Binary Search Tree (3A)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using LibreOffice and Octave.

Binary Search Tree (1)

Binary search trees (BST),
ordered binary trees
sorted binary trees

are a particular type of **container**:
data structures that store "items"
(such as numbers, names etc.) in memory.

They allow fast **lookup**, **addition** and **removal** of items
can be used to implement either dynamic sets of items
lookup tables that allow finding an item by its **key**
(e.g., finding the phone number of a person by name).

https://en.wikipedia.org/wiki/Binary_search_tree

Binary Search Tree (2)

keep their **keys** in sorted order
lookup operations can use
the principle of **binary search**

allowing to skip searching half of the tree
each operation (**lookup**, **insertion** or **deletion**)
takes time proportional to **log n**

much better than the **linear time**
but slower than the corresponding operations
on **hash tables**.

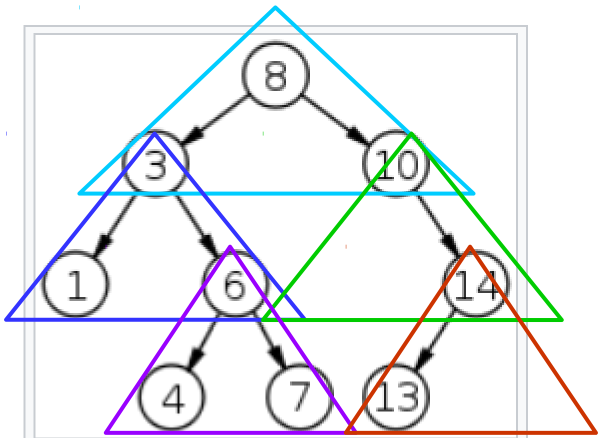
https://en.wikipedia.org/wiki/Binary_search_tree

Binary Search Tree (3)

when **looking** for a **key** in a tree
or **looking** for a **place** to insert a new key,
they traverse the tree from root to leaf,
making comparisons to keys stored in the nodes
deciding to continue in the **left** or **right subtrees**,
on the basis of the comparison.

https://en.wikipedia.org/wiki/Binary_search_tree

Node, Left Child, Right Child



A binary search tree of size 9 and depth 3, with 8 at the root. The leaves are not drawn.

$3 < 8 < 10$

$1 < 3 < 6$

$10 < 14$

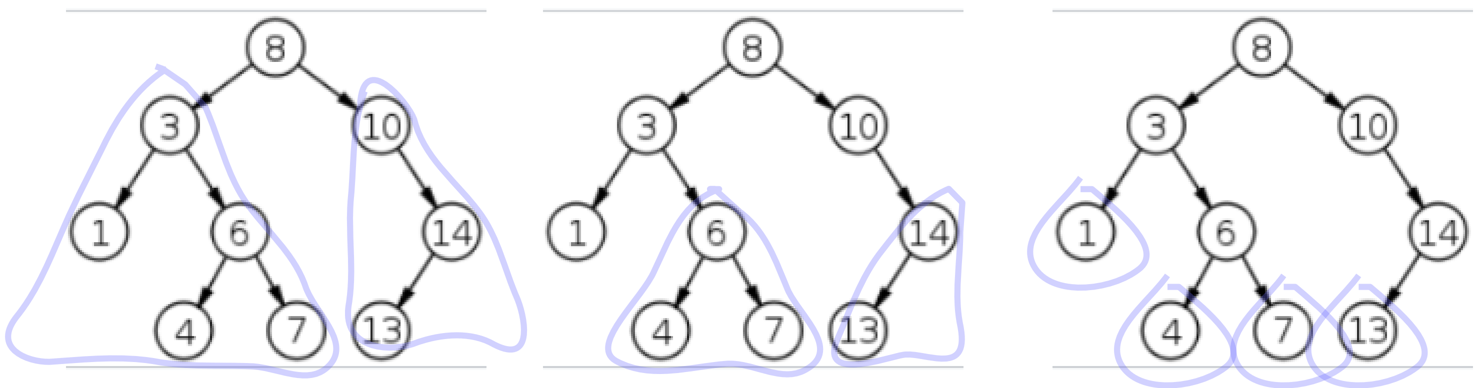
$4 < 6 < 7$

$13 < 14$

1, 3, 4, 6, 7, 8, 10, 13, 14

https://www.tutorialspoint.com/data_structures_algorithms/expression_parsing.html

Subtrees



1, 3, 4, 6, 7, 8, 10, 13, 14

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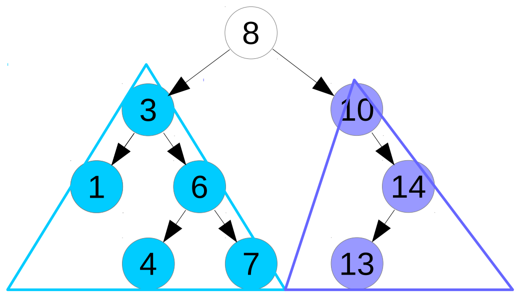
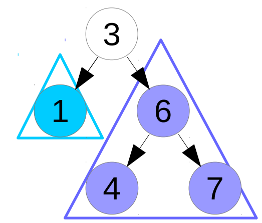
Node, Left Subtree, Right Subtree

$1,3,4,6,7 < 8 < 10,13,14$

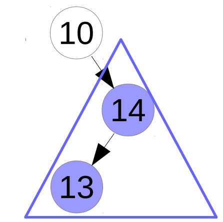


$1,\ 3,\ 4,\ 6,\ 7,\ 8,\ 10,\ 13,\ 14$

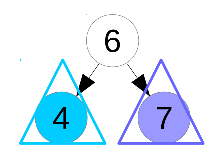
$1 < 3 < 4,6,7$



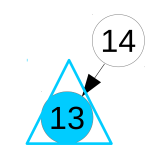
$10 < 13,14$



$4 < 6 < 7$



$13 < 14$



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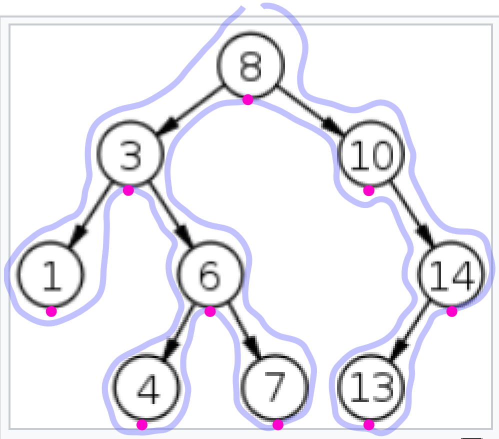
In-Order Traversal



A binary search tree of size 9 and depth 3, with 8 at the root. The leaves are not drawn.

1, 3, 4, 6, 7, 8, 10, 13, 14

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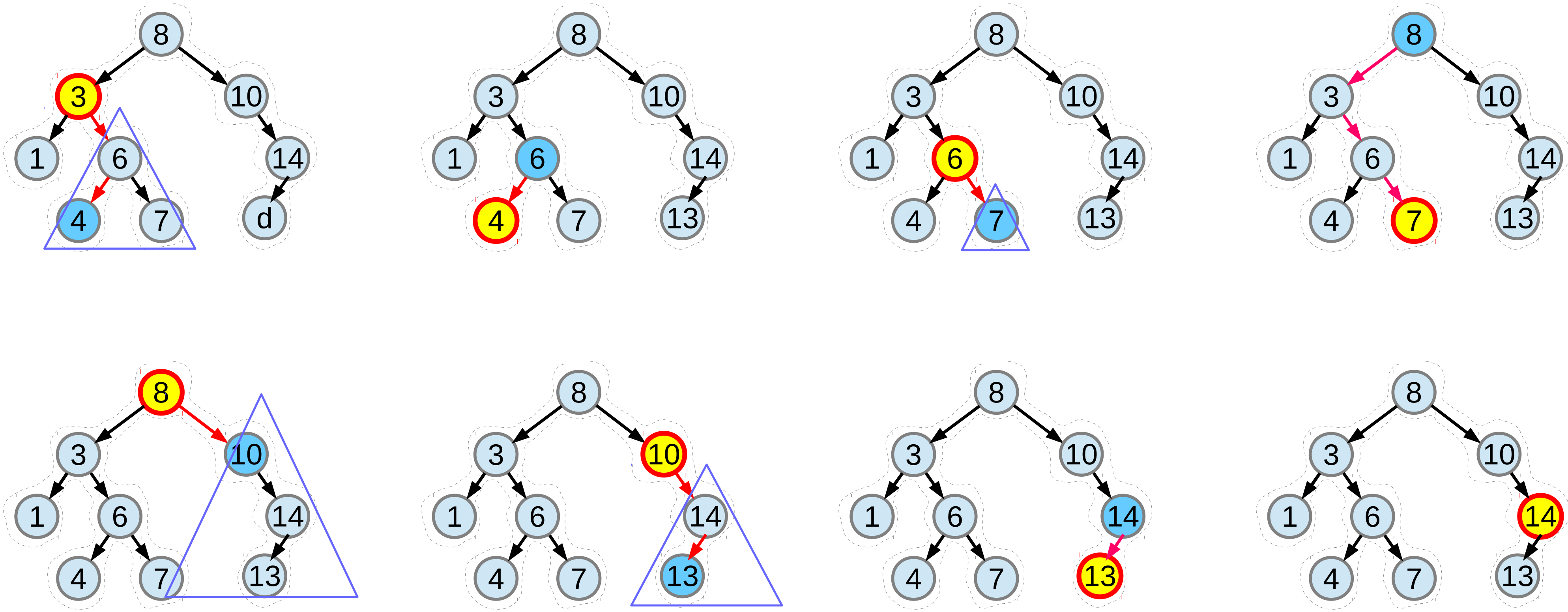
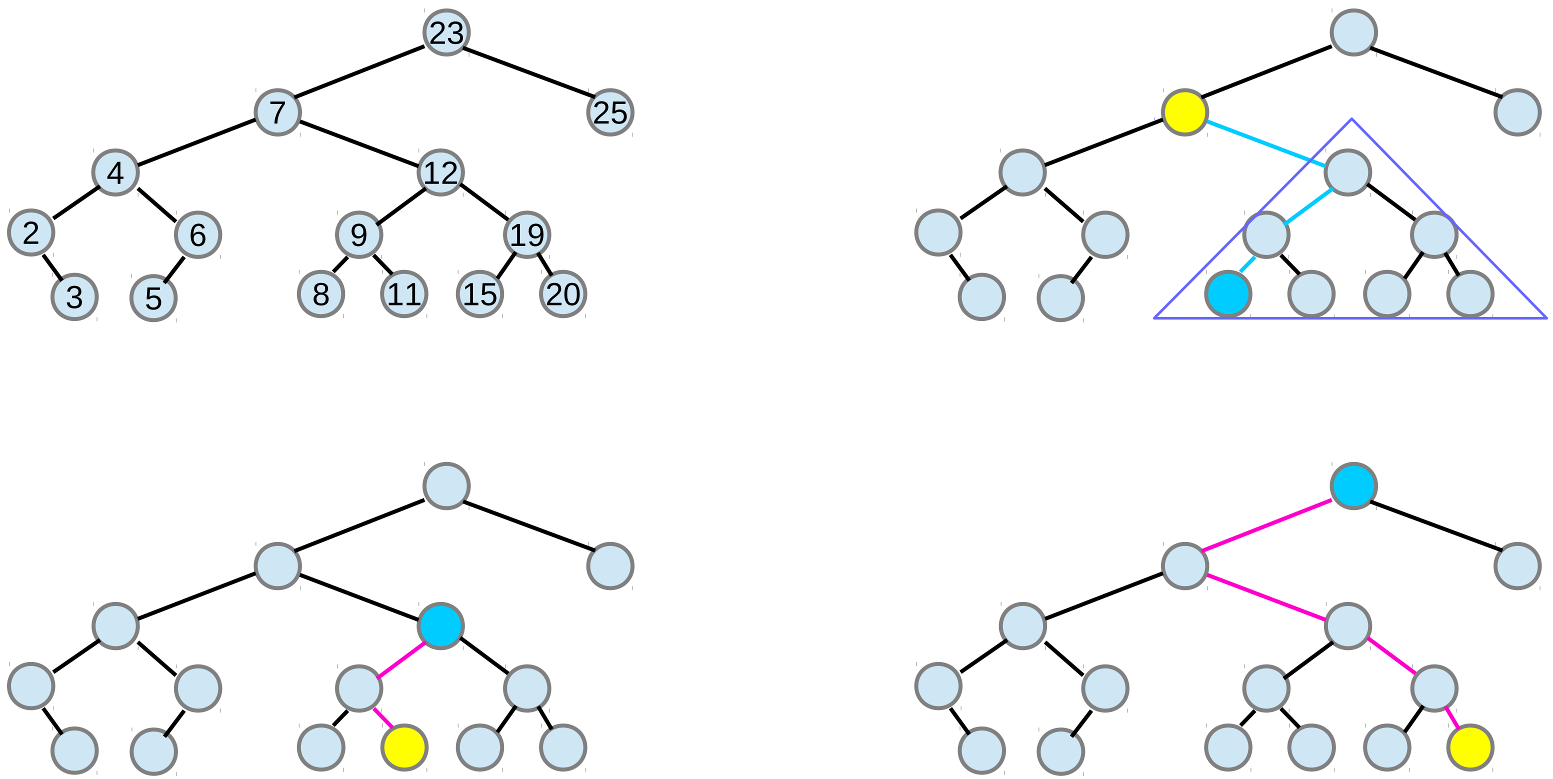
Successor Examples (1)



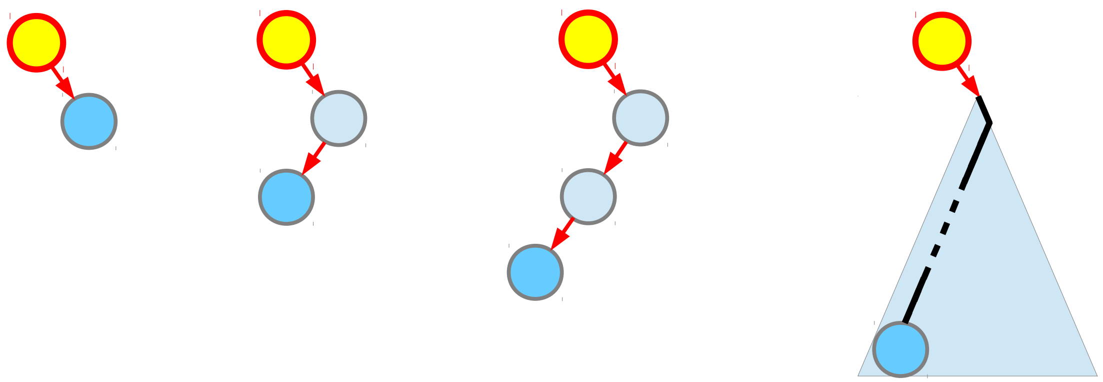
https://www.tutorialspoint.com/data_structures_algorithms/expression_parsing.html

Successor Examples (2)

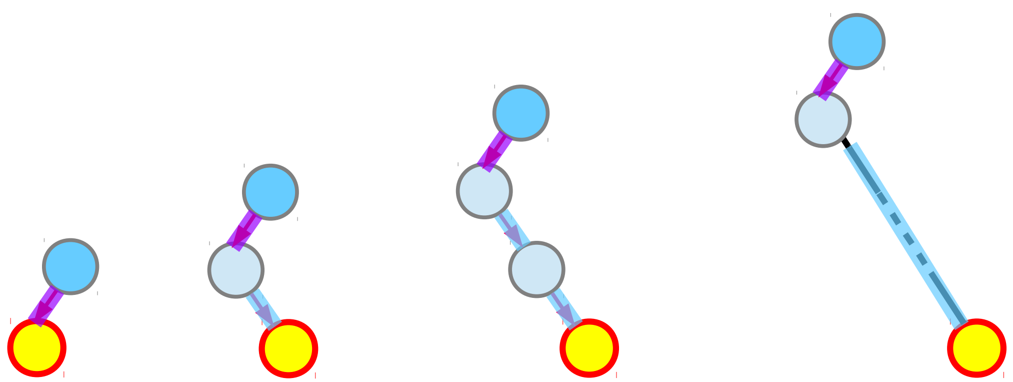


https://www.cs.rochester.edu/~gildea/csc282/slides/C12-bst.pdf

Successor Cases



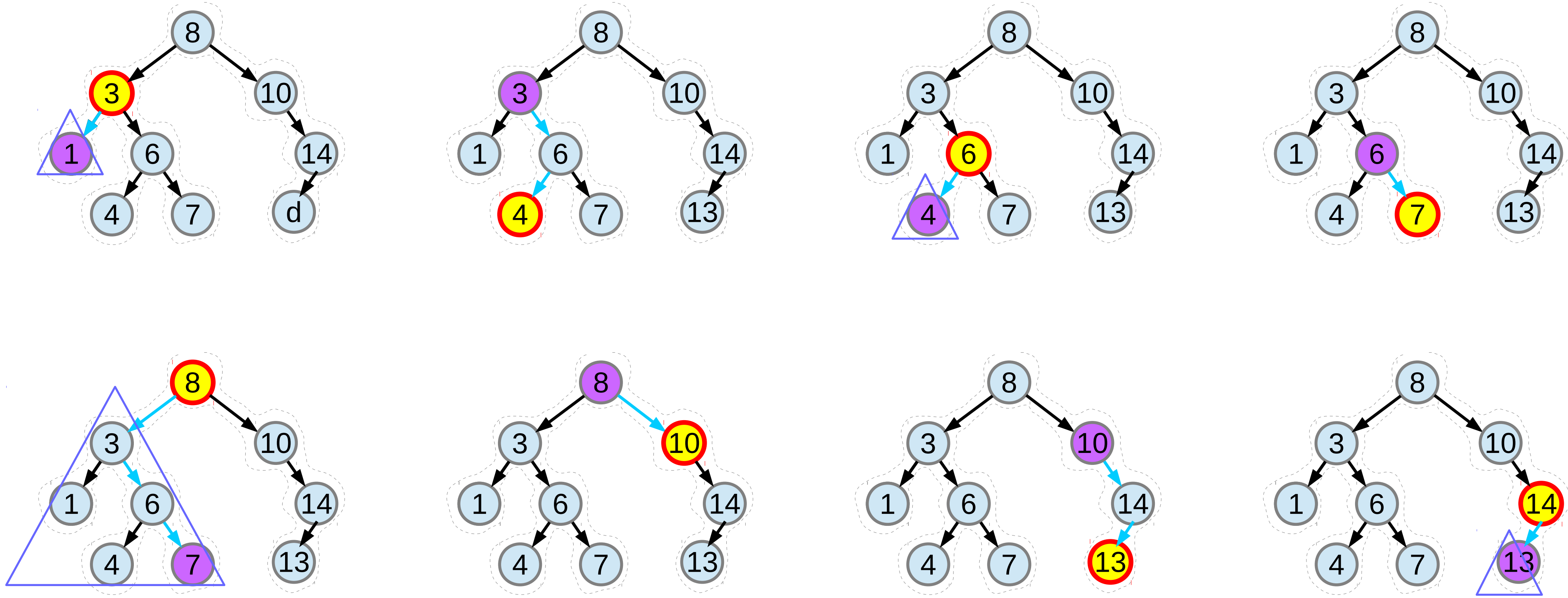
If the right child exists,
then the minimum
in the **right** subtree
– the **leftmost** node



the **parent** of the farthest
node that can be reached
by following only **right**
edges **backward**.

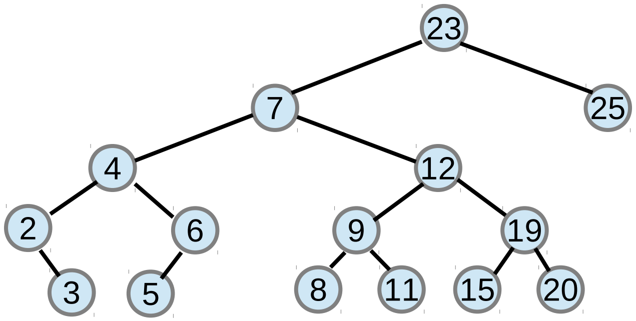
https://www.tutorialspoint.com/data_structures_algorithms/expression_parsing.html

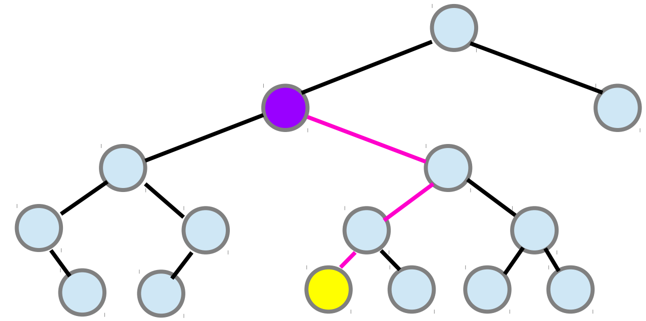
Predecessor Examples (1)

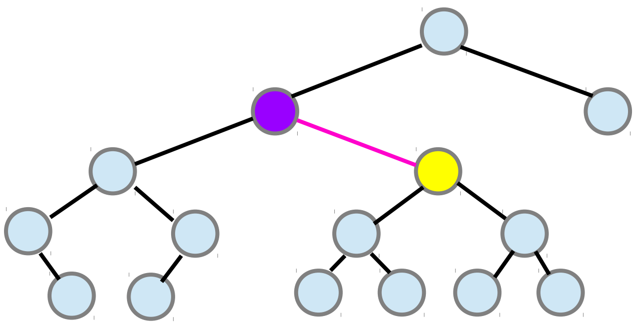


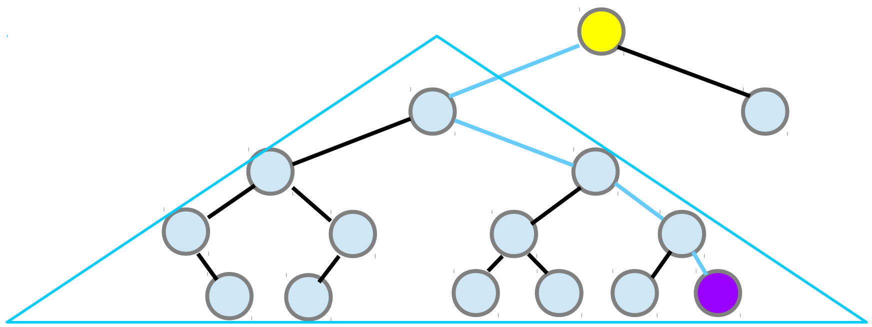
https://www.tutorialspoint.com/data_structures_algorithms/expression_parsing.html

Predecessor Examples (2)



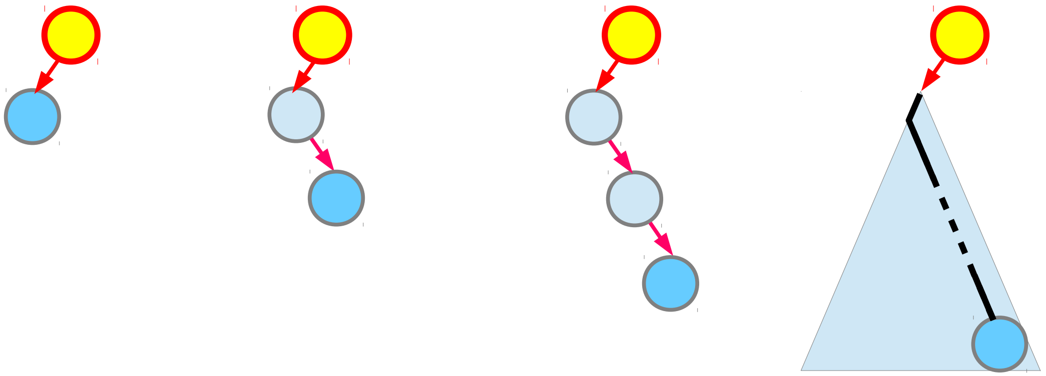




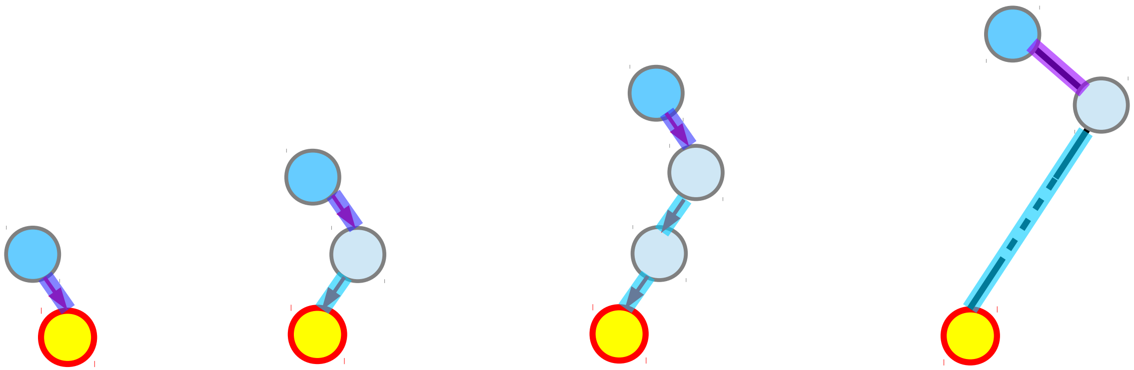


https://www.cs.rochester.edu/~gildea/csc282/slides/C12-bst.pdf

Predecessor Cases



If the left child exists, then the maximum
in the **left** subtree
– the **rightmost** node

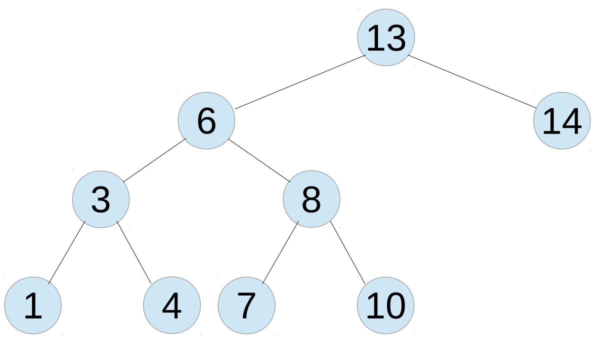


the **parent** of the farthest node that can be reached by following only **left** edges **backward**.

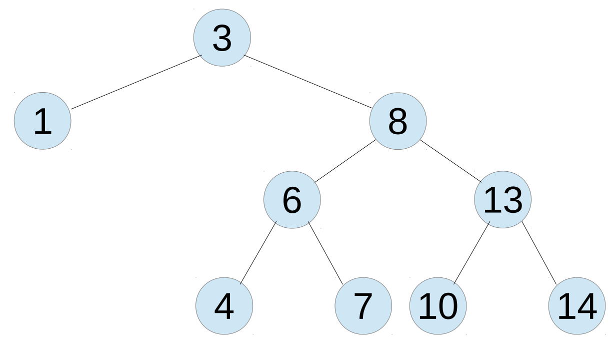
https://www.tutorialspoint.com/data_structures_algorithms/expression_parsing.html

Different BST’s with the same data

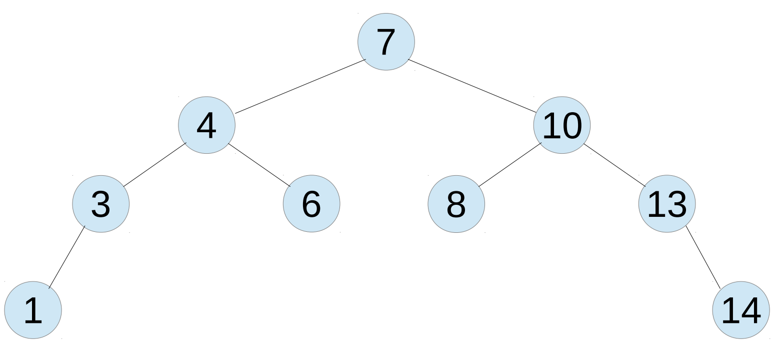
1, 3, 4, 6, 7, 8, 10, 13, 14



1, 3, 4, 6, 7, 8, 10, 13, 14

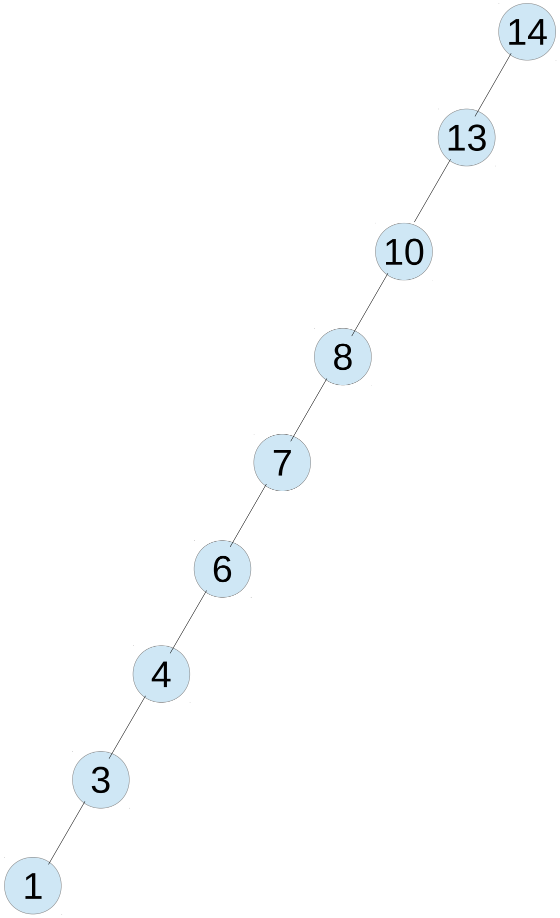


1, 3, 4, 6, 7, 8, 10, 13, 14

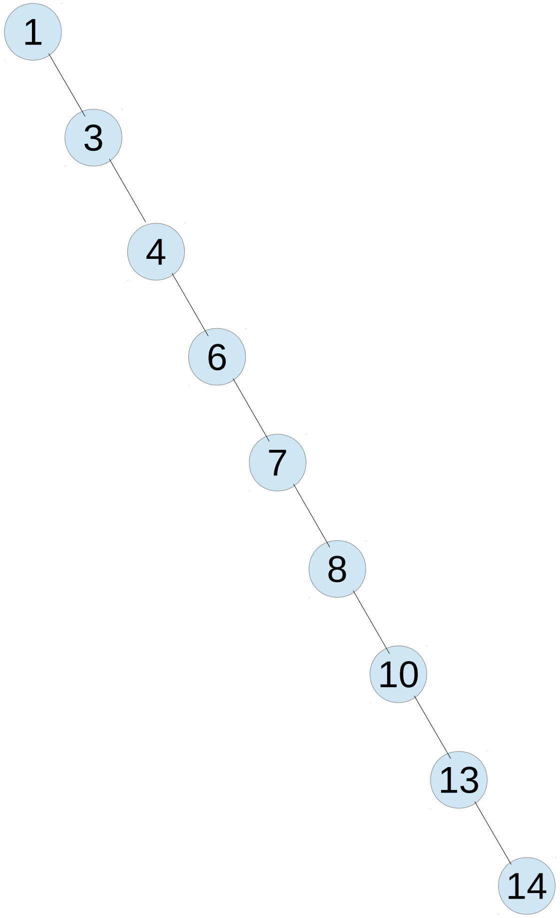


Unbalanced BSTs

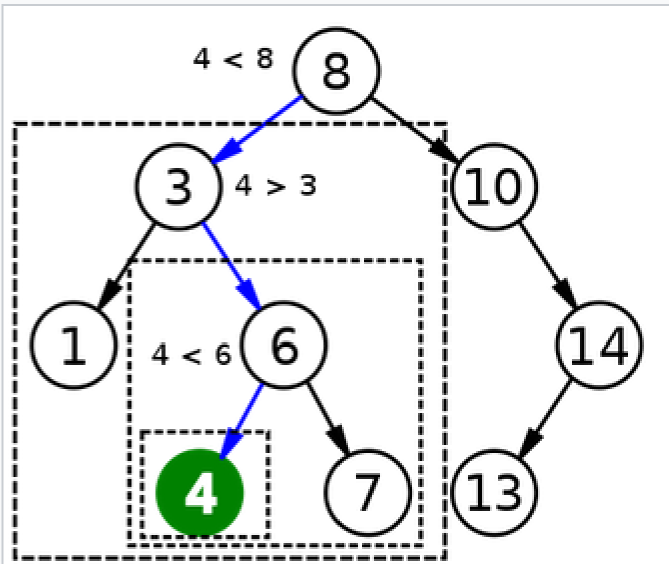
1, 3, 4, 6, 7, 8, 10, 13, 14



1, 3, 4, 6, 7, 8, 10, 13, 14



Binary Search on a Binary Search Tree



Binary search trees are searched using an algorithm similar to binary search.

1, 3, 4, 6, 7, 8, 10, 13, 14

1, 3, 4, 6, 7, 8, 10, 13, 14

1, 3, 4, 6, 7, 8, 10, 13, 14

https://en.wikipedia.org/wiki/Binary_search_algorithm

Insertion

Insertion begins as a **search** would begin;
if the key is not equal to that of the **root**,
we search the **left** or **right** subtrees as before.

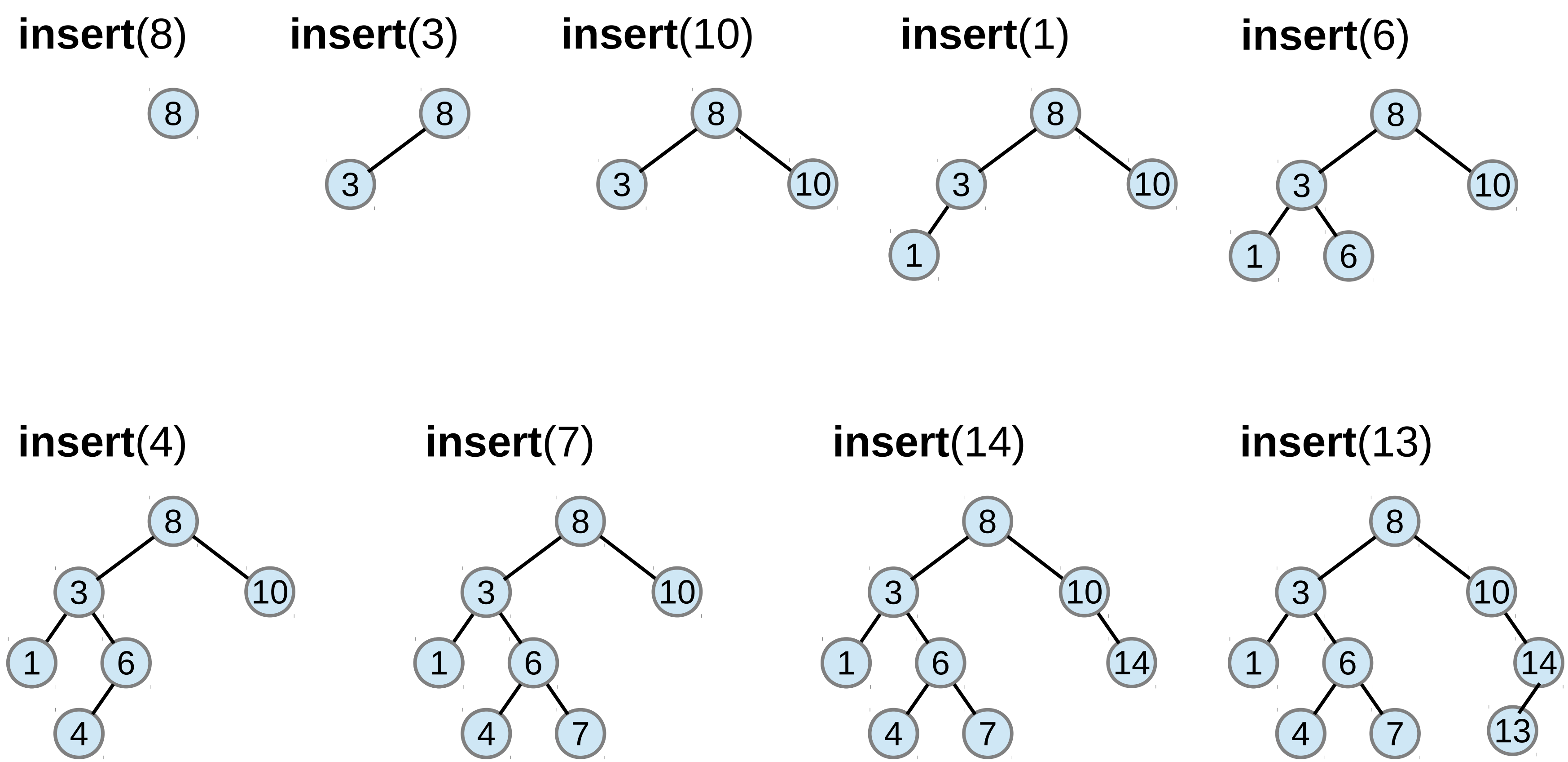
at an **leaf node**, **add** the new key-value pair
as its **right** or **left child**,
depending on the node's **key**.

first examine the **root**
and recursively insert the new node
to the **left** subtree if its key is less than that of the **root**,
or the **right** subtree if its key is greater than or equal to the **root**.

https://en.wikipedia.org/wiki/Binary_search_tree

Insertion Example (1)

Insert(8 → 3 → 10 → 1 → 6 → 4 → 7 → 14 → 13)

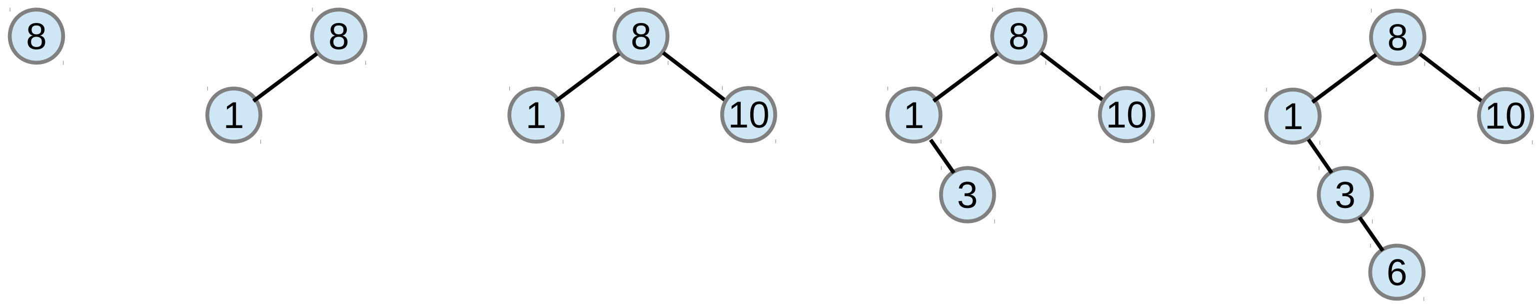


Insertion Example (2)

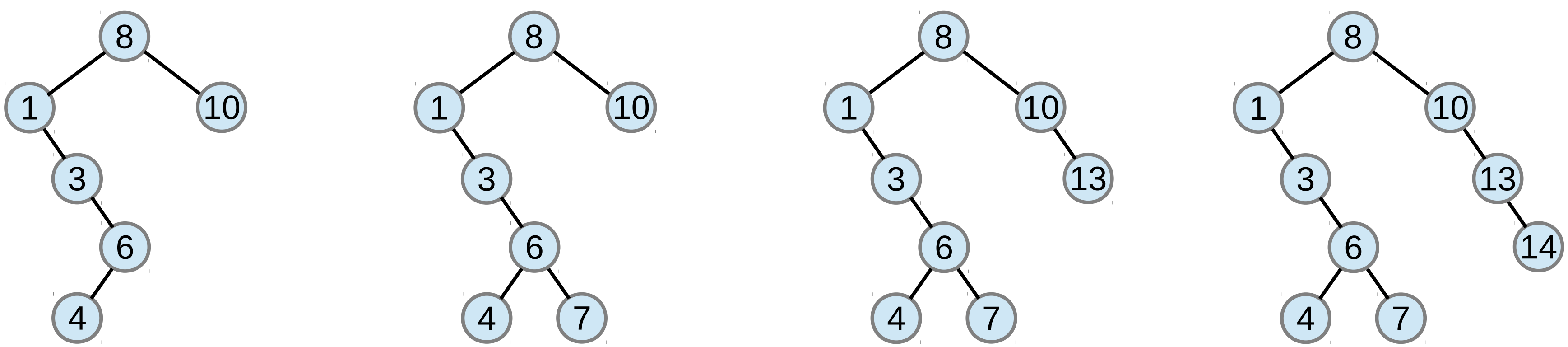
Insert(8 → 3 → 10 → 1 → 6 → 4 → 7 → 14 → 13)

Insert(8 → 1 → 10 → 3 → 6 → 4 → 7 → 13 → 14)

insert(8) **insert**(1) **insert**(10) **insert**(3) **insert**(6)



insert(4) **insert**(7) **insert**(13) **insert**(14)



Deletion

1. Deleting a **node** with no **children**:
   simply remove the node from the tree.

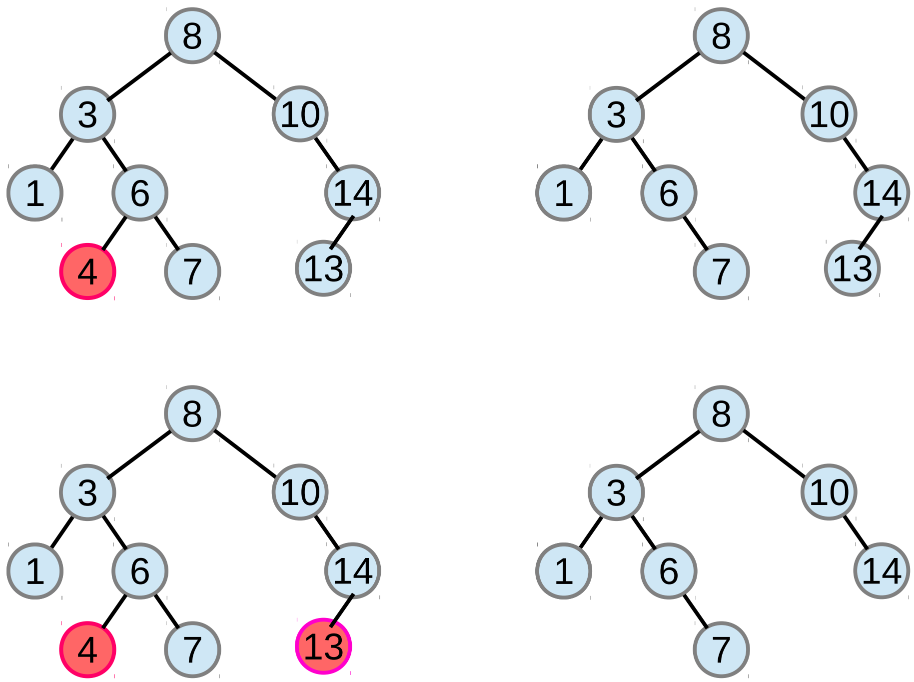
2. Deleting a **node** with one **child**:
   remove the node and replace it with its child.

3. Deleting a **node** with two **children**:
   call the **node** to be deleted D.
   Do not delete D.
   Instead, choose either its in-order **predecessor node**
   or its in-order **successor node** as replacement node E.
   Copy the user values of E to D
   If E does not have a **child**
      simply remove E from its previous parent G.
   If E has a **child**, say F, it is a right child.
      Replace E with F at E's parent.

https://en.wikipedia.org/wiki/Binary_search_tree

Deletion – Case 1

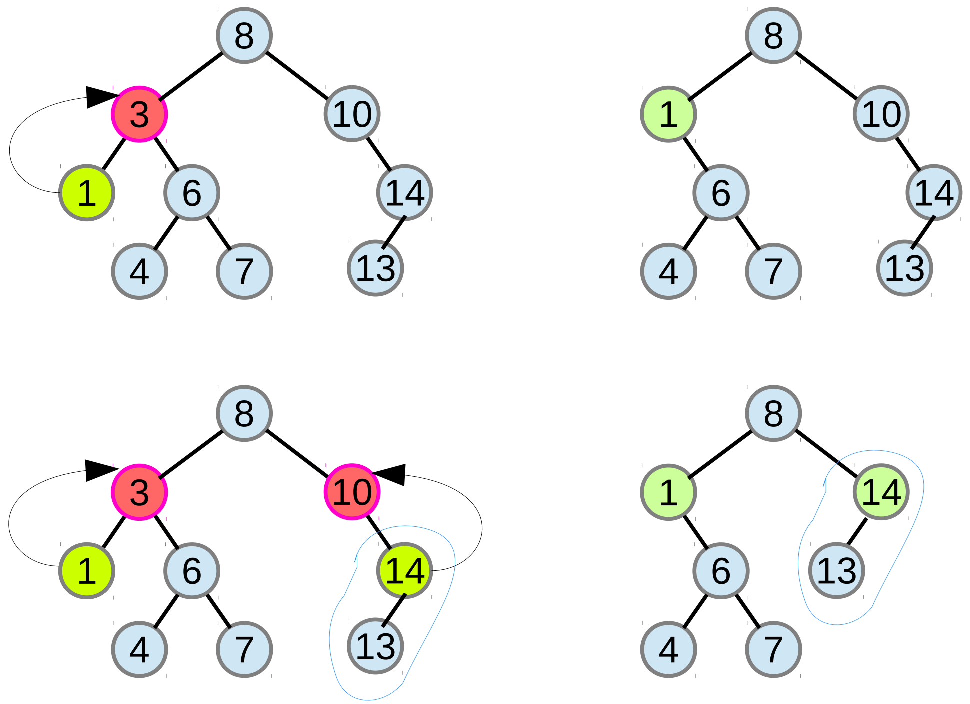
1. <u>Deleting</u> a **node** with <u>no</u> **children**:
   simply remove the node from the tree.



https://en.wikipedia.org/wiki/Binary_search_tree

Deletion – Case 2

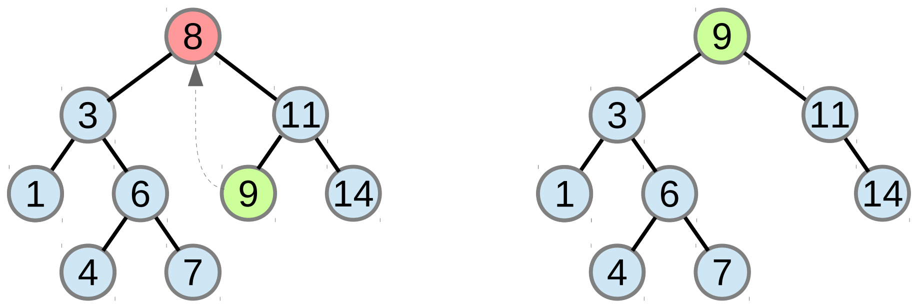
2. Deleting a **node** with one **child**:
   remove the node and replace it with its child.



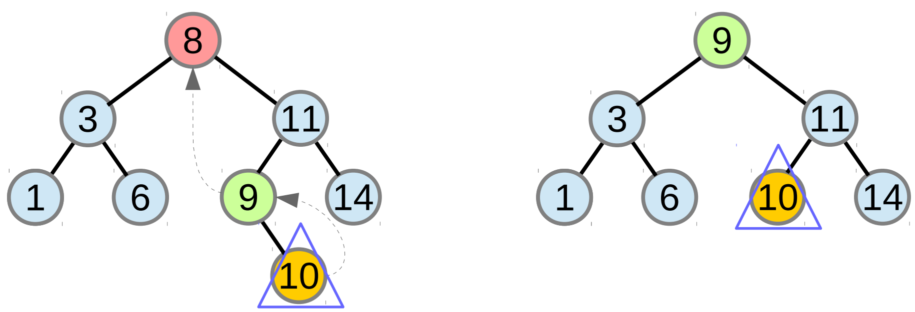
https://en.wikipedia.org/wiki/Binary_search_tree

Deletion – Case 3(a)

3. Deleting a **node** with two **children**:
call the **node** to be deleted **D**.
its in-order **successor node** as **E**. Copy **E** to **D**



Leftmost
E has no **child**
simply remove **E**
from its parent **G**.

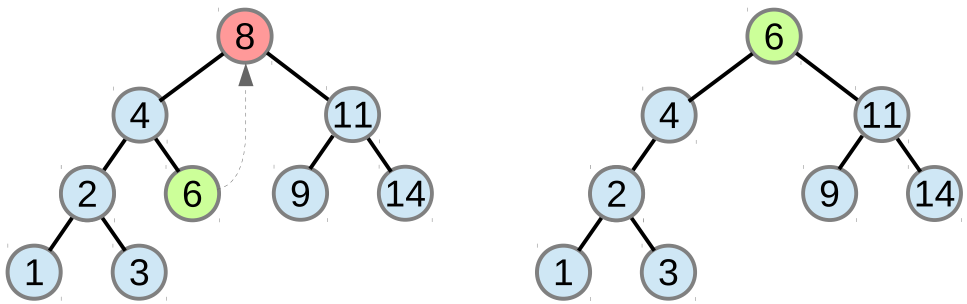


Leftmost
E has a child **F**
it is a **right** child
replace **E** with **F**
at **E**'s parent.

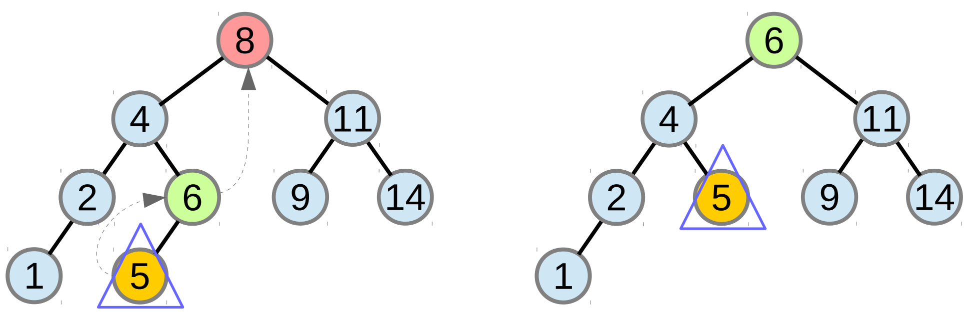
https://en.wikipedia.org/wiki/Binary_search_tree

Deletion – Case 3(b)

3. Deleting a **node** with two **children**:
call the **node** to be deleted **D**.
its in-order **precessor node** as **E**. Copy **E** to **D**



Rightmost
E has no **child**
simply remove **E**
from its parent **G**.



Rightmost
E has a child **F**
it is a **left** child
replace **E** with **F**
at **E**'s parent.

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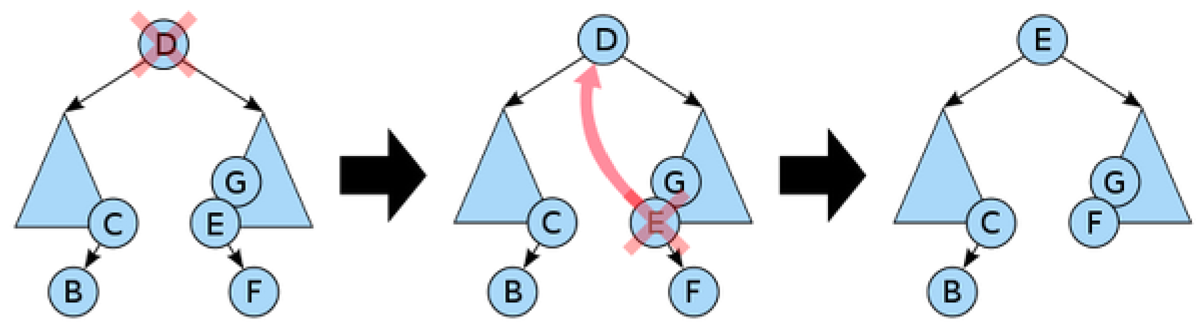
Deletion



Deleting a **node** with **two children** from a binary search tree.
First the **leftmost** node in the **right** subtree,
the in-order **successor E**, is identified.
Its value is **copied** into the **node D** being deleted.
The in-order successor can then be easily deleted
because it has at most **one child**.
The same method works symmetrically
using the in-order **predecessor** C.

https://en.wikipedia.org/wiki/Binary_search_tree

References

[1] http://en.wikipedia.org/
[2]