Sec.8

Axial loading, deformation

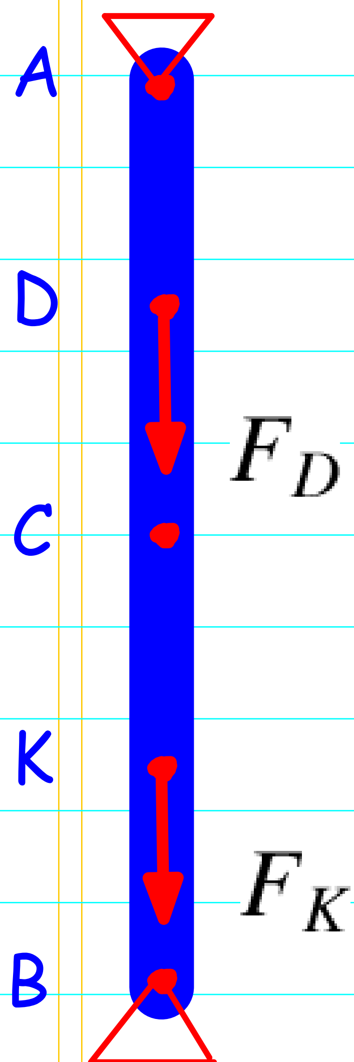
Example 2.04: Static indeterminacy

Beer 2012 p.80

Superposition method

Problem formulation (analytical derivation)

Beer et al. 2012, Mechanics of Materials, McGraw-Hill.



Data

Length

$$L_{AD} = L_{DC} = L = 150\,mm$$

L_{AD} = L_{DC} = L = 150 \, mm

$$L_{CK} = L_{KB} = L = 150\,mm$$

L_{CK} = L_{KB} = L = 150 \, mm

Area

$$A_{AD} = A_{DC} = A_{AC} = 250\,mm^2$$

A_{AD} = A_{DC} = A_{AC} = 250 \, mm^2

$$A_{CK} = A_{KB} = A_{CB} = 400\,mm^2$$

A_{CK} = A_{KB} = A_{CB} = 400 \, mm^2

Young's modulus

$$E_{AD} = E_{DC} = E_{CK} = E_{KB} = E$$

E_{AD} = E_{DC} = E_{CK} = E_{KB} = E

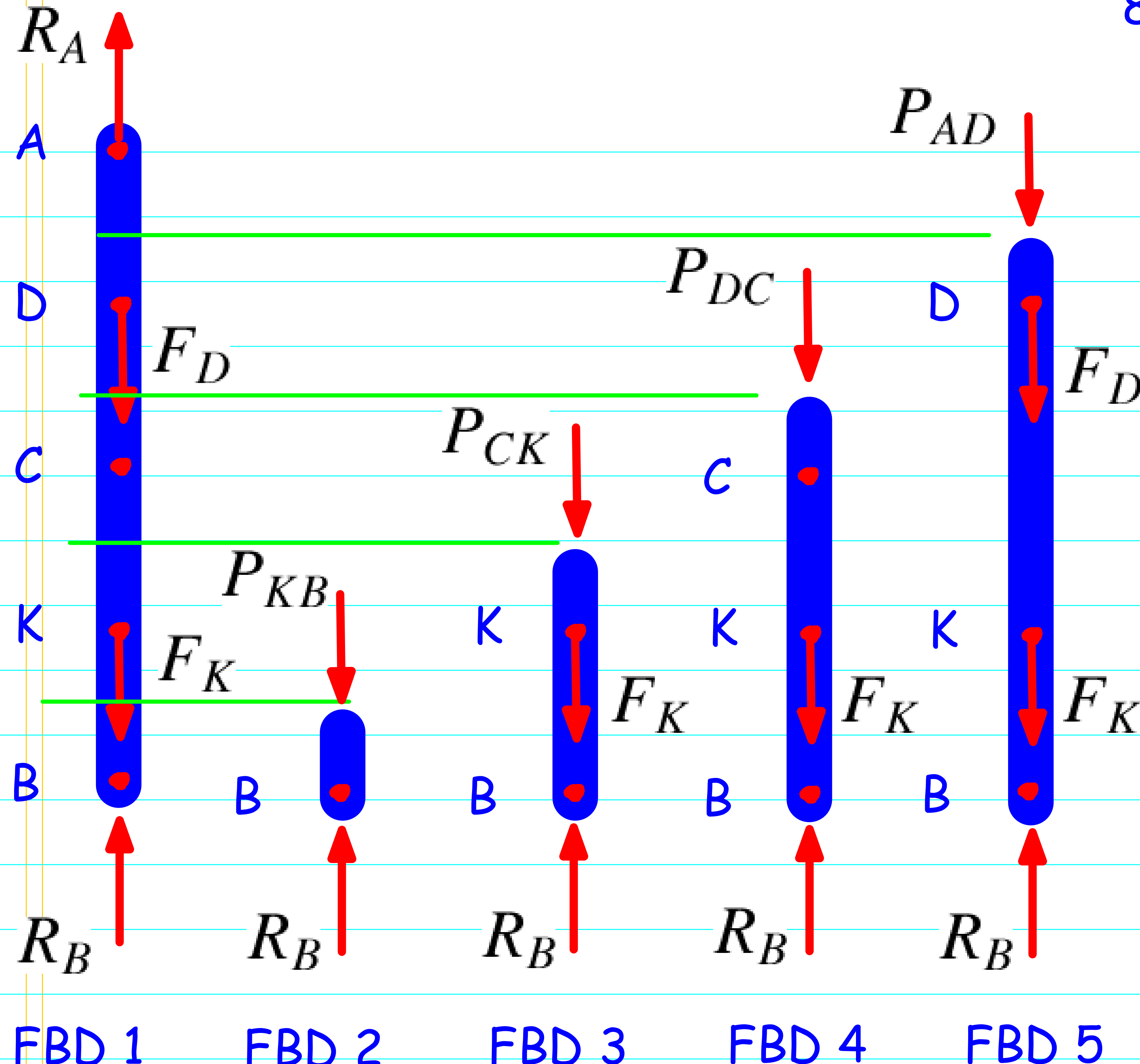
Applied forces

$$F_D = 300\,kN$$

F_D = 300 \, kN

$$F_K = 600\,kN$$

F_K = 600 \, kN


R_A
A
D
F_D
C
K
F_K
B
R_B
P_{KB}
P_{CK}
P_{DC}
P_{AD}
FBD 1
FBD 2
FBD 3
FBD 4
FBD 5

Method

2 unknown reactions R_A R_B

FBD 1 1 equilibrium eq.

$$\sum_i F_{i,y} = 0 \quad (1)$$

\sum_i F_{i,y} = 0

Need to use deformation to find 1 more eq.

FBD 2 $P_{KB},\ \delta_{KB}$ in terms of R_B P_{KB} , \ \delta_{KB}

FBD 3 $P_{CK},\ \delta_{CK}$ in terms of R_B

FBD 4 $P_{DC},\ \delta_{DC}$ in terms of R_B

FBD 5 $P_{AC},\ \delta_{AD}$ in terms of R_B

Point B is fixed; thus

$$\delta_B = 0 = \delta_{KB} + \delta_{CK} + \delta_{DC} + \delta_{AD} \quad (2)$$

\delta_B = 0 = \delta_{KB} + \delta_{CK} + \delta_{DC} + \delta_{AD}

(2) has one unknown R_B ; solve for R_B

(1)-(2) are 2 eqs for 2 unknowns R_A R_B

Recall:

$$\sigma = E\epsilon \Rightarrow \frac{P}{A} = E\frac{\delta}{L} \Rightarrow \delta = \frac{P\,L}{A\,E} \qquad (1)$$

\sigma = E \epsilon \Rightarrow \frac{P}{A} = E \frac{\delta}{L} \Rightarrow \delta = \frac{P \, L}{A \, E}

For multiple bar segments

$$\delta_i = \frac{P_i\,L_i}{A_i\,E_i} \qquad (2)$$

\delta_i = \frac{P_i \, L_i}{A_i \, E_i}

$$\delta = \sum_i \delta_i = \sum_i \frac{P_i\,L_i}{A_i\,E_i} \qquad (3)$$

\delta = \sum_i \delta_i = \sum_i \frac{P_i \, L_i}{A_i \, E_i}

Pause video NOW !

Work out the next step

→ on your own first

→ discuss with teammates

if you get stuck

then continue to watch the video

Computation

FBD 2 $$P_{KB} = R_B \quad (1)$$

P_{KB} = R_B

$$\delta_{KB} = -\frac{P_{KB}\,L_{KB}}{A_{KB}\,E_{KB}} = -\frac{R_B\,L}{A_{CB}\,E} \quad (2)$$

\delta_{KB} = - \frac{P_{KB} \, L_{KB}}{A_{KB} \, E_{KB}} = - \frac{R_B \, L}{A_{CB} \, E}

FBD 3 $$P_{CK} = R_B - F_K \quad (3)$$

P_{CK} = R_B - F_K

$$\delta_{CK} = -\frac{P_{CK}\,L_{CK}}{A_{CK}\,E_{CK}} = -\frac{(R_B - F_K)\,L}{A_{CB}\,E} \quad (4)$$

\delta_{CK} = - \frac{P_{CK} \, L_{CK}}{A_{CK} \, E_{CK}} = - \frac{(R_B - F_K) \, L}{A_{CB} \, E}

FBD 4 $$P_{DC} = R_B - F_K \quad (5)$$

P_{DC} = R_B - F_K

$$\delta_{DC} = -\frac{P_{DC}\,L_{DC}}{A_{DC}\,E_{DC}} = -\frac{(R_B - F_K)\,L}{A_{AC}\,E} \quad (6)$$

\delta_{DC} = - \frac{P_{DC} \, L_{DC}}{A_{DC} \, E_{DC}} = - \frac{(R_B - F_K) \, L}{A_{AC} \, E}

FBD 5 $$P_{AD} = R_B - F_K - F_D \quad (7)$$

P_{AD} = R_B - F_K - F_D

$$\delta_{AD} = -\frac{P_{AD}\,L_{AD}}{A_{AD}\,E_{AD}} = -\frac{(R_B - F_K - F_D)\,L}{A_{AC}\,E} \quad (8)$$

\delta_{AD} = - \frac{P_{AD} \, L_{AD}}{A_{AD} \, E_{AD}} = - \frac{(R_B - F_K - F_D) \, L}{A_{AC} \, E}

Total displacement at B

(2) p.8-3: $\delta_B = 0 = \delta_{KB} + \delta_{CK} + \delta_{DC} + \delta_{AD}$

Rearrange to put (a) all terms with the known applied forces together, and (b) all terms with the unknown reaction at B together.

$$\delta_B = 0 = \delta_L + \delta_R \qquad (1)$$

\delta_B = 0 = \delta_L + \delta_R

δ_L all terms with the known applied forces (put all these terms on the Left, thus the subscript "L")

δ_R all terms with the unknown reaction at B (put all these terms on the Right, thus the subscript "R")

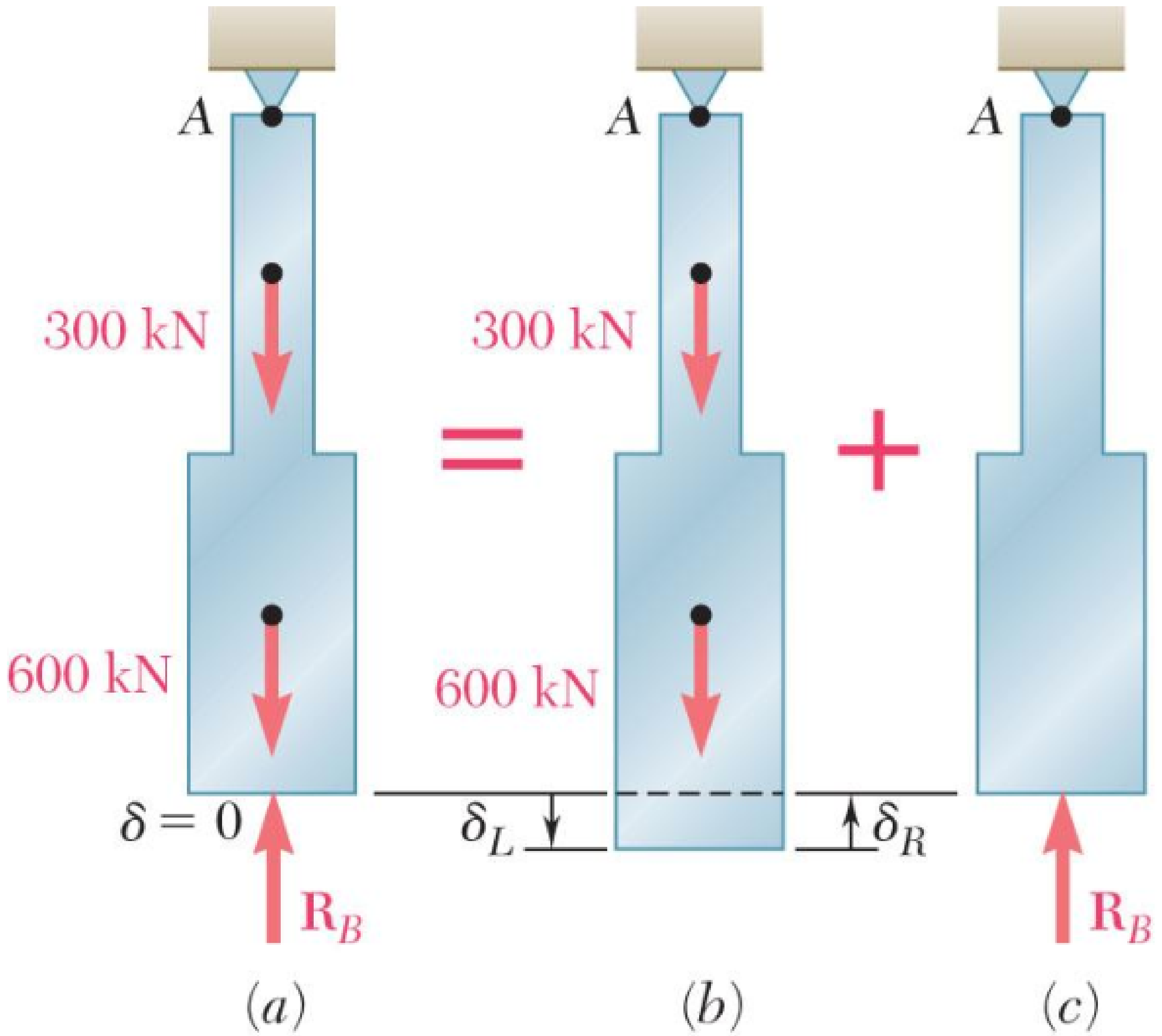
$$\delta_L = \frac{F_K \, L}{A_{CB} \, E} + \frac{F_K \, L}{A_{AC} \, E} + \frac{(F_K + F_D) \, L}{A_{AC} \, E} \qquad (2)$$

\delta_L = \frac{F_K \, L}{A_{CB} \, E} + \frac{F_K \, L}{A_{AC} \, E} + \frac{(F_K + F_D) \, L}{A_{AC} \, E}

$$\delta_R = - \frac{R_B \, L}{A_{CB} \, E} - \frac{R_B \, L}{A_{CB} \, E} - \frac{R_B \, L}{A_{AC} \, E} - \frac{R_B \, L}{A_{AC} \, E} \qquad (3)$$

\delta_R = - \frac{R_B \, L}{A_{CB} \, E} - \frac{R_B \, L}{A_{CB} \, E} - \frac{R_B \, L}{A_{AC} \, E} - \frac{R_B \, L}{A_{AC} \, E}

(1)-(3) p.8-6 can be interpreted pictorially as a "super-position" of 2 cases:



Beer 2012 p.80

Cancel the common factor L / E in (1)-(3) p.8-6; this the problem is independent of length L and Young's modulus E.

(1)-(3) p.8-6:

$$0 = \frac{F_K}{A_{CB}} + \frac{(2 F_K + F_D)}{A_{AC}} - 2 R_B \left[\frac{1}{A_{CB}} + \frac{1}{A_{AC}} \right] \quad (1)$$

0 = \frac{F_K}{A_{CB}} + \frac{(2 F_K + F_D)}{A_{AC}} - 2 R_B \left[\frac{1}{A_{CB}} + \frac{1}{A_{AC}} \right]

$$R_B = \frac{\displaystyle \frac{F_K}{A_{CB}} + \frac{(2 F_K + F_D)}{A_{AC}}}{\displaystyle 2 \left[\frac{1}{A_{CB}} + \frac{1}{A_{AC}} \right]} \quad (2)$$

R_B = \frac{\displaystyle \frac{F_K}{A_{CB}} + \frac{(2 F_K + F_D)}{A_{AC}}}{\displaystyle 2 \left[\frac{1}{A_{CB}} + \frac{1}{A_{AC}} \right]}

Additional simplification before computation

$$F_K = 2 F_D = 2 \times 300 \, kN \quad (3)$$

F_K = 2 F_D = 2 \times 300 \, kN

$$R_B = F_D \frac{\displaystyle \frac{2}{A_{CB}} + \frac{5}{A_{AC}}}{\displaystyle 2 \left[\frac{1}{A_{CB}} + \frac{1}{A_{AC}} \right]} \quad (4)$$

R_B = F_D \frac{\displaystyle \frac{2}{A_{CB}} + \frac{5}{A_{AC}}}{\displaystyle 2 \left[\frac{1}{A_{CB}} + \frac{1}{A_{AC}} \right]}

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A little more simplification before computation:

$$R_B = \frac{F_D}{2} \, \frac{2 A_{AC} + 5 A_{CB}}{A_{AC} + A_{CB}} \qquad (1)$$

R_B = \frac{F_D}{2} \, \frac{2 A_{AC} + 5 A_{CB}}{A_{AC} + A_{CB}}

Note that the 2nd factor in (1) is dimensionless, i.e., you don't need to convert mm^2 to m^2 (waste of time !).

Also note that (1) satisfies the principle of dimensional homogeneity, i.e., dimension of lhs is the same as the dimension of the rhs.

So you don't have to convert kN into N (waste of time !)

Now plug in the numbers and verify for yourself that you obtain the same result as in Beer et al 2012 p.80.

As in delay gratification, **delay the computation** as much as possible until the very end of the problem formulation.