://markkarpov.com/post/existential-quantification.html

Generalized Algebraic Data Type (1)

Generalised Algebraic Data Types

generalise ordinary algebraic data types

by allowing you to give the **type signatures** of **constructors** explicitly.

```
data Term a where
    Lit    :: Int                  -> Term Int
    Succ   :: Term Int             -> Term Int
    IsZero :: Term Int             -> Term Bool
    If     :: Term Bool            -> Term a -> Term a -> Term a
    Pair   :: Term a -> Term b     -> Term (a,b)
```

https://downloads.haskell.org/~ghc/6.6/docs/html/users_guide/gadt.html

Generalized Algebraic Data Type (2)

Notice that the **return type** of the constructors is not always **Term a**,

as is the case with ordinary vanilla data types.

Now we can write a well-typed **eval** function for these Terms:

```
eval :: Term a -> a
eval (Lit i)           = i
eval (Succ t)          = 1 + eval t
eval (IsZero t)        = eval t == 0
eval (If b e1 e2)      = if eval b then eval e1 else eval e2
eval (Pair e1 e2)      = (eval e1, eval e2)
```

https://downloads.haskell.org/~ghc/6.6/docs/html/users_guide/gadt.html

Existential Quantification

https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do

Existentials

Existential types, or

Existentials for short,

provide a way of

squashing a group of types

into one, single type.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Existentials

Existentials are part of GHC's type system **extensions**.

But not part of **Haskell98**

have to either compile with a command-line parameter of

-XExistentialQuantification,

or put at the top of your sources that use existentials.

{-# LANGUAGE ExistentialQuantification #-}

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

forall and type variables

The **forall** keyword is to explicitly bring fresh **type variables** into scope

type variables :

those variables that begin with a **lowercase** letter

the compiler allows **any type** to fill these variables

those variables that are **universally quantified**

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Type variables in a polymorphic function

Example: A polymorphic function

map :: (a -> b) -> [a] -> [b]

a **lowercase type parameter**

implicitly begins with a **forall** keyword,

Example: Explicitly quantifying the type variables

map :: forall a b. (a -> b) -> [a] -> [b]

two type declarations for map are equivalent

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Instantiating type variables

Example: A polymorphic function

map :: (a -> b) -> [a] -> [b]

Example: Explicitly quantifying the type variables

map :: forall a b. (a -> b) -> [a] -> [b]

instantiating the general type of **map**

to a more specific type

a = Int

b = String

(Int -> String) -> [Int] -> [String]

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Hiding a type variable

https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do

A rule for creating a new type

Normally when creating a new type

using **type**, **newtype**, **data**, etc.,

every **type variable** that appears on the right-hand side

must also appear on the left-hand side.

```
newtype ST s a = ST (State# s -> (# State# s, a #))
```

Existential types are a way of escaping this rule

Existential types can be used for several different purposes.

But what they do is to **hide** a **type variable** on the right-hand side.

https://wiki.haskell.org/Existential_type

Not specifying a type variable

Normally, any **type variable** appearing on the right
must also appear on the left:

data Worker x y = Worker {buffer :: b, input :: x, output :: y}

This is an **error**, since the **type b** of the **buffer**
is not specified on the right
(**b** is a **type variable** rather than a **type**)
but also is not specified on the left
(there's no **b** in the left part).

In **Haskell98**, you would have to write

data Worker b x y = Worker {buffer :: b, input :: x, output :: y}

Record Access Functions

```
buffer   :: Worker x y -> b
input    :: Worker x y -> x
output   :: Worker x y -> y
```

https://wiki.haskell.org/Existential_type

A type variable and a class

```
data Worker b x y = Worker {buffer :: b, input :: x, output :: y}
```

However, suppose that a **Worker** can use any type **b**

so long as it belongs to some particular **class**.

Then every **function** that uses a **Worker** will have a type like

```
foo :: (Buffer b) => Worker b Int Int
```

In particular, failing to write an **explicit type signature** **(Buffer b)**

will invoke the dreaded **monomorphism restriction**.

Using **existential types**, we can avoid this:

https://wiki.haskell.org/Existential_type

Explicit types and Existential types

Explicit type signature :

```
data Worker b x y = Worker {buffer :: b, input :: x, output :: y}
foo :: (Buffer b) => Worker b Int Int
```

Existential type :

```
data Worker x y =  forall b. Buffer b =>  Worker {buffer :: b, input :: x, output :: y}
foo :: Worker Int Int
```

The **type** of the **buffer** (**Buffer**) now does not appear in the **Worker** type at all. **Worker x y**

https://wiki.haskell.org/Existential_type

Monomorphism restriction

The **monomorphism restriction** is a counter-intuitive rule
in Haskell type inference.

If you *forget to provide* a **type signature**,
sometimes this rule will fill the free type variables
with specific types using **type defaulting** rules.

always less polymorphic than you'd expect,
so often this results in **type errors**
when you expected it to infer a perfectly sane type
for a polymorphic expression.

https://wiki.haskell.org/Existential_type

Monomorphism restriction example

A simple example is **plus = (+)**.

Without an explicit signature for **plus**,

the compiler will not infer the type for **plus**

(+) :: (Num a) => a -> a -> a

but will apply **defaulting rules** to specify

plus :: Integer -> Integer -> Integer

When applied to **plus 3.5 2.7**, GHCi will then produce

the somewhat-misleading-looking error,

No instance for (Fractional Integer) arising from the literal ‘3.5’.

https://wiki.haskell.org/Existential_type

Existential types and forall

func is a function with the same **type** for its **input** and **output**
so we could compose it with itself, for example.

the only things you can do with something
that has an **existential type** are
the things you can do based on the **non-existential parts** of the **type**.

Similarly, given something of type **exists a. [a]**
we can find its length, or concatenate it to itself,
or drop some elements, or anything else we can do to **any list**.

```
func :: exists a. a -> a
func True = False
func False = True
```

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Existential types and forall

an example of an **existentially quantified type**

data Sum = forall a. Constructor a

forall a. (Constructor_a:: a -> Sum) ≅ Constructor:: (exists a. a) -> Sum

data Sum = int | char | bool | ....

an example of a **universally quantified type**

data Product = Constructor (forall a. a)

data Product = int char bool ....

https://stackoverflow.com/questions/14299638/existential-vs-universally-quantified-types-in-haskell

Hiding a type variable (5)

- it is now impossible for a function
  to demand a **Worker** having a specific type of **buffer**.

- the **type** of **foo** can now be derived automatically
  without needing an explicit **type signature**.
  (No **monomorphism** restriction.)

- since code now has no idea
  what **type** the **buffer** function returns,
  you are more limited in what you can do to it.

```
data Worker x y =  forall b. Buffer b =>  Worker {buffer :: b, input :: x, output :: y}
foo :: Worker Int Int
```

https://wiki.haskell.org/Existential_type

Hiding a type variable (6)

you will usually want a **hidden type** to belong to a **specific class**,
or you will want to **pass some functions** along
that can work on that type.

Otherwise you'll have some value belonging
to a **random unknown type**,
and you won't be able to do anything to it!

```
data Worker x y = forall b. Buffer b => Worker {buffer :: b, input :: x, output :: y}
foo :: Worker Int Int
```

https://wiki.haskell.org/Existential_type

Hiding a type variable (7)

This illustrates **creating a heterogeneous list**,
all of whose members implement **Show**
and progressing through that list to show these items:

```
data Obj = forall a. (Show a) => Obj a

    xs :: [Obj]
    xs = [Obj 1, Obj "foo", Obj 'c']

    doShow :: [Obj] -> String
    doShow [] = ""
    doShow ((Obj x):xs) = show x ++ doShow xs
```

With output: **doShow xs ==> "1\"foo\"'c'"**

https://wiki.haskell.org/Existential_type

Hiding a type variable (7)

In Haskell, an existential data type is one
that is defined in terms not of a concrete type,
but in terms of a quantified type variable,
introduced on the right-hand side of the data declaration.

https://blog.sumtypeofway.com/posts/existential-haskell.html

Hiding a type variable (7)

an existential type provides
a well-typed "box" around an unspecified type.

The box does "hide" the type in a sense,
which allows you to make a heterogeneous list of such boxes,
ignoring the types they contain.

It turns out that an unconstrained existential pretty useless,
but a constrained type allows you to pattern match
to peek inside the "box" and make the type class facilities available:

https://blog.sumtypeofway.com/posts/existential-haskell.html

Less specific types

Note: You can use **existential types**

to **convert** a **more specific type**

into a **less specific one**.

constrained type variables

There is no way to perform the reverse conversion!

https://wiki.haskell.org/Existential_type

Existentials in terms of **forall** (1)

It is also possible to express existentials with **RankNTypes**
as **type expressions** directly (without a **data** declaration)

forall r. (forall a. Show a => a -> r) -> r

(the leading **forall r.** is optional
unless the expression is part of another expression).

the equivalent type **Obj** :

data Obj = forall a. (Show a) => Obj a

https://wiki.haskell.org/Existential_type

Existentials in terms of **forall** (2)

The conversions are:

```
fromObj ::  Obj -> forall r. (forall a. Show a => a -> r) -> r
fromObj (Obj x) k = k x

toObj :: (forall r. (forall a. Show a => a -> r) -> r)  ->  Obj
toObj f = f Obj
```

https://wiki.haskell.org/Existential_type

Heterogeneous Lists

https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do

Type hider

Suppose we have a group of values.

they may not be all the same **type**,

but they are all members of some **class**

thus, they have a certain **property**

It might be useful to throw all these values into a list.

normally this is impossible because lists elements

must be of the same type

(homogeneous with respect to types).

existential types allow us to loosen this requirement

by defining a **type hider** or **type box**:

```
data ShowBox = forall s. Show s => SB s
heteroList :: [ShowBox]
heteroList = [SB (), SB 5, SB True]
```

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Heterogeneous list example (1)

```
data ShowBox = forall s. Show s => SB s          -- type hider
heteroList :: [ShowBox]
heteroList = [SB (), SB 5, SB True]
```

[SB (), SB 5, SB True] calls the **constructor**

on three values of different types,

to place them all into a single list

virtually the same type for each one.

Use the **forall** in the **constructor**

SB :: forall s. Show s => s -> ShowBox.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Heterogeneous list example (2)

```
data ShowBox = forall s. Show s => SB s            -- type hider
heteroList :: [ShowBox]
heteroList = [SB (), SB 5, SB True]
```

When passing **heteroList type parameters** to a function

we cannot take out the **values** inside the **SB**

because their type might **Bool**. **Int**, **Char**, …

But each of the elements can be

converted to a **string** via **show**.

In fact, that's the only thing we know about them.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Heterogeneous list example (3)

```
instance Show ShowBox where
  show (SB s) = show s
```

In the definition of **show** for **ShowBox**
we don't know the **type** of **s**.

But we do know that the **type** is an **instance** of **Show**
due to the **constraint** on the **SB** **constructor**.

Therefore, it's legal to use the function **show** on **s**,
as seen in the right-hand side of the function definition.

ShowBox data type made into
an instance of the **Show** class
by this **instance declaration**:

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Heterogeneous list example (4)

```
instance Show ShowBox where
  show (SB s) = show s

 f :: [ShowBox] -> IO ()
 f xs = mapM_ print xs

main = f heteroList

heteroList :: [ShowBox]
heteroList = [SB (), SB 5, SB True]
```

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Heterogeneous list example (5)

Example: Using our heterogeneous list

```
instance Show ShowBox where
  show (SB s) = show s
f :: [ShowBox] -> IO ()
f xs = mapM_ print xs
main = f heteroList
```

Example: Types of the functions involved

```
print :: Show s => s -> IO ()        -- print x = putStrLn (show x)
mapM_ :: (a -> m b) -> [a] -> m ()
mapM_ print :: Show s => [s] -> IO ()
```

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

mapM, **mapM_**, and **map** (1)

mapM maps an "**action**" (ie function of type **a -> m b**)
over a **list [a]** and gives you all the results as **m [b]**

mapM_ does the same thing,
but never collects the results, returning a **m ()**.

If you care about the results
of your **a -> m b** function, use **mapM**.
If you only care about the effect,
but not the resulting value,
use **mapM_**, because it can be more efficient

https://stackoverflow.com/questions/27609062/what-is-the-difference-between-mapm-and-mapm-in-haskell/27609146

mapM, **mapM_**, and **map** (2)

Always use **mapM_** with functions of the type **a -> m ()**,

like **print** or **putStrLn**.

these functions return **()** to signify that only the effect matters.

If you used **mapM**, you'd get a list of **()** (ie **[(), (), ()]**),

which would be completely useless

but waste some memory.

If you use **mapM_**, you would just get a **()**,

but it would still print everything.

https://stackoverflow.com/questions/27609062/what-is-the-difference-between-mapm-and-mapm-in-haskell/27609146

mapM, **mapM_**, and **map** (3)

Normal **map** is something different:

it takes a normal function **(a -> b)**

instead of one using a monad **(a -> m b)**.

This means that it cannot have any sort of effect

besides returning the changed list.

You would use it if you want to transform a list

using a normal function.

map_ doesn't exist because, since you don't have any effects,

you always care about the results of using **map**.

https://stackoverflow.com/questions/27609062/what-is-the-difference-between-mapm-and-mapm-in-haskell/27609146

Quantified types

as products and sums

https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do

Quantified Types as Products and Sums

A universally **quantified type** may be interpreted

as an **infinite product** of **types**.

a **polymorphic function** can be understood

as a **product**, or a **tuple**, of **individual functions**,

one per every possible **type a**.

To construct a **value** of such **type**, we have

to provide all the **components** of the **tuple** at once.

-- one formula generating an infinity of functions

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Quantified Types as Products and Sums

Example: Identity function

id :: forall a. a -> a

id a = a

a **polymorphic function** can be understood

as a **product**, or a **tuple**, of **individual functions**,

one per every possible **type a**.

Int -> Int,

Double -> Double,

Char -> Char,

[Char] -> [Char],

…

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Quantified Types as Products and Sums

To construct a value of such type, we have

to provide all the components of the tuple at once.

in case of numeric types, one numeric constant

may be used to initialize many types at once.

Example: Polymorphic value

```
x :: forall a. Num a => a
x = 0
```

x may be conceptualized as a **tuple** consisting

of an **Int value**, a **Double value**, etc.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Quantified Types as Products and Sums

Similarly, an **existentially** **quantified type** may be interpreted as an **infinite sum**.

Example: Existential type

```
data ShowBox = forall s. Show s => SB s                -- type hider
```

may be conceptualized as a **sum**:

Example: Sum type

```
data ShowBox = SBUnit | SBInt Int | SBBool Bool | SBIntList [Int] | ...
```

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Quantified Types as Products and Sums

Example: Existential type

data ShowBox = forall s. Show s => SB s -- type hider

Example: Sum type

data ShowBox = SBUnit | SBInt Int | SBBool Bool | SBIntList [Int] | ...

to construct a **value** of this **type**,

we only have to pick one of the constructors

(**SBUnit**, **SBInt**, **SBBool**, **SBIntList** ...)

A **polymorphic constructor SB**

combines all those constructors into one.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Quantification as a primitive

https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do

Pair type example (1)

Existential quantification is useful

for defining **data types** that aren't already defined.

Suppose there was no such thing as **pairs** built into haskell.

Existential quantification could be used to define them.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (2)

```
{-# LANGUAGE ExistentialQuantification, RankNTypes #-}

newtype Pair a b = Pair (forall c. (a -> b -> c) -> c)

makePair :: a -> b -> Pair a b
makePair a b = Pair $ \f -> f a b
```

Defining a **data type c** that is not already defined

Pair $ \f -> f a b :: Pair a b

f :: a -> b -> c

f a b :: c

f is not yet defined

c can be any type (**forall c**)

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (3)

```
newtype Pair a b = Pair (forall c. (a -> b -> c) -> c)
```

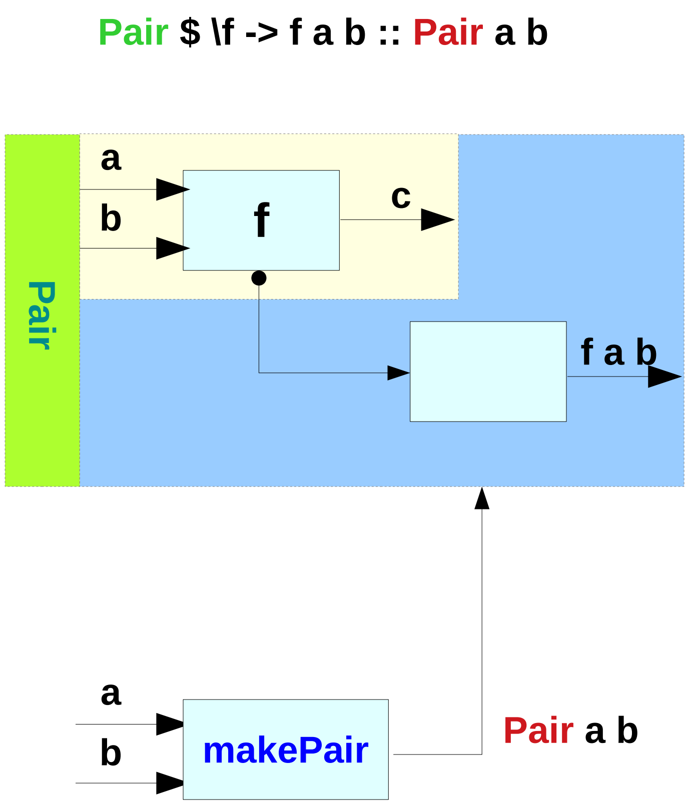
every type variable that appears on the right-hand side must also appear on the left-hand side.

Existential type hides a type variable c on the right-hand side.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (4)

```
newtype Pair a b = Pair (forall c. (a -> b -> c) -> c)


makePair :: a -> b -> Pair a b
makePair a b = Pair $ \f -> f a b
```



https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (5)

```
newtype Pair a b = Pair (forall c. (a -> b -> c) -> c)

makePair :: a -> b -> Pair a b
makePair a b = Pair $ \f -> f a b
```

using a **record type** with a **single field**

```
newtype Pair a b = Pair {runPair :: forall c. (a -> b -> c) -> c}
```

runPair is an **access function**

takes an input of the type **Pair a b**

returns an output of the type **forall c. (a -> b -> c) -> c**

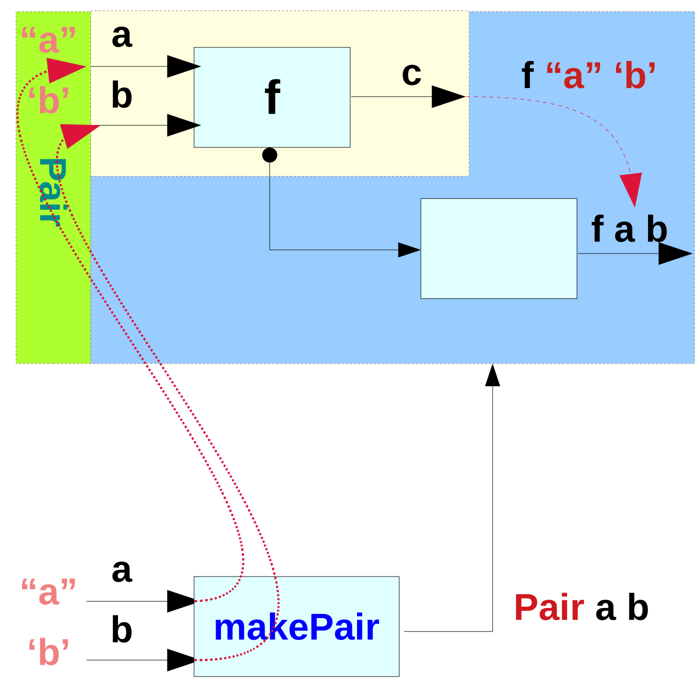
https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (6)

In GHCI

```
λ> :set -XExistentialQuantification
λ> :set -XrankNTypes
λ> newtype Pair a b = Pair {runPair :: forall c. (a -> b -> c) -> c}
λ> makePair a b = Pair $ \f -> f a b
λ> pair = makePair "a" 'b'

λ> :t pair
 pair :: Pair [Char] Char
λ> runPair pair (\x y -> x)     -- unwrap (a -> b -> c) -> c then apply
 "a"
λ> runPair pair (\x y -> y)    -- unwrap (a -> b -> c) -> c then apply
 'b'
```

Pair $ \f -> f a b :: Pair a b



makePair "a" 'b'

Pair $ \f -> f "a" 'b' :: Pair a b

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (7)

λ> newtype Pair a b = Pair {runPair :: forall c. (a -> b -> c) -> c}

λ> makePair a b = Pair $ \f -> f a b

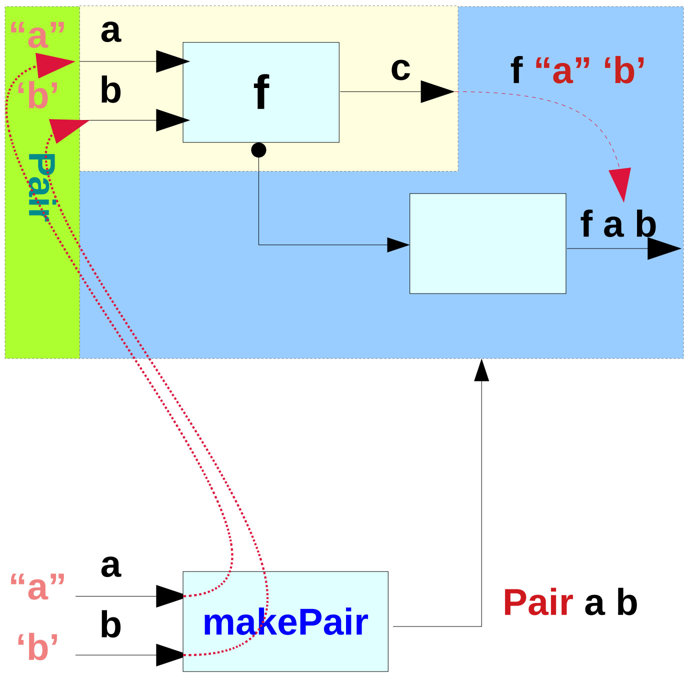
λ> pair = makePair "a" 'b'

Pair $ \f -> f "a" 'b'

\f : function itself   f :: a -> b -> c

f "a" 'b' : the result of applying the function

Pair $ \f -> f a b :: Pair a b



makePair "a" 'b'

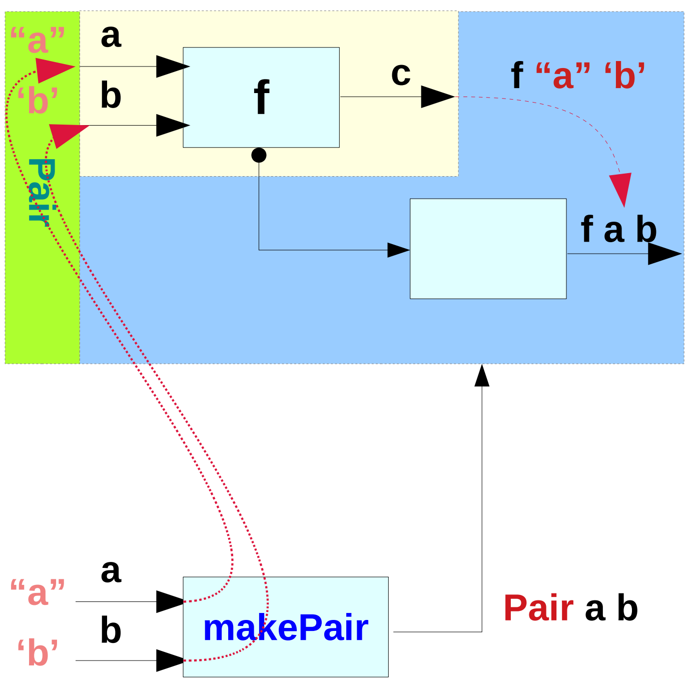
Pair $ \f -> f "a" 'b' :: Pair a b

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (8)

```
newtype Pair a b = Pair {runPair :: forall c. (a -> b -> c) -> c}
runPair ::  Pair a b -> forall c. (a -> b -> c) -> c

makePair a b = Pair $ \f -> f a b
runPair makePair a b = \f -> f a b              -- unwrapping

makePair "a" 'b'  = Pair $ \f -> f "a" 'b'
runPair makePair "a" 'b' = \f -> f "a" 'b'

pair = makePair                          :: Pair [Char] Char

runPair  pair  (\x y -> x) = (\x y -> x) "a" 'b'
runPair  pair  (\x y -> y) = (\x y -> y) "a" 'b'
```

Pair $ \f -> f a b :: Pair a b



makePair "a" 'b'

Pair $ \f -> f "a" 'b' :: Pair a b

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

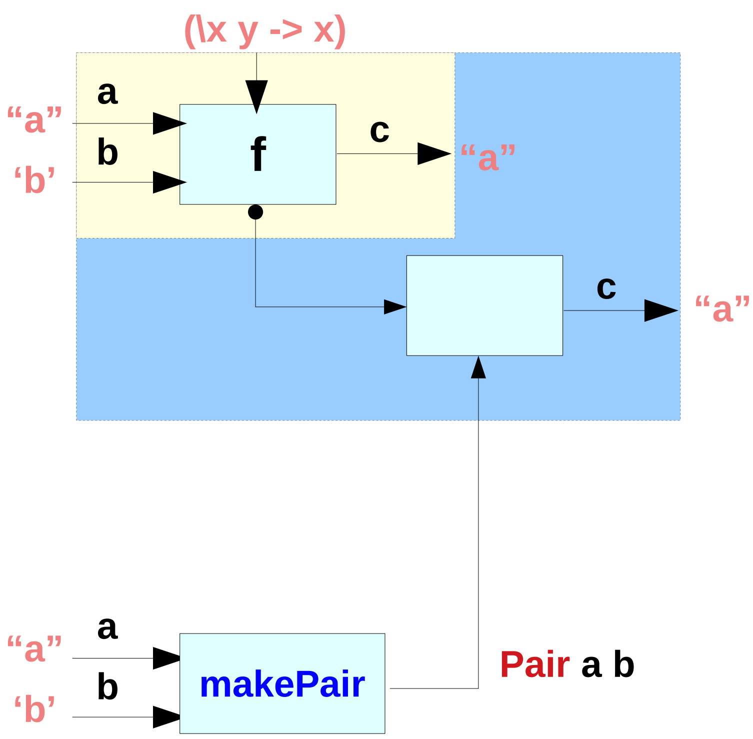
Pair type example (9)

```
runPair  pair  (\x y -> x) = (\x y -> x) "a" 'b'
runPair  pair  (\x y -> y) = (\x y -> y) "a" 'b'

runPair makePair "a" 'b' (\x y -> x)
(\x y -> x) "a" 'b'
 "a"

runPair makePair "a" 'b' (\x y -> y)
(\x y -> y) "a" 'b'
 'b'
```

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (10)

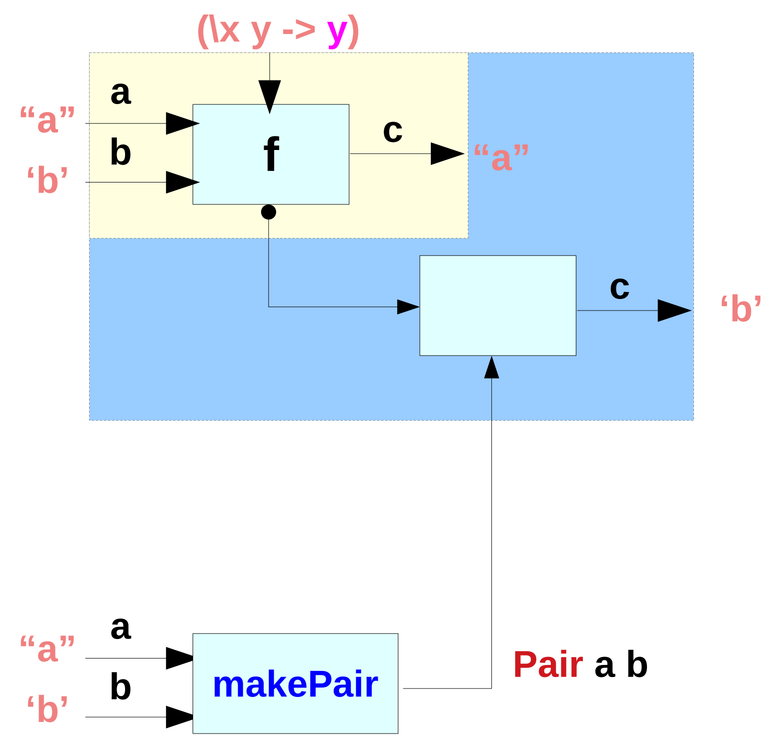
Pair $ \f -> f a b :: Pair a b



pair (\x y -> x)

makePair "a" 'b' (\x y -> x)

Pair $ \f -> f a b :: Pair a b



pair (\x y -> y)

makePair "a" 'b' (\x y -> y)

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

newtype and an access function

https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do

newtype can have a named function (1)

newtype Parser a = Parser { parse :: String -> Maybe (a,String) }

1) A **type** named **Parser**.

2) A **term level constructor** of **Parser's** named **Parser**.

The **type** of this (constructor) function is

Parser :: (String -> Maybe (a, String)) -> Parser a

You give it a function of the type

(String -> Maybe (a, String))

and it wraps it inside a **Parser**

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

newtype can have a named function (2)

```
newtype Parser a = Parser { parse :: String -> Maybe (a,String) }
```

3) A **function** named **parse** to remove the **Parser** wrapper and get your function back. The type of this function is:

parse :: Parser a -> String -> Maybe (a, String)

A **term level constructor** named **Parser**

Parser :: (String -> Maybe (a, String)) -> Parser a

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

newtype – constructor and unwrap functions (1)

```
Prelude> newtype
            Parser a = Parser { parse :: String -> Maybe (a,String) }

Prelude> :t Parser
Parser :: (String -> Maybe (a, String)) -> Parser a

Prelude> :t parse
parse :: Parser a -> String -> Maybe (a, String)
```

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

newtype – constructor and unwrap functions (2)

```
newtype Parser a = Parser { parse :: String -> Maybe (a,String) }
```

the **term level constructor** (**Parser**)

the **function** to remove the wrapper (**parse**)

Both can have arbitrary names

No need to match the type name.

It's common to write:

```
newtype Parser a = Parser { unParser :: String -> Maybe (a,String) }
```

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

newtype – constructor and unwrap functions (3)

```
newtype Parser a = Parser { unParser :: String -> Maybe (a,String) }
```

this name makes it clear **unParser** removes

the **wrapper** around the parsing function.

```
unParser :: Parser a -> String -> Maybe (a, String)
```

however, it is recommended that the **type** and **constructor**

have the same name when using **newtypes**.

(**Parser**, **Parser**)

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

newtype – instantiation

```
newtype Parser a = Parser { parser :: String -> Maybe (a,String) }
```

1) **Parser** is declared as a **type** with a **type parameter a**

2) can instantiate **Parser** by providing a **parser** function

```
p = Parser  (\s -> Nothing)
```

3) a function name **parser** defined and

it is capable of *running Parser's*.

unwrap the function

then apply the function

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

newtype – unwrapping

```
newtype Parser a = Parser { parser :: String -> Maybe (a,String) }

parser :: Parser a -> String -> Maybe (a, String)
parser (Parser  (\s -> Nothing)) "my input"
(\s -> Nothing)) "my input"
Nothing
```

You are unwrapping the function using **parse** and
then calling the unwrapped function with "myInput".

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

newtype – without record syntax (1)

First, let's have a look at a parser **newtype** <u>without</u> **record** syntax:

```
newtype Parser' a = Parser' (String -> Maybe (a,String))
```

it <u>stores</u> a **function** **String -> Maybe (a,String)**.

To <u>run</u> this parser, we will need to make an **extra function:**

```
runParser' :: Parser' a -> String -> Maybe (a,String)
runParser' (Parser' f) i = f i
```

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

newtype – without record syntax (2)

```
runParser' :: Parser' a -> String -> Maybe (a,String)
runParser' (Parser' f) i = f i

runParser' (Parser' $ \s -> Nothing) "my input".
```

But now note that, since Haskell functions are curried,

we can simply remove the reference to the input **i** to get:

```
runParser'' :: Parser' -> (String -> Maybe (a,String))
runParser'' (Parser' f') = f'
```

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

newtype – without record syntax (3)

```
runParser'' :: Parser' -> (String -> Maybe (a,String))
runParser'' (Parser' f’) = f’
```

This function is exactly equivalent to **runParser'**,

but you could think about it differently:

instead of applying the parser function to the value explicitly,

it simply takes a parser and extracts the parser function from it;

(Parser' f’) -> f’

however, thanks to **currying**, **runParser''**

can still be used with two arguments.

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

newtype – with record syntax (1)

```
newtype Parser a = Parser { parse :: String -> Maybe (a,String) }
newtype Parser' a = Parser' (String -> Maybe (a,String))
```

difference : record syntax with only one field

this record syntax automatically defines a function

```
parse :: Parser a  -> (String -> Maybe (a,String)),
```

which extracts the **String -> Maybe (a,String)** function
from the **Parser a**.

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

newtype – with record syntax (2)

```
newtype Parser a = Parser { parse :: String -> Maybe (a,String) }
```

parse can be used with two arguments thanks to **currying**,
and this simply has the effect of **running** the function stored
within the **Parser a**.

equivalent definition to the following code:

```
newtype Parser a = Parser (String -> Maybe (a,String))
```

```
parse :: Parser a -> (String -> Maybe (a,String))
parse (Parser p) = p
```

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Access functions in a record type (1)

```
data Person = Person {  firstName  :: String  ,
                        lastName   :: String  ,
                        age        :: Int     ,
                        height     :: Float   ,
                        phoneNo    :: String  ,
                        flavor     :: String
                     } deriving (Show)

ghci> :t flavor
flavor :: Person -> String
ghci> :t firstName
firstName :: Person -> String
```

return types of
access functions

Person ::
the input type of
access functions

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

Access functions in a record type (2)

```
data Car = Car String String Int deriving (Show)

ghci> Car "Ford" "Mustang" 1967
Car "Ford" "Mustang" 1967



data Car = Car {company :: String,
                model :: String,
                year :: Int} deriving (Show)

ghci> Car {company="Ford", model="Mustang", year=1967}
Car {company = "Ford", model = "Mustang", year = 1967}
```

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf

[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf