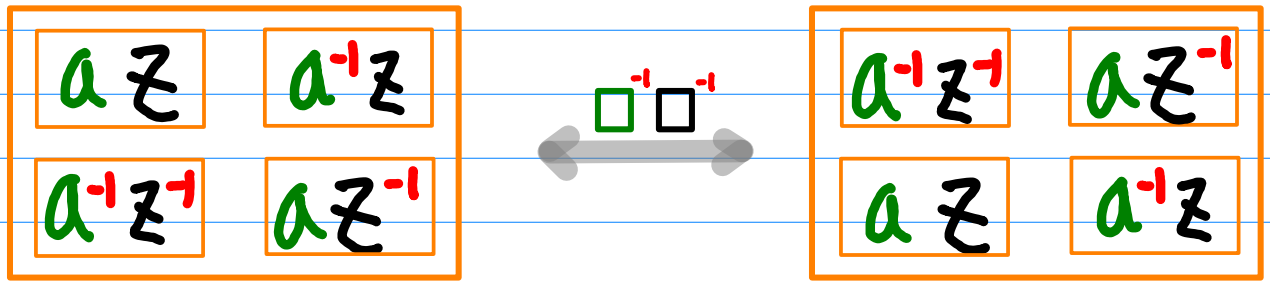
Laurent Series and z-Transform
- Geometric Series
Double Pole Examples A

20220630 Thr

Common ratios in geometric series

Assume $a \geq 1$



considered geometric series forms

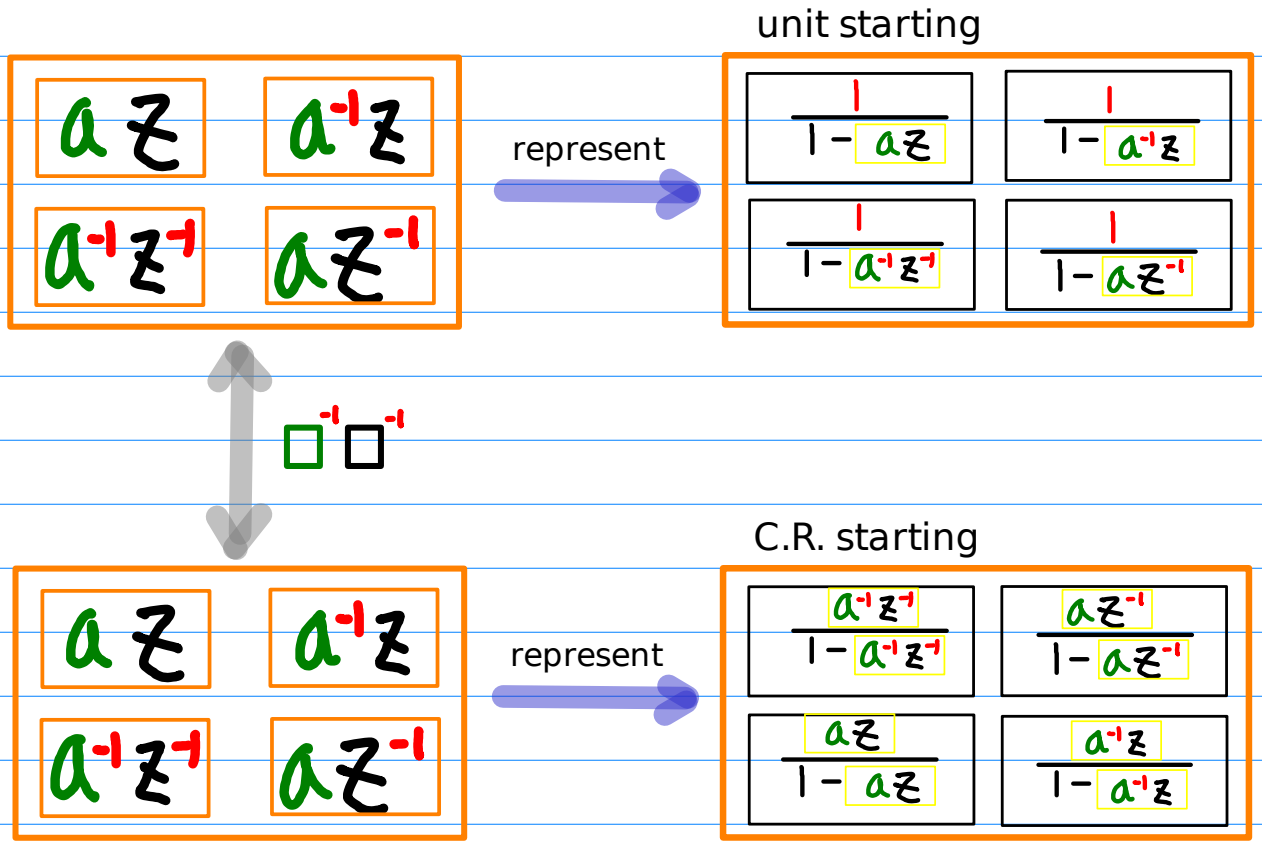
unit starting

$$\frac{1}{1-\text{C.R.}}$$

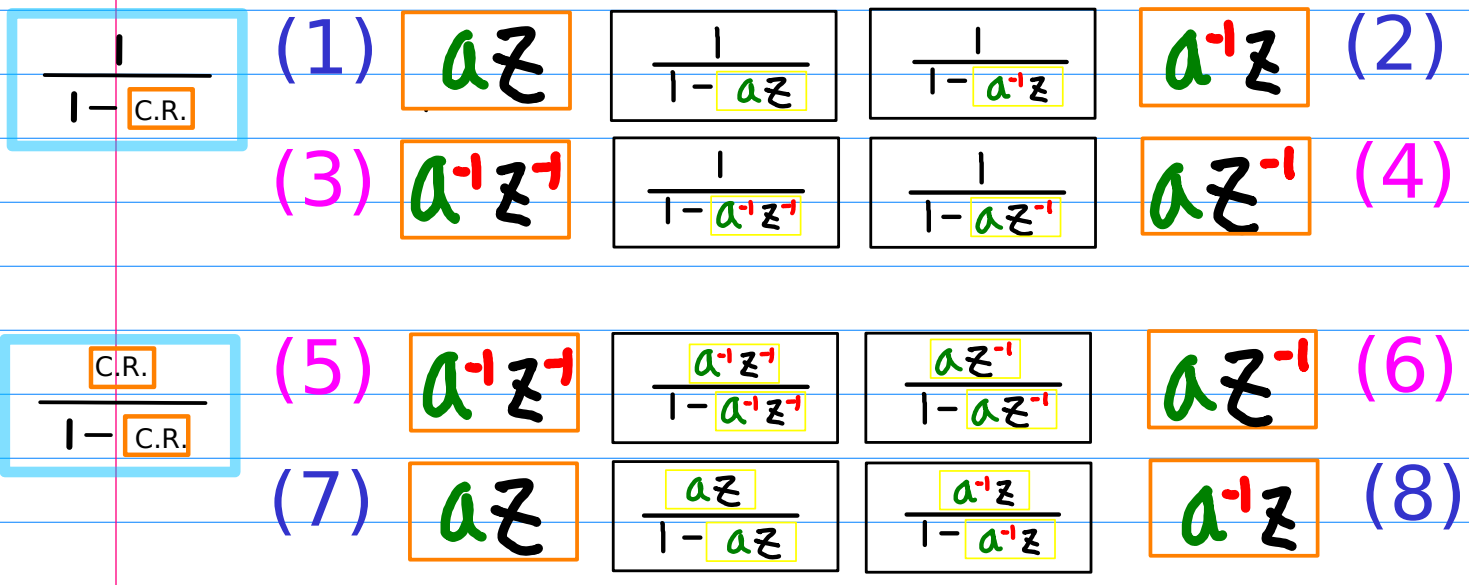
C.R. starting

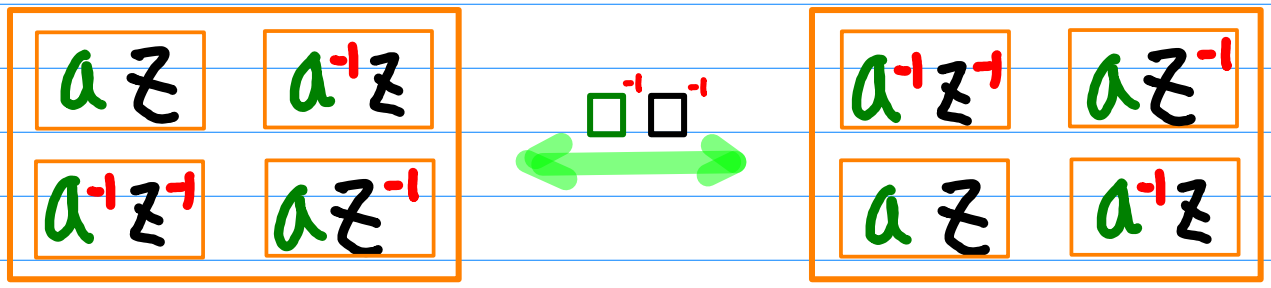
$$\frac{\text{C.R.}}{1-\text{C.R.}}$$

Representing geometric series



Numbering combinations





unit starting

(1), (2)

(3), (4)

C.R. starting

(5), (6)

(7), (8)

Decoding Geometric Series

Positive Exponent

$a z, a^{-1} z^{-1} \Rightarrow a^{n}$

even number of -1 exponent

Negative Exponent

$a^{-1} z, a z^{-1} \Rightarrow a^{-n}$

odd number of -1 exponent

Causal

$\square z \Rightarrow u(n), u(n-1)$

Anti-causal

$\square z^{-1} \Rightarrow u(-n), u(-n-1)$

$\frac{1}{1-\square z} \Rightarrow u(n)$

$\frac{\square z}{1-\square z} \Rightarrow u(n-1)$

$\frac{1}{1-\square z^{-1}} \Rightarrow u(-n)$

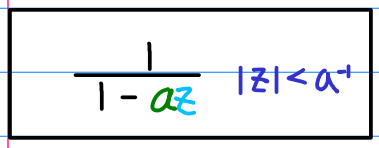
$\frac{\square z^{-1}}{1-\square z^{-1}} \Rightarrow u(-n-1)$

Decoding examples

Positive Exponent

Negative Exponent

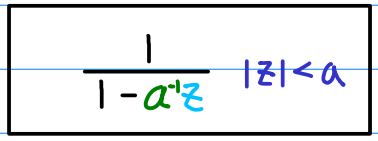
(1) **o-including**



$az \Rightarrow a^n$

$1, az \Rightarrow u(n)$

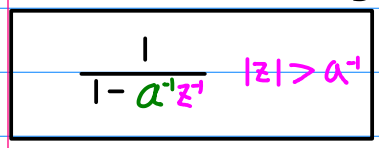
(2) **o-including**



$a^{-1}z \Rightarrow a^{-n}$

$1, a^{-1}z \Rightarrow u(n)$

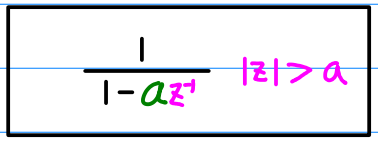
(3) **o-including**



$a^{-1}z^{-1} \Rightarrow a^n$

$1, a^{-1}z^{-1} \Rightarrow u(-n)$

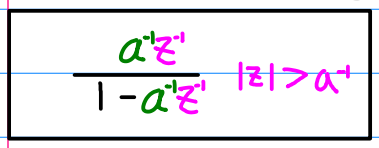
(4) **o-including**



$az^{-1} \Rightarrow a^{-n}$

$1, az^{-1} \Rightarrow u(-n)$

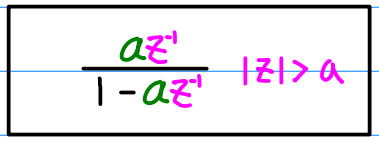
(5) **o-execluding**



$a^{-1}z^{-1} \Rightarrow a^n$

$a^{-1}z^{-1}, a^{-1}z^{-1} \Rightarrow u(-n-1)$

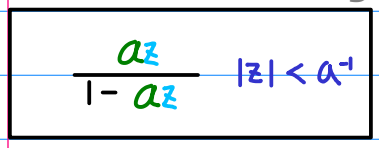
(6) **o-excluding**



$az^{-1} \Rightarrow a^{-n}$

$az^{-1}, az^{-1} \Rightarrow u(-n-1)$

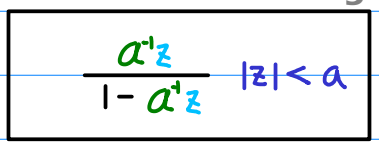
(7) **o-excluding**



$az \Rightarrow a^n$

$az, az \Rightarrow u(n-1)$

(8) **o-excluding**



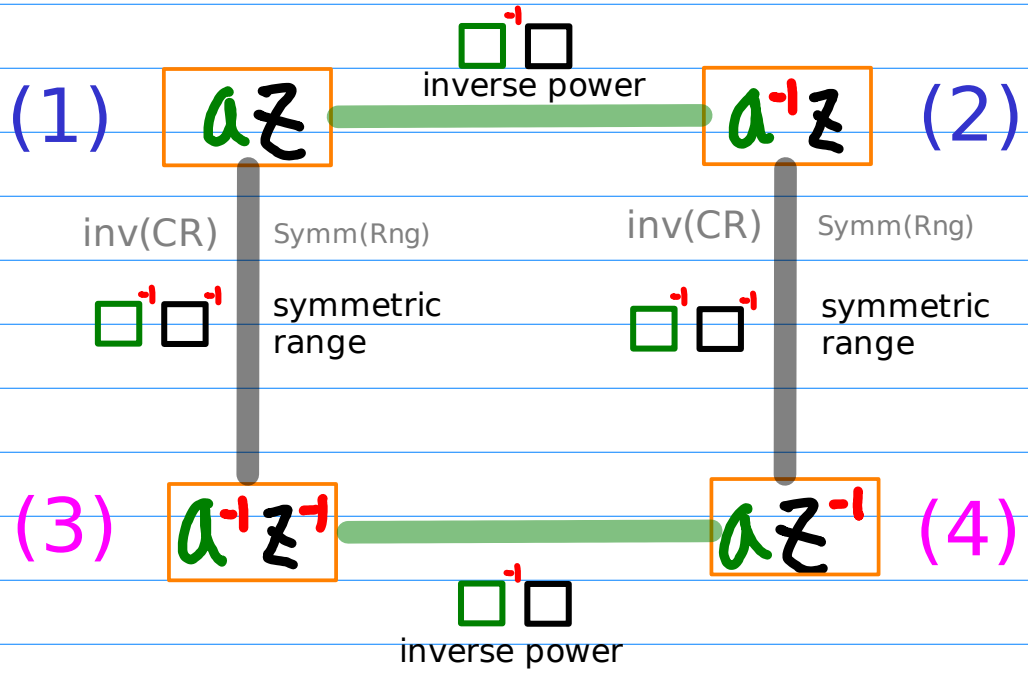
$a^{-1}z \Rightarrow a^{-n}$

$a^{-1}z, a^{-1}z \Rightarrow u(n-1)$

Inverse power, Symmetric range relations

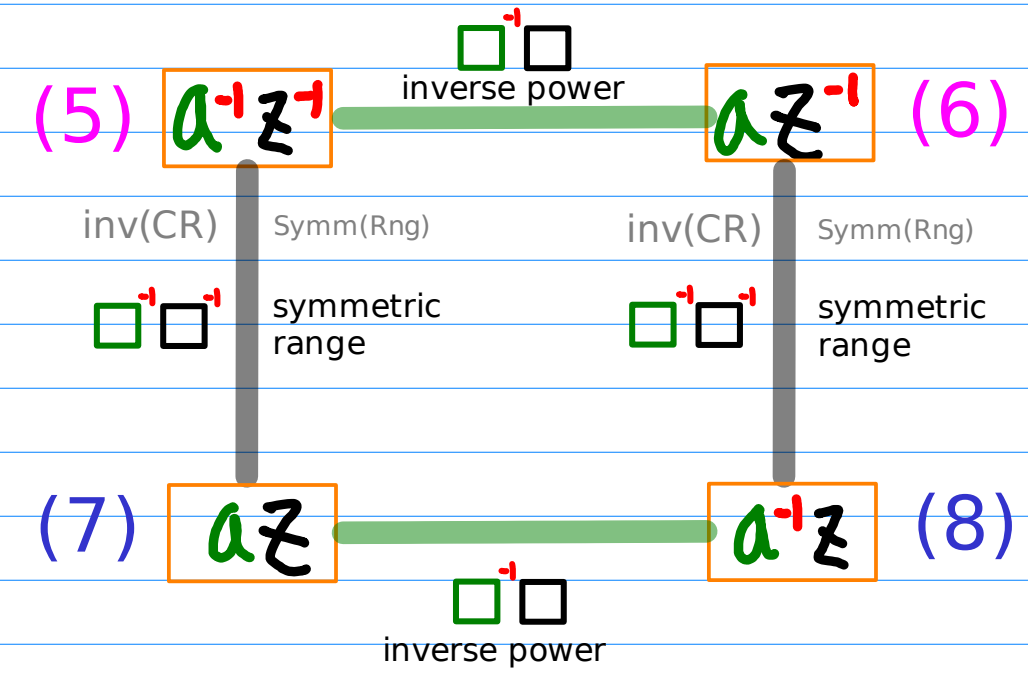
origin including geometric sequences

$$\frac{1}{1-\square}$$



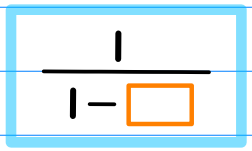
origin excluding geometric sequences

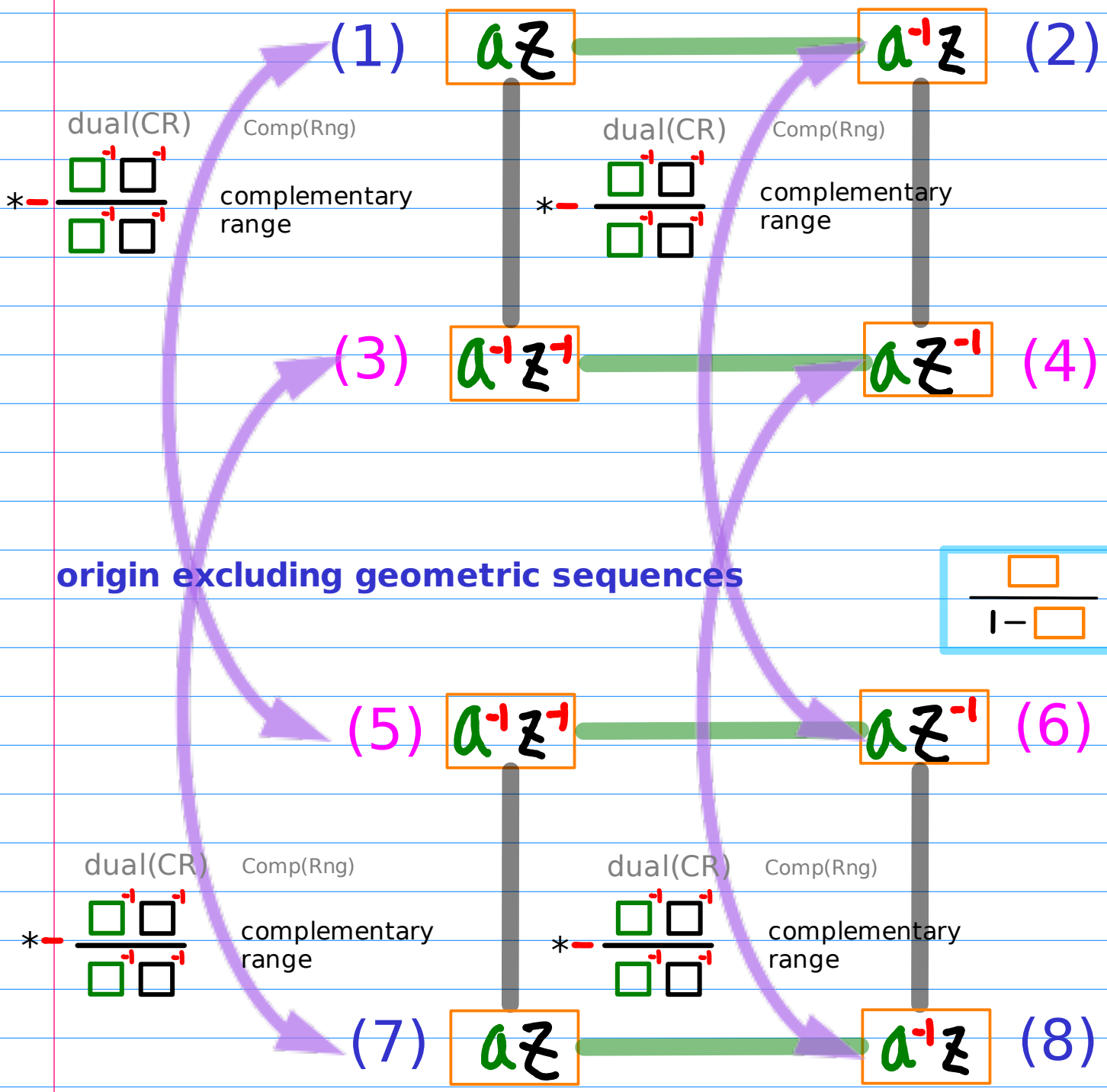
$$\frac{\square}{1-\square}$$



Complementary range relations

origin including geometric sequences

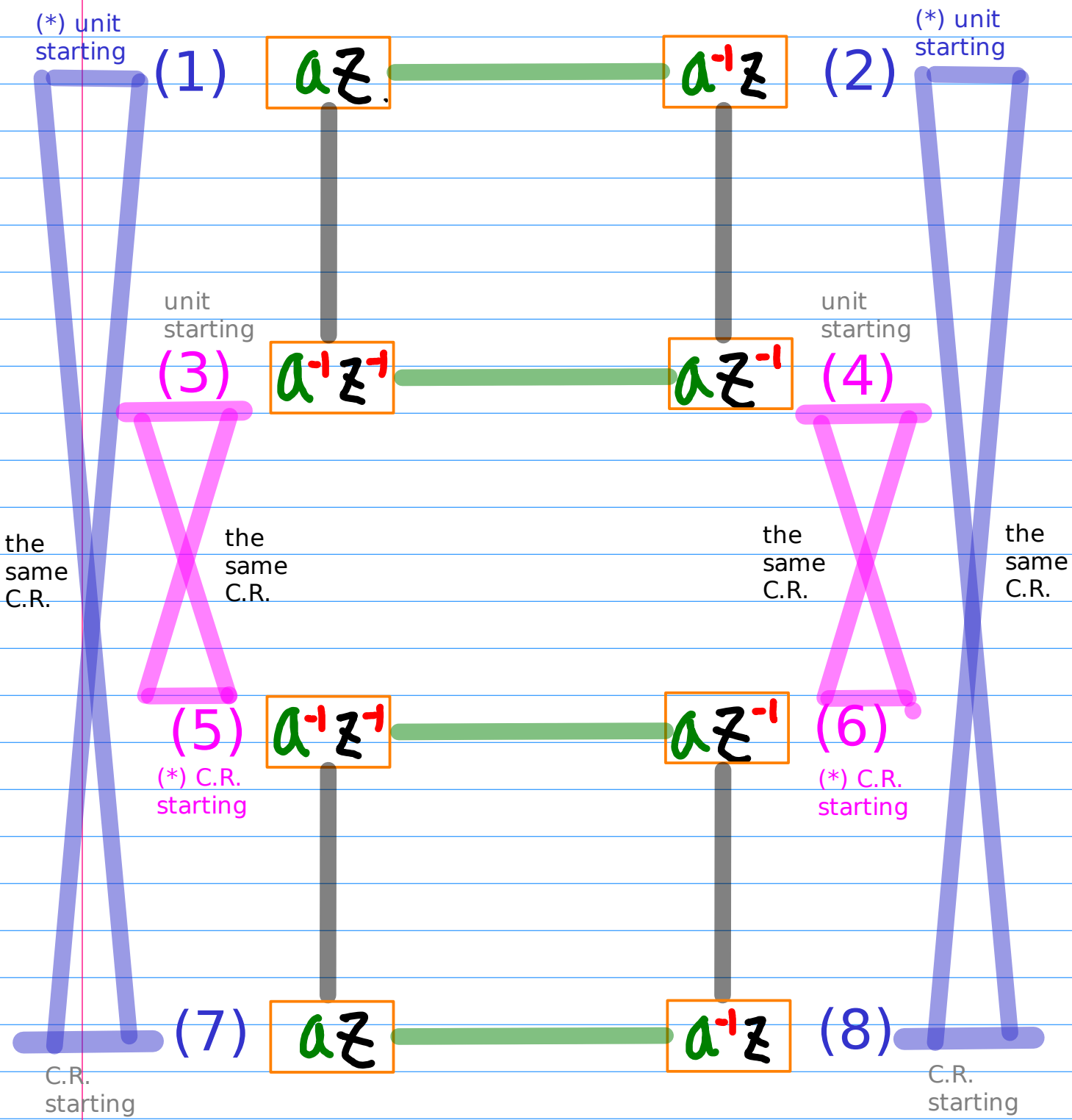




origin excluding geometric sequences

Shifting relations (a)

(1) (2)
(7) (8)
(5) (6)
(3) (4)

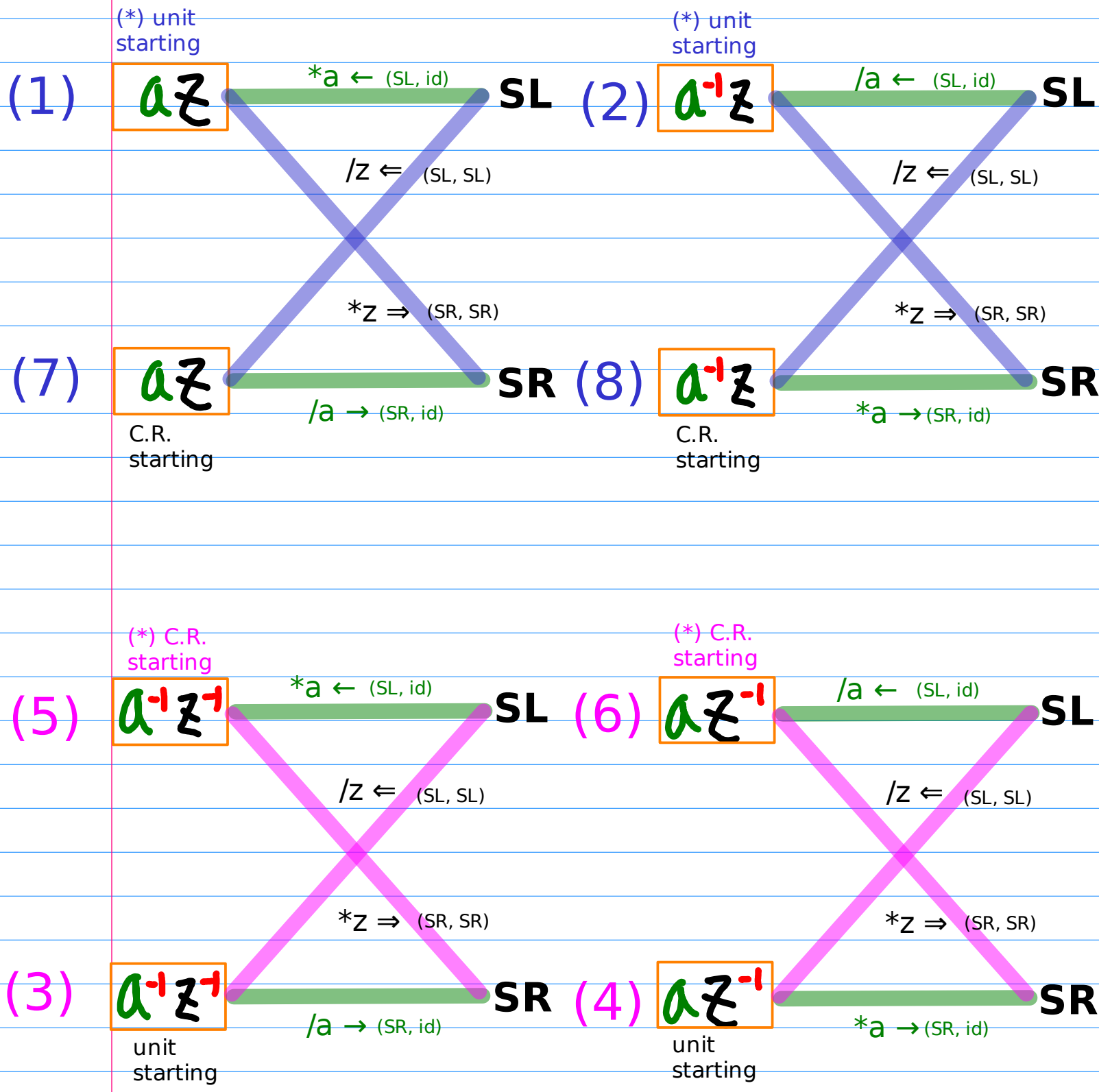


Shifting relations (b)

(exponent, range)

{SL, SR} {SL, SR, id}

(1) (2) (7) (8) (5) (6) (3) (4)



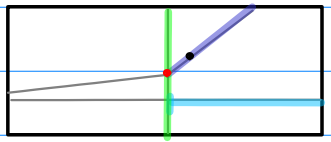
Unit starting **origin including**

(1) $\frac{1}{1 - az}$ a^n u(n)

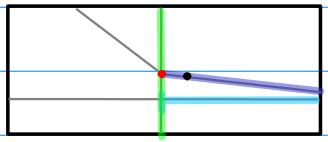
(2) $\frac{1}{1 - a^{-1}z}$ a^{-n} u(n)

(3) $\frac{1}{1 - a^{-1}z^{-1}}$ $-a^n$ u(-n)

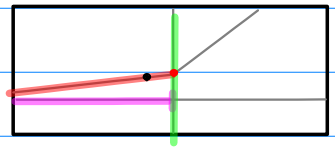
(4) $\frac{1}{1 - az^{-1}}$ $-a^{-n}$ u(-n)



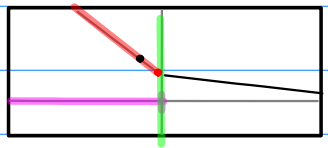
a^n u(n)



a^{-n} u(n)



a^n u(-n)



a^{-n} u(-n)

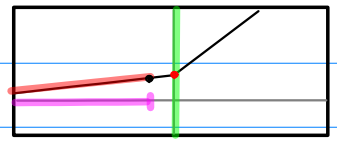
C.R. starting **origin excluding**

(5) $\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $-a^n\,u(-n-1)$

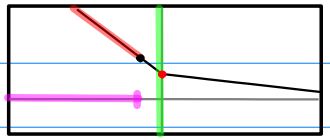
(6) $\dfrac{az^{-1}}{1-az^{-1}}$ $-a^{-n}u(-n-1)$

(7) $\dfrac{az}{1-az}$ $a^n\,u(n-1)$

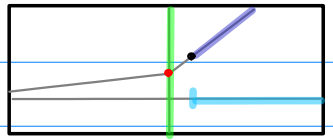
(8) $\dfrac{a^{-1}z}{1-a^{-1}z}$ $a^{-n}\,u(n-1)$



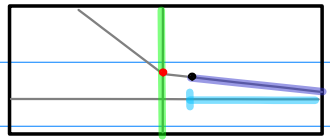
$a^n\,u(-n-1)$



$a^{-n}\,u(-n-1)$



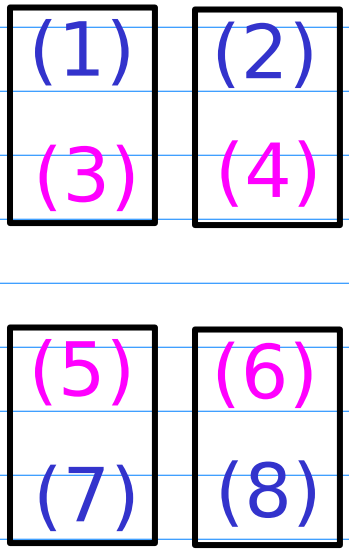
$a^n\,u(n-1)$



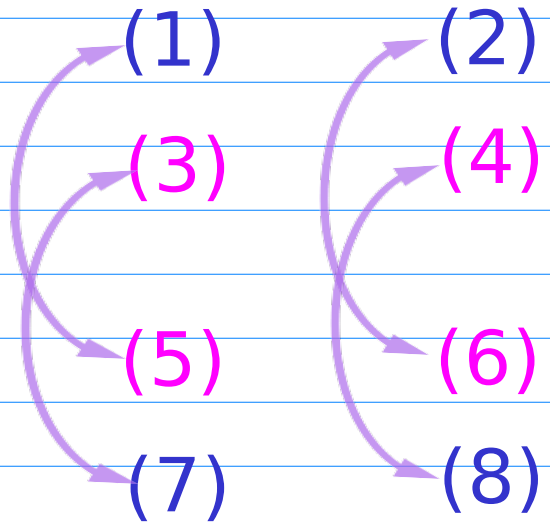
$a^{-n}\,u(n-1)$

Range Combinations (1)

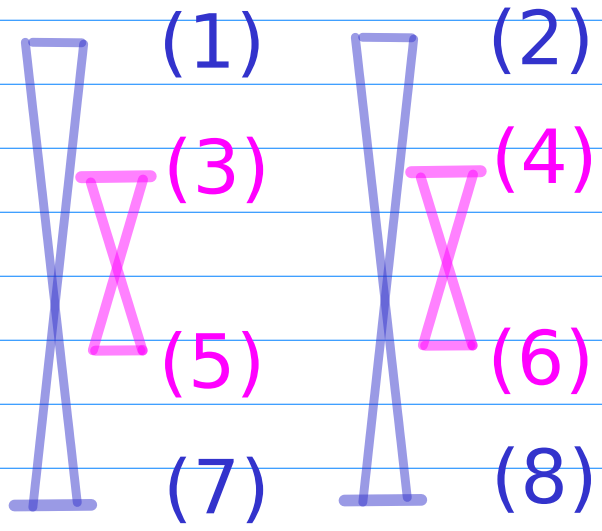
Symmetric Range



Complementary Range

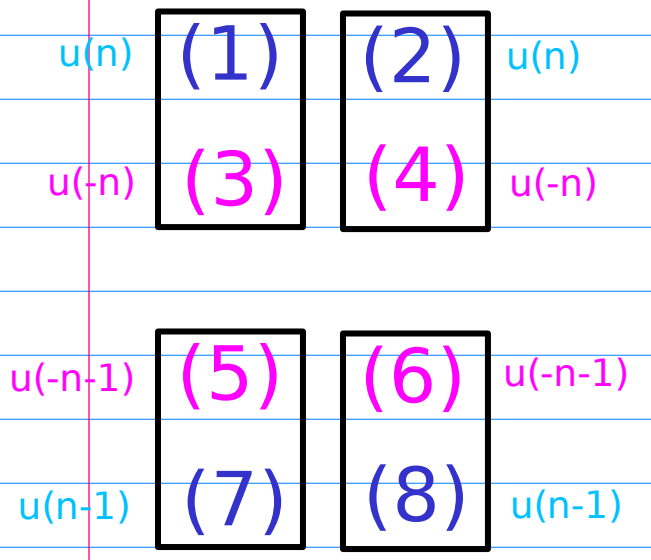


Shifted Range

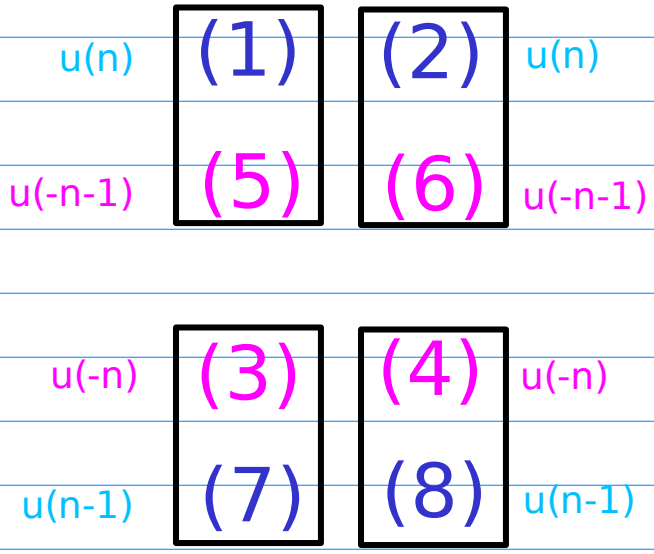


Range Combinations (2)

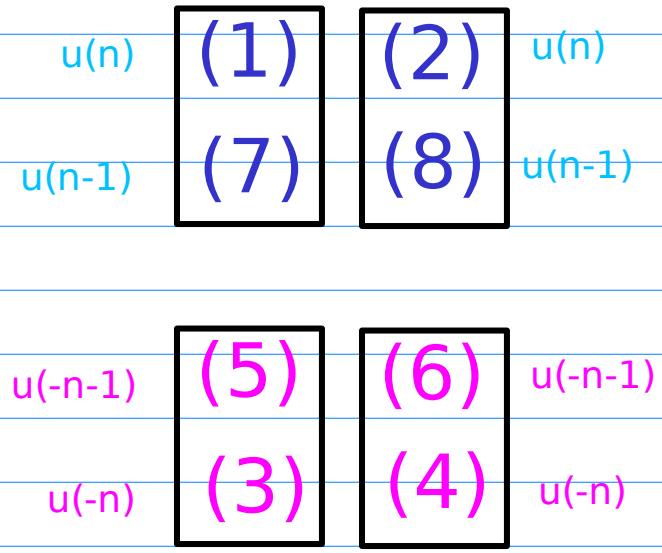
Symmetric Range



Complementary Range



Shifted Range



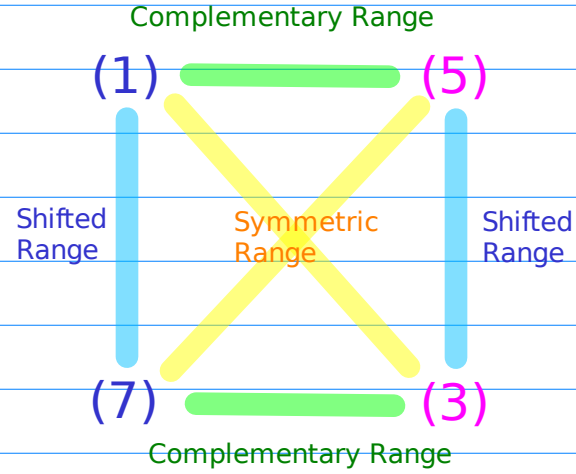
(1) (5)
(7) (3)

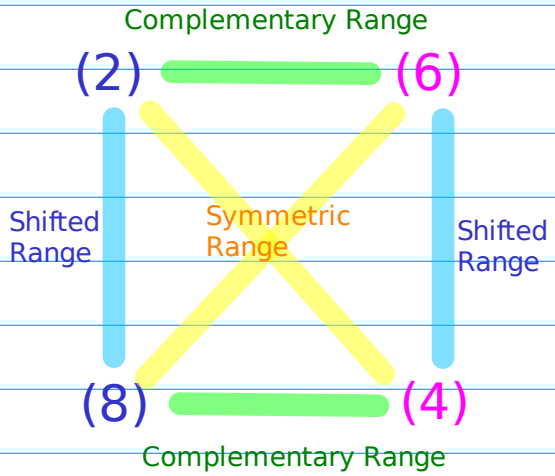
(2) (6)
(8) (4)

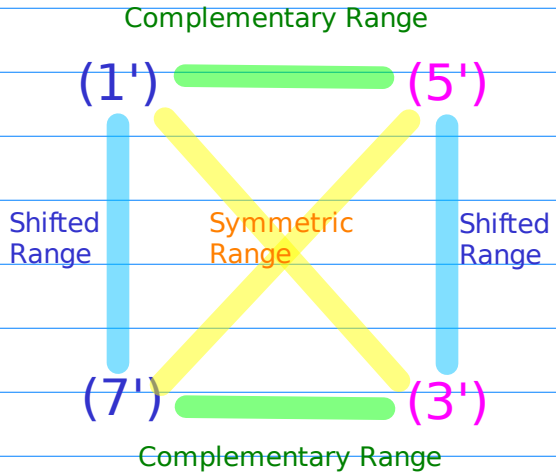
(1') (5')
(7') (3')

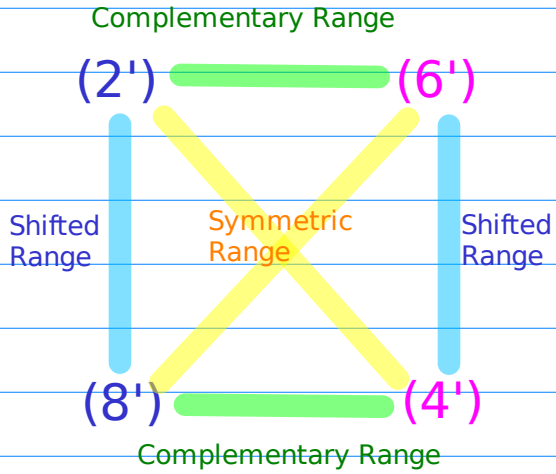
(2')(6')
(8')(4')

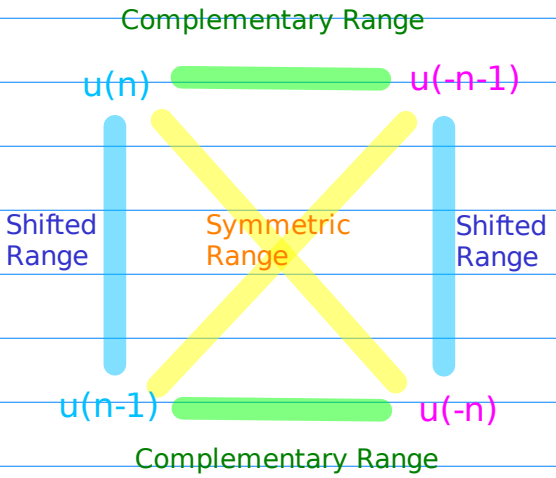
Complementary, Shifted, Symmetric Ranges

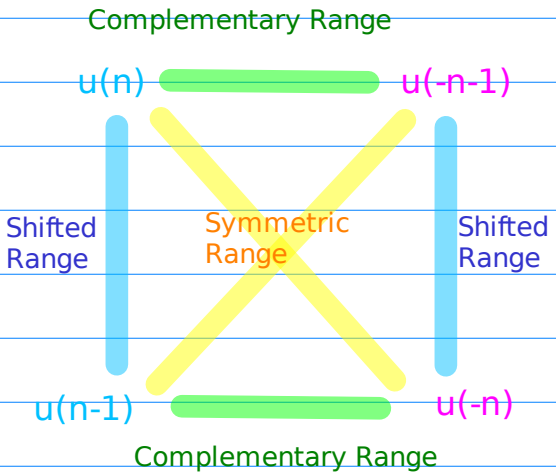












Partial Fraction Decompositions

$p_1 = 0.5$
$p_2 = 2$

$$-\frac{3}{2}\frac{1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$= \frac{(z-2)-(z-0.5)}{(z-0.5)(z-2)}$$

$$= \frac{-1.5}{(z-0.5)(z-2)}$$

$$-\frac{3}{2}\frac{z^2}{(z-0.5)(z-2)} = \left(\frac{0.5z}{z-0.5} - \frac{2z}{z-2}\right)$$

$$= \frac{0.5z^2 - z - 2z^2 + z}{(z-0.5)(z-2)}$$

$$= \frac{-1.5z^2}{(z-0.5)(z-2)}$$

$$-3\frac{z}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{4}{z-2}\right)$$

$$= \frac{(z-2)-4(z-0.5)}{(z-0.5)(z-2)}$$

$$= \frac{-3z}{(z-0.5)(z-2)}$$

Partial fractions as geometric power series

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$P_1 = 0.5$
$P_2 = 2$

Simple Pole Form

$$\frac{1}{z-0.5}$$

/0.5 /z

$$-\frac{2}{1-2z} \qquad \frac{z^{-1}}{1-0.5z^{-1}}$$

Geometric Power Series Forms

Simple Pole Form

$$\frac{1}{z-2}$$

/2 /z

$$-\frac{0.5}{1-0.5z} \qquad \frac{z^{-1}}{1-2z^{-1}}$$

Geometric Power Series Forms

$$-\frac{3}{2}\frac{z^2}{(z-0.5)(z-2)} = \left(\frac{0.5z}{z-0.5} - \frac{2z}{z-2}\right)$$

$P_1 = 0.5$
$P_2 = 2$

Simple Pole Form

$$\frac{0.5z}{z-0.5}$$

/0.5 /z

$$-\frac{z}{1-2z} \qquad \frac{0.5}{1-0.5z^{-1}}$$

Geometric Power Series Forms

Simple Pole Form

$$\frac{2z}{z-2}$$

/2 /z

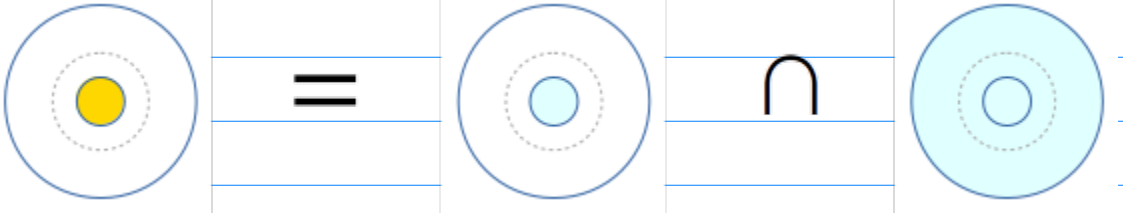
$$-\frac{z}{1-0.5z} \qquad \frac{2}{1-2z^{-1}}$$

Geometric Power Series Forms

ROC Cases
for irreducible polynomials (z-0.5), (z-2)

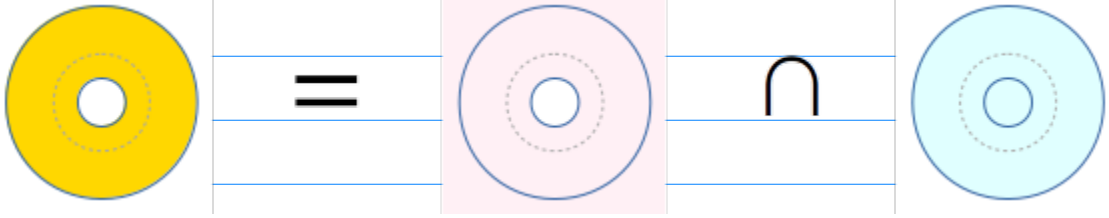
$$\frac{\square}{(z-0.5)(z-2)}$$

$|z| < 0.5$



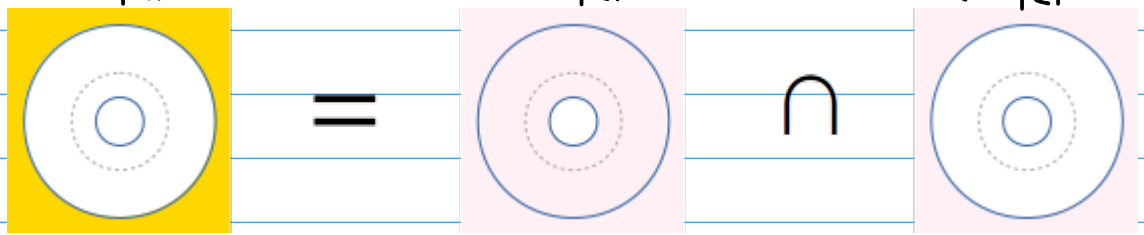
$|z| < 0.5$ $\quad \cap \quad$ $|z| < 2$

$0.5 < |z| < 2$



$0.5 < |z|$ $\quad \cap \quad$ $|z| < 2$

$2 < |z|$



$0.5 < |z|$ $\quad \cap \quad$ $2 < |z|$

Finding a common ratio

$P_1 = 0.5$

$P_2 = 2$

$$-\frac{3}{2}\,\frac{1}{(z-0.5)(z-2)} \qquad -\frac{3}{2}\,\frac{z^2}{(z-0.5)(z-2)} \qquad -3\,\frac{z}{(z-0.5)(z-2)}$$

Irreducible Polynomials

$(z-0.5)$ $(z-2)$

Geometric Power Series From

$$\frac{1}{(z-0.5)} \qquad \frac{1}{(z-2)}$$

$$\Downarrow \qquad \Downarrow$$

$$-\frac{2}{1-2z} \qquad -\frac{2^{-1}}{1-2^{-1}z}$$

$$\Downarrow \qquad \Downarrow$$

(1) $2z$ $\qquad$ $2^{-1}z$ (2)

$2^n\,u(n)$ $\qquad$ $2^{-n}\,u(n)$

Finding a geometric series

$p_1 = 0.5$

$p_2 = 2$

Irreducible Polynomials

Geometric Power Series From

$(z-0.5)$ $(z-2)$

$\frac{1}{(z-0.5)}$ $\frac{1}{(z-2)}$

$-\frac{2}{1-2z}$ $-\frac{2^{-1}}{1-2^{-1}z}$

$2 \cdot \frac{1}{1-2z}$ $2^{-1} \cdot \frac{1}{1-2^{-1}z}$

$a = 2$ $a^{-1} = 2^{-1}$

(1)	az	$+\frac{1}{1-az}$	$+\frac{1}{1-a^{-1}z}$	$a^{-1}z$	(2)
(3)	$a^{-1}z^{-1}$	$-\frac{1}{1-a^{-1}z^{-1}}$	$-\frac{1}{1-az^{-1}}$	az^{-1}	(4)
(5)	$a^{-1}z^{-1}$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$	$-\frac{az^{-1}}{1-az^{-1}}$	az^{-1}	(6)
(7)	az	$+\frac{az}{1-az}$	$+\frac{a^{-1}z}{1-a^{-1}z}$	$a^{-1}z$	(8)

$2^n u(n)$ $2^{-n} u(n)$

(1) (1')
(7) (7')

(2) (2')
(8) (8')

(5) (5')
(3')

(6) (6')
(4) (4')

$P_1 = 0.5$

$P_2 = 2$

Irreducible Polynomials

Geometric Power Series From

$\frac{1}{(z-0.5)}$ $\frac{1}{(z-2)}$

$-\frac{2}{1-2z}$ $-\frac{2^{-1}}{1-2^{-1}z}$

$2 \cdot \frac{1}{1-2z}$ $2^{-1} \cdot \frac{1}{1-2^{-1}z}$

$a = 2$ $a^{-1} = 2^{-1}$

(1) $\frac{1}{1-az}$ *a ← $\frac{a}{1-az}$ — a^n u(n) — a^{n+1} u(n)

(2) $\frac{1}{1-a^{-1}z}$ /a ← $\frac{a^{-1}}{1-a^{-1}z}$ — a^{-n} u(n) — a^{-n-1} u(n)

(7) $\frac{az}{1-az}$ /a → $\frac{z}{1-az}$ — a^n u(n-1) — a^{n-1} u(n-1)

(8) $\frac{a^{-1}z}{1-a^{-1}z}$ *a → $\frac{z}{1-a^{-1}z}$ — a^{-n} u(n-1) — a^{-n+1} u(n-1)

(5) $-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ *a ← $-\frac{z^{-1}}{1-a^{-1}z^{-1}}$ — a^n u(-n-1) — a^{n+1} u(-n-1)

(6) $-\frac{az^{-1}}{1-az^{-1}}$ /a ← $-\frac{z^{-1}}{1-az^{-1}}$ — a^{-n} u(-n-1) — a^{-n-1} u(-n-1)

(3) $-\frac{1}{1-a^{-1}z^{-1}}$ /a → $-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ — a^n u(-n) — a^{n-1} u(-n)

(4) $-\frac{1}{1-az^{-1}}$ *a → $-\frac{a}{1-az^{-1}}$ — a^{-n} u(-n) — a^{-n+1} u(-n)

Decoding Geometric Series

Positive Exponent

$2z, 2^{-1}z^{-1} \Rightarrow 2^{n}$

Negative Exponent

$2^{-1}z, 2z^{-1} \Rightarrow 2^{-n}$

Causal

$\square z \Rightarrow u(n), u(n-1)$

Anti-causal

$\square z^{-1} \Rightarrow u(-n), u(-n-1)$

$$\frac{1}{1-\square z} \Rightarrow u(n)$$

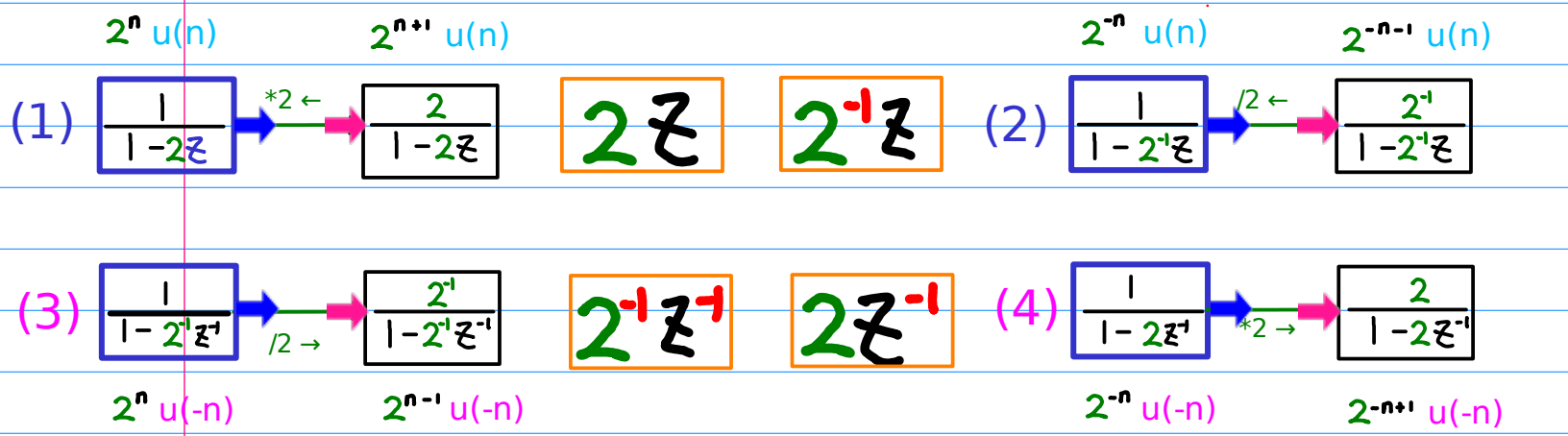
$$\frac{\square z}{1-\square z} \Rightarrow u(n-1)$$

$$\frac{1}{1-\square z^{-1}} \Rightarrow u(-n)$$

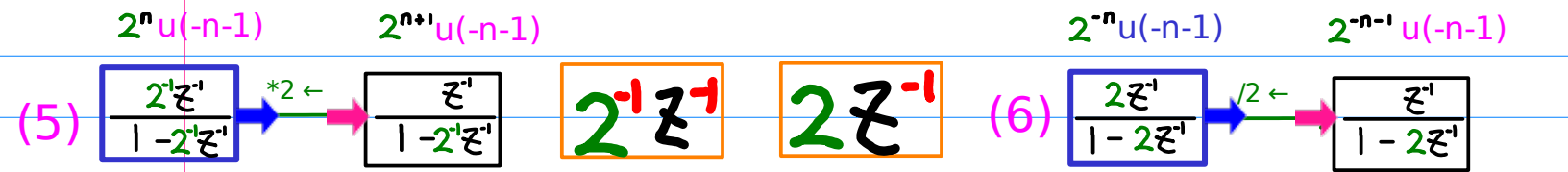
$$\frac{\square z^{-1}}{1-\square z^{-1}} \Rightarrow u(-n-1)$$

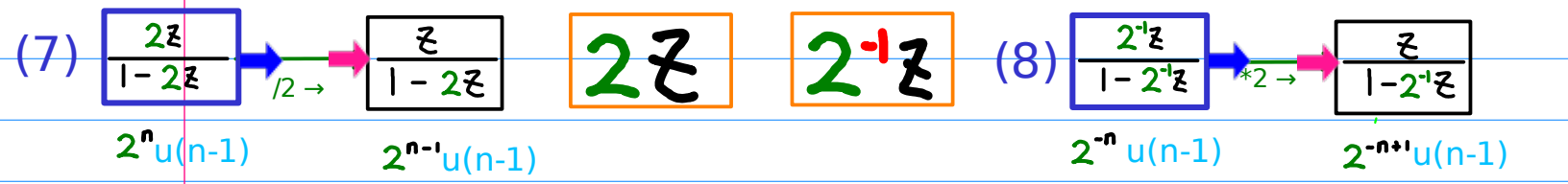
$a=2$

(1) (2)
(3) (4)
(5) (6)
(7) (8)



$2^n u(-n)$ $2^{n-1} u(-n)$ $2^{-n} u(-n)$ $2^{-n+1} u(-n)$





$2^n u(n-1)$ $2^{n-1} u(n-1)$ $2^{-n} u(n-1)$ $2^{-n+1} u(n-1)$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$P_1 = 0.5$

$P_2 = 2$

$\frac{1}{z-0.5}$ → /0.5 → $-\frac{2}{1-2z}$ (1')

$\frac{1}{z-0.5}$ → /z → $\frac{z^{-1}}{1-0.5z^{-1}}$ (5')

$\frac{1}{z-2}$ → /2 → $-\frac{0.5}{1-0.5z}$ (2')

$\frac{1}{z-2}$ → /z → $\frac{z^{-1}}{1-2z^{-1}}$ (6')

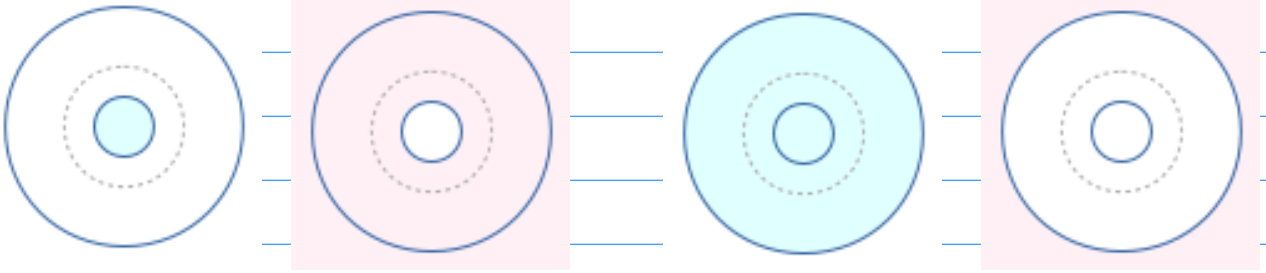
Complementary Ranges

Complementary Ranges

$-2\cdot\frac{1}{1-2z}$ $\quad$ $2\cdot\frac{0.5z^{-1}}{1-0.5z^{-1}}$ $\quad$ $-0.5\cdot\frac{1}{1-0.5z}$ $\quad$ $0.5\cdot\frac{2z^{-1}}{1-2z^{-1}}$

$-2^{n+1}\,u(n)$ $\quad$ $2^{n+1}\,u(-n-1)$ $\quad$ $-\left(\frac{1}{2}\right)^{n+1}u(n)$ $\quad$ $\left(\frac{1}{2}\right)^{n+1}u(-n-1)$

$|z| < 0.5$ $\quad$ $|z| > 0.5$ $\quad$ $|z| < 2$ $\quad$ $|z| > 2$



(A) (B) (C) (A) (B) (C)

Case A	$\lvert z\rvert < 0.5$	(1')	(2')
Case B	$0.5 < \lvert z\rvert < 2$	(5')	(2')
Case C	$2 < \lvert z\rvert$	(5')	(6')

$$\frac{3}{2}\frac{-z^2}{(z-0.5)(z-2)} = \left(\frac{0.5z}{z-0.5} - \frac{2z}{z-2}\right)$$

$p_1 = 0.5$

$p_2 = 2$

$\frac{0.5z}{z-0.5}$ /0.5 /z

$-\frac{z}{1-2z}$ (7') $\quad \frac{0.5}{1-0.5z^{-1}}$ (3')

Complementary Ranges

$\frac{2z}{z-2}$ /2 /z

$-\frac{z}{1-0.5z}$ (8') $\quad \frac{2}{1-2z^{-1}}$ (4')

Complementary Ranges

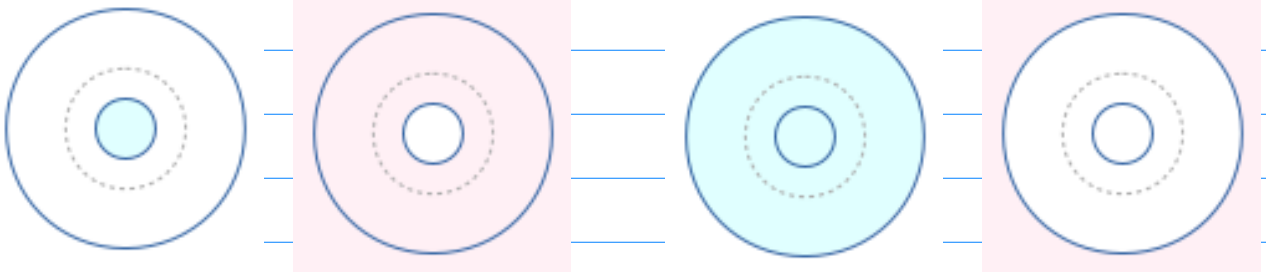
$-0.5 \cdot \frac{2z}{1-2z}$ $\quad 0.5 \cdot \frac{1}{1-0.5z^{-1}}$

$-2^{n-1}\,u(n-1)$ $\quad 2^{n-1}\,u(-n)$

$-2 \cdot \frac{0.5z}{1-0.5z}$ $\quad 2 \cdot \frac{1}{1-2z^{-1}}$

$-\left(\frac{1}{2}\right)^{n-1}u(n-1)$ $\quad \left(\frac{1}{2}\right)^{n-1}u(-n)$

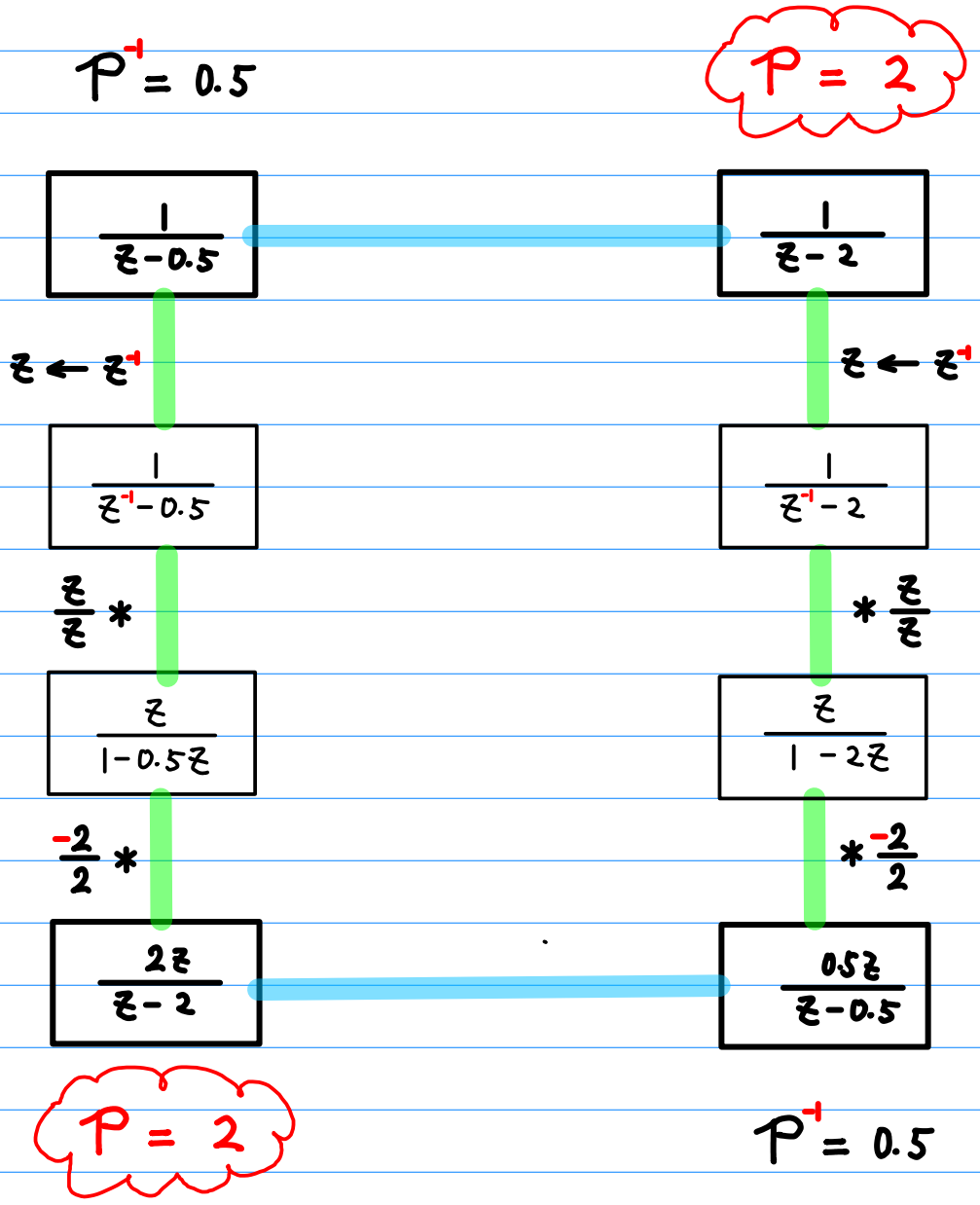
$|z| < 0.5$ $\quad |z| > 0.5$ $\quad |z| < 2$ $\quad |z| > 2$



(A) (B) (C) (A) (B) (C)

Case A	$\lvert z\rvert < 0.5$	(7')	(8')
Case B	$0.5 < \lvert z\rvert < 2$	(3')	(8')
Case C	$2 < \lvert z\rvert$	(3')	(4')

Reciprocal Pole Relation (p=2)



Reciprocal Pole Partial Fractions (p=2)

$p^{-1} = 0.5$

$p = 2$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$z \leftarrow z^{-1}$

$$\frac{3}{2}\frac{-1}{(z^{-1}-0.5)(z^{-1}-2)} = \left(\frac{1}{z^{-1}-0.5} - \frac{1}{z^{-1}-2}\right)$$

$* \frac{z}{z}$

$-1, z^{-1}$

$$\frac{3}{2}\frac{-z^2}{(1-0.5z)(1-2z)} = \left(\frac{z}{(1-0.5z)} - \frac{z}{(1-2z)}\right)$$

$* \frac{2}{2}$

$$\frac{3}{2}\frac{-z^2 \cdot 2 \cdot 0.5}{(2-z)(0.5-z)} = \left(\frac{2z}{(2-z)} - \frac{0.5z}{(0.5-z)}\right)$$

$* -1$

$$\frac{3}{2}\frac{-z^2}{(z-0.5)(z-2)} = \left(\frac{0.5z}{z-0.5} - \frac{2z}{z-2}\right)$$

Reciprocal Pole Relation : Partial Fractions and Geometric Power Series

$p^{-1} = 0.5$

$p = 2$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$-1, z^{-1}$

$$\frac{3}{2}\frac{-z^2}{(z-0.5)(z-2)} = \left(\frac{0.5z}{z-0.5} - \frac{2z}{z-2}\right)$$

$\frac{1}{z-0.5}$

/0.5 /z

$-\frac{2}{1-2z}$ $\frac{z^{-1}}{1-0.5z^{-1}}$

$-(1')$ $(5')$

$-2^{n+1}\,u(n)$ $2^{n+1}\,u(-n-1)$

$\frac{1}{z-2}$

/2 /z

$-\frac{0.5}{1-0.5z}$ $\frac{z^{-1}}{1-2z^{-1}}$

$-(2')$ $(6')$

$-\left(\frac{1}{2}\right)^{n+1}u(n)$ $\left(\frac{1}{2}\right)^{n+1}u(-n-1)$

$\frac{0.5z}{z-0.5}$

/0.5 /z

$-\frac{z}{1-2z}$ $\frac{0.5}{1-0.5z^{-1}}$

$-(7')$ $(3')$

$-2^{n-1}\,u(n-1)$ $2^{n-1}\,u(-n)$

$\frac{2z}{z-2}$

/2 /z

$-\frac{z}{1-0.5z}$ $\frac{2}{1-2z^{-1}}$

$-(8')$ $(4')$

$-\left(\frac{1}{2}\right)^{n-1}u(n-1)$ $\left(\frac{1}{2}\right)^{n-1}u(-n)$

Reciprocal poles in Simple Pole Forms (p=2)

$p^{-1} = 0.5$

$p = 2$

$p^{-1} = 0.5$

$$\frac{1}{z-0.5}$$

$$\frac{1}{z^{-1}-0.5}$$

$$\frac{z}{1-0.5z}$$

$$-\frac{2z}{z-2}$$

$p = 2$

$$\frac{1}{z-2}$$

$$\frac{1}{z^{-1}-2}$$

$$\frac{z}{1-2z}$$

$$-\frac{0.5z}{z-0.5}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$-1, z^{-1}$

$$\frac{1}{z-0.5}$$

$$\frac{1}{z-2}$$

$-1, z^{-1}$

$-1, z^{-1}$

$$\frac{0.5z}{z-0.5}$$

$$\frac{2z}{z-2}$$

$$\frac{3}{2}\frac{-z^2}{(z-0.5)(z-2)} = \left(\frac{0.5z}{z-0.5} - \frac{2z}{z-2}\right)$$

Reciprocal poles in Geometric Series Forms (p=2)

Causal

$p^{-1} = 0.5$ | $p = 2$

$$-\frac{2}{1-2z} \qquad -\frac{0.5}{1-0.5z}$$

$$-\frac{2}{1-2z^{-1}} \qquad -\frac{0.5}{1-0.5z^{-1}}$$

$$-\frac{2z}{z-2} \qquad -\frac{0.5z}{z-0.5}$$

$$\frac{z}{1-0.5z} \qquad \frac{z}{1-2z}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$-1, z^{-1}$

$-2^{n+1}\,u(n)$ | $-\left(\frac{1}{2}\right)^{n+1}u(n)$

(1') $-\frac{2}{1-2z}$ — $-1, z^{-1}$ — $-\frac{0.5}{1-0.5z}$ (2')

$-1, z^{-1}$

(7') $-\frac{z}{1-2z}$ — $-\frac{z}{1-0.5z}$ (8')

$-2^{n-1}\,u(n-1)$ | $-\left(\frac{1}{2}\right)^{n-1}u(n-1)$

$$\frac{3}{2}\frac{-z^2}{(z-0.5)(z-2)} = \left(\frac{0.5z}{z-0.5} - \frac{2z}{z-2}\right)$$

Reciprocal poles in Geometric Series Forms (p=2)

$P^{-1} = 0.5$ Anti-causal $P = 2$

$P^{-1} = 0.5$ | $P = 2$

$$\frac{z^{-1}}{1-0.5z^{-1}} \qquad \frac{z^{-1}}{1-2z^{-1}}$$

$$\frac{z^{+1}}{1-0.5z^{+1}} \qquad \frac{z^{+1}}{1-2z^{+1}}$$

$$\frac{1}{z^{-1}-0.5} \qquad \frac{1}{z^{-1}-2}$$

$$-\frac{2}{1-2z^{-1}} \qquad -\frac{0.5}{1-0.5z^{-1}}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$-1, z^{-1}$

$2^{n+1}\,u(-n-1)$ $\qquad$ $\left(\frac{1}{2}\right)^{n+1}u(-n-1)$

(5') $\frac{z^{-1}}{1-0.5z^{-1}}$ $\qquad$ $\frac{z^{-1}}{1-2z^{-1}}$ (6')

$-1, z^{-1}$ $\qquad$ $-1, z^{-1}$

(3') $\frac{0.5}{1-0.5z^{-1}}$ $\qquad$ $\frac{2}{1-2z^{-1}}$ (4')

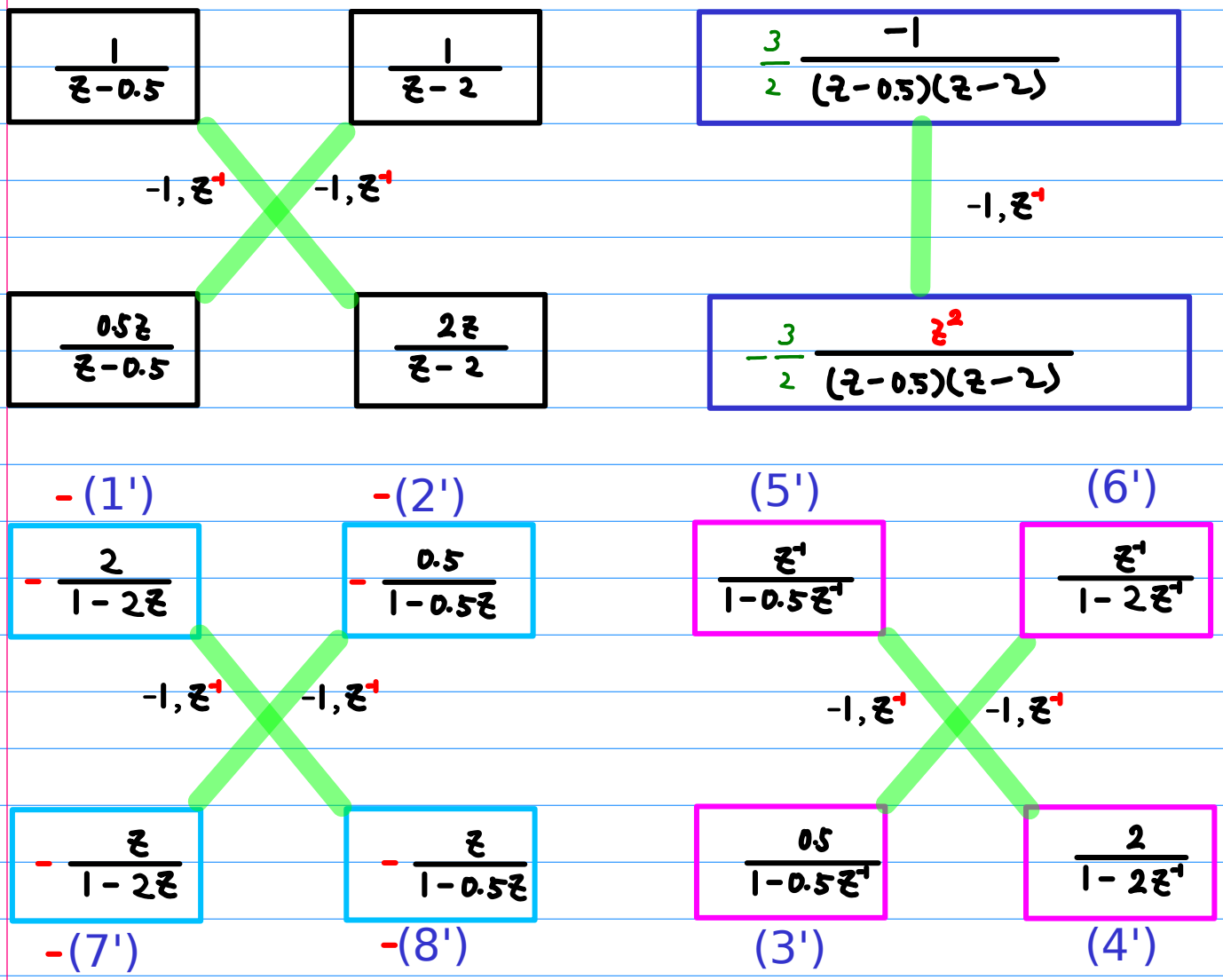
$2^{n-1}\,u(-n)$ $\qquad$ $\left(\frac{1}{2}\right)^{n-1}u(-n)$

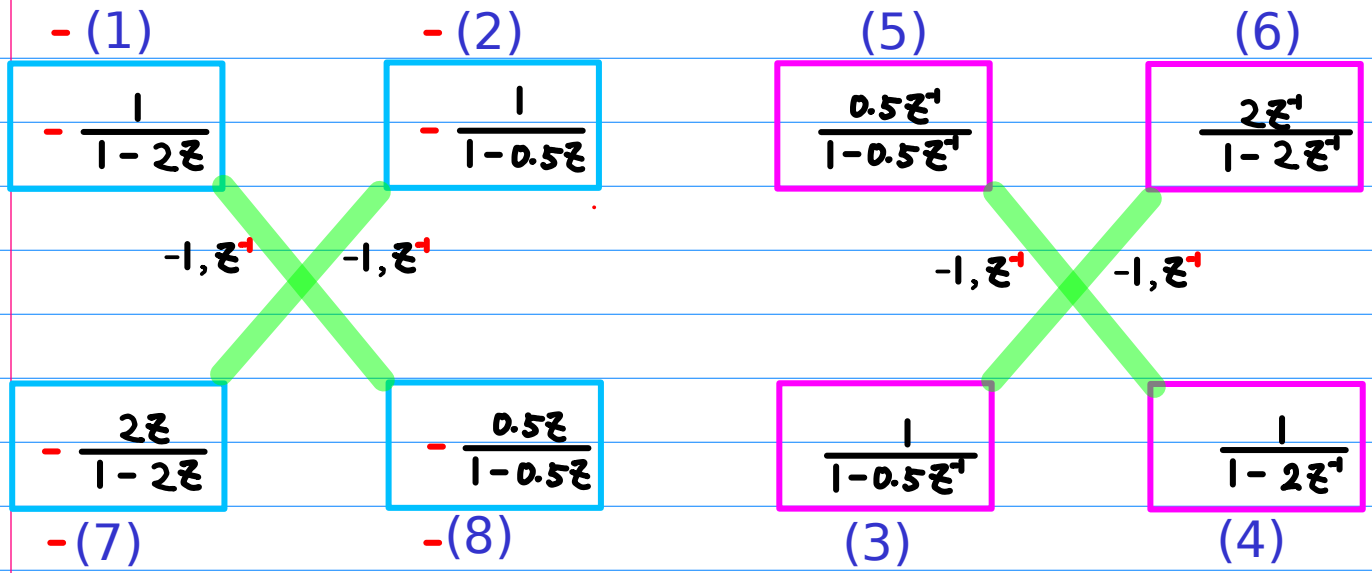
$$\frac{3}{2}\frac{-z^2}{(z-0.5)(z-2)} = \left(\frac{0.5z}{z-0.5} - \frac{2z}{z-2}\right)$$

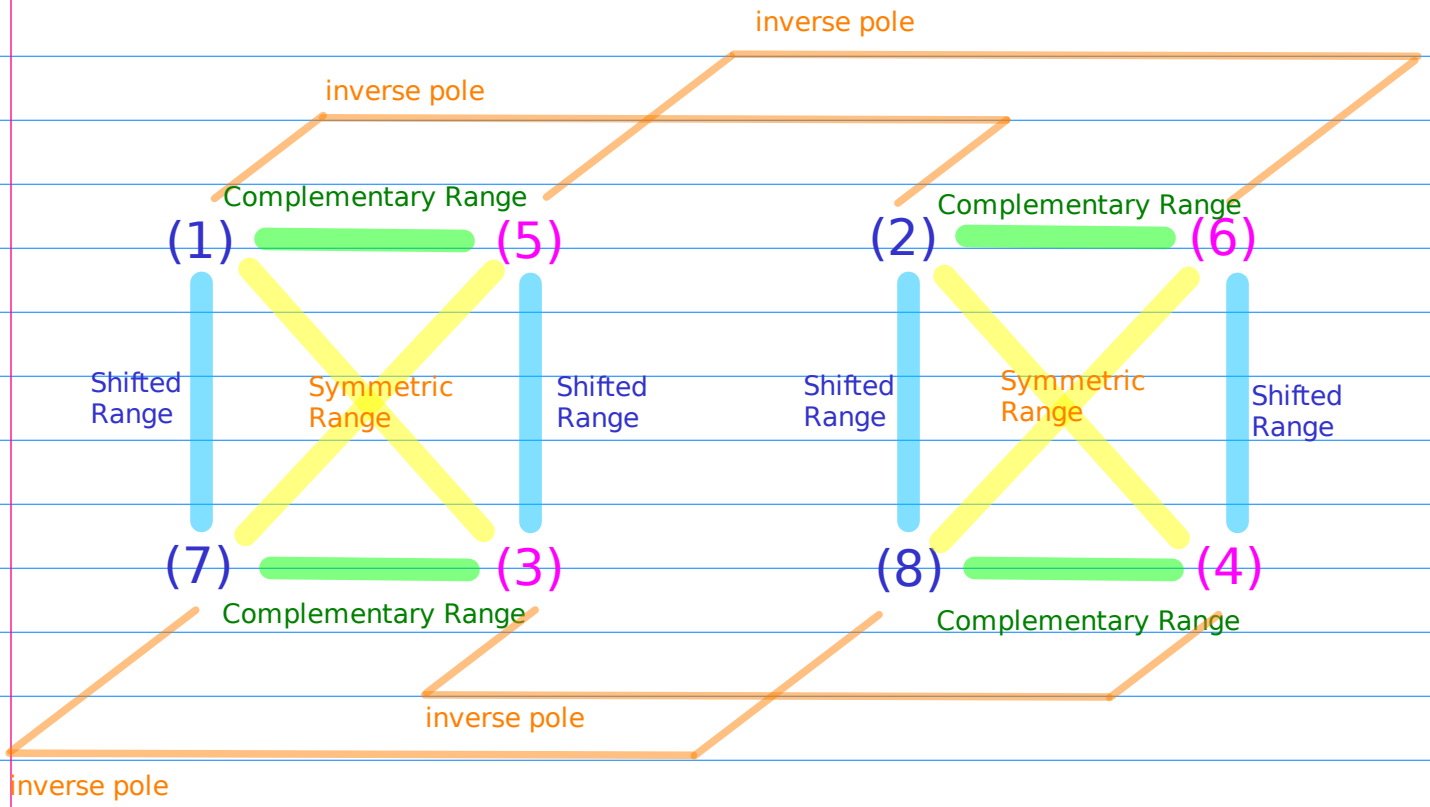
Reciprocal Pole Relation :
Shifted version and
Unshifted version

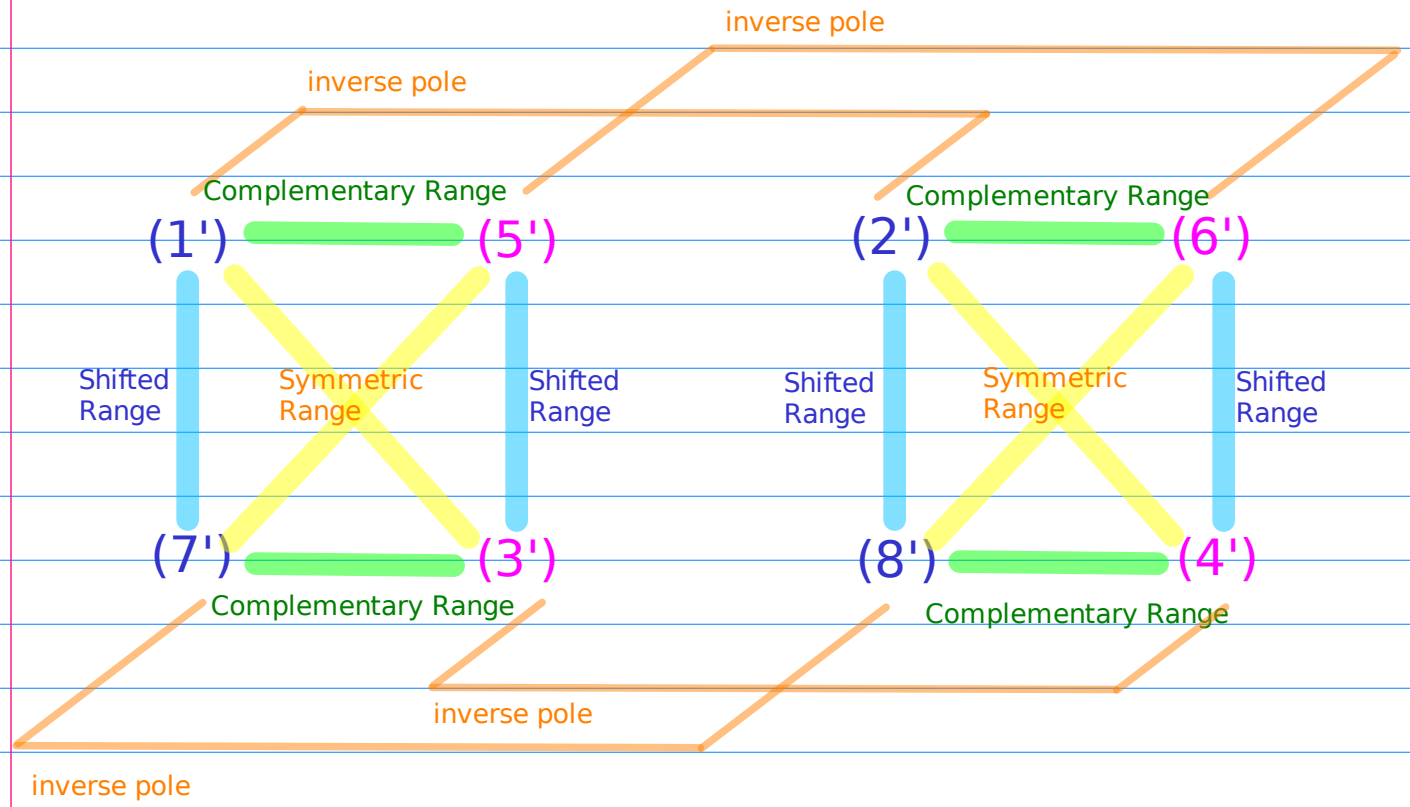
$P^{-1} = 0.5$

$P = 2$






inverse pole
inverse pole
Complementary Range
(1)
(5)
Shifted Range
Symmetric Range
Shifted Range
(7)
(3)
Complementary Range
inverse pole
inverse pole
Complementary Range
(2)
(6)
Shifted Range
Symmetric Range
Shifted Range
(8)
(4)
Complementary Range


inverse pole
inverse pole
Complementary Range
(1')
(5')
Shifted Range
Symmetric Range
Shifted Range
(7')
(3')
Complementary Range
inverse pole
inverse pole
Complementary Range
(2')
(6')
Shifted Range
Symmetric Range
Shifted Range
(8')
(4')
Complementary Range

-(1) $-\frac{1}{1-2z}$ $-2^n\ u(n)$

(5) $\frac{0.5z^{-1}}{1-0.5z^{-1}}$ $2^n\ u(-n-1)$

-(2) $-\frac{1}{1-0.5z}$ $-2^{-n}\ u(n)$

(6) $\frac{2z^{-1}}{1-2z^{-1}}$ $2^{-n}\ u(-n-1)$

-(7) $-\frac{2z}{1-2z}$ $-2^n\ u(n-1)$

(3) $\frac{1}{1-0.5z^{-1}}$ $2^n\ u(-n)$

-(8) $-\frac{0.5z}{1-0.5z}$ $-2^{-n}\ u(n-1)$

(4) $\frac{1}{1-2z^{-1}}$ $2^{-n}\ u(-n)$

-(1') $-\frac{2}{1-2z}$

(5') $\frac{z^{-1}}{1-0.5z^{-1}}$

-(2') $-\frac{0.5}{1-0.5z}$

(6') $\frac{z^{-1}}{1-2z^{-1}}$

-(7') $-\frac{z}{1-2z}$

(3') $\frac{0.5}{1-0.5z^{-1}}$

-(8') $-\frac{z}{1-0.5z}$

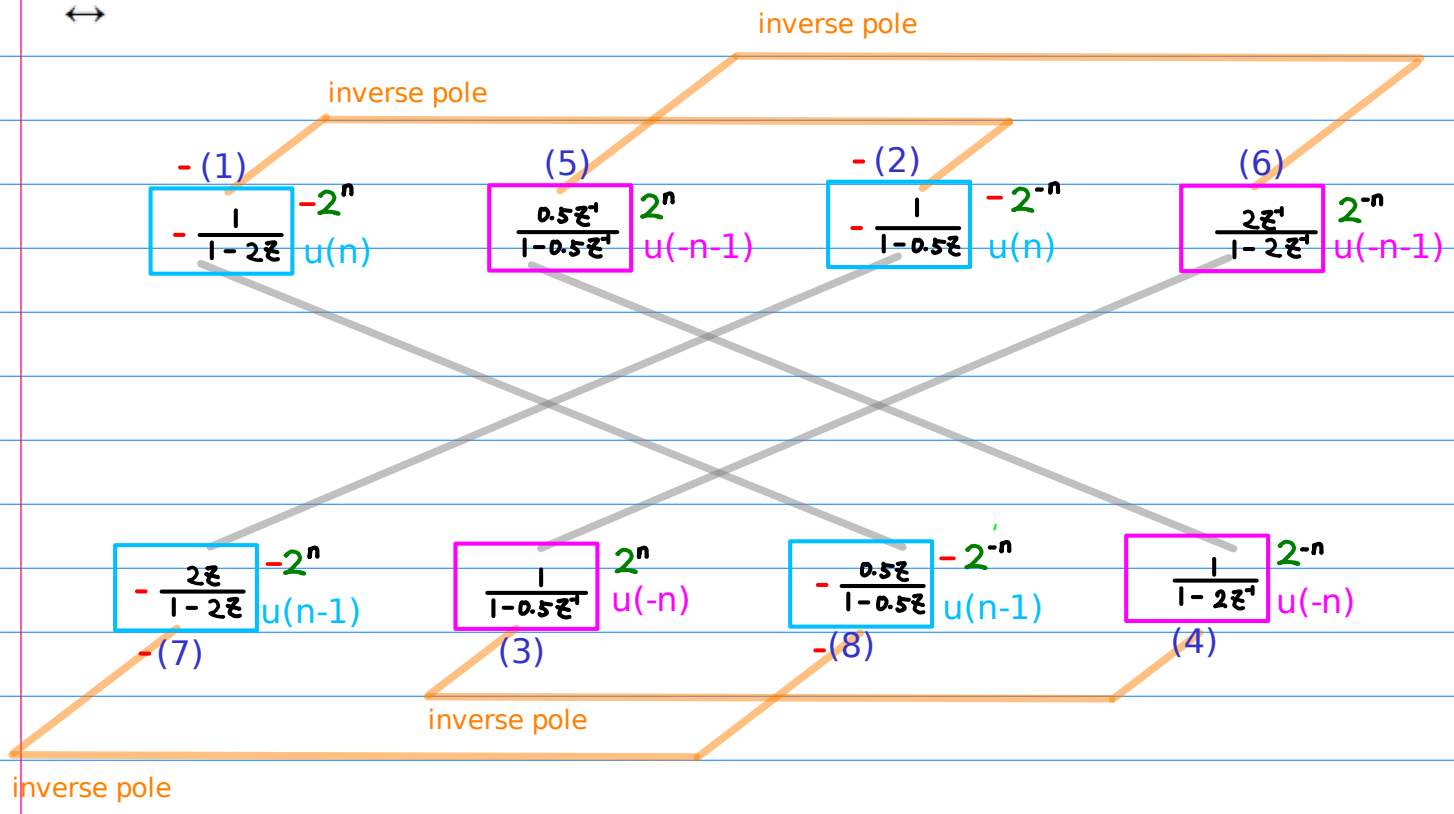
(4') $\frac{2}{1-2z^{-1}}$

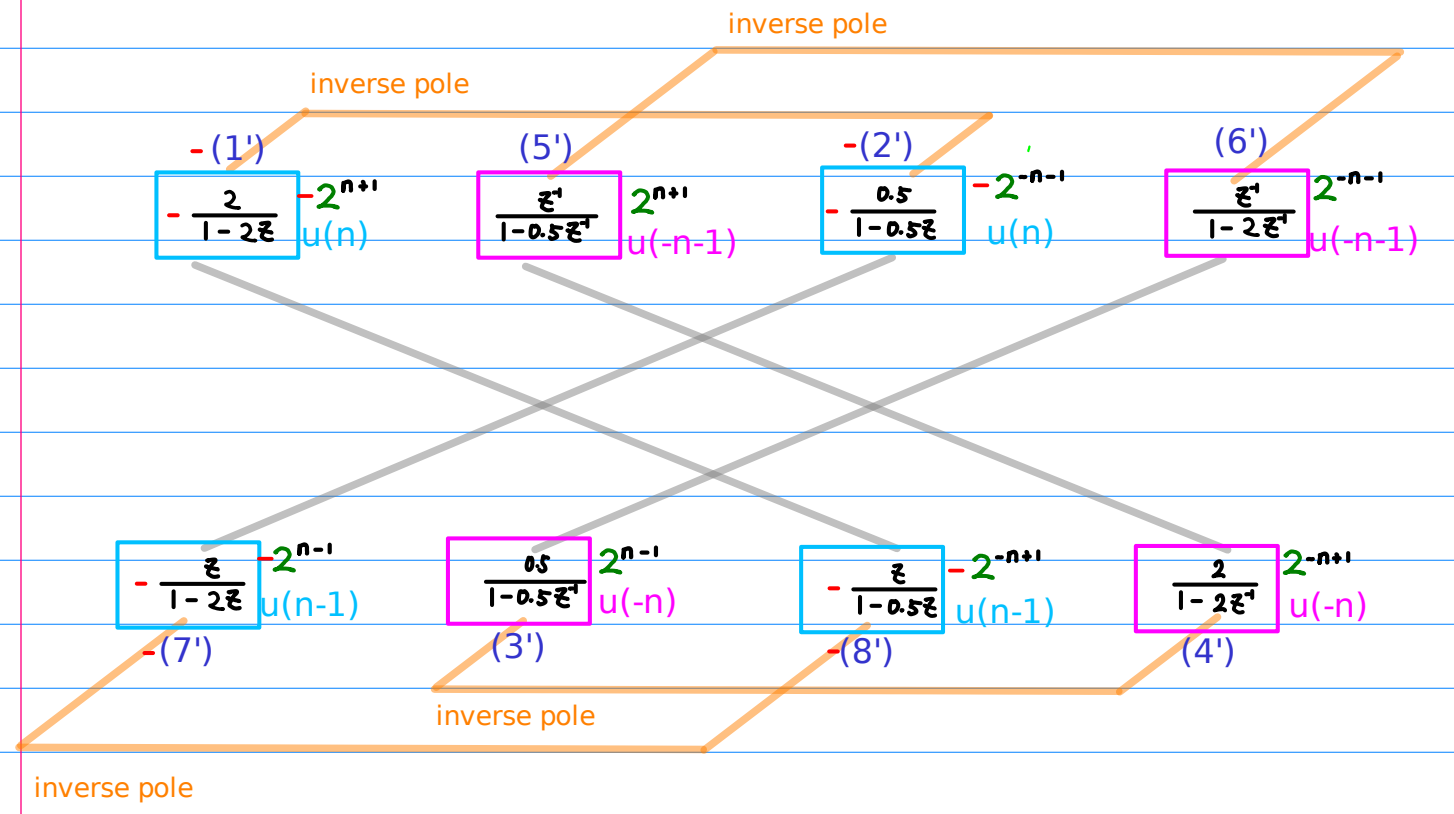
$n \leftarrow -n$

$u(n) \leftrightarrow u(n-1)$

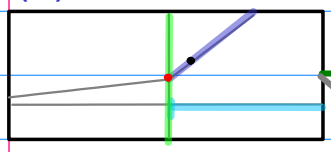
$u(-n) \leftrightarrow u(-n-1)$

$\leftrightarrow$





(1)

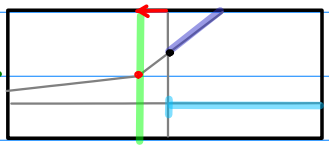


a^n u(n)

*a ←

/z ⇐

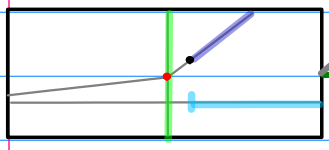
(1')



a^{n+1} u(n)

Left Shifted

(7)

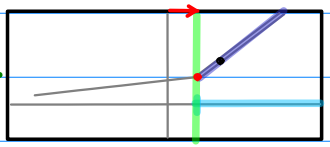


a^n u(n-1)

*z ⇒

/a →

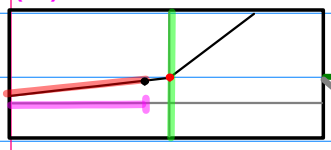
(7')



a^{n-1} u(n-1)

Right Shifted

(5)

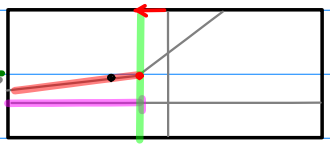


a^n u(-n-1)

*a ←

/z ⇐

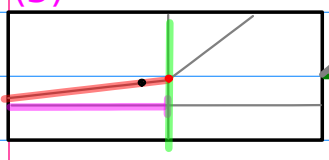
(5')



a^{n+1} u(-n-1)

Left Shifted

(3)

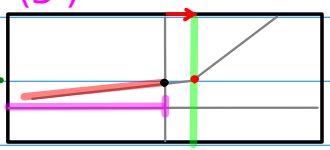


a^n u(-n)

*z ⇒

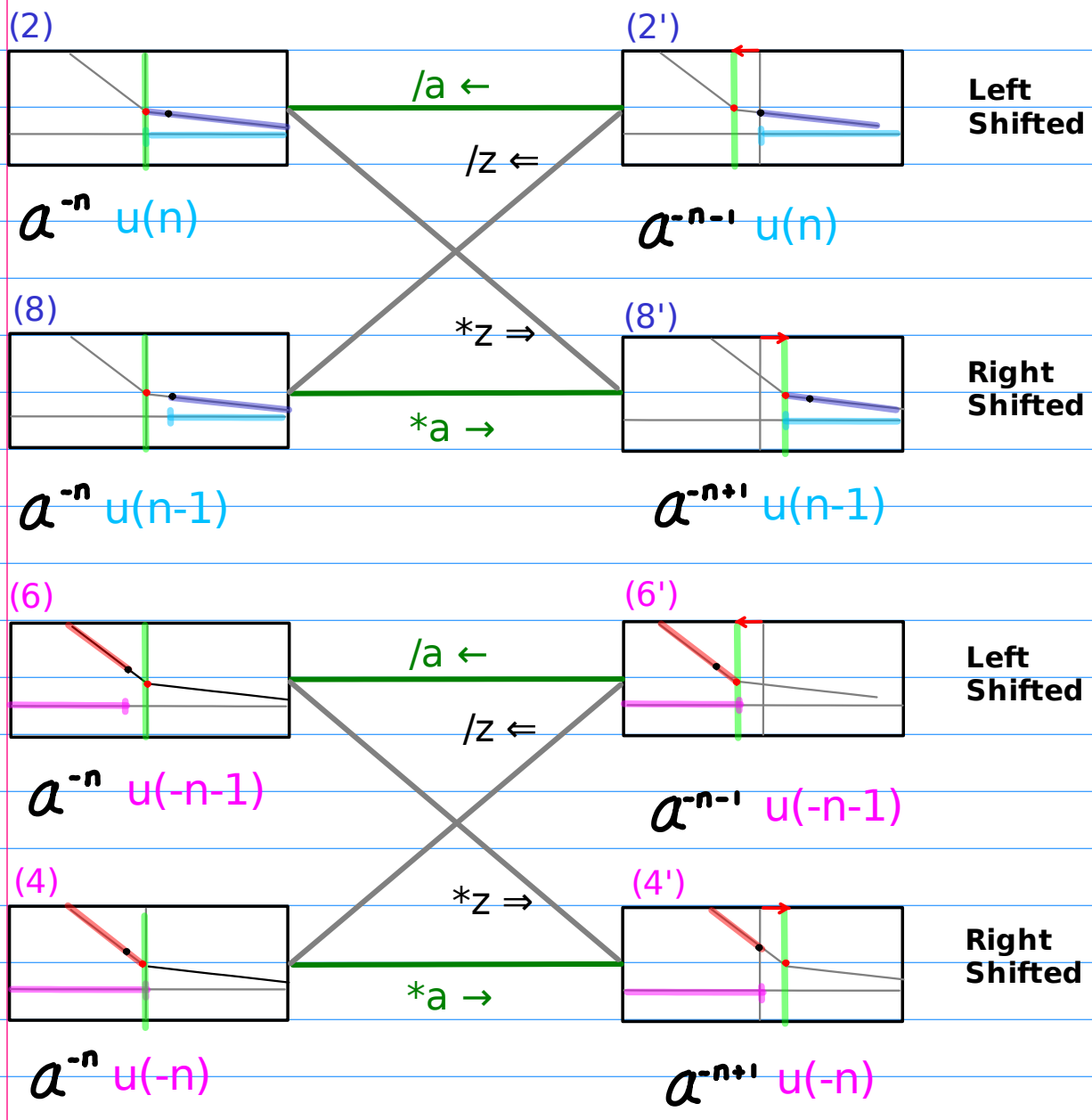
/a →

(3')

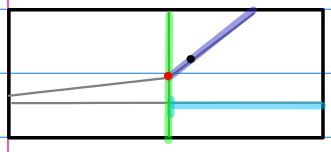


a^{n-1} u(-n)

Right Shifted

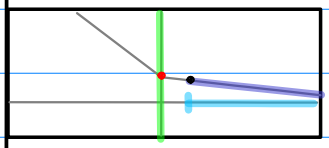

(2)
(2')
/a ←
/z ⇐
Left
Shifted
a^{-n} u(n)
a^{-n-1} u(n)
(8)
(8')
*z ⇒
*a →
Right
Shifted
a^{-n} u(n-1)
a^{-n+1} u(n-1)
(6)
(6')
/a ←
/z ⇐
Left
Shifted
a^{-n} u(-n-1)
a^{-n-1} u(-n-1)
(4)
(4')
*z ⇒
*a →
Right
Shifted
a^{-n} u(-n)
a^{-n+1} u(-n)

(1)



a^n u(n)

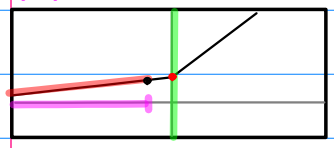
(8)



Left Shifted

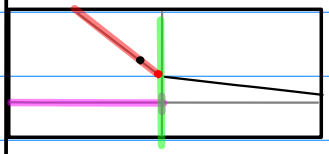
a^{-n} u(n-1)

(5)



a^n u(-n-1)

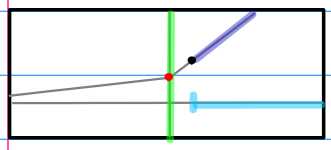
(4)



Left Shifted

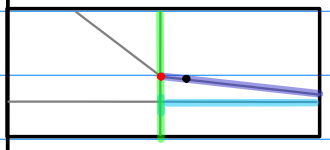
a^{-n} u(-n)

(7)



a^n u(n-1)

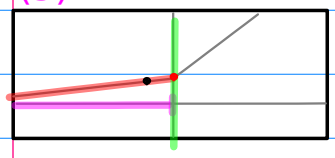
(2)



Right Shifted

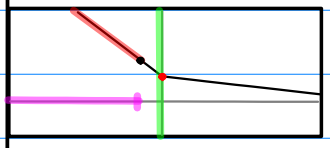
a^{-n} u(n)

(3)



a^n u(-n)

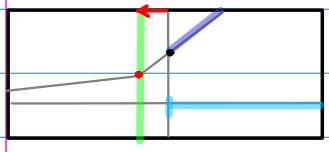
(6)



Right Shifted

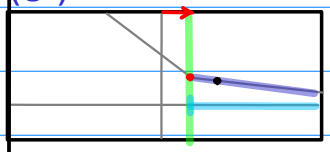
a^{-n} u(-n-1)

(1')



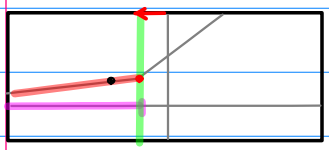
a^{n+1} u(n)

(8')



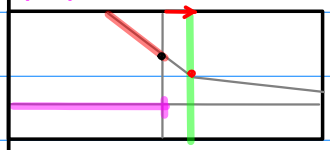
a^{-n+1} u(n-1)

(5')



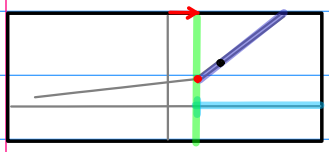
a^{n+1} u(-n-1)

(4')



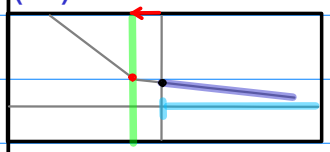
a^{-n+1} u(-n)

(7')



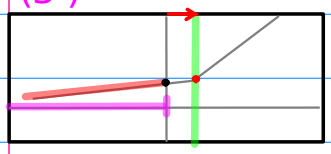
a^{n-1} u(n-1)

(2')



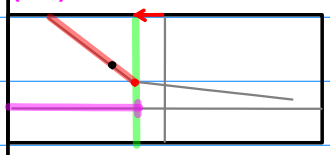
a^{-n-1} u(n)

(3')



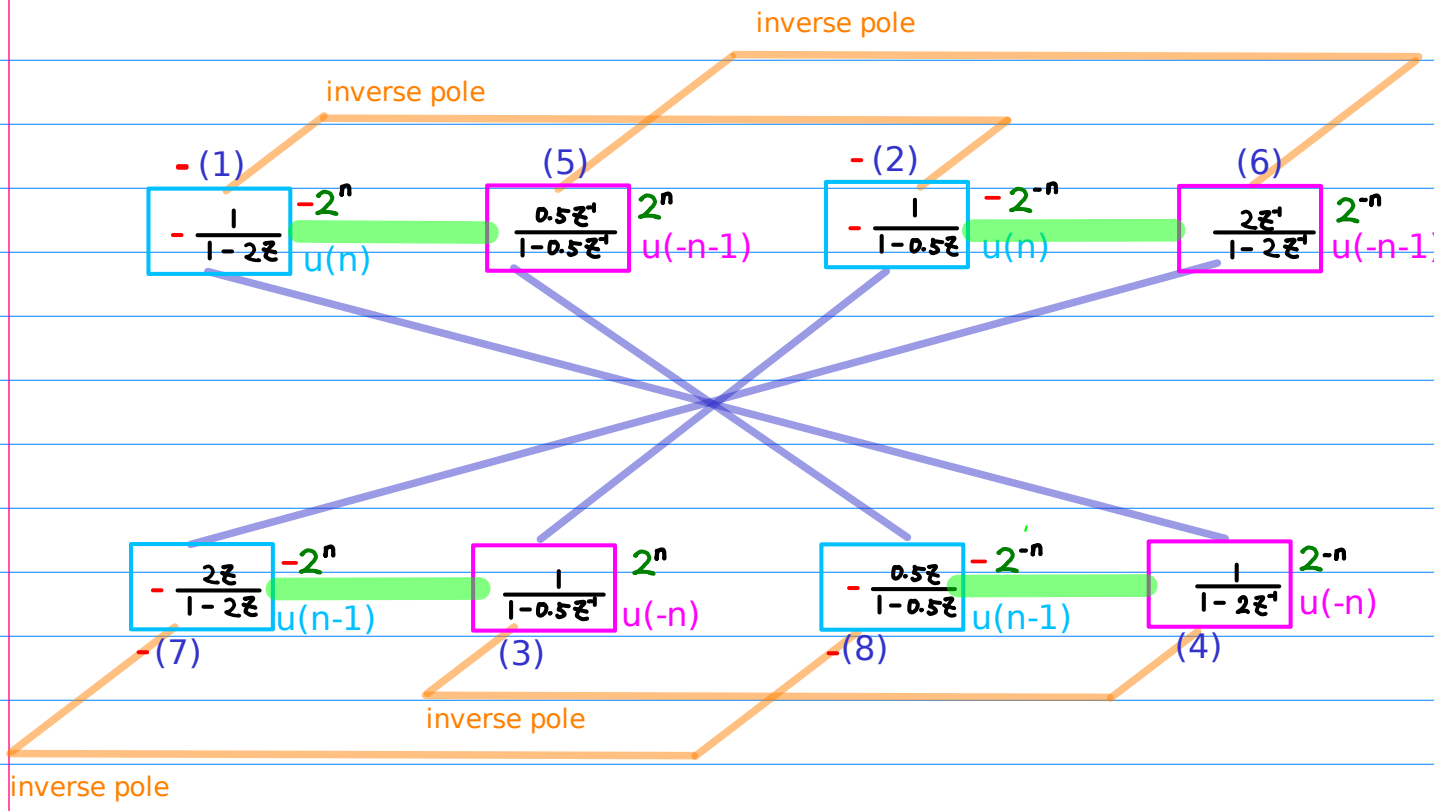
a^{n-1} u(-n)

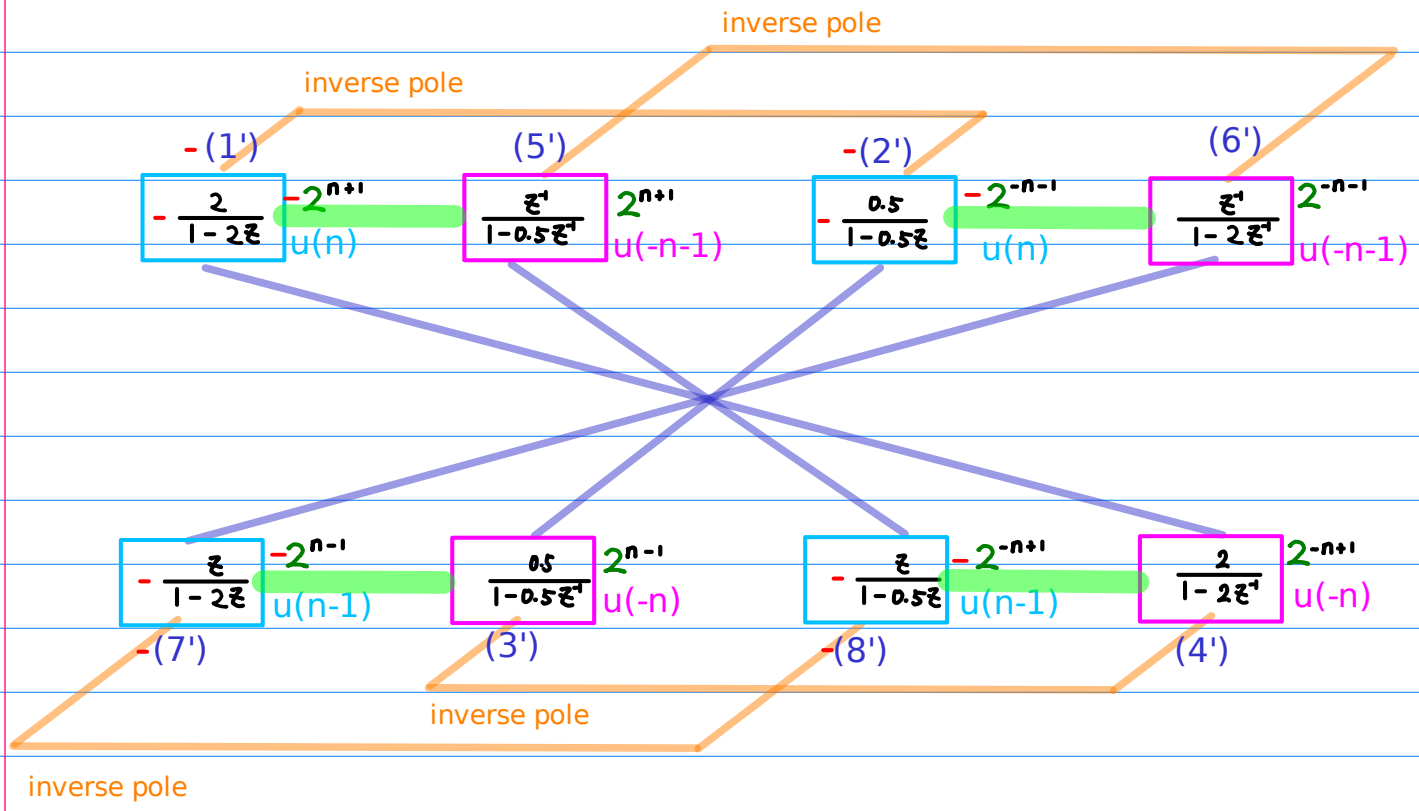
(6')



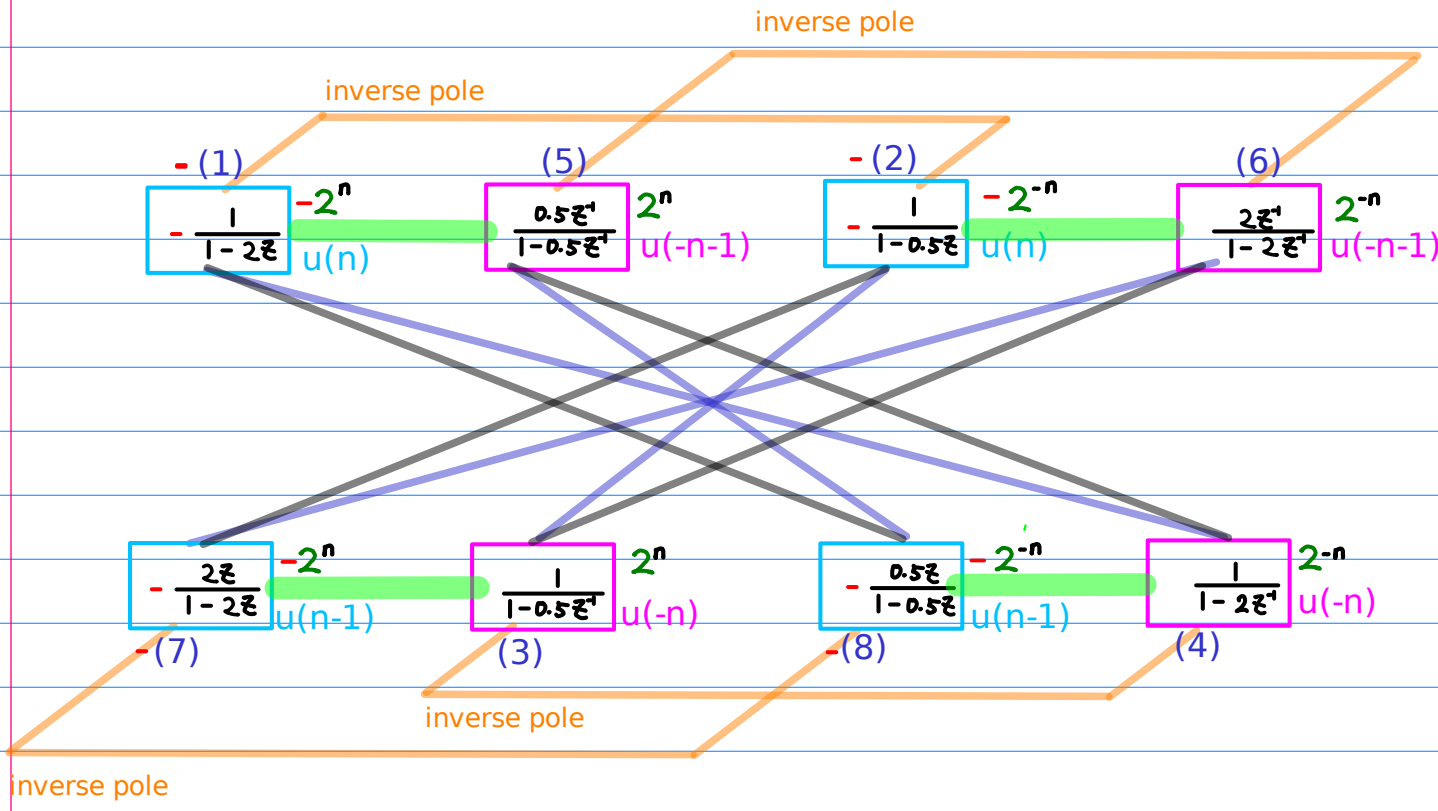
a^{-n-1} u(-n-1)

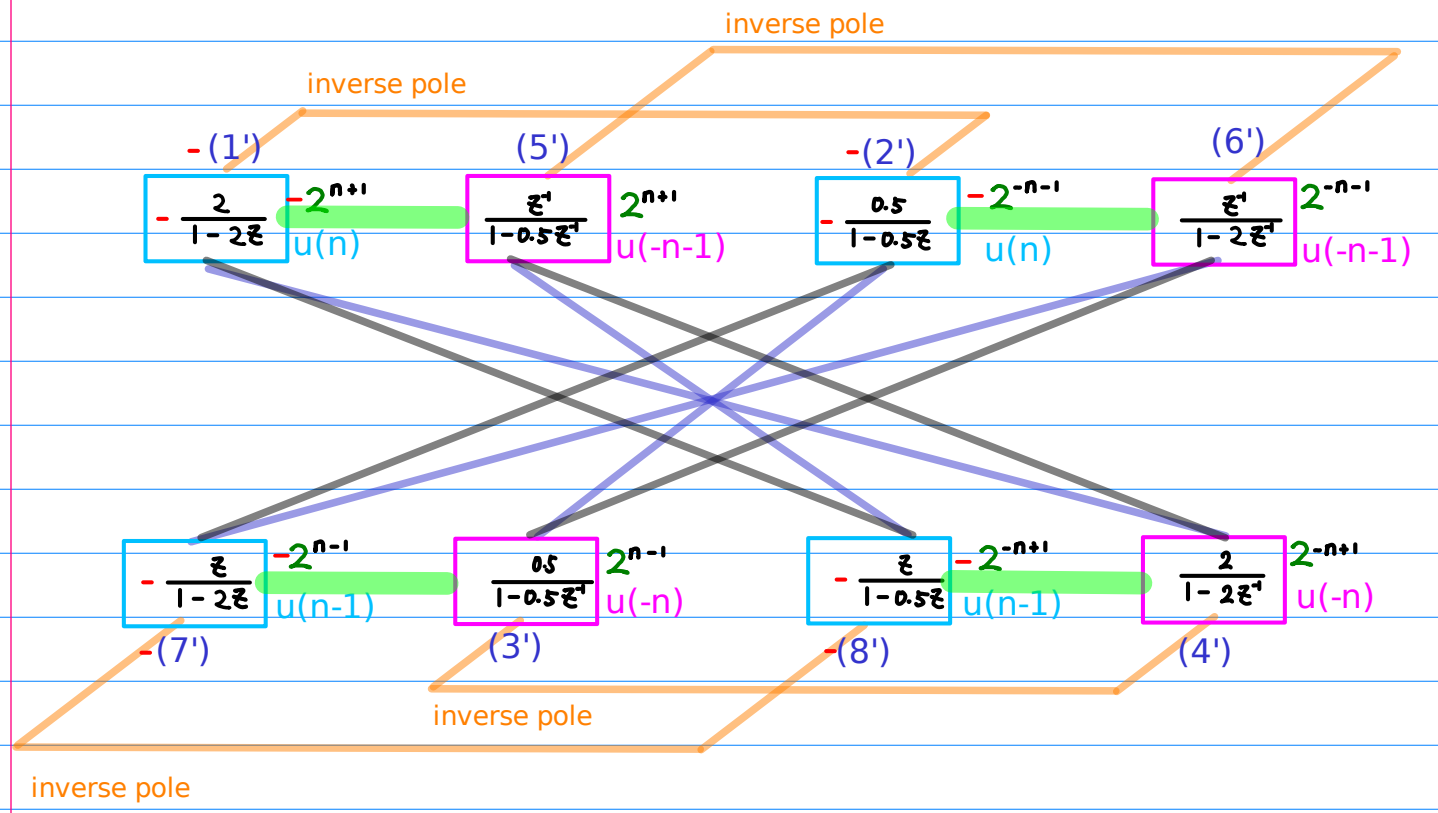
Time Reversal $-1, z^{-1}$



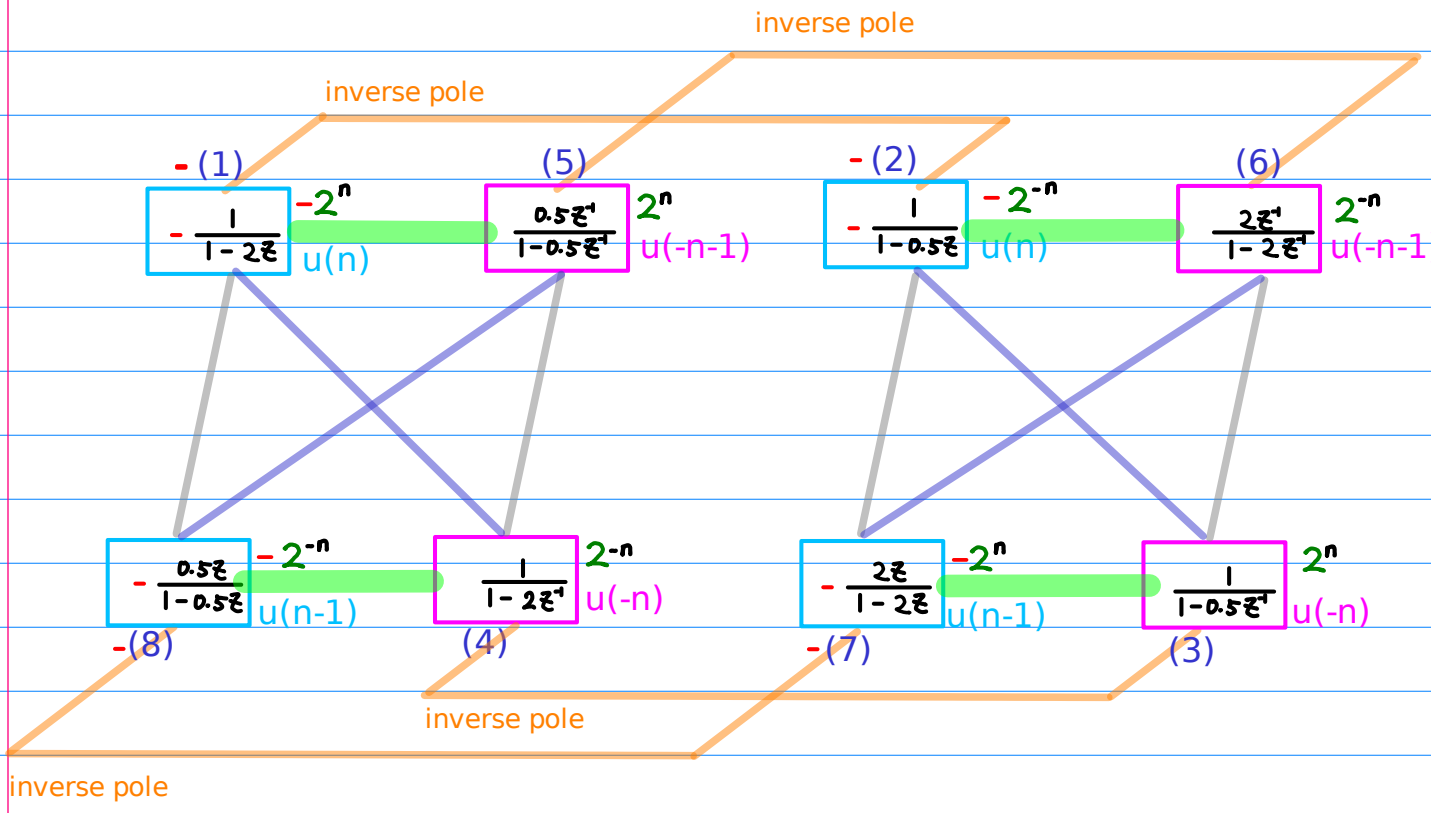


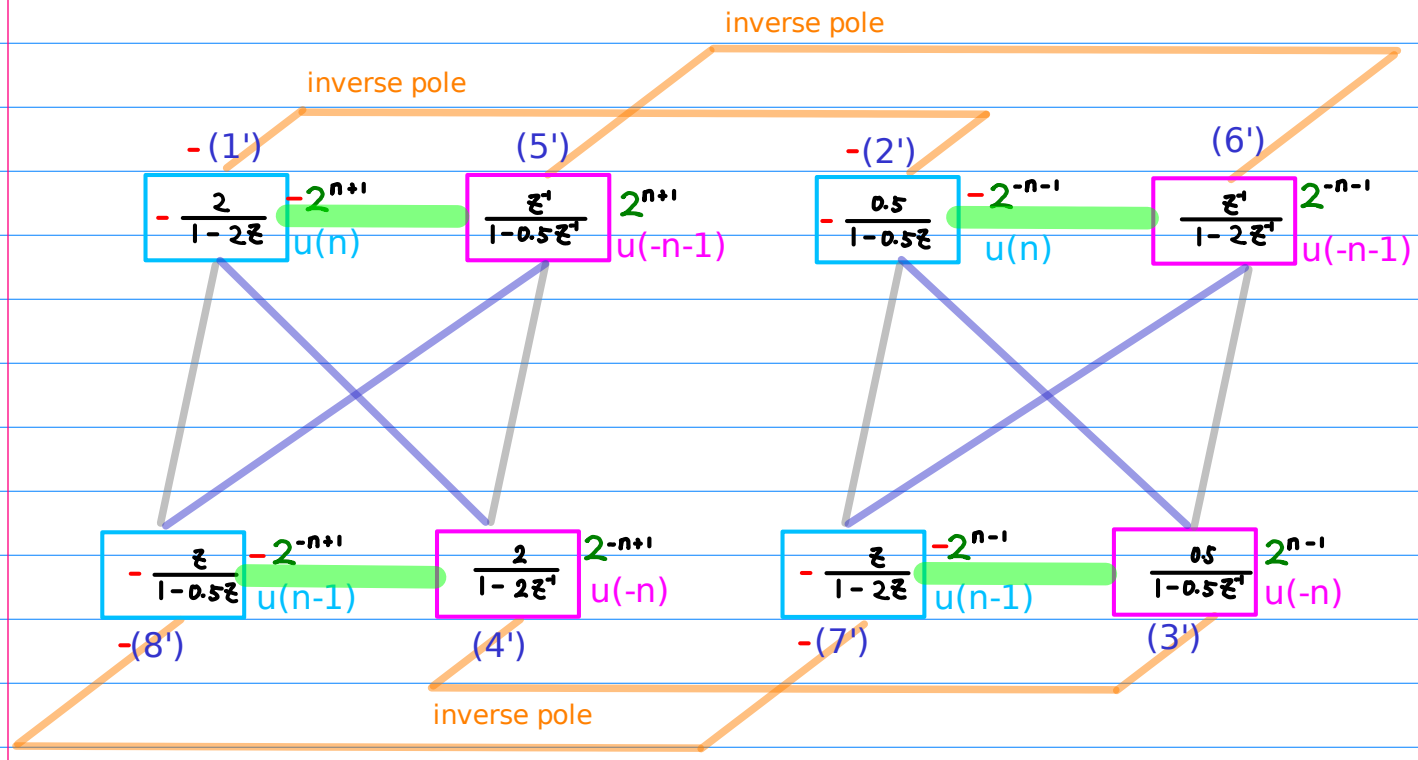
Time Reversal $-1, z^{-1}$





Time Reversal $-1, z^{-1}$





Time Reversal in z-Transform

Time reversal x[-n] X(z^-1)

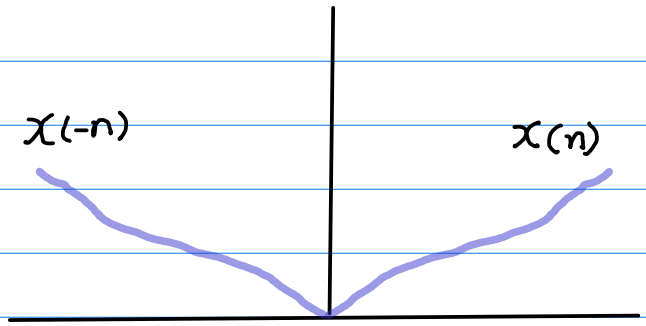
.

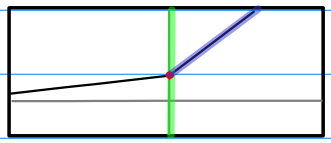
$$\mathcal{Z}\{x(-n)\} = \sum_{n=-\infty}^{\infty} x(-n)\, z^{-n}$$

$$= \sum_{m=-\infty}^{\infty} x(m)\, z^{m}$$

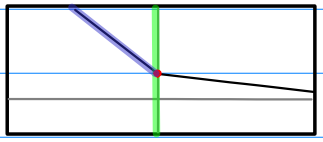
$$= \sum_{m=-\infty}^{\infty} x(m)\left(z^{-1}\right)^{-m}$$

$$= X(z^{-1})$$





2^n
u(n)

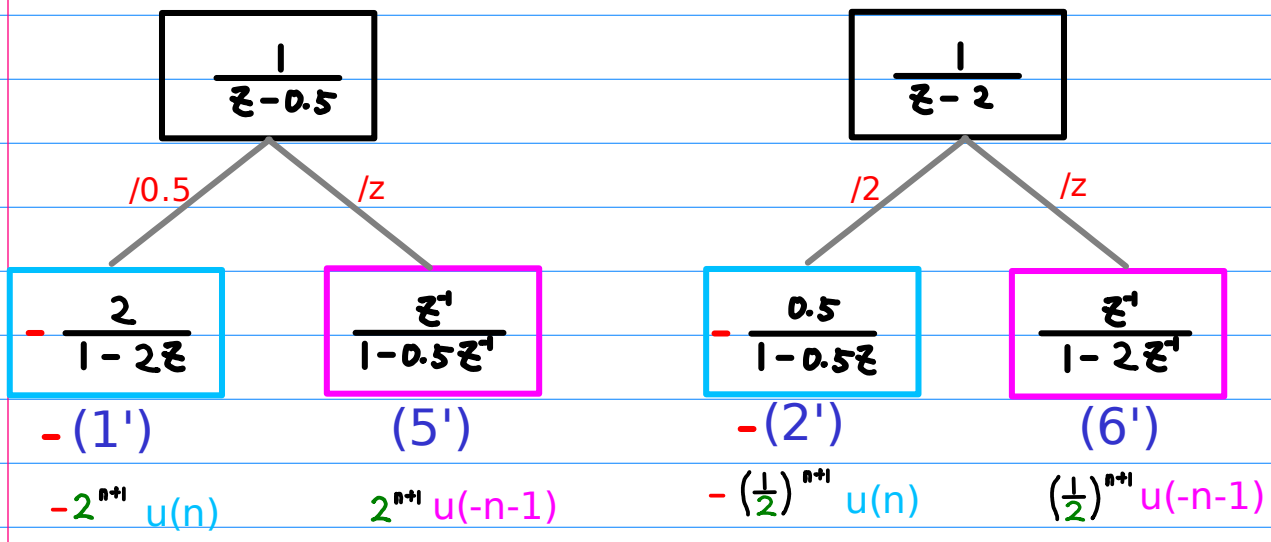


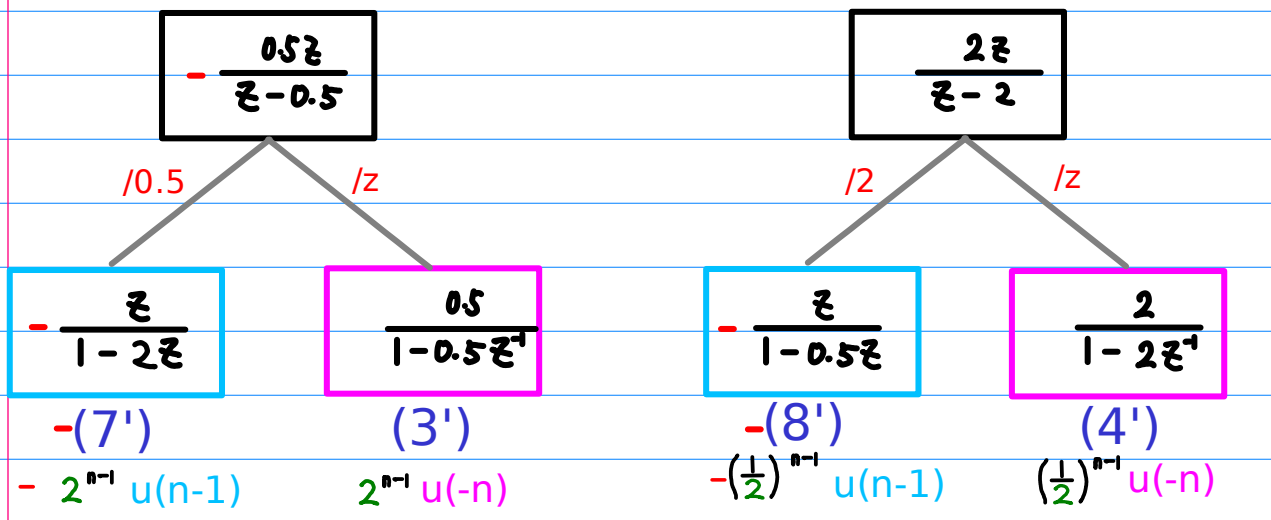
2^{-n}
u(-n)

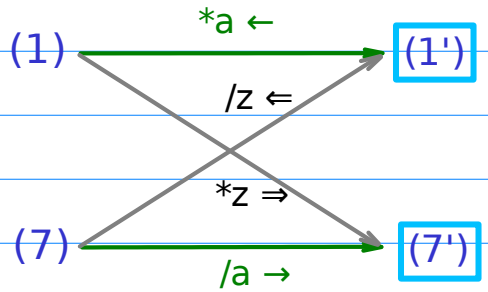
Reciprocal Pole and Shift Relations

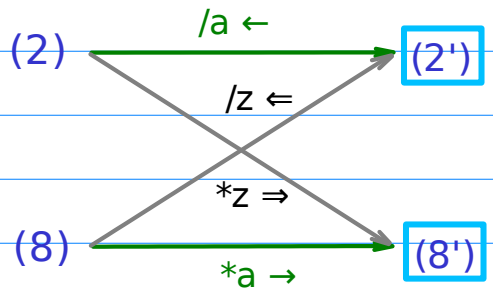
$p^{-1} = 0.5$

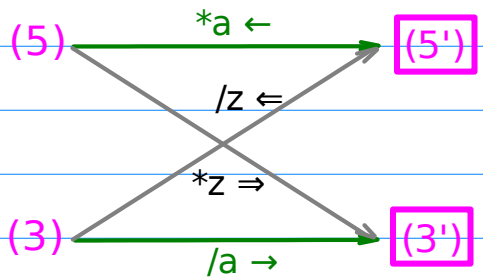
$p = 2$

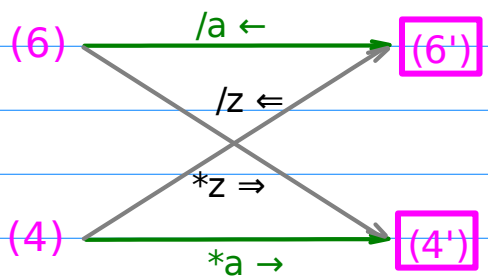












(1) (5)
(7) (3)

(2) (6)
(8) (4)

(1') (5')
(7') (3')

(2')(6')
(8')(4')

Partial fractions and geometric power series

$P^{-1} = 0.5$

$P = 2$

-(1) $-\frac{1}{1-2z}$

$-2^n \; u(n)$

(5) $\frac{0.5z^{-1}}{1-0.5z^{-1}}$

$2^n \; u(-n-1)$

-(2) $-\frac{1}{1-0.5z}$

$-2^{-n} \; u(n)$

(6) $\frac{2z^{-1}}{1-2z^{-1}}$

$2^{-n} \; u(-n-1)$

-(7) $-\frac{2z}{1-2z}$

$-2^n \; u(n-1)$

(3) $\frac{1}{1-0.5z^{-1}}$

$2^n \; u(-n)$

-(8) $-\frac{0.5z}{1-0.5z}$

$-2^{-n} \; u(n-1)$

(4) $\frac{1}{1-2z^{-1}}$

$2^{-n} \; u(-n)$

-(1') $-\frac{2}{1-2z}$

$-2^{n+1} \; u(n)$

(5') $\frac{z^{-1}}{1-0.5z^{-1}}$

$2^{n+1} \; u(-n-1)$

-(2') $-\frac{0.5}{1-0.5z}$

$-2^{-n-1} \; u(n)$

(6') $\frac{z^{-1}}{1-2z^{-1}}$

$2^{-n-1} \; u(-n-1)$

-(7') $-\frac{z}{1-2z}$

$-2^{n-1} \; u(n-1)$

(3') $\frac{0.5}{1-0.5z^{-1}}$

$2^{n-1} \; u(-n)$

-(8') $-\frac{z}{1-0.5z}$

$-2^{-n+1} u(n-1)$

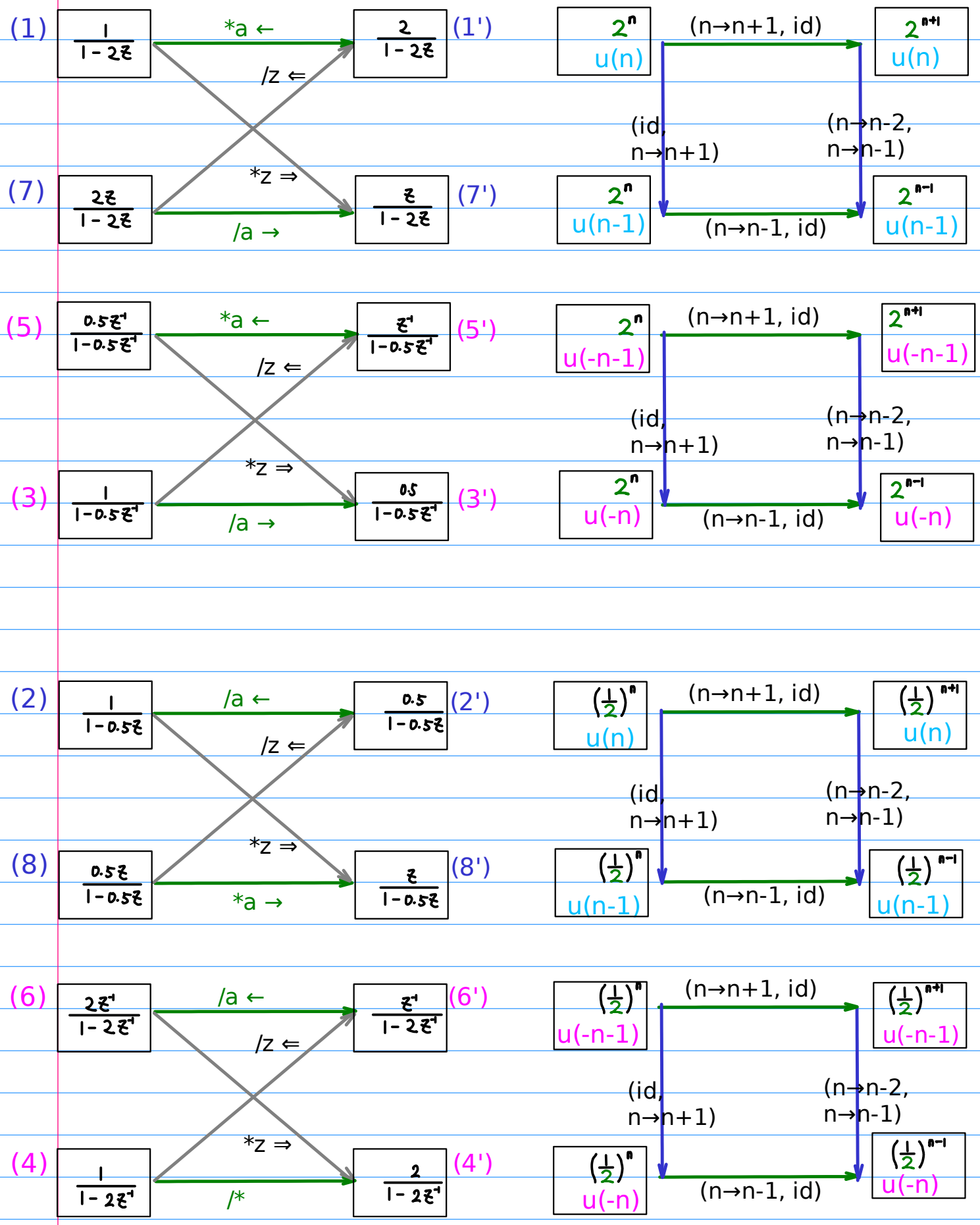
(4') $\frac{2}{1-2z^{-1}}$

$2^{-n+1} \; u(-n)$

(Exp, Range)

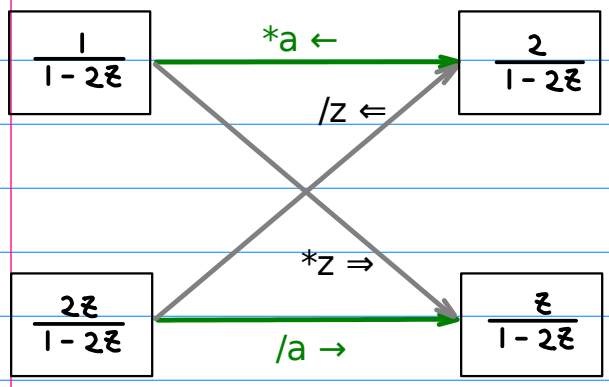
$P^{-1} = 0.5$

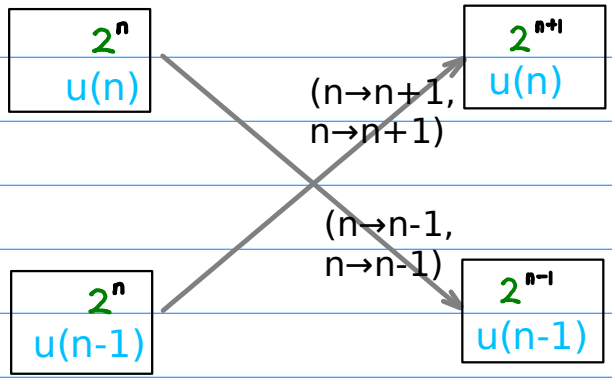
$P = 2$

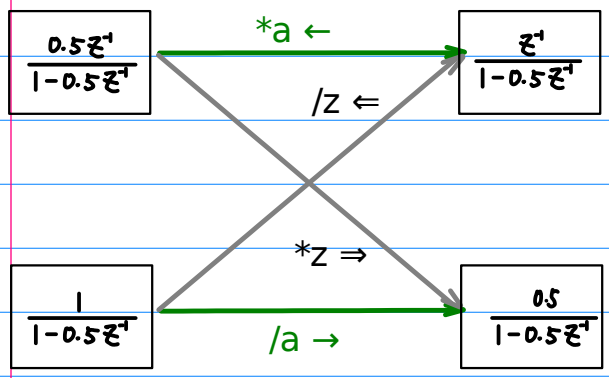


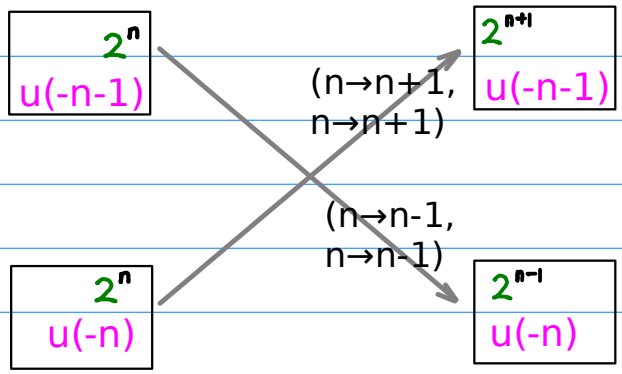
$P^{-1} = 0.5$

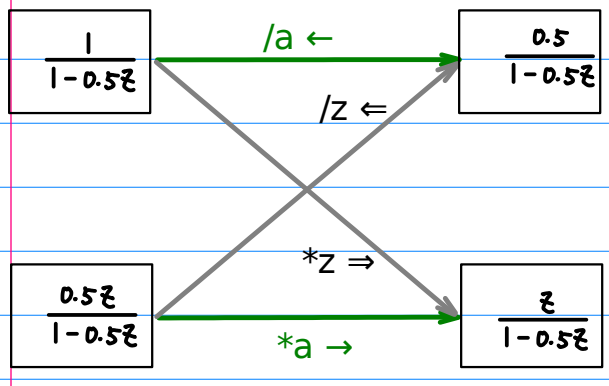
$P = 2$

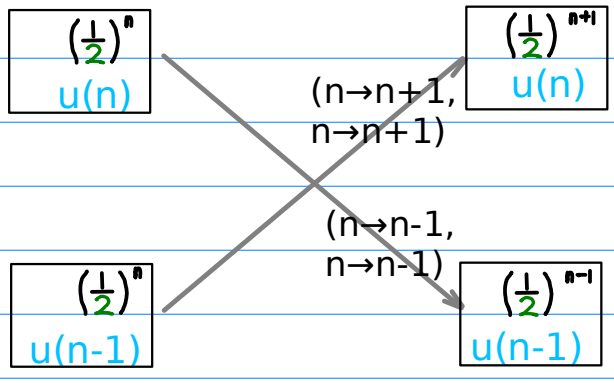

1 / 1 − 2z
*a ←
2 / 1 − 2z
/z ⇐
*z ⇒
2z / 1 − 2z
/a →
z / 1 − 2z

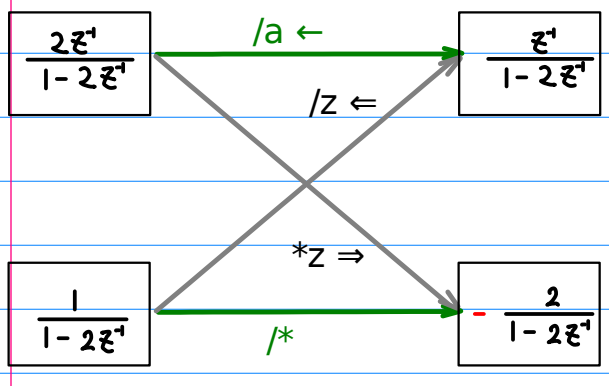

2^n u(n)
(n→n+1, n→n+1)
2^{n+1} u(n)
(n→n-1, n→n-1)
2^n u(n-1)
2^{n-1} u(n-1)

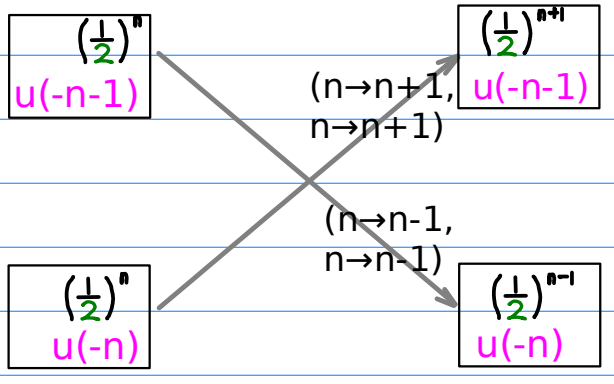

0.5z^{-1} / 1 − 0.5z^{-1}
*a ←
z^{-1} / 1 − 0.5z^{-1}
/z ⇐
*z ⇒
1 / 1 − 0.5z^{-1}
/a →
0.5 / 1 − 0.5z^{-1}


2^n u(-n-1)
(n→n+1, n→n+1)
2^{n+1} u(-n-1)
(n→n-1, n→n-1)
2^n u(-n)
2^{n-1} u(-n)


1 / 1 − 0.5z
/a ←
0.5 / 1 − 0.5z
/z ⇐
*z ⇒
0.5z / 1 − 0.5z
*a →
z / 1 − 0.5z

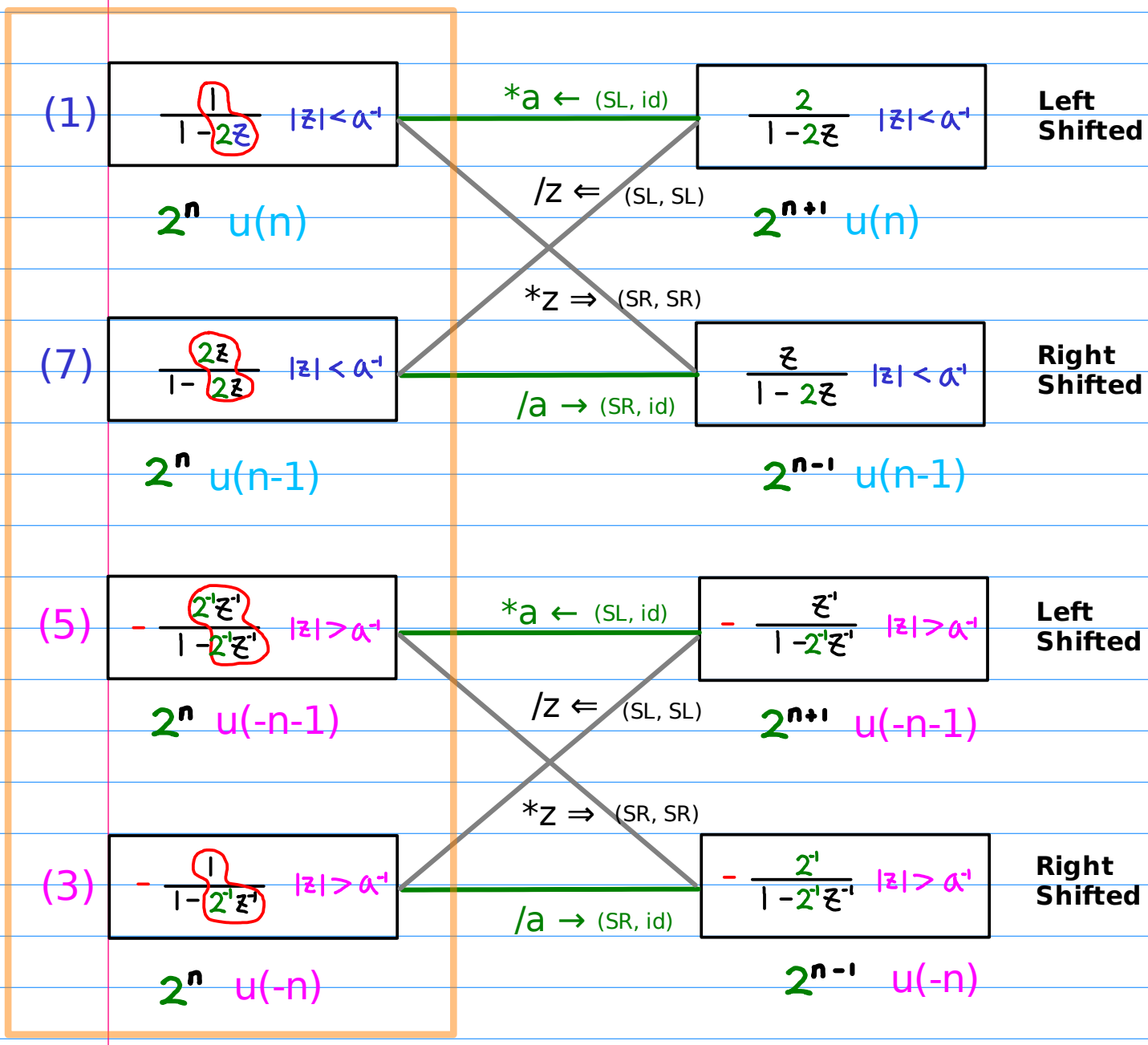

(1/2)^n u(n)
(n→n+1, n→n+1)
(1/2)^{n+1} u(n)
(n→n-1, n→n-1)
(1/2)^n u(n-1)
(1/2)^{n-1} u(n-1)


2z^{-1} / 1 − 2z^{-1}
/a ←
z^{-1} / 1 − 2z^{-1}
/z ⇐
*z ⇒
1 / 1 − 2z^{-1}
/*
- 2 / 1 − 2z^{-1}


(1/2)^n u(-n-1)
(n→n+1, n→n+1)
(1/2)^{n+1} u(-n-1)
(n→n-1, n→n-1)
(1/2)^n u(-n)
(1/2)^{n-1} u(-n)

Shifting Geometric Series (1) positive exponent

Negative Exponent $a = 2$ /z $n \leftarrow n+1$ *z $n \leftarrow n-1$

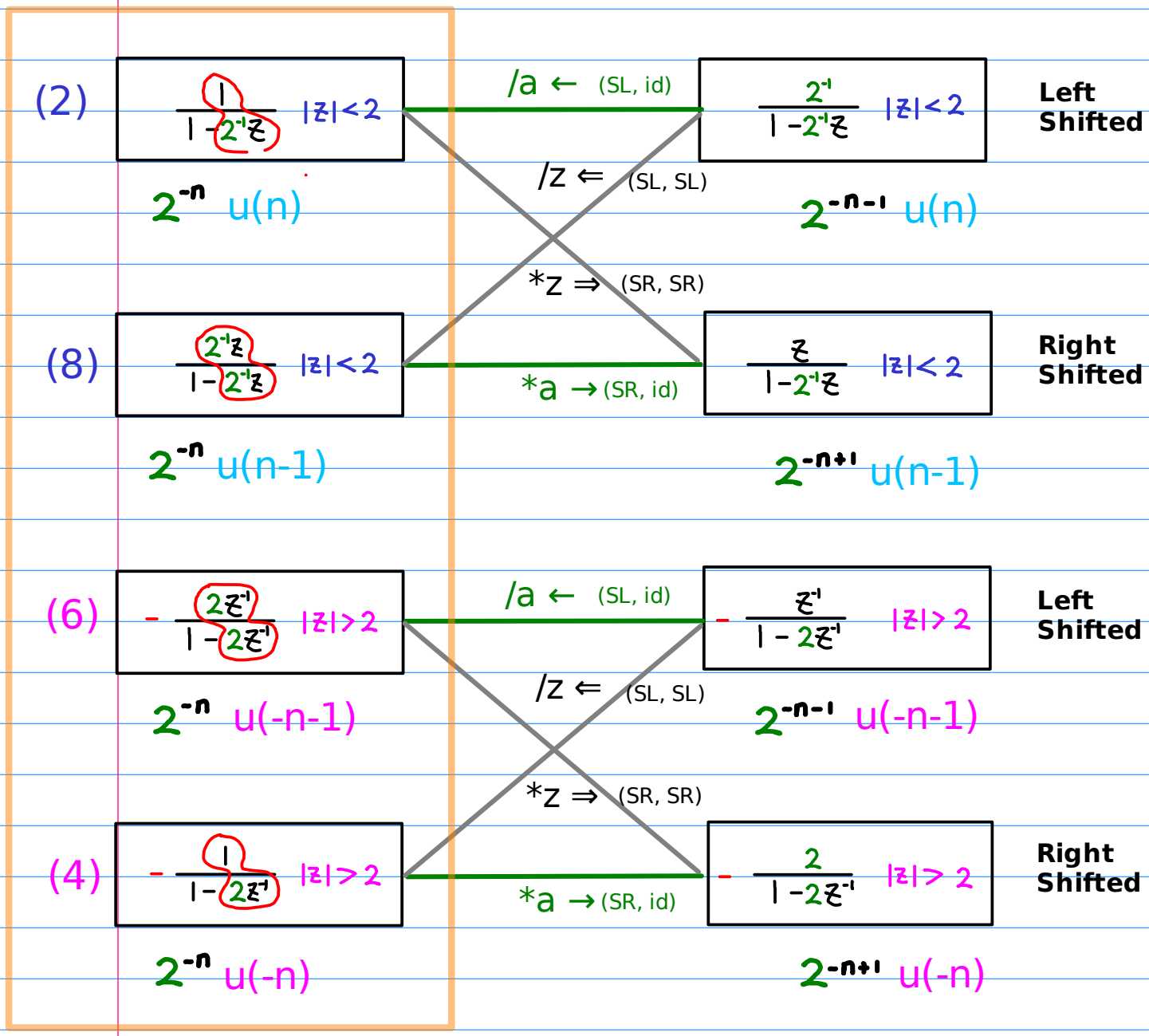


Causal	**u(n)**	(1)	(2)
	u(n-1)	(7)	(8)
Anti-Causal	**u(-n-1)**	(5)	(6)
	u(-n)	(3)	(4)

butterfly pair ordering

Shifting Geometric Series (2) negative exponent

Negative Exponent $a^{-1}=2^{-1}$ /z $n \leftarrow n+1$ *z $n \leftarrow n-1$



Causal	**u(n)**	(1)	(2)
	u(n-1)	(7)	(8)
Anti-Causal	**u(-n-1)**	(5)	(6)
	u(-n)	(3)	(4)

butterfly pair ordering

$a = 2$ (1) (2) (3) (4)

$a^{-1} = 2^{-1}$ (5) (6) (7) (8)

Unshifted combinations

including the origin

$P_1 = 0.5$

$P_2 = 2$

$a = 2$

(1)

$f(z) = \frac{1}{1-2z}$	$\lvert z \rvert < 2^{-1}$
$2^n\ u(n)$	$(n \geq 0)$

(3)

$\bar{f}_1(z) = \frac{1}{1-2^{-1}z^{-1}}$	$\lvert z \rvert > 2^{-1}$
$2^n\ u(-n)$	$(n < 1)$

(5)

$\bar{f}(z) = \frac{2^{-1}z^{-1}}{1-2^{-1}z^{-1}}$	$\lvert z \rvert > 2^{-1}$
$2^n\ u(-n-1)$	$(n < 0)$

(7)

$f_1(z) = \frac{2z}{1-2z}$	$\lvert z \rvert < 2^{-1}$
$2^n\ u(n-1)$	$(n \geq 1)$

$a^{-1} = 2^{-1}$

(2)

$g(z) = \frac{1}{1-2^{-1}z}$	$\lvert z \rvert < 2$
$(\frac{1}{2})^n\ u(n)$	$(n \geq 0)$

(4)

$\bar{g}_1(z) = \frac{1}{1-2z^{-1}}$	$\lvert z \rvert > 2$
$(\frac{1}{2})^n\ u(-n)$	$(n < 1)$

(6)

$\bar{g}(z) = \frac{2z^{-1}}{1-2z^{-1}}$	$\lvert z \rvert > 2$
$(\frac{1}{2})^n\ u(-n-1)$	$(n < 0)$

(8)

$g_1(z) = \frac{2^{-1}z}{1-2^{-1}z}$	$\lvert z \rvert < 2$
$(\frac{1}{2})^n\ u(n-1)$	$(n \geq 1)$

$a=2$ (1') (2') (3') (4')

$a^{-1}=2^{-1}$ (5') (6') (7') (8')

Shifted combinations

excluding the origin

$P_1 = 0.5$

$P_2 = 2$

$a=2$

(1')

$2z$

$f_2(z) = \frac{2}{1-2z}$	$\lvert z\rvert < 2^{-1}$
$2^{n+1}\,u(n)$	$(n \geq 0)$

(3')

$2^{-1}z^{-1}$

$\bar{f}_3(z) = \frac{2^{-1}}{1-2^{-1}z^{-1}}$	$\lvert z\rvert > 2^{-1}$
$2^{n-1}\,u(-n)$	$(n < 1)$

(5')

$2^{-1}z^{-1}$

$\bar{f}_2(z) = \frac{z^{-1}}{1-2^{-1}z^{-1}}$	$\lvert z\rvert > 2^{-1}$
$2^{n+1}\,u(-n-1)$	$(n < 0)$

(7')

$2z$

$f_3(z) = \frac{z}{1-2z}$	$\lvert z\rvert < 2^{-1}$
$2^{n-1}u(n-1)$	$(n \geq 1)$

$a^{-1}=2^{-1}$

(2')

$2^{-1}z$

$g_2(z) = \frac{2^{-1}}{1-2^{-1}z}$	$\lvert z\rvert < 2$
$(\frac{1}{2})^{n+1}\,u(n)$	$(n \geq 0)$

(4')

$2z^{-1}$

$\bar{g}_3(z) = \frac{2}{1-2z^{-1}}$	$\lvert z\rvert > 2$
$(\frac{1}{2})^{n-1}\,u(-n)$	$(n < 1)$

(6')

$2z^{-1}$

$\bar{g}_2(z) = \frac{z^{-1}}{1-2z^{-1}}$	$\lvert z\rvert > 2$
$(\frac{1}{2})^{n+1}\,u(-n-1)$	$(n < 0)$

(8')

$2^{-1}z$

$g_3(z) = \frac{z}{1-2^{-1}z}$	$\lvert z\rvert < 2$
$(\frac{1}{2})^{n-1}\,u(n-1)$	$(n \geq 1)$

(1') (2')
(1) (2)

(5') (6')
(5) (6)

Geometric series for unshifted combinations

$$-\frac{3}{2}\,\frac{1}{(z-0.5)(z-2)}$$

$a=2$

(1')

$f_2(z) = \frac{2}{1-2z}$	$\lvert z\rvert < 2^{-1}$
$2^{n+1}\,u(n)$	$(n \geq 0)$

(1)

$f(z) = \frac{1}{1-2z}$	$\lvert z\rvert < 2^{-1}$
$2^{n}\,u(n)$	$(n \geq 0)$

$a^{-1}=2^{-1}$

(2')

$g_2(z) = \frac{2^{-1}}{1-2^{-1}z}$	$\lvert z\rvert < 2$
$\left(\frac{1}{2}\right)^{n+1}\,u(n)$	$(n \geq 0)$

(2)

$g(z) = \frac{1}{1-2^{-1}z}$	$\lvert z\rvert < 2$
$\left(\frac{1}{2}\right)^{n}\,u(n)$	$(n \geq 0)$

(5')

$\bar{f}_2(z) = \frac{z^{-1}}{1-2^{-1}z^{-1}}$	$\lvert z\rvert > 2^{-1}$
$2^{n+1}\,u(-n-1)$	$(n < 0)$

(5)

$\bar{f}(z) = \frac{2^{-1}z^{-1}}{1-2^{-1}z^{-1}}$	$\lvert z\rvert > 2^{-1}$
$2^{n}\,u(-n-1)$	$(n < 0)$

(6')

$\bar{g}_2(z) = \frac{z^{-1}}{1-2z^{-1}}$	$\lvert z\rvert > 2$
$\left(\frac{1}{2}\right)^{n+1}\,u(-n-1)$	$(n < 0)$

(6)

$\bar{g}(z) = \frac{2z^{-1}}{1-2z^{-1}}$	$\lvert z\rvert > 2$
$\left(\frac{1}{2}\right)^{n}\,u(-n-1)$	$(n < 0)$

(7') (8')
(7) (8)

(3') (4')
(3) (4)

Geometric series for unshifted combinations

$$-\frac{3}{2}\,\frac{z^2}{(z-0.5)(z-2)}$$

$a = 2$

(7')

$f_3(z) = \dfrac{z}{1-2z}$	$\lvert z\rvert < 2^{-1}$
$2^{n-1}u(n-1)$	$(n \geq 1)$

(7)

$f_1(z) = \dfrac{2z}{1-2z}$	$\lvert z\rvert < 2^{-1}$
$2^{n}\,u(n-1)$	$(n \geq 1)$

(3')

$\bar{f}_3(z) = \dfrac{2^{-1}}{1-2^{-1}z^{-1}}$	$\lvert z\rvert > 2^{-1}$
$2^{n-1}\,u(-n)$	$(n < 1)$

(3)

$\bar{f}_1(z) = \dfrac{1}{1-2^{-1}z^{-1}}$	$\lvert z\rvert > 2^{-1}$
$2^{n}\,u(-n)$	$(n < 1)$

$a^{-1} = 2^{-1}$

(8')

$g_3(z) = \dfrac{z}{1-2^{-1}z}$	$\lvert z\rvert < 2$
$\left(\frac{1}{2}\right)^{n-1}u(n-1)$	$(n \geq 1)$

(8)

$g_1(z) = \dfrac{2^{-1}z}{1-2^{-1}z}$	$\lvert z\rvert < 2$
$\left(\frac{1}{2}\right)^{n}u(n-1)$	$(n \geq 1)$

(4')

$\bar{g}_3(z) = \dfrac{2}{1-2z^{-1}}$	$\lvert z\rvert > 2$
$\left(\frac{1}{2}\right)^{n-1}u(-n)$	$(n < 1)$

(4)

$\bar{g}_1(z) = \dfrac{1}{1-2z^{-1}}$	$\lvert z\rvert > 2$
$\left(\frac{1}{2}\right)^{n}u(-n)$	$(n < 1)$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

Case A $|z| < 0.5$

$$-\frac{3}{2}\frac{1}{(z-0.5)(z-2)}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

Simple Pole Form

Geometric Power Series Forms

$\frac{1}{z-0.5}$	$\frac{1}{z-2}$
$-\frac{2}{1-2z}$	$-\frac{0.5}{1-0.5z}$
$\lvert z\rvert < 0.5$	$\lvert z\rvert < 2$

$|z| < 0.5$ = $|z| < 0.5$ $\cap$ $|z| < 2$

$-2^{n+1}\,u(n)$ $\quad$ $-\left(\frac{1}{2}\right)^{n+1}u(n)$

$|z| < 0.5$ $\quad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad -2^{n+1}\,u(n) + \left(\frac{1}{2}\right)^{n+1}u(n)$

(1')

$f_2(z) = \frac{2}{1-2z}$	$\lvert z\rvert < 2^{-1}$
$2^{n+1}\,u(n)$	$(n \geq 0)$

(2')

$g_2(z) = \frac{2^{-1}}{1-2^{-1}z}$	$\lvert z\rvert < 2$
$\left(\frac{1}{2}\right)^{n+1}u(n)$	$(n \geq 0)$

(1)

$f(z) = \frac{1}{1-2z}$	$\lvert z\rvert < 2^{-1}$
$2^{n}\,u(n)$	$(n \geq 0)$

(2)

$g(z) = \frac{1}{1-2^{-1}z}$	$\lvert z\rvert < 2$
$\left(\frac{1}{2}\right)^{n}u(n)$	$(n \geq 0)$

Case B $0.5 < |z| < 2$

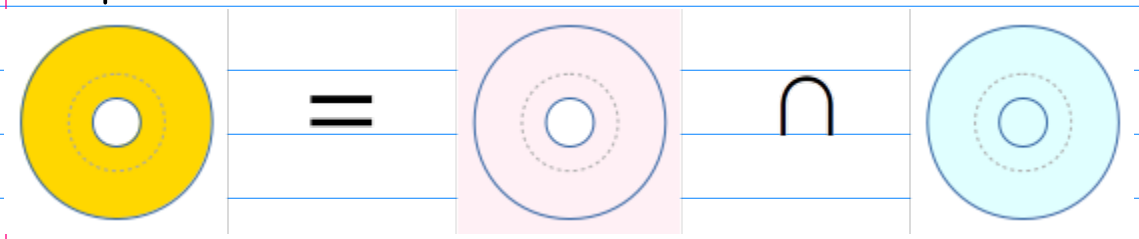
$$-\frac{3}{2}\frac{1}{(z-0.5)(z-2)}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

Simple Pole Form

Geometric Power Series Forms

	$\frac{1}{z-0.5}$	$\frac{1}{z-2}$
	$\frac{z^{-1}}{1-0.5z^{-1}}$	$-\frac{0.5}{1-0.5z}$
	$0.5 < \lvert z \rvert$	$\lvert z \rvert < 2$
$0.5 < \lvert z \rvert < 2$		



	$2^{n+1}\,u(-n-1)$	$-\left(\frac{1}{2}\right)^{n+1}u(n)$

$0.5 < |z| < 2$ $\quad f(z) = \frac{z^{-1}}{1-0.5z^{-1}} + \frac{0.5}{1-0.5z}$ $\quad 2^{n+1}\,u(-n-1) + \left(\frac{1}{2}\right)^{n+1}u(n)$

(5')

$\bar{f}_2(z) = \frac{z^{-1}}{1-2^{-1}z^{-1}}$	$\lvert z \rvert > 2^{-1}$
$2^{n+1}\,u(-n-1)$	$(n < 0)$

(5)

$\bar{f}(z) = \frac{2^{-1}z^{-1}}{1-2^{-1}z^{-1}}$	$\lvert z \rvert > 2^{-1}$
$2^{n}\,u(-n-1)$	$(n < 0)$

(2')

$g_2(z) = \frac{2^{-1}}{1-2^{-1}z}$	$\lvert z \rvert < 2$
$\left(\frac{1}{2}\right)^{n+1}u(n)$	$(n \geq 0)$

(2)

$g(z) = \frac{1}{1-2^{-1}z}$	$\lvert z \rvert < 2$
$\left(\frac{1}{2}\right)^{n}u(n)$	$(n \geq 0)$

Case C $2 < |z|$

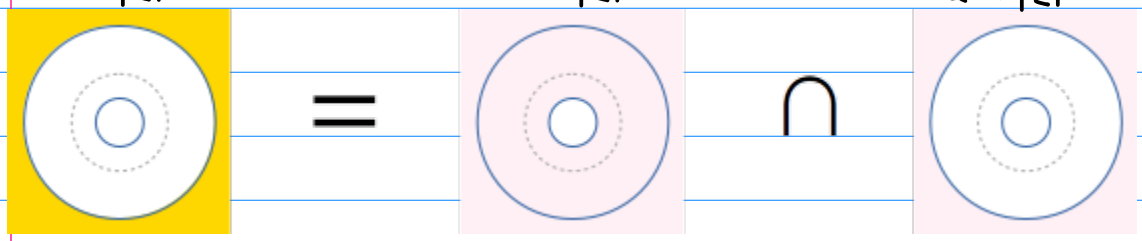
$$-\frac{3}{2}\frac{1}{(z-0.5)(z-2)}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

Simple Pole Form

Geometric Power Series Forms

$\frac{1}{z-0.5}$	$\frac{1}{z-2}$
$\frac{z^{-1}}{1-0.5z^{-1}}$	$\frac{z^{-1}}{1-2z^{-1}}$
$2 < \lvert z \rvert$	$0.5 < \lvert z \rvert$	$2 < \lvert z \rvert$
$2^{n+1}\,u(-n-1)$	$\left(\frac{1}{2}\right)^{n+1}u(-n-1)$



$2 < |z|$ $\quad f(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$ $\quad 2^{n+1}\,u(-n-1) - \left(\frac{1}{2}\right)^{n+1}u(-n-1)$

(5')

$\bar{f}_2(z) = \frac{z^{-1}}{1-2^{-1}z^{-1}}$	$\lvert z \rvert > 2^{-1}$
$2^{n+1}\,u(-n-1)$	$(n < 0)$

(6')

$\bar{g}_2(z) = \frac{z^{-1}}{1-2z^{-1}}$	$\lvert z \rvert > 2$
$\left(\frac{1}{2}\right)^{n+1}u(-n-1)$	$(n < 0)$

(5)

$\bar{f}(z) = \frac{2^{-1}z^{-1}}{1-2^{-1}z^{-1}}$	$\lvert z \rvert > 2^{-1}$
$2^{n}\,u(-n-1)$	$(n < 0)$

(6)

$\bar{g}(z) = \frac{2z^{-1}}{1-2z^{-1}}$	$\lvert z \rvert > 2$
$\left(\frac{1}{2}\right)^{n}u(-n-1)$	$(n < 0)$

Laurent Series and z Transform

$$-\frac{3}{2}\frac{1}{(z-0.5)(z-2)}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5}-\frac{1}{z-2}\right)$$

.

$|z|<0.5 \qquad f(z) = -\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$

Laurent Series $-2^{n+1}u(n)+\left(\frac{1}{2}\right)^{n+1}u(n)$

z Transform $-2^{-n+1}u(-n)+\left(\frac{1}{2}\right)^{-n+1}u(-n)$

$-\left(\frac{1}{2}\right)^{n-1}u(-n)+2^{n-1}u(-n)$

$0.5<|z|<2 \qquad f(z) = \frac{z^{-1}}{1-0.5z^{-1}}+\frac{0.5}{1-0.5z}$

Laurent Series $2^{n+1}u(-n-1)+\left(\frac{1}{2}\right)^{n+1}u(n)$

z Transform $2^{-n+1}u(n-1)+\left(\frac{1}{2}\right)^{-n+1}u(-n)$

$\left(\frac{1}{2}\right)^{n-1}u(n-1)+2^{n-1}u(-n)$

$2<|z| \qquad f(z) = \frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$

Laurent Series $2^{n+1}u(-n-1)-\left(\frac{1}{2}\right)^{n+1}u(-n-1)$

z Transform $2^{-n+1}u(n-1)-\left(\frac{1}{2}\right)^{-n+1}u(n-1)$

$\left(\frac{1}{2}\right)^{n-1}u(n-1)+2^{n-1}u(n-1)$

$$-\frac{3}{2}\,\frac{z^2}{(z-0.5)(z-2)} = \left(\frac{0.5z}{z-0.5} - \frac{2z}{z-2}\right)$$

Case A $|z| < 0.5$

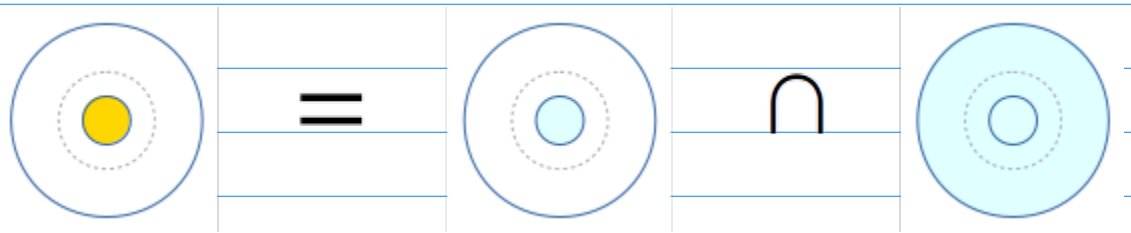
$$-\frac{3}{2}\frac{z^2}{(z-0.5)(z-2)}$$

$$-\frac{3}{2}\frac{z^2}{(z-0.5)(z-2)} = \left(\frac{0.5z}{z-0.5} - \frac{2z}{z-2}\right)$$

Simple Pole Form

Geometric Power Series Forms

$\frac{0.5z}{z-0.5}$	$\frac{2z}{z-2}$
$-\frac{z}{1-2z}$	$-\frac{z}{1-0.5z}$
$\lvert z\rvert < 0.5$	$\lvert z\rvert < 0.5$	$\lvert z\rvert < 2$



	$-2^{n-1}\,u(n)$	$-\left(\frac{1}{2}\right)^{n-1}u(n)$

$|z| < 0.5$ $\quad f(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$ $\quad -2^{n-1}\,u(n) + \left(\frac{1}{2}\right)^{n-1}u(n)$

(7')

$f_3(z) = \frac{z}{1-2z}$	$\lvert z\rvert < 2^{-1}$
$2^{n-1}u(n-1)$	$(n \geq 1)$

(8')

$g_3(z) = \frac{z}{1-2^{-1}z}$	$\lvert z\rvert < 2$
$\left(\frac{1}{2}\right)^{n-1}u(n-1)$	$(n \geq 1)$

(7)

$f_1(z) = \frac{2z}{1-2z}$	$\lvert z\rvert < 2^{-1}$
$2^{n}\,u(n-1)$	$(n \geq 1)$

(8)

$g_1(z) = \frac{2^{-1}z}{1-2^{-1}z}$	$\lvert z\rvert < 2$
$\left(\frac{1}{2}\right)^{n}u(n-1)$	$(n \geq 1)$

Case B $0.5 < |z| < 2$

$$-\frac{3}{2}\frac{z^2}{(z-0.5)(z-2)}$$

$$-\frac{3}{2}\frac{z^2}{(z-0.5)(z-2)} = \left(\frac{0.5z}{z-0.5} - \frac{2z}{z-2}\right)$$

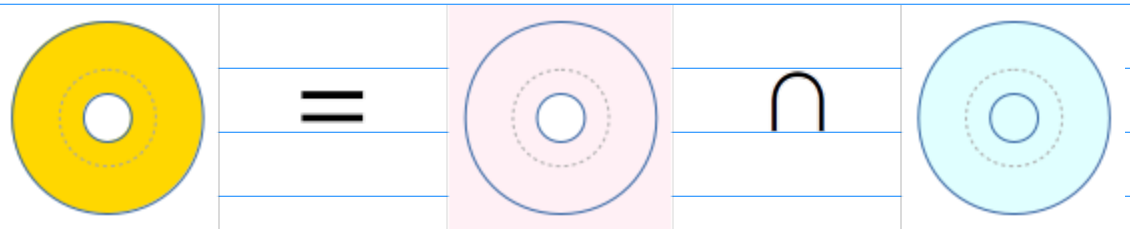
Simple Pole Form

$\frac{0.5z}{z-0.5}$ $\qquad$ $\frac{2z}{z-2}$

Geometric Power Series Forms

$\frac{0.5}{1-0.5z^{-1}}$ $\qquad$ $-\frac{z}{1-0.5z}$

$0.5 < |z| < 2$ $\qquad$ $0.5 < |z|$ $\qquad$ $|z| < 2$



$2^{n-1}\,u(-n-1)$ $\qquad$ $-\left(\frac{1}{2}\right)^{n-1}u(n)$

$0.5 < |z| < 2$ $\qquad f(z) = \frac{0.5}{1-0.5z^{-1}} + \frac{z}{1-0.5z}$ $\qquad 2^{n-1}\,u(-n-1) + \left(\frac{1}{2}\right)^{n-1}u(n)$

(3')

$\bar{f}_3(z) = \frac{2^{-1}}{1-2^{-1}z^{-1}}$	$\lvert z\rvert > 2^{-1}$
$2^{n-1}\,u(-n)$	$(n < 1)$

(3)

$\bar{f}_1(z) = \frac{1}{1-2^{-1}z^{-1}}$	$\lvert z\rvert > 2^{-1}$
$2^{n}\,u(-n)$	$(n < 1)$

(8')

$g_3(z) = \frac{z}{1-2^{-1}z}$	$\lvert z\rvert < 2$
$\left(\frac{1}{2}\right)^{n-1}u(n-1)$	$(n \geq 1)$

(8)

$g_1(z) = \frac{2^{-1}z}{1-2^{-1}z}$	$\lvert z\rvert < 2$
$\left(\frac{1}{2}\right)^{n}u(n-1)$	$(n \geq 1)$

Case C $2 < |z|$

$$-\frac{3}{2}\frac{z^2}{(z-0.5)(z-2)}$$

$$-\frac{3}{2}\frac{z^2}{(z-0.5)(z-2)} = \left(\frac{0.5z}{z-0.5} - \frac{2z}{z-2}\right)$$

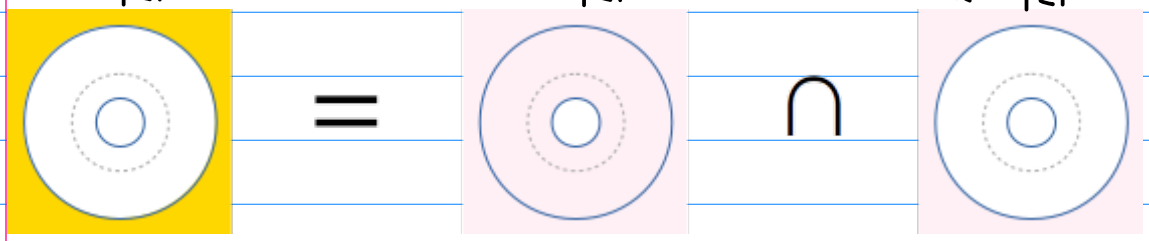
Simple Pole Form

$$\frac{0.5z}{z-0.5} \qquad \frac{2z}{z-2}$$

Geometric Power Series Forms

$$\frac{0.5}{1-0.5z^{-1}} \qquad \frac{2}{1-2z^{-1}}$$

$2 < |z|$ = $0.5 < |z|$ ∩ $2 < |z|$



$2^{n-1}\,u(-n-1)$ $\qquad$ $\left(\frac{1}{2}\right)^{n-1}u(-n-1)$

$2 < |z|$ $\qquad f(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$ $\qquad 2^{n-1}\,u(-n-1) - \left(\frac{1}{2}\right)^{n-1}u(-n-1)$

(3')

$\bar{f}_3(z) = \frac{2^{-1}}{1-2^{-1}z^{-1}}$	$\lvert z\rvert > 2^{-1}$
$2^{n-1}\,u(-n)$	$(n < 1)$

(3)

$\bar{f}_1(z) = \frac{1}{1-2^{-1}z^{-1}}$	$\lvert z\rvert > 2^{-1}$
$2^{n}\,u(-n)$	$(n < 1)$

(4')

$\bar{g}_3(z) = \frac{2}{1-2z^{-1}}$	$\lvert z\rvert > 2$
$\left(\frac{1}{2}\right)^{n-1}u(-n)$	$(n < 1)$

(4)

$\bar{g}_1(z) = \frac{1}{1-2z^{-1}}$	$\lvert z\rvert > 2$
$\left(\frac{1}{2}\right)^{n}u(-n)$	$(n < 1)$

Laurent Series and z Transform

$$-\frac{3}{2}\frac{z^2}{(z-0.5)(z-2)}$$

$$-\frac{3}{2}\frac{z^2}{(z-0.5)(z-2)} = \left(\frac{0.5z}{z-0.5} - \frac{2z}{z-2}\right)$$

$|z| < 0.5$ $\quad f(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$

Laurent Series $\quad -2^{n-1}u(n) + \left(\frac{1}{2}\right)^{n-1}u(n)$

z Transform $\quad -2^{-n-1}u(-n) + \left(\frac{1}{2}\right)^{-n-1}u(-n)$

$-\left(\frac{1}{2}\right)^{n+1}u(-n) + 2^{n+1}u(-n)$

$0.5 < |z| < 2$ $\quad f(z) = \frac{0.5}{1-0.5z^{-1}} + \frac{z}{1-0.5z}$

Laurent Series $\quad 2^{n-1}u(-n-1) + \left(\frac{1}{2}\right)^{n-1}u(n)$

z Transform $\quad 2^{-n-1}u(n-1) + \left(\frac{1}{2}\right)^{-n-1}u(-n)$

$\left(\frac{1}{2}\right)^{n+1}u(n-1) + 2^{n+1}u(-n)$

$2 < |z|$ $\quad f(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$

Laurent Series $\quad 2^{n-1}u(-n-1) - \left(\frac{1}{2}\right)^{n-1}u(-n-1)$

z Transform $\quad 2^{-n-1}u(n-1) - \left(\frac{1}{2}\right)^{-n-1}u(n-1)$

$\left(\frac{1}{2}\right)^{n+1}u(n-1) + 2^{n+1}u(n-1)$

$$-3\,\frac{z}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{4}{z-2}\right)$$

$$= \frac{(z-2)-4(z-0.5)}{(z-0.5)(z-2)}$$

$$= \frac{-3z}{(z-0.5)(z-2)}$$