Capacitor and Inductor

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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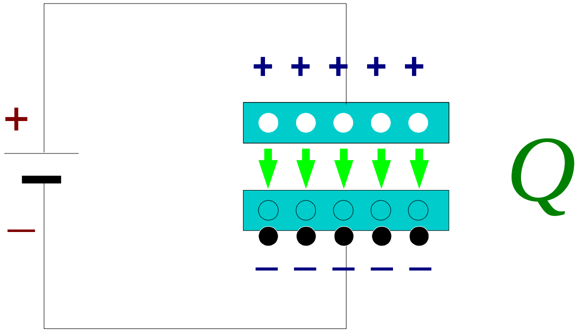
Energy Storage

Final



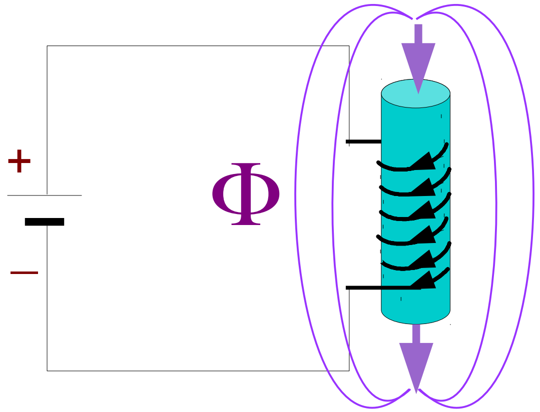
energy stored in **electric** field

potential energy of accumulated electrons $f(v_c)$

$$Q = CV$$

$$W = \frac{1}{2} C V^2$$

Final



energy stored in **magnetic** field

kinetic energy of moving electrons $f(i_L)$

$$N\Phi = LI \qquad \Phi = BA$$

$$W = \frac{1}{2} L I^2$$

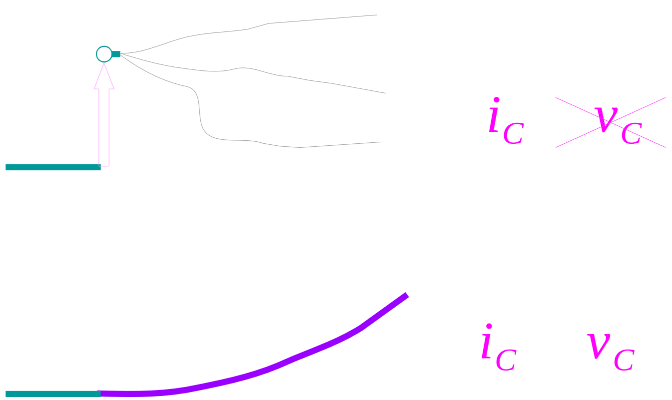
Newton's First Law of Motion

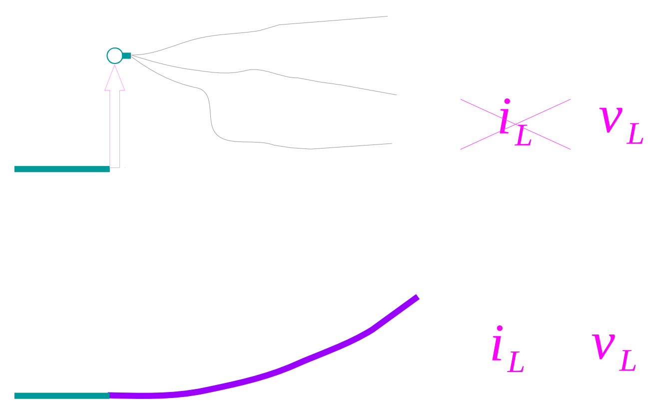
tendency to try
to maintain voltage
at a constant level

a capacitor *resists*
changes in voltage drop v_c

tendency to try
to maintain current
at a constant level

an inductor *resists*
changes in current flow i_L





potential energy $f(v_c)$

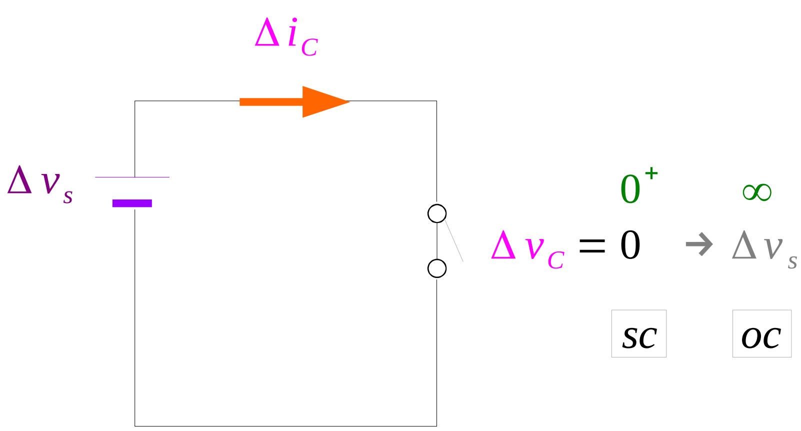
kinetic energy $f(i_L)$

Inertia

when v_c is increased
the capacitor *resists* the increase

by drawing <u>current</u> i_c
from the source of the voltage change

in opposition to the change

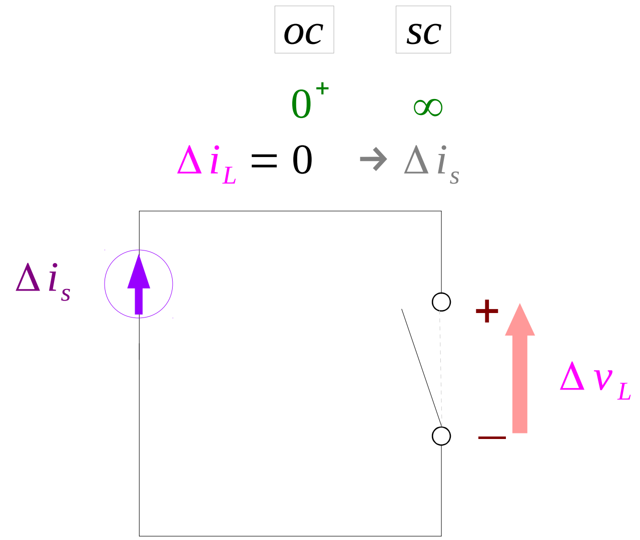


temporarily short circuit

when i_L is increased
the inductor *resists* the increase

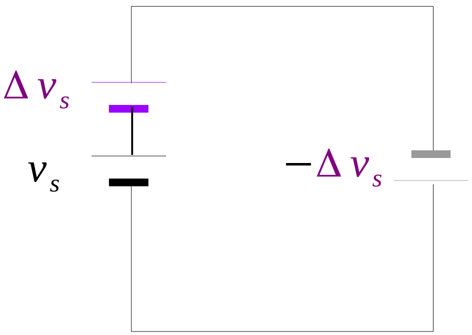
by dropping <u>voltage</u> v_L
from the source of the current change

in opposition to the change

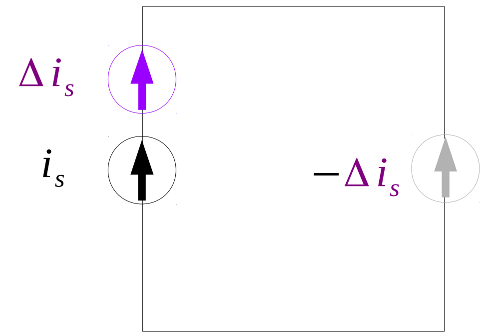


temporarily open circuit

Back voltage drop and current



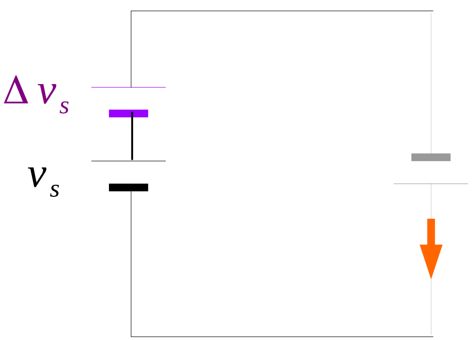
$\Delta v_s - \Delta v_s = 0$

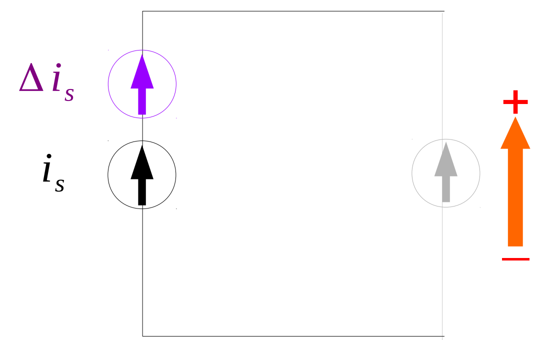


$\Delta i_s - \Delta i_s = 0$

back voltage drop $-\Delta v_s$

back current $-\Delta i_s$





opposing current Δi_C

opposing emf Δv_L

Resisting changes

resists the change of voltage

increasing v_c =
increasing electric field :

opposing **current** →
back voltage drop

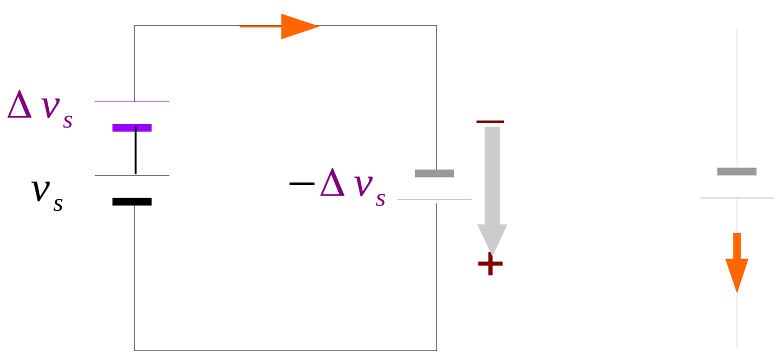
acting as reducing the voltage drop :
resists the change of voltage

resists the change of current

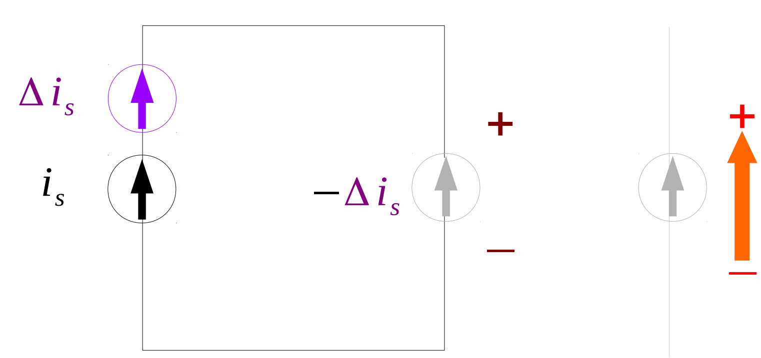
increasing i_L =
increasing magnetic field :

opposing **emf** →
back current

acting as reducing the current :
resists the change of current



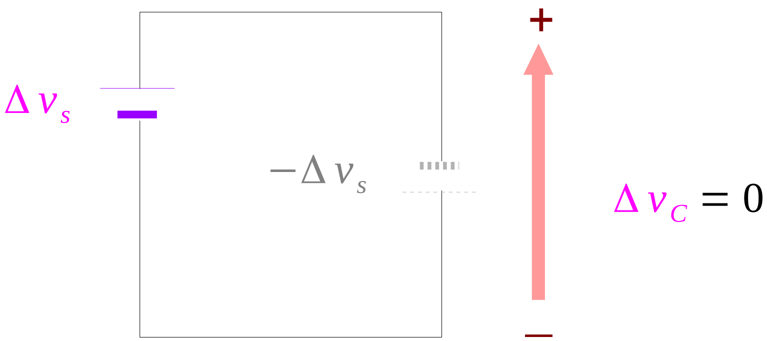
tries to cancel Δv_s



tries to cancel Δi_s

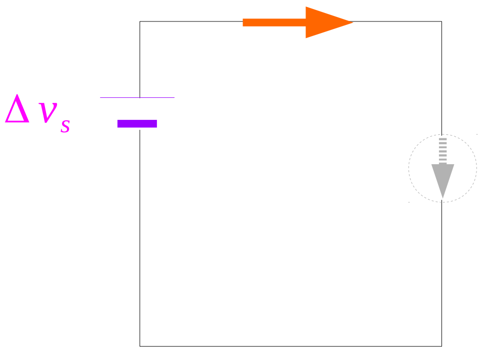
Opposition to the change

Δv_s change at $t = 0$

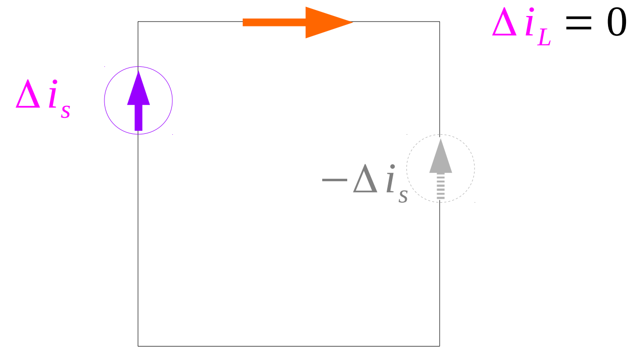


temporarily short circuit

by drawing current Δi_C

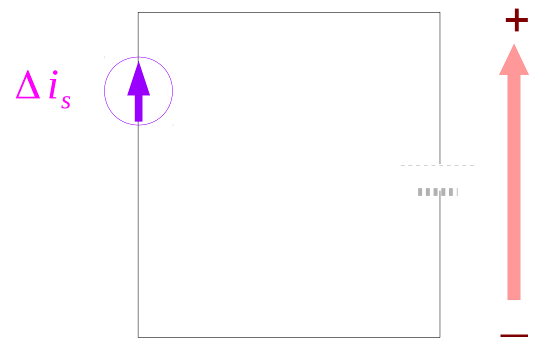


Δi_s change at $t = 0$

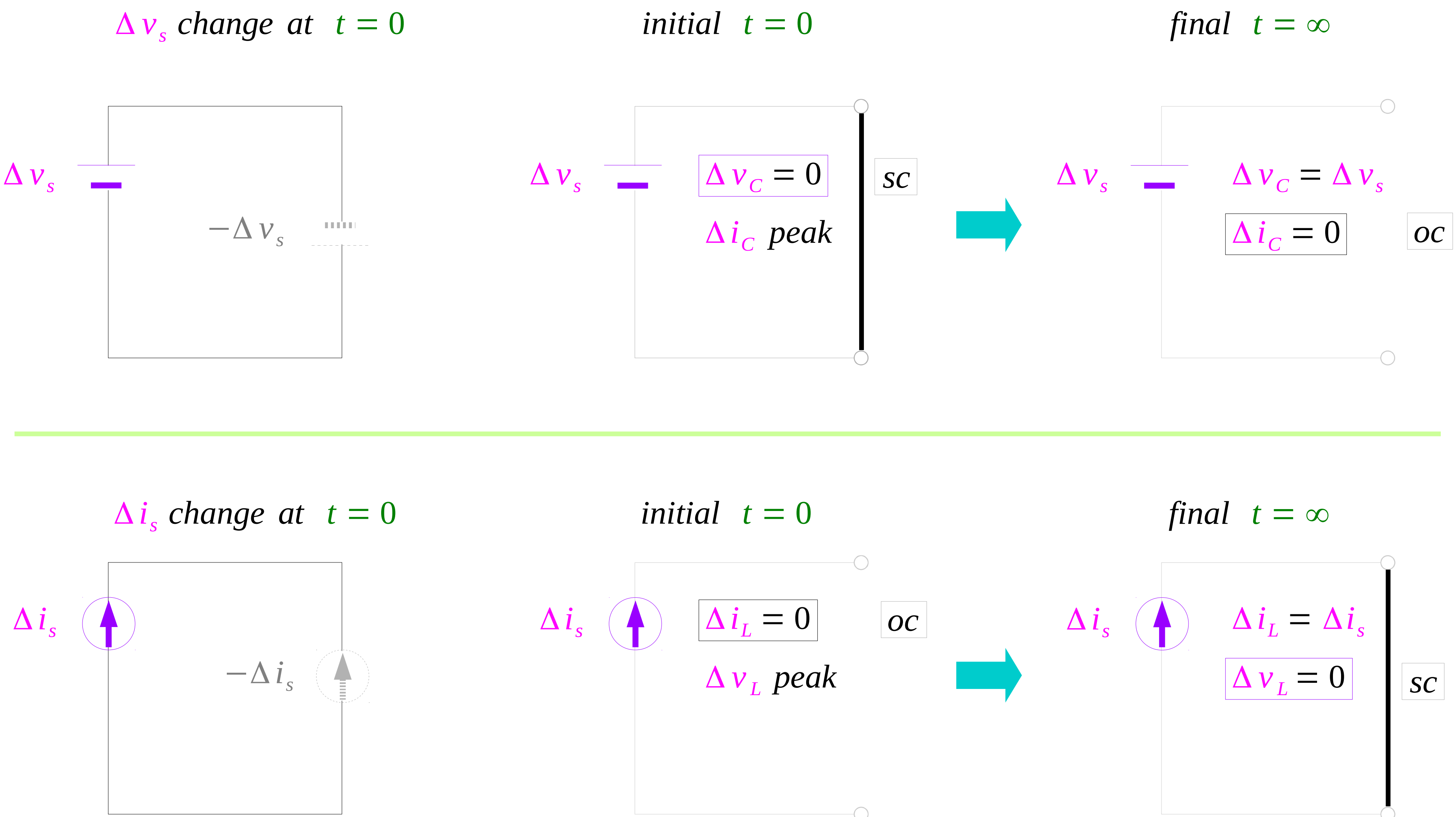


temporarily open circuit

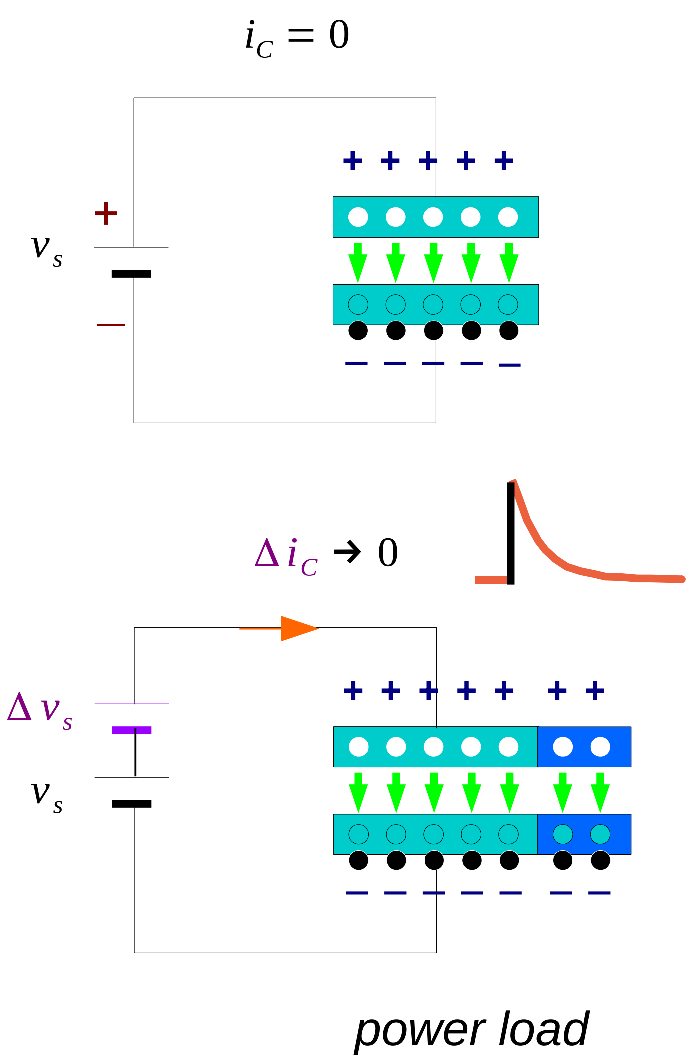
by inducing voltage Δv_L



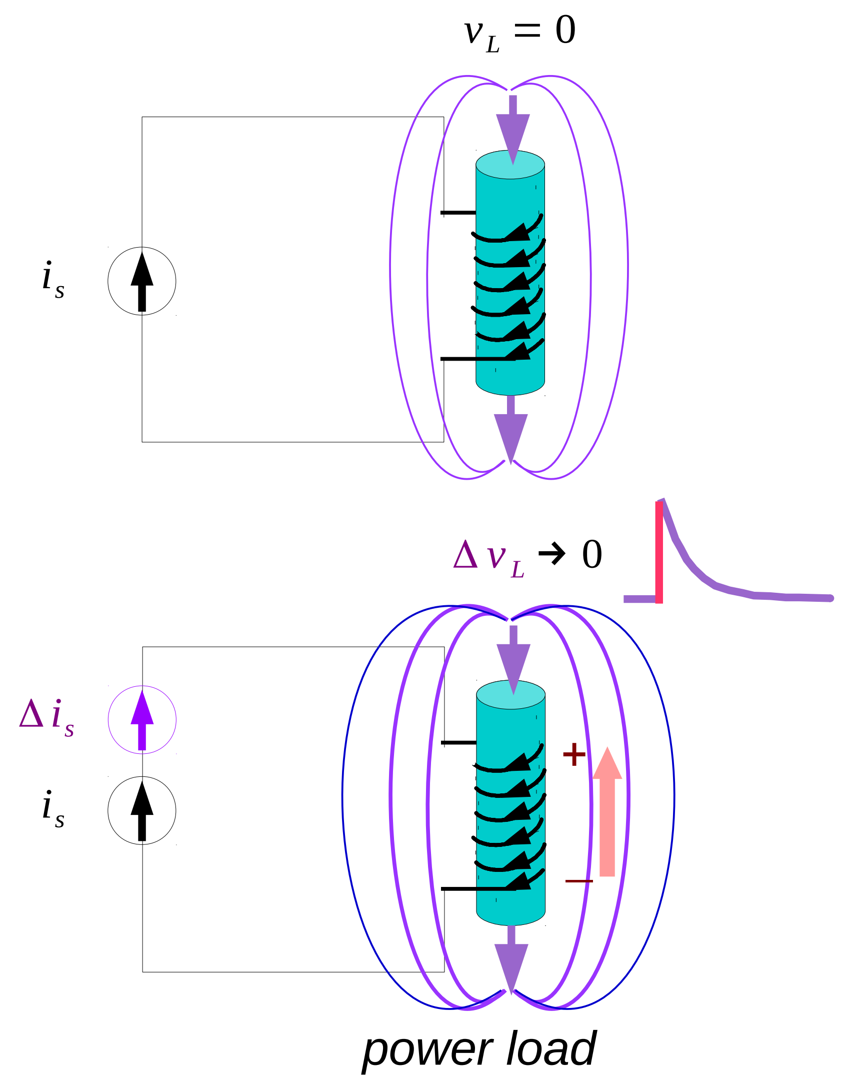
Opposition to the change



Charging

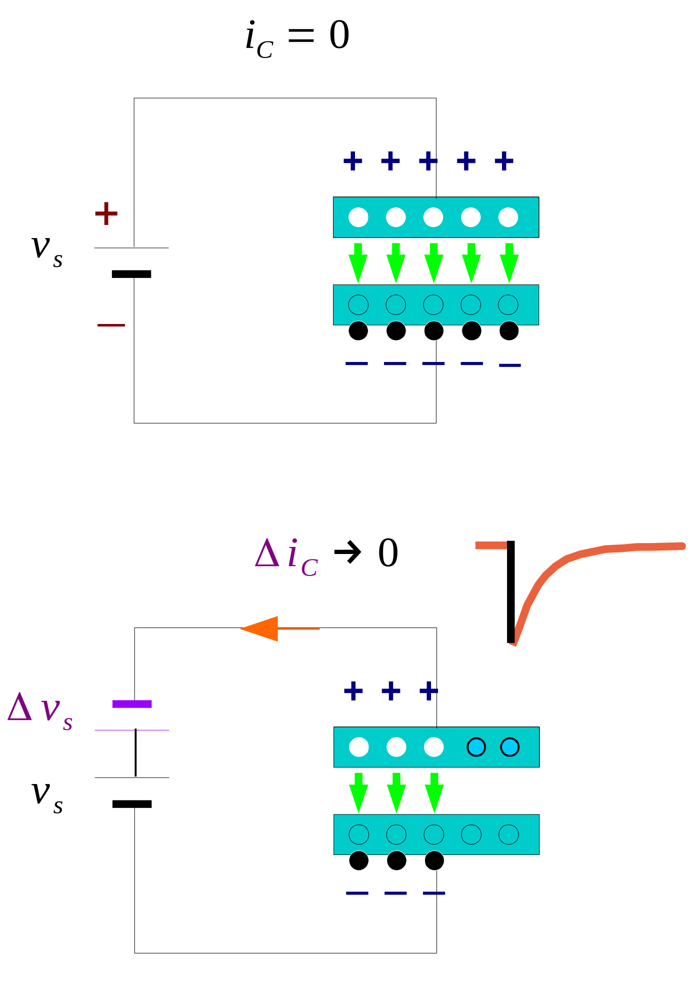


$i_C = 0$
v_s
+
−
$\Delta i_C \rightarrow 0$
Δv_s
v_s
power load

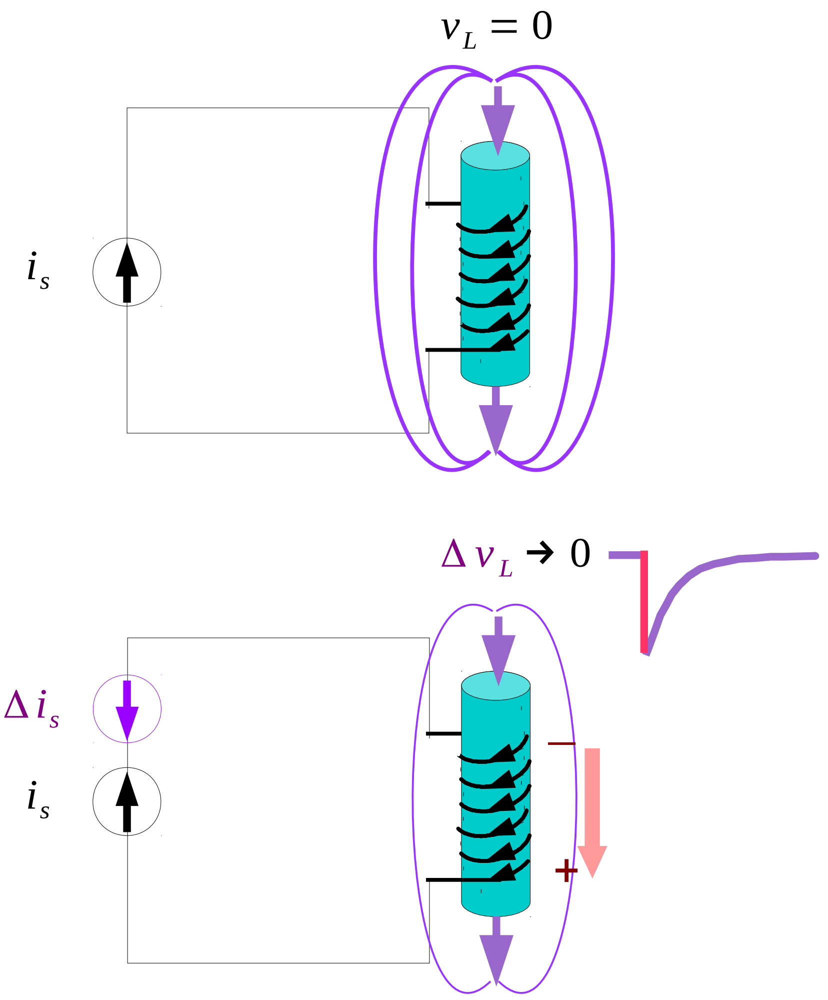


$v_L = 0$
i_s
$\Delta v_L \rightarrow 0$
Δi_s
i_s
+
−
power load

Discharging



power source



power source

Dissipating Energy

when v_c is decreased
the capacitor resists the decrease

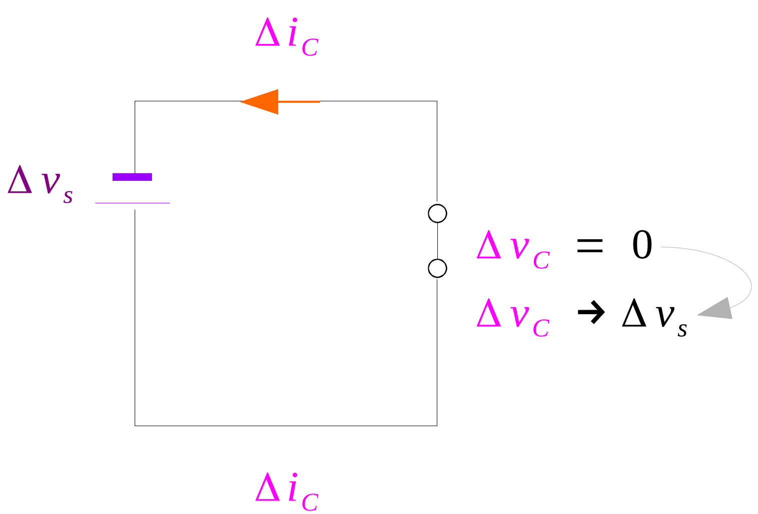
by supplying <u>current</u> i_c
to the source of the voltage change

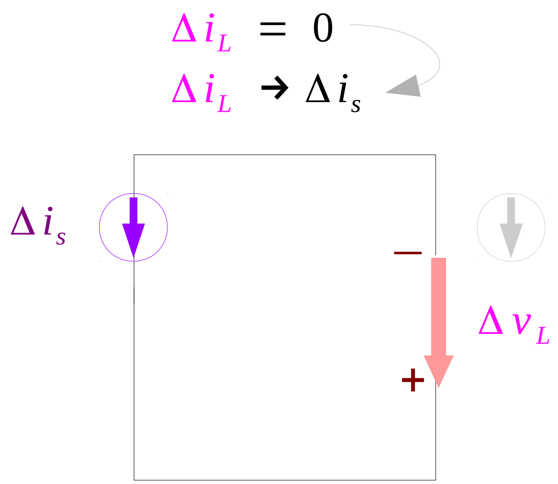
in opposition to the change

when i_L is decreased
the inductor resists the decrease

by producing <u>voltage</u> v_L
to the source of the current change

in opposition to the change





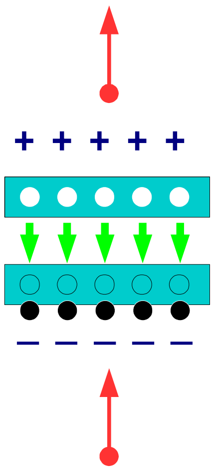
To store more energy

- *static nature*
- *a function of V*

to store more energy
v_c *must be increased* →

increasing charges on both sides
produces electron movement :
a way of resisting change of V

$$Q = C \cdot V$$



more
charges

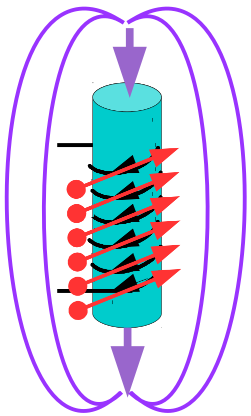
- *dynamic nature*
- *a function of I*

to store more energy
i_L *must be increased* →

increasing magnetic flux induces emf to
prohibit electron movement :
a way of resisting change of I

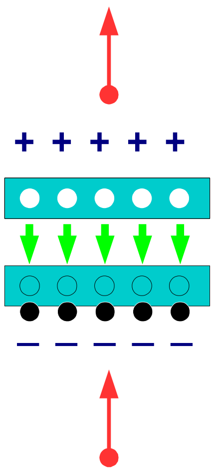
$$\Phi = L \cdot I$$

more
magnetic flux



Fast Change

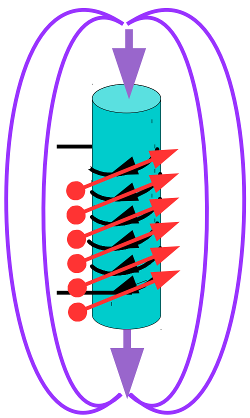
$Q = C \cdot V$



more electron imbalance

$\Phi = L \cdot I$

more moving electrons



fast change Δv ➡ *fast change* Δq

large $\frac{\Delta v}{\Delta t}$ ➡ *large* $\frac{\Delta q}{\Delta t} \propto i$

fast change Δi ➡ *fast change* $\Delta \phi$

large $\frac{\Delta i}{\Delta t}$ ➡ *large* $\frac{\Delta \phi}{\Delta t} \propto e$

Ohm's Law

whenever voltage v_C changes
there is a flowing current i_c

the magnitude of the current is proportional to the rate of change

$$i_C = C \cdot \frac{d\,v_C}{dt}$$

whenever current I_L changes
there is an induced voltage v_L

the magnitude of the voltage is proportional to the rate of change

$$v_L = L \cdot \frac{d\,i_L}{dt}$$

Stored Energy

resists to the change of voltage

to increase v_c,
***energy** should be*
transferred to capacitors
by the flowing current i_c

The stored energy is a function of voltage

$$Q = C \cdot V$$

$$\frac{dq}{dt} = C \cdot \frac{dv}{dt}$$

$$i_C = C \cdot \frac{dv_C}{dt}$$

resists to the change of current

to increase i_L,
***energy** should be*
transferred to the inductors
by the induced voltage v_L

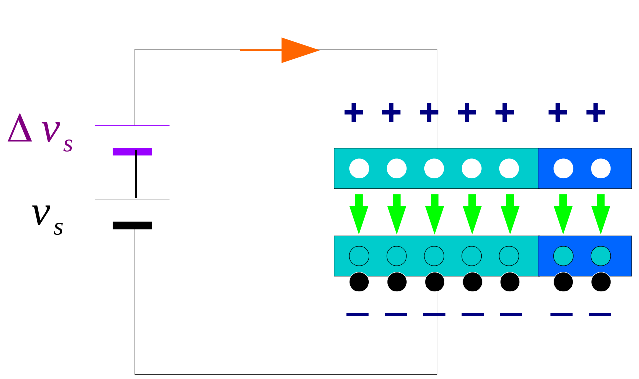
The stored energy is a function of current

$$\Phi = L \cdot I$$

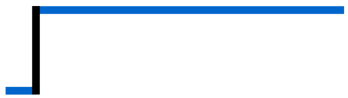
$$\frac{d\phi}{dt} = L \cdot \frac{di}{dt}$$

$$v_L = L \cdot \frac{di_L}{dt}$$

Short term effects



v_s

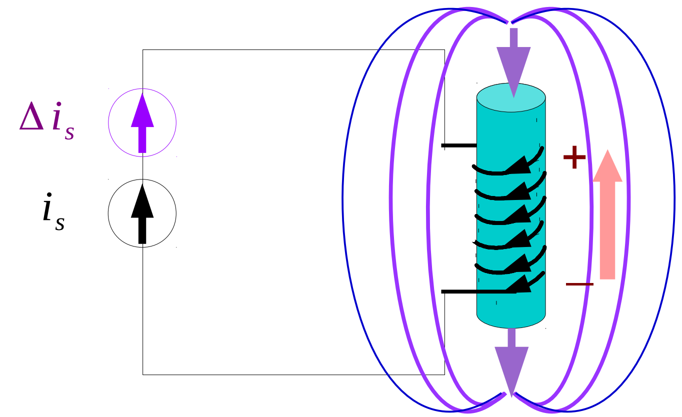


the short term effect: a way of resisting

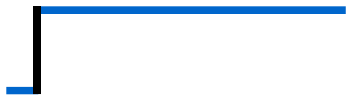
$i_C \rightarrow 0$

the long term result: voltage change

$v_C \rightarrow v + \Delta v$

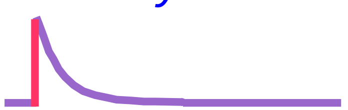


i_s



the short term effect: a way of resisting

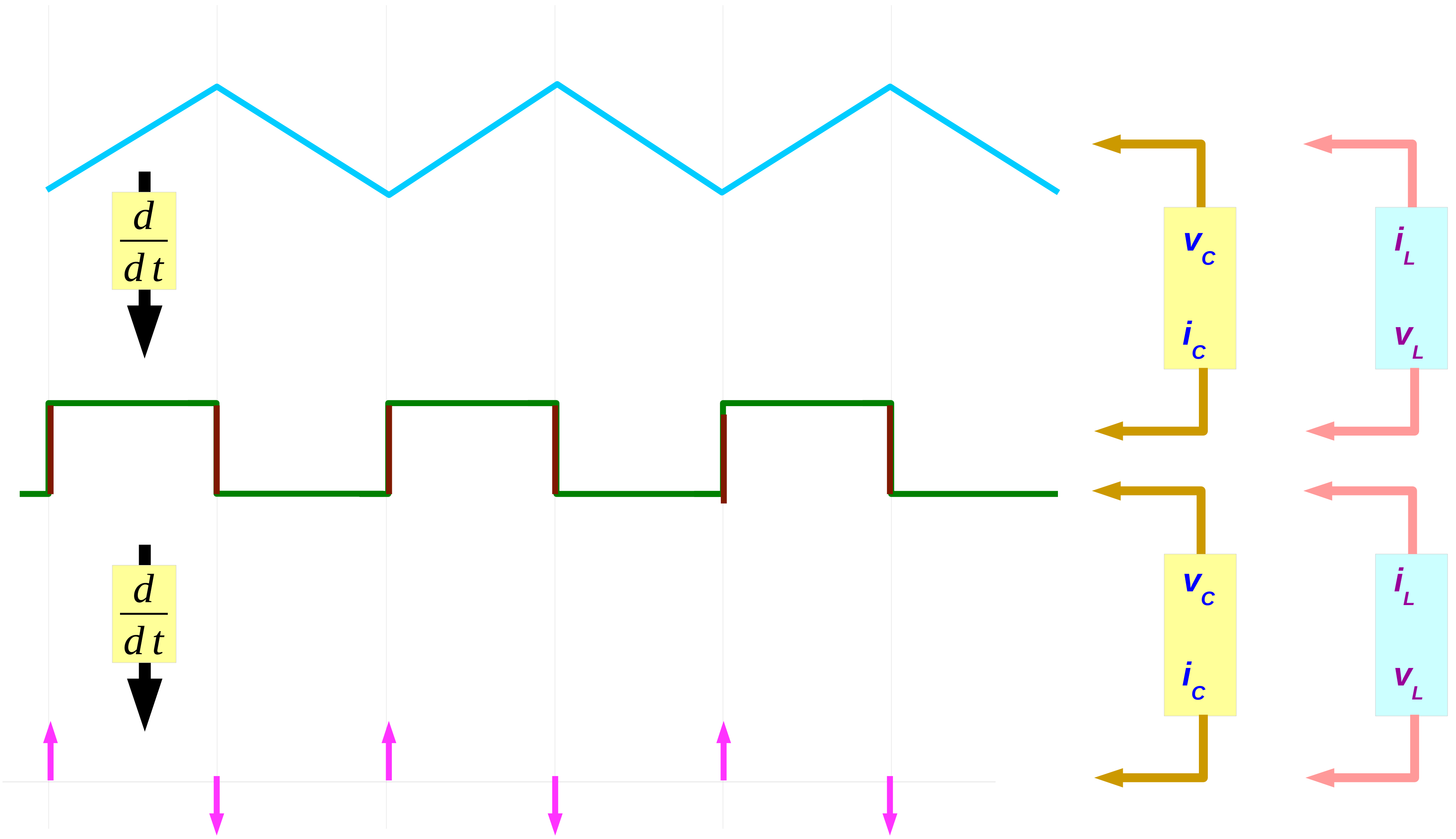
$v_L \rightarrow 0$



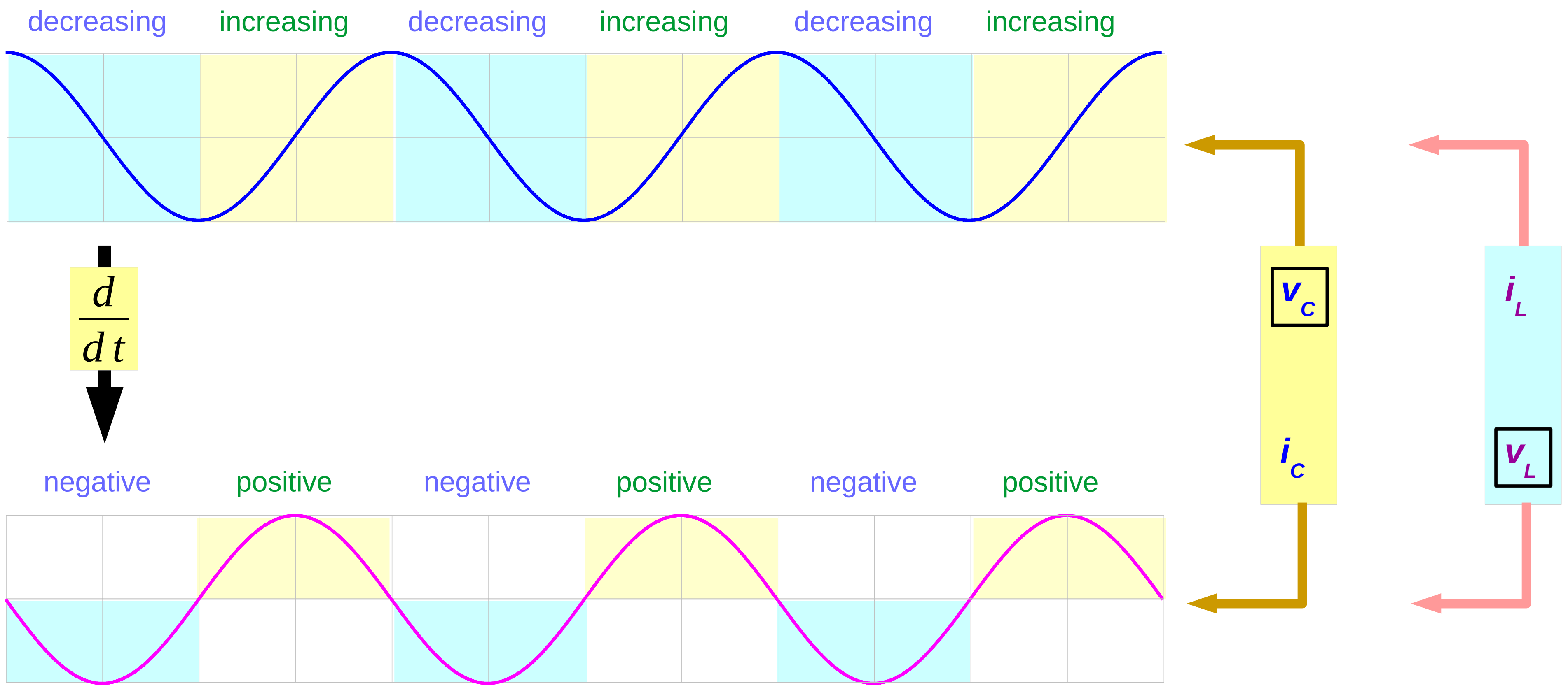
the long term result: current change

$i_L \rightarrow i + \Delta i$

Some (*i*,*v*) signal pair examples



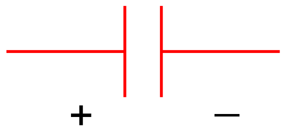
Everchanging signal pairs



Current & Voltage Changes

constant v

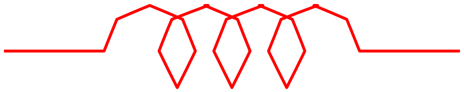
$$\frac{dv}{dt} = 0$$



$i = 0$

constant i

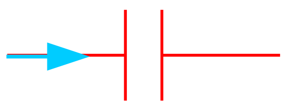
$$\frac{di}{dt} = 0$$



$v = 0$

increasing v

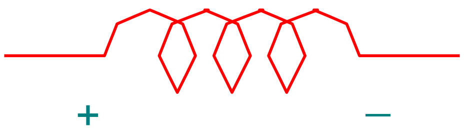
$$\frac{dv}{dt} > 0$$



$i > 0$

increasing i

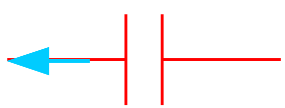
$$\frac{di}{dt} > 0$$



$v > 0$

decreasing v

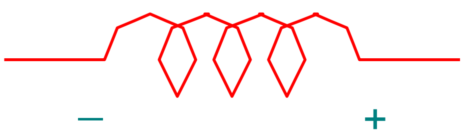
$$\frac{dv}{dt} < 0$$



$i < 0$

increasing i

$$\frac{di}{dt} < 0$$



$v < 0$

Sinusoidal voltage and current

$$i_C = C \cdot \frac{d\,v_C}{dt}$$

$$v_L = L \cdot \frac{d\,i_L}{dt}$$

$$v_C = A\cos(\omega t)$$
$$i_C = -\omega C A \sin(\omega t)$$
$$= \omega C A \cos(\omega t + \pi/2)$$

$$i_L = A\sin(\omega t)$$
$$= A\cos(\omega t - \pi/2)$$
$$v_L = \omega L A \cos(\omega t)$$

$$\frac{v_C}{i_C} = \frac{1}{\omega C}\frac{\cos(\omega t)}{\cos(\omega t + \pi/2)}$$

$$\frac{v_L}{i_L} = \omega L \frac{\cos(\omega t)}{\cos(\omega t - \pi/2)}$$

$$\boldsymbol{V}_C = A\angle\, 0$$
$$\boldsymbol{I}_C = \omega C A \angle\, \pi/2$$

$$\boldsymbol{I}_L = A\angle -\pi/2$$
$$\boldsymbol{V}_L = \omega L A \angle 0$$

$$\boldsymbol{Z}_C = \frac{\boldsymbol{V}_C}{\boldsymbol{I}_C} = \frac{1}{\omega C}\angle -\pi/2$$
$$= \frac{-j}{\omega C} = \frac{1}{j\omega C}$$

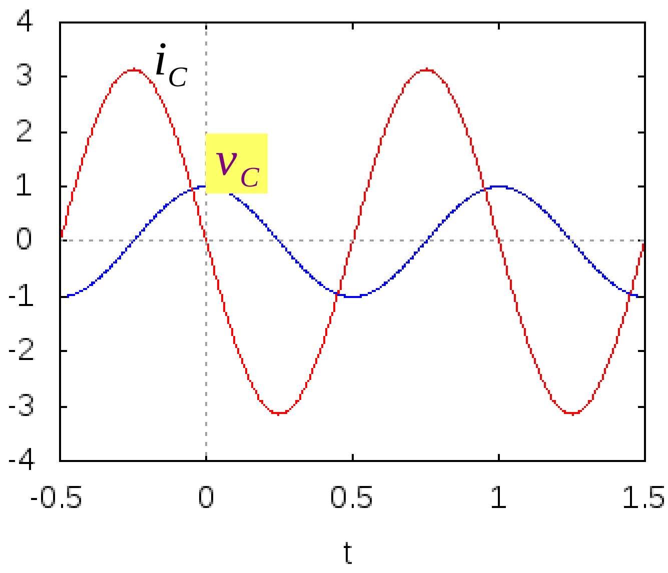
$$\boldsymbol{Z}_L = \frac{\boldsymbol{V}_L}{\boldsymbol{I}_L} = \omega L \angle +\pi/2$$
$$= j\omega L$$

Leading and Lagging Current

$$i_C = C \cdot \frac{d\,v_C}{dt}$$

$$v_C = \cos(2\pi t)$$
$$i_C = -\pi \sin(2\pi t)$$
$$= \pi \cos(2\pi t + \pi/2)$$

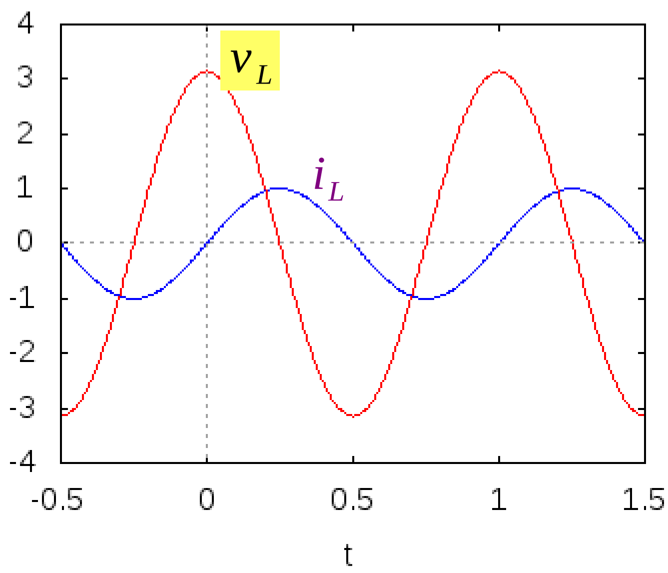
$C = 0.5$



$$v_L = L \cdot \frac{d\,i_L}{dt}$$

$$i_L = \sin(2\pi t)$$
$$= \cos(2\pi t - \pi/2)$$
$$v_L = \pi \cos(2\pi t)$$

$L = 0.5$



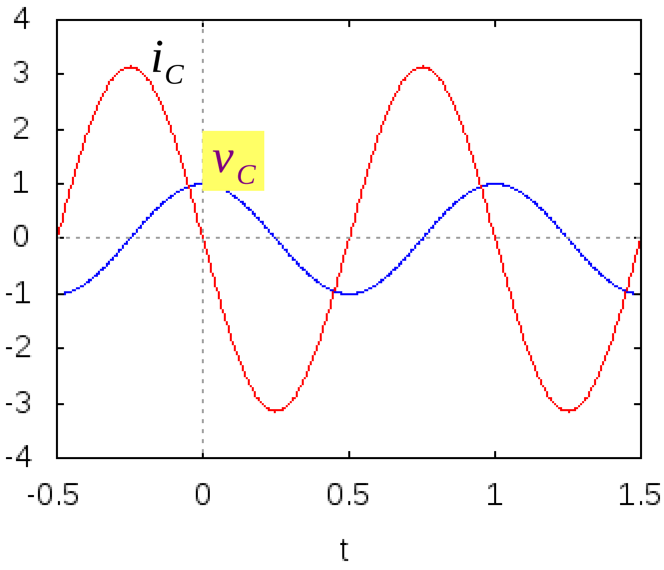
Ohm's Law

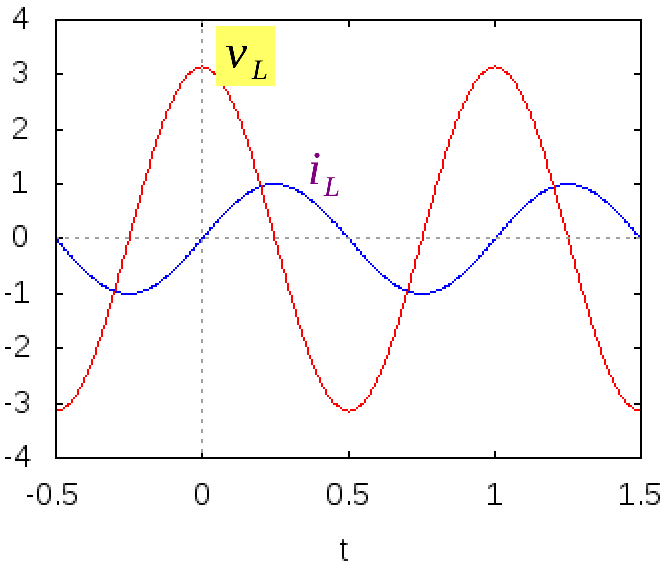
$$V_C = A\angle 0$$
$$I_C = \omega C A\angle \pi/2$$

$$I_L = A\angle -\pi/2$$
$$V_L = \omega L A\angle 0$$

$$Z_C = \frac{V_C}{I_C} = \frac{1}{\omega C}\angle -\pi/2$$
$$= \frac{-j}{\omega C} = \frac{1}{j\omega C}$$

$$Z_L = \frac{V_L}{I_L} = \omega L\angle +\pi/2$$
$$= j\omega L$$





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05/19/2017

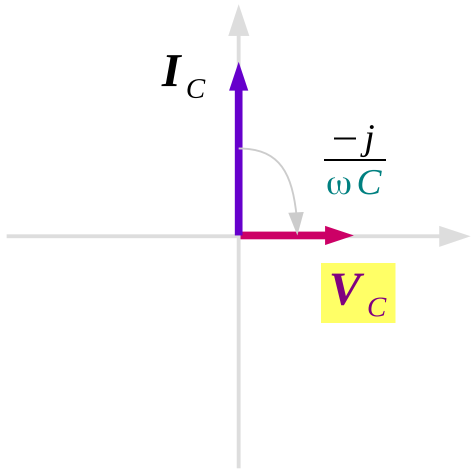
Phasor and Ohm's Law

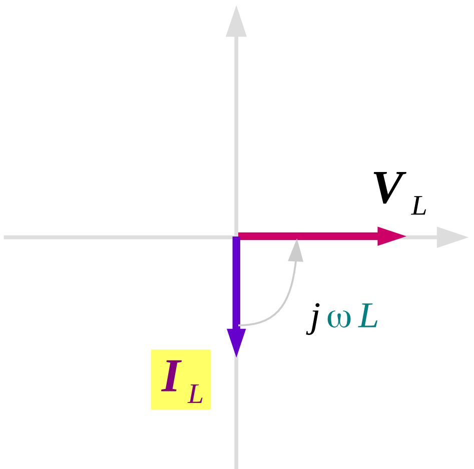
$$\boldsymbol{V}_C = A\angle\, 0$$
$$\boldsymbol{I}_C = \omega C\, A\angle\, \pi/2$$

$$\boldsymbol{I}_L = A\angle -\pi/2$$
$$\boldsymbol{V}_L = \omega L\, A\angle\, 0$$

$$\boldsymbol{Z}_C = \frac{\boldsymbol{V}_C}{\boldsymbol{I}_C} = \frac{1}{\omega C}\angle -\pi/2 = \frac{-j}{\omega C} = \frac{1}{j\omega C}$$

$$\boldsymbol{Z}_L = \frac{\boldsymbol{V}_L}{\boldsymbol{I}_L} = \omega L \angle +\pi/2 = j\omega L$$





Phase Lags and Leads

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003