Background – Functions (1C)

Young Won Lim

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using LibreOffice.

Based on

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Haskell in 5 steps
https://wiki.haskell.org/Haskell_in_5_steps

First-Class Functions

a programming language is said to have **first-class functions**
if it treats **functions** as **first-class citizens**.

the language supports
- passing functions as arguments to other functions,
- returning them as the values from other functions,
- and assigning them to variables or
- storing them in data structures.

support for **anonymous functions** (function literals) as well

the names of functions do not have any special status
they are treated like ordinary variables with a function type.

https://en.wikipedia.org/wiki/First-class_function

First-Class Functions

first-class functions are a necessity
in the **functional programming style**
where **higher**-**order functions** are widely used

A simple example of a **higher-ordered function**
is the **map** function,

which takes a function and a list, as its arguments,
and returns the list formed
by applying the function to each member of the list.

For a language to support **map**, (higher-ordered function)
it must support passing a function as an argument.

https://en.wikipedia.org/wiki/First-class_function

Higher-order Functions

a **higher-order function** (**functional**, **functional form** or **functor**)
is a function that does at least one of the following:

- takes one or more functions as arguments (i.e. procedural parameters),
- returns a function as its result.

All other functions are **first-order functions**.
In mathematics higher-order functions are also termed operators or functionals.
The differential operator in calculus is a common example,
since it maps a function to its derivative, also a function.
Higher-order functions should not be confused
with other uses of the word "functor" throughout mathematics

https://en.wikipedia.org/wiki/Functor

Function Definition

Function Definition I.

```
square x = x * x
```

- **function type** is <u>inferred</u> → not efficient **Type Inference**

Function Definition II.

```
square :: Double -> Double

square x = x * x
```

– **function type declaration**

– **function definition**

- **function type** **declaration**
- **function** **definition**

http://www.toves.org/books/hsfun/

Type Declaration

Type Declaration

the declaration of an identifier's type

identifier name :: type name ...

identifier names (including function identifiers) must always begin with a lower-case letter

type names in Haskell always begin with a capital letter

http://www.toves.org/books/hsfun/

Function Types and Type Classes

Function Definition I.

square x = x * x

Function Definition II.

square :: Double -> Double

square x = x * x

function definition

=

function definition

- function **type declaration**

=

type class – a set of types

- function **type** 1
- function **type** 2
-
- function **type** n

Requirements

Subclasses

http://www.toves.org/books/hsfun/

Curry & Uncurry

f :: a -> b -> c   the curried form of **g :: (a, b) -> c**

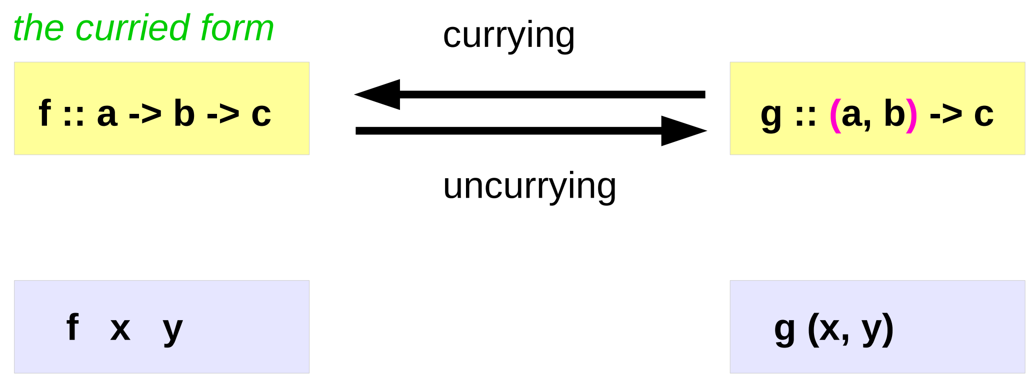
f = curry g
g = uncurry f

f x y = g (x,y)

the curried form is usually more convenient because it allows partial application.

all functions are considered curried

all functions take just one argument



https://wiki.haskell.org/Currying

Functions : First-class Data Types

functions are **first-class data types**

Haskell treats functions as regular data,

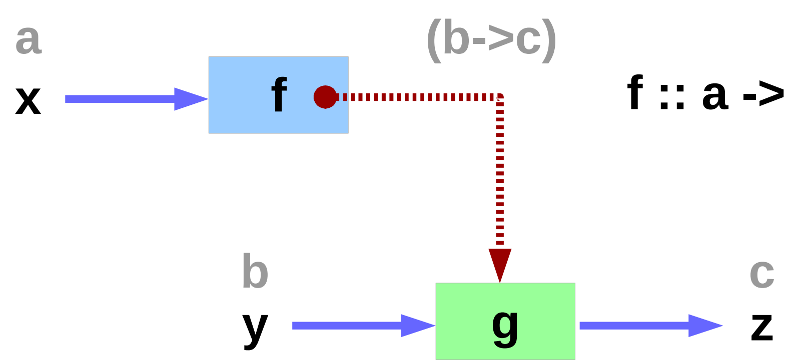
just like integers, or floating-point values, or other types.

- a function can take other functions as parameters
- a function takes a parameter and produces another function (curried function)

f x y	**f :: a -> b -> c**
(f x) y	**f :: a -> (b -> c)**

f x returns a function of type **b -> c**

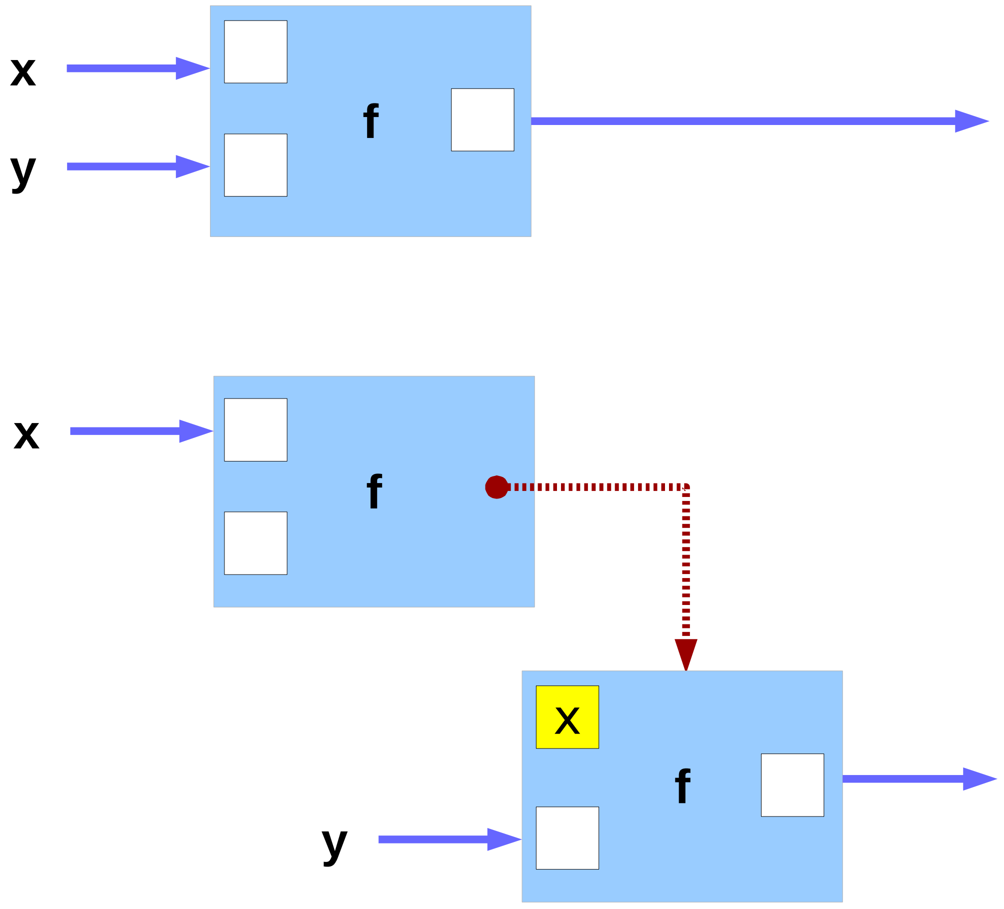
g y	**g :: b -> c**



f :: a -> b -> c

http://www.toves.org/books/hsfun/

Currying Examples



http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Polymorphic Functions

specific types vs. arbitrary types

a **polymorphic** functions – an abstract type

each type variable is generally a lower-case letter.

Example) A translate function

takes a function **f** and a distance **d**

returns a new function **g**

that is **f** "translated" **d** units to the right

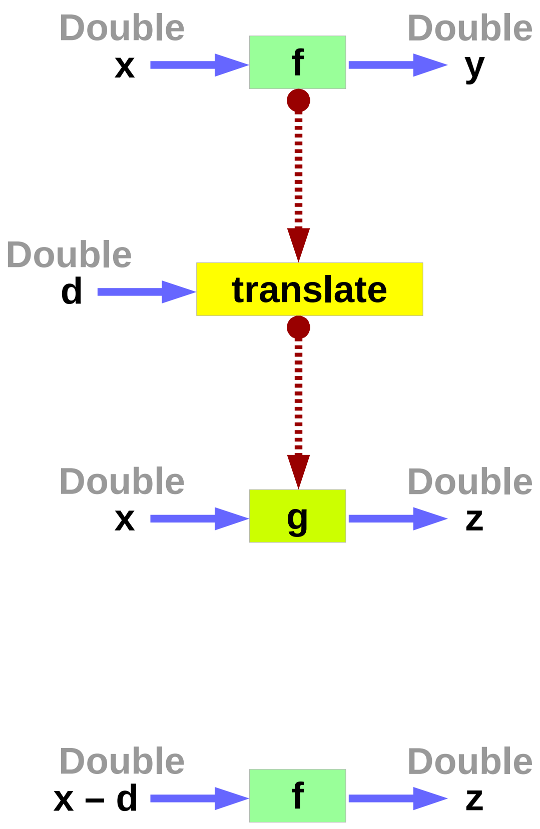
http://www.toves.org/books/hsfun/

Polymorphic Function Examples

translate :: (Double -> Double) -> Double -> (Double -> Double)

translate f d = **g where g** x = **f** (x – d)

translate :: (Double -> a) -> Double -> (Double -> a)



http://www.toves.org/books/hsfun/

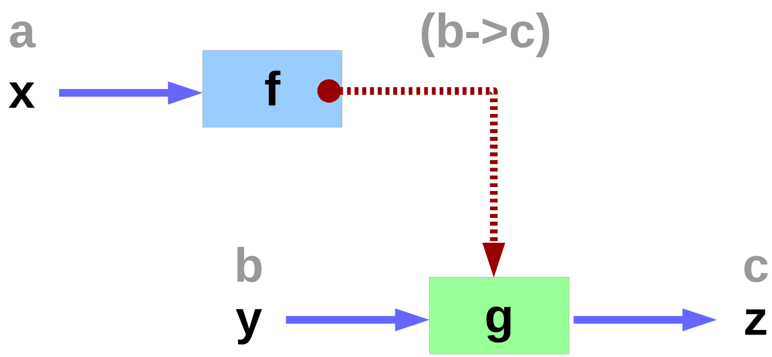
Currying

Currying recursively transforms
a function that takes multiple arguments
into a function that takes just a single argument and
returns another function if any arguments are still needed.

f :: a -> b -> c

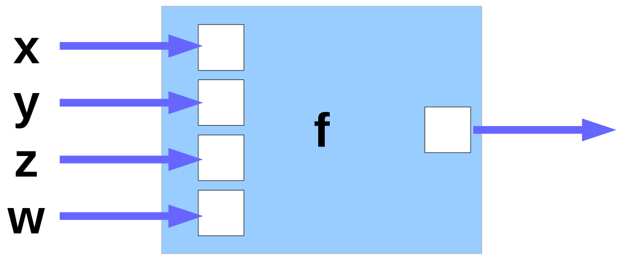
f x y	**f :: a -> b -> c**
(f x) y	**f :: a -> (b -> c)**
g y	**g :: b -> c**

f :: a -> b -> c



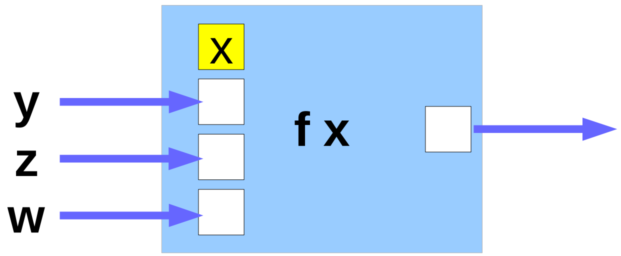
https://wiki.haskell.org/Currying
http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Partially Applied Functions – f, (f x)



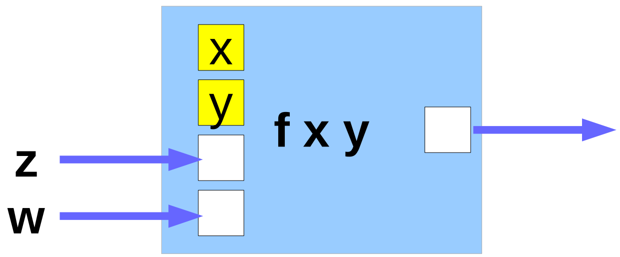
f :: a -> b -> c -> d -> e
f x y z w = …

(f x) y z w



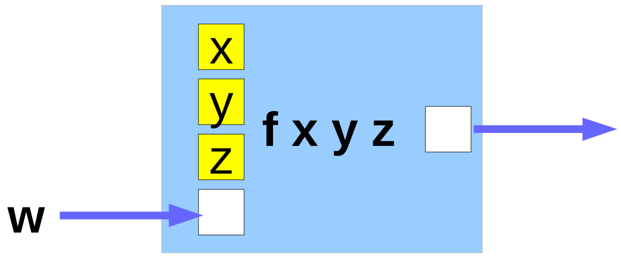
g1 :: b -> c -> d -> e
g1 y z w = …

Partially Applied Functions – (f x y), (f x y z)



(f x y) z w

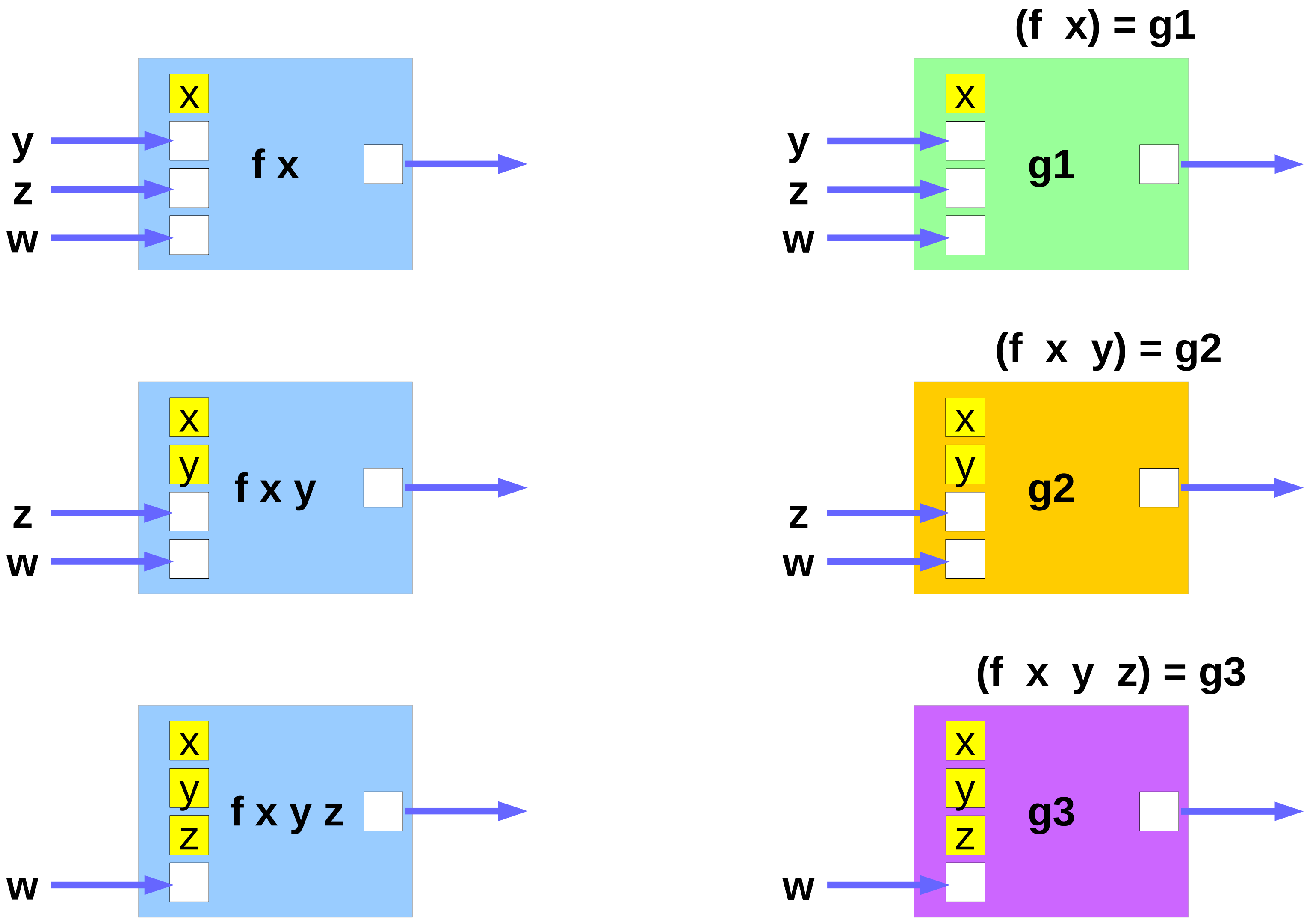
g2 :: c -> d -> e
g2 z w = …



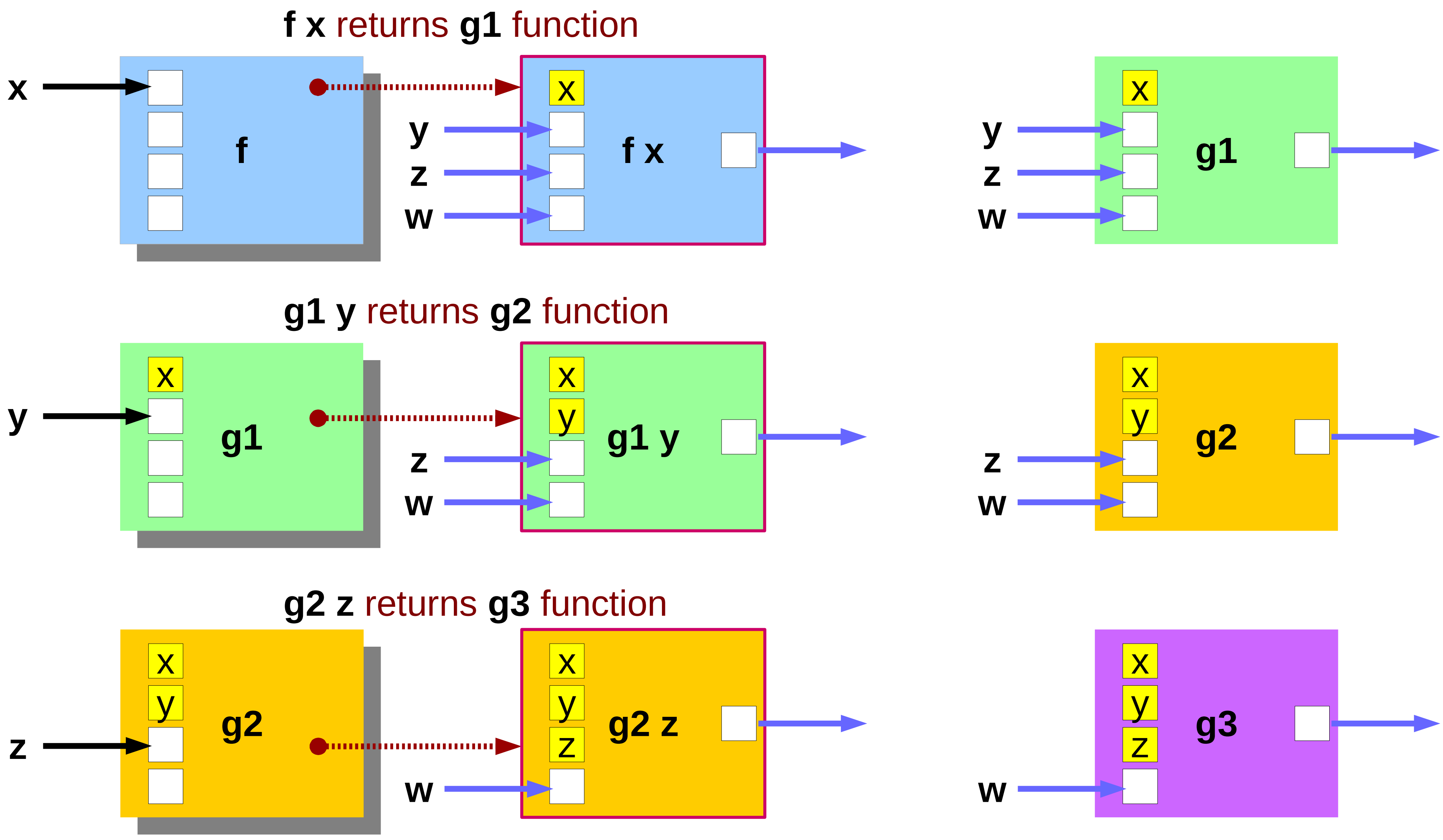
(f x y z) w

g3 :: d -> e
g3 w = …

Partially Applied Functions – g1, g2, g3



Returning Functions

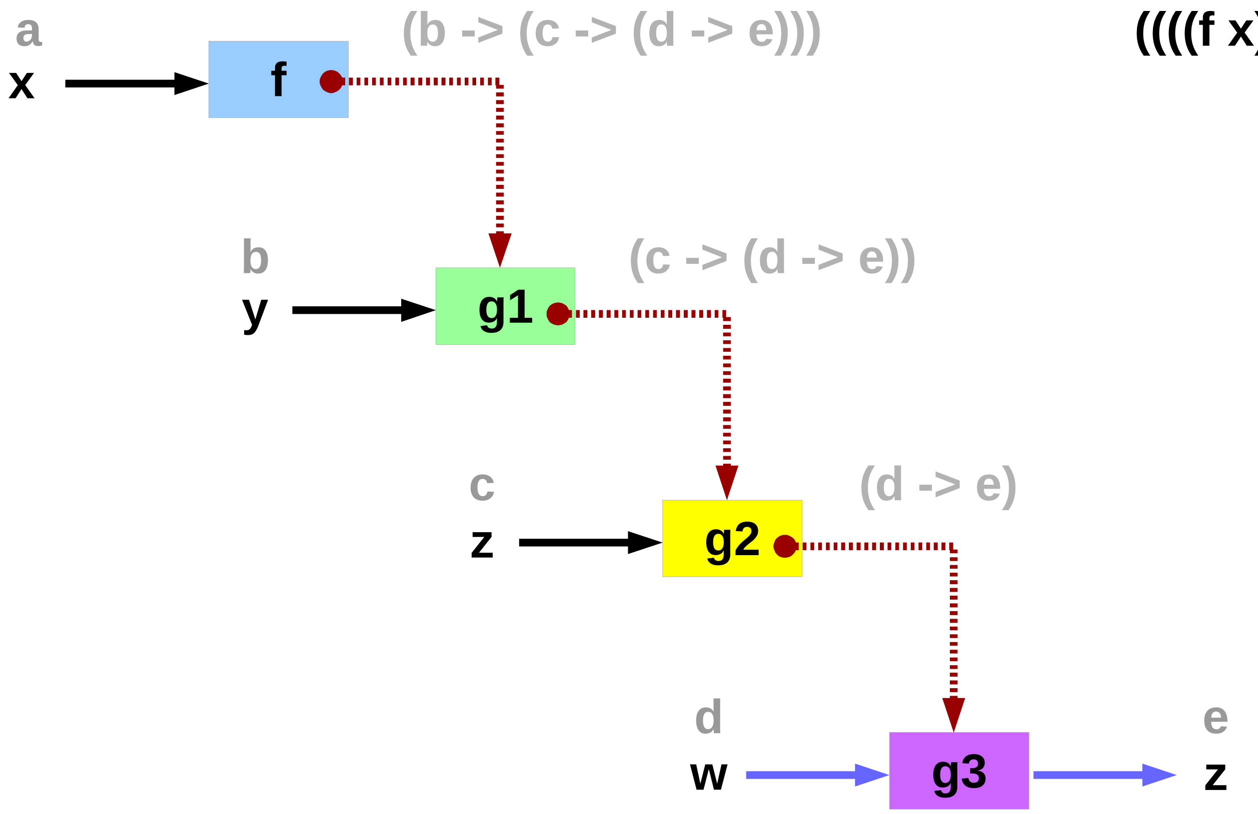


Currying Examples

f :: a -> b -> c -> d -> e

f :: a -> (b -> (c -> (d -> e)))

((((f x) y) z) w)



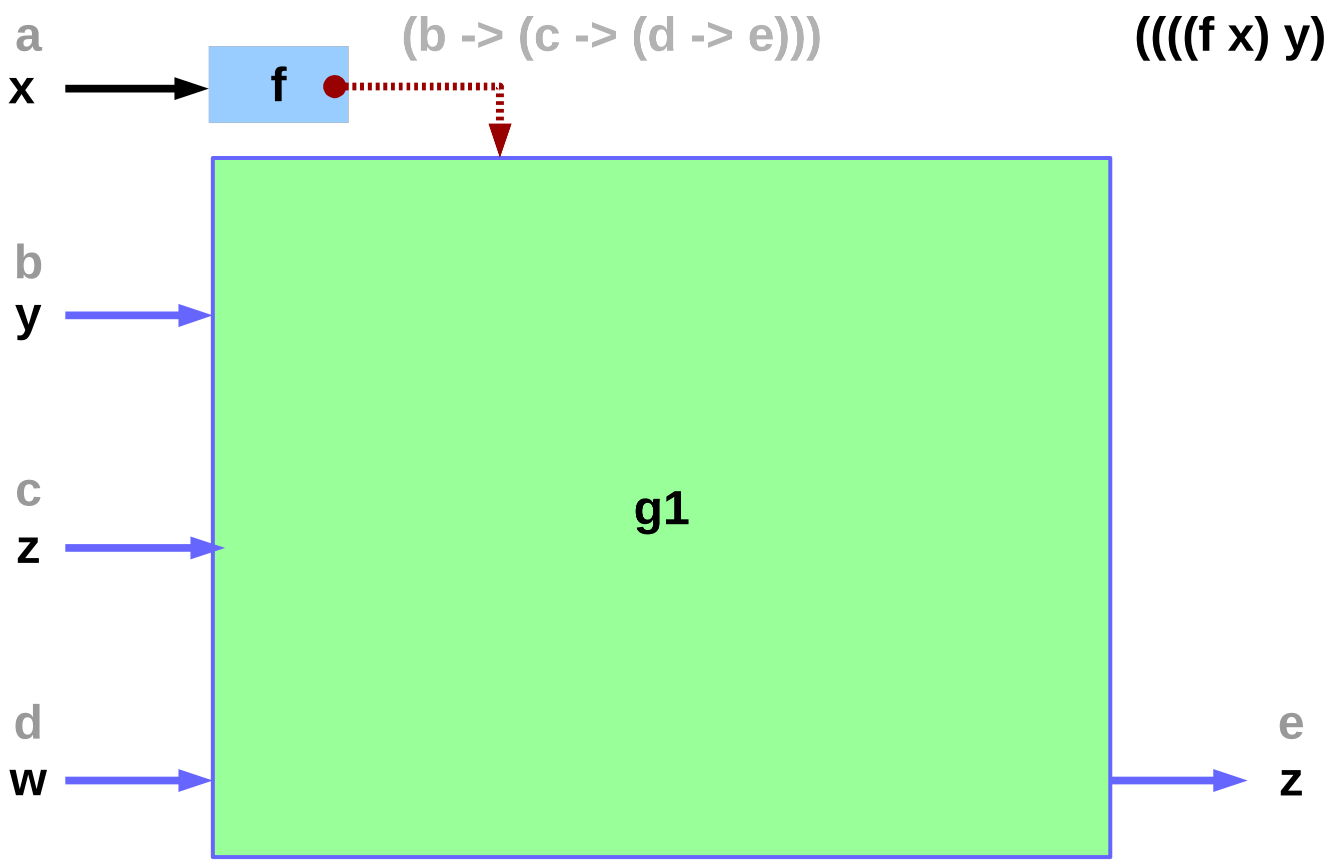
http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Currying Examples

f :: a -> b -> c -> d -> e

f :: a -> (b -> (c -> (d -> e)))

((((f x) y) w) z)



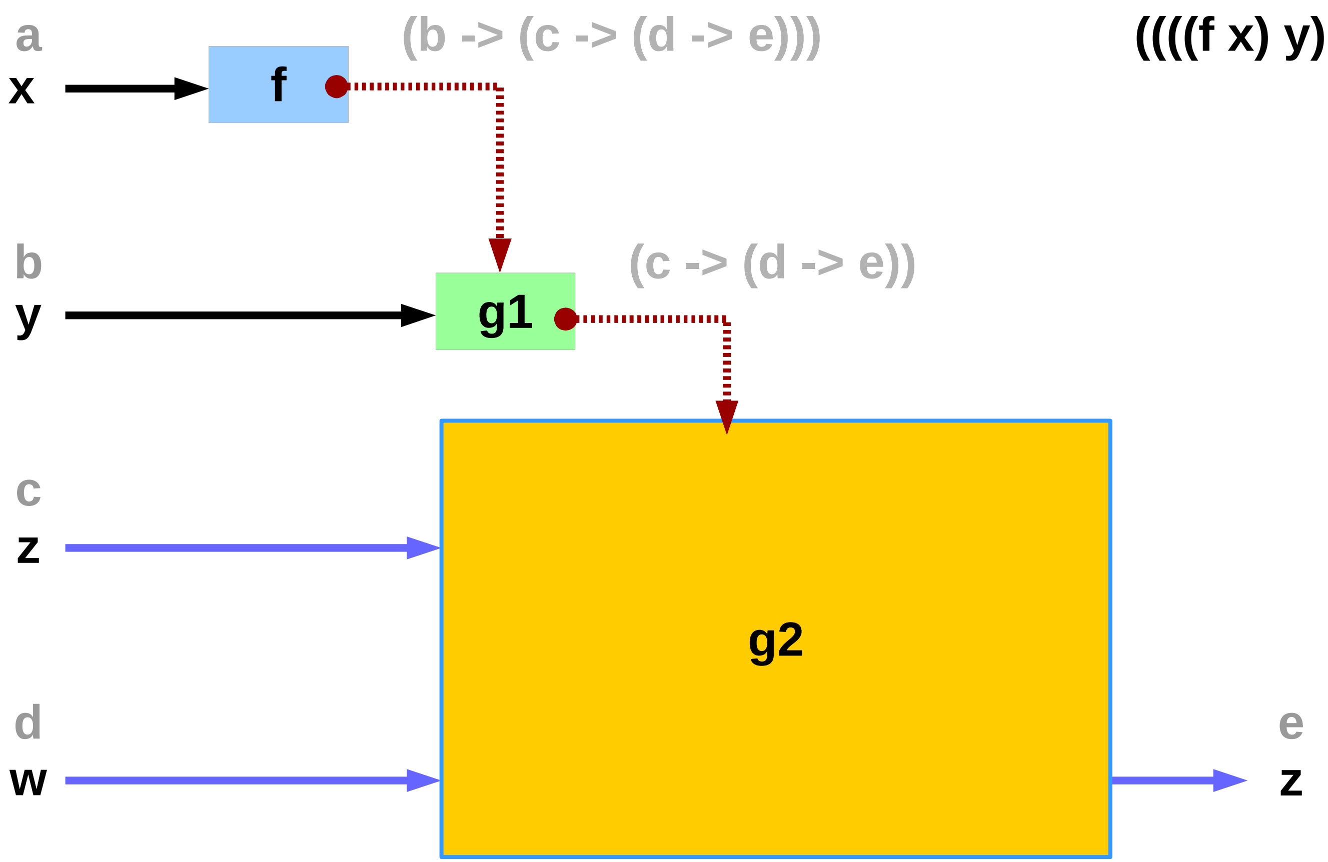
http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Curying Examples

f :: a -> b -> c -> d -> e

f :: a -> (b -> (c -> (d -> e)))

((((f x) y) z) w)



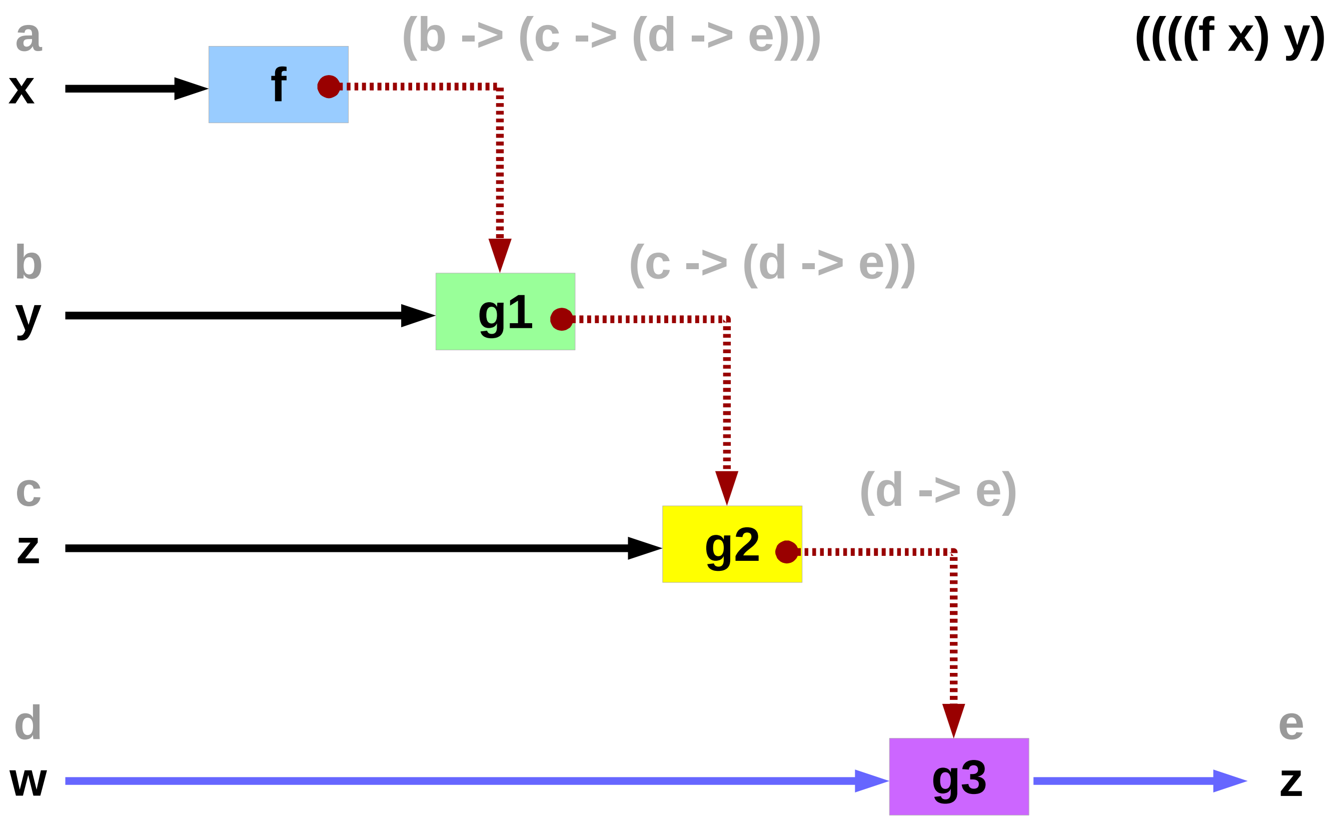
http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Currying Examples

f :: a -> b -> c -> d -> e

f :: a -> (b -> (c -> (d -> e)))

((((f x) y) z) w)



http://learnyouahaskell.com/functors-applicative-functors-and-monoids

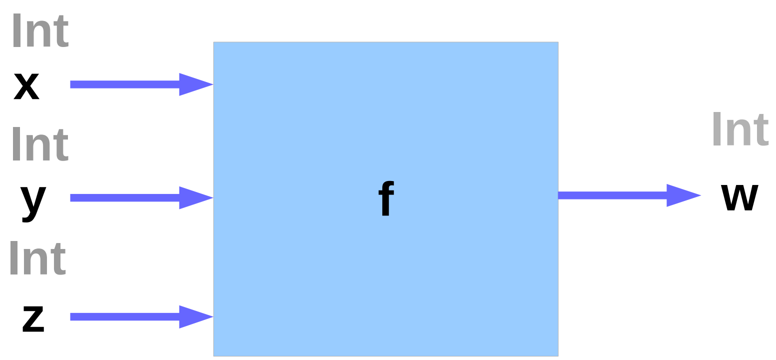
Currying Examples

mult :: Int -> Int -> Int -> Int

(((mult x) y) z)

f :: a -> (b -> (c -> d))

(((f x) y) z)



http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Partial Applications

mult :: Int -> Int -> Int -> Int

mult x y z

mult a_1 y z = g1 y z

mult a_1 a_2 z = g2 z

mult a_1 a_2 a_3 constants

f :: Int -> (Int -> (Int -> Int))

f :: Int -> (Int -> (Int -> Int))

f x y z

f x :: Int -> (Int -> Int)

g1 :: Int -> (Int -> Int)

g1 y z

f x y :: Int -> Int

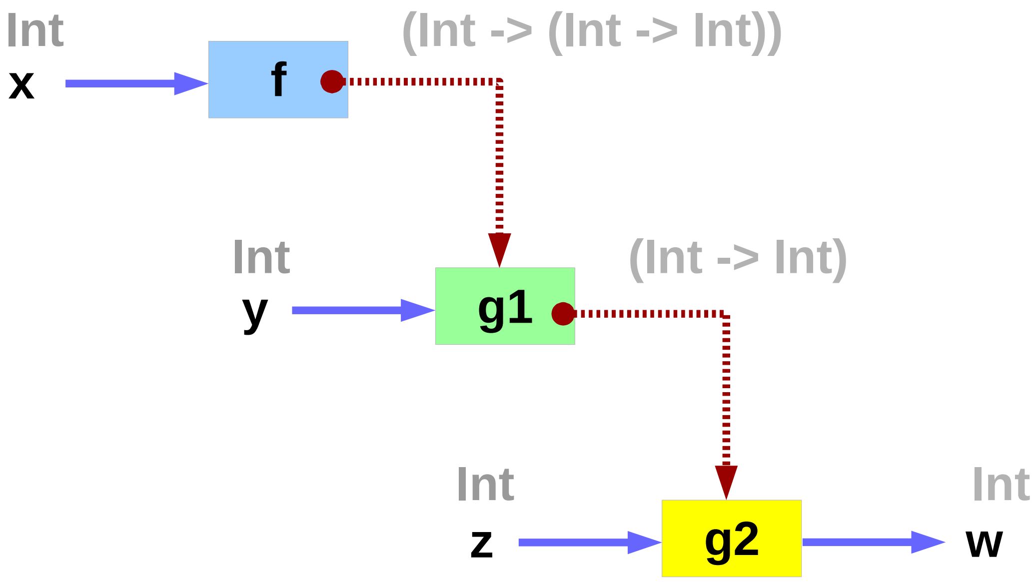
g2 :: Int -> Int

g2 z

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Returning Functions

mult :: Int -> Int -> Int -> Int



mult x y z

mult a_1 y z

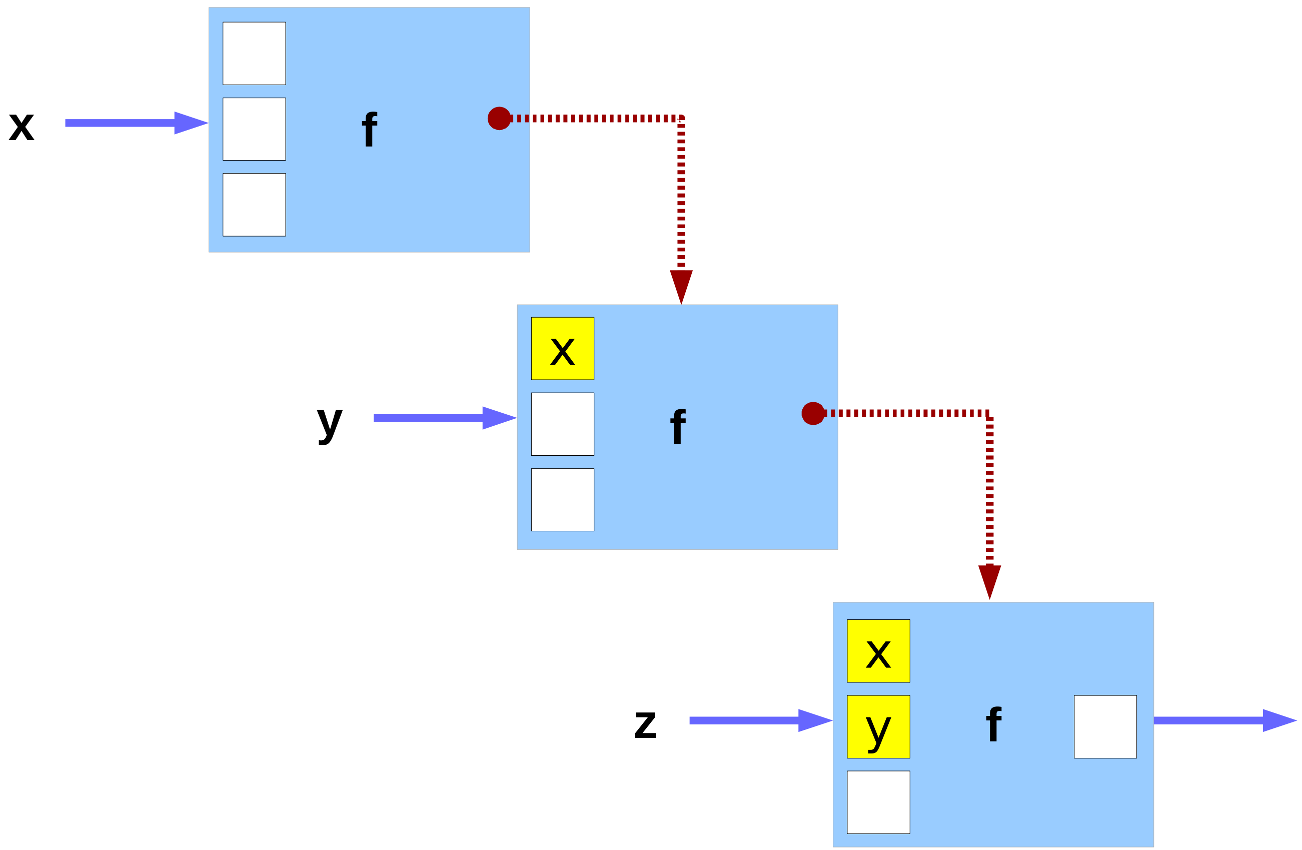
mult a_1 a_2 z

mult a_1 a_2 a_3

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Currying Examples

mult :: Int -> Int -> Int -> Int



mult x y z

mult a_1 y z

mult a_1 a_2 z

mult a_1 a_2 a_3

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Anonymous Function

```
\x -> x + 1

(\x -> x + 1) 4
5 :: Integer


(\x y -> x + y) 3 5
8 :: Integer

addOne = \x -> x + 1
```

Lambda Expression

https://wiki.haskell.org/Anonymous_function

let … in …

```
cylinder :: (RealFloat a) => a -> a -> a
cylinder r h =
    let sideArea = 2 * pi * r * h
        topArea = pi * r ^2
    in  sideArea + 2 * topArea
```

The form is **let** <bindings> **in** <expression>.

The names that you define in the **let** part
are **accessible** to the expression after the **in** part.

Notice that the names are also aligned in a single column.

For now it just seems that **let** puts the bindings first
and the expression that uses them later
whereas where is the other way around.

http://learnyouahaskell.com/syntax-in-functions

$ Function Application

($) :: (a -> b) -> a -> b
f $ x = f x

(a -> b) : left function
a : right value
b : result

Function application with a **space** — **f x**
- high precedence
- left-associative — **f a b c = ((f a) b) c)**

Function application with **$** — **f $ x**
- the lowest precedence
- right associative — **f $ a $ b $ c = f (a (b c))**

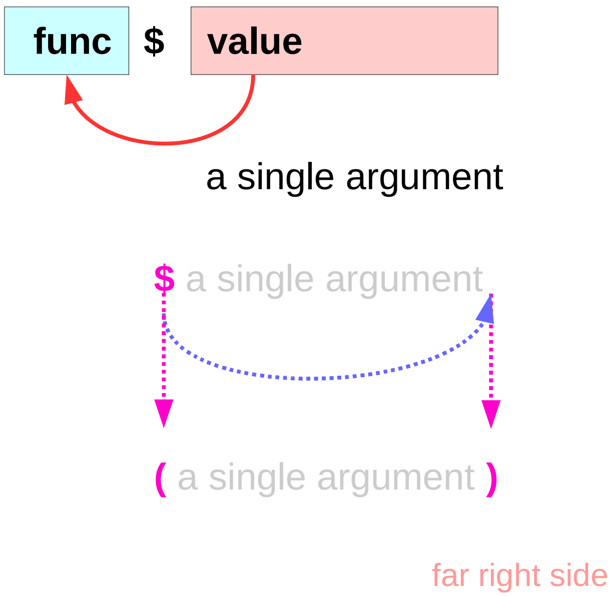
http://learnyouahaskell.com/higher-order-functions

$ a single argument

$ a convenience function that eliminates many parentheses.

When a **$** is encountered, the expression on its right is applied as the parameter to the function on its left.

writing an opening parentheses **(**
and then writing a closing one **)**
on the far right side of the expression.



http://learnyouahaskell.com/higher-order-functions

$ Function Application Examples

```
sum (map sqrt [1..130])
```

due to a low precedence

```
sum $ map sqrt [1..130]
```

```
sqrt 3 + 4 + 9
((sqrt 3) + (4 + 9))
```

```
sqrt (3 + 4 + 9)
sqrt $ 3 + 4 + 9
```

http://learnyouahaskell.com/higher-order-functions

$ Right Associative Examples

because $ is right-associative

f (g (z x))
f $ g $ z x

sum (filter (> 10) (map (*2) [2..10]))
sum $ filter (> 10) $ map (*2) [2..10]

http://learnyouahaskell.com/higher-order-functions

$ Map Function Application Examples

But apart from getting rid of parentheses,
$ means that function application
can be treated just like another function.

map function application over a list of functions.

map ($ 3) [(4+), (10*), (^2), sqrt]

[(4+ $ 3), (10* $ 3), (^2 $ 3), sqrt $ 3]

[7.0, 30.0, 9.0, 1.7320508075688772]

http://learnyouahaskell.com/higher-order-functions

const function

const x _ = x

Prelude> **const** 3 333

3

Prelude> **const** 3 99999

3

useful for passing to higher-order functions
when you don't need all their flexibility.

For example, the monadic sequence operator >>
can be defined in terms of the monadic bind operator as

x **>>** y = x **>>= const** y

(>>) = (. **const**) . (>>=)

https://stackoverflow.com/questions/7402528/whats-the-point-of-const-in-the-haskell-prelude

read function

Prelude> **let x = read "True"**

Prelude> **:t x**

x :: Read a => a

x doesn't have a concrete type.

x is sort of an expression

that can provide a value of a concrete type,

when we ask for it.

ask **x** to be an **Int** or a **Bool** or anything

Prelude> **x :: Bool**

True

Input: read "12"::Int

Output: 12

Input: read "12"::Double

Output: 12.0

Input: read "1.22"::Double

Output: 1.22

https://stackoverflow.com/questions/7402528/whats-the-point-of-const-in-the-haskell-prelude
http://zvon.org/other/haskell/Outputprelude/read_f.html

replicate, take, repeat, cycle, iterate

replicate **Int -> a -> [a]**
creates a list of **length** given by the first argument
and the items having **value** of the second argument

take **Int -> [a] -> [a]**
creates a list, the first argument determines,
how many **items** should be taken from the list passed
as the second argument

repeat **a -> [a]**
it creates an **infinite** list where all items are the first argument

cycle **[a] -> [a]**
it creates a **circular list** from a finite one

Iterate **(a -> a) -> a -> [a]**
creates an **infinite** list where the first item is calculated
by applying the function on the second argument, the second item
by applying the function on the previous result and so on.

http://zvon.org/other/haskell/Outputprelude/cycle_f.html

replicate, take, repeat, cycle, iterate examples

Input: replicate 3 5
Output: [5,5,5]

Input: replicate 4 "aa"
Output: ["aa","aa","aa","aa"]

Input: replicate 5 'a'
Output: "aaaaa"

Input: take 5 [1,2,3,4,5,6,7]
Output: [1,2,3,4,5]

Input: take 5 [1,2]
Output: [1,2]

Input: take 0 [1,2,3,4,5,6,7]
Output: []

Input: take 5 (repeat 3)
Output: [3,3,3,3,3]

Input: take 7 (iterate (2*) 1)
Output: [1,2,4,8,16,32,64]

Input: take 10 (cycle [1,2,3])
Output: [1,2,3,1,2,3,1,2,3,1]

Input: take 4 (repeat 3)
Output: [3,3,3,3]

Input: take 6 (repeat 'A')
Output: "AAAAAA"

Input: take 5 (repeat "A")
Output: ["A","A","A","A","A"]

Input: take 10 (cycle [1,2,3])
Output: [1,2,3,1,2,3,1,2,3,1]

Input: take 10 (cycle "ABC")
Output: "ABCABCABCA"

http://zvon.org/other/haskell/Outputprelude/cycle_f.html

flip

```
flip        :: (a -> b -> c) -> b -> a -> c
flip f x y   =  f y x
```

flip f takes its (first) two arguments in the reverse order of f.

https://www.haskell.org/hoogle/?hoogle=flip

flip

```
flip        :: (a -> b -> c) -> b -> a -> c
flip f x y   =  f y x
```

```
flip        :: (a -> b -> c) -> b -> a -> c
flip f x y   =  g
  where
    g = f y x
```

```
flip        :: (a -> b -> c) -> b -> a -> c
flip f x y   =  g x y
  where
    g a b = f b a
```

```
flip f x y   =  g x y
flip f x   =  g x
flip f   =  g
```

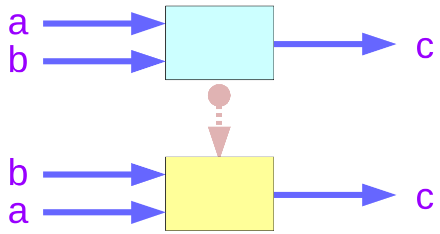
```
flip     :: (a -> b -> c) -> b -> a -> c
flip f   =  g
  where
    g a b = f b a
```

```
flip     :: (a -> b -> c) -> b -> a -> c
flip f   =  g
  where
    g x y = f y x
```

https://stackoverflow.com/questions/14397128/how-does-the-flip-function-work

flip

flip :: (a -> b -> c) -> b -> a -> c
flip f x y = f y x



flip f takes its (first) two arguments
in the reverse order of **f**.

f :: (a -> b -> c)
flip f :: (b -> a -> c)

https://www.haskell.org/hoogle/?hoogle=flip

flip implementation

```
flip        :: (a -> b -> c) -> b -> a -> c
flip f x y  =  f y x
```

```
flip        :: (a -> b -> c) -> b -> a -> c
flip f x y  =  g
  where
    g = f y x
```

```
flip        :: (a -> b -> c) -> b -> a -> c
flip f x y  =  g x y
  where
    g a b = f b a
```

```
flip f x y  =  g x y
flip f x  =  g x
flip f  =  g
```

```
flip     :: (a -> b -> c) -> b -> a -> c
flip f   =  g
  where
    g a b = f b a
```

```
flip     :: (a -> b -> c) -> b -> a -> c
flip f   =  g
  where
    g x y = f y x
```

https://stackoverflow.com/questions/14397128/how-does-the-flip-function-work

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf